

## Abstract

Using 43,000 public construction contracts in Chile procured employing open calls for proposals, I study the effect of firm experience on the likelihood of winning a contract in the future. To address endogeneity of experience (better firms tend to win more contracts in the past and in the future), I instrument firm experience with the number of past contracts won in closely contested auctions, where close auctions are defined as either i) having close monetary bids and price as an important awarding factor ii) involving closely ranked firms (via a modified ELO algorithm) . The IV estimates indicate that firm experience increases the proportion of contracts won by seven percentage points (roughly a third of the winning rate of firms with no experience). I investigate possible mechanisms that could explain this increase in market success by improvements along i) cost measures and ii) quality variables. I find that experienced firms submit bids which are three percentage points lower than firms with no experience, which is correlated with an increase in winning probability. Additionally, experienced firms increase in ten percentage points the approval rate of their proposals in the first stage of the awarding process. I discuss the magnitude of the findings and possible implications for public auction design.



# 1. Experience and Outcomes

This chapter addresses the main research question of whether public experience improves future prospects for firms in the market of public construction projects. The rationale behind the hypothesis is that through experience firms become more efficient and delivering better products; and get familiarized with the bidding process and the bureaucracy of the public sector.

The empirical strategy proceeds by slicing the data in specific points in time and examining how past experience for a firm is related to the proportion of future contracts that it wins out of the future contracts that it bids for. The focus is on the existence of a discontinuity in the outcomes of firms with strictly positive experience and the outcomes of firms with no experience.

Section 1.1 presents the empirical strategy, Section 1.2.1 presents the data and the construction of the regression sample, Section 1.3 shows the main results and finally Section 1.4 the robustness checks.

## 1.1 Empirical Strategy

Our empirical strategy consists in a Regression Discontinuity design in which we compare the bidding outcomes for firms with different levels of previous experience in the market. This section presents the main OLS specifications and the variables of the regression. The next section deals with the causal interpretation of results.

Our two main OLS specification are presented in equations 1.1 and 1.2. Here,  $S_{it2}$  is the share of contracts won in period 2 of slice  $t$ ,  $EXP_{it1}^k$  and  $EXP_{it1}^k > 0$  are the experience treatment variables, and  $T_t$  are period fixed effects. We employ

indexes 1 and 2 to make explicit that each time slice  $t$  involves two periods: period 1 of experience computation and period 2 of outcome computation. Also, the slice is indexed by time  $t$  which is the date in between the two periods. Period fixed effects are added for each period of outcomes to control for changes in the market environment throughout the sample.

$$S_{it2} = \alpha + \beta_k(EXP_{it1}^k > 0) + T_t + \varepsilon_{it} \quad (1.1)$$

$$S_{it2} = \alpha + \gamma_k EXP_{it1}^k + T_t + \varepsilon_{it} \quad (1.2)$$

The outcome variable  $S_{it2}$  is the share of contracts won out of total contracts bid for, in the second period of a given slice  $t$ . That is, for slice  $t$ , the outcome variable for firm  $i$  is  $\frac{W_{it}}{B_{it}}$  where  $B_{it}$  are the bids submitted by firm  $i$  on the period  $[t, t + \tau]$ ,  $W_{it}$  are the contracts won in period  $[t, t + \tau]$  and  $\tau$  is a parameter that controls the length of the periods where we compute the outcomes. In our initial specification, we consider each  $\tau =$  two years.

The treatment variables are either i) an indicator of past experience  $EXP_{it1}^k > 0$  and ii) total experience  $EXP_{it1}^k$ . Moreover, we consider two ways of *computing* the total experience  $EXP_{it1}^k$  for a firm  $i$ , which we index by  $k$ ,  $k \in \{1, 2\}$ . The first measure of experience computes experience acquired in a fixed period immediately before  $t$  (*rolling experience*). The second measure of experience computes experience by adding up all the contracts won for a firm before  $t$  and dividing by the number of years in which it was accrued (*annualized cumulative experience*). Details are given in the data section.

### 1.1.1 Endogeneity and Identification

A structural interpretation of equations 1.1 and 1.2 must recognize the presence of endogeneity and heterogenous effects, which prevents from considering the OLS estimates as causal treatment effects directly. In this section, we present the problem of endogeneity, propose an IV strategy to address it, and then discuss the causal interpretations of the estimates obtained with it.

First we discuss endogeneity. Unobserved cost variables, specific to each firm, are omitted in the OLS regressions above and expectedly endogenous. If there are highly efficient firms who are able to bid more aggressively or submit better proposals, they should win more projects, and in turn accumulate more experience over time. We thus expect our estimates  $\hat{\beta}$  and  $\hat{\gamma}$  in 1.1 and 1.2 to be biased upwards due to correlation (expectedly positive) between omitted cost variables or efficiency variables (which we call  $W_i$ ) and the amount of past experience. That is,  $E(\varepsilon \cdot EXP) \neq 0$  in the structural interpretation of Equations 1.1.

To estimate consistently the parameters of interest, we employ variation in experience uncorrelated with firm's internal characteristics to instrument the experience of a firm in an Instrumental Variables (IV) approach. We propose to employ close wins as an instrument for total wins. If we are able to find wins where the success of a firm is less or not at all attributable to unobserved cost factors, or other efficiency advantages, but instead attributable to random differences (e.g. the conservativeness of each firms' engineers' estimates), we can estimate consistently the coefficient of interest by instrumenting total wins with close wins.

In this approach, our first stage takes the form of Equation 1.3. Here  $EXP > 0_{it1}^k$  is an indicator for contracts won in period 1 of slice  $t$  for firm  $i$ , while  $EXPCLOSE > 0_{it1}$  is an indicator for a close win in the same period, and  $\nu_{it}$  is an error term uncorrelated with  $EXPCLOSE_{it}$ . The second stage is shown in Equation 1.4.

$$EXP_{it2} > 0 = \delta EXPCLOSE_{it} > 0 + T_t + \nu_{it} \quad (1.3)$$

$$S_{it2} = \tilde{\beta} EXP_{it2} > 0 + T_t + \varepsilon_{it} \quad (1.4)$$

The consistency of the estimates obtained require as the assumptions of i) validity and ii) rank conditions. The validity condition is satisfied if  $E((EXPCLOSE_i > 0) \cdot W_i) = 0$ , i.e. close wins are uncorrelated with the omitted advantage variables in the OLS regression. As we argued, a close win, if correctly identified, can be attributed to noise, rather than fundamental differences between firms. For example,

risk-aversion differences between firms, random approximation differences between engineering teams. The Data section examines how close wins are identified and labeled as such in practice with the data.

The rank condition, on the other side, requires correlation between experience and close experience. Indeed, by construction, we expect this condition to be fulfilled. The data section show results that prove that this condition is fulfilled.

If conditions i) and ii) are fulfilled the estimates  $\hat{\beta}$  and  $\hat{\gamma}$  are expected to be consistent. However, estimates should not to be interpreted as a single treatment effect. The treatment effect should be heterogeneous, mainly because:

- Experience itself is heterogenous given the complexity, length and size of a project, so it is expected that treatment effects are also heterogenous.
- Firm's absorptive capacity and learning ability depends on internal skill, financial strength and other organizational variables.
- More experienced firms should see diminishing returns to experience.

Following the discussion of (Angrist and Imbens, 1995) as presented in (Hansen, 2009), we argue that the estimation strategy identifies the Local Average Treatment Effect (LATE) for our treatment. This is a weighted average treatment effect for the firms who only get the extra experience due to a close wins (i.e. "compliers"). For example, a firm in this sample could be a firm not efficient enough to win a contract through its cost advantages alone.

This interpretation, additionally to rank and validity, also requires a monotonicity condition, that here is equivalent to having no firms negatively impacted in their experience by experiencing a close win. This condition is satisfied in our setting, since a close win belongs by construction to the set of all wins.

Having discussed the theoretical rationale and consequences of employing close wins as instruments, the problem remains of how to successfully find close wins and label them as such, which is analyzed in subsection 1.2.2. Two alternatives are proposed: first, find contracts with very close wins where price was heavily weighted, and second, develop a ranking measure of firms to find "balanced" auctions.

## 1.2 Data

### 1.2.1 Construction of the regression dataset

Our starting dataset consists in a set of bids submitted by firms in auctions developed by the government in Chile between 2010 and 2020 for construction projects. The source and main characteristics of the dataset employed in the investigation were detailed in the previous chapters and the Table 1.1 repeats some relevant descriptive statistics. The main purpose of this section is to describe the process employed to create the dataset employed in the estimation of equations 1.1 and 1.2.

Table 1.1: Sample Descriptive Statistics

| name                                   | N      | Complete Cases | mean     | std      | max      | min  |
|--|--------|----------------|----------|----------|----------|------|
| Bid (all)                              | 153000 | 1              | 7.92e+10 | 2.61e+13 | 1e+16    | 0    |
| Winning Bid                            | 38500  | 1              | 2.52e+08 | 2.39e+09 | 2.47e+11 | 0.6  |
| Difference between 1st bid and 2nd (%) | 38500  | 0.707          | 0.0933   | 0.162    | 1        | 0    |
| Number of Bidders per Contract         | 49400  | 1              | 3.1      | 3.09     | 466      | 1    |
| Year                                   | 49400  | 1              | 2016     | 3.19     | 2021     | 2010 |
| Offers made by Firm                    | 15500  | 1              | 9.83     | 27.9     | 1980     | 1    |
| Win prob. by Firm                      | 15500  | 1              | 0.216    | 0.3      | 1        | 0    |
| Offers won by Firm                     | 15500  | 1              | 2.48     | 6.13     | 146      | 0    |

The creation of the analysis sample requires defining distinct periods of time to compute experience and outcomes. We proceed by fixing several points in time  $t$ 's in the sample, spaced by one year each, starting one year after the earliest date in the dataset. For each  $t$ , we employ a period immediately before it to compute experience (Period 1) and a period immediately after to compute outcomes (Period 2). We call a pair Period 1 - Period 2 a *slice* and we index it by  $t$ . For every firm in a slice, we link experience computed with method one or two in the first period (period 1) to the outcomes in the next period (period 2), to form an individual observation.

The period of outcome computation is always two years. On the other side, the period of experience computation depends on the experience measure employed. Recall that we compute experience as either rolling experience or annualized cumulative experience. Rolling experience computes experience as the total amount of contracts won in a fixed period of length  $\sigma$  previous to  $t$ , comprising the period  $[t - \sigma, t]$  before

the outcomes period  $[t, t + \tau]$ . As our baseline, we set  $\sigma =$  two years.

The second alternative (annualized cumulative experience) computes experience cumulatively by summing contracts developed from the start of our bid dataset (i.e. the year 2010) up until time  $t$  and dividing this number by the number of years since the firm's first win. That is, the experience computation period is  $[2010, t]$  for every  $t$ . Instead of restricting our measure of past experience to two years before the outcomes' period, as in the previous method, we consider all the previous years when counting contracts won. We call this computation strategy annualized cumulative experience.

We end up with two datasets (one for each  $k$ ) where each observation is the linked experience-outcomes pair for a firm  $i$  at a given  $t$ , the dependent variable is a measure of the firm's outcomes in Period 2 (i.e.  $S_{it2}$ ), and the independent variable is a measure of the (past) experience of the firm in Period 1 (i.e.  $EXP_{it}^k$ ,  $EXP_{it}^k > 0$ ,  $k = 1, 2$ ).

We make an important filtering step before computing outcomes, as we only consider contracts for which previous experience is not among the awarding criteria to choose the winner. This is because including contracts for which experience is among the awarding criteria would i) render (expectedly) trivially positive and significant results and ii) confound the true effect of learning by doing among contracts which do not include experience as awarding criteria. Note that this filtering step is only carried out for outcomes' computation and not for experience computation.

The diagram in Figure 1-1 shows a toy example of how we transform the data from per-firm/period to a per firm/slice dataset. The original firm-period level dataset has, for every period, the contracts bid for and contracts won. The second dataset aggregates these results by slice. Note that this diagram assumed that no contracts had experience as part of awarding criteria.

