# Part II

# Supplement for "Optimal Convergence Rates, Bahadur Representation, and Asymptotic Normality of Partitioning Estimators"

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#### Abstract

This is a supplemental appendix for "Optimal Convergence Rates, Bahadur Representation, and Asymptotic Normality of Partitioning Estimators". We first present detailed proofs of all the theoretical results from the main text. Lemmas and other claims are restated before proof, and as such this section may replace the appendix contained in the main manuscript. Note that equation numbers may change. Next, for the special case of the piecewise constant partitioning estimator we characterize the leading terms of an *unconditional* integrated mean-square error expansion. The result agrees with Theorem 3, specialized to the same case. Finally, numerous additional simulation results are presented, vastly expanding the discussion in Section 5.

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## A Proof of Theorems

Let C denote a generic positive constant that may take different values in different places. If specific constants are needed they will be numbered consecutively. For scalars, vectors, or matrixes, let  $|\cdot|$  be the Euclidean norm. To denote the various products, we use  $\times$  for Cartesian,  $\otimes$  for Kronecker, and  $\prod$  for usual multiplication over arguments: any may be repeated, as in  $\times_{\ell=1}^d$ . Any use of the "times" symbol  $\times$  will be clear from the context; examples include matrix dimensions and line breaks in displayed equations. Matrix inequalities are understood to be in the positive definite sense. Consecutive uses of the symbol  $\times$  are to be interpreted pairwise. All results below hold for the partitioning schemes described in the text. For a generic cell  $P_j$ , let  $p_{j*}$ ,  $\overline{p}_j$ , and  $p_j^*$  be the vectors in  $\mathbb{R}^d$  giving the start, mid-point, and end of the cell, respectively, where the start and end are defined in distance to the origin. For a multi-index k, we define the additional notation:  $k! = k_1! \cdots k_d!$ ,  $k \leq \tilde{k} \Leftrightarrow k_1 \leq \tilde{k}_1, \dots, k_d \leq \tilde{k}_d$ , and  $\sum_{[k] \leq K} \sum_{L=0}^K \sum_{[k]=L}$  for  $K \geq 0$ .

Prior to proving the main results, it is convenient to take a nonsingular linear transformation of the polynomial basis. The estimator  $\hat{\mu}(x)$  is invariant to such rotations, thus without loss of generality we may take the basis to be centered at the midpoint of each cell and scaled by the length of the cell. Observe that centering the polynomial basis around the center of each cell avoids issues of differentiability at the boundary of each cell and the support  $\mathcal{X}$ . Recall that R(x) is ordered ascendingly in  $k \in \mathbb{Z}_+^d$  and  $\ell = 1, \ldots, d$ . Define the one-to-one function  $g(k) : \mathbb{Z}_+^d \to \mathbb{N}$  that gives the index position of R(x) corresponding to entry  $x^k$ . Let  $g^* = \max_k \{g(k) : k \in \mathbb{Z}_+^d, [k] \le K - 1\}$ . Then R(x) is a  $g^* \times 1$  vector with element g(k) equal to  $x^k$  for all  $\{k \in \mathbb{Z}_+^d : [k] \le K - 1\}$ . As R(x) excludes terms with degree exceeding K - 1, it follows that  $g^* \le K^d$ . To fix ideas, consider the two simple cases from the text: if K = 1, then (for any d) R(x) = (1) and hence  $K^d = g^* = 1$ ; if K = 2 and d = 2 say, then  $R(x) = (1, x_1, x_2)'$  and  $K^d = 4$ ,  $g^* = 3$ .

Recall from the text that the interval endpoints  $p_{\ell,j-1}$  and  $p_{\ell,j}$ , for  $j=1,\ldots,J_n$ , define the partition of the  $\ell$ -dimension of  $\mathcal{X}$ , and let  $\overline{p}_{\ell,j}=(p_{\ell,j}+p_{\ell,j-1})/2\in\mathbb{R}$  be the midpoint of each interval. Define the matrix functions D(a) to be the  $K\times K$  diagonal matrix with entries given by  $a^{-(v-1)}$ ,  $v=1,\ldots,K$  and L(b) to be the  $K\times K$  lower triangular matrix with typical element  $\binom{u-1}{v-1}(-b)^{u-v}$ ,  $(u,v)\in\{1,\ldots,K:u\geq v\}$ . We then take the (rotated) polynomial basis to be given by

$$\tilde{R}_{j}(x) \equiv \mathbb{I}_{P_{j}}(x)\tilde{R}(x) = \mathbb{I}_{P_{j}}(x)S_{K}\bigotimes_{\ell=1}^{d} \left\{ D\left(p_{\ell,j} - \overline{p}_{\ell,j}\right)L(\overline{p}_{\ell,j})r(x_{\ell}) \right\}.$$

Each element of the product  $L(\bar{p}_{\ell,j})r(x_{\ell})$  is (the binomial expansion of)  $(x_{\ell}-\bar{p}_{\ell,j})^{k_{\ell}}$ ,  $0 \le k_{\ell} \le K-1$ , and premultiplication by  $D\left(p_{\ell,j}-\bar{p}_{\ell,j}\right)$  rescales appropriately. To be explicit, for the  $\ell$ -dimension,

with  $\underline{p}_{\ell,j} = 1/(p_{\ell,j} - \overline{p}_{\ell,j})$  the product  $D\left(p_{\ell,j} - \overline{p}_{\ell,j}\right) L(\overline{p}_{\ell,j}) r(x_{\ell})$  is given by:

$$\begin{pmatrix} 1 & & & \\ & \underline{p}_{\ell,j} & & & \\ & & \underline{p}_{\ell,j}^2 & & \\ & & \ddots & \\ & & & \underline{p}_{\ell,j}^{K-1} \end{pmatrix} \begin{pmatrix} 1 & & & \\ & \overline{p}_{\ell,j}^2 & -2\overline{p}_{\ell,j} & 1 & \\ & \vdots & & \\ & & (-\overline{p}_{\ell,j})^{K-1} & \dots & -(K-1)\overline{p}_{\ell,j} & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x_r \\ x_r^2 \\ \vdots \\ x_r^{K-1} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & \\ & \underline{p}_{\ell,j} & & \\ & & \underline{p}_{\ell,j}^2 & & \\ & & & \ddots & \\ & & & \underline{p}_{\ell,j}^{K-1} \end{pmatrix} \begin{pmatrix} \sum_{k_\ell=0}^0 \binom{0}{k_\ell} (-\overline{p}_{\ell,j})^{k_\ell} x_r^{0-k_\ell} \\ \sum_{k_\ell=0}^1 \binom{1}{k_\ell} (-\overline{p}_{\ell,j})^{k_\ell} x_r^{1-k_\ell} \\ \sum_{k_\ell=0}^2 \binom{2}{k_\ell} (-\overline{p}_{\ell,j})^{k_\ell} x_r^{K-1-k_\ell} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & \\ & \underline{p}_{\ell,j} & & & \\ & & \underline{p}_{\ell,j}^{K-1} & & \\ & & \ddots & & \\ & & & \underline{p}_{\ell,j}^{K-1} \end{pmatrix} \begin{pmatrix} 1 & & & \\ (x_r - \overline{p}_{\ell,j}) & & & \\ (x_r - \overline{p}_{\ell,j})^2 & & & \\ \vdots & & & \vdots & \\ (x_r - \overline{p}_{\ell,j})^{K-1} \end{pmatrix} = r \begin{pmatrix} \underline{x}_\ell - \overline{p}_{\ell,j} \\ \underline{p}_{\ell,j} - \overline{p}_{\ell,j} \end{pmatrix}.$$

The matrix  $S_K$  is a  $g^* \times K^d$  selection matrix which removes terms of degree exceeding K-1, with the properties that

$$S_K S_K' = I_{g^*} \quad \text{ and } \quad S_K' S_K = \begin{bmatrix} I_{g^*} & Z_{g^*,K^d-g^*} \\ Z_{K^d-g^*,g^*} & Z_{K^d-g^*,K^d-g^*} \end{bmatrix},$$

where  $Z_{z_1,z_2}$  is a  $z_1 \times z_2$  matrix of zeros.

To see that this is equivalent to a transformation of the full basis, define  $T_{\ell,j} = D\left(p_{\ell,j} - \overline{p}_{\ell,j}\right) L(\overline{p}_{\ell,j})$  and observe that

$$\left[S_K\left(T_{1,j}\otimes\cdots\otimes T_{d,j}\right)S_K'\right]\tilde{R}_j(x) = \mathbb{I}_{P_j}(x)S_K\left(T_{1,j}\otimes\cdots\otimes T_{d,j}\right)S_K'S_K\left(r(x_1)\otimes\cdots\otimes r(x_d)\right)$$
$$= \mathbb{I}_{P_j}(x)S_K\left(T_{1,j}r(x_1)\otimes\cdots\otimes T_{d,j}r(x_d)\right),$$

as required, relying upon R(x) being ordered ascendingly in  $k \in \mathbb{Z}_+^d$ .

Finally let  $\tilde{R}_j = (\tilde{R}_j(X_1), \dots, \tilde{R}_j(X_n))'$  and (globally) redefine  $\Omega_j = \mathbb{E}\left[\tilde{R}_j(X)\tilde{R}_j(X)'\right]/q_j$  and  $\hat{\Omega}_j = \tilde{R}_j'\tilde{R}_j/(nq_j)$ , maintaining the same notation for the latter two for simplicity.

#### A.1 Preliminary Lemmas

Several intermediate lemmas are required before proving the main results. These lemmas establish properties of partitioning estimators which may be of independent interest for other applications.

**Lemma A.1.** Under Assumption 1(b), for  $s \leq K-1$  the polynomial basis satisfies:

$$\max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \left\| \partial^m \tilde{R}_j(\cdot) \right\|_{\infty} = O\left(J_n^s\right).$$

Proof. The proof below can be read for either d=1 or d>1. The difference notational: in the d=1 case the indexes k and m are treated as simple integers, rather than multi-indexes (and [k] is read as simply k), whereas for the d>1 case the multi-index notation is maintained. Recall the definitions of the vectors  $\bar{p}_j$  and  $p_j^*$  above. By construction of the partition, for  $x \in P_j$ ,  $\left|x-\bar{p}_j\right| \leq \left|p_j^*-\bar{p}_j\right| \asymp J_n^{-1}$ . Following rotation, each element of the basis is of the form  $\frac{(x-\bar{p}_j)^k}{(p_j^*-\bar{p}_j)^k}$  for some  $k \in \mathbb{Z}_+^d$ . Hence for fixed  $x \in \mathcal{X}$  and a multi-index m such that  $[m] \leq K-1$ , the norm (squared) of  $\partial^m \tilde{R}_j(x)$  is the sum of squares over all such elements with  $[k] \leq K-1$ , restricted to the cell  $P_j$ :

$$\begin{split} \left| \partial^m \tilde{R}_j(x) \right|^2 &= \mathbb{I}_{P_j}(x) \sum_{[k] \leq K-1} \left\{ \partial^m \frac{(x - \overline{p}_j)^k}{(p_j^* - \overline{p}_j)^k} \right\}^2 \\ &= \mathbb{I}_{P_j}(x) \sum_{[k] \leq K-1} \mathbb{I}\{m \leq k\} \left\{ \frac{k!}{(k - m)!} \frac{(x - \overline{p}_j)^{k - m}}{(p_j^* - \overline{p}_j)^k} \right\}^2 \\ &= \left( \frac{1}{(p_j^* - \overline{p}_j)^m} \right)^2 \mathbb{I}_{P_j}(x) \sum_{[k] \leq K-1} \mathbb{I}\{m \leq k\} \left\{ \frac{k!}{(k - m)!} \frac{(x - \overline{p}_j)^{k - m}}{(p_j^* - \overline{p}_j)^{k - m}} \right\}^2 \\ &\leq C \left( \frac{1}{(p_j^* - \overline{p}_j)^m} \right)^2 = O\left(J_n^{2[m]}\right), \end{split}$$

uniformly in  $1 \leq j \leq J_n^d$ ,  $x \in P_j$ , and  $\{m : [m] \leq K - 1\}$ , and in particular for those satisfying  $[m] \leq s \leq K - 1$ , for any such s.

**Lemma A.2.** Define  $\mu_j(x) \equiv \mathbb{I}_{P_j}(x)\mu(x)$ , and following the definition in Eqn. (2),  $\partial^m \mu_j(x) \equiv \mathbb{I}_{P_j}(x)\partial^m \mu(x)$ . Under Assumptions 1(b) and 1(e), there is a non-random vector  $\beta_j^0$ , depending only on K and j, such that for  $s \leq S \wedge (K-1)$ :

$$\max_{1 \le j \le J_n^d} \max_{m: [m] \le s} \left\| \partial^m \mu_j(\cdot) - \partial^m \tilde{R}_j(\cdot)' \beta_j^0 \right\|_{\infty} = O\left(J_n^{-((S+\alpha) \wedge K - s)}\right).$$

*Proof.* The proof consists of two steps. The first is to show that  $\partial^m \mu_j(x)$  admits a Taylor series ap-

proximation with remainder of the appropriate order. Second, we show that  $\beta_j^0$  may be constructed so that  $\partial^m \tilde{R}_j(x)' \beta_j^0$  is that Taylor series. Critical to this is that differentiation only operates on the basis, not the non-random vector  $\beta_j^0$ . We first present complete details for d=1, to keep the notation simple. The extension to higher dimensions follows along the same lines and a more terse proof is given.

Take d=1. For the first step of the proof we derive the order of the remainder using the integral representation. By Assumption 1(e),  $\partial^m \mu_j(x)$  admits a Taylor series approximation of order  $S \wedge (K-1) - m$  (at least). To save notation, let  $\tilde{\mathbf{s}} = S \wedge (K-1)$ . For  $x \in P_j$ , the remainder is given by:

$$\begin{split} \left| \partial^m \mu_j(x) - \sum_{k=0}^{\tilde{\mathbf{s}}-m} \frac{\partial^{k+m} \mu_j(\overline{p}_j)}{k!} (x - \overline{p}_j)^k \right| \\ &= \left| \partial^m \mu_j(x) - \sum_{k=0}^{\tilde{\mathbf{s}}-m-1} \frac{\partial^{k+m} \mu_j(\overline{p}_j)}{k!} (x - \overline{p}_j)^k - \frac{\partial^{\tilde{\mathbf{s}}} \mu_j(\overline{p}_j)}{(\tilde{\mathbf{s}}-m)!} (x - \overline{p}_j)^{(\tilde{\mathbf{s}}-m)} \right| \\ &= \left| \frac{1}{(\tilde{\mathbf{s}}-m-1)!} \int_{\overline{p}_j}^x [\partial^{\tilde{\mathbf{s}}} \mu_j(z)] (x - z)^{(\tilde{\mathbf{s}}-m-1)} dz - \frac{\partial^{\tilde{\mathbf{s}}} \mu_j(\overline{p}_j)}{(\tilde{\mathbf{s}}-m)!} (x - \overline{p}_j)^{(\tilde{\mathbf{s}}-m)} \right| \\ &= \left| \frac{1}{(\tilde{\mathbf{s}}-m-1)!} \int_{\overline{p}_j}^x [\partial^{\tilde{\mathbf{s}}} \mu_j(t)] (x - z)^{(\tilde{\mathbf{s}}-m-1)} dz - \frac{1}{(\tilde{\mathbf{s}}-m-1)!} [\partial^{\tilde{\mathbf{s}}} \mu_j(\overline{p}_j)] \int_{\overline{p}_j}^x (x - z)^{(\tilde{\mathbf{s}}-m-1)} dz \right| \\ &= \frac{1}{(\tilde{\mathbf{s}}-m-1)!} \left| \int_{\overline{p}_j}^x (x - z)^{(\tilde{\mathbf{s}}-m-1)} \left[ \partial^{\tilde{\mathbf{s}}} \mu_j(z) - \partial^{\tilde{\mathbf{s}}} \mu_j(\overline{p}_j) \right] dz \right|. \end{split}$$

For notational purposes, define  $\tilde{\alpha} = \alpha \mathbb{I}\{K \geq S+1\} + (1)\mathbb{I}\{K < S+1\}$ . Under Assumption 1(e), there is a constant  $C_1$  depending only on  $\mu(\cdot)$  and  $\tilde{\mathbf{s}}$  such that  $\left|\partial^{\tilde{\mathbf{s}}}\mu_j(z) - \partial^{\tilde{\mathbf{s}}}\mu_j(\bar{p}_j)\right| \leq C_1 \left|z - \bar{p}_j\right|^{\tilde{\alpha}}$ , for all j. The notation of  $\tilde{\alpha}$  is introduced because if K < S+1, then  $\partial^{\tilde{\mathbf{s}}}\mu_j(z)$  is Lipschitz continuous (i.e. Hölder continuous with  $\alpha = 1$ ). Hence the above display is:

$$\leq \frac{C_1}{(\widetilde{\mathbf{s}}-m-1)!} \int_{\overline{p}_j}^x |x-z|^{(\widetilde{\mathbf{s}}-m-1)} |z-\overline{p}_j|^{\widetilde{\alpha}} dz,$$

which by the construction of the partition and the range of integration is:

$$\begin{split} & \leq \frac{C_1}{(\tilde{\mathbf{s}} - m - 1)!} \left| x - \overline{p}_j \right|^{\tilde{\mathbf{s}} - m + \tilde{\alpha}} \\ & \leq \frac{C_1}{(\tilde{\mathbf{s}} - m - 1)!} \left( \frac{p_{J_n}^* - p_{1*}}{J_n} \right)^{\tilde{\mathbf{s}} - m + \tilde{\alpha}}. \end{split}$$

