Payoffs, Beliefs and Cooperation in Infinitely Repeated Games*

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Abstract

This paper studies the interaction of beliefs, payoff parameters, and the cooperation rate in the infinitely repeated prisoner's dilemma. We show formally that a player's belief about the probability of cooperation by their opponent moderates the effect of changes in the payoff parameters on cooperation. If beliefs are high, increasing the gain from unilateral defection has a large negative effect on cooperation, while increasing the loss from unilateral cooperation has a negligible effect. However, if beliefs are low, this relationship is reversed: increasing the gain has only a negligible effect, while increasing the loss has a large negative effect on cooperation. The negative effect of both payoff parameters on cooperation becomes even larger when the belief is a function of the payoff parameters.

JEL-codes: C72, C73, D81, D83

Keywords: infinitely repeated prisoner's dilemma, equilibrium-selection, beliefs, payoffs, cooperation

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1 Introduction

Social dilemmas are a perpetual challenge for the social sciences. The economic contribution to addressing this challenge is to examine how institutions can solve the dilemma by changing the incentives to cooperate or defect.

This paper adds an essential piece to this literature: the *effect size* of incentive changes on cooperation depends inherently on the degree of strategic uncertainty that agents face. Thus, while recent literature has highlighted that cooperation must be robust to maximum strategic uncertainty in order to prevail (Dal Bó and Fréchette, 2018), we argue that the effect of changes in the incentive structure of a social dilemma can only be fully understood once we consider the complete range of strategic uncertainty.

We provide a formal proof of this proposition, based on the theory of equilibrium-selection in the infinitely repeated prisoner's dilemma, where both cooperation and defection might be equilibria (Fudenberg and Maskin, 1986). Building on seminal contributions by Blonski et al. (2011) and Dal Bó and Fréchette (2011), we introduce a critical discount factor that captures both the incentive structure of the game, i.e. the payoffs from cooperation and defection, and a measure of varying strategic uncertainty: a player's belief about the probability of cooperation by their opponent. This critical discount factor enables us to analyze how the effect of changes in the payoffs on cooperation depends on players' beliefs. Further, it enables us to consider beliefs as endogenous to the payoff parameters, which is a reasonable assumption once players consider not only their own strategy choice based on the payoff matrix but also their opponent's.

While the literature has repeatedly emphasized the importance of beliefs for cooperation (Dal Bó and Fréchette, 2018; Dvorak and Fehrler, 2020; Kartal and Müller, 2022; Andres, 2023), and has also empirically shown that cooperation decreases in the payoffs from unilateral defection as well as cooperation (Mengel, 2018; Dal Bó and Fréchette, 2018; Gill and Rosokha, 2020; Baader et al., 2022; Gächter et al., 2022; Aoyagi et al., 2023; Andres, 2023), these two aspects have not been linked comprehensively in a way that allows to study how the payoffs affect equilibrium-selection depending on the beliefs. It was, so far, only possible to show that both the gain from unilateral defection *and* the loss from unilateral cooperation affect cooperation if players are maximally uncertain (risk-dominance) or that both payoffs increase the range of beliefs for which defection is optimal (the size of the basin of attraction), see Dal Bó and Fréchette (2018) and the literature therein.[1] Thus, we add a more comprehensive understanding to the literature of how cooperation in social dilemmas can be fostered or hindered through changes in the

[1]The meta-study by Dal Bó and Fréchette (2018) analyzes data by Andreoni and Miller (1993); Cooper et al. (1996); Dal Bó (2005); Dreber et al. (2008); Aoyagi and Fréchette (2009); Duffy and Ochs (2009); Dal Bó et al. (2010); Dal Bó and Fréchette (2011); Blonski et al. (2011); Fudenberg et al. (2012); Bruttel and Kamecke (2012); Sherstyuk et al. (2013); Kagel and Schley (2013); Fréchette and Yuksel (2017) and Dal Bó and Fréchette (2019).

payoffs.

We find that if beliefs are *high*, increasing the gain from unilateral defection has a large negative effect on cooperation while increasing the loss from unilateral cooperation has only a negligible effect. However, if beliefs are *low*, increasing the gain only has a negligible effect while increasing the loss has a large effect. Further, if players form their beliefs endogenously by reacting to the payoffs, increasing either payoff has a larger negative effect than if the beliefs were exogenous.

These findings are intuitive once we consider that players react to the *expected* payoffs of their choices. For example, if beliefs are high, players consider it to be relatively likely that their opponent will cooperate. Thus, if they defect, the gain from unilateral defection might very well be the realized payoff. Hence, changes in the gain payoff are relatively important to the players if their beliefs are high, while changes in the loss are relatively unimportant.[2] However, if beliefs are low, players consider it to be relatively likely that their opponent will defect. Thus, if they cooperate, the loss from unilateral cooperation will likely be the realized payoff. Hence, changes in the loss payoff are relatively important to the player if their beliefs are low. In a nutshell, players' beliefs are at the core of their strategy choices, and, thus determine how changes in the incentive structure affect cooperation.

The remainder of this paper is organized as follows. We begin by introducing the theoretical setup in Section 2, which includes the setup of the stage game, the infinite repetition, and the critical discount factor as a function of the payoffs and the belief. We then present our results in Section 3 by demonstrating how the critical discount factor behaves in the payoffs conditional on the beliefs, first with exogenous beliefs, then with beliefs that are endogenous to the payoffs. Section 4 concludes.

2 The Infinitely Repeated Prisoner's Dilemma

To set the stage, this section first describes the stage game and its parametrization in detail. We will then continue with the infinite repetition of the game and equilibrium-selection criteria discussed in the literature so far. In the next section, we will analyze how beliefs and payoff parameters interact and how this may affect cooperation.

Stage Game The standard prisoner's dilemma Γ is a symmetric game of two players $i \in \{X, Y\}$ that face the same choice of action a simultaneously: to cooperate or to defect, $a_i \in \{C, D\}$. The left part of Table 1 summarizes the payoffs in the game. If both players cooperate, both receive the reward payoff R. If both defect, both receive

[2] First experimental evidence, for example, shows that changes in the Sucker's payoff have a smaller effect on cooperation in games with than in games without communication. Since players hold higher beliefs in games with communication, this is consistent with our proposition. See Andres (2023).

the punishment payoff P. If one player defects unilaterally, they receive the temptation payoff T. The player who cooperates unilaterally, while the other player defects, receives the sucker's payoff S. In the prisoner's dilemma Γ, two conditions must be met (Rapoport et al., 1965): First, $T > R > P > S$ ensures that mutual cooperation is Pareto-superior to mutual defection ($2R > 2P$) and that there is an incentive to defect because the individual payoff from unilateral defection is larger than from mutual cooperation ($T > R$) and the individual payoff from mutual defection is larger than from unilateral cooperation ($P > S$). Second, $2R > T + S$ ensures that mutual cooperation is also Pareto-superior to the asymmetric outcome of unilateral defection.

