Max Gebski - Answers to Test Flight Problem Set

Question 3

Say whether the following is true or false and support your answer by a proof: For any integer n, the number n 2 + n + 1 is odd.

True. Suppose *n* is an even number. Then *n* is a number of the form *2k* where $k \in$ Z. Then filling in:

(1) $$(2k)^2 + 2k + 1 = 4k^2 + 2k + 1$$

For any k, $4k^2$ will always be even because k^2 multiplied by 4. Because of this $4k^2$ is divisible by 4. Because (2 | 4), $4k^2$ will always be an even number. In (1) 2k + 1 is added to $4k^2$. 2k + 1 is an odd number and $4k^2$ is an even number. Because the sum of an odd number an and even number is always an odd number, $4k^2 + 2k + 1$ will always be odd. Thus the statement is true.