This bound is uniform in  $x \in P_j$  and  $1 \le j \le J_n$ . The difference  $(p_{J_n}^* - p_{1*})$  represents the length

of the support  $\mathcal{X}$ , which under Assumption 1(b) is a bounded constant. Hence, as m appears only in the denominator and the exponent:

$$\begin{aligned} \max_{1 \leq j \leq J_n^d} \max_{1 \leq m \leq s} \left| \partial^m \mu_j(x) - \sum_{k=0}^{\tilde{\mathbf{s}} - m} \frac{\partial^{k+m} \mu_j(\overline{p}_j)}{k!} (x - \overline{p}_j)^k \right| &\leq \max_{1 \leq j \leq J_n^d} \max_{1 \leq m \leq s} \frac{C_1}{(\tilde{\mathbf{s}} - m - 1)!} \left( \frac{p_{J_n}^* - p_{1*}}{J_n} \right)^{\tilde{\mathbf{s}} - m + \tilde{\alpha}} \\ &= O\left(J_n^{-((S + \alpha) \wedge K - s)}\right). \end{aligned}$$

To complete the proof for d=1, we now construct  $\beta_j^0$  such that  $\partial^m \tilde{R}_j(x)' \beta_j^0$  is the Taylor series approximation for  $\partial^m \mu(x)$ . Differentiation operates only on the vector  $\tilde{R}_j(x)$ . The first m-1 entries of  $\partial^m \tilde{R}_j(x)$  are zero. Thus, element k of  $\partial^m \tilde{R}_j(x)$  is given by

$$\mathbb{I}\{m \le k\} \frac{k!}{(k-m)!} \frac{(x-\overline{p}_j)^{k-m}}{(p_j^* - \overline{p}_j)^k}.$$

Define the coefficient vector  $\beta_i^0$  with entry k equal to

$$\frac{1}{k!} [\partial^k \mu_j(\overline{p}_j)] (p_j^* - \overline{p}_j)^k.$$

Then we have

$$\partial^{m} \tilde{R}_{j}(x)' \beta_{j}^{0} = \sum_{k=0}^{S \wedge (K-1)} \mathbb{I}\{m \leq k\} \frac{k!}{(k-m)!} \frac{(x-\overline{p}_{j})^{k-m}}{(p_{j}^{*}-\overline{p}_{j})^{k}} \frac{1}{k!} [\partial^{k} \mu_{j}(\overline{p}_{j})] (p_{j}^{*}-\overline{p}_{j})^{k}$$

$$= \sum_{k>m}^{S \wedge (K-1)} \frac{1}{(k-m)!} (x-\overline{p}_{j})^{k-m} [\partial^{k} \mu_{j}(\overline{p}_{j})],$$

and re-indexing the sum by changing variables using  $\tilde{k} = k - m$  this is equal to

$$=\sum_{\tilde{k}=0}^{S\wedge (K-1)-m}\frac{\partial^{\tilde{k}+m}\mu_j(\overline{p}_j)}{\tilde{k}!}(x-\overline{p}_j)^{\tilde{k}}.$$

This expression now exactly matches the Taylor approximation of  $\partial^m \mu_j(x)$  given above. This completes the proof for d=1.

Now consider any  $d \geq 1$ . Just as above, Assumption 1(e) implies that  $\partial^m \mu_j(x)$  satisfies the Taylor expansion for  $x \in P_j$  given by:

$$\partial^{m} \mu_{j}(x) = \sum_{[k] \leq S \wedge (K-1) - [m]} \frac{1}{k!} \left( \partial^{k+m} \mu_{j}(\overline{p}_{j}) \right) \left( x - \overline{p}_{j} \right)^{k} + O\left( \left| x - \overline{p}_{j} \right|^{(S+\alpha) \wedge K - [m]} \right), \tag{A.1}$$

with constants which can be made uniform in the multi-index m, s, and j. The terms of the summation are assumed to be ordered ascendingly in g(k) as defined above. It remains to construct  $\beta_j^0$  appropriately so that  $\partial^m \tilde{R}_j(x)' \beta_j^0$  is the Taylor approximation given as the first term on the right hand side of (A.1). Recall the multi-index notational conventions defined earlier. For fixed  $m \in \mathbb{Z}_+^d$ ,  $[m] \leq s$ , any entry of  $\partial^m \tilde{R}_j(x)$  with  $k \leq m$  is zero. Thus, entry g(k) of  $\partial^m \tilde{R}_j(x)$  is given by:

$$\mathbb{I}\{m \le k\} \frac{k!}{(k-m)!} \frac{(x-\overline{p}_j)^{k-m}}{(p_j^* - \overline{p}_j)^k}.$$

Next, for  $k \in \mathbb{Z}_+^d$  define the function  $\beta_i^0(k)$  as:

$$\beta_j^0(k) = \frac{1}{k!} \left( \partial^k \mu_j(\overline{p}_j) \right) \left( p_j^* - \overline{p}_j \right)^k.$$

As g(k) is one-to-one and returns the index position of the entry corresponding to multi-index k, we can define the coefficient vector  $\beta_j^0$  as the  $g^* \times 1$  vector with entry e equal to  $\beta_j^0(g^{-1}(e))$ , for all entries  $e = 1, \ldots, g^*$ , where we note that  $g^{-1}(e) \in \mathbb{Z}_+^d$  is a multi-index valued function. Therefore:

$$\partial^{m} \tilde{R}_{j}(x)' \beta_{j}^{0} = \sum_{[k] \leq S \wedge (K-1)} \mathbb{1}\{m \leq k\} \frac{k!}{(k-m)!} \frac{(x-\overline{p}_{j})^{k-m}}{(p_{j}^{*}-\overline{p}_{j})^{k}} \frac{1}{k!} \left(\partial^{k} \mu_{j}(\overline{p}_{j})\right) \left(p_{j}^{*}-\overline{p}_{j}\right)^{k}$$

$$= \sum_{[k] \leq S \wedge (K-1)} \mathbb{1}\{m \leq k\} \frac{1}{(k-m)!} \left(x-\overline{p}_{j}\right)^{k-m} \partial^{k} \mu_{j}(\overline{p}_{j}).$$

By definition, the multi-index satisfies  $[k + \tilde{k}] = [k] + [\tilde{k}]$ , and so re-indexing the above sum by changing variables  $\tilde{k} = k - m$ , we obtain

$$\partial^m \tilde{R}_j(x)' \beta_j^0 = \sum_{[\tilde{k}+m] < S \wedge (K-1)} \frac{1}{\tilde{k}!} \left( \partial^{\tilde{k}+m} \mu_j(\overline{p}_j) \right) \left( x - \overline{p}_j \right)^{\tilde{k}}.$$

This matches the Taylor series, hence subtracting from Eqn. (A.1) gives:

$$\max_{1 \le j \le J_n^d} \max_{m:[m] \le s} \left\| \partial^m \mu_j(x) - \partial^m \tilde{R}_j(x)' \beta_j^0 \right\|_{\infty} = O\left( \max_{1 \le j \le J_n^d} \max_{m:[m] \le s} \sup_{x \in P_j} \left| x - \overline{p}_j \right|^{(S+\alpha) \wedge K - [m]} \right)$$

$$= O\left( J_n^{-((S+\alpha) \wedge K - [s])} \right),$$

completing the proof.

**Lemma A.3.** Recall that  $q_j = \mathbb{P}[X \in P_j]$  and  $\Omega_j = \mathbb{E}\left[\tilde{R}_j(X)\tilde{R}_j(X)'\right]/q_j$ . Under Assumption 1,  $\Omega_j \times I_{g^*}$ , the identity matrix, uniformly in j.

*Proof.* By Assumption 1(d) and the construction of the partition,  $q_j = \int_{P_i} f(x) dx \approx C \int_{P_i} dx =$ 

 $C \operatorname{vol}(P_j) \simeq J_n^{-d}$ . Applying this result and Assumption 1(d) again, we have:

$$\Omega_j = \frac{1}{q_j} \int_{\mathcal{X}} \tilde{R}_j(x) \tilde{R}_j(x)' f(x) dx \approx J_n^d \int_{\mathcal{X}} \tilde{R}_j(x) \tilde{R}_j(x)' f(x) dx \approx J_n^d \int_{\mathcal{X}} \tilde{R}_j(x) \tilde{R}_j(x)' dx.$$

Now, by Assumption 1(b), properties of the Kronecker product, and the construction of the transformed basis,

$$\Omega_{j} \simeq J_{n}^{d} S_{K} \int_{P_{j}} \left\{ \bigotimes_{\ell=1}^{d} r \left( \frac{x_{\ell} - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}} \right) r \left( \frac{x_{\ell} - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}} \right)' \right\} dx S_{K}' 
\simeq J_{n}^{d} S_{K} \bigotimes_{\ell=1}^{d} \left\{ \int_{p_{\ell,j-1}}^{p_{\ell,j}} r \left( \frac{x_{\ell} - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}} \right) r \left( \frac{x_{\ell} - \overline{p}_{\ell,j}}{p_{\ell,j} - \overline{p}_{\ell,j}} \right)' dx_{\ell} \right\} S_{K}'.$$

A change of variables using  $z=(x_{\ell}-\overline{p}_{\ell,j})/(p_{\ell,j}-\overline{p}_{\ell,j})$ , followed by the fact that  $|p_{\ell,j}-p_{\ell,j-1}| \approx J_n^{-1}$  shows that:

$$\Omega_{j} \simeq J_{n}^{d} \left( \prod_{\ell=1}^{d} \left| p_{\ell,j} - \overline{p}_{\ell,j} \right| \right) S_{K} \left\{ \bigotimes_{\ell=1}^{d} \int_{-1}^{1} r(z) r(z)' dz \right\} S_{K}'$$
$$\simeq S_{K} \left\{ \bigotimes_{\ell=1}^{d} \int_{-1}^{1} r(z) r(z)' dz \right\} S_{K}',$$

For the change of variables t = (z+1)/2 we have  $\int_{-1}^{1} z^k dz = 2 \int_{0}^{1} (2t-1)^k dt$ . Applying this change of variables to the entire basis is equivalent to the inversion of the centering and scaling performed by the matrixes  $L(\cdot)$  and  $D(\cdot)$  defined earlier. Therefore:

$$\int_{-1}^{1} r(z)r(z)'dz = \int_{0}^{1} \left[D(2)L(-1)\right]^{-1} r(t)r(t)' \left[L(-1)D(2)\right]^{-1} dt = \left[D(2)L(-1)\right]^{-1} H \left[L(-1)D(2)\right]^{-1},$$

where H denotes the Hilbert matrix of order K, which is positive definite. Collecting these results, where consecutive uses of the symbol  $\approx$  are interpreted pairwise, we have:

$$\Omega_j \simeq S_K \left\{ \bigotimes_{\ell=1}^d \left[ D(2)L(-1) \right]^{-1} H \left[ L(-1)D(2) \right]^{-1} \right\} S_K' \simeq I_{g^*}.$$

**Lemma A.4.** Let  $a_n = n^{-1}J_n^d \log(J_n^d)$ , and recall  $\hat{\Omega}_j = \tilde{R}_j' \tilde{R}_j/(nq_j)$ . Under Assumption 1 and the rate restriction of Theorem 1:  $\max_{1 \leq j \leq J_n^d} |\hat{\Omega}_j - \Omega_j|^2 = O_p(a_n)$ . If, in addition,  $J_n^d \approx (n/\log(n))^{\gamma}$ ,  $\gamma \in (0,1)$ , the same is true almost surely.

*Proof.* For  $k, \tilde{k} \in \mathbb{Z}_+^d$  with  $[k], [\tilde{k}] \leq K - 1$ , let the  $(g(k), g(\tilde{k}))$  element of  $(\hat{\Omega}_j - \Omega_j)$  be denoted

 $\sum_{i=1}^{n} W_{ij}(k, \tilde{k})/(nq_j)$ , where

$$W_{ij}(k,\tilde{k}) = \left[\tilde{R}_j(X_i)\tilde{R}_j(X_i)'\right]_{g(k),g(\tilde{k})} - \left[\mathbb{E}\left[\tilde{R}_j(X_i)\tilde{R}_j(X_i)'\right]\right]_{g(k),g(\tilde{k})}.$$

By Lemma A.1 (taking s=0) and the triangle inequality,  $\left|W_{ij}(k,\tilde{k})\right| \leq 2\|\tilde{R}_{j}(\cdot)\|_{\infty}^{2} < C$  and  $\mathbb{E}\left[W_{ij}(k,\tilde{k})^{2}\right] \leq C\|\tilde{R}_{j}(\cdot)\|_{\infty}^{4}\mathbb{E}\left[\mathbb{I}_{P_{j}}(X)\right] \leq Cq_{j}$ , for any  $k,\tilde{k}$ . Thus by Boole's inequality, K being fixed, Bernstein's inequality, and finally applying  $q_{j} \times J_{n}^{-d}$  and canceling where possible:

$$\mathbb{P}\left[\max_{1\leq j\leq J_n^d} \left| \hat{\Omega}_j - \Omega_j \right| > (a_n)^{1/2} \varepsilon\right] \leq J_n^d \max_{1\leq j\leq J_n^d} \mathbb{P}\left[\left| \hat{\Omega}_j - \Omega_j \right| > (a_n)^{1/2} \varepsilon\right] \\
\leq C J_n^d \max_{1\leq j\leq J_n^d [k], [\tilde{k}]\leq K-1} \mathbb{P}\left[\left| \sum_{i=1}^n W_{ij}(k,\tilde{k}) \right| > q_j \sqrt{n J_n^d \log(J_n^d)} \varepsilon\right] \\
\leq C J_n^d \max_{1\leq j\leq J_n^d [k], [\tilde{k}]\leq K-1} \exp\left\{ -C \frac{q_j^2 n J_n^d \log(J_n^d) \varepsilon^2}{n q_j + q_j \sqrt{n J_n^d \log(J_n^d)} \varepsilon} \right\} \\
\leq C \exp\left\{ \log(J_n^d) \left[ 1 - C \frac{\varepsilon^2}{1 + \sqrt{a_n} \varepsilon} \right] \right\},$$

which is arbitrarily small for  $\varepsilon$  large enough by the rate restriction of Theorem 1.

When  $J_n^d \simeq (n/\log(n))^{\gamma}$ , we use the above bound to write:

$$\sum_{n=1}^{\infty} \mathbb{P}\left[\max_{1 \leq j \leq J_n^d} \left| \hat{\Omega}_j - \Omega_j \right| > (a_n)^{1/2} \varepsilon\right] \leq \sum_{n=1}^{\infty} C\left(\frac{n}{\log(n)}\right)^{\gamma - C\gamma \varepsilon^2/(1 + \sqrt{a_n}\varepsilon)} < \infty,$$

where summability is ensured by choosing  $\varepsilon$  large enough and  $a_n \to 0$  by the rate restriction in Theorem 1. The conclusion follows by the Borel-Cantelli Lemma.

**Lemma A.5.** Let the conditions of Theorem 2 hold, and for  $\xi$  therein let  $r_n^2 = n^{-1}J_n^{d(2-\xi)}\log(J_n^d)^{\xi}$ . Then for  $G = (\mu(X_1), \dots, \mu(X_n))'$ , we have  $\max_{1 \leq j \leq J_n^d} |\tilde{R}'_j(Y-G)/(nq_j)| = O_p(r_n)$ . If, in addition,  $J_n^d \asymp (n/\log(n))^{\gamma}$ ,  $\gamma \in (0,1)$ , the same is true almost surely.