Following Stahl II (1991), we normalize the payoffs to reduce the prisoner's dilemma to a function $\Gamma(g, l)$ of the gain from unilateral defection g and the loss from unilateral cooperation $-l$, with $g, l > 0$. The normalization subtracts the punishment payoff P from the original payoff R, S, T or P and then divides by $R - P$. The right part of Table 1 presents the resulting payoffs after the normalization: The gain from unilateral defection is $\frac{T-P}{R-P} = 1 + g$, and the loss from unilateral cooperation is $\frac{S-P}{R-P} = -l$, while the reward and punishment payoffs are $\frac{R-P}{R-P} = 1$ and $\frac{P-P}{R-P} = 0$, respectively. Thus, in the normalized version of the game, $T > R > P > S$ holds since $1 + g > 1 > 0 > -l$, and $2 \cdot R > T + S$ holds as long as $2 > 1 + g - l \Leftrightarrow l > g - 1$.

	C	D
C	R, R	S, T
D	T, S	P, P

(a) Original

	C	D
C	$1, 1$	$-l, 1 + g$
D	$1 + g, -l$	$0, 0$

(b) Normalized

Table 1: Stage-Game Payoffs in the Prisoner's Dilemma Γ

Infinite Repetition In the infinitely repeated prisoner's dilemma, players discount future payoffs by a factor of δ, where $0 < \delta < 1$. This understanding is equivalent to a setup with an indefinite number of repetitions, where δ is the probability that the game will be played again after a particular round. Thus, the expected number of rounds is given by $\frac{1}{1-\delta}$ (Roth and Murnighan, 1978).

Following Dal Bó and Fréchette (2018) and the literature therein, we can reduce the strategy set as far as possible and assume that players choose between two strategies at the game's inception: Grim and Always-Defect (AD). In AD, players defect in each round, forever. In Grim, players start by cooperating and cooperate as long as the other player has cooperated in the previous round. Once the other player defects, they defect forever. Hence, Grim is the strongest possible and completely unforgiving retaliation for defection by the other player.

Multiple Equilibria and Equilibrium-Selection In the infinitely repeated prisoner's dilemma, $\{\mathrm{AD}, \mathrm{AD}\}$ is always a subgame-perfect Nash-equilibrium. In addition, $\{\mathrm{Grim}, \mathrm{Grim}\}$ is a subgame-perfect Nash-equilibrium if

$$\delta \geq \delta^{SPE}(g) \quad \text{where} \quad \delta^{SPE}(g) \equiv \frac{g}{1+g} \tag{1}$$

Thus, if players are sufficiently patient or if the continuation probability is sufficiently high, multiple equilibria exist, including cooperative equilibria. Note that $\delta^{SPE}(g)$ is increasing in g, i.e. a higher gain parameter g makes cooperation less likely. Furthermore note that $\delta^{*}(g)$ does not depend on the loss parameter l. This implies that the subgame-perfect Nash-equilibrium excludes the possibility that variations in the sucker's payoff S might affect players' strategy choices, too.

With multiple equilibria, the literature has recognized the need for equilibrium-selection criteria to determine the likely outcome of an infinitely repeated prisoner's dilemma. Two equilibrium-selection criteria stand out: Pareto-dominance (Friedman, 1971; Fudenberg and Maskin, 1986) and risk-dominance (Harsanyi and Selten, 1988; Blonski et al., 2011; Dal Bó and Fréchette, 2011; Blonski and Spagnolo, 2015).

According to Pareto-dominance, players always choose the Pareto-superior equilibrium if multiple equilibria exist. Thus, $\{\mathrm{Grim}, \mathrm{Grim}\}$ is Pareto-dominant as soon as $\{\mathrm{Grim}, \mathrm{Grim}\}$ is an equilibrium, i.e. if

$$\delta \geq \delta^{PD}(g) \quad \text{where} \quad \delta^{PD}(g) \equiv \frac{g}{1+g} \tag{2}$$

Theories of equilibrium-selection can also incorporate a measure of *strategic uncertainty*: a player's belief about the probability $p \in [0,1]$ that the other player plays a cooperative strategy. If a player holds a belief of $p = 1$, they are perfectly certain that their opponent will play Grim. If they hold a belief of $p = 0$, they are certain that their opponent will defect. Then, Always-Defect is a dominant strategy for all δ and all $\Gamma(g, l)$. A cooperative equilibrium is risk-dominant if it is individually optimal to cooperate given that players are maximally uncertain, believing that their opponent randomizes between Grim and AD with a probability of 50% ($p = 0.5$) (see Blonski and Spagnolo, 2015). Thus, $\{\mathrm{Grim}, \mathrm{Grim}\}$ is risk-dominant if

$$\delta \geq \delta^{RD}(g,l) \quad \text{where} \quad \delta^{RD}(g,l) \equiv \frac{g+l}{1+g+l} \tag{3}$$

The critical value $\delta^{RD}(g,l)$ is increasing in g and l. Thus, both a higher gain parameter g and a larger loss parameter l make cooperation less likely. Moreover, for any prisoner's dilemma, it holds that $\delta^{RD}(g,l) > \delta^{PD}(g)$, i.e. if mutual cooperation is risk-dominant, it is also Pareto-dominant (Blonski et al., 2011). Note that in this line of thinking, assuming that both players choose their strategy according to the Pareto-dominance criterion would be equivalent to assuming $p = 1$.

Thus, equilibrium-selection based on Pareto- or risk-dominance focuses on two specific beliefs, $p = 1$ or $p = 0.5$. In this paper, we consider beliefs as a continuous measure of varying degrees of strategic uncertainty and, thus, derive a continuous measure of equilibrium-selection based on the expected payoffs of cooperation and defection. For all players, it is individually optimal to play Grim if and only if the expected payoff from Grim is larger than or equal to the expected payoff of Always-Defect:

$$p \cdot \left(\frac{1}{1-\delta}\right) + (1-p) \cdot (-l) \geq p \cdot (1+g) \tag{4}$$

where, for each player, $p \cdot \left(\frac{1}{1-\delta}\right)$ is the expected discounted payoff from mutual cooperation, $(1-p) \cdot (-l)$ is the expected payoff from unilateral cooperation, and $p \cdot (1+g)$ is the expected payoff from unilateral defection. Rearranging (4) for δ yields the condition on δ for which mutual cooperation is individually optimal. Thus, the critical discount factor $\delta^*(p, g, l)$ is a continuous function of p, g, and l.

Definition. *Let $\Gamma(g, l)$ be an infinitely repeated prisoner's dilemma of two players with strategy set $\omega \equiv \{Grim, AD\}$, discount factor $\delta \in (0, 1)$, belief $p \in [0, 1]$ and domain $\{g, l \in \mathbb{R} | g, l > 0 \wedge l > g - 1\}$. Then the continuous function $\delta^* : D \mapsto R$ with domain $D \equiv \{p, g, l \in \mathbb{R} | g, l > 0 \wedge l > g - 1 \wedge 0 \leq p \leq 1\}$, range $R \in \mathbb{R}$, image $I = [0, 1]$ and formula*

$$\delta^*(p, g, l) = \frac{p \cdot (g - l) + l}{p \cdot (1 + g - l) + l} \tag{5}$$

is called the critical discount factor of $\Gamma(g, l)$ for any given p.