After the transformation steps, we obtain ten slice-firm datasets for each measure of experience. Tables 1.2 and 1.3 show the dates of period 1 and period 2 for each slice, their lengths in years, and the number of observations in each slice. Recall that every observation has the pair firm-slice as a unique identifier and contains as key variable the past experience and a summary of future outcomes, having the form of the rightmost table in Figure 1-1.



| A | Firm Period Dataset |          |          |          |          |    | Firm Slice Dataset :<br>Two Year Past Experience |            |         |
|---|---------------------|----------|----------|----------|----------|----|--|------------|---------|
|   | Time                | 1        | 2        | 3        | 4        | 5  | Slice  | Experience | Outcome |
|   | Bids Made           | 0        | 5        | 10       | 10       | 10 | 1  | 5 (5+0)    | 10/20   |
|   | Bids Won            | 0        | 5        | 5        | 5        | 0  | 2  | 10 (5+5)   | 5/20    |
|   | Slice 1             | Period 1 |          | Period 2 |          |    |  |            |         |
|   | Slice 2             |          | Period 1 |          | Period 2 |    |  |            |         |

| B | Firm Period Dataset |          |   |          |          |    | Firm Slice Dataset :<br>Cumulative Yearly Experience |            |         |
|---|---------------------|----------|---|----------|----------|----|--|------------|---------|
|   | Time                | 1        | 2 | 3        | 4        | 5  | Slice  | Experience | Outcome |
|   | Bids Made           | 0        | 5 | 10       | 10       | 10 | 1  | 0 (0/1)    | 10/15   |
|   | Bids Won            | 0        | 5 | 5        | 5        | 0  | 2  | 2.5 (5/2)  | 10/20   |
|   | Slice 1             | Period 1 |   | Period 2 |          |    | 3  | 3.3 (10/3) | 5/20    |
|   | Slice 2             | Period 1 |   | Period 2 |          |    |  |            |         |
|   | Slice 3             | Period 1 |   |          | Period 2 |    |  |            |         |

Figure 1-1: Example computation of slice-firm dataset, employing two-year fixed periods of past experience (A), and cumulative yearly experience (B).

Table 1.2: Analysis dataset characteristics for experience computed in rolling periods of two years

| Slice | Period 1 dates        | Period 2 dates        | Observations | Length Period 1 | Length Period 2 | Contracts in Period 1 | Contracts in Period 2 |
|-------|-----------------------|-----------------------|--------------|-----------------|-----------------|-----------------------|-----------------------|
| 1     | 2010-01-04/2012-01-04 | 2012-01-04/2014-01-04 | 2485         | 2               | 2               | 6056                  | 2994                  |
| 2     | 2011-01-04/2013-01-04 | 2013-01-04/2015-01-04 | 2391         | 2               | 2               | 8360                  | 2465                  |
| 3     | 2012-01-04/2014-01-04 | 2014-01-04/2016-01-04 | 2515         | 2               | 2               | 8470                  | 2771                  |
| 4     | 2013-01-04/2015-01-04 | 2015-01-04/2017-01-04 | 2682         | 2               | 2               | 7870                  | 2993                  |
| 5     | 2014-01-04/2016-01-04 | 2016-01-04/2018-01-04 | 2585         | 2               | 2               | 9425                  | 2588                  |
| 6     | 2015-01-04/2017-01-04 | 2017-01-04/2019-01-04 | 2300         | 2               | 2               | 9978                  | 2061                  |
| 7     | 2016-01-04/2018-01-04 | 2018-01-04/2020-01-04 | 2183         | 2               | 2               | 9007                  | 1806                  |
| 8     | 2017-01-04/2019-01-04 | 2019-01-04/2021-01-04 | 2230         | 2               | 2               | 8637                  | 1900                  |
| 9     | 2018-01-04/2020-01-04 | 2020-01-04/2022-01-04 | 1577         | 2               | 2               | 9212                  | 1198                  |

Table 1.3: Analysis dataset characteristics for experience computed as cumulative annualized

| Slice | Period 1 dates        | Period 2 dates        | Observations | Length Period 1 | Length Period 2 | Contracts in Period 1 | Contracts in Period 2 |
|-------|-----------------------|-----------------------|--------------|-----------------|-----------------|-----------------------|-----------------------|
| 0     | 2010-01-04/2011-01-04 | 2011-01-04/2013-01-04 | 2334         | 1               | 2               | 2393                  | 2892                  |
| 1     | 2010-01-04/2012-01-04 | 2012-01-04/2014-01-04 | 2485         | 2               | 2               | 6056                  | 2994                  |
| 2     | 2010-01-04/2013-01-04 | 2013-01-04/2015-01-04 | 2391         | 3               | 2               | 10753                 | 2465                  |
| 3     | 2010-01-04/2014-01-04 | 2014-01-04/2016-01-04 | 2515         | 4               | 2               | 14526                 | 2771                  |
| 4     | 2010-01-04/2015-01-04 | 2015-01-04/2017-01-04 | 2682         | 5               | 2               | 18623                 | 2993                  |
| 5     | 2010-01-04/2016-01-04 | 2016-01-04/2018-01-04 | 2585         | 6               | 2               | 23951                 | 2588                  |
| 6     | 2010-01-04/2017-01-04 | 2017-01-04/2019-01-04 | 2300         | 7               | 2               | 28601                 | 2061                  |
| 7     | 2010-01-04/2018-01-04 | 2018-01-04/2020-01-04 | 2183         | 8               | 2               | 32958                 | 1806                  |
| 8     | 2010-01-04/2019-01-04 | 2019-01-04/2021-01-04 | 2230         | 9               | 2               | 37238                 | 1900                  |
| 9     | 2010-01-04/2020-01-04 | 2020-01-04/2022-01-04 | 1577         | 10              | 2               | 42170                 | 1198                  |

The next section describes how to identify and label close wins, which is a key part of the identification strategy.

### 1.2.2 Definition of a close win

We discuss what would be the optimal way of finding close wins, and, since the data does not allow us to employ this strategy, we propose two second-best alternatives. The optimal way to identify close wins would be to single out auctions for which the winning firm had a final weighted score which was marginally superior to the ones of its competitors. Recall that, for each contract, the firm's proposals are scored in several criteria. The scores are then weighted and added up to produce the total score for that firm. Unfortunately, the optimal strategy described above is unfeasible with the data we have available, since our data only allows us to see the criteria employed in each contract and the weight of each factor, but not the individual scores for each firm. We attempt two alternative methods detailed in the subsections below.

#### Close wins by price

This method of identifying close wins relies on the fact that price is a major awarding criteria, as was shown in the Data section. Wherever price is a major awarding criteria, cost advantages play a part in the outcome of the bidding procedure as more efficient contractors can submit lower bids than its competitors. The strategy is the to single out auctions where price is a major factor and bids where close. This way, cost advantages can plausibly be disregarded as the cause behind a win. This strategy should indeed capture a subset of the random wins, namely, random wins in projects where price is the major awarding criteria.

Close wins are operationally identified as the wins where i) the winning bid was not more than .05% below the second lowest win, if the winner had the lowest bid, ii) the winning bid was not more than 0.05% below the lowest bid, if the winner did not submit the lowest bid and iii) the weight of the price item in the awarding decision was more than 50%.

This definition of close wins leads to approximately 2% of winning bids being classified as a close one. In Table 1.4 we examine whether close wins defined as above are different from the population in several types of metrics. We can see that in

most aspects these bids have less dispersion in variables such as participants and less size. These might because of fat tails in the distributions of sizes and participants. However, the size and number of participants are both relatively close.

Table 1.4: Comparison between close and non-close wins, by price

| Variable                               | Mean (Not close win) | Mean (Close win) | Sd (Not close win) | Sd (Close win) |
|--|----------------------|------------------|--------------------|----------------|
| Bid (all)                              | 8.06e+10             | 2.14e+08         | 2.63e+13           | 7.45e+08       |
| Winning Bid                            | 2.53e+08             | 1.87e+08         | 2.41e+09           | 7.21e+08       |
| Difference between 1st bid and 2nd (%) | 0.0957               | 0.00216          | 0.164              | 0.00155        |
| Number of Bidders per Contract         | 3.08                 | 3.96             | 3.1                | 2.36           |
| Year                                   | 2016                 | 2015             | 3.19               | 3.08           |

The rank condition, required for consistent estimates, is verified via a regression of experience on close experience. The F-Statistic of this regression is 118.2 for the indicator treatment and 1,500 for the continuous measure. Also, we find positive and significant coefficients of close experience on experience. The details are included in Appendix A.1.

## Close wins by rank

The second strategy to identify close wins does not rely in prices or any other aspect of the bid itself. Instead, we label a winning bid as a close win if all the firms involved in the auction were close in ranking. The argument here is that, given a well constructed ranking, winning a contract against closely placed opponents should be attributable to random factors.

Obviously, the main issue is how to construct a good ranking measure. We proceed by modeling each auction as a multi-player game event (in the non-economic sense of the term) in which firms gain points by winning the project and lose points by not winning it. We award and subtract points based on a modified Elo algorithm suited for multi-player games.

Each firm has its ranking initialized at a pre-specified level (1,500 for the main analysis). Then, it is awarded 25 points for winning against a similar opponent and subtracted 8 by losing. The implementation of the algorithm recommends that points awarded and subtracted sum to zero, so we fix awarded points and choose subtracted points so that on average (given the number of players in an auction) this condition

holds. Against non-similar opponents, the algorithm makes a correction on points awarded and subtracted based on the ranking of the players and the outcome of the game.

Proceeding from the oldest to the most recent auction, we update the initial rankings for each firm and obtain for each firm its ranking at any point in time. Next, we label a win as a "close win" when the highest rank among the bidders for the auction was not more than 3% higher than the lowest rank among the same set of bidders. This yields around 6,900 closely won contracts (13.7% of the wins in the analysis sample) which corresponds to 20,924 observations (14% of the observations in the analysis sample). In Table 1.5 we present summary statistics for close wins identified via rank.

Table 1.5: Comparison between close and non-close wins

| Variable                               | Mean (Not close win) | Mean (Close win) | Sd (Not close win) | Sd (Close win) |
|--|----------------------|------------------|--------------------|----------------|
| Bid (all)                              | 9.17e+10             | 4.61e+08         | 2.81e+13           | 1.08e+10       |
| Winning Bid                            | 2.52e+08             | 2.5e+08          | 2.22e+09           | 3.03e+09       |
| Difference between 1st bid and 2nd (%) | 0.0914               | 0.0987           | 0.164              | 0.157          |
| Number of Bidders per Contract         | 3.11                 | 3.03             | 3.29               | 1.42           |
| Year                                   | 2016                 | 2014             | 3.14               | 3.19           |

In the analysis, we drop the first two years of data to allow for a period of rank adjustment. This is necessary since the algorithm does not work well when the average rank in the population is not clearly defined. The way ranks evolve as time progresses can be seen in Figure 1-2. Note that ranks appear highly concentrated at the end of the first year of data, while they are much more dispersed at the end. In the robustness checks we analyze both i) different values for the won/lost points after an auction and ii) the threshold in ranking for a close win.

More details for this strategy, regarding the theory, algorithm and results, can be found on Appendix ??.

The rank condition, required for consistent estimates, is again verified via a regression of experience on close experience, now defined via rank. The F-Statistic of this regression is over 3,000 for the indicator treatment and over 2,000 for the continuous measure. Also, we find positive and significant coefficients of close experience on experience. The details are included in Appendix A.1.

