Extracting Landscape Utility from Zebra Distributions

For Dan Rubenstein et al.

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Consider that zebras move across the landscape in a stochastic way, influenced by both short- and long-term environmental pressures, and guided by the prospect of water, grasses and plants, and scared away by predators. We can imagine that zebras are small statistically mechanical particles in a system equipped with a (conservative, i.e. path-independent) force that tends to move these zebra-particles towards certain areas of good grazing, drinking, and protection, and away areas of high-risk of predation, intraspecific competition for resources, and disruption from cattle.

This conservative force F has an associated potential energy E, namely,

$$-\nabla E = F$$

Let's assume that the zebras follow a Boltzmann distribution according to this energy:

$$\mathbb{P}(\text{zebra at } (x, y)) \propto e^{-E(x,y)/\beta}$$

For β a constant of the system. If there are a fixed number of zebras on the landscape at any given point in time (which there are), then we can write:

$$P(x, y) := \#\text{ of zebras at } (x, y) = \alpha e^{-E(x,y)/\beta}$$

For α and β constants of the system. Simple algebra reorders this expression to:

$$E = -\beta \ln(P/\alpha) = \beta \ln \alpha - \beta \ln P$$

Consider now that the higher energy an area is, the lower utility it is for the zebras (by the definition of the force from which E is derived). Consequently, we know that utility $U \sim -E$ is a monotonic (strictly) increasing relationship. If we let U range from 0 to 1, then an appropriate definition is:

$$U(x, y) = \frac{\max_{(x,y)} E(x, y) - E(x, y)}{\max_{(x,y)} E(x, y) - \min_{(x,y)} E(x, y)} = \frac{\text{Max} - E}{\Delta}$$

From here, we can generate a relationship between population and utility, as we do below:

$$U = \frac{\beta}{\Delta} \ln P + (\text{Max} - \beta \ln \alpha) = \eta \ln P + \gamma$$

So now we have a monotonic strictly increasing function $U(P)$. However, there are two parameters (η and γ that we need to examine before we can use this relationship. To understand this, we will use the edge conditions on U and E. When P is maximal, we want U to be just below 1, and when P is minimal (but > 0), we want U to be just above 0. This is using the intuition for energy and population that we established above. So,

$$\frac{0.99 - \gamma}{\eta} = \ln \text{Max} \text{ and } \frac{0.01 - \gamma}{\eta} = \ln \text{Min}$$

This is a system of equations in γ and η, which is enough to derive their values:

$$\gamma = \frac{0.01 \ln \text{Max} - 0.99 \ln \text{Min}}{\ln \Big(\text{Max}/\text{Min}\Big)} \text{ and } \eta = \frac{0.98}{\ln \Big(\text{Max}/\text{Min}\Big)}$$