Following the literature (see, e.g., Athey et al., 2004; Gilo et al., 2006; Bruttel, 2009; Blonski et al., 2011), we interpret the critical discount factor $\delta^*(p, g, l)$ as a measure of the (inverse of the) likelihood of cooperation. If the critical discount factor increases, cooperation becomes *less* likely. If the critical discount factor decreases, cooperation becomes *more* likely. Thus, policy interventions designed to hinder or foster cooperation can be analyzed by their potential to manipulate the critical discount factor into the desired direction (Blonski et al., 2011). Note that inverting the inequality sign in (4) and rearranging for p yields the size of the basin of attraction of Always-Defect against Grim, which is the set of beliefs that make Always-Defect optimal (Dal Bó and Fréchette, 2011). Furthermore, note that plugging in $p = 0.5$ or $p = 1$ into $\delta^*(p, g, l)$ yields $\delta^{RD}(g, l)$ or $\delta^{PD}(g)$, respectively.

3 Interaction of Beliefs, Payoffs and Cooperation

In this section, we will show that the effect size of changes in the payoff parameters on cooperation depends inherently on a player's degree of strategic uncertainty. We will show that if beliefs are *high*, increasing the gain g from unilateral defection has a large negative effect on cooperation while increasing the loss l from unilateral cooperation has a negligible effect. If beliefs are *low*, however, increasing g only has a negligible effect, while increasing l has a large effect.

We start by assuming that the belief p is exogenous to the payoff parameters of the game. The exogenous belief p may, for example, be interpreted as the player's belief over the distribution of cooperative players in the population. Later, we will relax this exogeneity-assumption by allowing for "endogenous" beliefs, which vary in the parameters g and l.

For ease of presentation, we consider only interior $p \in (0,1)$ in the proposition. Note that for $p = 1$, $\delta^*(p,g,l)$ increases in the gain g and is independent of the loss l while for $p = 0$, $\delta^*\big(p,g,l\big)$ is independent of the gain g and the loss l.

Proposition 1. *Let $\Gamma(g,l)$ be an infinitely repeated prisoner's dilemma of two players with strategy set $\omega \equiv \{Grim, AD\}$, discount factor $\delta \in (0,1)$, belief $p \in (0,1)$, domain $\{g,l \in \mathbb{R} | g,l > 0 \wedge l > g-1\}$ and critical discount factor $\delta^*(p,g,l)$. Then:*

(a) The positive effect of an increase in g on δ^* increases in p.

(b) For any given l, the maximum-possible positive effect of an increase in l on δ^* decreases in p.

Proof. Let us first consider how the critical discount factor behaves in the payoff parameters. Deriving $\delta^*\big(p,g,l\big)$ with respect to g and l—see A.2.1 and A.2.2—yields:

$$\frac{\partial \delta^*\big(p,g,l\big)}{\partial g} = \frac{p^2}{\big(l - l \cdot p + p + g \cdot p\big)^2} \tag{6}$$

$$\frac{\partial \delta^*\big(p,g,l\big)}{\partial l} = \frac{p - p^2}{\big(l - l \cdot p + p + g \cdot p\big)^2} \tag{7}$$

For $0 < p < 1$, the nominator in both derivatives is positive. The denominator in both derivatives is positive because of the square. Thus, both derivatives are positive, i.e. the critical discount factor increases in both g and l:

$$\frac{\partial \delta^*\big(p,g,l\big)}{\partial g} > 0 \tag{8}$$

$$\frac{\partial \delta^*\big(p,g,l\big)}{\partial l} > 0 \tag{9}$$

Deriving (6) and (7) with respect to p—see A.2.3 and A.2.4—demonstrates how these effects depend on p:

$$\frac{\partial\delta^*\big(p,g,l\big)}{\partial g\partial p} = \frac{2\cdot l\cdot p}{\big(l-l\cdot p+p+g\cdot p\big)^3} \tag{10}$$

$$\frac{\partial\delta^*\big(p,g,l\big)}{\partial l\partial p} = \frac{l-p\cdot(1+g+l)}{\big(l-l\cdot p+p+g\cdot p\big)^3} \tag{11}$$

The denominator in both derivatives is positive since $l > l\cdot p$ for $0 < p < 1$. The nominator in Equation (10) is positive since $l, p > 0$. Thus, the positive effect of an increase in the gain g on the critical discount factor $\delta^*\big(p,g,l\big)$ increases in the belief p:

$$\frac{\partial\delta^*\big(p,g,l\big)}{\partial g\partial p} > 0 \tag{12}$$

This establishes part (a) of the proposition.

The nominator in Equation (11) is negative

$$\frac{\partial\delta^*\big(p,g,l\big)}{\partial l\partial p} < 0 \tag{13}$$

for

$$p > \frac{l}{1+g+l} \tag{14}$$

Thus, the positive effect of an increase in the loss l on the critical discount factor $\delta^*\big(p,g,l\big)$ decreases in the belief p if and only if (14) is true. If the condition is reversed, the positive effect of an increase in the loss l on the critical discount factor $\delta^*\big(p,g,l\big)$ increases in the belief p. Thus, there exists a maximum of Equation (7) in p, in which

$$p^* = \frac{l}{1+g+l} \tag{15}$$

Rearranging Equation (15) for l yields:

$$l(p,g) = \frac{p\cdot(1+g)}{1-p} \tag{16}$$

Taking Equation (16) as the solution to maximizing $\delta^*_l(p,g,l) \equiv \frac{\partial\delta^*(p,g,l)}{\partial l}$ with respect to p, and plugging in yields:

$$\delta^*_l\big(p,g,l=l(p,g)\big) = \frac{p-p^2}{\Big(\frac{p\cdot(1+g)}{1-p} - \frac{p\cdot(1+g)}{1-p}\cdot p+p+g\cdot p\Big)^2} \tag{17}$$

Equation (17) describes the maximum-possible effect of l on the critical discount factor

$\delta^*\big(p, g, l\big)$, that is reachable for any given l, as a function of the belief p and the gain g. Deriving Equation (17) with respect to p—see A.2.5—yields:

$$\frac{\partial \delta_l^*\big(p, g, l = l(p, g)\big)}{\partial p} = -\frac{1}{4 \cdot (1+g)^2 \cdot p^2} < 0 \tag{18}$$

Thus, for any given l, the maximum-possible positive effect of an increase in the loss l on the critical discount factor $\delta^*\big(p, g, l\big)$ decreases in p. This establishes part (b) of the proposition. ■