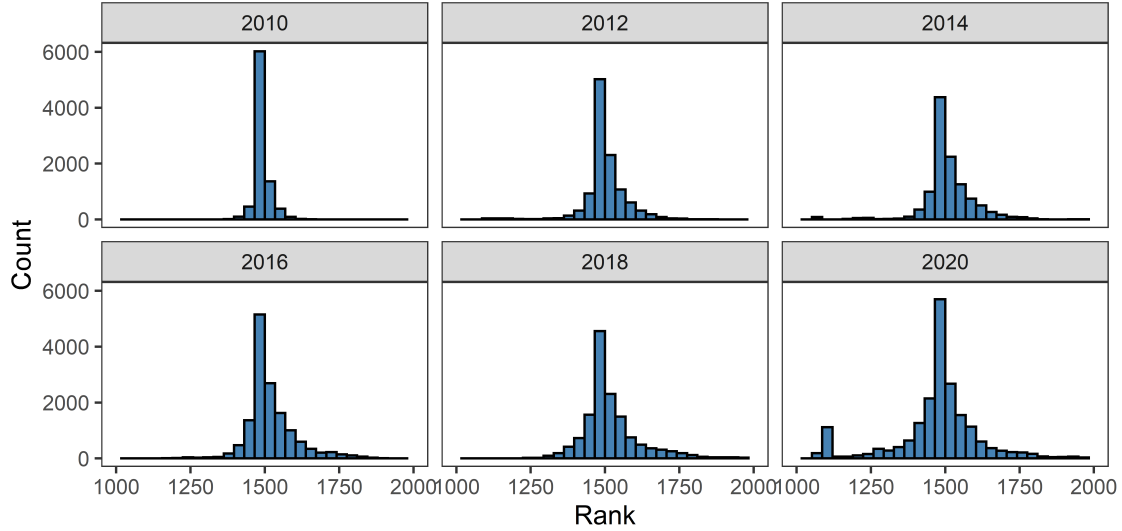


Figure 1-2: Evolution of ranks by selected years

### 1.3 Main Results

First we explore graphically the relationship between experience and outcomes. Figure 1-3 shows the relationship between rolling (top row) and annualized (bottom row) measures of experience and outcomes. Each column represents a different subsample and dependent variable. The first column (panels A and D) selects all firms and displays past experience in the  $x$ -axis. The second column (panels B and E) contains only firms with equal experience and close experience (including zero). The  $x$ -axis displays the close wins. The third column (panels C and F) is analogous to column two but employs the definition of a close win as close win by firm rank.

We observe that average winning shares increase with more experience. The effect appears to be close to linear, although for experiences higher than ten contracts performed (rolling) or five contracts performed (annualized) we have wide error bars or no observations available. In the case of our "reduced form" graphs, we observe that almost always the close wins seem to improve average winning shares, although we observe wide error bars in the second column, caused by the low amount of observations that fulfill the conditions imposed.

Next we show the results from our regression analysis. Table 1.6 shows the results

for OLS and IV regressions for our first experience measure (i.e. rolling two year periods) while Table 1.7 shows the results for our second measure of experience (i.e. annualized experience). The first three panels in each table employ as treatment the binary indicator of experience, whereas the last three panels employ total experience.

The OLS estimate of the effect of having experience on winning proportion is 0.07 for rolling experience and 0.06 for annualized experience. IV estimates of the coefficient are very close to OLS counterparts or even higher, for the case of annualized experience. The specification with linear returns on experience shows that experience renders a 0.01 and 0.03 increase in winning share per extra contract developed (for rolling and annualized experience respectively). IV estimates of linear effect of experience are again close to OLS counterparts. Finally, all the estimates for the experience treatments are significant at  $p < 0.01$  with robust standard errors.

A concerning result is the low  $R^2$  of the regressions, which shows that although the effect of experience on the mean outcome is significant, there is much variability among firms' outcomes which is not explained by the increase in experience.

Given the average winning shares (approximately 0.23), depending on the sample), the effect of having experience is equivalent to an increase of around 30% of the winning share of a firm (i.e. around 7 percentage points out of 23 percentage points). This points towards significant importance of previous experience in future outcomes.

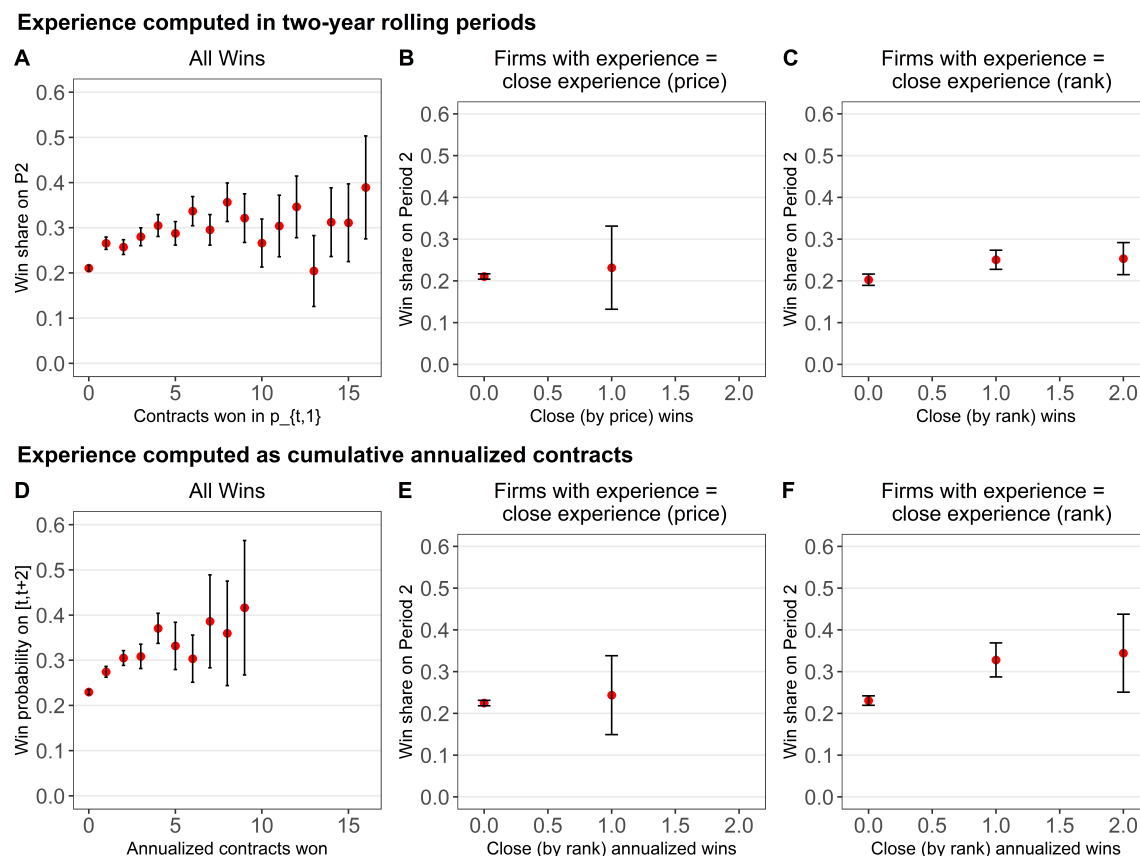


Figure 1-3: Relationship between contracts won on  $t-1$  and mean winning probability across contractors in  $t$ .

Note: The plots show the mean across firms of the number of contracts won out of the number of contracts bid for in period  $t$  (in the  $y$ -axis), against experience accrued in period  $(t-1)$  in the  $x$ -axis.  $t$  and  $t-1$  correspond to two periods of two years each for the top row, for the bottom row  $t$  is also a period of two years, but  $t-1$  are all years in the interval  $[2010, t]$ . Error bars correspond to means plus/minus two standard errors. First column: all sample observations are considered. Second column: only contractors with experience = close experience. Third column: analogous to second column employing the rank definition of close win. The first row definition of experience is rolling experience while the second row employs cumulative annualized experience.

Table 1.6: Regression for OLS and IV specifications with Experience computed in rolling 2-year periods

|                              | <i>Dependent variable:</i>  |                     |                     |                     |                     |                     |
|------------------------------|-----------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
|                              | Share of Contracts won in t |                     |                     |                     |                     |                     |
|                              | OLS<br>(1)                  | IV (Price)<br>(2)   | IV (Rank)<br>(3)    | OLS<br>(4)          | IV (Price)<br>(5)   | IV (Rank)<br>(6)    |
| Experience in (t-1) (Binary) | 0.074***<br>(0.005)         | 0.063***<br>(0.019) | 0.082***<br>(0.007) |                     |                     |                     |
| Experience in (t-1) (Linear) |                             |                     |                     | 0.010***<br>(0.001) | 0.007***<br>(0.002) | 0.017***<br>(0.001) |
| Constant                     | 0.258***<br>(0.007)         | 0.262***<br>(0.010) | 0.237***<br>(0.008) | 0.273***<br>(0.007) | 0.277***<br>(0.008) | 0.245***<br>(0.007) |
| Fixed effects By period      | Yes                         | Yes                 | Yes                 | Yes                 | Yes                 | Yes                 |
| Observations                 | 20,948                      | 20,948              | 16,072              | 20,948              | 20,948              | 16,072              |
| R <sup>2</sup>               | 0.018                       | 0.017               | 0.017               | 0.015               | 0.014               | 0.010               |

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

The regression analyzes the relationship between past experience (dependent var.) and future winning shares in the market (indep. var.). The winning share is contracts bid / contracts won in the next two years. Experience is contracts won in the past two years. The instrument for panels 2,3,5 and 6 is a binary indicator for a closely won contract among the past contracts won. Robust (HC1) error are on parenthesis.

Table 1.7: Regression for OLS and IV specifications with Experience computed as annualized cumulative experience

|                              | <i>Dependent variable:</i>  |                     |                     |                     |                     |                     |
|------------------------------|-----------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
|                              | Share of Contracts won in t |                     |                     |                     |                     |                     |
|                              | OLS<br>(1)                  | IV (Price)<br>(2)   | IV (Rank)<br>(3)    | OLS<br>(4)          | IV (Price)<br>(5)   | IV (Rank)<br>(6)    |
| Experience in (t-1) (Binary) | 0.061***<br>(0.005)         | 0.079***<br>(0.016) | 0.084***<br>(0.013) |                     |                     |                     |
| Experience in (t-1) (Linear) |                             |                     |                     | 0.027***<br>(0.002) | 0.024***<br>(0.005) | 0.032***<br>(0.005) |
| Constant                     | 0.282***<br>(0.008)         | 0.278***<br>(0.009) | 0.251***<br>(0.013) | 0.284***<br>(0.008) | 0.286***<br>(0.008) | 0.264***<br>(0.012) |
| Fixed effects By period      | Yes                         | Yes                 | Yes                 | Yes                 | Yes                 | Yes                 |
| Observations                 | 21,705                      | 21,705              | 12,327              | 21,705              | 21,705              | 12,327              |
| R <sup>2</sup>               | 0.016                       | 0.016               | 0.012               | 0.016               | 0.016               | 0.015               |

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

The regression analyzes the relationship between past experience (dependent var.) and future winning shares in the market (indep. var.). The winning share is contracts bid / contracts won in the next two years. Experience is contracts won since the beginning of the sample, divided by number of years since the first win. The instrument for panels 2,3,5 and 6 is a binary indicator for a closely won contract among the past contracts won. Robust (HC1) error are on parenthesis.

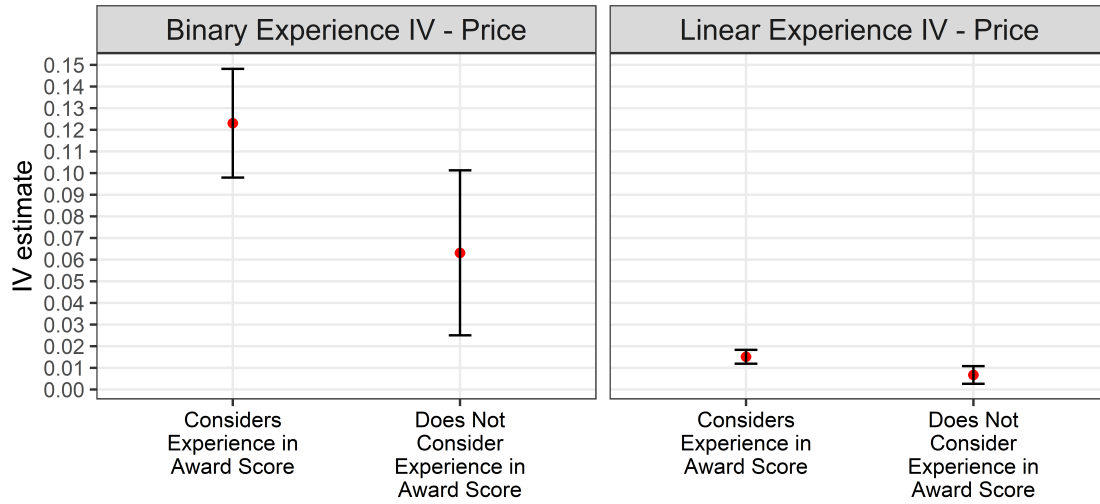


### 1.3.1 Comparing with contracts that do include experience in awarding score

We compare the main results obtained in the previous section with the results obtained by considering for outcome computation only contracts which *do* require experience in the awarding criteria. This helps to put the results in context and also serves as a validation check of the empirical strategy. We expect to find greater estimates for the effect of experience on outcomes among contracts which explicitly reward experience.

