Université Paris Cité

2023/2024

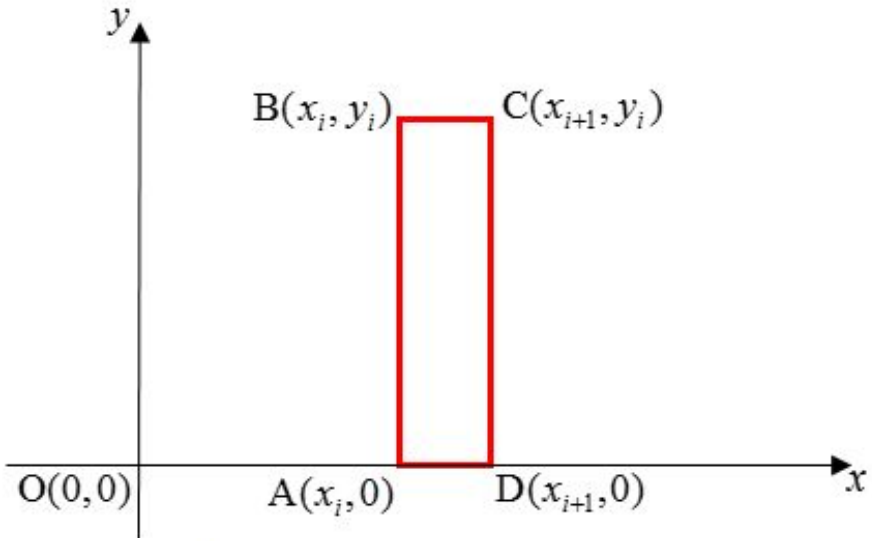
Programmation et traitement numérique

Python 3 : pour la physique

Courbe et intégration

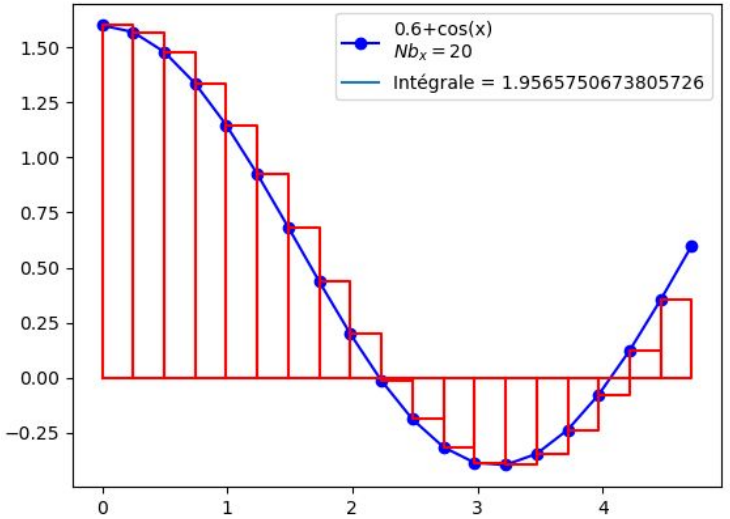
FRERE Maxime

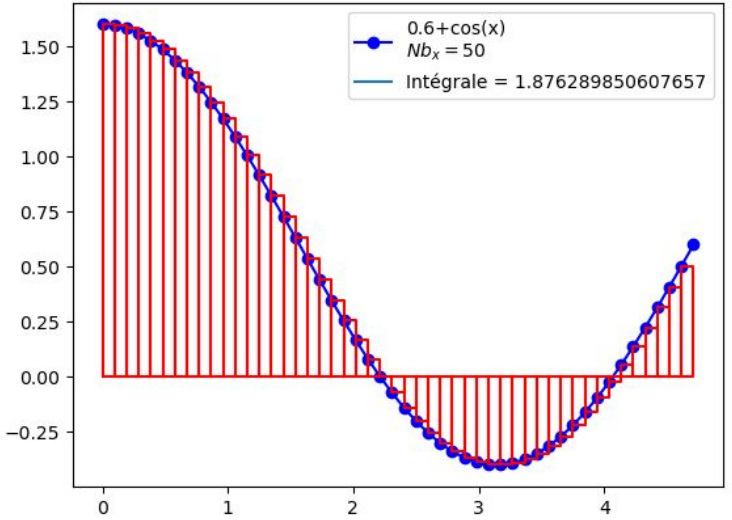
Méthode des rectangles



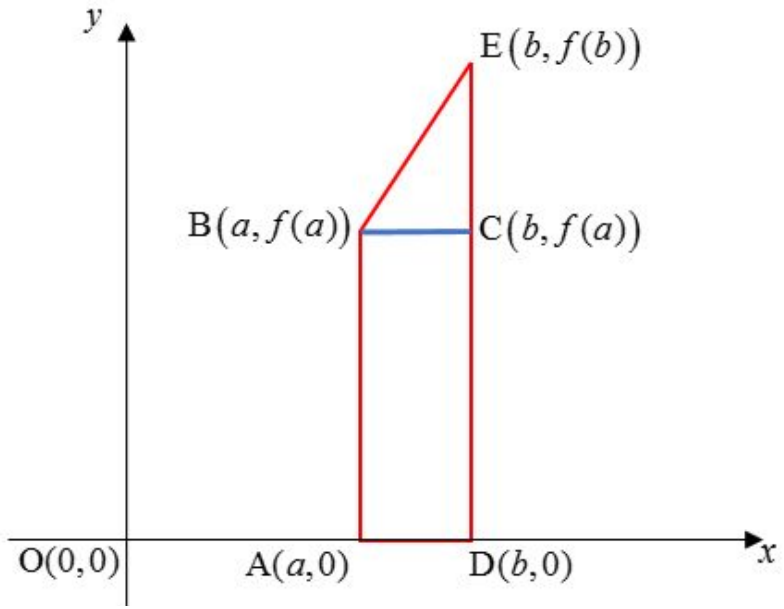
$$\int_a^b f(x)dx \approx (b-a)f(\alpha)$$