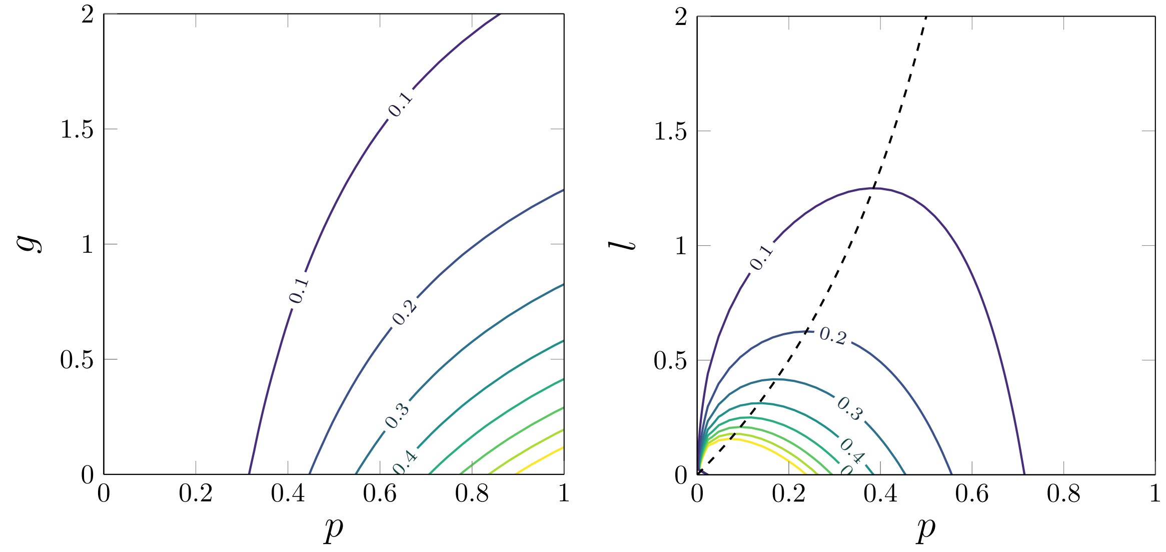
Let us provide an intuition for Proposition 1. Consider how the gain g affects the critical discount factor $\delta^*\big(p, g, l\big)$ for varying beliefs p. If the belief p is relatively low, players find it unlikely that their opponent will cooperate. An increase in the payoff from unilateral defection, i.e., an increase in the gain g will be relatively inconsequential because players do not expect to enter an outcome where they *defect unilaterally.* If, however, the belief p is relatively high, players find it likely that their opponent will cooperate. Then, an increase in the gain g will matter substantially because defecting unilaterally is a relatively probable outcome. Players react to the *expected* payoffs of their actions, not the payoffs themselves. Thus, the positive effect of an increase in the gain g on the critical discount factor δ^* increases in the belief p.

And thus, the opposite must be true for the effect of the loss l on the critical discount factor $\delta^*\big(p, g, l\big)$: the effect of the loss l will be negligible if players find it very likely that the other players will cooperate. Consequentially, suppose cooperation by other players seems unlikely. In that case, players will attach considerable weight to the payoff from unilateral cooperation, i.e. the loss l, because this is a relatively probable outcome if they cooperate in such an environment. However, defecting oneself also becomes more probable with decreasing p. In the extreme case of $p = 0$, players will certainly defect and hence, not react to any changes in either payoff. This opposing effect, however, loses relevance with decreasing l because cooperation becomes more likely with decreasing l. Thus, increasing the loss l has large effects in situations where the belief p and the loss l are low to start with.[3]

In Figure 1, we visualize Proposition 1. We draw contour-lines of the first derivatives of the critical discount factor $\delta^*\big(p, g, l\big)$ with respect to the gain g (Figure 1a, Equation 6) and the loss l (Figure 1b, Equation 7) as a function of the belief p and initial values of the gain g (Figure 1a) and the loss l (Figure 1b). In Figure 1a, the contour lines show the value of the derivative of the critical discount factor $\delta^*\big(p, g, l\big)$ with respect to the gain g

[3] It is worth mentioning that if we include an additional restriction on the domain of a prisoner's dilemma—cooperation always increases efficiency, i.e., $T + S > 2 \cdot P$ or $1 + g > l$, see Friedman and Sinervo (2016)—the positive effect of an increase in the loss l on the critical discount factor $\delta^*(p, g, l)$ always decreases in p for $p > 0.5$. This is because plugging in Equation (16) into Equation (7) yields $p > 0.5$, for which $\frac{\partial \delta^*(p,g,l)}{\partial l \partial p}$ is always negative.

Figure 1: The Partial Derivatives of δ^*, Conditional on p



(a) The Effect of the Gain g (b) The Effect of the Loss l

Note: The derivative of the critical discount factor $\delta^*(p, g, l)$ (contour lines and corresponding lighter color) with respect to (a) the gain g and (b) the loss l, for different beliefs p. Figure (a) additionally varies g, while holding $l = 1$ constant. Figure (b) additionally varies l, while holding $g = 1$ constant. This satisfies $l > g - 1$ (mutual cooperation is pareto-superior to unilateral defection), ensuring that every coordinate represents a true prisoner's dilemma. Figure 2 in the appendix shows the contour-plots for $l, g \in \{0.5, 1.5\}$, respectively. To the right of the dashed line in Figure (b), the condition in (14) holds.

as a 3-dimensional function of the belief p (abscissa) and the gain g (ordinate). In this figure, the positive effect of the gain g on the critical discount factor $\delta^*(p, g, l)$ clearly increases in the belief p: the further we go to the right, the higher the contour lines. Additionally, for any given p, there is a decreasing marginal effect of g. In Figure 1b, the contour lines show the value of the derivative of the critical discount factor $\delta^*(p, g, l)$ with respect to the loss l as a 3-dimensional function of the belief p (abscissa) and the loss l (ordinate). It is apparent from this figure that the positive effect of the loss l on the critical discount factor $\delta^*(p, g, l)$ is highest for low beliefs p, given that the loss l is low: moving towards the bottom-left of the graph corresponds to upwards movement on the contour lines. Additionally, for any given p, there is a decreasing marginal effect of l.

Note that there exists an interesting special-case of Proposition 1: risk-dominance. For $p = 0.5$, the critical discount factor $\delta^*(p, g, l)$ increases in the gain g and the loss l by the same magnitudes. Plugging in $p = 0.5$ into Equations (6) and (7) yields

$$\frac{\partial \delta^*(p, g, l)}{\partial g} = \frac{0.25}{\left(l - l \cdot 0.5 + 0.5 + g \cdot 0.5\right)^2} \tag{19}$$

$$\frac{\partial \delta^*(p, g, l)}{\partial l} = \frac{0.25}{\left(l - l \cdot 0.5 + 0.5 + g \cdot 0.5\right)^2} \tag{20}$$

Thus, in the special case of risk-dominance with $p = 0.5$ the marginal effects of changes in the two payoff parameters on cooperation are of the same size. This implies that for $p > 0.5$, the risk-dominance criterion underestimates the effect of the gain and overestimates the effect of the loss, while for $p < 0.5$, it underestimates the effect of the loss and overestimates the effect of the gain.

Endogenous Beliefs So far, we have followed previous theoretical literature in assuming that beliefs are exogenously given. We will now show how changes in the payoff parameters affect the critical discount factor if beliefs are endogenous to the payoff parameters g and l. We consider beliefs endogenous if players form their beliefs only after they have seen the game's payoff matrix and then form their beliefs depending on the payoffs. To our knowledge, this has not been considered theoretically in the literature, although very recent experimental literature finds evidence for this: players are more pessimistic for subgame payoff parameters which are less conducive to cooperation (Aoyagi et al., 2023; Andres, 2023). This is in line with our results above, showing that cooperation decreases in both g and l. Accordingly, if players hold accurate beliefs, then their beliefs should decrease in both g and l. All else equal, games with higher g and l are less conducive to cooperation. Based on this consideration, we can define the belief p as a continuous function of the payoff-parameters g and l, without assuming a specific functional form.