Figure 1-4 shows the estimate from the IV specifications, both with linear and binary functional forms of experience, by the type of contract considered to compute outcomes (we only employed rolling experience). It can be seen that the effect of experience on outcomes is about twice as big in contracts which do consider experience as a factor in the awarding criteria with respect to those who do not.

Figure 1-4: Comparison between estimates obtained in contracts with and without experience in the awarding criteria employed by the government



Note: The plot shows a comparison between the estimate of a regression of winning shares on various forms of experience (instrumented with close wins  $>0$ ), considering on the x-axis two types of analysis samples. The first considers only contracts that did not include experience in the awarding criteria (left of each panel). The second considers only contracts without experience in the awarding criteria (right of each panel). Close wins are defined by price (see text). Error bars are confidence intervals at 95% with robust standard errors.

## 1.4 Robustness checks

Several of the parameters in the empirical strategy of the previous section admit more than one reasonable choice. This section considers alternatives for them. Robustness checks are studied for the following parameters:

1. Periods of outcome computation.
2. Definition of a close win (by price).
3. Definition of a close win (by rank).

### 1.4.1 Periods of outcomes

In the main analysis, we computed outcomes across a period of two years for each of our slices. This choice is sensibled by computing outcomes in one and three year periods as well. While varying the length of the period where outcomes are computed, the procedures to compute experience are kept the same as before.

A shorter timeframe would be a better parameter choice if: firms bid frequently, so their true outcomes manifest quickly; learning is itself instantaneous, so past experience immediately influences outcomes; or the learning effect is short lived, which would make much more important for the outcomes the recent history. Conversely, a longer time frame is better in the case of infrequent bidding, slow learning, and long lasting knowledge.

For construction projects, it is expected that the better parameter would be more close to a longer timeframe than to a shorter one. Construction projects, especially complex ones, can be less frequently auctioned than in simpler, undifferentiated products. More importantly, since construction projects take longer to perform than regular purchases, it is reasonable to expect a longer learning process.

Table 1.8 shows estimated experience coefficients where outcomes were computed in periods of 1, 2 (the original specification) and 3 years. The rows correspond to OLS, IV (by price) and IV (by rank) specifications. Notably, i) all results are significant with  $p < 0.01$  and ii) estimates are close to each other across different values of the parameter. Standard errors decrease with the number of years considered because of the increase in sample size. In almost every case, estimates remain within one and a half a standard error of the original estimates, and in all cases they remain within two standard errors.

### 1.4.2 Definition of a close win - Price IVs

In the main section, close wins by price were defined as those in which the winning contractor submitted a bid that i) was not more than .05% below the second lowest win, if he had the lowest bid, ii) was not more than 0.05% below the lowest bid, if he

Table 1.8: Robustness analysis for the coefficient on Experience (Rolling) by length of outcome computation period

| Experience Computation | Specification | 1 year outcomes   | 2 year outcomes (Main) | 3 year outcomes   |
|------------------------|---------------|-------------------|------------------------|-------------------|
| Indicator              | IV-Price      | 0.098 (0.028) *** | 0.063 (0.019) ***      | 0.07 (0.017) ***  |
| Indicator              | IV-Ranks      | 0.075 (0.014) *** | 0.082 (0.01) ***       | 0.081 (0.009) *** |
| Indicator              | OLS           | 0.076 (0.006) *** | 0.074 (0.005) ***      | 0.07 (0.004) ***  |
| Linear                 | IV-Price      | 0.009 (0.003) *** | 0.007 (0.002) ***      | 0.008 (0.002) *** |
| Linear                 | IV-Ranks      | 0.014 (0.003) *** | 0.017 (0.002) ***      | 0.017 (0.002) *** |
| Linear                 | OLS           | 0.009 (0.001) *** | 0.01 (0.001) ***       | 0.012 (0.001) *** |

*Note:* \*\*\*p<0.01,\*\*p<0.05

did not submit the lowest bid and iii) the weight of the price item in the awarding decision is more than 50%. In this section the main estimates are sensibilized to different values of the threshold parameter and the weight parameter.

We first sensibilize the threshold for bid differences for the linear estimate of experience in the rolling experience measure. The plot in Figure 1-5 displays the coefficient of interest and 95% confidence as we vary the threshold for a close win. For thresholds below .025%, we obtain much wider standard errors. The reduction in sample size for the instrument is significant below .05%, since this percentage is already at around the 15th percentile of bid differences in the sample. However, we keep significant outcomes at p=0.05 for all values analyzed.

### Estimates of IV treatment effects by threshold for close wins by price

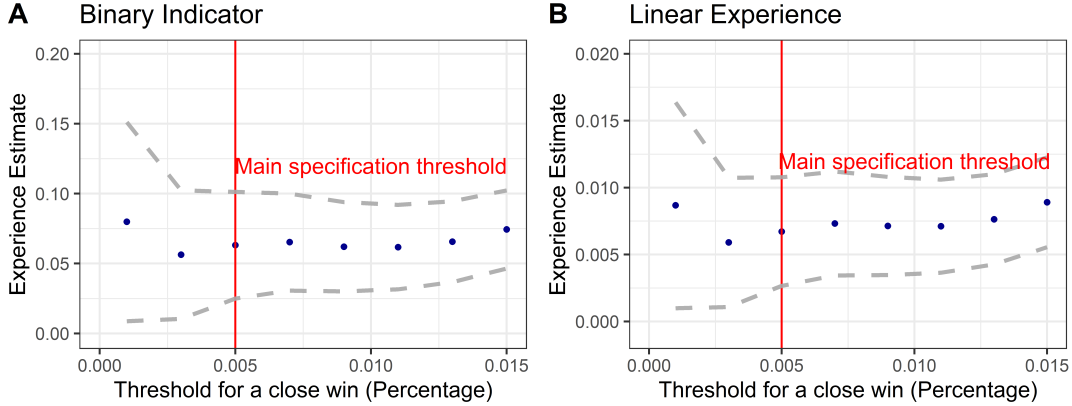


Figure 1-5: Robustness analysis for threshold of close wins

Note: The plot shows the coefficient on experience as in the specification of Panels 4 (left) and 5 (right) of table 1.6, that is, the dependent variable is the share of contracts won in period  $t$  and the independent variable is an indicator of experience or linear experience. Experience is instrumented with close wins  $> 0$  in period  $(t - 1)$ . The  $x$ -axis shows how the coefficient varies with the threshold for the maximum percentage difference between the winner's bid and the closest bid of interest.

Next we examine the parameter for the weight of the price component in the total score. We replicate our main IV-price results but consider weights of 60%, 70%, and 80% as the minimum weights of the price component in the factors considered to evaluate proposals. Table 1.9 shows the results. At 60%, most results remain significant, but beyond 70% almost all results are not. Since 60% is the 80th percentile of the score weight across contracts, we have again a sample size problem for the instrument when there are higher requirements for the threshold of the price weight.

Table 1.9: Robustness analysis for the price weight parameter in the IV Regression by price

| Experience Computation | Functional Form  | 50                | 60                | 70                | 80            |
|------------------------|------------------|-------------------|-------------------|-------------------|---------------|
| Annualized             | Binary Indicator | 0.079 (0.016) *** | 0.079 (0.019) *** | 0.059 (0.023) *** | 0.051 (0.031) |
| Annualized             | Linear           | 0.024 (0.005) *** | 0.024 (0.006) *** | 0.017 (0.007) *** | 0.015 (0.009) |
| Rolling                | Binary Indicator | 0.063 (0.019) *** | 0.059 (0.024) **  | 0.028 (0.028)     | 0.045 (0.04)  |
| Rolling                | Linear           | 0.007 (0.002) *** | 0.006 (0.003) **  | 0.003 (0.003)     | 0.004 (0.004) |

*Note:* \*\*\* $p < 0.01$ , \*\* $p < 0.05$ . Numeric columns show the weight required for the price parameter to be considered a close win.

### 1.4.3 Definition of a close win - Rank IVs

The IV-Rank estimates are sensibilized by choosing alternative thresholds for the maximum difference between the highest and lowest bidder's rank (bandwidth) and different values for the points awarded for a win. Recall that an auction is labeled as close in the main specification if the difference in rank between the highest and lowest ranked in the auction is less than 3%. In the main specifications, 25 points are awarded for a win and eight are subtracted for a loss.

We analyze bandwidths of 1%, 2%, 3% and 4%. Regarding points for a win, we analyze as alternatives 10, 15, 25, 35 and 50 points. Again, to preserve stability, points subtracted for a loss are approximately a third of the points awarded for a win. Since average bidders are close to three, we divide awarded points by three to obtain subtracted points

Given the amount of possible parameter combinations, results are shown in graphic form in Figure 1-6 and they only consider the first type of experience computation (rolling). Results show that IV estimates are robust to all the alternatives considered. Considering a lower thresholds for the difference in ranks does increase the standard errors. However, estimates do not vary much, staying close to .075 for a binary indicator of experience as treatment and to .012 for the total experience treatment.

### Robustness analysis for threshold and points awarded - close wins by rank

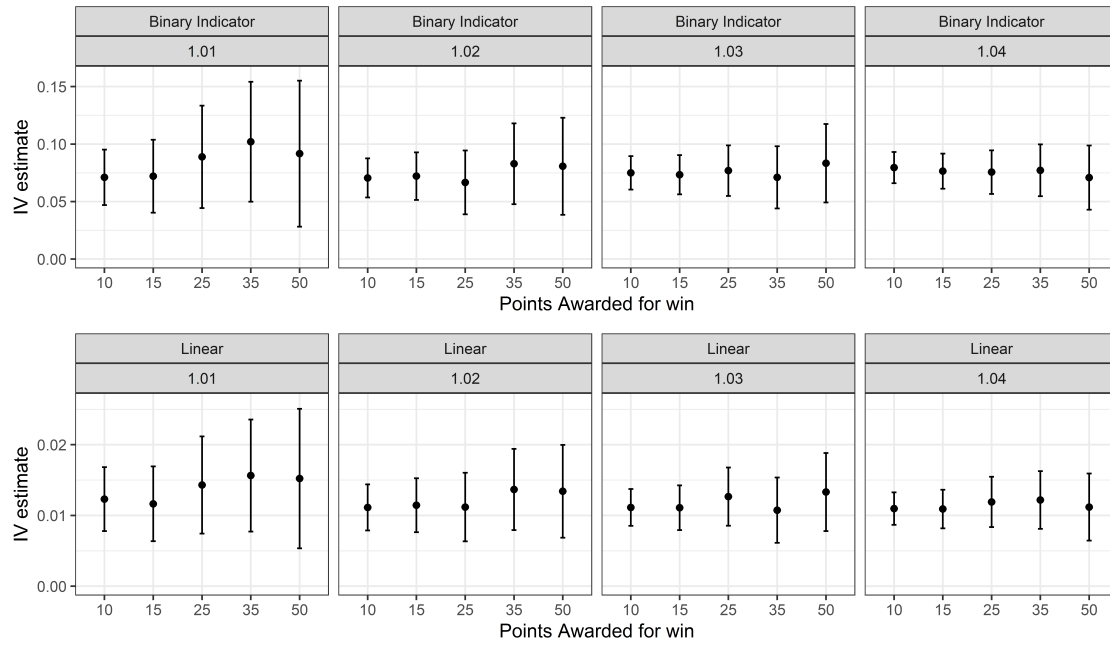


Figure 1-6: Robustness analysis for parameters in the IV-Rank strategy

Note: The plot shows the robustness analysis for the parameters of the rank instrument strategy. The first parameter sensibilized is the bandwidth allowed between the highest ranked and lowest ranked participants in the auction (secondary top box). The second parameter (in the x-axis) are the amount of points awarded for a win. The primary top box shows the type of treatment (i.e. binary or continuous).