$$E(f) = \frac{(b-a)^2}{2} f'(\eta), \quad \eta \in [a,b].$$



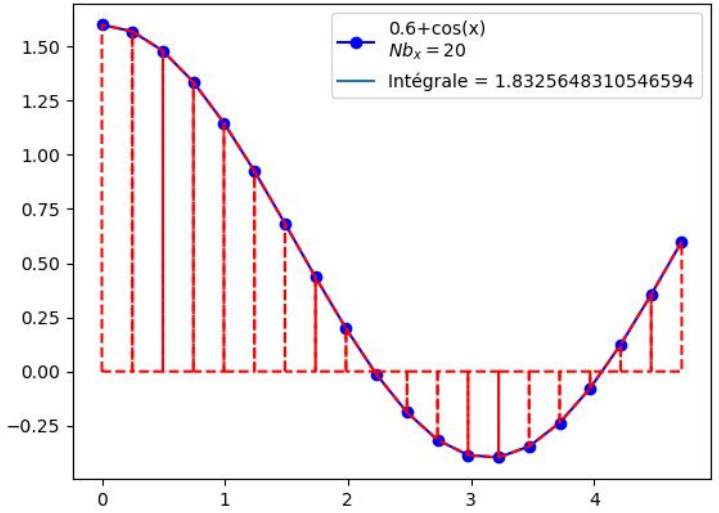


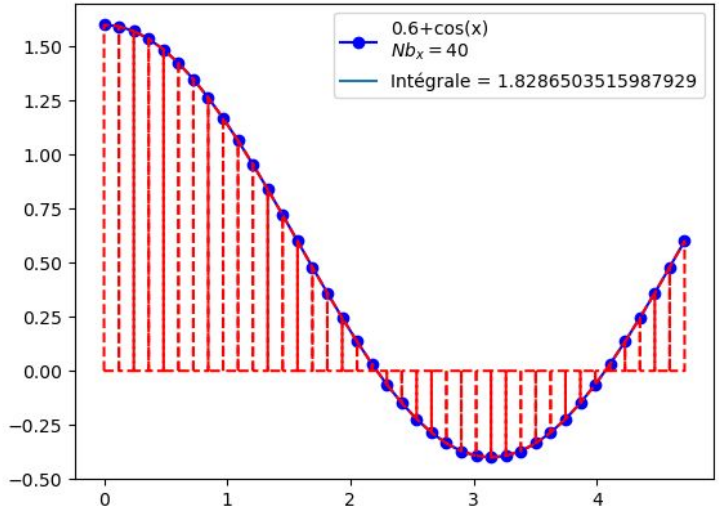
Méthode des trapèzes



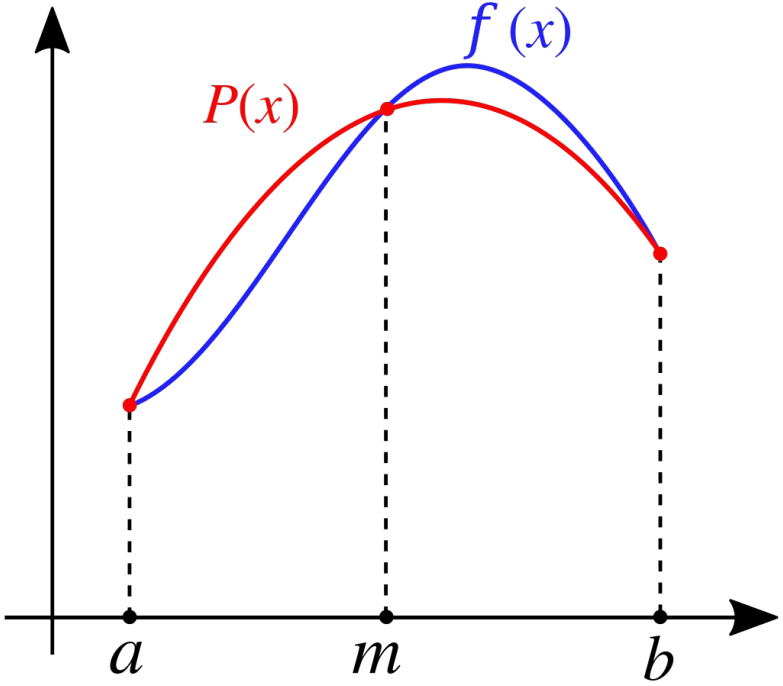
$$\int_a^b f(x)dx \approx \frac{b-a}{2}[f(a)+f(b)]$$

