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Procedia Computer Science 114 (2017) 481–490

Procedia Computer Science

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Complex Adaptive Systems Conference with Theme: Engineering Cyber Physical Systems, CAS October 30 – November 1, 2017, Chicago, Illinois, USA

Daily Stock Returns Characteristics and Forecastability

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Abstract

While stock prices and economic activity are interrelated in a nation, they "are not coincident" with each other. Stock prices are a leading economic indicator of the United States of America's (U.S.A.'s) economy. An economic variable that influences stock market prices is interest rates through an inverse relationship. The changes in stock prices (or stock returns) are generally caused by the demand for stocks. This paper reports on a study that investigates the underlying spectral and time-frequency characteristics of daily Standard and Poor's (S&P) 500, Dow Jones Industrial Average (DJIA), and National Association of Securities Dealers Automated Quotations (NASDAQ) composite stock returns, and changes in interest rate (namely, inverted 3-month Treasury bill). The study thereafter compared these findings with those obtained in a previous study by Joseph et al, which focused on monthly stock returns and interest rate data. Subsequent to studying stock returns and changes in interest rate that showed relatively similar spectral and frequency-time characteristics, this study investigated the forecastability of stock returns (in S&P 500, DJIA, and NASDAQ composite) by inverted interest rate (in 3-month Treasury bills) over prediction horizons of five and 30 days with the forecasting period covering the last 13 years. The measures of forecast accuracy used were root mean square error and correlation. The forecasts were favorable in all cases even with simpler neural network models.

Peer-review under responsibility of the scientific committee of the Complex Adaptive Systems Conference with Theme: Engineering Cyber Physical Systems.

Keywords: Stock market returns; Treasury bill interest rate; Forecasting models; Forecastability; Nonparametric spectral analysis; Time-frequency analysis.

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Peer-review under responsibility of the scientific committee of the Complex Adaptive Systems Conference with Theme: Engineering Cyber Physical Systems.
10.1016/j.procs.2017.09.033

1. Introduction

Stock prices and economic activity in a nation are interrelated. However, they "are not coincident" with each other [1]. Stock prices are a leading economic indicator of the United States' (U.S.') economy. One of the economic variables that influences stock prices is interest rates, but through an inverse relationship [1], [2], [3], [4], [5]. The changes in stock prices (or stock returns) are generally caused by the demand for stocks [3]. This paper investigated the underlying spectral and time-frequency characteristics of daily Standard and Poor's (S&P) 500, Dow Jones Industrial Average (DJIA), and National Association of Securities Dealers Automated Quotations (NASDAQ) composite stock returns as well as changes in interest rates (namely in inverted 3-month Treasury bill rate). Thereafter, the findings were compared with each other and those obtained in the study by Joseph et al [4], which focused on monthly stock returns and 3-month Treasury bill (T-bill) interest rate data. Subsequent to studying the stock market returns of the three prominent indices and changes in T-bill interest rate, this study proceeded to examine the forecastability of stock returns (in S&P 500, DJIA, and NASDAQ composite) using their past values and inverted 3-month T-bill interest rate as the predictor variables. Forecasting period covered almost 13 years out-of-sample stock return data, July/August 2004 to March 2017. The measures used to determine forecast accuracy were root mean square error and correlation, and the forecasted results were overall better than those obtained for the equivalent monthly data in [4] by Joseph et al.

Interest rates affect the economy through both short-term effects in the money market and the long-term effects in the loanable funds market [3]. The Federal government regulates the economy in the short-term with interest rates (namely, Federal funds rate) through the supply of and demand for money [3]. The rise and fall in the Federal funds rate correspondingly results in the rise and fall of all other interest rates including T-bill interest rate [2], [3]. The changes in interest rates are inversely related to the changes in stock prices, and consequentially stock returns [2], [3], [4], [5].

In [4], Joseph et al confirmed that monthly T-bill interest rate and stock returns time series data were nonGaussian, persistent, and time varying with smooth compact support over a very narrow band of low frequencies. They also showed that the correlation between pairs of S&P 500, DJIA, and NASDAQ were strong -- moderately high to high: lowest between DJIA and NASDAQ and highest between S&P 500 and DJIA with values of 0.73 and 0.95, respectively. Moreover, they said that the models used for forecasting were designed with a prediction horizon of one month (1-step ahead) and that the forecasts for S&P 500 stock returns were discernably the best of the three stock market returns forecasts. Fama in [6] stated that stock returns were examples of "stable" nonGaussian distributions, but that daily returns were more nonGaussian than monthly ones. Jondeau et al [7] provided further evidence to show that stock returns and interest rates were nonGaussian and time-varying with stock returns being negatively skewed and interest rates being positively skewed, and both stock returns and interest rates have excess kurtosis suggestive of fat tails, which are not present in Gaussian distributions that have skewness of 0 and kurtosis of 3. They further said that "correlation is not a valid measure for dependence" for non-Gaussian stock returns. Cont [8] discussed and elaborated further on the properties of financial asset returns including stock returns. In addition, Cont addressed the persistence and nonlinearity that are inherent in stock returns. Starica and Granger [9] stated that by acknowledging the nonstationarity inherent in stock returns and taking advantage of it in the design of the forecasting models yielded better forecasts than those produced by models designed for stationarity of stock returns.