*Proof.* With the convention 0/0 = 0, define  $t_n = J_n^{d\xi/\eta} \log(J_n^d)^{-\xi/\eta}$ . Following the same notation as in Lemma A.4, let

$$H_{ij}(k) = \mathbb{I}_{P_j}(X_i) \left[ \tilde{R}_j(X_i) \right]_{g(k)} (Y_i \mathbb{I} \{ Y_i \le t_n \} - \mathbb{E} \left[ Y_i \mathbb{I} \{ Y_i \le t_n \} | X_i \right]),$$

$$T_{ij}(k) = \mathbb{I}_{P_j}(X_i) \left[ \tilde{R}_j(X_i) \right]_{g(k)} (Y_i \mathbb{I} \{ Y_i > t_n \} - \mathbb{E} \left[ Y_i \mathbb{I} \{ Y_i > t_n \} | X_i \right]).$$

For the truncated term, since  $|H_{ij}(k)| \leq t_n$  by construction and  $\mathbb{E}\left[H_{ij}(k)^2\right] \leq Cq_j$ , applying

Bernstein's inequality and  $q_j \simeq J_n^{-d}$ , we find that for fixed  $k \in \mathbb{Z}_+^d$ :

$$J_n^d \max_{1 \le j \le J_n^d} \mathbb{P}\left[\left|\sum_{i=1}^n H_{ij}(k)\right| > nq_j r_n \varepsilon\right] \le C J_n^d \max_{1 \le j \le J_n^d} \exp\left\{-C \frac{(nq_j r_n \varepsilon)^2}{nq_j + t_n nq_j r_n \varepsilon}\right\}$$

$$\le C \exp\left\{\log(J_n^d) \left[1 - C \frac{nr_n^2 (J_n^d \log(J_n^d))^{-1} \varepsilon^2}{1 + t_n r_n \varepsilon}\right]\right\}.$$

By  $\xi \in [0, 1]$  and the rate restriction of the Theorem, the above probability can be made arbitrarily small for  $\varepsilon$  large enough, as:

$$\frac{n}{J_n^d \log(J_n^d)} r_n^2 = \frac{J_n^{d(1-\xi)}}{\log(J_n^d)^{1-\xi}} \ge 1, \quad \text{and,} \quad \frac{t_n}{r_n} \frac{J_n^d \log(J_n^d)}{n} = \left(\frac{J_n^{d\xi(1+2/\eta)} \log(J_n^d)^{2-\xi(1+2/\eta)}}{n}\right)^{1/2} = O(1).$$

For the tails, by Markov's inequality,  $\mathbb{E}[T_{ij}(k)] = 0$ , Lemma A.1, Assumption 1(c), and  $q_j \approx J_n^{-d}$ :

$$\begin{split} J_{n}^{d} \max_{1 \leq j \leq J_{n}^{d}} \mathbb{P} \left[ \left| \sum_{i=1}^{n} T_{ij}(k) \right| > nq_{j}r_{n}\varepsilon \right] &\leq C J_{n}^{d} \max_{1 \leq j \leq J_{n}^{d}} \frac{1}{(nq_{j}r_{n}\varepsilon)^{2}} \mathbb{E} \left[ \left| \sum_{i=1}^{n} T_{ij}(k) \right|^{2} \right] \\ &\leq C \frac{J_{n}^{d}}{nr_{n}^{2}\varepsilon^{2}} \max_{1 \leq j \leq J_{n}^{d}} \frac{1}{q_{j}^{2}} \mathbb{E} \left[ \mathbb{I}_{P_{j}}(X_{i}) \left| \left[ \tilde{R}_{j}(X_{i}) \right]_{g(k)} Y_{i} \mathbb{I} \{Y_{i} > t_{n}\} \right|^{2} \right] \\ &\leq C \frac{J_{n}^{d}}{nr_{n}^{2}t_{n}^{\eta}\varepsilon^{2}} \max_{1 \leq j \leq J_{n}^{d}} \frac{1}{q_{j}^{2}} \mathbb{E} \left[ \mathbb{I}_{P_{j}}(X_{i}) \mathbb{E} \left[ |Y_{i}|^{2+\eta} |X_{i}| \right] \right] \\ &\leq C \frac{J_{n}^{2d}}{nr_{n}^{2}t_{n}^{\eta}\varepsilon^{2}}. \end{split}$$

This is arbitrarily small for large enough  $\varepsilon$ , since:

$$\frac{J_n^{2d}}{nr_n^2t_n^{\eta}} = \frac{J_n^{2d}}{n} \frac{n}{J_n^{d(2-\xi)}\log(J_n^d)^{\xi}} \frac{\log(J_n^d)^{\xi}}{J_n^{d\xi}} = 1.$$

The two bounds do not depend on k, and hence by Boole's inequality and K constant,

$$\mathbb{P}\left[\max_{1\leq j\leq J_n^d} \left| \tilde{R}_j'(Y-G)/(nq_j) \right| > r_n \varepsilon\right] \leq C J_n^d \max_{1\leq j\leq J_n^d} \max_{[k]\leq K-1} \mathbb{P}\left[ \left| \sum_{i=1}^n H_{ij}(k) \right| > nq_j r_n \varepsilon \right] + C J_n^d \max_{1\leq j\leq J_n^d} \max_{[k]\leq K-1} \mathbb{P}\left[ \left| \sum_{i=1}^n T_{ij}(k) \right| > nq_j r_n \varepsilon \right],$$

which is arbitrarily small for  $\varepsilon$  large enough.

The conclusion will hold almost surely by the Borel-Cantelli Lemma if we find sequences  $r_n \to 0$ 

and  $t_n \to \infty$  such that

$$\frac{n}{J_n^d \log(J_n^d)} r_n^2 \neq 0, \qquad \frac{t_n}{r_n} \frac{J_n^d \log(J_n^d)}{n} \neq \infty, \quad \text{and,} \quad \sum_{n=1}^{\infty} \frac{J_n^{2d}}{n r_n^2 t_n^{\eta}} < \infty.$$

For  $r_n$  in the statement of the Lemma, the first requirement is satisfied as above. For  $J_n^d \simeq (n/\log(n))^{\gamma}$  and  $t_n = n^{\tau}$ ,  $\tau > 0$ , the second and third conditions above require  $(1 + \xi \gamma)/\eta < \tau < (1 - \xi \gamma)/2$ . This interval is nonempty since by assumption  $\eta > 2\left(\frac{1+\xi \gamma}{1-\xi \gamma}\right)$ .

## A.2 Convergence Rates

Proof of Theorem 1. Define  $\mathbb{I}_{n,j} = \mathbb{I}\{\lambda_{\min}(\hat{\Omega}_j) \geq C\}$  for some positive constant C, where  $\lambda_{\min}(\hat{\Omega}_j)$  is the smallest eigenvalue. In the proofs that follow we will redefine the notation  $\hat{\mu}(x) = \sum_{j=1}^{J_n^d} \mathbb{I}_{n,j} \tilde{R}_j(x)' \hat{\beta}_j$  (cf. Eqn. (1)). As  $\min_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} = 1$  w.p.a. 1 by Lemma A.4, this distinction vanishes asymptotically. For  $\beta_j^0$  as in Lemma A.2 and  $G = (\mu(X_1), \dots, \mu(X_n))'$ :

$$\max_{m:[m] \leq s} \left\| \partial^{m} \hat{\mu} - \partial^{m} \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} \mu_{j} \right\|_{2}^{2} = \max_{m:[m] \leq s} \left\| \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} \left[ (\partial^{m} \tilde{R}_{j}(\cdot))' \hat{\Omega}_{j}^{-1} \tilde{R}_{j}' Y / (nq_{j}) - \partial^{m} \mu_{j}(\cdot) \right] \right\|_{2}^{2} \\
\leq \max_{m:[m] \leq s} 3 \sum_{j=1}^{J_{n}^{d}} \left\| \mathbb{I}_{n,j} (\partial^{m} \tilde{R}_{j}(\cdot))' \hat{\Omega}_{j}^{-1} \tilde{R}_{j}' (Y - G) / (nq_{j}) \right\|_{2}^{2} \qquad (T_{n1}) \\
+ \max_{m:[m] \leq s} 3 \sum_{j=1}^{J_{n}^{d}} \left\| \mathbb{I}_{n,j} (\partial^{m} \tilde{R}_{j}(\cdot))' \hat{\Omega}_{j}^{-1} \tilde{R}_{j}' (G - \tilde{R}_{j} \beta_{j}^{0}) / (nq_{j}) \right\|_{2}^{2} \qquad (T_{n2}) \\
+ \max_{m:[m] \leq s} 3 \sum_{j=1}^{J_{n}^{d}} \left\| \mathbb{I}_{n,j} \left[ (\partial^{m} \tilde{R}_{j}(\cdot))' \beta_{j}^{0} - \partial^{m} \mu_{j}(\cdot) \right] \right\|_{2}^{2} \qquad (T_{n3})$$

The proof proceeds by bounding  $T_{n1}$ – $T_{n3}$ . To begin, observe that by properties of the trace operator, Assumption 1(c),  $\tilde{R}_j(\tilde{R}'_j\tilde{R}_j)^{-1}\tilde{R}'_j$  idempotent, K fixed, and  $q_j \approx J_n^{-d}$ ,

$$\mathbb{E}\left[\left|\mathbb{I}_{n,j}\hat{\Omega}_{j}^{-1/2}\tilde{R}_{j}'(Y-G)/(nq_{j})\right|^{2}\left|\{X_{i}\}\right] = \frac{\mathbb{I}_{n,j}}{nq_{j}}\operatorname{tr}\left\{\mathbb{E}\left[\left(Y-G\right)'\tilde{R}_{j}\left(\tilde{R}_{j}'\tilde{R}_{j}\right)^{-1}\tilde{R}_{j}'(Y-G)\right|\left\{X_{i}\}\right]\right\}$$

$$= \frac{\mathbb{I}_{n,j}}{nq_{j}}\operatorname{tr}\left\{\tilde{R}_{j}\left(\tilde{R}_{j}'\tilde{R}_{j}\right)^{-1}\tilde{R}_{j}'\mathbb{E}\left[\left(Y-G\right)(Y-G)'\right|\left\{X_{i}\}\right]\right\}$$

$$\leq C\frac{\mathbb{I}_{n,j}}{nq_{j}}\operatorname{tr}\left\{\tilde{R}_{j}\left(\tilde{R}_{j}'\tilde{R}_{j}\right)^{-1}\tilde{R}_{j}'\right\}$$

$$= C\frac{\mathbb{I}_{n,j}}{nq_{j}}\operatorname{tr}\left\{\left(\tilde{R}_{j}'\tilde{R}_{j}\right)^{-1}\tilde{R}_{j}'\tilde{R}_{j}\right\}$$

$$\leq \frac{Cg^*}{nq_j} \leq \frac{CJ_n^d}{n}.$$
(A.2)

This bound is uniform in  $1 \le j \le J_n^d$ , and hence:

$$\mathbb{E}\left[\sum_{j=1}^{J_n^d} q_j \mathbb{I}_{n,j} \left| \frac{\hat{\Omega}_j^{-1/2} \tilde{R}_j'(Y-G)}{nq_j} \right|^2 \right] \leq \max_{1 \leq j \leq J_n^d} \mathbb{E}\left[\mathbb{I}_{n,j} \left| \frac{\hat{\Omega}_j^{-1/2} \tilde{R}_j'(Y-G)}{nq_j} \right|^2 \right] \sum_{j=1}^{J_n^d} q_j = O\left(J_n^d/n\right).$$

So by Markov's inequality  $\sum_{j=1}^{J_n^d} q_j \mathbb{I}_{n,j} \left| \hat{\Omega}_j^{-1/2} \tilde{R}_j'(Y-G)/(nq_j) \right|^2 = O_p \left( J_n^d/n \right)$ . Using this result, that  $q_j = \int_{P_j} f(x) dx$ , Lemmas A.1 and A.4, and because the differentiation only affects the basis at the point of evaluation, we have the following bound:

$$T_{n1} \leq \left( \max_{1 \leq j \leq J_n^d \ m:[m] \leq s} \left\| \partial^m \tilde{R}_j(\cdot) \right\|_{\infty}^2 \right) \left( \max_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} \left| \hat{\Omega}_j^{-1} \right| \right) \sum_{j=1}^{J_n^d} \mathbb{I}_{n,j} \left| \hat{\Omega}_j^{-1/2} \frac{\tilde{R}'_j(Y - G)}{nq_j} \right|^2 \int_{P_j} f(x) dx$$

$$= O\left( J_n^{2s} \right) O_p(1) O_p\left( J_n^d / n \right) = O_p\left( J_n^{d+2s} / n \right). \quad (A.3)$$

By Boole's and Bernstein's inequality and the condition of Theorem 1, the random variables  $\mathbb{1}_{P_i}(X_i)$  satisfy the following, as  $\mathbb{1}_{P_i}(X_i) \leq 1$  and  $\mathbb{E}\left[\mathbb{1}_{P_i}(X)^2\right] = q_j$ :

$$\mathbb{P}\left[\max_{1\leq j\leq J_n^d} \sum_{i=1}^n (\mathbb{I}_{P_j}(X_i) - q_j) > nq_j\varepsilon\right] \leq CJ_n^d \max_{1\leq j\leq J_n^d} \exp\left\{-C\frac{nq_j\varepsilon^2}{1+\varepsilon}\right\} \\
\leq C\exp\left\{\log(J_n^d)\left[1 - C\frac{n}{J_n^d\log(J_n^d)}\frac{\varepsilon^2}{1+\varepsilon}\right]\right\} \to 0.$$
(A.4)

Therefore, by  $\tilde{R}_j(\tilde{R}_j'\tilde{R}_j)^{-1}\tilde{R}_j'$  idempotent and Lemma A.2:

$$\max_{1 \leq j \leq J_{n}^{d}} \left\| \mathbb{I}_{n,j} \hat{\Omega}_{j}^{-1/2} \tilde{R}_{j}' \left( G - \tilde{R}_{j} \beta_{j}^{0} \right) / (nq_{j}) \right\|^{2}$$

$$= \max_{1 \leq j \leq J_{n}^{d}} \mathbb{I}_{n,j} \left| \left( G - \tilde{R}_{j} \beta_{j}^{0} \right)' \tilde{R}_{j} (\tilde{R}_{j}' \tilde{R}_{j})^{-1} \tilde{R}_{j}' \left( G - \tilde{R}_{j} \beta_{j}^{0} \right) / (nq_{j}) \right|$$

$$\leq \max_{1 \leq j \leq J_{n}^{d}} \left| \left( G - \tilde{R}_{j} \beta_{j}^{0} \right)' \left( G - \tilde{R}_{j} \beta_{j}^{0} \right) / (nq_{j}) \right|$$

$$= \max_{1 \leq j \leq J_{n}^{d}} \frac{1}{nq_{j}} \sum_{i=1}^{n} \mathbb{I}_{P_{j}} (X_{i}) \left( \mu(X_{i}) - \tilde{R}_{j}(X_{i})' \beta_{j}^{0} \right)^{2}$$

$$\leq \max_{1 \leq j \leq J_{n}^{d}} \left\| \mathbb{I}_{P_{j}} (\cdot) \left( \mu(\cdot) - \tilde{R}_{j}(\cdot)' \beta_{j}^{0} \right) \right\|_{\infty}^{2} \max_{1 \leq j \leq J_{n}^{d}} \frac{1}{nq_{j}} \sum_{i=1}^{n} \mathbb{I}_{P_{j}} (X_{i}) = O_{p} \left( J_{n}^{-2((S+\alpha) \wedge K)} \right). \quad (A.5)$$

The first inequality follows by the fact that for a vector v and idempotent matrix P, v'Pv

 $v'v - v'(I - P)v = v'v - |(I - P)v| \le v'v$ , since norms are nonnegative. And so for  $T_{n2}$ , by the above result, Lemmas A.1 and A.4, and  $\sum_{j=1}^{J_n^d} \int_{P_j} f(x) dx = \int_{\mathcal{X}} f(x) dx = 1$ , we have

$$T_{n2} \leq \left(\max_{1 \leq j \leq J_n^d \ m:[m] \leq s} \left\| \partial^m \tilde{R}_j(\cdot) \right\|_{\infty}^2 \right) \left(\max_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} \left| \hat{\Omega}_j^{-1} \right| \right)$$

$$\times \left(\max_{1 \leq j \leq J_n^d} \left| \mathbb{I}_{n,j} \hat{\Omega}_j^{-1/2} \tilde{R}_j' \left( G - \tilde{R}_j \beta_j^0 \right) / (nq_j) \right|^2 \right) \sum_{j=1}^{J_n^d} \int_{P_j} f(x) dx$$

$$\leq O\left(J_n^{2s}\right) O_p(1) O_p\left(J_n^{-2((S+\alpha)\wedge K)}\right) = O_p\left(J_n^{-2((S+\alpha)\wedge K-s)}\right). \tag{A.6}$$

Finally, Lemma A.2 immediately gives:

$$T_{n3} \leq \max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \left\| \mathbb{I}_{n,j} \left( (\partial^m \tilde{R}_j(\cdot))' \beta_j^0 - \partial^m \mu_j(\cdot) \right) \right\|_{\infty}^2 \sum_{j=1}^{J_n^d} \int_{P_j} f(x) dx = O\left( J_n^{-2((S+\alpha) \wedge K - s)} \right),$$
(A.7)

again using that  $\sum_{j=1}^{J_n^d} \int_{P_j} f(x) dx = 1$ .

Combining the bounds (A.3), (A.6), and (A.7), the result follows from  $\min_{1 \le j \le J_n^d} \mathbb{I}_{n,j} = 1$  w.p.a. 1 by Lemma A.4.

Proof of Theorem 2. For  $\beta_j^0$  as in Lemma A.2:

$$\max_{m:[m] \leq s} \left\| \partial^{m} \hat{\mu} - \partial^{m} \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} \mu_{j} \right\|_{\infty}^{2} = \max_{1 \leq j \leq J_{n}^{d}} \max_{m:[m] \leq s} \left\| \mathbb{I}_{n,j} \left( (\partial^{m} \tilde{R}_{j}(\cdot))' (\tilde{R}'_{j} \tilde{R}_{j})^{-1} \tilde{R}'_{j} Y - \partial^{m} \mu_{j}(\cdot) \right) \right\|_{\infty}^{2} \\
\leq \max_{1 \leq j \leq J_{n}^{d}} \max_{m:[m] \leq s} 3 \left\| \mathbb{I}_{n,j} \left( (\partial^{m} \tilde{R}_{j}(\cdot))' \hat{\Omega}_{j}^{-1} \tilde{R}'_{j} (Y - G)/(nq_{j}) \right) \right\|_{\infty}^{2} \\
+ \max_{1 \leq j \leq J_{n}^{d}} \max_{m:[m] \leq s} 3 \left\| \mathbb{I}_{n,j} \left( (\partial^{m} \tilde{R}_{j}(\cdot))' \hat{\Omega}_{j}^{-1} \tilde{R}'_{j} (G - \tilde{R}_{j} \beta_{j}^{0})/(nq_{j}) \right) \right\|_{\infty}^{2} \\
+ \max_{1 \leq j \leq J_{n}^{d}} \max_{m:[m] \leq s} 3 \left\| \mathbb{I}_{n,j} \left( (\partial^{m} \tilde{R}_{j}(\cdot))' \hat{\beta}_{j}^{0} - \partial^{m} \mu_{j}(\cdot) \right) \right\|_{\infty}^{2}.$$

Using Lemmas A.1, A.4, and A.5, the first term is bounded by:

$$\begin{split} C\left(\max_{1\leq j\leq J_n^d}\max_{m:[m]\leq s}\left\|\partial^m\tilde{R}_j(\cdot)\right)\right\|_{\infty}^2\right)\left(\max_{1\leq j\leq J_n^d}\left|\mathbb{I}_{n,j}\hat{\Omega}_j^{-1}\right|\right)\left\|\tilde{R}_j'(Y-G)/(nq_j)\right\|_{\infty}^2\\ &=O\left(J_n^{2s}\right)O_p(1)O_p\left(\frac{J_n^{d(2-\xi)}\log(J_n^d)^\xi}{n}\right). \end{split}$$

Using Lemmas A.1 and A.4 and Eqn. (A.5), the second term is bounded by:

$$\begin{split} \left( \max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \left\| \partial^m \tilde{R}_j(\cdot) \right\|_{\infty}^2 \right) \left( \max_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} \left| \hat{\Omega}_j^{-1} \right| \right) \left( \max_{1 \leq j \leq J_n^d} \left| \mathbb{I}_{n,j} \hat{\Omega}_j^{-1/2} \tilde{R}_j' \left( G - \tilde{R}_j \beta_j^0 \right) / (nq_j) \right|^2 \right) \\ &= O\left( J_n^{2s} \right) O_p(1) O_p\left( J_n^{-2((S+\alpha) \wedge K)} \right). \end{split}$$

Finally, the rate for the third term is given in Lemma A.2, since  $\mathbb{1}_{n,j} \leq 1$ . Adding these three rates completes the proof, as  $\min_{1 \leq j \leq J_n^d} \mathbb{1}_{n,j} = 1$  w.p.a. 1 by Lemma A.4.