Definition. *Let $\Gamma(g, l)$ be an infinitely repeated prisoner's dilemma of two players with strategy set $\omega \equiv \{Grim, AD\}$, discount factor $\delta \in (0, 1)$ and domain $\{g, l \in \mathbb{R} | g, l > 0 \wedge l > g - 1\}$. Assume that players form their beliefs at the inception of the game after observing the payoff-matrix. Further, assume that players get more pessimistic in increasing g and l. Then the continuous function $p : D \mapsto R$ with domain $D \equiv \{g, l \in \mathbb{R} | g, l > 0 \wedge l > g - 1\}$, range $R \in \mathbb{R}$, image $I = (0, 1)$, and first derivatives $\frac{\partial p(g,l)}{\partial g} < 0$ and $\frac{\partial p(g,l)}{\partial l} < 0$ is called the endogenous belief $p(g, l)$.*[4]

Assuming that beliefs are fully determined by the payoff parameters has two implications. First, it is now meaningless to ask how the effects of the payoff parameters on cooperation depend on the belief since the belief itself depends on the payoff parameters. Second, increases in the payoff parameters have a larger negative effect on cooperation if beliefs are endogenous than if they were exogenous. This is because two effects add up: a direct and an indirect effect. Increases in the payoff parameters have a *direct* negative effect on cooperation because they make defection more tempting or cooperation more risky, as in Proposition 1. But they also have an *indirect* effect: The belief decreases in both payoff-parameters, while cooperation increases in the belief, see A.2.8. Thus, an

[4]This implies further that the second derivatives must be positive since the limits must be $\lim_{g\to\infty} p(g, l) = \lim_{l\to\infty} p(g, l) = 0$ because the belief cannot fall below zero. Thus, there must be a decreasing marginal effect of the parameters on the belief (if $p(g, l)$ is continuous and twice-differentiable).

increase in either payoff-parameter leads to a decrease in the belief, which leads to an additional decrease in cooperation.

Let us show this formally. Based on the definition of the endogenous belief $p(g,l)$ above, we can raise the following proposition:

Proposition 2. *Let $\Gamma(g,l)$ be an infinitely repeated prisoner's dilemma of two players with strategy set $S \equiv \{Grim, AD\}$, discount factor $\delta \in (0,1)$ and domain $\{g,l \in \mathbb{R} | g,l > 0 \wedge l > g-1\}$. Let players have endogenous beliefs $p(g,l) \in (0,1)$. For comparison, assume that there exists an exogenous p^{exo} such that $p(g,l) = p^{exo}$. Then, the derivatives of $\delta^*\big(p(g,l),g,l\big)$ with respect to g and l are positive and larger than the derivatives of $\delta^*\big(p^{exo},g,l\big)$ with respect to g and l, respectively.*

Proof. Deriving $\delta^*\big(p(g,l),g,l\big)$ with respect to g and l—see A.2.6 and A.2.7—yields:

$$\frac{\partial\delta^*\big(p(g,l),g,l\big)}{\partial g} = \frac{\big(p(g,l)\big)^2 - l \cdot \frac{\partial p(g,l)}{\partial g}}{\Big(l - l\cdot p(g,l) + p(g,l) + g \cdot p(g,l)\Big)^2} \tag{21}$$

$$\frac{\partial\delta^*\big(p(g,l),g,l\big)}{\partial l} = \frac{p(g,l) - \big(p(g,l)\big)^2 - l \cdot \frac{\partial p(g,l)}{\partial l}}{\Big(l - l\cdot p(g,l) + p(g,l) + g \cdot p(g,l)\Big)^2} \tag{22}$$

In both derivatives, the denominators are clearly positive, because they are squared. By assumption, namely by $p(g,l) \in (0,1)$, $l > 0$ and $\frac{\partial p(g,l)}{\partial g}, \frac{\partial p(g,l)}{\partial l} < 0$, the nominators in both derivatives are positive. Thus, the critical discount factor $\delta^*\big(p(g,l),g,l\big)$ increases in both g and l, also if the belief is endogenous.[5]

Since $p(g,l) \in (0,1)$, $l > 0$ and $\frac{\partial p(g,l)}{\partial g}, \frac{\partial p(g,l)}{\partial l} < 0$, the terms $-l \cdot \frac{\partial p(g,l)}{\partial g}$ and $-l \cdot \frac{\partial p(g,l)}{\partial l}$ in Equations (21) and (22), respectively, are positive. Thus,

$$\frac{\partial\delta^*\big(p(g,l),g,l\big)}{\partial g} > \frac{\partial\delta^*\big(p^{exo},g,l\big)}{\partial g} > 0 \tag{23}$$

$$\frac{\partial\delta^*\big(p(g,l),g,l\big)}{\partial l} > \frac{\partial\delta^*\big(p^{exo},g,l\big)}{\partial l} > 0 \tag{24}$$

Thus, with endogenous beliefs $p(g,l)$, the derivatives of $\delta^*\big(p(g,l),g,l\big)$ with respect to g and l are positive, and larger than the derivatives of $\delta^*\big(p^{exo},g,l\big)$ with respect to g and l, respectively. This difference increases in the size of the first derivatives of $p(g,l)$ with respect to g and l, respectively. ∎

Thus, if beliefs are endogenous to the payoff parameters, it is still possible to affect cooperation in the infinitely repeated prisoner's dilemma via changes in the payoffs. The effect of changes in the payoff parameters will even be larger than if the beliefs were

[5] Note that it is impossible to derive Equations (21) and (22) with respect to p because this would include the derivatives $\frac{\partial p(g,l)}{\partial g \partial p(g,l)}$ and $\frac{\partial p(g,l)}{\partial l \partial p(g,l)}$, respectively.

exogenous. Again, we see that allowing for varying beliefs, and thus varying degrees of strategic uncertainty, is necessary to fully capture the effects of the payoff parameters on cooperation.

4 Conclusion

This paper has studied how changes in the payoffs of the infinitely repeated prisoner's dilemma affect cooperation. We have argued that the effect size of changes in the payoffs on cooperation depends inherently on a player's belief about the probability of cooperation by their opponent. To show this formally, we derived a novel critical discount factor by allowing for varying and endogenous beliefs. This reflects that equilibrium-selection is most likely context-dependent: In social dilemmas, players will react differently to changes in the incentive structure depending on whether they are optimistic or pessimistic and to what degree. Thus, we contribute a more comprehensive understanding of the determinants of cooperation in social dilemmas to the literature.