In addition to stock returns being nonGaussian, time-varying with relatively low frequency spectral content on smooth compact support, negatively skewed with kurtosis exceeding 3, nonlinear, and persistent, they are noisy and volatile [5], [10], [11], [12], 13, 14]. Nonetheless, there is an abundance of evidence demonstrating that stock returns are forecastable with both conventional parametric statistical tools and more sophisticated nonparametric statistical learning tools (such as neural network, neuro-fuzzy, support vector machine models) [4], [9], [10], [12], [14], [15], [16], [17], [18], [19], [20], [21], [22] with statistical learning models performing generally better than classical statistical models [10]. In spite of this evidence, there is no consensus between academics and practitioners on the validity of stock returns forecasting on the basis of the efficient market hypothesis [14], [16], [18] [19], [23]. It may prove difficult to resolve the issue of stock returns' forecastability among academics and practitioners because as reported by Balvers et al [16], forecastability, within certain contexts, may necessarily be consistent with the efficient market hypothesis, Zhou [14] reported on the existent misalignment of financial practice and academic theory supporting stock returns' predictability, and Elliott and Timmermann [15] stated that economic models are "coarse approximations to a far more complex and evolving reality with biases that shift over time." Nevertheless, in forecasting stock returns, preprocessing [4], [10], 12] and out-of-sample forecasts [4], [9], [15], [18] are said to yield more meaningful results in terms of quality and usefulness.

2. Materials and Methods

"The central idea to successful stock market prediction is achieving best results using minimum required input data and the least complex stock market model ... , and an important observation is that almost all articles referring to data preprocessing find it useful and necessary" [10]. The data used for this study were comprised of daily stock market adjusted closing prices of the S&P 500, DJIA, ands NASDAQ composite obtained from the Yahoo! Finance website (https://finance.yahoo.com/) and 3-month T-bill interest rate obtained from Federal Reserve Economic Data of Federal Reserve Bank of St. Louis. These stock prices were closing prices adjusted for splits and dividends. The 3-month T-bill interest rate was a secondary market rate not adjusted for seasonality. The 3-month T-bill was used in this study because it is among the most widely traded interest rate financial instruments and unlike the one-month T-bill it covers the entire period of the study, which starts January 29, 1985. Hence, 3-month T-bill rate is best at representing the depth, breath and resilience of yield markets, and it is typically used as a proxy for risk-free rate of return on an investment. It also has moderate volatility compared to the 1-month bill and other rates below a 5 and 30 day time horizon. Stock prices are inversely and nonlinearly related to interest rates [2], [4], [5], and interest rates and corporate earnings are among the variables that directly determine stock prices [2], [4], [10], [12], [17], [18], [19], [21], but interest rates affect stock prices through price per earnings ratios; interest rates and price per earnings "ratios are inversely related to each other and interest rates are used 'as a proxy' for the [price per earnings] ratio" [2], [4]. However, Davis et al [17] found in "predicting historical stock returns" price per earnings ratios were more useful than any other variable in forecasting stock returns; it could explain "approximately 40% of the time variation in real stock returns" over a long horizon. Since price per earnings ratios and interest rates are inversely related to each other, and that there are other myriad of influences on stock prices, we anticipate that inverse interest rates as proxies of price per earning should also be a reasonable predictor of stock prices and stock returns.

To get the data in a form for analyses and forecasting, the data were filtered, transformed, and detrended using the following tools: Microsoft Excel 2010 and MathWorks Matlab R2017a. The analyses and forecasting of the preprocessed data were done with the aid of Microsoft Excel 2010 in MathWorks Matlab 2017a and NeuroDimension NeuroSolutions 7.