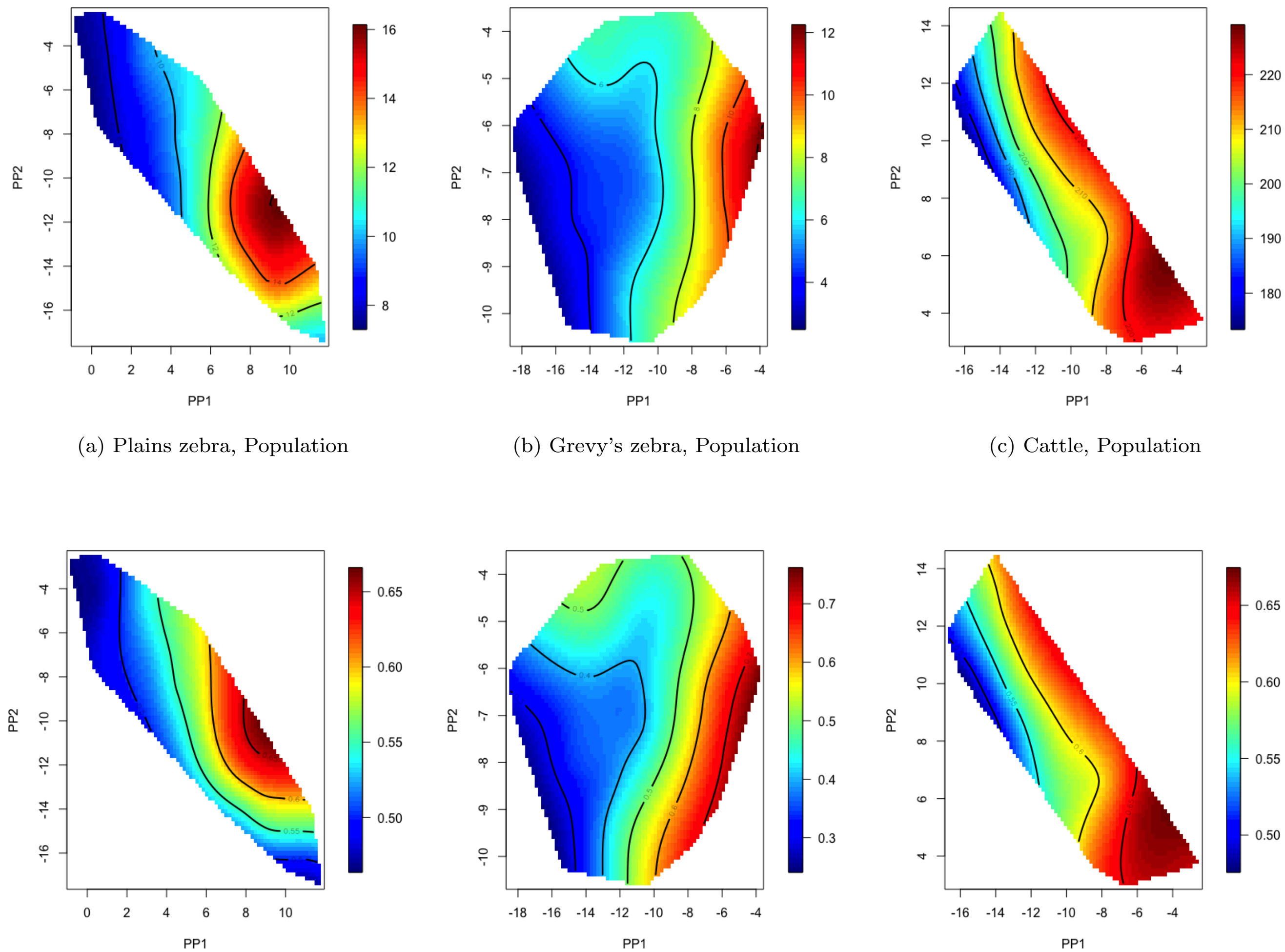
Which gives us a final solution of:

$$U(x, y) = \frac{0.98 \ln P(x, y) + \ln\left(\text{Max}^{0.01}/\text{Min}^{0.99}\right)}{\ln\left(\text{Max}/\text{Min}\right)}$$

We can use this utility as a fitness metric and produce an adaptive (i.e. fitness) landscape from it. The following figures and tables show how this change alters the results from PPR and the shape of the fitness landscape in general.

It is perhaps not a surprise that the overall topography of the fitness landscape does not change when we use utility instead of population as a fitness metric (Fig. 1), since utility and population are related by monotonically (strictly) increasing functions, and so the only differences are likely to be in the steepness of different faces of the landscape. However, it is worth noting that PPR will select a different plane into which we project all of the data points, and as such we should not expect the fitness landscape to look identical.

One interesting result from testing this model is that the PPR loading score (Table 1) for plains zebras using utility as fitness was significantly lower than the score using population as fitness. This seems to suggest that using potential energy or conservative forces is much less accurate for plains zebras that we would expect. On the other hand, the loading score for Grevy's zebras increased marginally. Assuming this is a relevant change, this suggests that the Boltzmann model suits Grevy's zebras. These results can be seen in the Fig. 2: histograms of dazzle and mob populations. While the Grevy's dazzles appear almost exponential, the plains dazzles demonstrate an up-down histogram, which shows a peak to the right of where we would expect. The cattle show a completely non-exponential histogram, so it is entirely expected that the Boltzmann assumption causes the loading score to drop dramatically when using utility as a fitness metric instead of population.



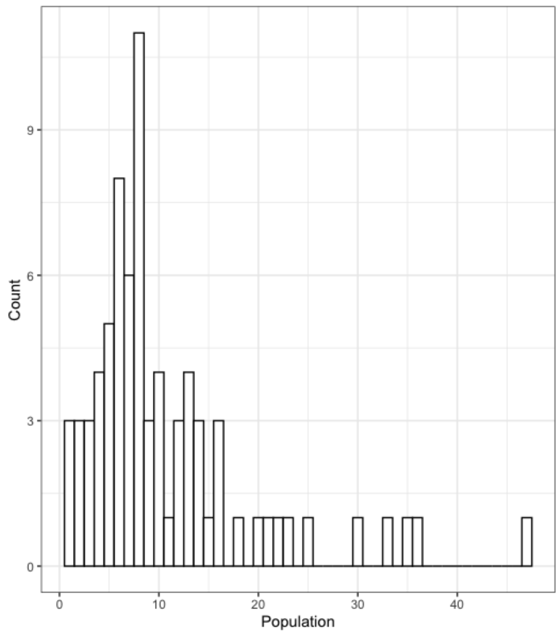
(a) Plains zebra, Population (b) Grevy's zebra, Population (c) Cattle, Population