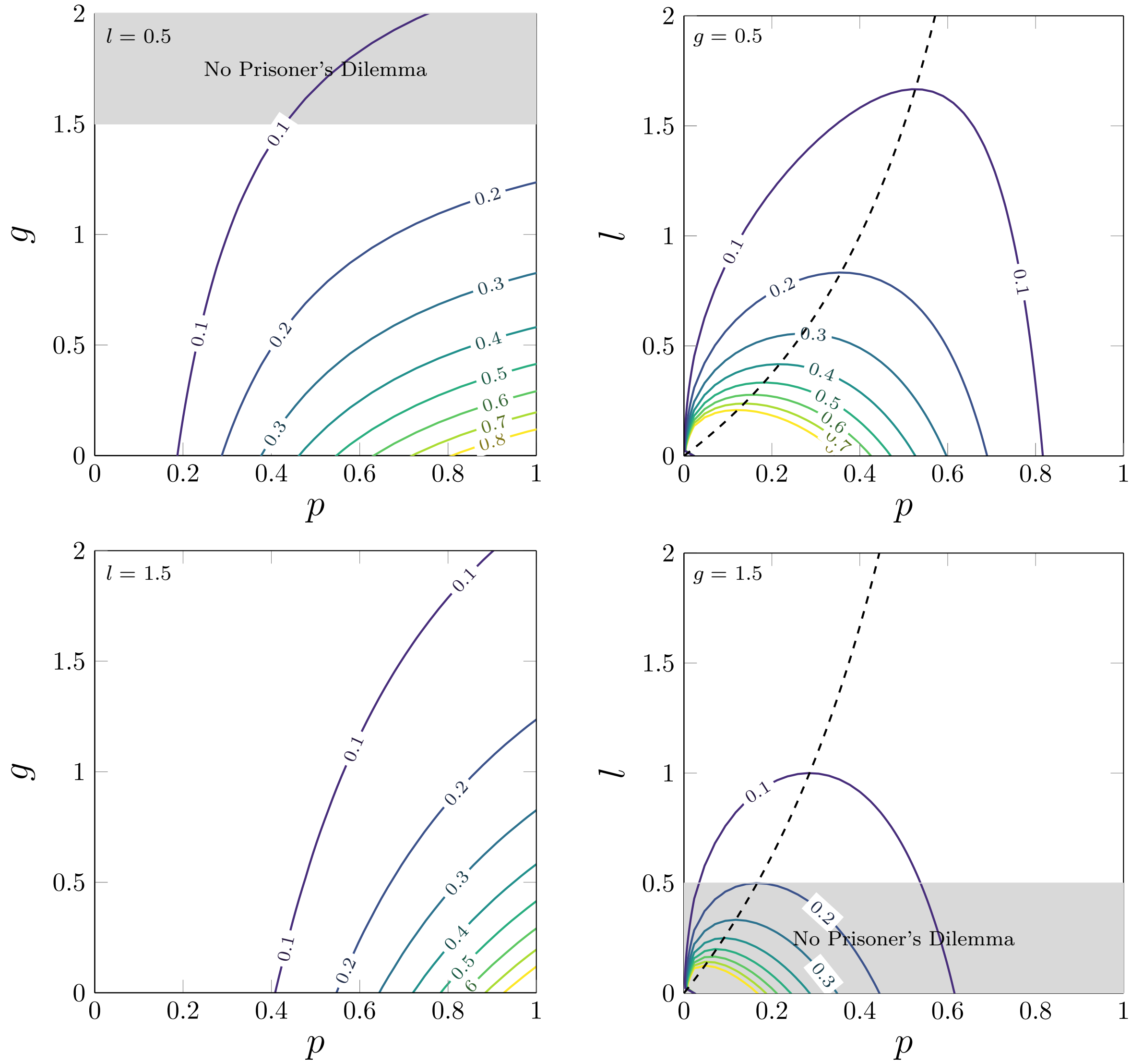
Our results imply that policy interventions which aim to change the incentive-structure of a social dilemma should take the context of the strategic interaction into account. In a rather trustworthy context, players find it likely that their opponent will cooperate, and, thus, changes in the gain from unilateral defection are more effective than changes in the loss from unilateral cooperation. In a rather untrustworthy context, however, changes in the gain from unilateral defection are much less effective than changes in the loss from unilateral cooperation. If a player's belief itself depends on the incentive-structure, then the effect of changes in the gain and the loss are larger than if beliefs were exogenous. This additional effect increases in the degree to which beliefs are affected by changes in the incentive-structure. Interventions into social dilemmas could make use of this by explicitly fostering empathetic thinking among the players.

Comparisons of effect sizes *across* the two payoff-parameters hold as long as we assume that the payoffs posses a cardinal interpretation. We can only state that, e.g., for a certain belief, a change in the gain will be more effective than a change in the loss, if we assume that a given change in the gain is quantitatively comparable to a given change in the loss. However, if we assume that the payoffs posses only an ordinal interpretation, we can still make comparisons for changes *within* the same payoff-parameter: A given qualitative change in the gain will have a larger effect if beliefs are high, rather than low, and a given qualitative change in the loss will have a smaller effect if beliefs are high, rather than low. Thus, our results are applicable to social dilemmas where the payoffs posses a clear quantitative or even monetary characteristic *and* social dilemmas where the incentives differ ordinally. This is the core advantage of the prisoner's dilemma as a very general model of social dilemmas.

A Appendix

A.1 Additional Figures

Figure 2: The Partial Derivatives of δ^*, Conditional on p, for Different l and g



(a) The Effect of the Gain g for $l \in \{0.5, 1.5\}$

(b) The Effect of the Loss l for $g \in \{0.5, 1.5\}$

Note: See the note to Figure 1.

A.2 Additional Details to the Proofs

A.2.1 The partial derivative of $\delta^*(p, g, l)$ with respect to g

$$\frac{\partial \delta^*\big(p, g, l\big)}{\partial g} = \frac{\partial}{\partial g}\left(\frac{p \cdot (g - l) + l}{p \cdot (1 + g - l) + l}\right) \tag{25}$$

Use the quotient rule.

$$\Leftrightarrow \frac{\Big(p \cdot (1 + g - l) + l\Big) \cdot \Big(\frac{\partial}{\partial g}\big(p \cdot (g - l) + l\big)\Big) - \Big(p \cdot (g - l) + l\Big) \cdot \Big(\frac{\partial}{\partial g}\big(p \cdot (1 + g - l) + l\big)\Big)}{\Big(p \cdot (1 + g - l) + l\Big)^2} \tag{26}$$

Apply the derivatives with respect to g.

$$\Leftrightarrow \frac{\Big(p \cdot (1 + g - l) + l\Big) \cdot p - \Big(p \cdot (g - l) + l\Big) \cdot p}{\Big(p \cdot (1 + g - l) + l\Big)^2} \tag{27}$$

Simplify.

$$\Leftrightarrow \frac{p^2}{\Big(l - l \cdot p + p + g \cdot p\Big)^2} \tag{28}$$

Thus, the partial derivative of $\delta^*\big(p, g, l\big)$ with respect to g is:

$$\frac{\partial \delta^*\big(p, g, l\big)}{\partial g} = \frac{p^2}{\Big(l - l \cdot p + p + g \cdot p\Big)^2} \tag{29}$$

A.2.2 The partial derivative of $\delta^*(p, g, l)$ with respect to l

$$\frac{\partial \delta^*\big(p, g, l\big)}{\partial l} = \frac{\partial}{\partial l}\left(\frac{p \cdot (g - l) + l}{p \cdot (1 + g - l) + l}\right) \tag{30}$$