$$-\frac{1}{12}h^3 f^{(2)}(\xi)$$



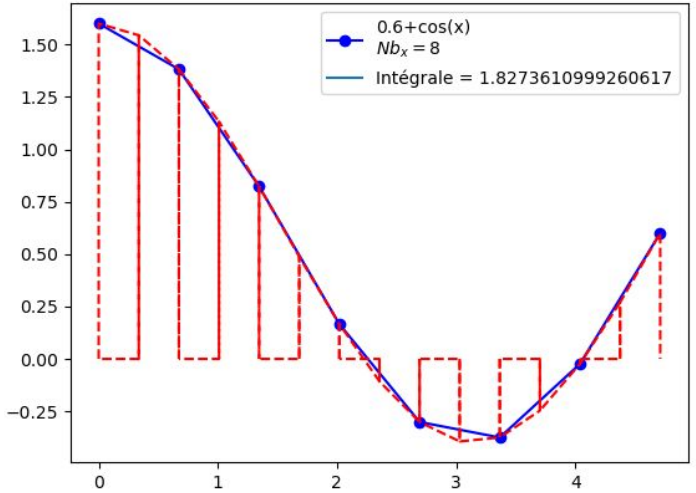


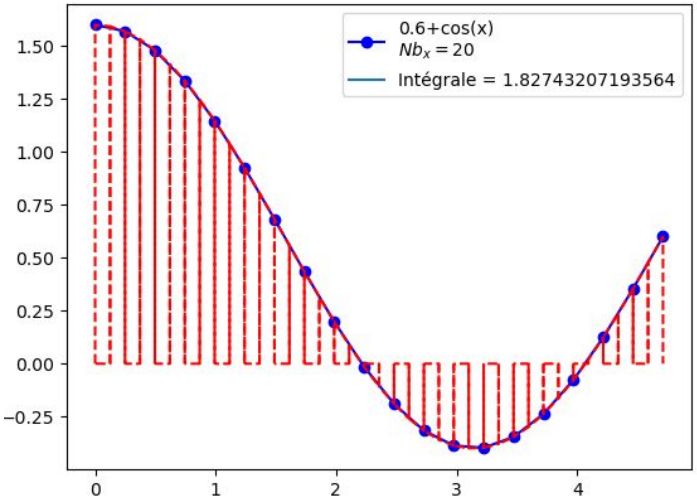
Méthode de Simpson



$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right].$$

$$-\frac{1}{90}h^5 f^{(4)}(\xi)$$





Méthodes plus avancées

Simpson's 3/8

$$\int_a^b f(x)\,dx \approx \frac{3h}{8}\left[f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b)\right]$$
$$= \frac{(b-a)}{8}\left[f(a) + 3f\left(\frac{2a+b}{3}\right) + 3f\left(\frac{a+2b}{3}\right) + f(b)\right], \quad -\frac{(b-a)^5}{6480} f^{(4)}(\xi),$$

Boole's rule