## 2. Operational Mechanisms of Experience Improvement

Having established positive and significant treatment effects of experience on outcomes in the market for public construction projects, we seek to investigate how does experience operate in practice to produce improved outcomes in the treated firms. The objective is to provide evidence of some of the changes that might have taken place within firms and helped them achieving a higher rate of success.

We start presenting the following working hypothesis regarding the benefits of experience among firms. Each details one way in which a firm might have experienced improvements. The chapter objective is to test these hypothesis as well as possible with the data available.

First we present our hypothesis:

1. H1: experience produces improvements in cost measures in the firm, keeping constant the type of project. This improvement in cost operates either via economies of scale, since after winning the project the firm is bigger than before; or via adjustments in the production function itself, for example, by changing the relative inputs employed to produce a unit of the product.
2. H2: experience allows the firm to produce at higher quality than before, constant the cost of the works. This improvement operates because the firm, having performed certain tasks once, is able to better predict potential problems, and adapt accordingly. For our purposes, we hypothesize that the technical quality of the firm's *proposal* improves, and we assume that this is in direct correlation with executed quality.



Section 2.1 investigates the first hypothesis while Section 2.2 investigates the second. In each section the data is characterized and the empirical strategy discussed, before showing the results. Most of these elements are very similar to their previous chapter counterparts so the expositions are brief.

## 2.1 Bids and experience

This section investigates whether experience causes improvements in cost levels for treated firms. We approach this hypothesis by examining how do firm's bids evolve after the firm has been treated, i.e. after it has acquired experience. We assume that bid amounts are a non-decreasing function of bids' costs, which seems a plausible assumption.

The relationship between bids and several firms characteristics has been investigated several times in the construction and economics literature, which is discussed in the Literature Review. Previous studies have generally found aggressiveness in new entrants, but also reduced bids from incumbents. The identification strategy employed is, to our knowledge, novel.

### 2.1.1 Data

Our main dataset is the same as in the previous chapter, i.e. a set of bids submitted by firms in auctions for public construction projects. However, instead of aggregating firm's experience and outcomes in time slices, our observations are the bids themselves, so the original unit of observation (i.e. the bid) is kept. We still employ aggregation to compute previous experience at each point in time for every firm. Similarly as before, contracts where experience is employed in the awarding factors of the contract are filtered out from the analysis sample, but not from experience computation).

Furthermore, the first year in the data is excluded from the regression sample, since all firms have zero experience at this point and keeping it would introduce noise in the estimates due to spurious treatments set to zero. All the available years in the

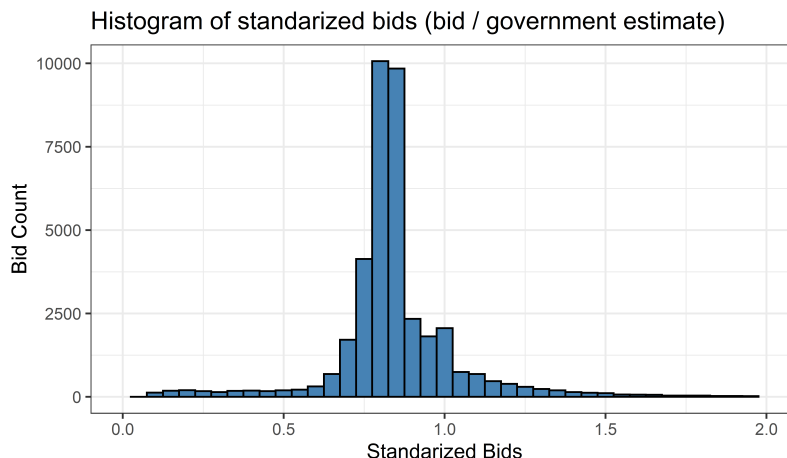


Figure 2-1: Histogram of standardized bids

data are employed to compute experience, as in the previous section.

The data includes two key variables for this section: bid amounts and a government estimate of how much the project "should" cost, called the official estimate. The estimate is prepared by the government unit in charge of the auction and usually disclosed after the auction has taken place. It is of interest for the government to produce a reasonable estimate, since if the winning bid is below a certain fraction of the official estimate, the government unit must undergo additional administrative steps to justify the awarding decision.

We produce comparable bid amounts across different contracts by dividing each bid by the corresponding government estimate, obtaining a new variable which we call standardized bid. This procedure helps to prevent some heteroskedastic effects, and also reflects that most effects in our regression are expected to act "per-dollar" unit of a contract (Bajari, Houghton, and Tadelis, 2014). We filter from the dataset standardized bids less than 0.1 and over 5.0, since they could correspond to outlier cases and not to a regular auctioning procedure or project, or could be a symptom of a very bad initial estimate from the government. This last step eliminates around 1,000 contracts. Figure 2-1 shows a histogram of standardized bid amounts (we restrict the visualization range for convenience).

Table 2.1 shows descriptive statistics of the observations employed in the analysis sample for this section. Note that there are modifications with respect to Table 2.1,

Table 2.1: Sample descriptive statistics for bid analysis

| name                                   | N     | mean     | std      | max      | min     |
|--|-------|----------|----------|----------|---------|
| Bid (all)                              | 38700 | 7.52e+08 | 6.74e+09 | 2.54e+11 | 2500000 |
| Winning Bid                            | 10100 | 4.13e+08 | 4.44e+09 | 2.47e+11 | 4940000 |
| Difference between 1st bid and 2nd (%) | 10100 | 0.0735   | 0.0956   | 0.912    | 0       |
| Number of Bidders per Contract         | 12500 | 3.2      | 2.42     | 33       | 1       |
| Year                                   | 12500 | 2015     | 2.85     | 2021     | 2011    |
| Offers made by Firm                    | 7430  | 5.21     | 9.89     | 265      | 1       |
| Win prob. by Firm                      | 7430  | 0.232    | 0.325    | 1        | 0       |
| Offers won by Firm                     | 7430  | 1.36     | 3.12     | 64       | 0       |

given by the extra filtering steps employed for this analysis. .

### 2.1.2 Empirical Strategy

Our empirical strategy relies on a regression of the form:

$$BID_{ijt} = \alpha + \beta EXP > 0_{ijt} + X_j + FIRST_{ijt} + \varepsilon_{ijt} \quad (2.1)$$

$$BID_{ijt} = \alpha + \beta EXP_{ijt} + X_j + FIRST_{ijt} + \varepsilon_{ijt} \quad (2.2)$$

Here, the outcome variable  $BID_{ijt}$  is the standardized bid submitted by firm  $i$  at time  $t$  to contract  $j$ . Our treatment variable is experience, either in binary form  $EXP > 0$  or continuous form  $EXP$ . We compute experience by summing all contracts won up to  $t$ . Each bid in our main dataset (after the filtering steps detailed above) is an observation in the regression. We add controls  $X_j$  corresponding to the region and year of the contract. Finally, we add an indicator variable  $FIRST_{ij}$  which is 1 if firm  $i$  is on its first year in the market when bidding for contract  $j$ , because from the theoretical analysis and empirical literature we expect a positive effect due to "aggressiveness" of first entrants.

Similarly as before, we expect to have unobserved cost variables, specific to each firm, which might bias estimates upwards due to positive correlation with experience. We repeat the same strategy as before to produce consistent estimates, using closely won bids to produce random variation in total experience. The setting is an IV regression where we instrument  $EXP_{it}$  with  $EXPCLOSE_{it} > 0$ , the presence of close wins by a firm up to time  $t$ . Wins are labeled as close wins if they fulfill the conditions established in the previous chapter. For brevity, we only employ rank

instruments in this section.

Our consistency strategy relies in validity and relevance assumptions. The first one requires uncorrelatedness between close wins and cost measures. The second requires that our instrument does produce variation in the independent variable. We test this assumption by developing a regression of bids won on bids closely won (by price). The regression on wins on close wins by rank shows instead an F-statistic of 814 for the binary indicator and 631 for linear experience. Finally, to interpret our indicator estimate as the LATE, we again require a monotonicity condition, which is satisfied by construction.

### **2.1.3 Results**

We show graphical results in Figure 2-2. Panel A shows standardized bids against experience, employing all bids and firms in the sample. It can be seen that the average bid for firms without experience (0.89) is higher than the average of firms with any amount of positive experience. Panel B shows only firms close win (by rank) equal to wins. Firms with one close wins (and no regular wins) submit bids that are slightly lower than those firms without experience, although firms with two close wins have bids around three percentage points lower.

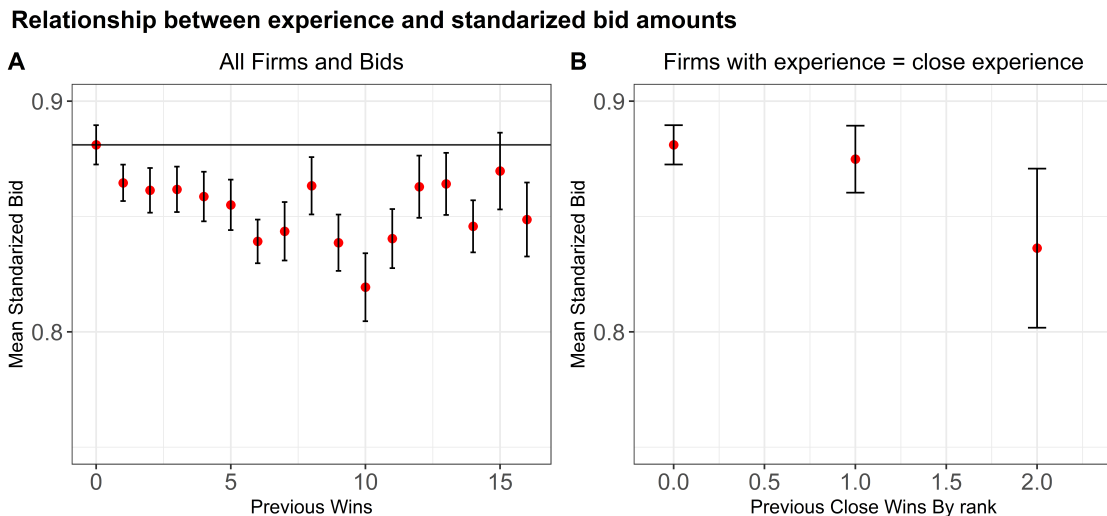


Figure 2-2: Relationship between experience and standardized bid amounts

Note: The plots show the relationship between experience and standardized bids (monetary bid amount/government's estimate of the cost). The x-axis show experience, either as total previous wins (left) or close wins(right). Range of the close wins in the right panel is cut at two.

We perform four regressions between experience and standardized bids. The first two are the OLS and IV results employing binary experience as treatment; while the third and fourth are the OLS and IV regressions employing total experience as treatment. Table 2.2 presents our main results. The OLS estimates of the effect of having experience on bid amounts is around -0.040 for OLS estimates and -0.038 for IV estimates. Although this is only around 15% of the standard deviation of the standardized bid, given that the average difference between the lowest and second lowest bid is around eight percentage points, the effect is relevant to auction outcomes.

The continuous OLS and IV estimates are small but significant. Notably, the IV estimate is twice as big as the OLS one at 0.001. We advance a possible explanation of this result based on our empirical strategy. This is further analyzed in the Discussion section.

Our main hypothesis of interest, which was that experience produces cost advantages among treated firms, seems to be substantiated by the results. Although we cannot speak with certainty about the levels of the effect, we can conclude that experience does allow firms to submit lower bids as a source of competitive advantage.