We now demonstrate a version of Theorem 2 that holds with probability one.

**Theorem A.1.** Suppose the conditions of Theorem 1 hold. If, in addition, for some  $\xi \in [0, 1 \land \eta]$  the partition satisfies  $J_n^d \approx (n/\log(n))^{\gamma}$ ,  $\gamma \in (0, 1)$  and  $\eta > 2(1 + \xi \gamma)/(1 - \xi \gamma)$ , then for  $s \leq S \land (K - 1)$ :

$$\max_{m:[m] \le s} \|\partial^m \hat{\mu} - \partial^m \mu\|_{\infty}^2 = O_{as} \left( \frac{J_n^{(2-\xi)d+2s} \log(J_n^d)^{\xi}}{n} + J_n^{-2((S+\alpha) \wedge K - s)} \right).$$

Proof of Theorem A.1. First observe that the rate restriction on  $J_n$  given implies that of Theorem 2. This holds because by the assumption on  $\eta$ ,

$$\eta > 2\frac{1+\gamma\xi}{1-\gamma\xi} > 2\frac{\gamma\xi}{1-\gamma\xi},$$

and hence  $\gamma \xi(1+2/\eta) \leq 1$ . The exponential bound of (A.4) and  $n^{-1}J_n^d \log(J_n^d) \to 0$  gives

$$\max_{1 \le j \le J_n^d} \frac{1}{nq_j} \sum_{i=1}^n \mathbb{I}_{P_j}(X_i) = O_{as}(1).$$

Hence Eqn. (A.5) and the steps of Eqn. (A.6) hold almost surely. Coupled with the second conclusion in Lemma A.5, the proof of Theorem 2 can be strengthened to hold with probability one, as  $\min_{1 \le j \le J_n^d} \mathbb{I}_{n,j} = 1$  w.p.a. 1 using the almost sure Lemma A.4.

### A.3 ASYMPTOTIC MEAN-SQUARE ERROR

We first give three lemmas necessary for results.

**Lemma A.6.** Let the conditions of Theorem 1 hold and  $g(\cdot)$  be continuous on  $\mathcal{X}$ . Then for  $h_j(x) = \mathbb{I}_{P_j}(x)h(x)$ , with remainder uniform in  $1 \leq j \leq J_n^d$ :

$$\int_{P_j} h(z)g(z)dz = g(\overline{p}_j) \int_{P_j} h(z)dz + \max_{1 \le j \le J_n^d} ||h_j(\cdot)||_{\infty} (o(J_n^{-d})).$$

Further, if  $g(\cdot)$  is Lipschitz continuous, then

$$\int_{P_j} h(z)g(z)dz = g(\overline{p}_j)\int_{P_j} h(z)dz + \max_{1\leq j\leq J_n^d} \|h_j(\cdot)\|_{\infty}(O(J_n^{-d-1})).$$

*Proof.* First, write

$$\int_{P_j} h(z)g(z)dz = g(\overline{p}_j) \int_{P_j} h(z)dz + \int_{P_j} h(z)[g(z) - g(\overline{p}_j)]dz.$$

By continuity, the remainder is bounded by:

$$\begin{split} \max_{1 \leq j \leq J_n^d} \int_{P_j} |h(z)| \left| g(z) - g(\overline{p}_j) \right| dz &\leq \max_{1 \leq j \leq J_n^d} \sup_{x \in P_j} \left| g(\overline{p}_j) - g(x) \right| \max_{1 \leq j \leq J_n^d} \|h_j(\cdot)\|_{\infty} \int_{P_j} dz \\ &= o(1) \max_{1 \leq j \leq J_n^d} \|h_j(\cdot)\|_{\infty} \max_{1 \leq j \leq J_n^d} \operatorname{vol}\left(P_j\right) = \max_{1 \leq j \leq J_n^d} \|h_j(\cdot)\|_{\infty} o\left(J_n^{-d}\right). \end{split}$$

The second conclusion follows from the same steps, but the rate is obtained from the Lipschitz continuity because  $\max_{1 \leq j \leq J_n^d} \sup_{x \in P_j} \left| g(\overline{p}_j) - g(x) \right| \leq C |p_j^* - \overline{p}_j| = O(J_n^{-1}).$ 

**Lemma A.7.** Let the conditions of Theorem 1 hold. If  $g(\cdot)$  is continuous on  $\mathcal{X}$ , then:

$$\sum_{j=1}^{J_n^d} g(\overline{p}_j) \operatorname{vol}(P_j) = \int_{\mathcal{X}} g(z) dz + o(1).$$

Further, if  $g(\cdot)$  is Lipschitz continuous, then the remainder is  $O(J_n^{-1})$ .

*Proof.* This is a standard Riemann sum argument: the result follows as  $g(\cdot)$  is continuous and  $\mathcal{X}$  is compact.

$$\begin{split} \sum_{j=1}^{J_n^d} g(\overline{p}_j) \operatorname{vol}\left(P_j\right) - \int_{\mathcal{X}} g(z) dz &= \sum_{j=1}^{J_n^d} \left( g(\overline{p}_j) \operatorname{vol}\left(P_j\right) - \int_{P_j} g(z) dz \right) \\ &= \sum_{j=1}^{J_n^d} \left( g(\overline{p}_j) \int_{P_j} dz - \int_{P_j} g(z) dz \right) \\ &= \sum_{j=1}^{J_n^d} \int_{P_j} (g(\overline{p}_j) - g(z)) dz. \end{split}$$

Then, as  $\sum_{j=1}^{J_n^d} \int_{P_j} dz = \int_{\mathcal{X}} dz = \operatorname{vol}(\mathcal{X})$ , and  $\mathcal{X}$  is compact, we have by continuity that the

magnitude of this remainder is bounded by

$$\sum_{j=1}^{J_n^d} \int_{P_j} \left| g(\overline{p}_j) - g(z) \right| dz \le \max_{1 \le j \le J_n^d} \sup_{x \in P_j} \left| g(\overline{p}_j) - g(x) \right| \sum_{j=1}^{J_n^d} \int_{P_j} dz$$
$$= o(1) \operatorname{vol}(\mathcal{X}) = o(1).$$

The second conclusion follows from the same steps, but the rate is obtained from the Lipschitz continuity because  $\max_{1 \leq j \leq J_n^d} \sup_{x \in P_j} \left| g(\overline{p}_j) - g(x) \right| \leq C|p_j^* - \overline{p}_j| = O(J_n^{-1}).$ 

**Lemma A.8.** Under the conditions of Theorem 3, for  $\Gamma_i$  defined Eqn. (4) and any  $k \in \mathbb{Z}^d_+$ :

(a) 
$$\max_{1 \le j \le J_n^d} \left| \frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \sigma^2(X_i) - \Gamma_j \right|^2 = O_p\left(\frac{J_n^d \log(J_n^d)}{n}\right);$$

(b) 
$$\max_{1 \le j \le J_n^d} \left| \frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i) \frac{(X_i - \overline{p}_j)^k}{(p_j^* - \overline{p}_j)^k} - \frac{1}{q_j} \mathbb{E} \left[ \tilde{R}_j(X) \frac{(X_i - \overline{p}_j)^k}{(p_j^* - \overline{p}_j)^k} \right] \right|^2 = O_p \left( \frac{J_n^d \log(J_n^d)}{n} \right).$$

*Proof.* Both results follow identically to Lemma A.4, the conditions of which are met as  $\sigma^2(x)$  is bounded by Assumption 2(a) and  $|(X_i - \overline{p}_j)^k/(p_j^* - \overline{p}_j)^k| \leq 1$  by construction. The proof goes through essentially as written, with the appropriate notational changes to  $W_{ij}(k, \tilde{k})$ .

Proof of Theorem 3. We first give some notation and facts used repeatedly throughout. With a slight abuse notation, let  $|\mathcal{X}|^k = \prod_{\ell=1}^d |\mathcal{X}_\ell|^{k_\ell}$ ; this is distinct from  $\operatorname{vol}(\mathcal{X})$  unless  $k_\ell = 1$  for all  $\ell = 1, \ldots, d$ . Let  $\mathcal{U} = \times_{\ell=1}^d [-1, 1]$  be the Cartesian product of d copies of the interval [-1, 1]. Under the conditions placed on the partition and Assumption 1(b),  $\operatorname{vol}(P_j) = \operatorname{vol}(\mathcal{X})/J_n^d$ . We frequently use the change of variables  $z_\ell = (x_\ell - \overline{p}_{\ell,j})/(p_{\ell,j} - \overline{p}_{\ell,j})$ ,  $\ell = 1, \ldots, d$ , the Jacobian of which is  $\prod_{\ell=1}^d (p_{\ell,j} - \overline{p}_{\ell,j}) = 2^{-d} \operatorname{vol}(P_j)$ . Recall from Lemma A.2 that entry g(k) of  $\partial^m \tilde{R}_j(x)$  is given by:

$$\mathbb{1}\{m \le k\} \frac{k!}{(k-m)!} \frac{(x-\overline{p}_j)^{k-m}}{(p_j^* - \overline{p}_j)^k},$$

whereas the same entry of  $\partial R_j(x) = (k!/(k-m)!)x^{k-m}$ . Finally, because the partition is evenly spaced, for any  $k \in \mathbb{Z}_+^d$ :

$$(p_j^* - \overline{p}_j)^k = \prod_{\ell=1}^d \frac{(p_{\ell,j} - p_{\ell,j-1})^{k_\ell}}{2^{k_\ell}} = \prod_{\ell=1}^d (|\mathcal{X}_\ell|/(2J_n))^{k_\ell} = (2J_n)^{-[k]} |\mathcal{X}|^k.$$

Using Lemma A.6 and the change of variables above, we get the following results which are

used below: first,

$$\int_{\mathcal{X}} (\partial^m \tilde{R}_j(x))(x - \overline{p}_j)^{k-m} w(x) dx = w(\overline{p}_j) \int_{P_j} (\partial^m \tilde{R}_j(x))(x - \overline{p}_j)^{k-m} dx + o(J_n^{-d-K})$$

$$= 2^{-d} w(\overline{p}_j)(p_j^* - \overline{p}_j)^{k-2m} \operatorname{vol}(P_j) \int_{\mathcal{U}} (\partial^m R(z)) z^k dz + o(J_n^{-d-K}),$$

which also holds for w(x) = f(x) or m = 0; second,

$$\Omega_j = \frac{2^{-d}}{q_j} f(\overline{p}_j) \operatorname{vol}(P_j) \int_{\mathcal{U}} R(z) R(z)' dz + o(J_n^{-d});$$

and finally,

$$\int_{X} \left( \partial^{m} \tilde{R}_{j}(x) \right) \left( \partial^{m} \tilde{R}_{j}(x) \right)' w(x) dx = \frac{2^{-d} w(\overline{p}_{j}) \operatorname{vol}\left(P_{j}\right)}{(p_{j}^{*} - \overline{p}_{j})^{2m}} \int_{\mathcal{U}} \left( \partial^{m} R(z) \right) \left( \partial^{m} R(z) \right)' dz + o(J_{n}^{-d-2[m]}),$$

where we have applied Lemma A.1 to bound  $\partial^m \tilde{R}_j(x)$ .

Recall that  $\mathbf{X}_n = (X_1, \dots, X_n)'$  and expand as follows:

$$\int_{X} \mathbb{E}\left[\left(\partial^{m}\hat{\mu}(x) - \partial^{m}\mu(x)\right)^{2} | \mathbf{X}_{n}\right] w(x) dx = \int_{\mathcal{X}} \left\{ \mathbb{V}\left[\partial^{m}\hat{\mu}(x) \mid \mathbf{X}_{n}\right] + \left(\mathbb{E}\left[\partial^{m}\hat{\mu}(x) | \mathbf{X}_{n}\right] - \partial^{m}\mu(x)\right)^{2} \right\} w(x) dx.$$

First consider the variance term. By Lemma A.6,  $\Gamma_j = \sigma^2(\overline{p}_j)\Omega_j + o(J_n^{-d})$ . Applying this result and Lemmas A.1, A.4, and A.8(a), we have:

$$\begin{split} & \mathbb{V}\left[\sum_{j=1}^{J_n^d} \left(\partial^m \tilde{R}_j(x)\right)' \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j Y/(nq_j) | \mathbf{X}_n \right] \\ &= \sum_{j=1}^{J_n^d} \frac{1}{nq_j} \left(\partial^m \tilde{R}_j(x)\right)' \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \left(\frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \sigma^2(X_i)\right) \hat{\Omega}_j^{-1} \left(\partial^m \tilde{R}_j(x)'\right) \\ &= \sum_{j=1}^{J_n^d} \frac{1}{nq_j} \left(\partial^m \tilde{R}_j(x)\right)' \Omega_j^{-1} \Gamma_j \Omega_j^{-1} \left(\partial^m \tilde{R}_j(x)\right) + O_p \left(\frac{J_n^{d+2[m]}}{n} \frac{J_n^d \log(J_n^d)}{n}\right) \\ &= \sum_{j=1}^{J_n^d} \frac{1}{nq_j} \sigma^2(\overline{p}_j) \left(\partial^m \tilde{R}_j(x)\right)' \Omega_j^{-1} \Omega_j \Omega_j^{-1} \left(\partial^m \tilde{R}_j(x)\right) + o_p \left(\frac{J_n^{d+2[m]}}{n}\right) \\ &= \sum_{j=1}^{J_n^d} \frac{1}{nq_j} \sigma^2(\overline{p}_j) \left(\partial^m \tilde{R}_j(x)\right)' \Omega_j^{-1} \left(\partial^m \tilde{R}_j(x)\right) + o_p \left(\frac{J_n^{d+2[m]}}{n}\right) \end{split}$$

$$= \sum_{j=1}^{J_n^d} \frac{1}{nq_j} \sigma^2(\overline{p}_j) \operatorname{tr} \left\{ \left( \partial^m \tilde{R}_j(x) \right)' \Omega_j^{-1} \left( \partial^m \tilde{R}_j(x) \right) \right\} + o_p \left( \frac{J_n^{d+2[m]}}{n} \right)$$

$$= \sum_{j=1}^{J_n^d} \frac{1}{nq_j} \sigma^2(\overline{p}_j) \operatorname{tr} \left\{ \Omega_j^{-1} \left( \partial^m \tilde{R}_j(x) \right) \left( \partial^m \tilde{R}_j(x) \right)' \right\} + o_p \left( \frac{J_n^{d+2[m]}}{n} \right).$$

Integrating the above expression, applying Lemma A.6, the above facts and change of variables, and Lemma A.7 (under Assumption 2(a)), we have:

$$\begin{split} \sum_{j=1}^{J_n^d} \frac{1}{nq_j} \sigma^2(\overline{p}_j) \operatorname{tr} \left\{ \Omega_j^{-1} \int_{X} \left( \partial^m \tilde{R}_j(x) \right) \left( \partial^m \tilde{R}_j(x) \right)' w(x) dx \right\} + o_p \left( \frac{J_n^{d+2[m]}}{n} \right) \\ &= \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{w(\overline{p}_j)}{f(\overline{p}_j)} \sigma^2(\overline{p}_j) \frac{1}{(p_j^* - \overline{p}_j)^{2m}} \operatorname{tr} \left\{ \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} \left( \partial^m R(z) \right) \left( \partial^m R(z) \right)' dz \right\} \\ &\quad + o_p \left( \frac{J_n^{d+2[m]}}{n} \right) \\ &= \frac{1}{n} \frac{J_n^d}{\operatorname{vol}(\mathcal{X})} \frac{(2J_n)^{2[m]}}{|\mathcal{X}|^{2m}} \sum_{j=1}^{J_n^d} \frac{w(\overline{p}_j)}{f(\overline{p}_j)} \sigma^2(\overline{p}_j) \operatorname{vol}(P_j) \operatorname{tr} \left\{ \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} \left( \partial^m R(z) \right) \left( \partial^m R(z) \right)' dz \right\} \\ &\quad + o_p \left( \frac{J_n^{d+2[m]}}{n} \right) \\ &= \frac{J_n^{d+2[m]}}{n} \frac{2^{2[m]}}{|\mathcal{X}|^{2m} \operatorname{vol}(\mathcal{X})} \left( \int_{\mathcal{X}} \frac{\sigma^2(x)}{f(x)} w(x) dx \right) \operatorname{tr} \left\{ \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} \left( \partial^m R(z) \right) \left( \partial^m R(z) \right)' dz \right\} \\ &\quad \times [1 + o(1)] + o_p \left( \frac{J_n^{d+2[m]}}{n} \right). \end{split}$$

