PRINCIPLES OF COMPLEX SYSTEMS

ASSIGNMENT 1

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0.1 Problem 1

Use a back of an envelope scaling argument to show that maximal rowing speed V increases as the number of oarspeople N as V $\propto N^{1/2}$.
Assuming the following:
(a) Rowing shells are geometrically similar (isometric). The table below taken from McMahon and Bonner shows that shell width is roughly proportional to shell length .
(b) The resistance encountered by a shell is due largely to drag on its wetted surface.
(c) $V^2 \propto l^2$
(d) $P \propto D_f \propto V$
(f) Depth of water displacement by the shell grows isometrically with boat length l.
(g) $P \propto N$

Goal: Show that $V \propto N^{1/9}$

Derived Relations: l^3 = V
Solution Steps:

$$V \propto P/D \text{ From assumption C} \tag{1}$$
$$D \propto V^2 l^2 \text{ From assumption b} \tag{2}$$
$$D \propto V^2 N^{2/3} \text{ From derived relation} \tag{3}$$
$$V \propto N/V^2 * N^{2/3} \tag{4}$$
$$V \propto N^{1/3}/V^2 \tag{5}$$
$$V \propto N^{1/9} \tag{6}$$

0.2 Question Two

Find the modern world record times for the 2000 meter races and see if this scaling still holds up.

Data was found at : https://en.wikipedia.org/wiki/List_of_world_records_in_rowing
Modern record times for the 2000 meter race are displayed in the table below.

Number Oarsmen	Time (min)
1	6.51
2	5.99
4	5.63
4	5.85
8	5.50

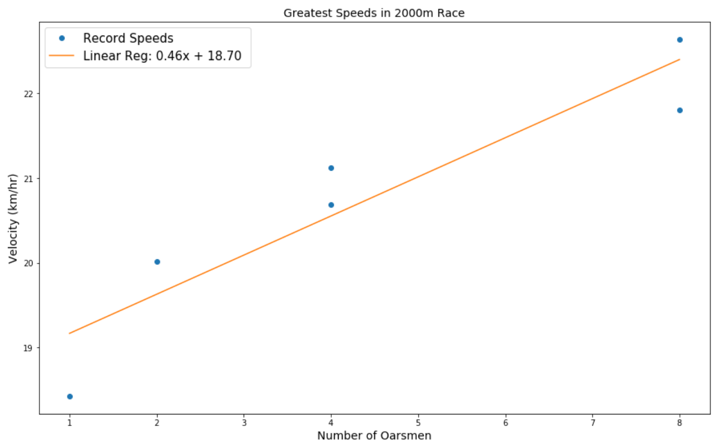


Figure 1: Scaling in Boat Performance

0.3 Problem 3

Check the current weight lifting records for the snatch, clean and jerk, and the total for scaling with body mass (Three regressions).

(a) Does the 2/3 scaling hold up?

Records from all three disciplines are plotted against their respective weight classes. Linear Least Squares Regressions are included.The 2/3 rule did not hold up. The slopes of the log log plots does not hold up.

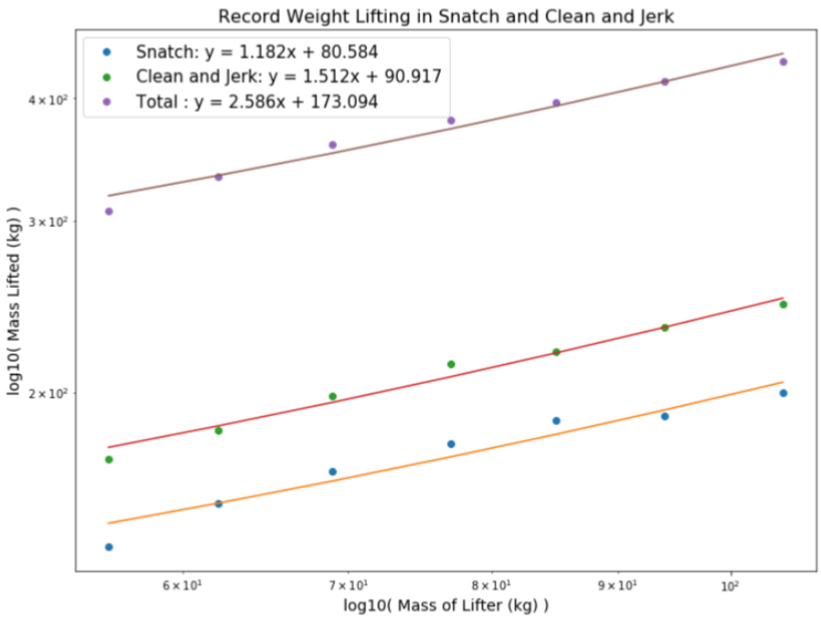


Figure 2: Scaling in Olympic Weight Lifting

(b) Normalizing by the appropriate scaling, who holds the overall, rescaled world record?