The raw unfiltered daily data span the duration from January 29, 1985 to March 17, 2017. While the stock prices consisted of 8101 samples, the 3-month T-bill interest rate consisted of 8384 entries including valid samples and not available (#N/A). The 3-month T-bill interest rate data set was cleansed of the #N/As and excess valid entries to bring it to the length of the stock prices. The N/As were removed when they did not coincide with stock prices, and when they did coincide with stock prices the mean values of the respective two adjacent T-bill interest rate values were used in place of the N/As. The valid T-bill interest rate values that did not coincide with the stock price values were also removed. The T-bill interest rate variable was then multiplied by minus one (-1) to get the additive inverse of T-bill interest rate. The unfiltered 8101 daily samples of stock prices and inverted T-bill interest rate have respective mean and standard deviation values of 985.41 and 555.52 for S&P 500, 8540.8 and 4954.1 for DJIA, 1923.3 and 1375.8 for NASDAQ, and -3.42 and 2.59 for inverted 3-month T-bill interest rate. Furthermore, the Spearman's correlation between unfiltered pairs of stock prices and stock prices and inverted T-bill rate were 0.9864 between S&P 500 and DJIA, 0.9874 between S&P 500 and NASDAQ, 0.9772 between DJIA and NASDAQ, 0.6770 between S&P 500 and inverted T-bill interest rate, 0.7465 between DJIA and inverted T-bill interest rate, and 0.7074 between NASDAQ and inverted T-bill interest rate.

The stock price and T-bill rate data were filtered, transformed, and detrended in the Matlab environment. These data were filtered with the *smoothdata* function setup for a 100-day moving average window. The corresponding respective mean and standard deviation values were 985.22 and 553.77 for S&P 500, 8539.0 and 4939.6 for DJIA, 1922.7 and 1368.3 for NASDAQ, -3.42 and 2.58 for inverted T-bill rate while the Spearman's correlation values consistent with pairs of these variables were 0.9884 between S&P 500 and DJIA, 0.9878 between S&P 500 and NASDAQ, 0.6864 between S&P 500 and inverted T-bill rate, 0.9812 between DJIA and NASDAQ, 0.7504 between DJIA and inverted T-bill rate, and 0.7172 between NASDAQ and inverted T-bill rate. Subsequent to the filtering process, the stock prices and inverted T-bill rate were transformed into stock returns using 121-day relative changes and 121-day backward first difference respectively [4], [7], thereby reducing these datasets to 7980 samples, which spanned July 23, 1985 to March 17, 2017. The corresponding respective mean and standard deviation values for the stock returns and the backward differenced inverted 3-month T-bill interest rate (or simply T-bill rate or interest rate henceforth) were 0.0430 and 0.0924 for S&P 500, 0.0460 and 0.8900 for DJIA, 0.0553 and 0.1382 for NASDAQ, and 0.1103 and 0.6875 for T-bill rate whereas the Spearman's correlation values corresponding to pairs of these four variables were 0.9255 between S&P 500 and DJIA, 0.8615 between S&P 500 and NASDAQ, -0.0984 between S&P 500 and T-bill rate, 0.7633 between DJIA and NASDAQ, -0.0610 between DJIA and T-bill rate, and -0.0843 between NASDAQ and T-bill rate. Following the transformation process, the stock returns and T-bill rate were detrended using the Matlab *detrend* function. The general form of the removed trend line was $y(t) = a + bt$, where for S&P 500 returns the specific equation for the removed trend line was $y_{sp}(t) = 0.0641 - 5.27\times10^{-6}t$ and DJIA returns it was $y_{dj}(t) = 0.0761 - 7.54\times10^{-6}t$. Following detrending, T-bill rate was scaled to within ±1 by dividing it by its maximum absolute value, which was 2.1127. With the stock returns and T-bill rate detrended, their means became essentially zero and their standard deviations were 0.0916, 0.0873, 0.1381, and 0.3249 corresponding to S&P 500, DJIA, and NASDAQ stock returns and T-bill rate, respectively (see Table 1). Moreover, the Spearman's correlation values between respective pairs of detrended stock returns were 0.9363 between S&P 500 and DJIA, 0.8604 between S&P 500 and NASDAQ, and 0.7665 between DJIA and NASDAQ. The correlation values between the detrended stock returns and T-bill rate were very weak. The correlation values between pairs of the detrended stock returns and between stock returns and T-bill rate were examined via coherence. Coherence is a form of correlation defined in the frequency domain on the range between 0 and 1 [24]. The coherence values between stock returns and T-bill rate were similarly very weak, generally ranging on average from 0.0 to 0.2 over the frequency range, but their values were weak to moderate: 0.329 between NSADAQ and T-bill at 4.29μHz, 0.495 between DJIA and T-bill at 1.910μHz, and 0.500 between S&P 500 and T-bill rate at 1.91μHz. The coherence values between detrended stock returns (or simply stock returns henceforth) were generally more moderate to strong: between S&P 500 and DJIA the coherence generally ranged from 0.85 to 0.95, between S&P 500 and NASDAQ the coherence generally ranged from 0.55 to 0.85, and between DJIA and NASDAQ the coherence generally ranged from 0.40 to 0.75.