Use the quotient rule.

$$\Leftrightarrow \frac{\Big(p \cdot (1 + g - l) + l\Big) \cdot \Big(\frac{\partial}{\partial l}\big(p \cdot (g - l) + l\big)\Big) - \Big(p \cdot (g - l) + l\Big) \cdot \Big(\frac{\partial}{\partial l}\big(p \cdot (1 + g - l) + l\big)\Big)}{\Big(p \cdot (1 + g - l) + l\Big)^2} \tag{31}$$

Apply the derivatives with respect to l.

$$\Leftrightarrow \frac{\Big(p \cdot (1 + g - l) + l\Big) \cdot \Big(1 - p\Big) - \Big(p \cdot (g - l) + l\Big) \cdot \Big(1 - p\Big)}{\Big(p \cdot (1 + g - l) + l\Big)^2} \tag{32}$$

Simplify.

$$\Leftrightarrow \frac{p - p^2}{\left(l - l \cdot p + p + g \cdot p\right)^2} \tag{33}$$

Thus, the partial derivative of $\delta^*\left(p, g, l\right)$ with respect to l is:

$$\frac{\partial \delta^*\left(p, g, l\right)}{\partial l} = \frac{p - p^2}{\left(l - l \cdot p + p + g \cdot p\right)^2} \tag{34}$$

A.2.3 The partial derivative of $\frac{\partial \delta^*(p,g,l)}{\partial g}$ with respect to p

$$\frac{\partial \delta^*\left(p, g, l\right)}{\partial g \partial p} = \frac{\partial}{\partial p}\left(\frac{p^2}{\left(l - l \cdot p + p + g \cdot p\right)^2}\right) \tag{35}$$

Use the product rule.

$$\Leftrightarrow \frac{\frac{\partial}{\partial p}\left(p^2\right)}{\left(l - l \cdot p + p + g \cdot p\right)^2} + p^2 \cdot \left(\frac{\partial}{\partial p}\left(\frac{1}{\left(l - l \cdot p + p + g \cdot p\right)^2}\right)\right) \tag{36}$$

Apply the derivatives with respect to p.

$$\Leftrightarrow \frac{2 \cdot p}{\left(l - l \cdot p + p + g \cdot p\right)^2} + p^2 \cdot \left(\frac{(-2) \cdot (g - l + 1)}{\left(l - l \cdot p + p + g \cdot p\right)^3}\right) \tag{37}$$

Simplify.

$$\Leftrightarrow \frac{2 \cdot p}{\left(l - l \cdot p + p + g \cdot p\right)^2} - \frac{2 \cdot p^2 \cdot (g - l + 1)}{\left(l - l \cdot p + p + g \cdot p\right)^3} \tag{38}$$

Simplify.

$$\Leftrightarrow \frac{2 \cdot l \cdot p}{\left(l - l \cdot p + p + g \cdot p\right)^3} \tag{39}$$

Thus, the partial derivative of $\frac{\partial \delta^*(p,g,l)}{\partial g}$ with respect to p is:

$$\frac{\partial \delta^*\left(p, g, l\right)}{\partial g \partial p} = \frac{2 \cdot l \cdot p}{\left(l - l \cdot p + p + g \cdot p\right)^3} \tag{40}$$

A.2.4 The partial derivative of $\frac{\partial \delta^*(p,g,l)}{\partial l}$ with respect to p

$$\frac{\partial \delta^*\big(p,g,l\big)}{\partial l \partial p} = \frac{\partial}{\partial p}\left(\frac{p-p^2}{\big(l-l\cdot p+p+g\cdot p\big)^2}\right) \tag{41}$$

Use the product rule.

$$\Leftrightarrow \frac{\frac{\partial}{\partial p}\big(p-p^2\big)}{\big(l-l\cdot p+p+g\cdot p\big)^2} + \big(p-p^2\big)\cdot\left(\frac{\partial}{\partial p}\left(\frac{1}{\big(l-l\cdot p+p+g\cdot p\big)^2}\right)\right) \tag{42}$$

Apply the derivatives with respect to p.

$$\Leftrightarrow \frac{1-2\cdot p}{\big(l-l\cdot p+p+g\cdot p\big)^2} + \big(p-p^2\big)\cdot\left(\frac{(-2)\cdot(g-l+1)}{\big(l-l\cdot p+p+g\cdot p\big)^3}\right) \tag{43}$$

Simplify.

$$\Leftrightarrow \frac{1-2\cdot p}{\big(l-l\cdot p+p+g\cdot p\big)^2} - \frac{2\cdot(p-p^2)\cdot(g-l+1)}{\big(l-l\cdot p+p+g\cdot p\big)^3} \tag{44}$$

Simplify.

$$\Leftrightarrow \frac{l-p\cdot(1+g+l)}{\big(l-l\cdot p+p+g\cdot p\big)^3} \tag{45}$$

Thus, the partial derivative of $\frac{\partial \delta^*(p,g,l)}{\partial l}$ with respect to p is:

$$\frac{\partial \delta^*\big(p,g,l\big)}{\partial l \partial p} = \frac{l-p\cdot(1+g+l)}{\big(l-l\cdot p+p+g\cdot p\big)^3} \tag{46}$$

A.2.5 The partial derivative of $\delta_l^*(p,g,l=l(p,g))$ with respect to p

$$\frac{\partial \delta_l^*\big(p,g,l=l(p,g)\big)}{\partial p} = \frac{\partial}{\partial p}\left(\frac{p-p^2}{\Big(\frac{p\cdot(1+g)}{1-p} - \frac{p\cdot(1+g)}{1-p}\cdot p+p+g\cdot p\Big)^2}\right) \tag{47}$$

Simplify the denominator.

$$\Leftrightarrow \frac{\partial}{\partial p}\left(\frac{p-p^2}{4\cdot(g+1)^2\cdot p^2}\right) \tag{48}$$

Use the product rule.

$$\Leftrightarrow \frac{\frac{\partial}{\partial p}\big(p-p^2\big)}{4\cdot(g+1)^2\cdot p^2} + \big(p-p^2\big)\cdot\left(\frac{\partial}{\partial p}\left(\frac{1}{4\cdot(g+1)^2\cdot p^2}\right)\right) \tag{49}$$

Apply the derivatives with respect to p.

$$\Leftrightarrow \frac{1-2\cdot p}{4\cdot(g+1)^2\cdot p^2}+\big(p-p^2\big)\cdot\left(\frac{-2}{4\cdot(g+1)^2\cdot p^3}\right) \tag{50}$$

Simplify.

$$\Leftrightarrow \frac{1-2\cdot p}{4\cdot(g+1)^2\cdot p^2}-\frac{2\cdot(p-p^2)}{4\cdot(g+1)^2\cdot p^3} \tag{51}$$

Simplify.