Next, recall that K = S + 1 and that  $\tilde{R}_j(x)$  is of degree K - 1. From Lemma A.2, under Assumption 2(b), we have that  $\partial^m \mu_j(x)$  satisfies the Taylor expansion for  $x \in P_j$ :

$$\partial^{m} \mu_{j}(x) - \partial^{m} \tilde{R}_{j}(x)' \beta_{j}^{0} = \sum_{k: [k] = K} \left( \partial^{k} \mu_{j}(\overline{p}_{j}) \right) \frac{(x - \overline{p}_{j})^{k - m}}{(k - m)!} + o(J_{n}^{-(K - [m])}) \equiv T_{K, j, m}(x) + o(J_{n}^{-(K - [m])}),$$

where  $\beta_j^0$  does not depend on m and the remainder is uniform over  $1 \leq j \leq J_n^d$ . The final equality defines  $T_{K,j,m}(x)$  as the leading term. Therefore, by Lemmas A.4 and A.8,

$$\sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbb{I}_{n,j} (\tilde{R}'_j \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \mu(X_i) - \sum_{j=1}^{J_n^d} \partial^m \mu_j(x)$$

$$\begin{split} &= \sum_{j=1}^{J_n^d} \left( \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \beta_j^0 - \partial^m \mu_j(x) \right) \\ &+ \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \left( T_{K,j,0}(X_i) + o(J_n^{-K}) \right) \\ &= \sum_{j=1}^{J_n^d} \left( \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \left( T_{K,j,0}(X_i) + o(J_n^{-K}) \right) \right. \\ &+ \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \left( T_{K,j,0}(X_i) + o(J_n^{-K}) \right) \\ &+ \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) \left( T_{K,j,0}(X_i) + o(J_n^{-K}) \right) \\ &= \sum_{j=1}^{J_n^d} \mathbf{I}_{n,j} \left( \partial^m \tilde{R}_j(x)' \beta_j^0 - \partial^m \mu_j(x) \right) + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} (\tilde{R}_j' \tilde{R}_j)^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) T_{K,j,0}(X_i) + o\left( J_n^{-(K-[m])} \right) \\ &= - \sum_{j=1}^{J_n^d} \mathbf{I}_{n,j} \mathbf{I}_{P_j}(x) T_{k,j,m}(x) + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} \tilde{\Omega}_j^{-1} \sum_{i=1}^n \tilde{R}_j(X_i) T_{K,j,0}(X_i) \\ &+ o\left( J_n^{-(K-[m])} \right) \\ &= - \sum_{j=1}^{J_n^d} \mathbf{I}_{P_j}(x) T_{k,j,m}(x) + \sum_{j=1}^{J_n^d} \partial^m \tilde{R}_j(x)' \mathbf{I}_{n,j} \tilde{\Omega}_j^{-1} \sum_{k:[k]=K} \frac{\partial^k \mu_j(\bar{p}_j)}{k!} \frac{1}{nq_j} \sum_{i=1}^n \tilde{R}_j(X_i)(X_i - \bar{p}_j)^k \\ &+ o\left( J_n^{-(K-[m])} \right) \\ &= - \sum_{j=1}^{J_n^d} \mathbf{I}_{P_j}(x) T_{k,j,m}(x) + \sum_{j=1}^{J_n^d} \frac{1}{q_j} \partial^m \tilde{R}_j(x)' \Omega_j^{-1} \sum_{k:[k]=K} \frac{\partial^k \mu_j(\bar{p}_j)}{k!} \mathbb{E} \left[ \tilde{R}_j(X)(X - \bar{p}_j)^k \right] \\ &+ O_p \left( J_n^{-(K-[m])} \frac{J_n^d \log(J_n^d)}{n} \right) + o\left( J_n^{-(K-[m])} \right) \\ &= \sum_{k:[k]=K} \sum_{j=1}^{J_n^d} \mathbf{I}_{P_j}(x) \left( \partial^k \mu_j(\bar{p}_j) \right) \left( - \frac{1}{(k-m)!} (x - \bar{p}_j)^{k-m} + \frac{1}{k!} \frac{1}{q_j} \partial^m \tilde{R}_j(x)' \Omega_j^{-1} \mathbb{E} \left[ \tilde{R}_j(X)(X - \bar{p}_j)^k \right] \right) \\ &+ o_p \left( J_n^{-(K-[m])} \right). \end{split}$$

Then since  $\min_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} = 1$  w.p.a. 1 by Lemma A.4, the integrated, squared bias becomes:

$$\int_{\mathcal{X}} \left( \mathbb{E} \left[ \hat{\mu}(x) | \mathbf{X}_n \right] - \mu(x) \right)^2 w(x) dx$$

$$\begin{split} &= \sum_{j=1}^{J_n^d} \sum_{\substack{k,\tilde{k} \\ [k] = [\tilde{k}] = K}} \left( \partial^k \mu_j(\overline{p}_j) \right) \left( \partial^{\tilde{k}} \mu_j(\overline{p}_j) \right) \left\{ \frac{1}{(k-m)!(\tilde{k}-m)!} \int_{P_j} (x-\overline{p}_j)^{k+\tilde{k}-2m} w(x) dx \right. \\ &+ \frac{1}{k!\tilde{k}!} \frac{1}{q_j^2} \int_{P_j} \partial^m \tilde{R}_j(x)' \Omega_j^{-1} \mathbb{E} \left[ \tilde{R}_j(X)(X-\overline{p}_j)^k \right] \mathbb{E} \left[ (X-\overline{p}_j)^{\tilde{k}} \tilde{R}_j(X)' \right] \Omega_j^{-1} \partial^m \tilde{R}_j(x) w(x) dx \\ &- \frac{1}{k!(\tilde{k}-m)!} \frac{1}{q_j} \int_{P_j} (x-\overline{p}_j)^{\tilde{k}-m} \partial^m \tilde{R}_j(x)' w(x) dx \Omega_j^{-1} \mathbb{E} \left[ \tilde{R}_j(X)(X-\overline{p}_j)^k \right] \\ &- \frac{1}{\tilde{k}!(k-m)!} \frac{1}{q_j} \int_{P_j} (x-\overline{p}_j)^{k-m} \partial^m \tilde{R}_j(x)' w(x) dx \Omega_j^{-1} \mathbb{E} \left[ \tilde{R}_j(X)(X-\overline{p}_j)^{\tilde{k}} \right] \right\} + o_p \left( J_n^{-2(K-[m])} \right) \\ &= \sum_{j=1}^{J_n^d} \sum_{\substack{k,\tilde{k} \\ [k] = [\tilde{k}] = K}} \left( \partial^k \mu_j(\overline{p}_j) \right) \left( \partial^{\tilde{k}} \mu_j(\overline{p}_j) \right) \left\{ B_1 + B_2 - B_3 - B_4 \right\} + o_p \left( J_n^{-2(K-[m])} \right), \end{split}$$

where the final equality defines the terms  $B_1$ – $B_4$ . We examine each in order, applying Lemma A.6 and the change of variables above. For the first term,

$$B_{1} = \frac{w(\overline{p}_{j})}{(k-m)!(\tilde{k}-m)!} \int_{P_{j}} (x-\overline{p}_{j})^{k+\tilde{k}-2m} dx + o(J_{n}^{-d})O\left(J_{n}^{-2(K-[m])}\right)$$

$$= \frac{w(\overline{p}_{j})(p_{j}^{*}-\overline{p}_{j})^{k+\tilde{k}-2m}}{(k-m)!(\tilde{k}-m)!} \int_{P_{j}} \frac{(x-\overline{p}_{j})^{k+\tilde{k}-2m}}{(p_{j}^{*}-\overline{p}_{j})^{k+\tilde{k}-2m}} dx + o(J_{n}^{-d})O\left(J_{n}^{-2(K-[m])}\right)$$

$$= \frac{(p_{j}^{*}-\overline{p}_{j})^{k+\tilde{k}-2m}w(\overline{p}_{j})\operatorname{vol}(P_{j})}{2^{d}(k-m)!(\tilde{k}-m)!} \int_{\mathcal{U}} z^{k+\tilde{k}-2m} dz + o(J_{n}^{-d})O\left(J_{n}^{-2(K-[m])}\right).$$

For the second, applying the results given above,

$$\begin{split} B_2 &= \frac{1}{k!\tilde{k}!} \frac{1}{q_j^2} \int_{P_j} \operatorname{tr} \left\{ (\partial^m \tilde{R}_j(x))' \Omega_j^{-1} \mathbb{E} \left[ \tilde{R}_j(X) (X - \overline{p}_j)^k \right] \mathbb{E} \left[ (X - \overline{p}_j)^{\tilde{k}} \tilde{R}_j(X)' \right] \Omega_j^{-1} (\partial^m \tilde{R}_j(x)) \right\} w(x) dx \\ &= \frac{1}{k!\tilde{k}!} \frac{1}{q_j^2} \operatorname{tr} \left\{ \Omega_j^{-1} \mathbb{E} \left[ \tilde{R}_j(X) (X - \overline{p}_j)^k \right] \mathbb{E} \left[ (X - \overline{p}_j)^{\tilde{k}} \tilde{R}_j(X)' \right] \Omega_j^{-1} \int_{P_j} (\partial^m \tilde{R}_j(x)) (\partial^m \tilde{R}_j(x))' w(x) dx \right\} \\ &= \frac{(p_j^* - \overline{p}_j)^{k + \tilde{k} - 2m} w(\overline{p}_j) \operatorname{vol}(P_j)}{2^d k! \tilde{k}!} \operatorname{tr} \left\{ \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} R(z) z^k dz \right. \\ &\qquad \times \int_{\mathcal{U}} R(z)' z^{\tilde{k}} dz \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} (\partial^m R(z)) (\partial^m R(z))' dz \right\} + o(J_n^{-d}) O\left(J_n^{-2(K - [m])}\right). \end{split}$$

Similarly,

$$B_3 = \frac{(p_j^* - \overline{p}_j)^{k + \tilde{k} - 2m} w(\overline{p}_j) \operatorname{vol}(P_j)}{2^d k! (\tilde{k} - m)!} \int_{\mathcal{U}} (\partial^m R(z))' z^{\tilde{k} - m} dz \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1}$$

$$\times \int_{\mathcal{U}} R(z) z^k dz + o(J_n^{-d}) O\left(J_n^{-2(K-[m])}\right).$$

Identical steps apply to  $B_4$ , with k and  $\tilde{k}$  reversed.

All four terms have the common factor  $(p_j^* - \overline{p}_j)^{k+\tilde{k}-2m}w(\overline{p}_j)$  vol  $(P_j)$ , which contains all dependence on the partition. By Lemma A.7, the facts at the outset, and that  $[k] = [\tilde{k}] = K$ ,

$$\begin{split} \sum_{j=1}^{J_n^a} \left( \partial^k \mu_j(\overline{p}_j) \right) \left( \partial^{\tilde{k}} \mu_j(\overline{p}_j) \right) (p_j^* - \overline{p}_j)^{k+\tilde{k}-2m} w(\overline{p}_j) \operatorname{vol}(P_j) \\ &= J_n^{-2(K-[m])} \frac{|\mathcal{X}|^{k+\tilde{k}-2m}}{2^{2(K-[m])}} \int_{\mathcal{X}} \left( \partial^k \mu_j(x) \right) \left( \partial^{\tilde{k}} \mu_j(x) \right) w(x) dx [1 + o(1)]. \end{split}$$

Combining all the above steps, if we define the two constants

$$\mathcal{Y}_{K,d,m} = \frac{2^{2[m]}}{\operatorname{vol}(\mathcal{X})} \left( \prod_{\ell=1}^{d} |\mathcal{X}_{\ell}|^{-2m_{\ell}} \right) \left( \int_{\mathcal{X}} \frac{\sigma^{2}(x)}{f(x)} w(x) dx \right) \\
\times \operatorname{tr} \left\{ \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} \left( \partial^{m} R(z) \right) \left( \partial^{m} R(z) \right)' dz \right\}$$
(A.8)

and

$$\mathcal{B}_{K,d,m} = 2^{-2(K+d-[m])} \sum_{\substack{k,\tilde{k} \\ [k]=[\tilde{k}]=K}} \left( \prod_{\ell=1}^{d} |\mathcal{X}_{\ell}|^{k_{\ell}+\tilde{k}_{\ell}-2m_{\ell}} \right) \left( \int_{\mathcal{X}} \left( \partial^{k}\mu(x) \right) \left( \partial^{\tilde{k}}\mu(x) \right) w(x) dx \right)$$

$$\times \left\{ \frac{1}{(k-m)!(\tilde{k}-m)!} \int_{\mathcal{U}} z^{k+\tilde{k}-2m} dz \right.$$

$$+ \frac{1}{k!\tilde{k}!} \operatorname{tr} \left[ \left( \int_{\mathcal{U}} R(x)R(x)'dz \right)^{-1} \int_{\mathcal{U}} R(z)z^{k} dz \right.$$

$$\times \int_{\mathcal{U}} R(z)'z^{\tilde{k}} dz \left( \int_{\mathcal{U}} R(z)R(z)'dz \right)^{-1} \int_{\mathcal{U}} (\partial^{m}R(z)) \left( \partial^{m}R(z) \right)' dz \right]$$

$$- \frac{1}{k!(\tilde{k}-m)!} \int_{\mathcal{U}} (\partial^{m}R(z))'z^{\tilde{k}-m} dz \left( \int_{\mathcal{U}} R(z)R(z)'dz \right)^{-1} \int_{\mathcal{U}} R(z)z^{k} dz \right.$$

$$- \frac{1}{\tilde{k}!(k-m)!} \int_{\mathcal{U}} (\partial^{m}R(z))'z^{k-m} dz \left( \int_{\mathcal{U}} R(z)R(z)'dz \right)^{-1} \int_{\mathcal{U}} R(z)z^{\tilde{k}} dz \right\},$$

we obtain the final result, applying  $\min_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} = 1$  w.p.a. 1 by Lemma A.4.

Finally, we give demonstrate simplifications of the above constants in specials cases. First, let

[m] = 0. In this case,

$$\operatorname{tr}\left\{\left(\int_{\mathcal{U}} R_j(z) R_j(z)' dz\right)^{-1} \int_{\mathcal{U}} \left(\partial^m R_j(z)\right) \left(\partial^m R_j(z)\right)' dz\right\} = g^* = \dim(R(\cdot)).$$

Therefore

$$\mathcal{V}_{K,d,0} = \frac{\dim(R(\cdot))}{\operatorname{vol}(\mathcal{X})} \int_{\mathcal{X}} \frac{\sigma^2(x)}{f(x)} w(x) dx,$$

and

$$\mathcal{B}_{K,d,0} = \frac{1}{2^{2K+d}} \sum_{\substack{k,\tilde{k} \\ [k]=[\tilde{k}]=K}} \frac{1}{k!\tilde{k}!} \left( \prod_{\ell=1}^{d} |\mathcal{X}_{\ell}|^{k_{\ell}+\tilde{k}_{\ell}} \right) \left\{ \int_{\mathcal{X}} \left( \partial^{k}\mu(x) \right) \left( \partial^{\tilde{k}}\mu(x) \right) w(x) dx \right\}$$

$$\times \left\{ \int_{\mathcal{U}} z^{k+\tilde{k}} dz - \int_{\mathcal{U}} R(z)' z^{\tilde{k}} dz \left( \int_{\mathcal{U}} R(z) R(z)' dz \right)^{-1} \int_{\mathcal{U}} R(z) z^{k} dz \right\}.$$
(A.10)

The variance constant is already considerably simplified when estimating the level of  $\mu(x)$ . For the bias, we examine two further specializations. First, if m=0 and d=1, then  $k=\tilde{k}=K$ , and so  $\int_{-1}^1 z^{2K} = \frac{2}{1+2K}$  and  $\prod_{\ell=1}^d |\mathcal{X}_{\ell}|^{k_{\ell}+\tilde{k}_{\ell}} = \operatorname{vol}(\mathcal{X})^{2K}$ . Hence

$$\mathcal{B}_{K,1,0} = \frac{\text{vol}(\mathcal{X})^{2K}}{2^{2K+1}(K!)^2} \left\{ \int_{\mathcal{X}} \left( \partial^K \mu(x) \right)^2 w(x) dx \right\} \times \left( \frac{2}{1+2K} - \left( \int_{-1}^1 R(x) x^K dx \right)' \left( \int_{-1}^1 R(x) R(x)' dx \right)^{-1} \left( \int_{-1}^1 R(x) x^K dx \right) \right).$$

Alternatively, if [m] = 0 and K = 1, we have

$$\mathscr{B}_{1,d,0} = \frac{1}{12} \sum_{\ell=1}^{d} |\mathcal{X}_{\ell}|^2 \int_{\mathcal{X}} \left( \frac{\partial \mu(x)}{\partial x_{\ell}} \right)^2 w(x) dx,$$

using  $R_j(z) = 1$  and  $[k] = [\tilde{k}] = 1$ , so that  $\int_{\mathcal{U}} R_j(z)' z^{\tilde{k}} dz = 0$ , and further, if  $k \neq \tilde{k}$ , then  $\int_{\mathcal{U}} z^{k+\tilde{k}} dz = 0$ , whence the entire term in braces is zero; otherwise  $k_\ell + \tilde{k}_\ell = 2$  and  $2^{d-1} \int_{-1}^1 z_\ell^2 dz_\ell = 2^d/3$ .