Subsequent to filtering, transforming, and detrending the stock price/returns and T-bill rate data, these data were examined for their skewness, kurtosis, Jacque-Bera statistics, and Hurst exponents (see Table 1) to first determine if they were nonGaussian and forecastable. Thereafter, the stock returns and T-bill rate were subjected to spectral and time-frequency analyses through frequency analysis and nonparametric estimation (power spectral density and short-time Fourier transform) [24], [25]. The Matlab functions used for the power spectral density and short-time Fourier transform (and spectrogram) were *pwelch* and *spectrogram* designed with the Chebyshev window (*chebwin*) [4] overlapping 50% and 87% respectively. The sampling rate of and the maximum frequency in the daily stock returns and T-bill rate were 1 sample per day (or 11.6μHz) and 5.79 μHz respectively with minimum permissible frequency resolution of 1.45nHz.

Table 1. Attributes of the preprocessed data: filtered, transformed, and detrended.

Data Type	Data	Mean	Standard Deviation	Skewness	Kurtosis	Jarque-Bera Test (p-value)	Hurst Exponent	Correlation between 3-Month T-Bill Interest Rate and Stock Returns
Independent Variables	3-Month T-Bill Interest Rate	0.0000	0.3249	0.4119	3.7862	0.0010	0.7606	
Dependent Variables (Stock Returns)	S&P 500	0.0000	0.0916	-0.9843	4.9039	0.0010	0.8117	-0.1318
	DJIA	0.0000	0.0873	-0.7690	4.6362	0.0010	0.8042	-0.1062
	NASDAQ	0.0000	0.1381	-0.3311	4.4407	0.0010	0.8103	0.0932

In the forecasting phase of this study, stock returns were the response variables and T-bill rate and past values of stock returns were the predictor variables. The forecasting experiments were carried out with eight models designed using temporal gamma neural networks, a time-lagged recurrent network, designed in NeuroSolutions environment for 8, 4 and 2 processing elements [4], [26] in 5-day and 30-day ahead prediction (see Table 2). Four of these neural network models were used to forecast S&P 500 stock returns with two of them using two processing elements for multistep prediction at 5-day and 30-day ahead while the other two used eight and four processing elements for 5-day ahead prediction. In each case the number of taps was 6 and the trajectory length was 15, with the activation function and learning algorithm respectively being the *tanh* function and the resilient backpropagation (Rprop) [27], [28]. The forecast of DJIA and NASDAQ stock returns did not include models designed with 8 and 4 processing elements.

Table 2. Design parameters for focused gamma neural network models.

Parameters	Network Models	Inputs	Hidden Layer	Output Layer
Topology	TLRN: Focused Gamma		1	1
Prediction Mode Horizon	5 and 30 day-ahead			
No. of Taps	6			
Tap Delay	1			
Depth of Samples	10			
Trajectory Length	15			
Training Data Subsets	60% (4785 and 4770 samples)			
Testing data Subsets	40% (3190 and 3180 samples)			
Total No. of Samples	7980			
No. of Processing Elements			8, 4, & 2	1
No. of Weights + Biases		2	104, 52, & 26	9, 5, & 3
Activation Function			Tanh	Tanh
Weight Update Mode	Batch			
Learning Algorithm			Rprop	Rprop
Step Size Initial Value			1.00	0.10
Type of Learning	Supervised control			
Weights on Testing	Best Weights on Training			
Error Termination	MSE			
Epochs/Run	28, 36, 9, 94, 34, 23, 33, & 39			

Note: MSE termination threshold = 0.0001 minimum on training.

Moreover, for all the forecasting regimens, the data were subdivided into two subsets – one subset for training the models and the other for testing them with out-of-sample data that constituted 40% of the entire datasets of 7980 samples minus the 5 and 30 samples used for 5-day and 30-day ahead prediction that corresponded to 3190 and 3180 samples respectively.

