PRINCIPLES OF COMPLEX SYSTEMS

ASSIGNMENT 6

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0.1 Problem 1

The 1-d theoretical percolation problem:
Consider an infinite 1-d lattice forest with a tree present at any site with probability p.

(a) Find the distribution of forest sizes as a function of p. Do this by moving along the 1-d world and figuring out the probability that any forest you enter will extend for a total length l.

A forest is defined by a section of trees nested between non tree sites. The probability that we enter a forest of size l is p^l. However we need to consider the requirement for non tree sites on the outside of the space. These appear with probability (1-p).

Thus: The distribution can be written as the following:

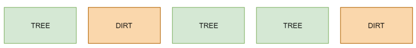


Figure 1: Distribution of Forest Sizes

$$(1-p)^2 p^l \tag{1}$$

(b) Find p_c, the critical probability for which a giant component exists.Hint: One way to find critical points is to determine when certain average quantities explode. Computeland find p such that this expression goes boom (if it does).:

A giant component means that the forest has no breaks. We're looking at one massive grove of trees covering the surface of our imaginary world. This means that the distribution of our forest length will be 1.

$$(1-p)^2 p^l = 1 \tag{2}$$
$$p^l = \frac{1}{(1-p)^2} \tag{3}$$
$$l ln(p) = \frac{1}{(1-p)^2} \tag{4}$$
$$l = \frac{-2ln(1-p)}{ln(p)} \tag{5}$$

We see that as p–>1, l approaches its maximum conceivable size. In this case, a spark would be disastrous.

0.2 Problem 2

Show analytically that the critical probability for site percolation on a triangular lattice is $p_c = 1/2$.
All we need to do here is show that 1/2 is a fixed point of function P(p). Given that each node in the lattice has a probability of being permeable p. The probability distribution can be written as:

$$p' = p^3 + 3(p^2(1-p)) \tag{6}$$
$$\text{And in-fact:} \tag{7}$$
$$p'(1/2) = 1/2 \tag{8}$$

0.3 Problem 3

Percolation in two dimensions!

(i) For each L, run 100 tests for occupation probability p moving from 0 to 1 in increments of .01.
Below are are a hundred matrices when L = 20 for rising p. It is now obvious how the clusters form for higher p.