$$x_1, \quad x_2 = x_1 + h, \quad x_3 = x_1 + 2h, \quad x_4 = x_1 + 3h, \quad x_5 = x_1 + 4h.$$
$$\int_{x_1}^{x_5} f(x)\,dx = \frac{2h}{45}\left(7f(x_1) + 32f(x_2) + 12f(x_3) + 32f(x_4) + 7f(x_5)\right) + \text{error term}, \quad -\frac{8}{945} h^7 f^{(6)}(c)$$

documentation **scipy.integrate** :
https://docs.scipy.org/doc/scipy/tutorial/integrate.html

Interpolation

Interpolation de Lagrange

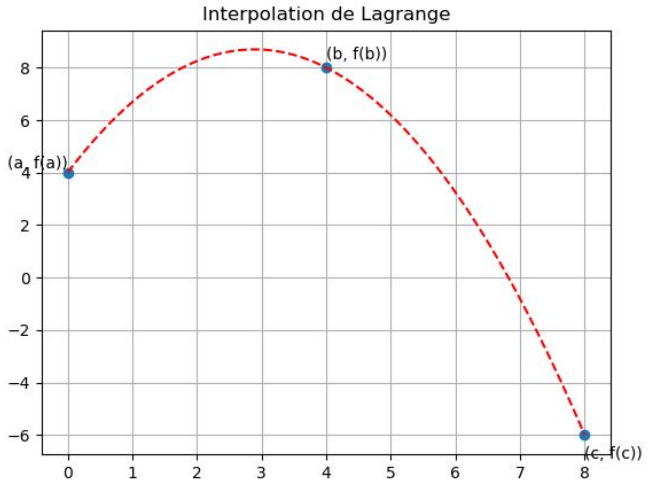
Méthode pour trois points

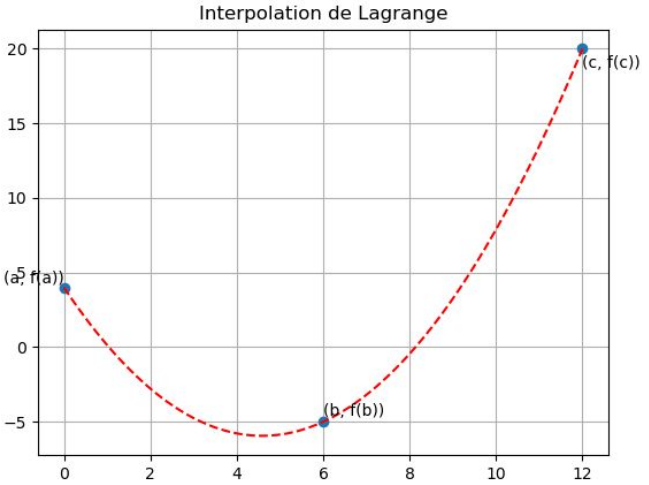
$$P(x) = f(a)\frac{(x-m)(x-b)}{(a-m)(a-b)} + f(m)\frac{(x-a)(x-b)}{(m-a)(m-b)} + f(b)\frac{(x-a)(x-m)}{(b-a)(b-m)}$$