3. Results and Discussion

The correlation between pairs of stock prices and returns were reasonably strong to very strong. It was strongest between S&P 500 and DJIA with the correlation for their filtered and unfiltered prices being higher than the correlation for their filtered returns. The correlation between S&P 500 and DJIA filtered stock prices and returns were 0.9884 and 0.9255 respectively. The correlation values were lowest between DJIA and NASDAQ composite filtered stock prices and returns with values of 0.9812 and 0.7633 respectively. In all cases the correlation values between pairs of filtered stock prices were higher than those for unfiltered stock prices by difference values ranging from as low as 0.0004 between S&P 500 and NASDAQ to as high as 0.0040 between DJIA and NASDAQ stock prices. These findings show that stock market prices and returns are highly correlated but that some are more highly correlated than others, and that this very strong correlation is even evident among noisy stock prices. They further suggest that S&P 500 and DJIA stock market behavior as reflected in stock prices and returns are not as highly correlated with NASDAQ behavior as they are with each other. While filtered and unfiltered stock prices and inverted T-bill rate show reasonably strong correlation between pairs of stock prices and T-bill rate with the correlation values between filtered stock prices and T-bill rate being higher than those corresponding values between unfiltered stock prices and T-bill rate by correlation values of 0.0094 (S&P 500 and T-bill rate), 0.0039 (DJIA and T-bill rate), and 0.0098 (NASDAQ and T-bill rate). However, the correlation values between filtered stock returns and backward differenced T-bill rate were very weak. For example, the correlation between S&P 500 stock returns and T-bill rate was -0.0984 and that between DJIA stock returns and T-bill rate was -0.0610. This appears to suggest that the transformation of stock prices into stock returns and levels of T-bill rate into differences in T-bill rate practically removed all the linear relationship between stock returns and T-bill rate. After the stock returns and T-bill rate were detrended to remove any remaining trends in them, the correlation values between pairs of stock returns remained very strong between S&P 500 and DJIA (0.9363) and between S&P 500 and NASDAQ (0.8604) and reasonably strong between DJIA and NASDAQ (0.7665), Fig 1. The correlation values between stock returns and T-bill rate remained very weak (see Table 1). These very weak correlation values between stock returns and T-bill rate suggest that any relationship that exists between stock returns and T-bill rate must be nonlinear.

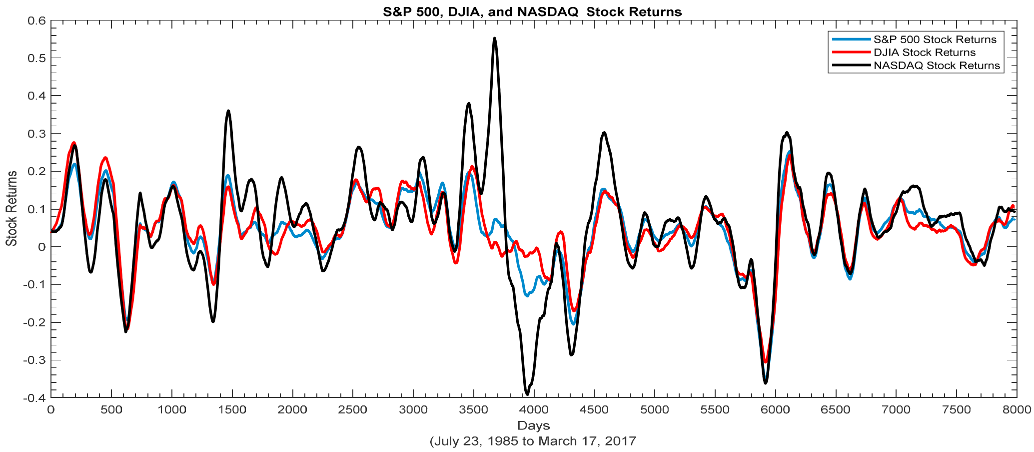

Fig. 1. The relationship between S&P 500, DJIA, and NASDAQ Composite daily stock returns over 32 trading years.

The power spectral density, spectrogram and the short-time Fourier transform of stock prices and T-bill rate were examined. The overall power spectral density estimates supports are bounded over a very narrow band of frequencies concentrated to within 0.2 μHz, see Fig.2. This compares well with the finding of Joseph et al [4] for