Table 2.2: Regression of bid amounts to experience

|                                    | <i>Dependent variable:</i> |                              |                        |                              |
|------------------------------------|----------------------------|------------------------------|------------------------|------------------------------|
|                                    | Standardized Bid           |                              |                        |                              |
|                                    | <i>OLS</i>                 | <i>instrumental variable</i> | <i>OLS</i>             | <i>instrumental variable</i> |
|                                    | (1)                        | (2)                          | (3)                    | (4)                          |
| Experience in (t-1) (Binary)       | −0.040***<br>(0.005)       | −0.038***<br>(0.010)         |                        |                              |
| Experience in (t-1) (Linear)       |                            |                              | −0.0005***<br>(0.0001) | −0.001***<br>(0.0002)        |
| IndFirstYear                       | −0.019***<br>(0.003)       | −0.018***<br>(0.005)         | −0.009***<br>(0.003)   | −0.013***<br>(0.004)         |
| Constant                           | 0.858***<br>(0.011)        | 0.856***<br>(0.014)          | 0.825***<br>(0.009)    | 0.828***<br>(0.010)          |
| Fixed effects By Period and Region | Yes                        | Yes                          | Yes                    | Yes                          |
| Observations                       | 38,714                     | 38,714                       | 38,714                 | 38,714                       |
| R <sup>2</sup>                     | 0.025                      | 0.025                        | 0.023                  | 0.023                        |

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

The regression analyzes the relationship between past experience (dependent var.) and standardized bid amount (indep. var.). The standardized bid amount is the monetary bid amount divided by the government estimate of the cost of the project. We include an indicator IndFirstYear for firms that are bidding on their first year in the market. Robust standard errors are on parenthesis.

Results show treatment effects implying bids at least two percentage points higher on average for firms without experience compared with firms with strictly positive experience.

## 2.2 Quality and Experience

In order to test hypothesis number two, in this section we study if experience treatments causes firms to submit higher quality proposals. We proceed by analyzing whether experienced firms have higher proposal acceptance rates in the first stage of the awarding process, in which government units in charge of the auction discard proposals that do not fulfill basic formal requirements and/or technical specifications.

Recall that, for each auction, firm proposals are analyzed in two steps. The first step examines mostly if the proposals fulfill formal requirements. Formal requirements include the inclusion of required legal documents, submitting each of the technical documents asked for in the bidding documents, etc<sup>1</sup>. In essence, the first stage verifies that all proposals can be evaluated in equal terms and that the minimum legal requirements are fulfilled. Clearly, whether a proposal was accepted is a measure of its quality, albeit an imperfect one. Although it leaves out a significant part of the variation that would be expected in proposal's qualities, it is nonetheless an interesting measure of quality because formal acceptance is a necessary condition to win a contract.

Quality is explicitly evaluated in many contracts by including an item in the awarding criteria labeled as "technical specifications" or just "quality of the proposal". Employing string pattern matching, we estimate that around 30% of contracts include some measure of technical evaluation in the awarding criteria. Ideally, we would test the hypothesis that experience improves the quality of a firm's proposals by employing the score that each firm obtained in the technical or quality item of the evaluation criteria of the project. However, since our data has not this item available by firm, we must employ this alternative strategy.

Our research design, detailed below, tests whether experienced firms have a higher formal acceptance rate than unexperienced firms at the first stage of the awarding

---

<sup>1</sup>In some units/contracts, the first step can be a time-consuming and important part of the process. For example, in contracts from the Ministry of Public Works, the first step is called "Technical Evaluation" and examines thoroughly the bids to analyze if the technical standard of the proposal is adequate.



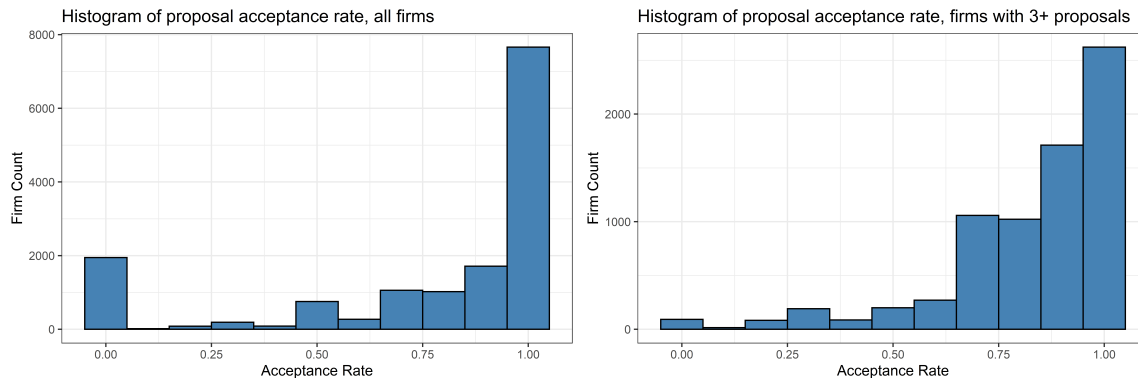


Figure 2-3: Histograms of proposal acceptance rate by firms in the dataset

process.

## 2.2.1 Data

We employ our bid dataset similarly as in the previous chapter. We create time slices exactly as detailed in Section 1.1 so we do not repeat the explanation of the full process. Each observation consists in the outcomes of a firm in period 2 of slice  $t$  and experience acquired during period 1 of the same slice  $t$ . Due to possible self-selection effects for firms with experience, contracts which include experience in the awarding factor are again we still filtered out for outcome computation.

To compute outcomes an indicator variable  $INDACC_{ijt}$  is employed, which is 1 if the proposal submitted by firm  $i$  at time  $t$  for contract  $j$  is accepted or not. The aggregated outcome is the mean of this indicator variable across the proposals submitted during the outcome period.

We show a histogram of the acceptance rates in Figure 2-3. We can already see that the fraction of firms getting all proposals rejected decreases if we consider firms with more than one proposal, which could be caused by the effect of learning about the formal revision stage after the first few bidding processes.

## 2.2.2 Empirical Strategy

We test whether experience leads to a higher rate of formal proposal acceptance employing the following regression:

$$ACCRATE_{it2} = \alpha + \beta EXP_{it1} + T_t + \varepsilon_{it} \quad (2.3)$$

Here,  $ACCRATE_{it2}$  is the share of proposals accepted out of proposals submitted in period 2 of slice  $t$ ,  $EXP_{it1}$  is the measure of experience employed for firm  $i$  in slice  $t$  (gained in period 1), and  $T_t$  are period fixed effects. We employ indexes 1 and 2 to make explicit that each slice has two periods: one of experience computation and one of outcome computation, and every slice is indexed by  $t$ , which is date in between the two periods.

To be more explicit, let  $C_{itk}$  be the set of contracts where firm  $i$  submitted a proposal at period  $k$  of slice  $t$ . Then, the outcome variable  $ACCRATE_{it2}$  can also be written as:

$$ACCRATE_{it2} = \frac{\sum_{j \in C_{it2}} INDACC_{ijt}}{|C_{it2}|}$$

We again expect unobserved cost advantages that are endogenous to experience, so we repeat our instrumentation of experience with the existence of strictly positive close wins the same as the previous chapter and section. Since we apply the same sample procedure as in the previous chapter, the same discussion and results regarding validity and rank apply.

We perform six regressions between proposal acceptance rates and experience. The first three are the OLS and IV results employing our binary treatment; and the third to sixth employ a continuous experience treatment. We employed our first alternative (rolling experience) to compute experience, i.e. we employ two year periods to compute experience and subsequent two year periods to compute outcomes.

### 2.2.3 Results

Figure 2-4 displays graphic results. Panel A displays a clear discontinuity between the mean of the acceptance indicator variable for proposals sent by firms without experience and firms with any amount of positive experience. The mean acceptance rate for firms with no experience is .68, whereas it is equal or above .79 for proposals belonging to firms with positive experience.

To be more stringent with the sample, panel B displays the same analysis but here we leave out all firms except those which have only one previous proposal (won or lost), so they are new entrants to the market which may have won or lost their first contract (we analyze their next submitted proposal). Notably, mean acceptance rates increase from .74 ( $N = 4,490$ ) for firms which lost their first auction to .87 ( $N = 1,024$ ) for firms which won their first auction.

Furthermore, we find that, for observations in the fifth quintile of acceptance rate, 40% of them correspond to firms with strictly positive experience. On the other side, only 26% of the observations in the first quintile of acceptance come from firms with no experience (at the point of observation, since a firm can be in both quintiles at different points in time).

Panels C and D show the mean acceptance rate against close experience as per the instrument level. We consider only firms having equal experience to close experience. In Panel C, the instrument is close experience by price and in D the instrument is close experience by rank. In both panels, we see an increase in the mean acceptance rate, although the sample is so reduced in panel C that we obtain very big standard errors.

Our regression results are shown in Table 2.3. The first three panels show the results for binary experience as treatment and the last three the treatment is total experience. We find positive and significant treatment effects of experience on outcomes: having positive experience results in almost 11 percentage points higher mean acceptance rates in future proposals (next two years). This means that having experience increases acceptance rates in around a third of a standard deviation of the outcome variable (.33). The IV results are close to OLS estimates (between one and two percentage points higher), but the price instrument specification has high standard errors.

Regarding the treatment effect per unit of experience, we find that each new contract performed increases mean acceptance rates by around 1.2 percentage points. Again, the IV results are higher than the OLS results for the two alternative instruments.

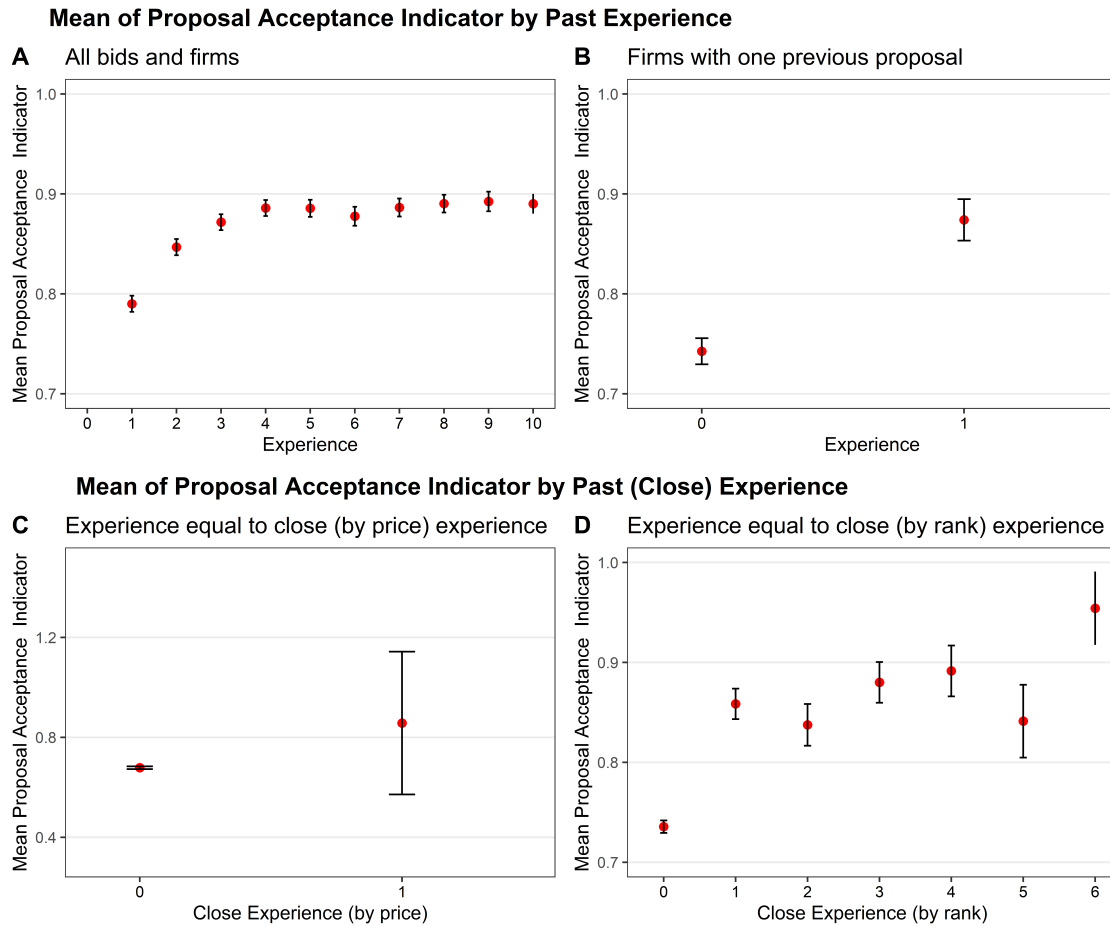


Figure 2-4: Acceptance rate for proposals sent by firms to auctions for public construction projects.