The best normalized lifting scores were held by the lightest contestants. These data are displayed below.

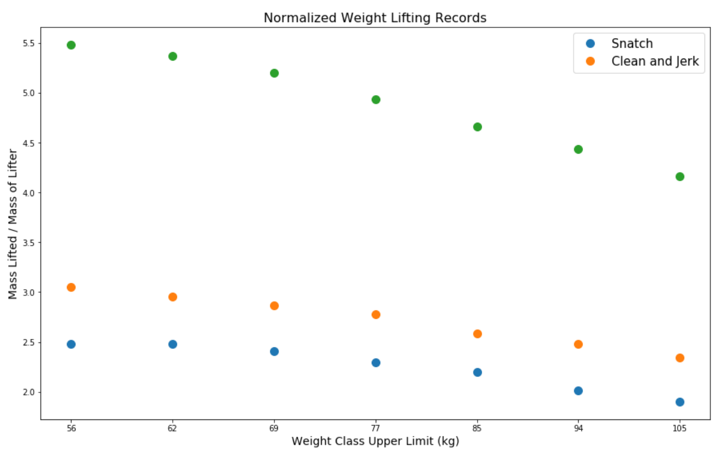


Figure 3: Scaling in Olympic Weight Lifting

0.4 Question 4

Finish the calculation for the platypus on a pendulum problem to show that a simple pendulum's period τ is proportional to $\sqrt{l/g}$

$$A\vec{x} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} * \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If we define the following parameters of the pendulum model as their dimensions, we can use the Buckingham Pi Theorem to find a fundamental scaling relationship.
$[l]$ = L, $[m]$ = M, $[g] = L * T^{-2}$, $[\tau]$ = T
We develop $[\pi_1] = L^{X_1} M^{X_2} LT^{-2X_3} T^{X_4}$
By setting X_1 = 1, we solve: X_2 = 0, X_3 = -1, x_4 = -1. Thus:

$$[\pi_1] = L(LT^{-2})^{-2} T^{-2} \tag{7}$$
$$\pi_1 = \frac{l}{g} t^2 \tag{8}$$
$$\pi_1 * t^2 = \frac{l}{g} \tag{9}$$
$$t \propto \sqrt{l/g} \tag{10}$$

0.5 Question 5

Show that the maximum speed of animals V_{max} is proportional to their length L using five dimensional parameters.

V_{max} : Max speed
l : animal length
p : organismal density
σ : max applied force per unit mass

β : max metabolic rate per unit mass
Express $\frac{V_{max}}{l}$ in terms of p,σ,β.
We first determine the dimensions of the parameters

$$[p] \propto ML^{-3} \tag{11}$$
$$[\sigma] \propto ML^{-1}T^{-2} \tag{12}$$
$$[\beta] \propto L^2T^{-3} \tag{13}$$
$$[V_{max} \propto T^{-1}] \tag{14}$$

Some basic algebra yeilds:

$$[\sigma M \frac{1}{L}\frac{1}{T^2}] \tag{15}$$
$$T = \sqrt{\frac{M}{L\sigma}} \tag{16}$$
$$T = \sqrt{\frac{pl^3}{L\sigma}} \tag{17}$$
$$T^2 = \frac{pl^2}{\sigma} \tag{18}$$
$$T^{-1} = \frac{p\beta}{\sigma} \tag{19}$$

0.6 Question Six

Use the B.P. Theorem to reproduce G.I. Taylors finding the energy of an atom bomb E is related to the density of air p and the radius of the blast wave R at time t.
Show that E = constant X $^5/t^2$.

In constructing the matrix, order parameters as E,p,R, and t and dimensions as L,T,M.
We show that:

$$[p] = M/L^3, [R] = L, [t] = T, [E] = \frac{ML^2}{T^2} \tag{20}$$

$$[\pi_1] = \frac{ML^2}{T^2}^{X_1} \frac{M}{L^3}^{X_2} L^{X_3} T^{X_3} \tag{21}$$

$$[\pi_1] = M^{X_1+X_2} L^{-3X_2+X_3+2X_1} T^{X_4-2X_1} \tag{22}$$

Setting $X_1 = 1$, yields $X_2 = -1$,$x_3 = -5$,$X_4 = 2$.

$$\pi_1 = EP_{-1}R^{-5}T^2 \tag{23}$$

$$E \propto \frac{pR^5}{T^2} \tag{24}$$

0.7 Question 7

Use the B.P. Theorem to derive Keplers third law. Find $T^2 \propto r^3$. Given parameters include: Planets Mass m,Suns Mass M, Orbital Period T,Orbital Radius r,Gravitational Constant G.