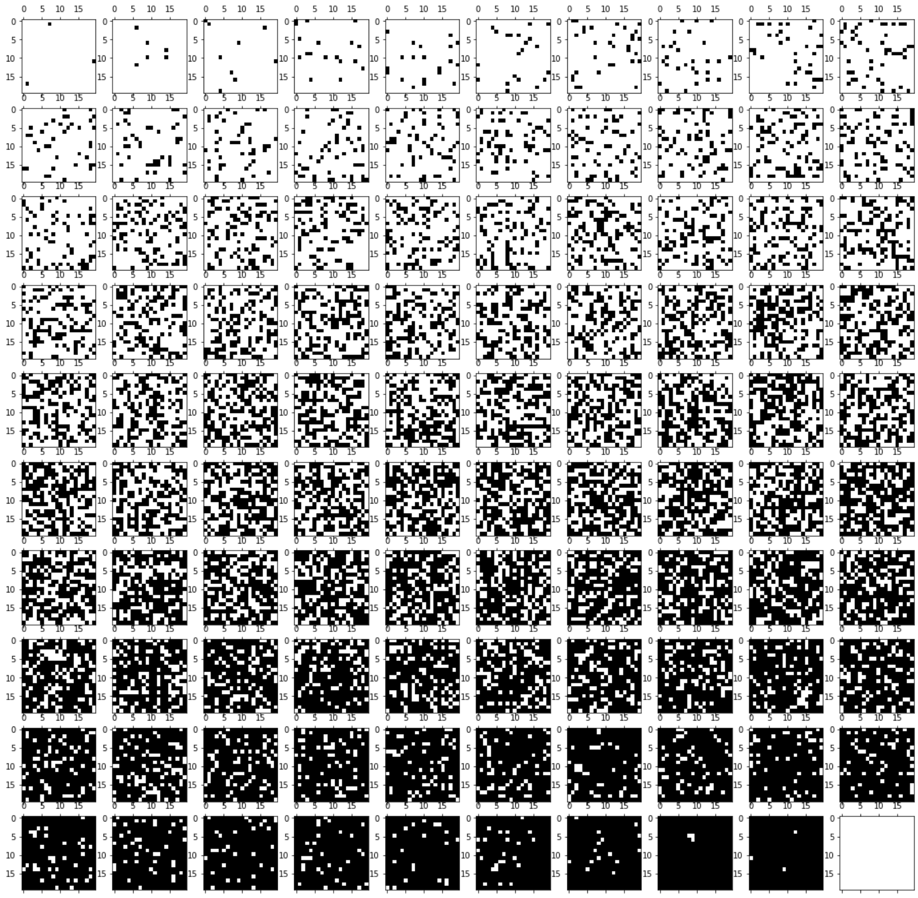


Figure 2: Forest on 20x20 Grid for changing p-val

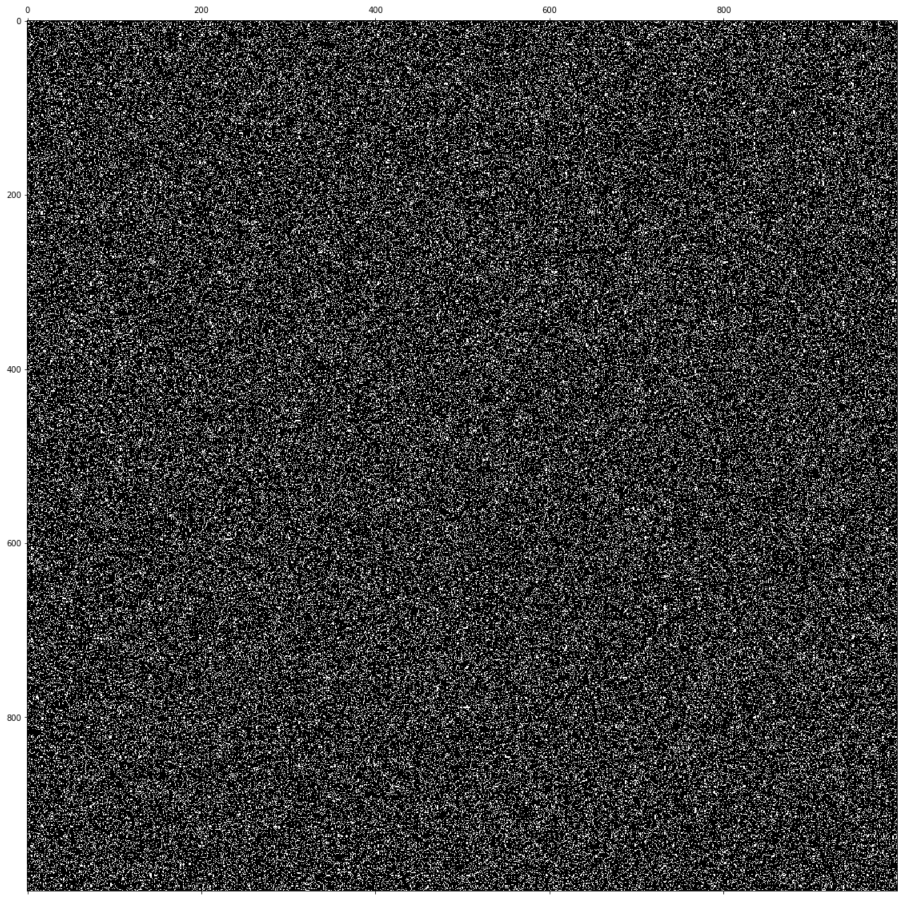


Figure 3: Forest on 1000X1000 Grid for p = .80

(ii- iii)

After finding the largest connected component from each forest generated, a average was taken over the hundred trials for each p value for each forest size L.

For each L, the average largest component size is plotted as a function of p,the probability of a tree being planted at location i, j. We observe a the critical density pc at a little below 0.6.