Table 2.3: Regression of proposal acceptance on experience

|                                  | <i>Dependent variable:</i> |                      |                     |                     |                      |                     |
|----------------------------------|----------------------------|----------------------|---------------------|---------------------|----------------------|---------------------|
|                                  | Proposal Acceptance Rate   |                      |                     |                     |                      |                     |
|                                  | OLS<br>(1)                 | IV (by price)<br>(2) | IV (by rank)<br>(3) | OLS<br>(4)          | IV (by price)<br>(5) | IV (by rank)<br>(6) |
| Experience in (t-1) > 0 (Binary) | 0.105***<br>(0.004)        | 0.127***<br>(0.014)  | 0.111***<br>(0.006) |                     |                      |                     |
| Experience in (t-1) (Continuous) |                            |                      |                     | 0.012***<br>(0.001) | 0.014***<br>(0.002)  | 0.023***<br>(0.001) |
| Constant                         | 0.787***<br>(0.007)        | 0.779***<br>(0.009)  | 0.787***<br>(0.007) | 0.811***<br>(0.007) | 0.809***<br>(0.007)  | 0.799***<br>(0.007) |
| Fixed effects By Period          | Yes                        | Yes                  | Yes                 | Yes                 | Yes                  | Yes                 |
| Observations                     | 20,948                     | 20,948               | 13,559              | 20,948              | 20,948               | 13,559              |
| R <sup>2</sup>                   | 0.025                      | 0.024                | 0.024               | 0.013               | 0.013                | -0.001              |

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\*p<0.01

Results of a Regression of proposal acceptance on experience. The independent variable is contracts won in the previous two years(rolling experience). The dependent variable is the rate of proposal acceptance over the next two years, i.e. (proposals accepted by the government unit in charge of the auction) / (proposals sent by the firm) . The proposals are accepted or rejected at the first stage of the awarding decision, where formal and/or technical requirements are checked.

# A. Appendix

## A.1 Rank Regressions

The following regressions have as dependent variable experience and as independent variable close experience, which is our instrument in the main results section. Table A.2 shows the results for the first measure of experience (rolling), while Table A.2 shows the results for the second measure of experience (annualized cumulative).

Table A.1: Regressions for rank condition verification

|                              | <i>Dependent variable:</i> |                              |                            |                              |
|------------------------------|----------------------------|------------------------------|----------------------------|------------------------------|
|                              | Rolling Experience > 0     |                              | Rolling Experience         |                              |
|                              | OLS<br>(1)                 | IV (Price)<br>(2)            | IV (Rank)<br>(3)           | OLS<br>(4)                   |
| Close Experience > 0 (Price) | 0.619***<br>(0.004)        |                              |                            |                              |
| Close Experience (Price)     |                            |                              | 5.187***<br>(0.231)        |                              |
| Close Experience > 0 (Rank)  |                            | 0.828***<br>(0.004)          |                            |                              |
| Close Experience (Rank)      |                            |                              |                            | 1.825***<br>(0.027)          |
| Constant                     | 0.345***<br>(0.009)        | 0.145***<br>(0.006)          | 0.986***<br>(0.045)        | 0.241***<br>(0.032)          |
| Fixed effects By period      | Yes                        | Yes                          | Yes                        | Yes                          |
| Observations                 | 20,948                     | 16,072                       | 20,948                     | 16,072                       |
| R <sup>2</sup>               | 0.048                      | 0.573                        | 0.117                      | 0.475                        |
| F Statistic                  | 118.209*** (df = 9; 20938) | 3,085.328*** (df = 7; 16064) | 307.129*** (df = 9; 20938) | 2,073.281*** (df = 7; 16064) |
| <i>Note:</i>                 |                            |                              |                            | *p<0.1; **p<0.05; ***p<0.01  |

Table A.2: Regressions for rank condition verification

|                              | <i>Dependent variable:</i>              |                             |   |                            |
|------------------------------|---|-----------------------------|---|----------------------------|
|                              | Annualized Experience > 0<br>OLS<br>(1) | IV (Price)<br>(2)           | Annualized Experience<br>IV (Rank)<br>(3) | OLS<br>(4)                 |
| Close Experience > 0 (Price) | 0.578***<br>(0.004)                     |                             |   |                            |
| Close Experience > 0 (Rank)  |   | 0.457***<br>(0.006)         |   | 1.202***<br>(0.021)        |
| Close Experience (Price)     |   |                             | 5.022***<br>(0.255)                       |                            |
| Close Experience (Rank)      | 0.264***<br>(0.009)                     | 0.421***<br>(0.013)         | 0.457***<br>(0.024)                       | 0.706***<br>(0.039)        |
| Fixed effects By period      | Yes                                     | Yes                         | Yes                                       | Yes                        |
| Observations                 | 21,705                                  | 12,327                      | 21,705                                    | 12,327                     |
| R <sup>2</sup>               | 0.085                                   | 0.297                       | 0.120                                     | 0.229                      |
| F Statistic                  | 223.339*** (df = 9; 21695)              | 579.188*** (df = 9; 12317)  | 327.225*** (df = 9; 21695)                | 405.665*** (df = 9; 12317) |
| <i>Note:</i>                 |   | *p<0.1; **p<0.05; ***p<0.01 |   |                            |

## A.2 Additional Sample Characterization

This section expands on the sample characterization of the Data chapter. Regional and market participants descriptions are included in the form of tables or graphs.

### A.2.1 Regional Representation

Table A.3: Sample coverage by Region

| Region                                    | Contracts | Bids (N) | Bids/Contract | Unique Gov. Units | Unique Firms |
|---|-----------|----------|---------------|-------------------|--------------|
| Arica y Parinacota                        | 830       | 2600     | 3.1           | 37                | 410          |
| Tarapacá                                  | 1400      | 3900     | 2.8           | 39                | 640          |
| Antofagasta                               | 1400      | 4300     | 3.0           | 59                | 830          |
| Atacama                                   | 1300      | 3500     | 2.8           | 47                | 710          |
| Coquimbo                                  | 2600      | 8400     | 3.2           | 64                | 1400         |
| Valparaíso                                | 5100      | 15000    | 2.9           | 130               | 2400         |
| Metropolitana de Santiago                 | 8600      | 30000    | 3.5           | 260               | 4500         |
| Libertador General Bernardo O'Higgins     | 3500      | 10000    | 2.9           | 69                | 1600         |
| Maule                                     | 4100      | 14000    | 3.4           | 78                | 1600         |
| Ñuble                                     | 570       | 2100     | 3.7           | 44                | 530          |
| Biobío                                    | 7000      | 22000    | 3.2           | 150               | 2900         |
| Araucanía                                 | 3900      | 12000    | 3.1           | 88                | 1800         |
| Los Ríos                                  | 2100      | 5700     | 2.8           | 52                | 1100         |
| Los Lagos                                 | 4000      | 10000    | 2.5           | 90                | 1600         |
| Aysén del General Carlos Ibáñez del Campo | 1400      | 3700     | 2.6           | 120               | 850          |
| Magallanes y de la Antártica              | 1600      | 4300     | 2.7           | 49                | 520          |
| NA  | 5         | 17       | 3.4           | 2                 | 16           |

*Note:* The table displays key variables for the data sample available for each geographic region in Chile. Note that Ñuble was created as a separate political division in 2018. Previously, it was a part of the Biobio Región, and its contracts were labeled as such.



## A.2.2 Top Market Participants

Table A.4: Top Gov.Units

| Government Unit  | Contracts Held | Bids  | Firms Related |
|--|----------------|-------|---------------|
| Ministerio De Obras Publicas Direc Cion Gral De Oo Pp Dcyf | 2780           | 11850 | 706           |
| Junta Nacional De Jardines Infantiles                      | 1334           | 5953  | 1169          |
| Dirección De Obras Hidráulicas - Moptt                     | 510            | 1986  | 318           |
| I.municipalidad De Concepcion                              | 441            | 1604  | 344           |
| Ilustre Municipalidad De Osorno                            | 405            | 1115  | 264           |
| Municipalidad De Temuco                                    | 403            | 1328  | 413           |
| Direccion De Logistica De Carabineros                      | 394            | 974   | 381           |
| Serviu Vii Región  | 382            | 1581  | 133           |
| Ilustre Municipalidad De Rancagua                          | 356            | 1088  | 251           |
| Municipalidad De Ovalle                                    | 338            | 1502  | 340           |

Table A.5: Top Firms

| Firm Name                              | Contracts Won | Bids | Gov. Units Related |
|--|---------------|------|--------------------|
| Bitumix S.a.                           | 640           | 140  | 97                 |
| Constructora Santa Sofia               | 260           | 120  | 48                 |
| Empresa Constructora Ecmovial Limitada | 320           | 120  | 53                 |
| Ingecom                                | 250           | 120  | 45                 |
| Patricia López Romero                  | 170           | 100  | 21                 |
| Idr Chile Spa                          | 410           | 93   | 120                |
| Constructora Beltrán Y Venegas Ltda.   | 230           | 91   | 69                 |
| Trinjos                                | 160           | 91   | 16                 |
| Constructora Salfa (punta Arenas)      | 260           | 84   | 21                 |
| Ingetalk                               | 200           | 74   | 31                 |

### A.3 Bids and Winning Probability

This section explores the relationship between bids and winning probability. We develop two simple regressions of the form:

$$INDWIN_{ij} = \alpha + BID_{ij} + FIRSTYEAR_{ij} + X_j\varepsilon_{ij} \quad (A.1)$$

Where *INDWIN* is an indicator for winning the firm *i* submitting a winning bid for contract *j*, *BID* is a standardized bid, and *X<sub>j</sub>* are controls for the contract. In the first specification, *X<sub>j</sub>* are just fixed effects for Year and Region. In the second specification, we employ per-contract Fixed Effects, which means that we identify the parameter of interest through the variation in bids and outcomes for the participants in the same auction. The second specification omits the intercept as well.

Our sample consists in contracts for which i) there were two or more valid participants and ii) had a winner chosen. This renders 29,608 observations. The regressions are at the firm-bid level.

Results are shown in Table A.6. Notably, the results imply that a three percentage point decrease in the standardized bid is correlated with an increase of ten percentage points in winning probability. Given that previous discussions showed that price is an awarding component in most contracts, there seems to be enough basis to establish a causal nexus between submitting lower bids and winning contracts, although there is still much uncertainty about the actual magnitude. For similar example, probably all firms' measures of quality are probably correlated, so we cannot attribute the win to smaller bids alone.

Table A.6

|                                    | <i>Dependent variable:</i>  |
|------------------------------------|-----------------------------|
|                                    | Indicator of Contract Won   |
| Standarized Bid                    | −0.318***<br>(0.012)        |
| Number of firms in Auction         | −0.034***<br>(0.001)        |
| IndFirstYear                       | −0.065***<br>(0.006)        |
| Constant                           | 0.681***<br>(0.025)         |
| Fixed effects By Period and Region | Yes                         |
| Fixed effects By Contract          | No                          |
| Observations                       | 29,608                      |
| R <sup>2</sup>                     | 0.072                       |
| Residual Std. Error                | 0.416 (df = 29579)          |
| <i>Note:</i>                       | *p<0.1; **p<0.05; ***p<0.01 |

## A.4 ELO algorithm : theory and implementation

### A.4.1 Introduction

<sup>1</sup> The Elo Ranking is a system to place players in a numerical rating scale, in which differences among players can be converted into scoring or winning probabilities. Similarly, scoring percentages (over time or competitions) can be converted into ranking differences. Ratings in the Elo system are points in a scale which, for historical reasons, has been chosen to have its midpoint at 2000. However, the importance is on difference of ratings rather than in absolute number, since a Elo ranking is only valid within a specific pool of players.

The first basic principle of the Elo system is that performances from an individual are normally distributed, when evaluated on an appropriate scale. Let the expectancy score be the expected number of points that a player is expected to win, out of the total possible, in a match or matches. According to Elo, the percentage expectancy score for a player is a function of the difference in rating with the opponent. For example, a player one standard deviation in ranking above another has an approximate percentage expectancy score. This follows from a standard computation of the probability that a draw from a random variable of mean  $\mu_1$  and standard deviation  $\sigma$  is higher than a random variable of mean  $\mu_2$  and standard deviation  $\sigma$  as well.