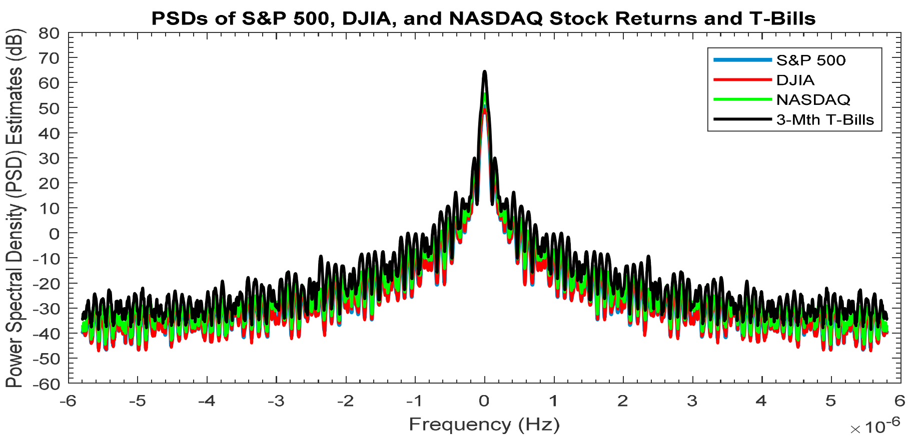

Fig. 2. The Power spectral density estimates of stock returns and T-bill rate.

monthly stock returns and T-bill rate. This low frequency narrow band support of stock returns and T-bill rate was also evident in the spectrogram and the short-time Fourier transform of these financial variables, which showed different views of the same thing. The stock returns had very similar spectrograms and short-time Fourier transforms with most of their energies concentrated over the very lowest frequencies, within 0.2μHz, for the about 20 calendar years; the trading year for stocks is about 252 days. An example of the spectrogram of stock returns is shown in Fig. 3 for S&P 500. This spectrogram does not contain the levels of time-frequency fluctuations in energy found in the one shown in Joseph et al [4]. In [4], the changes in the intensity of the energy of monthly S&P 500 stock returns

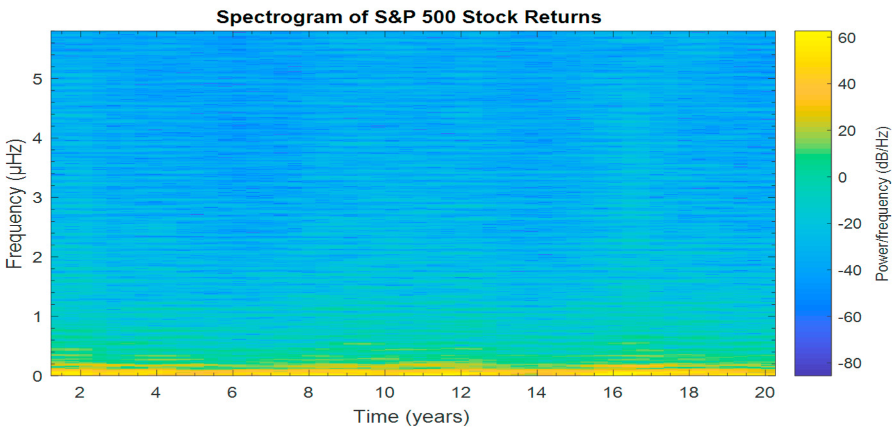

Fig.3. The spectrogram of the S&P 500 stock returns illustrative of DJIA and NASDAQ stock returns.

were clearly evident over different time frequency bins. The T-bill rate showed somewhat more changes in energy over the different time frequency bins, see Fig. 4. The energy intensity in the time slot of years 15 to 17 appears to

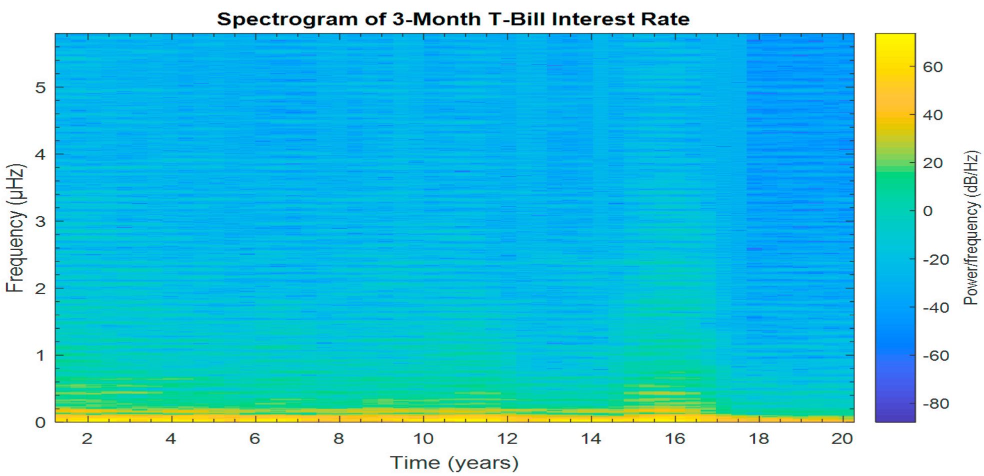

Fig. 4. The spectrogram of 3-month T-bill rate.

exist over several frequencies. It is strongest within 0.2μHz, but easily discernible up to about 2μHz.