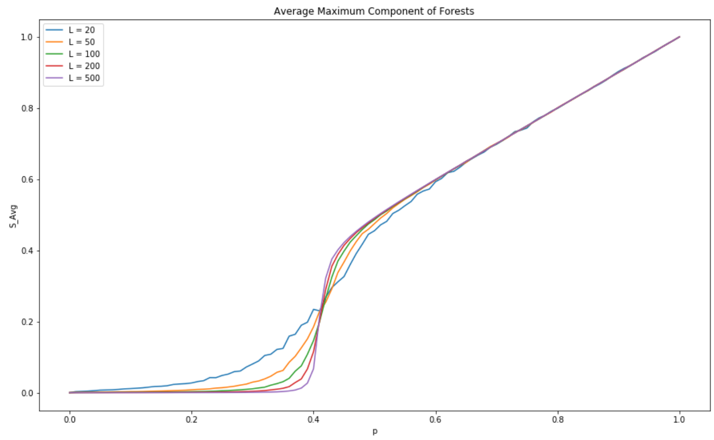


Figure 4: Linear Scale

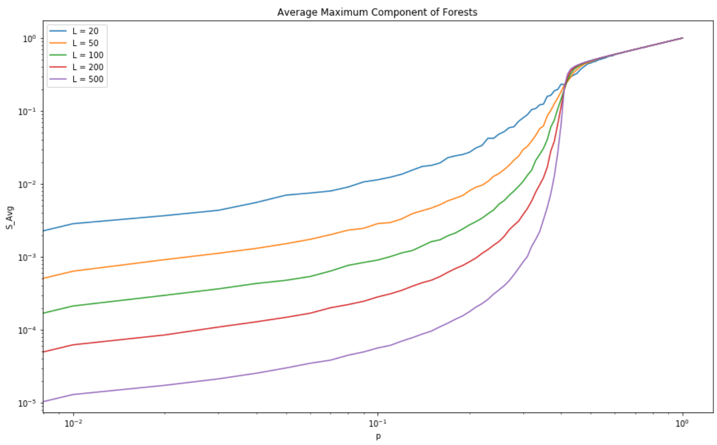
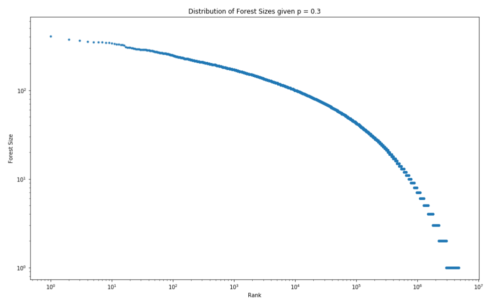


Figure 5: Log Scale

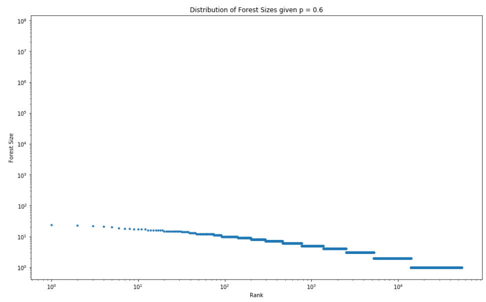
It's worth examining the log-log plot as we can see how the average component sizes changes as a function of L for small P. We see that for low probability values, smaller forests have larger average component sizes. However, as the probability of a trees growing in the forest increases, the largest component sizes converge for all forest sizes. This convergence happens at about p = 0.6. This would be my estimate of pc.

0.4 Problem 4

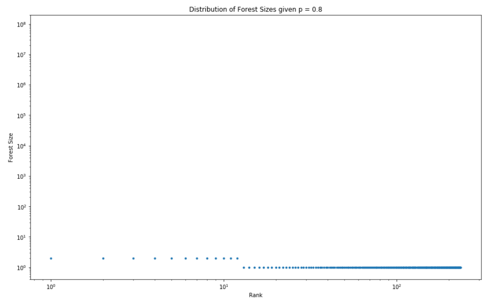
Using the model, plot the distribution of forest sizes for $P = pc$ and for values of p well above and below pc



(a)



(b)



(c)