The performance of a player is evaluated on the basis of how do his or her actual performance compare to the expected score, both of his own and his opponents. The *Performance Rating* is a measure of performance evaluated over a number of encounters which combines the average rating of the competition and the percent score achieved. A more stable measure over time is the *Player rating* which ought to vary less than the performance rating. In the context of chess, where it was developed, a performance rating would be obtained for a chess tournament, while the rating of the player would be his overall ranking. The player rating can be updated periodically through Performance Ratings as follows.

Over intervals, rankings for players can be calculated using the Performance Rating formula:

$$R_p = R_c + D_p \tag{A.2}$$

---

<sup>1</sup>This section is based on Arpad Elo's book introduction to the Elo system in (Elo, 1978)

Where  $R_p$  is the performance rating,  $R_c$  is the average rating and  $D_p$  is the difference based on the percentage score  $P$ , which is obtained from the curve or table. This formula can be employed to update rankings over periods of time. However, to maintain a continuous ranking (i.e. at every point in time), the following formula is used:

$$R_n = R_0 + K(W - W_e) \quad (\text{A.3})$$

Where  $R_n$  is the rating after the event,  $R_0$  is the rating pre-event,  $K$  is the rating point value of a single game score,  $W$  is the actual game score, and  $W_e$  is the expected game score based on  $R_0$ . The parameter  $K$  adjusts the relative weight of newer and older performances. A higher  $K$  gives a higher weight to newer performances, and vice versa.

### A.4.2 Current Implementation

The algorithm employed in the current investigation is a modified Elo system suited for multiplayer games with variable player numbers. The implementation used is contained the function *elom* of the R package *PlayerRatings*, which implements several types of ranking algorithms (Stephenson and Sonas., 2020). The adaptation of the canonical Elo algorithm to allow for a variable number of (multiple) players requires to i) implement a computation of expected scores that considers the wider range of competitors, and ii) change the way points are awarded and subtracted depending on the actual number of players for a game.

For algorithmic purposes, we consider a win as being awarded a contract and a lose as bidding for but not winning a specific contract. All losing players are considered "tied" in their loss, so base points subtracted are the same. However, actual points subtracted might differ depending on the expected score, as shown in Equation ??.

The initial rating of a player was chosen to be 1,500. Also, since an adaptation time is needed to construct a reliable average rank, for all the analysis employing the ranking measure the first year of the data is discarded (after constructing the ranks). The documentation of the implemented algorithm recommends to make it so the net number of awarded points after a game is zero. Although keeping this recommendation and keeping the same number of points for every contest constant is impossible, we choose awarded and subtracted points such that this is true for the average contract<sup>2</sup>.

---

<sup>2</sup>It would be possible the assign scores based on number of players in the contest. However, for

To construct the ranking, the following computation steps were performed:

1. The data was filtered to contain only contracts for which there were two or more opponents and which had a winner contractor. This renders approx. 29,000 contracts.
2. The relevant competition datasets were constructed. In these datasets, every observation is an auction with two or more players. Columns are participants. The columns contain the names of the participants in the auction. A similar dataset contains the outcomes for each player participating.
3. The ranking algorithm was run with the match history as the key input.

Results consists in a ranking for every firm at every contract in the sample constructed in point 2. For interpretation purposes, rankings for contracts filtered out from the main sample are filled by i) getting the ranking of the closest past contract which performed a ranking update or ii) imputing the base rank, for contractors who did not have any contracts with ranking updates. The latter can happen if a contractor bids only in contracts where he is the only bidder <sup>3</sup>.

Later, in the main estimation step, the rankings are employed to identify close wins, as described in the empirical strategy section. Note that the algorithm itself employs the assumption that, for (two) equally ranked opponents, winning chances are 50 - 50. Wins against close opponents are thus attributed in both schemes mainly to random factors.

### A.4.3 Analysis of algorithm results

In this section we describe some of the results to validate and interpret the rankings obtained. Figure A-1 shows the progression of ranks over the history of firms. Every panel is the distribution of firms at a point in their bidding history. For example, panel two shows the distribution of ranks between firms who are bidding for the second time. Note that ranks split in two after the first bidding by the win or a lose. Successive biddings "fill in" the gaps. Given that points awarded for wins are more than points subtracted by loses, the distributions are right-skewed.

---

parsimony and ease of validation purposes we take the approach to fix constant the scores.

<sup>3</sup>Note that for instrument construction, any contract with an imputed ranking cannot, by definition, be labeled as a close win, since this requires two or more players and a winner as a necessary condition

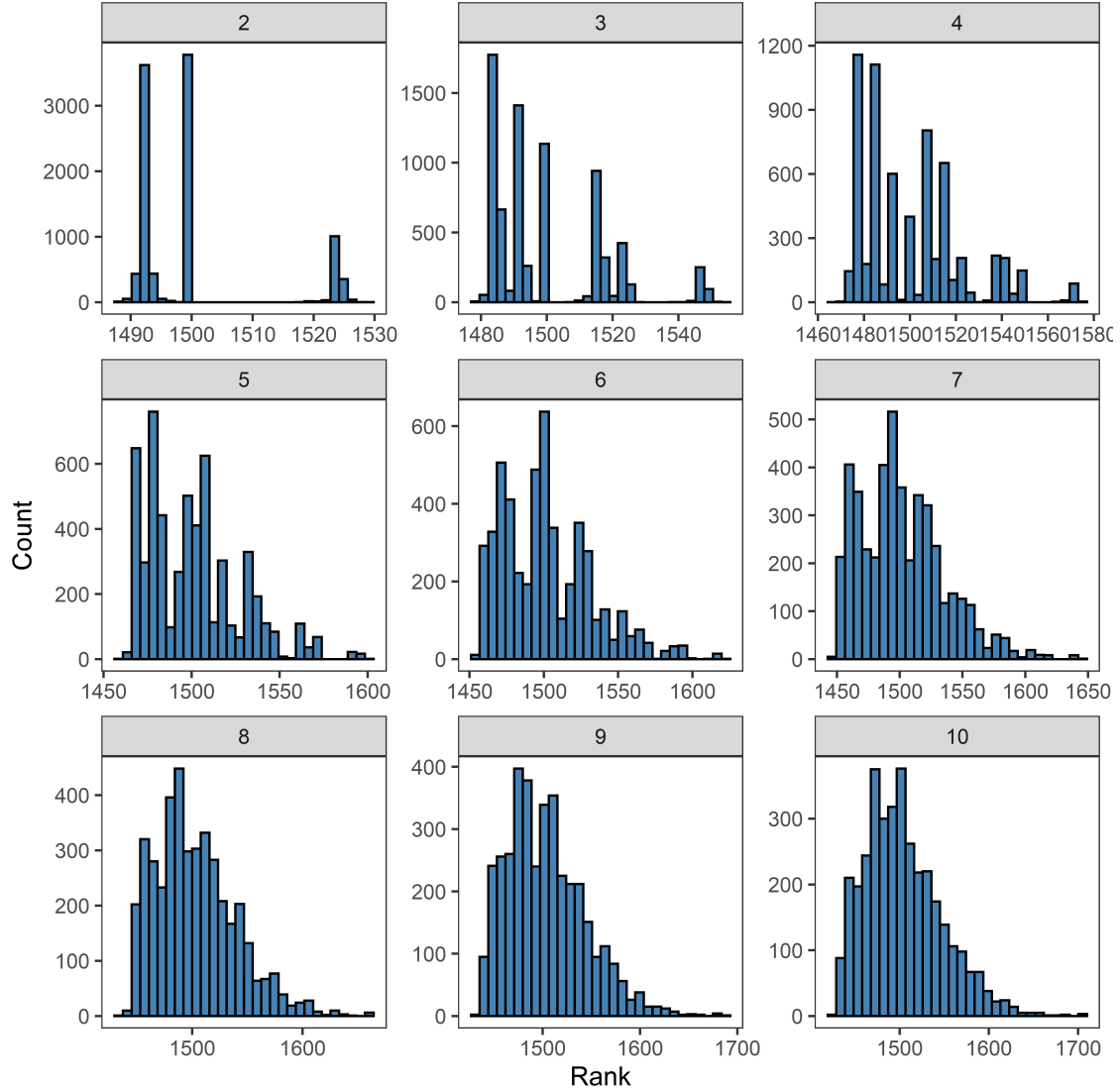


Figure A-1: Rank progression over firm's bidding history

Note: each panel contains the firms bidding for their  $i$ -th contract. The graph only displays the ranks for the first 10 bidding events. Ranks displayed are ranks *previous* to the contract, i.e. they are not adjusted by the outcome of the  $i$ -th bidding event.

The progression of rankings over time is further illustrated with two firms which end higher and lower respectively in ranking than their starting points at 1,500. We call the first firm "A" and the second firm "B". The next tables show the contracts participated in, the ranking at each contract, the average ranking of opponents, the result of the bidding (i.e. win or lose) and the net effect of the "game" of the firm's ranking (measured in points). Table A.7 shows a firm which won all but its last contracts. While all winning contracts resulted in points added to ranking, the first wins rendered more points, as the advantage

of the firm was lower. Table A.8 shows firm B, which mostly experienced defeats. Note that points subtracted by losing are less than the ones granted for winning, so the ranking still remains close to 1,500. Also, this firm faced similar opponents, as measured by the similar average ranks.

Table A.7: Evolution of Firm A

| Date       | Participants | Firm | Rank | Win | Opponent Mean Rank | Points Adjustment |
|------------|--------------|------|------|-----|--------------------|-------------------|
| 2010-04-07 | 8            | A    | 1500 | 1   | 1496               | 23.88             |
| 2010-09-21 | 15           | A    | 1524 | 1   | 1532               | 23.85             |
| 2010-09-21 | 15           | A    | 1548 | 1   | 1555               | 23.75             |
| 2011-08-11 | 10           | A    | 1571 | 1   | 1555               | 22.69             |
| 2011-08-11 | 11           | A    | 1594 | 1   | 1601               | 22.78             |
| 2011-08-11 | 11           | A    | 1617 | 1   | 1624               | 22.63             |
| 2011-08-11 | 9            | A    | 1640 | 1   | 1570               | 22.07             |
| 2011-08-11 | 9            | A    | 1662 | 1   | 1581               | 21.80             |
| 2012-09-13 | 4            | A    | 1683 | 0   | 1693               | NA                |

Table A.8: Evolution of Firm B

| Date       | Participants | Firm | Rank | Win | Opponent Mean Rank | Points Adjustment |
|------------|--------------|------|------|-----|--------------------|-------------------|
| 2016-11-29 | 11           | B    | 1500 | 0   | 1482               | 0.00              |
| 2016-11-29 | 11           | B    | 1500 | 0   | 1478               | -15.05            |
| 2017-02-23 | 11           | B    | 1485 | 0   | 1485               | -7.62             |
| 2017-06-08 | 7            | B    | 1477 | 1   | 1477               | 24.42             |
| 2017-07-05 | 8            | B    | 1502 | 1   | 1503               | 24.05             |
| 2017-07-18 | 4            | B    | 1526 | 0   | 1526               | -8.59             |
| 2017-12-20 | 8            | B    | 1517 | 0   | 1519               | -8.44             |
| 2018-08-24 | 11           | B    | 1509 | 0   | 1509               | NA                |



## A. Bibliography

- Angrist, Joshua and Guido Imbens (1995). “Identification and estimation of local average treatment effects”. In.
- Bajari, Patrick, Stephanie Houghton, and Steven Tadelis (2014). “Bidding for incomplete contracts: An empirical analysis of adaptation costs”. In: *American Economic Review* 104.4, pp. 1288–1319.
- Elo, Arpad E (1978). *The rating of chessplayers, past and present*. Arco Pub.
- Hansen, Bruce E (2009). *Econometrics*.
- Stephenson, Alec and Jeff Sonas. (2020). *PlayerRatings: Dynamic Updating Methods for Player Ratings Estimation*. R package version 1.1-0. URL: <https://CRAN.R-project.org/package=PlayerRatings>.