#### A.4 Bahadur Representation and Asymptotic Normality

For completeness, we give the explicit form of the random function  $\nu_n(x)$  from Eqn. (3). It is the remainder in the Bahadur representation of the identity functional,  $\theta_{1,0}$  of Example 1 or  $\theta_{2,0}$  of Example 2. Recall the definition of  $\psi_n(x,z)$  from the text and write:  $\hat{\mu}(x) - \mu(x) =$   $\frac{1}{n}\sum_{i=1}^n \psi_n(x,X_i)\varepsilon_i + \nu_n(x)$ , where the remainder  $\nu_n(x)$  is given by:

$$\nu_{n}(x) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{J_{n}^{d}} \tilde{R}_{j}(x)' \mathbb{I}_{n,j} \Omega_{j}^{-1} (\Omega_{j} - \hat{\Omega}_{j}) \hat{\Omega}_{j}^{-1} \tilde{R}_{j}(X_{i}) \varepsilon_{i}/q_{j}$$

$$+ \sum_{j=1}^{J_{n}^{d}} \tilde{R}_{j}(x)' \mathbb{I}_{n,j} \hat{\Omega}_{j}^{-1} \tilde{R}'_{j}(G - \tilde{R}_{j}\beta_{j}^{0})/(nq_{j})$$

$$+ \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} (\tilde{R}_{j}(x)'\beta_{j}^{0} - \mu_{j}(x))$$

$$+ \sum_{j=1}^{J_{n}^{d}} (\mathbb{I}_{n,j} - 1) \left[ \mu_{j}(x) + \tilde{R}_{j}(x)' \Omega_{j}^{-1} \tilde{R}'_{j}(Y - G)/(nq_{j}) \right].$$

Proof of Theorem 4. Recall from the text that  $\Theta_j = (\theta([R_j(\cdot)]_1), \dots, \theta([R_j(\cdot)]_{\dim(R(\cdot))}))'$ . Under the linearity condition on  $\theta(\cdot)$  in Assumption 3, we can write the remainder  $\theta(\nu_n)$  from Eqn. (3) as

$$\theta(\nu_n) = \sum_{j=1}^{J_n^d} \Theta_j' \mathbb{I}_{n,j} \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \hat{\Omega}_j^{-1} \tilde{R}_j' (Y - G) / (nq_j)$$
 (T<sub>n1</sub>)

$$+\sum_{j=1}^{J_n^d} \Theta_j' \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0) / (nq_j)$$
 (T<sub>n2</sub>)

$$+\sum_{j=1}^{J_n^d} \mathbb{I}_{n,j}(\Theta_j'\beta_j^0 - \theta(\mu_j))$$
 (T<sub>n3</sub>)

$$+ \sum_{j=1}^{J_n^d} (\mathbb{I}_{n,j} - 1) \left[ \theta(\mu_j) + \Theta_j' \Omega_j^{-1} \tilde{R}_j' (Y - G) / (nq_j) \right]. \tag{T_{n4}}$$

For  $T_{n1}$  write:

$$T_{n1} = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{J_n^d} \Theta_j' \mathbb{I}_{n,j} \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \hat{\Omega}_j^{-1} \tilde{R}_j(X_i) \varepsilon_i / q_j$$
 (T<sub>n11</sub>)

$$-\frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{J_n^a}\Theta_j'\mathbb{I}_{n,j}\Omega_j^{-1}(\hat{\Omega}_j-\Omega_j)\Omega_j^{-1}\tilde{R}_j(X_i)\varepsilon_i/q_j.$$
 (T<sub>n12</sub>)

Applying linearity and then continuity of the functional  $\theta(\cdot)$  from Assumption 3, followed by

Lemmas A.1, A.3, A.4, and A.5 we have the following bound on  $|T_{n11}|$ :

$$\begin{aligned} |T_{n11}| &= \left| \theta \left( \sum_{j=1}^{J_n^d} (\tilde{R}_j(\cdot))' \mathbb{I}_{n,j} \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \hat{\Omega}_j^{-1} \frac{\tilde{R}'_j(Y - G)}{nq_j} \right) \right| \\ &\leq C \max_{m:[m] \leq s} \left\| \sum_{j=1}^{J_n^d} (\partial^m \tilde{R}_j(\cdot))' \mathbb{I}_{n,j} \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \hat{\Omega}_j^{-1} \frac{\tilde{R}'_j(Y - G)}{nq_j} \right\|_{\infty} \\ &\leq C \left( \max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \|\partial^m \tilde{R}_j(\cdot)\|_{\infty} \right) \left( \max_{1 \leq j \leq J_n^d} \left| \Omega_j - \hat{\Omega}_j \right|^2 \right) \left( \max_{1 \leq j \leq J_n^d} \left| \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \right| \right) \\ &\times \left( \max_{1 \leq j \leq J_n^d} \left| \Omega_j^{-1} \right|^2 \right) \left( \max_{1 \leq j \leq J_n^d} \left| \frac{\tilde{R}'_j(Y - G)}{nq_j} \right| \right) \\ &= O_p \left( J_n^s \frac{J_n^d \log(J_n^d)}{n} \frac{J_n^{d-d\xi/2} \log(J_n^d)^{\xi/2}}{\sqrt{n}} \right) \\ &= O_p \left( \frac{J_n^{(2-\xi/2)d+s} \log(J_n^d)^{1+\xi/2}}{n^{3/2}} \right). \end{aligned}$$

For  $T_{n12}$ , begin by defining

$$W_j(i,l) = \mathbb{I}_{n,j}\Omega_j^{-1} \left( \tilde{R}_j(X_i)\tilde{R}_j(X_i)' - \mathbb{E}[\tilde{R}_j(X_i)\tilde{R}_j(X_i)'] \right) \Omega_j^{-1} \tilde{R}_j(X_l) \varepsilon_l,$$

so that we can express  $T_{n12}$  as

$$T_{n12} = \sum_{i=1}^{J_n^d} \frac{1}{(nq_j)^2} \sum_{i=1}^n \sum_{l=1}^n \Theta_j' W_j(i,l).$$

Observe that  $\mathbb{E}[T_{n12}] = 0$  and that unless i = h and l = m,  $\mathbb{E}[W_j(i, l)W_j(h, m)] = 0$ . By Lemmas A.1 and A.3, Assumption 1(c), and  $q_j \times J_n^{-d}$ , we have:

$$\max_{1 \le j \le J_n^d} \mathbb{E}\left[W_j(i,i)W_j(i,i)'\right] \le C\left(\max_{1 \le j \le J_n^d} \left|\Omega_j^{-1}\right|^4\right) \left(\left|\tilde{R}_j(\cdot)\right|_{\infty}^6\right) \left(\sup_{x \in \mathcal{X}} \sigma^2(x)\right) \max_{1 \le j \le J_n^d} \mathbb{E}[\mathbb{I}_{P_j}(X_i)]$$

$$= CO(1)O(1) \max_{1 \le j \le J_n^d} q_j = O(J_n^{-d}), \tag{A.11}$$

and similarly  $\max_{1 \leq j \leq J_n^d} \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] = O(J_n^{-2d})$ . Further note that Assumption 3 and Lemma A.1 give that:

$$\max_{1 \le j \le J_n^d} |\Theta_j| \le C \max_{1 \le j \le J_n^d} \left( \max_{m:[m] \le s} \|\partial^m \tilde{R}_j(\cdot)\|_{\infty} \right) = C \max_{1 \le j \le J_n^d} \max_{m:[m] \le s} \sup_{x \in P_j} |\partial^m \tilde{R}_j(x)| = O(J_n^s).$$
(A.12)

Therefore the variance of  $T_{n2}$  may be bounded as follows, using  $q_j \approx J_n^{-d}$ , Eqns. (A.11) and (A.12), linearity and continuity of  $\theta(\cdot)$ , and Lemma A.1:

$$\begin{split} \mathbb{E}[T_{n2}^2] &= \sum_{j=1}^{J_n^d} \frac{1}{(nq_j)^4} \sum_{i=1}^n \sum_{l=1}^n \Theta_j' \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] \Theta_j \\ &\leq \frac{CJ_n^{4d}}{n^4} \sum_{j=1}^{J_n^d} \Theta_j' \left\{ n \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1) \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] \right\} \Theta_j \\ &= \frac{J_n^{4d}}{n^4} \theta \left( \sum_{j=1}^{J_n^d} \tilde{R}_j(\cdot)' \left\{ n \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1) \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] \right\} \Theta_j \right) \\ &\leq \frac{CJ_n^{4d}}{n^4} \max_{m:[m] \leq s} \left\| \sum_{j=1}^{J_n^d} (\partial^m \tilde{R}_j(\cdot))' \left\{ n \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1) \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] \right\} \Theta_j \right\|_{\infty} \\ &\leq \frac{CJ_n^{4d}}{n^4} \left( \max_{1 \leq j \leq J_n^d} |\Theta_j| \right) \left( \max_{1 \leq j \leq J_n^d} n \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] + n(n-1) \mathbb{E}\left[W_j(i,l)W_j(i,l)'\right] \right) \\ &\times \left( \max_{m:[m] \leq s} \max_{1 \leq j \leq J_n^d} \sup_{x \in P_j} (\partial^m \tilde{R}_j(\cdot)) \right) \\ &= \frac{CJ_n^{4d}}{n^4} O(J_n^s) \left\{ nJ_n^{-d} + n^2J_n^{-2d} \right\} O(J_n^s) \\ &= O_p \left(J_n^{2d+2s}/n^2\right). \end{split}$$

Hence  $|T_{n2}| = O_p(J_n^{d+s}/n)$ , by Markov's inequality.

Following similar logic as  $T_{n11}$ , by linearity, continuity, Lemmas A.1 and A.4, and Eqn. (A.5):

$$\begin{aligned} |T_{n2}| &= \left| \theta \left( \sum_{j=1}^{J_n^d} \tilde{R}_j(\cdot)' \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0) / (nq_j) \right) \right| \\ &\leq C \max_{m:[m] \leq s} \left\| \partial^m \tilde{R}_j(x)' \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0) / (nq_j) \right\|_{\infty} \\ &\leq C \max_{m:[m] \leq s} \max_{1 \leq j \leq J_n^d} \sup_{x \in P_j} \left| \partial^m \tilde{R}_j(x)' \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \tilde{R}_j' (G - \tilde{R}_j \beta_j^0) / (nq_j) \right| \\ &\leq \left( \max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \| \partial^m \tilde{R}_j(\cdot) \|_{\infty} \right) \left( \max_{1 \leq j \leq J_n^d} \left| \mathbb{I}_{n,j} \hat{\Omega}_j^{-1/2} \right| \right) \left( \max_{1 \leq j \leq J_n^d} \left| \mathbb{I}_{n,j} \hat{\Omega}_j^{-1/2} \tilde{R}_j' \frac{(G - \tilde{R}_j \beta_j^0)}{nq_j} \right| \right) \\ &= O_p \left( J_n^{-((S + \alpha) \wedge K - s)} \right). \end{aligned}$$

Next, similar logic gives:

$$|T_{n3}| = \left| \theta \left( \sum_{j=1}^{J_n^d} \mathbb{I}_{n,j} (\tilde{R}_j(\cdot)' \beta_j^0 - \partial^m \mu_j(\cdot)) \right) \right|$$

$$\leq C \max_{m:[m] \leq s} \left\| \sum_{j=1}^{J_n^d} \mathbb{I}_{n,j} (\tilde{R}_j(\cdot)' \beta_j^0 - \partial^m \mu_j(\cdot)) \right\|_{\infty}$$

$$\leq C \max_{m:[m] \leq s} \max_{1 \leq j \leq J_n^d} \left\| \tilde{R}_j(\cdot)' \beta_j^0 - \partial^m \mu_j(\cdot)) \right\|_{\infty}$$

$$= O_p \left( J_n^{-((S+\alpha) \wedge K - s)} \right),$$

directly by Lemma A.2. Finally, from  $\min_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} = 1$  w.p.a. 1 it follows that  $T_{n4}$  is smaller order than the other terms. This completes the proof.

We now demonstrate a version of Theorem 4 that holds with probability one.

**Theorem A.2.** Let Assumption 3 hold with  $s \leq S \wedge (K-1)$ , and consider the representation in Eqn. (3). If the conditions of Theorem A.1 hold, then:

$$\theta(\nu_n) = O_{as} \left( \frac{J_n^{(3/2 - \xi/2)d + s} \log(J_n^d)^{(1+\xi)/2}}{n} + J_n^{-((S+\alpha) \wedge K - s)} \right).$$

Proof of Theorem A.2. Use the same expansion as in the proof of Theorem 4. Remainders  $T_{n2}$ ,  $T_{n3}$ , and  $T_{n4}$  are handled identically, applying the almost sure versions of the same steps. For  $T_{n1}$  we use similar steps as above for  $T_{n11}$ . Applying linearity and then continuity of the functional  $\theta(\cdot)$  from Assumption 3, followed by Lemmas A.1, A.3, A.4, and A.5 we have the following bound on  $|T_{n1}|$ :

$$|T_{n1}| = \left| \theta \left( \sum_{j=1}^{J_n^d} (\tilde{R}_j(\cdot))' \mathbb{I}_{n,j} \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \hat{\Omega}_j^{-1} \frac{\tilde{R}_j'(Y - G)}{nq_j} \right) \right|$$

$$\leq C \max_{m:[m] \leq s} \left\| \sum_{j=1}^{J_n^d} (\partial^m \tilde{R}_j(\cdot))' \mathbb{I}_{n,j} \Omega_j^{-1} (\Omega_j - \hat{\Omega}_j) \hat{\Omega}_j^{-1} \frac{\tilde{R}_j'(Y - G)}{nq_j} \right\|_{\infty}$$

$$\leq C \left( \max_{1 \leq j \leq J_n^d} \max_{m:[m] \leq s} \|\partial^m \tilde{R}_j(\cdot)\|_{\infty} \right) \left( \max_{1 \leq j \leq J_n^d} \left| \Omega_j - \hat{\Omega}_j \right| \right) \left( \max_{1 \leq j \leq J_n^d} \left| \mathbb{I}_{n,j} \hat{\Omega}_j^{-1} \right| \right)$$

$$\times \left( \max_{1 \leq j \leq J_n^d} \left| \Omega_j^{-1} \right| \right) \left( \max_{1 \leq j \leq J_n^d} \left| \frac{\tilde{R}_j'(Y - G)}{nq_j} \right| \right)$$

$$= O_{as} \left( J_n^s \sqrt{\frac{J_n^d \log(J_n^d)}{n}} \frac{J_n^{d - d\xi/2} \log(J_n^d)^{\xi/2}}{\sqrt{n}} \right)$$

$$= O_{as} \left( \frac{J_n^{d(3-\xi)/2+s} \log(J_n^d)^{(1+\xi)/2}}{n} \right),$$

where the second inequality holds because the functional only operates on  $\tilde{R}_{i}(\cdot)$ .

Proof of Theorem 5(a). Recall the definitions given in Eqn. (4) and that consecutive uses of the symbol  $\asymp$  are to be interpreted pairwise. By assumption  $\sigma^2(x)$  is bounded away from zero on  $\mathcal{X}$ , so for some  $\overline{\sigma}$ ,  $\sigma^2(\cdot) \asymp \overline{\sigma}$ . Then under Assumption 1(c) we have  $\Gamma_j \asymp \overline{\sigma} \mathbb{E}\left[\tilde{R}_j(X)\tilde{R}_j(X)'\right] \asymp \Omega_j$ . Again using  $\sigma^2(\cdot) \asymp \overline{\sigma}$  and  $\Gamma_j \asymp \Omega_j$ , and further by  $q_j \asymp J_n^{-d}$  and Lemma A.3 we have:

$$V_n = \mathbb{E}\left[\Psi_n(X)^2 \sigma^2(X)\right] \approx \mathbb{E}[\Psi_n(X)^2] = \|\Psi_n\|_2^2, \text{ and also}$$

$$V_n \approx \sum_{j=1}^{J_n^d} \Theta_j' \Omega_j^{-1} \Theta_j / q_j \approx J_n^d \sum_{j=1}^{J_n^d} |\Theta_j|^2.$$
(A.13)

The condition that  $\theta(\nu_n) = o_p(\sqrt{V_n}/\sqrt{n})$  and the result of Theorem 4 immediately give the triangular array representation of the Theorem. By construction,  $\mathbb{E}\left[\Psi_n(X_i)\varepsilon_i/\sqrt{nV_n}\right] = 0$  and  $\sum_{i=1}^n \mathbb{E}\left[\left(\Psi_n(X_i)\varepsilon_i/\sqrt{nV_n}\right)^2\right] = 1$ . It remains to verify the Lindeberg condition. For any  $\delta > 0$ , by the Hölder and Markov inequalities, Assumption 1(c),  $V_n \simeq \|\Psi_n\|_2^2$  by Eqn. (A.13), and the conditions of the Theorem,

$$\begin{split} \sum_{i=1}^{n} \mathbb{E} \left[ \left( \frac{\Psi_{n}(X_{i})\varepsilon_{i}}{\sqrt{nV_{n}}} \right)^{2} \mathbb{I} \left\{ \left| \frac{\Psi_{n}(X_{i})\varepsilon_{i}}{\sqrt{nV_{n}}} \right| > \delta \right\} \right] &\leq n \left( \mathbb{E} \left[ \left( \frac{\Psi_{n}(X_{i})\varepsilon_{i}}{\sqrt{nV_{n}}} \right)^{2+\eta} \right] \right)^{\frac{2}{2+\eta}} \left( \mathbb{P} \left[ \left| \frac{\Psi_{n}(X_{i})\varepsilon_{i}}{\sqrt{nV_{n}}} \right| > \delta \right] \right)^{\frac{\eta}{2+\eta}} \\ &\leq \frac{n}{\delta^{\eta}} \mathbb{E} \left[ \left| \frac{\Psi_{n}(X_{i})\varepsilon_{i}}{\sqrt{nV_{n}}} \right|^{2+\eta} \right] \\ &= \frac{1}{\delta^{\eta}} \frac{\mathbb{E} \left[ \left| \Psi_{n}(X_{i}) \right|^{2+\eta} \mathbb{E} \left[ \left| \varepsilon_{i} \right|^{2+\eta} \mid X_{i} \right] \right]}{n^{\eta/2} V_{n}^{1+\eta/2}} \\ &= O\left( \left( \frac{\|\Psi_{n}\|_{2+\eta}}{n^{\eta/(4+2\eta)} \|\Psi_{n}\|_{2}} \right)^{2+\eta} \right) \to 0. \end{split}$$

Convergence in distribution follows by the Lindeberg-Feller central limit theorem.