(a) What are the dimensions of these five quantities?

[m] = M, [M] = M,[T] = T, [r] = T, [G] = $L^3T^{-2}M^{-1}$

(b) When $\pi_2 = m/M$, what is π_1? π_i are a collection of solutions to the Ax=b problem formed by the exponent matrix.

$$\pi_i = M^{x_1+x_2-x_5} * T^{X_3-2x_5} * L^{x_4+3x_5}$$

$$A\vec{x} = \begin{bmatrix} 1 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix} * \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The second solution to the Ax =b problem gives us:

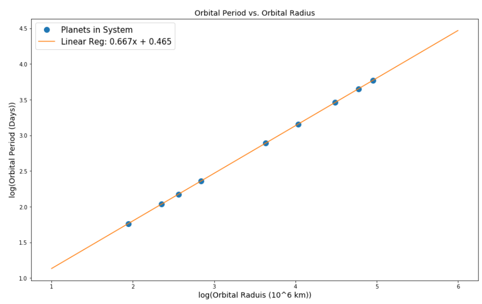
$$X_4 = -3, X_3 = 2, X_1 = X_2 = 1/2 \tag{25}$$

$$\text{Resulting in:} \tag{26}$$

$$\pi_i = m^{1/2}M^{1/2}T^2r^{-3}G \tag{27}$$

$$\text{As G,r,m and M are constants and thus} \tag{28}$$

$$\text{depend on which solar system is in question: } \pi_i = \frac{T^2}{r^3} \tag{29}$$



(c/d) Argue that $T^2 \propto r^3$ and use Linear Regression to see how the 2/3 scaling holds up. As the remaining terms of π_1 were constants. We can define:

$$\pi_1 = \frac{T^2}{r^3} T^2 \propto r^3 \tag{30}$$

The empirical slope of the log log plot is given by $\alpha = .667$ which confirms that the 2/3 scaling is still true and that the planets have not changed in this way.

0.8 Question 8

Surface areas of allometrically growing minecraft organisms. Let's consider animals as parallelepipeds (e.g., the well known box cow), with dimensions L1,L2,L3and volume V=L1*L2*L3. As we vary in scale of organism, lets assume the lengths scale with volume as Li=C1iViwhere the exponents satisfy 1+2+3= 1 and the c_i are prefactors such that c1*c2*c3= 1. Let's also arrange our organisms so that $\gamma_1 \leq \gamma_2 \leq \gamma_3$.

(a) Show that the scaling $L_i = C_i^{-1} V^{\gamma^i}$

$$L_1 * L_2 * L_3 = C_1^{-1} V^{\gamma 1} * C_2^{-1} V^{\gamma 2} * C_3^{-1} V^{\gamma 3} \tag{1}$$

$$L_1 * L_2 * L_3 = \frac{V^{\gamma 1+\gamma 2+\gamma 3}}{c_1 * c_2 * c_3} \tag{2}$$

$$L_1 * L_2 * L_3 = V \tag{3}$$

Write down the γ_i corresponding to isometric scaling.

Isometric scaling means that as the volume grows, surface area must grow at the same rate. Due to the constraints on γ_i , $\gamma_1 \leq \gamma_2 \leq \gamma_3$ and $\gamma_1 + \gamma_2 + \gamma_3 = 1, \gamma_i = 1/3$. This would be true if our shape was an isometrically growing square or prism.

(c) Calculate the surface area S of our imaginary beings.

$$S = 2(L_1 L_2) + 2(L_1 L_3) + 2(L_2 L_3) \tag{4}$$

$$*V = L_1 L_2 L_3 \tag{5}$$

$$S = 2V(\frac{1}{L_3} + \frac{1}{L_2} + \frac{1}{L_1}) \tag{6}$$

(d) Show the S behaves as V becomes large(i.e. which term dominates).

As the volume grows, surface area grows as a function of σ values. This behavior is explored in a graph included in the solution to part e. This was discovered experimentally by varying values of sigma around a known starting point.

(e) Which sets of λ_i give the fastest and slowest possible scaling of S as a function of V?

As gamma approaches zero, one of the planes of the cow will begin to collapse and flatten like a pancake pressed onto a grill top. The length of the collapsing side will approach one. This will result in the fastest possible scaling, such a $_i$ would be represented as (m,1/2-m,1/2+m) where m= machine epsilon. The slowest scaling would result from gammas such as λ_i= (1/3,1/3,1/3).Below is a plot that shows how surface area scale with volume varying theparameterwith respect to its constraints. The top green curve represents the scaling with the first combination of λ values and bottom the last.