The negative skewness and excess kurtosis values as well as Jacque-Bera inferential statistics along with Hurst exponent (see Table 1) provide evidence that stock returns are forecastable. The forecasted results for the out-of-sample stock returns were very good. They were, overall, better than those yielded for the related monthly stock returns reported in Joseph et al [4]. The correlation values were higher and the root mean square error (rmse) values were generally lower except in two cases relating to S&P 500 forecasts. The forecasts produced for DJIA and NASDAQ stock returns were generally better than those produced for S&P 500 stock returns. This is in spite of the fact that more powerful models were used in the forecast of S&P 500 returns; models 1 and 2 used 8 and 4 processing elements corresponding to 104 and 52 weights in the hidden layer, but their performances were not among the best two models, see Tables 2 and 3. The models that performed the best were models 5 and 7 for DJIA

Table 3. The out-of-sample forecasts of daily stock returns.

Response Variable	Models	Training				Testing (out-of-sample)			
		Time Span	Samples	RMSE	Correlation	Time Span	Samples	RMSE	Correlation
S&P 500	Model 1	7/ 23/1985-7/16/2004	4785	0.0548	0.9904	7/19/2004-3/17/2017	3190	0.1063	0.9744
	Model 2	7/23/1985-7/16/2004	4785	0.0424	0.9943	7/19/2004-3/17/2017	3190	0.0927	0.9830
	Model 3	7/23/1985-7/16/2004	4785	0.0557	0.9899	7/19/2004-3/17/2017	3190	0.1136	0.9711
	Model 4	7/23/1985-7/30/2004	4770	0.0748	0.9821	8/2/2004-3/17/2017	3180	0.1086	0.9744
DJIA	Model 5	7/23/1985-7/16/2004	4785	0.0566	0.9845	7/19/2004-3/17/2017	3190	0.0600	0.9849
	Model 6	7/23/1985-7/30/2004	4770	0.0889	0.9625	8/2/2004-3/17/2017	3180	0.0922	0.9623
NASDAQ	Model 7	7/23/1985-7/16/2004	4785	0.0616	0.9802	7/19/2004-3/17/2017	3190	0.0316	0.9883
	Model 8	7/23/1985-7/30/2004	4770	0.0640	0.9770	8/2/2004-3/17/2017	3180	0.0469	0.9761

Note:The prediction horizon of models 1, 2, 3, 5, and 7 is 5 whereas the prediction horizon for models 4, 6, and 8 is 30.

and NASDAQ stock returns respectively with model 7 performing the best with rmse value of 0.0316 and correlation value of 0.9883 between the desired and the forecasted values. No other model performed better in the two statistics of rmse and correlation. Both model 5 and model 7 used two processing elements and 26 weights in the hidden layer. The model that performed the worst in rmse values is model 3 with an rmse value of 0.1136 and the model that performed the worst in correlation is model 6 with a correlation value 0.9623 between the desired and the forecasted DJIA stock returns. The difference between the lowest (model 6) and the highest (model 7) correlation values was 0.028 while the difference between the highest (model 3) and the lowest (model 7) rmse values was 0.082. Models 4, 6, and 8 performed 30-day ahead out-of-sample forecasts while the other models performed 5-day ahead out-of-sample forecasts. Overall, the models performed well regardless of forecast horizon, but the 5-day ahead forecasts were generally better, at least in quality, than the 30 ahead forecasts that showed evidence of noise in the forecasts. The noisy forecasts were more pronounced in the S&P 500 and NASDAQ forecasts with it being most pronounced in the NASDAQ forecasts, see Figs. 5 & 6. In spite of the noisiness of 30-day ahead forecasts they followed the general pattern of all three stock returns reasonably well.

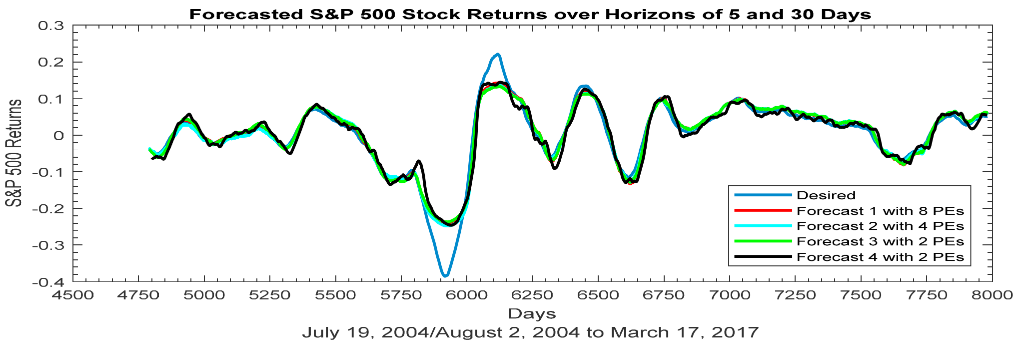

Fig. 5. S&P 500 stock returns forecasts with forecast 1, 2 & 3 being 5-day ahead and forecast 4 being 30-day ahead.