$$\Leftrightarrow -\frac{1}{4\cdot(1+g)^2\cdot p^2} \tag{52}$$

Thus, the partial derivative of $\delta_l^*\big(p,g,l=l(p,g)\big)$ with respect to p is:

$$\frac{\partial\delta_l^*\big(p,g,l=l(p,g)\big)}{\partial p}=-\frac{1}{4\cdot(1+g)^2\cdot p^2} \tag{53}$$

A.2.6 The partial derivative of $\delta^*(p(g,l),g,l)$ with respect to g

$$\frac{\partial\delta^*\big(p(g,l),g,l\big)}{\partial g}=\frac{\partial}{\partial g}\left(\frac{p(g,l)\cdot(g-l)+l}{l-l\cdot p(g,l)+p(g,l)+g\cdot p(g,l)}\right) \tag{54}$$

For simplicity, define $q\equiv p(g,l)$. Use the quotient rule and write as two separate fractions.

$$\begin{aligned}\Leftrightarrow & \frac{\Big(l-l\cdot q+q+g\cdot q\Big)\cdot\Big(\frac{\partial}{\partial g}\big(q\cdot(g-l)+l\big)\Big)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2}\\ & -\frac{\Big(q\cdot(g-l)+l\Big)\cdot\Big(\frac{\partial}{\partial g}\big(l-l\cdot q+q+g\cdot q\big)\Big)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2}\end{aligned} \tag{55}$$

Apply the derivatives with respect to g.

$$\begin{aligned}\Leftrightarrow & \frac{\Big(l-l\cdot q+q+g\cdot q\Big)\cdot\Big(\frac{\partial q}{\partial g}\cdot g+q-l\cdot\frac{\partial q}{\partial g}+0\Big)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2}\\ & -\frac{\Big(q\cdot(g-l)+l\Big)\cdot\Big(0-l\cdot\frac{\partial q}{\partial g}+\frac{\partial q}{\partial g}+\frac{\partial q}{\partial g}\cdot g+q\Big)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2}\end{aligned} \tag{56}$$

Simplify.

$$\Leftrightarrow \frac{q^2-l\cdot\frac{\partial q}{\partial g}}{\Big(l-l\cdot q+q+g\cdot q\Big)^2} \tag{57}$$

Thus, the partial derivative of $\delta^*\big(p(g,l),g,l\big)$ with respect to g is:

$$\frac{\partial \delta^*\big(p(g,l),g,l\big)}{\partial g} = \frac{\big(p(g,l)\big)^2 - l \cdot \frac{\partial p(g,l)}{\partial g}}{\Big(l - l \cdot p(g,l) + p(g,l) + g \cdot p(g,l)\Big)^2} \tag{58}$$

A.2.7 The partial derivative of $\delta^*(p(g,l),g,l)$ with respect to l

$$\frac{\partial \delta^*\big(p(g,l),g,l\big)}{\partial l} = \frac{\partial}{\partial l}\left(\frac{p(g,l)\cdot(g-l)+l}{l - l \cdot p(g,l) + p(g,l) + g \cdot p(g,l)}\right) \tag{59}$$

For simplicity, define $q \equiv p(g,l)$. Use the quotient rule and write as two separate fractions.

$$\begin{aligned}\Leftrightarrow\ &\frac{\Big(l - l\cdot q + q + g\cdot q\Big)\cdot\Big(\frac{\partial}{\partial l}\big(q\cdot(g-l)+l\big)\Big)}{\Big(l - l\cdot q + q + g\cdot q\Big)^2}\\ &-\frac{\Big(q\cdot(g-l)+l\Big)\cdot\Big(\frac{\partial}{\partial l}\big(l - l\cdot q + q + g\cdot q\big)\Big)}{\Big(l - l\cdot q + q + g\cdot q\Big)^2}\end{aligned} \tag{60}$$

Apply the derivatives with respect to l.

$$\begin{aligned}\Leftrightarrow\ &\frac{\Big(l - l\cdot q + q + g\cdot q\Big)\cdot\Big(\frac{\partial q}{\partial l}\cdot g - \frac{\partial q}{\partial l}\cdot l - q + 1\Big)}{\Big(l - l\cdot q + q + g\cdot q\Big)^2}\\ &-\frac{\Big(q\cdot(g-l)+l\Big)\cdot\Big(1 - \frac{\partial q}{\partial l}\cdot l - q + \frac{\partial q}{\partial l} + \frac{\partial q}{\partial l}\cdot g\Big)}{\Big(l - l\cdot q + q + g\cdot q\Big)^2}\end{aligned} \tag{61}$$

Simplify.

$$\Leftrightarrow \frac{q - q^2 - l\cdot\frac{\partial q}{\partial l}}{\Big(l - l\cdot q + q + g\cdot q\Big)^2} \tag{62}$$

Thus, the partial derivative of $\delta^*\big(p(g,l),g,l\big)$ with respect to l is:

$$\frac{\partial \delta^*\big(p(g,l),g,l\big)}{\partial l} = \frac{p(g,l) - \Big(p(g,l)\Big)^2 - l \cdot \frac{\partial p(g,l)}{\partial l}}{\Big(l - l \cdot p(g,l) + p(g,l) + g \cdot p(g,l)\Big)^2} \tag{63}$$

A.2.8 The partial derivative of $\delta^*(p(g,l),g,l)$ with respect to $p(g,l)$

$$\frac{\partial\delta^*\big(p(g,l),g,l\big)}{\partial p(g,l)} = \frac{\partial}{\partial p(g,l)}\left(\frac{p(g,l)\cdot(g-l)+l}{l-l\cdot p(g,l)+p(g,l)+g\cdot p(g,l)}\right) \tag{64}$$

For simplicity, define $q \equiv p(g,l)$. Use the quotient rule and write as two separate fractions.

$$\begin{aligned} \Leftrightarrow &\frac{\Big(l-l\cdot q+q+g\cdot q\Big)\cdot\left(\frac{\partial}{\partial q}\Big(q\cdot(g-l)+l\Big)\right)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2} \\ &-\frac{\Big(q\cdot(g-l)+l\Big)\cdot\left(\frac{\partial}{\partial q}\Big(l-l\cdot q+q+g\cdot q\Big)\right)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2} \end{aligned} \tag{65}$$

Apply the derivatives with respect to q.

$$\Leftrightarrow \frac{\Big(l-l\cdot q+q+g\cdot q\Big)\cdot\Big(g-l\Big)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2} - \frac{\Big(q\cdot(g-l)+l\Big)\cdot\Big(g-l+1\Big)}{\Big(l-l\cdot q+q+g\cdot q\Big)^2} \tag{66}$$

Simplify.

$$\Leftrightarrow -\frac{l}{\Big(l-l\cdot q+q+g\cdot q\Big)^2} \tag{67}$$

Thus, the partial derivative of $\delta^*\big(p(g,l),g,l\big)$ with respect to $p(g,l)$ is:

$$\frac{\partial\delta^*\big(p(g,l),g,l\big)}{\partial p(g,l)} = -\frac{l}{\Big(l-l\cdot p(g,l)+q+g\cdot p(g,l)\Big)^2} \tag{68}$$

Equation (68) is negative since the denominator is positive because of the square and the nominator is negative since $l > 0$. Thus, the critical discount factor decreases in the belief and hence, cooperation increases in the belief.

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