(d) Plains zebra, Utility (e) Grevy's zebra, Utility (f) Cattle, Utility

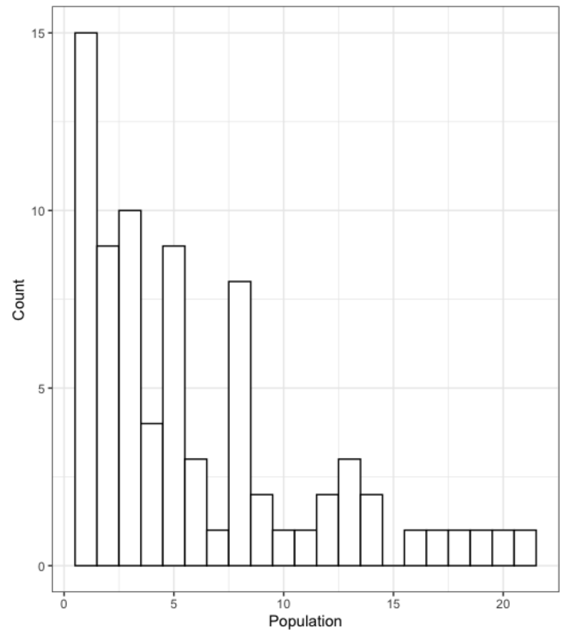
Figure 1: Figures to show the fitness landscapes of plains zebras, Grevy's zebras, and cattle using utility and population as fitness metrics. Notice that the topography of each pair of images are alike. This is due to the monotonic (strictly) increasing relationship between utility and population.

Species	Goodness of fit	β_1	β_2	PP1 main variable (weight)	PP2 main variable (weight)
Fitness = Population					
GZ	17.295	0.7175160	0.6208459	Hour (0.5189)	Other.species.1 (0.4931)
PZ	19.53792	0.6341586	0.6707800	Grass.height (0.7263)	Grass.color (0.5467)
Cattle	4.409146	0.9043666	0.4638345	Number.grasses (0.5836)	Sun (0.4549)
Fitness = Utility					
GZ	18.13077	0.7974306	0.3352370	Grass.spp.4 (0.5093)	Number.grasses (0.5672)
PZ	15.26962	0.8502065	0.5008751	Grass.spp.4 (0.6056)	Grass.spp.4 (0.6529)
Cattle	1.305095	0.8673195	0.3988726	Grass.spp.4 (0.5688)	Bush.type (0.4911)

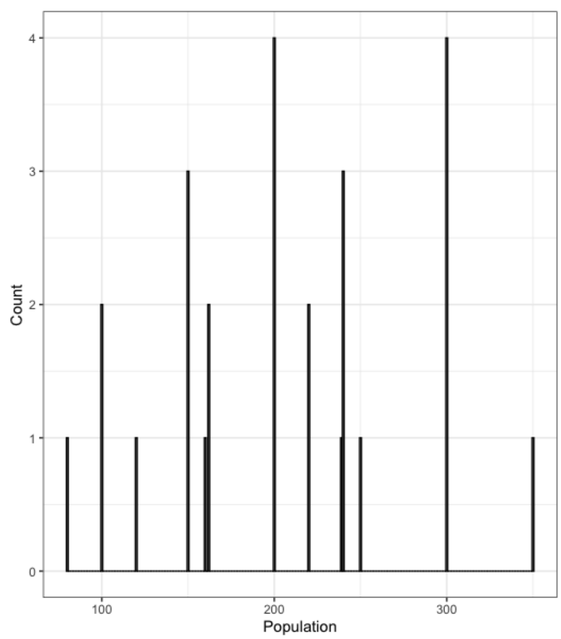
Table 1: Table to show projection pursuit regression (PPR) on plains zebras, Grevy's zebras, and cattle using utility and population as fitness metrics.



(a) Plains zebra



(b) Grevy's zebra



(c) Cattle

Figure 2: Histograms of dazzle and mob populations.