For the second conclusion of Theorem 5(a), observe that by  $\mathbb{1}_{n,j} = 1$  w.p.a. 1, uniformly in j, we have  $\hat{V}_n/V_n - 1 = T_{n1} + T_{n2} + T_{n3} + o_p(1)$ , where

$$T_{n1} = V_n^{-1} \hat{V}_n - V_n^{-1} \sum_{j=1}^{J_n^d} \mathbb{I}_{n,j} \Theta_j' \hat{\Omega}_j^{-1} \tilde{\Gamma}_j \hat{\Omega}_j^{-1} \Theta_j / q_j,$$

$$T_{n2} = V_n^{-1} \sum_{j=1}^{J_n^d} \mathbb{I}_{n,j} \Theta_j' (\hat{\Omega}_j^{-1} + \Omega_j^{-1}) \tilde{\Gamma}_j \left( \hat{\Omega}_j^{-1} - \Omega_j^{-1} \right) \Theta_j / q_j,$$

$$T_{n3} = V_n^{-1} \sum_{j=1}^{J_n^d} \Theta_j' \Omega_j^{-1} \left( \tilde{\Gamma}_j - \Gamma_j \right) \Omega_j^{-1} \Theta_j / q_j,$$

and  $\tilde{\Gamma}_j = \sum_{i=1}^n \tilde{R}_j(X_i) \tilde{R}_j(X_i)' \varepsilon_i^2/(nq_j)$ . First, expanding the squared terms,  $T_{n1}$  can be split into two terms, and upon applying Lemmas A.1 and A.4,  $q_j \approx J_n^{-d}$ , Eqns. (A.4) and (A.13), and the condition of the Theorem, we find that

$$T_{n1} = V_{n}^{-1} \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} \Theta_{j}' \hat{\Omega}_{j}^{-1} \left( \frac{1}{nq_{j}} \sum_{i=1}^{n} \tilde{R}_{j}(X_{i}) \tilde{R}_{j}(X_{i})' (\hat{\mu}(X_{i}) - \mu(X_{i}))^{2} \right) \hat{\Omega}_{j}^{-1} \Theta_{j}/q_{j}$$

$$- V_{n}^{-1} \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} \Theta_{j}' \hat{\Omega}_{j}^{-1} \left( \frac{1}{nq_{j}} \sum_{i=1}^{n} \tilde{R}_{j}(X_{i}) \tilde{R}_{j}(X_{i})' 2\varepsilon_{i} (\hat{\mu}(X_{i}) - \mu(X_{i})) \right) \hat{\Omega}_{j}^{-1} \Theta_{j}/q_{j}$$

$$\leq \left( \max_{1 \leq j \leq J_{n}^{d}} \mathbb{I}_{n,j} |\hat{\Omega}_{j}^{-1}|^{2} \right) \left( \max_{1 \leq j \leq J_{n}^{d}} ||\tilde{R}_{j}(\cdot)||_{\infty}^{2} \right) (||\hat{\mu} - \mu||_{\infty})$$

$$\times \left\{ ||\hat{\mu} - \mu||_{\infty} \frac{J_{n}^{d}}{V_{n}} \sum_{j=1}^{J_{n}^{d}} |\Theta_{j}|^{2} \frac{1}{nq_{j}} \sum_{i=1}^{n} \mathbb{I}_{P_{j}}(X_{i}) + \frac{J_{n}^{d}}{V_{n}} \sum_{j=1}^{J_{n}^{d}} |\Theta_{j}|^{2} \frac{1}{nq_{j}} \sum_{i=1}^{n} \mathbb{I}_{P_{j}}(X_{i}) |\varepsilon_{i}| \right\}$$

$$= O_{p} (||\hat{\mu} - \mu||_{\infty}) \times \{o_{p}(1)O(1)O_{p}(1) + O_{p}(1)\} = o_{p}(1),$$

where the final line additionally uses Assumption 1(c) and the final relation of Eqn. (A.13) to give:

$$\mathbb{E}\left[\frac{J_n^d}{V_n}\sum_{j=1}^{J_n^d}|\Theta_j|^2\frac{1}{nq_j}\sum_{i=1}^n\mathbb{I}_{P_j}(X_i)|\varepsilon_i|\right] \leq C\frac{J_n^d}{V_n}\sum_{j=1}^{J_n^d}|\Theta_j|^2\frac{\mathbb{E}\left[\mathbb{I}_{P_j}(X_i)\mathbb{E}\left[|\varepsilon_i|\mid X_i\right]\right]}{q_j} = O(1).$$

By Lemma A.1 and otherwise identical steps to the above, we get:

$$\mathbb{E}\left[\frac{1}{V_n}\sum_{j=1}^{J_n^d}|\Theta_j|^2|\tilde{\Gamma}_j|/q_j\right] \leq \frac{J_n^d}{V_n}\sum_{j=1}^{J_n^d}|\Theta_j|^2\frac{1}{nq_j}\sum_{i=1}^n\mathbb{E}\left[\left|\tilde{R}_j(X)\right|^2\varepsilon_i^2\right] = O(1).$$

Therefore, applying Lemmas A.3 and A.4:

$$\begin{split} |T_{n2}| &= V_n^{-1} \sum_{j=1}^{J_n^d} \mathbb{I}_{n,j} \Theta_j' (\hat{\Omega}_j^{-1} + \Omega_j^{-1}) \tilde{\Gamma}_j \Omega_j^{-1} \left( \hat{\Omega}_j - \Omega_j \right) \hat{\Omega}_j^{-1} \Theta_j / q_j \\ &\leq C \left( \max_{1 \leq j \leq J_n^d} \mathbb{I}_{n,j} |\hat{\Omega}_j^{-1}|^3 \vee \max_{1 \leq j \leq J_n^d} |\Omega_j^{-1}|^3 \right) \left( \max_{1 \leq j \leq J_n^d} |\hat{\Omega}_j - \Omega_j| \right) V_n^{-1} \sum_{j=1}^{J_n^d} |\Theta_j|^2 |\tilde{\Gamma}_j| / q_j \\ &= O_p \left( \sqrt{J_n^d \log(J_n^d) / n} \right) = o_p(1). \end{split}$$

Finally, referring to the definitions in Eqn. (3), observe that  $T_{n3} = \sum_{i=1}^{n} T_{n3}(i)/n$ , where  $T_{n3}(i) = V_n^{-1}(\Psi_n(X_i)^2 \varepsilon_i^2 - \mathbb{E}[\Psi_n(X_i)^2 \varepsilon_i^2])$ , so that  $\mathbb{E}[T_{n3}(i)] = 0$ . Consider two cases. First, suppose  $\eta < 2$ . Then by Burkholder's inequality, the fact that for  $\delta \in (0,1)$ ,  $(a+b)^{(1+\delta)/2} \le a^{(1+\delta)/2} + b^{(1+\delta)/2}$ , the  $c_r$  inequality, Jensen's inequality, Assumption 1(c), and the first relation of Eqn. (A.13):

$$\mathbb{E}\left[\left|\frac{1}{n}\sum_{i=1}^{n}T_{n3}(i)\right|^{1+\eta/2}\right] \leq \frac{C}{n^{1+\eta/2}}\mathbb{E}\left[\left|\sum_{i=1}^{n}T_{n3}(i)^{2}\right|^{(1+\eta/2)/2}\right] \\
\leq \frac{C}{n^{1+\eta/2}}\mathbb{E}\left[\sum_{i=1}^{n}|T_{n3}(i)|^{1+\eta/2}\right] \\
\leq \frac{C}{n^{\eta/2}}2^{\eta/2}\frac{\mathbb{E}\left[\left|\Psi_{n}(X_{i})^{2}\varepsilon_{i}^{2}\right|^{1+\eta/2}\right] + \left(\mathbb{E}\left[\Psi_{n}(X_{i})^{2}\varepsilon_{i}^{2}\right]\right)^{1+\eta/2}}{V_{n}^{1+\eta/2}} \\
\leq \frac{C}{n^{\eta/2}}\frac{\mathbb{E}\left[\left|\Psi_{n}(X_{i})\right|^{2+\eta}\mathbb{E}\left[\left|\varepsilon_{i}\right|^{2+\eta}\mid X\right]\right] + \left(\mathbb{E}\left[\Psi_{n}(X_{i})^{2}\sigma^{2}(X)\right]\right)^{1+\eta/2}}{V_{n}^{1+\eta/2}} \\
\leq \frac{C}{n^{\eta/2}}\frac{\mathbb{E}\left[\left|\Psi_{n}(X_{i})\right|^{2+\eta}\mathbb{E}\left[\left|\varepsilon_{i}\right|^{2+\eta}\mid X\right]\right] + \left(\mathbb{E}\left[\Psi_{n}(X_{i})^{2}\sigma^{2}(X)\right]\right)^{1+\eta/2}}{V_{n}^{1+\eta/2}} \\
= O\left(\left(\frac{\|\Psi_{n}\|_{2+\eta}}{n^{\eta/(4+2\eta)}\|\Psi_{n}\|_{2}}\right)^{2+\eta}\right) \to 0.$$

Next, for the case of  $\eta \geq 2$  we utilize only the fourth moment to find that:

$$\mathbb{E}\left[\left(\frac{1}{n}\sum_{i=1}^{n}T_{n3}(i)\right)^{2}\right] = \frac{1}{n}\mathbb{E}\left[T_{n3}(i)^{2}\right] = \frac{1}{n}\mathbb{E}\left[V_{n}^{-1}(\Psi_{n}(X_{i})^{2}\varepsilon_{i}^{2} - \mathbb{E}[\Psi_{n}(X_{i})^{2}\varepsilon_{i}^{2}])^{2}\right]$$

$$\leq \frac{1}{n}V_{n}^{-2}\mathbb{E}\left[\Psi_{n}(X_{i})^{4}\varepsilon_{i}^{4}\right]$$

$$= O\left(\left(\frac{\|\Psi_{n}\|_{4}}{n^{1/4}\|\Psi_{n}\|_{2}}\right)^{4}\right) \to 0,$$

again using Jensen's inequality, Assumption 1(c), and the first relation of Eqn. (A.13). In either case,  $T_{3n} = o_p(1)$  by Markov's inequality.

Proof of Theorem 5(b). By Assumption 1(c), the Cauchy-Schwarz and triangle inequalities, and the conditions of the Theorem:

$$\begin{split} V_n - V &= \mathbb{E}[(\Psi_n(X)^2 - \Psi(X)^2)\sigma^2(X)] \\ &= \mathbb{E}[(\Psi_n(X) - \Psi(X))(\Psi_n(X) + \Psi(X))\sigma^2(X)] \\ &= \mathbb{E}[(\Psi_n(X) - \Psi(X))(\Psi_n(X) - \Psi(X) + 2\Psi(X))\sigma^2(X)] \\ &< C\mathbb{E}[(\Psi_n(X) - \Psi(X))^2]^{1/2}\mathbb{E}[(\Psi_n(X) - \Psi(X) + 2\Psi(X))^2]^{1/2} \end{split}$$

$$= C\|\Psi_n - \Psi\|_2(\|\Psi_n - \Psi + 2\Psi\|_2)$$

$$\leq C\|\Psi_n - \Psi\|_2(\|\Psi_n - \Psi\|_2 + 2\|\Psi\|_2) \to 0,$$
(A.14)

whence the second conclusion.

Using the above result, the assumed mean-square convergence of  $\Psi_n(X)$ , and the remainder condition of the Theorem,

$$\frac{\sqrt{n}(\theta(\hat{\mu}) - \theta(\mu))}{\sqrt{V_n}} = \sum_{i=1}^n \left[ \frac{\Psi(X_i)\varepsilon_i}{\sqrt{nV}} + \frac{(\Psi_n(X_i) - \Psi(X_i))\varepsilon_i}{\sqrt{nV}} + \frac{\Psi_n(X_i)\varepsilon_i}{\sqrt{nV}} \left( \frac{\sqrt{V}}{\sqrt{V_n}} - 1 \right) \right] + \frac{\sqrt{n}\theta(\nu_n)}{\sqrt{V_n}}$$

$$= \frac{1}{\sqrt{n}} \sum_{i=1}^n \frac{\Psi(X_i)\varepsilon_i}{\sqrt{V}} + o_p(1).$$

Convergence in distribution now follows under the assumed moment condition on  $\Psi(X)$  and a standard central limit theorem.

For the final conclusion, as in the proof of Theorem 5(a) write  $\hat{V}_n/V_n-1=T_{n1}+T_{n2}+T_{n3}+o_p(1)$ , for  $T_{n1}$ ,  $T_{n2}$ , and  $T_{n3}$  defined there. As above,  $T_{n1}=o_p(1)$  and  $T_{n2}=o_p(1)$ . Next,

$$T_{n3} = \left(\frac{1}{V_n} - \frac{1}{V}\right) \frac{1}{n} \sum_{i=1}^n \Psi_n(X_i)^2 \varepsilon_i^2 + \frac{1}{n} \sum_{i=1}^n \frac{[\Psi_n(X_i)^2 - \Psi(X_i)^2]\varepsilon_i^2}{V} + \frac{1}{nV} \sum_{i=1}^n (\Psi(X_i)^2 \varepsilon_i^2 - V),$$

where the first two terms tend to zero in probability by Eqn. (A.14) (and the steps therein) and Markov's inequality, and the third by the law of large numbers.  $\Box$ 

# B Unconditional IMSE Expansion for K = 1

In the especial case of a piecewise constant fit (K = 1) the leading constants in the unconditional IMSE may also be computed explicitly. The special structure of the constant-fit partitioning estimator is crucial to obtaining this result. When K = 1,  $R'_j R_j = N_j$  is a binomial random variable whose inverse moments can be calculated or approximated accurately, as done in Lemma B.1 below. The Theorem below shows that for this special case, the unconditional IMSE has the same leading constants as the conditional IMSE.

**Theorem B.1.** Suppose the conditions of Theorem 3 hold with S = 0. Then, if w(x) is continuous, the piecewise-constant partitioning estimator (K = 1) satisfies:

$$\int_{\mathcal{X}} \mathbb{E}[(\hat{\mu}(x) - \mu(x))^2] w(x) dx = \frac{J_n^d}{n} [\mathcal{V}_{1,d,0} + o(1)] + \frac{1}{J_n^2} [\mathcal{B}_{1,d,0} + o(1)].$$

Prior to proving the Theorem, we give several results regarding binomial random variables.

**Lemma B.1.** Let the conditions of Theorem B.1 hold. Recall that for K = 1,  $\tilde{R}'_j \tilde{R}_j = \sum_{i=1}^n \mathbb{I}_{P_j}(X_i)$  is the number of observations in  $P_j$ . Call this  $N_j$ . Further define  $N_{j,-i} = \sum_{l \neq i} \mathbb{I}_{P_j}(X_l)$  and  $N_{j,-i-l} = \sum_{m \neq i,l} \mathbb{I}_{P_j}(X_m)$ . Then  $N_j \sim Bin(n,q_j)$ ,  $N_{j,-i} \sim Bin(n-1,q_j)$ , and  $N_{j,-i-l} \sim Bin(n-2,q_j)$ . All remainder terms are uniform in  $1 \leq j \leq J_n^d$ .

