Max Gebski - Answers to Test Flight Problem Set

Question 5

Prove that for any integer n, at least one of the integers n, n + 2, n + 4 is divisible by 3.

Proof: If an integer is divided by 3 we know that the remainder can be 0, 1 or 2. Because of this let us express *n* as:

$$n = 3p \lor n = 3p + 1 \lor n = 3p + 2 \quad where \ \ p \in \mathrm{Z}$$

If $n = 3p$, n is divisible by 3 meaning that the initial statement is true. If n is not 3p then the only other options could be that $n = 3p + 1$ or $n = 3p + 2$.

Remember that at least one of n, n + 2, n + 4 has to be divisible by 3. If $n = 3p + 1$, n + 2 becomes $(3p + 1) + 2 = 3(p + 1)$. Because p ∈ Z this means that $3(p + 1)$ is divisible by 3, meaning that the initial statement is true. When $n = 3p + 2$, n + 4 becomes $(3p + 2) + 4 = 3(p + 2)$. Because p ∈ Z this means that $3(p + 2)$ is also divisible by 3, meaning that the initial statement is true. Proof complete.