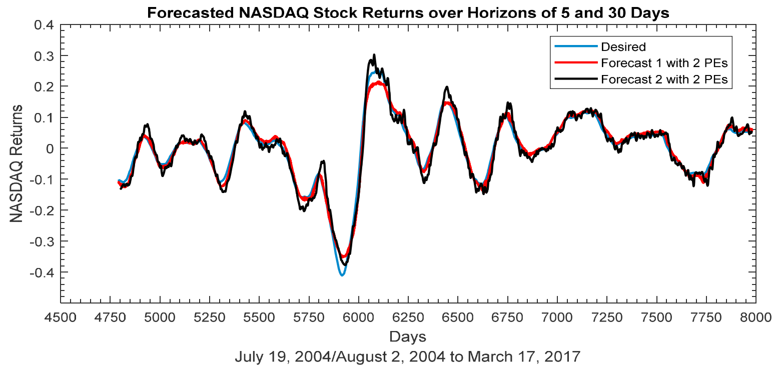

Fig. 6. NASDAQ Composite stock returns forecasts with forecast 1 being 5-day ahead and forecast 2 being 30-day ahead.

Because the models used interest rate as an independent variable with time leads of 5 and 30 days, there is no need to forecast the independent variable. In other words, given current rates we can apply the econometric coefficients of today's interest rate to forecast stock returns 5 days out, as well as use the second model to forecast 30 days into the future using the past 30 days' rates. The 30-day models can also be applicable in risk management, by giving warning of likely volatility. Additionally, there are over forty stock groupings by industry in 11 sectors reported by S&P. Examples of these groupings include such industries as automobiles, pharmaceuticals, and chemicals. These industry groupings have varying correlations with the S&P 500 index. Some industry groupings are highly correlated to the S&P 500 while others are less correlated to it. Therefore the highly correlated industries may also be forecasted with our models. The reliability of the forecast would depend on the level of the industry grouping's correlation with the S&P 500. Further significance of the results of this study lies in the use of the temporal neural network models that are more powerful statistical methods than those of regression analysis to relate interest rates to stock market equity indices.

4. Conclusion

Given the importance of economic and financial forecasting with regard to business decision making, understanding the characteristics of stock prices/returns, interest rates, and other economic and financial variables from different perspectives is paramount. For example, when this study is compared with Joseph et al [4] it is evident that higher frequencies' stock returns and interest rates behave differently especially in the time-frequency domain. This potentially suggests that at even higher frequencies there might be even greater differences in the behavior of these variables, thereby leading to new insights on how best to design models to effectively forecast stock returns and prices. In relationship to the work reported by Joseph et al [4] on the properties and forecasts of monthly stock returns as well as T-bill interest rate, this work compares well in some respects and not in others. The time-frequency analysis supported the power spectral density estimate analysis in both cases by showing the spectral support of the energy in stock returns and T-bill rate were concentrated in the very lowest frequencies. However, the spectrogram in [4] showed more distinct fluctuations in the energy of the stock returns and T-bill rate over the different time frequency cells. The forecasts in this work are generally better with higher correlation values and lower rmse values; the forecasts for DJIA and NASDAQ stock returns were generally better than those for S&P 500, albeit marginally. In addition, the models discussed in this paper are pertinent to the simulation and forecasting of the short-run trend of prices in the first offerings of stocks as well as the trends of the Dow Jones, S&P 500, and NASDAQ indices. They can help estimate the effect of the Federal Reserve weekly Treasury bill auctions on the 5 and 30 day equity markets. Moreover, the study of the relationship among stock prices and returns (S&P 500, DJIA, and NASDAQ composite) showed that reasonably strong correlation exists between them even when the stock prices are noisy and when the stock returns are detrended. However, the correlation values between levels of stock prices and T-bill rate are generally strong, but the correlation values between stock returns and differences in T-bill rate are very weak regardless of whether they were detrended. Future work on stock returns will include higher frequency stock returns: hourly, minute, or seconds stock returns data to determine if the attributes observed remain in terms of consistency in properties and forecastability using the appropriate predictor variables.

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