$$l_0(x) = \frac{(x-m)(x-b)}{(a-m)(a-b)}$$

$$l_1(x) = \frac{(x-a)(x-b)}{(m-a)(m-b)}$$

$$l_2(x) = \frac{(x-a)(x-m)}{(b-a)(b-m)}$$

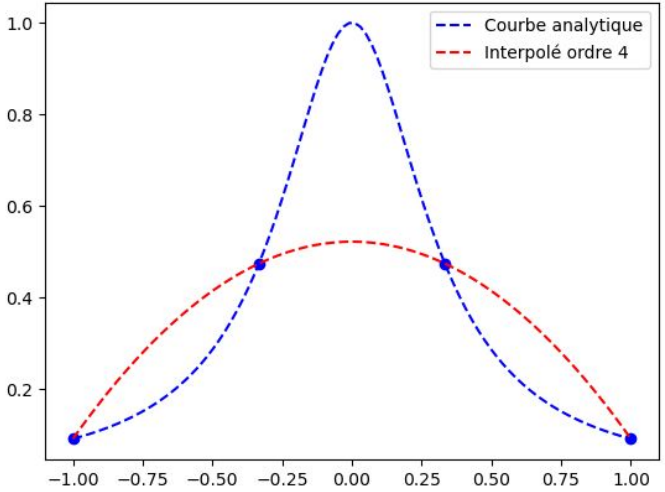


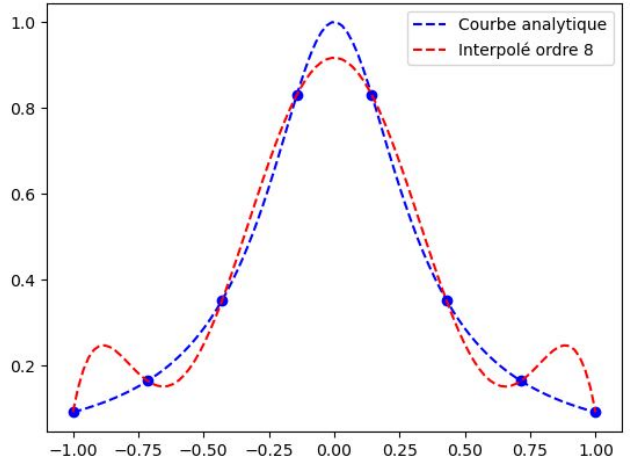


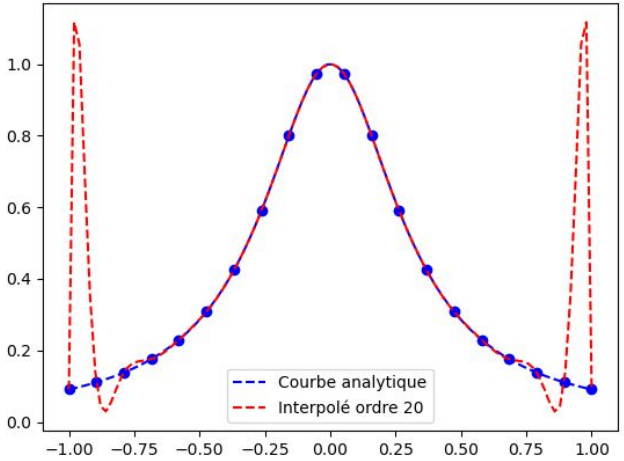
Généralisable pour n points

documentation **scipy.interpolate** :
https://docs.scipy.org/doc/scipy/reference/interpolate.html

Phénomène de Runge







Solutions :

- Choix des points
- Segmentation
- Choix échantillonnage
- Vérification par les moindres carrés

La prochaine fois

Les équations différentielles et leurs solutions :

- Euler
- Runge-Kutta

documentation Euler **scipy.integrate.ode** :
https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.ode.html
documentation Runge-Kutta 5 **scipy.integrate.RK5** :
https://docs.scipy.org/doc/scipy/reference/generated/scipy.integrate.RK45.html