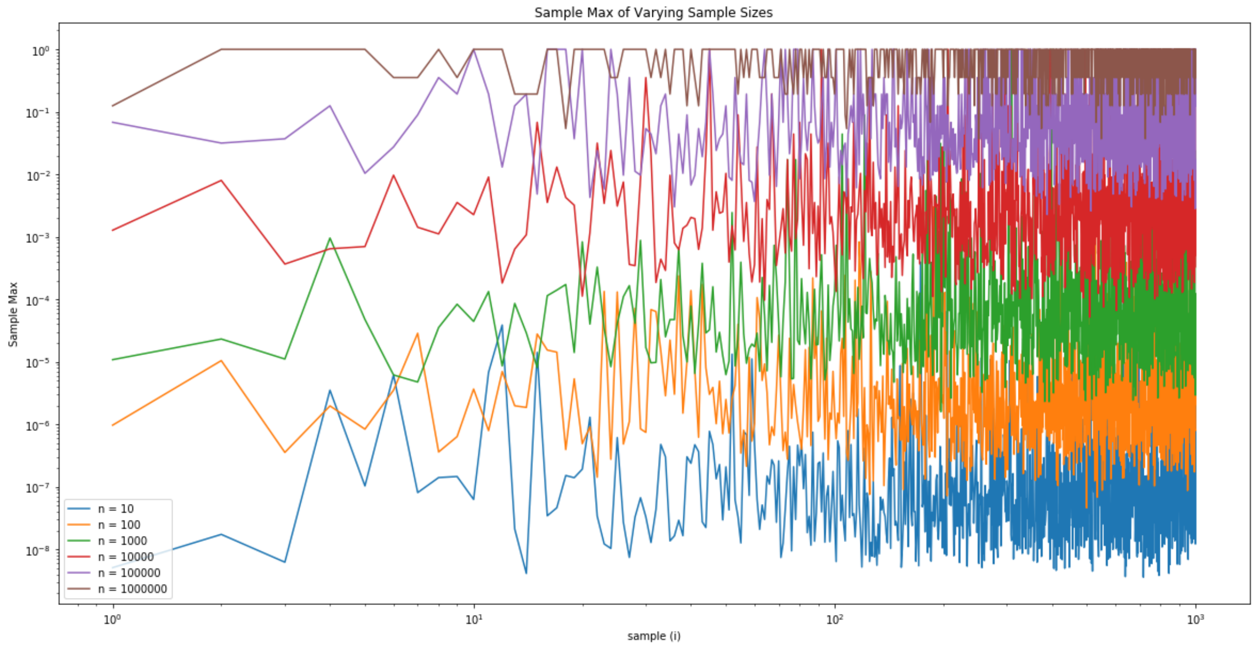
Figure 6: Values of p above, at and below the critical p level

The cluster size distribution for critical $p = .6$ shows a clear heavy tailed power law distribution. There are many many components of size one and two and than the number of larger clusters immediately taper off. When we twist the knob and set p above and below the critical point, we see different cluster size distributions. When $p > p_c$,there is an expanding cluster for which all cells have a probability of joining. All other clusters shrink, most to single treed forests. When $p < p_c$, we see a much more diverse spread of cluster sizes. There is a much slower decay of of cluster sizes.

0.5 Problem 5

Repeat the last question from assignment 4, changing from $\gamma = 5/2$ to $\gamma = 3/2$.

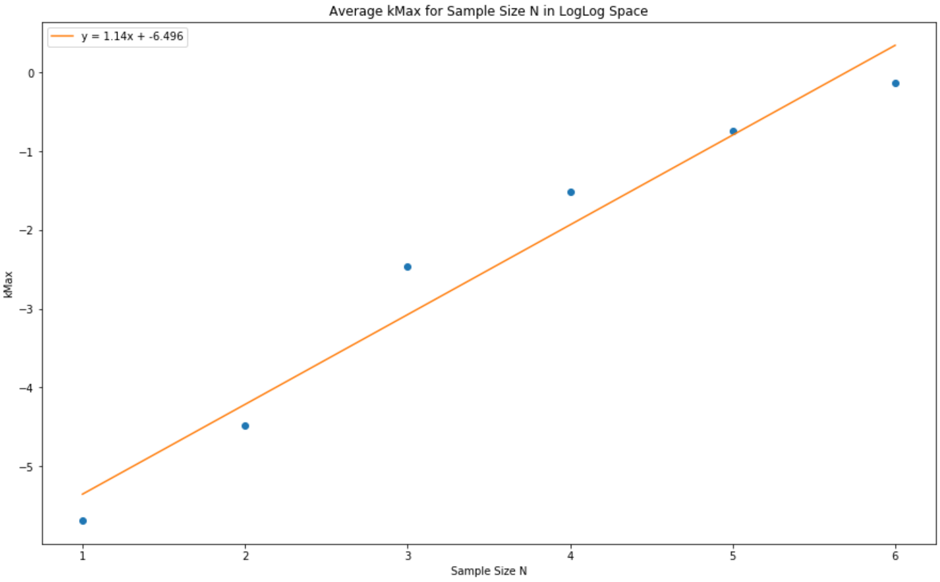
The graph shows us that each sample size produces a different maximum K regime. The smaller the sample size, the higher the probability. This is true for both γ



kMax for different sample sizes

(b) For each set, find the max value, then find the average max value for each N. Plot <Kmax> as a function of N and calculate the scaling using least squares.
Does your scaling match up?

The calculated scaling exponent is 1.14.


Average kMax for Sample Size N in LogLog Space
y = 1.14x + -6.496
kMax
Sample Size N
0
−1
−2
−3
−4
−5
1
2
3
4
5
6