1. 
$$\mathbb{E}\left[\mathbb{I}\{N_{j}>0\}\frac{1}{N_{j}}\right] = \frac{1}{nq_{j}} + o\left(J_{n}^{d}/n\right).$$
2. 
$$\mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] = \frac{1 - (1 - q_{j})^{n}}{nq_{j}}.$$
3. 
$$\mathbb{E}\left[\frac{1}{(N_{j,-i}+1)^{2}}\right] = \frac{1}{(nq_{j})^{2}}\left(1 + o\left(J_{n}^{d}/n\right)\right)$$
4. 
$$\mathbb{E}\left[\frac{1}{(N_{j,-i-l}+2)^{2}}\right] = \frac{1}{n(n-1)q_{j}^{2}}\left(1 - \frac{1}{nq_{j}}\left(1 + o\left(J_{n}^{d}/n\right)\right)\right).$$

Proof. The first result follows from Rempala (2003, Proceeds of the American Mathematical Society) or Znidaric (2009, The Open Statistics and Probability Journal), whose expansions remain valid if  $q_j \to 0$ ,  $nq_j \to \infty$ , and because  $q_j \asymp J_n^{-d}$ , the result holds uniformly in  $1 \le j \le J_n^d$ . The final three results are proven by direct calculation: an exact expression for each moment may be found in terms of n,  $q_j$ , and  $\mathbb{E}[\mathbb{I}\{N_j > 0\}N_j^{-1}]$ , and then the claims follow by substituting the first result. The calculations are as follows, where we make use of the facts that

$$\frac{n}{k} \binom{n-1}{k-1} = \binom{n}{k}$$
, and,  $\mathbb{E}\left[\mathbb{1}\{N_j > 0\} \frac{1}{N_j}\right] = \sum_{k=1}^n \frac{1}{k} \binom{n}{k} q_j^k (1 - q_j)^{n-k}$ .

For the second result:

$$\mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] = \sum_{k=0}^{n-1} \frac{1}{k+1} \binom{n-1}{k} q_j^k (1-q_j)^{n-1-k}$$

$$= \sum_{\tilde{k}=1}^n \frac{1}{\tilde{k}} \binom{n-1}{\tilde{k}-1} q_j^{\tilde{k}-1} (1-q_j)^{n-\tilde{k}}$$

$$= \frac{1}{nq_j} \sum_{\tilde{k}=1}^n \binom{n}{\tilde{k}} q_j^{\tilde{k}} (1-q_j)^{n-\tilde{k}}$$

$$= \frac{1}{nq_j} \mathbb{P}[N_j > 0] = \frac{1}{nq_j} (1 - \mathbb{P}[N_j = 0])$$

$$= \frac{1 - (1-q_j)^n}{nq_j}.$$

For the third result:

$$\mathbb{E}\left[\frac{1}{(N_{j,-i}+1)^2}\right] = \sum_{k=0}^{n-1} \frac{1}{(k+1)^2} \binom{n-1}{k} q_j^k (1-q_j)^{n-1-k}$$

$$= \sum_{\tilde{k}=1}^n \frac{1}{\tilde{k}^2} \binom{n-1}{\tilde{k}-1} q_j^{\tilde{k}-1} (1-q_j)^{n-\tilde{k}}$$

$$= \frac{1}{nq_j} \sum_{\tilde{k}=1}^n \frac{1}{\tilde{k}} \binom{n}{\tilde{k}} q_j^{\tilde{k}} (1-q_j)^{n-\tilde{k}}$$

$$= \frac{1}{nq_j} \mathbb{E}\left[\mathbb{I}\{N_j > 0\} \frac{1}{N_j}\right].$$

For the fourth result:

$$\begin{split} \mathbb{E}\left[\frac{1}{(N_{j,-i-l}+2)^2}\right] &= \sum_{k=0}^{n-2} \frac{1}{(k+2)^2} \binom{n-2}{k} q_j^k (1-q_j)^{n-2-k} \\ &= \sum_{\tilde{k}=2}^n \frac{1}{\tilde{k}^2} \binom{n-2}{\tilde{k}-2} q_j^{\tilde{k}-2} (1-q_j)^{n-\tilde{k}} \\ &= \frac{1}{n(n-1)q_j^2} \sum_{\tilde{k}=1}^n \frac{\tilde{k}-1}{\tilde{k}} \binom{n}{\tilde{k}} q_j^{\tilde{k}} (1-q_j)^{n-\tilde{k}} \\ &= \frac{1}{n(n-1)q_j^2} \left\{ \sum_{\tilde{k}=1}^n \binom{n}{\tilde{k}} q_j^{\tilde{k}} (1-q_j)^{n-\tilde{k}} - \sum_{\tilde{k}=1}^n \frac{1}{\tilde{k}} \binom{n}{\tilde{k}} q_j^{\tilde{k}} (1-q_j)^{n-\tilde{k}} \right\} \\ &= \frac{1}{n(n-1)q_j^2} \left( \mathbb{E}\left[\mathbb{I}\{N_j>0\}\right] - \mathbb{E}\left[\mathbb{I}\{N_j>0\} \frac{1}{N_j}\right] \right) \end{split}$$

$$= \frac{1}{n(n-1)q_j^2} \left( 1 - (1-q_j)^n - \mathbb{E}\left[\mathbb{I}\{N_j > 0\} \frac{1}{N_j}\right] \right)$$
$$= \frac{1}{n(n-1)q_j^2} \left( 1 - \mathbb{E}\left[\mathbb{I}\{N_j > 0\} \frac{1}{N_j}\right] \right) - o(J_n^{2d}/n^2),$$

where the final equality uses  $q_j \approx J_n^{-d}$ , hence  $(1 - q_j)^n \to 0$  exponentially.

Proof of Theorem B.1. Recall that  $\mathbf{X}_n = (X_1, \dots, X_n)'$  and expand as follows:

$$\int_{X} \mathbb{E}\left[\left(\hat{\mu}(x) - \mu(x)\right)^{2}\right] f(x) dx = \int_{\mathcal{X}} \left\{ \mathbb{E}\left[\mathbb{V}\left[\hat{\mu}(x) \mid \mathbf{X}_{n}\right]\right] + \mathbb{V}\left[\mathbb{E}\left[\hat{\mu}(x) \mid \mathbf{X}_{n}\right]\right] + \left(\mathbb{E}\left[\hat{\mu}(x)\right] - \mu(x)\right)^{2}\right\} f(x) dx.$$

We examine each term one at a time. The following two results will be used frequently. Recall that the volume of cell  $P_j$  is denoted  $\operatorname{vol}(P_j)$  and similarly for  $\operatorname{vol}(\mathcal{X})$ . First observe that  $q_j = \int_{P_j} f(z)dz = f(\overline{p}_j)\operatorname{vol}(P_j) + o(J_n^{-d})$ , by Lemma A.6. Further, under the conditions placed on the partition and Assumption 1(b),  $\operatorname{vol}(P_j) = \operatorname{vol}(\mathcal{X})/J_n^d$ .

Consider the first term. Using the two results above, as well as Lemmas B.1, A.6, and A.7 (under Assumption 2(a)), we have:

$$\begin{split} \int_{\mathcal{X}} \mathbb{E}\left[\mathbb{V}\left[\hat{\mu}(x) \mid \mathbf{X}_{n}\right]\right] w(x) dx &= \int_{\mathcal{X}} \mathbb{E}\left[\sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} \mathbb{I}_{P_{j}}(x) \frac{1}{N_{j}^{2}} \sum_{i=1}^{n} \mathbb{I}_{P_{j}}(X_{i}) \sigma^{2}(X_{i})\right] w(x) dx \\ &= \int_{\mathcal{X}} \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{P_{j}}(x) \sum_{i=1}^{n} \mathbb{E}\left[\mathbb{I}_{n,j} \frac{1}{N_{j}^{2}} \mathbb{I}_{P_{j}}(X_{i}) \sigma^{2}(X_{i})\right] w(x) dx \\ &= \int_{\mathcal{X}} \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{P_{j}}(x) \sum_{i=1}^{n} \mathbb{E}\left[\frac{1\{N_{j,-i} + \mathbb{I}_{P_{j}}(X_{i}) > 0\}}{(N_{j,-i} + \mathbb{I}_{P_{j}}(X_{i}))^{2}}\right] \mathbb{E}\left[\mathbb{I}_{P_{j}}(X_{i}) \sigma^{2}(X_{i})\right] w(x) dx \\ &= \sum_{j=1}^{J_{n}^{d}} \left(\int_{\mathcal{X}} \mathbb{I}_{P_{j}}(x) w(x) dx\right) \sum_{i=1}^{n} \mathbb{E}\left[\frac{1}{(N_{j,-i} + \mathbb{I}_{P_{j}}(X_{i}))^{2}}\right] \mathbb{E}\left[\mathbb{I}_{P_{j}}(X_{i}) \sigma^{2}(X_{i})\right] \\ &= \sum_{j=1}^{J_{n}^{d}} n \left(w(\overline{p}_{j}) \operatorname{vol}(P_{j}) + o(J_{n}^{-d})\right) \left(\frac{1}{(nq_{j})^{2}} + o\left((J_{n}^{d}/n)^{3}\right)\right) \left(\int_{P_{j}} \sigma^{2}(z) f(z) dz\right) \\ &= \sum_{j=1}^{J_{n}^{d}} n w(\overline{p}_{j}) \operatorname{vol}(P_{j}) \left(\frac{1}{n^{2}q_{j})^{2}} + o\left((J_{n}^{d}/n)^{2}\right)\right) \left(\sigma^{2}(\overline{p}_{j}) f(\overline{p}_{j}) \operatorname{vol}(P_{j}) + o(J_{n}^{-d})\right) \\ &= \sum_{j=1}^{J_{n}^{d}} \frac{\sigma^{2}(\overline{p}_{j}) f(\overline{p}_{j}) w(\overline{p}_{j}) \operatorname{vol}(P_{j})^{2}}{nq_{j}} + o\left(\frac{J_{n}^{d}}{n}\right) \\ &= \frac{J_{n}^{d}}{\operatorname{vol}(\mathcal{X})} \sum_{i=1}^{J_{n}^{d}} \frac{\sigma^{2}(\overline{p}_{j}) w(\overline{p}_{j})}{f(\overline{p}_{j})} \operatorname{vol}(P_{j}) + o\left(\frac{J_{n}^{d}}{n}\right) \end{aligned}$$

$$= \frac{J_n^d}{\operatorname{vol}(\mathcal{X}) n} \int_{\mathcal{X}} \frac{\sigma^2(x)}{f(x)} w(x) dx [1 + o(1)] + o\left(J_n^d/n\right). \tag{B.1}$$

For the second variance term, define the following:

$$\bar{w}_j = \int_{P_j} w(x) dx; \qquad \bar{\mu}_j \equiv \mathbb{E}\left[\mathbb{1}_{P_j}(X)\mu(X)\right]; \qquad \bar{N}_j \equiv \mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] = \frac{1-(1-q_j)^n}{nq_j}.$$

Then we have:

$$\int_{\mathcal{X}} \mathbb{V}\left[\mathbb{E}\left[\hat{\mu}(x) \mid \mathbf{X}_{n}\right]\right] w(x) dx = \int_{\mathcal{X}} \mathbb{V}\left[\sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{n,j} \mathbb{I}_{P_{j}}(x) \sum_{i=1}^{n} \mathbb{I}_{P_{j}}(X_{i}) \mu(X_{i}) / N_{j}\right] w(x) dx \\
= \int_{\mathcal{X}} \mathbb{E}\left[\sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{P_{j}}(x) \left\{\sum_{i=1}^{n} \frac{\mathbb{I}\left\{N_{j,-i}+1>0\right\}}{N_{j,-i}+1} \mathbb{I}_{P_{j}}(X_{i}) \mu(X_{i}) - \bar{\mu}_{j} \bar{N}_{j}\right\}^{2}\right] w(x) dx \\
= \mathbb{E}\left[\sum_{j=1}^{J_{n}^{d}} \bar{w}_{j} \left\{\sum_{i=1}^{n} \frac{\mathbb{I}\left\{N_{j,-i}+1>0\right\}}{N_{j,-i}+1} \mathbb{I}_{P_{j}}(X_{i}) \mu(X_{i}) - \bar{\mu}_{j} \bar{N}_{j}\right\}^{2}\right] \\
= \sum_{j=1}^{J_{n}^{d}} \bar{w}_{j} n \mathbb{E}\left[\mathbb{I}_{P_{j}}(X) \mu(X)^{2}\right] \mathbb{E}\left[\frac{1}{(N_{j,-i}+1)^{2}}\right] \qquad (V_{n1}) \\
+ \sum_{j=1}^{J_{n}^{d}} \bar{w}_{j} n(n-1) \bar{\mu}_{j}^{2} \mathbb{E}\left[\frac{1}{(N_{j,-i-l}+2)^{2}}\right] \qquad (V_{n2}) \\
- \sum_{j=1}^{J_{n}^{d}} \bar{w}_{j} (n \bar{\mu}_{j} \bar{N}_{j})^{2}. \qquad (V_{n3})$$

Now apply the two results above, as well as Lemmas B.1, A.6, and A.7 (under Assumption 2(a)), to get:

$$V_{1n} = \sum_{j=1}^{J_n^d} n \bar{w}_j \left( \int_{P_j} \mu(z)^2 f(z) dz \right) \frac{1}{(nq_j)^2} \left( 1 + o\left( J_n^d/n \right) \right)$$

$$= \sum_{j=1}^{J_n^d} \bar{w}_j \left( \mu(\bar{p}_j)^2 q_j + o(J_n^{-d}) \right) \left( \frac{1}{nq_j^2} + o\left( \left( J_n^d/n \right)^2 \right) \right)$$

$$= \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{\mu(\bar{p}_j)^2 \bar{w}_j}{q_j} + o\left( \frac{J_n^d}{n} \right). \tag{B.2}$$

Similarly:

$$V_{2n} = \sum_{j=1}^{J_n^d} \bar{w}_j \bar{\mu}_j^2 n(n-1) \frac{1}{n(n-1)q_j^2} \left( 1 - \frac{1}{nq_j} \left( 1 + o\left(J_n^d/n\right) \right) \right)$$

$$= \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} - \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^3} + o\left(J_n^d/n\right)$$

$$= \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} - \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{\bar{w}_j (\mu(\bar{p}_j)q_j + o(J_n^{-d}))^2}{q_j^3} + o\left(J_n^d/n\right)$$

$$= \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} - \frac{1}{n} \sum_{j=1}^{J_n^d} \frac{\bar{w}_j \mu(\bar{p}_j)^2}{q_j} + o\left(J_n^d/n\right). \tag{B.3}$$

For the final term, similar steps give:

$$V_{3n} = -\sum_{j=1}^{J_n^d} \bar{w}_j \bar{\mu}_j^2 n^2 \left( \frac{1 - (1 - q_j)^n}{nq_j} \right)^2$$

$$= -\sum_{j=1}^{J_n^d} \frac{\bar{w}_j \bar{\mu}_j^2}{q_j^2} + O((1 - q_j)^n). \tag{B.4}$$

Adding together Eqns. (B.2), (B.3), and (B.4) shows that

$$\int_{\mathcal{X}} \mathbb{V}\left[\mathbb{E}\left[\hat{\mu}(x) \mid \mathbf{X}_n\right]\right] w(x) dx = V_{1n} + V_{2n} + V_{3n} = o(J_n^d/n). \tag{B.5}$$

Finally, for the bias term, we first compute  $\mathbb{E}\left[\hat{\mu}(x)\right]$  using Lemmas B.1 and A.6.

$$\begin{split} \mathbb{E}\left[\hat{\mu}(x)\right] &= \sum_{j=1}^{J_n^d} \mathbb{I}_{P_j}(x) \mathbb{E}\left[\mathbb{I}_{n,j} \frac{1}{N_j} \sum_{i=1}^n \mathbb{I}_{P_j}(X_i) \mu(X_i)\right] \\ &= \sum_{j=1}^{J_n^d} \mathbb{I}_{P_j}(x) \mathbb{E}\left[\frac{1}{N_{j,-i}+1}\right] n \mathbb{E}\left[\mathbb{I}_{P_j}(X_i) \mu(X_i)\right] \\ &= \sum_{j=1}^{J_n^d} \mathbb{I}_{P_j}(x) \frac{1 - (1 - q_j)^n}{q_j} \int_{P_j} \mu(z) f(z) dz \\ &= \sum_{j=1}^{J_n^d} \mathbb{I}_{P_j}(x) \frac{\mu(\overline{p}_j) f(\overline{p}_j) \operatorname{vol}(P_j)}{f(\overline{p}_j) \operatorname{vol}(P_j)} \left(1 + o(J_n^{-d})\right) \end{split}$$

$$= \sum_{j=1}^{J_n^d} \mathbb{I}_{P_j}(x) \mu(\overline{p}_j) \left( 1 + o(J_n^{-d}) \right),$$

where all remainder terms are uniform in  $1 \le j \le J_n^d$  and  $x \in P_j$ . By Assumption 2(b), with S = 0,  $\mu(x)$  satisfies the Taylor expansion, with uniform (in j) remainder:

$$\mu(x) = \mu(\overline{p}_j) + \sum_{\ell=1}^d \frac{\partial \mu(\overline{p}_j)}{\partial x_\ell} (x_\ell - \overline{p}_{\ell,j}) + o(J_n^{-1}).$$

Next, by symmetry  $(p_{\ell,j} - \overline{p}_{\ell,j}) = -(p_{\ell,j-1} - \overline{p}_{\ell,j}) = (p_{\ell,j} - p_{\ell,j-1})/2 = |\mathcal{X}_{\ell}|/(2J_n)$ . Furthermore, Assumption 1(b) and symmetry, imply that  $m \neq \ell$ :

$$\int_{P_j} (x_\ell - \overline{p}_{\ell,j})(x_m - \overline{p}_{m,j})dx = 0, \text{ and,}$$

$$\int_{P_j} (x_{\ell} - \overline{p}_{\ell,j})^2 dx = \left( \prod_{\ell \neq m}^d (p_{\ell,j} - p_{\ell,j-1}) \right) \left( (p_{m,j} - \overline{p}_{m,j})^3 - (p_{m,j-1} - \overline{p}_{m,j})^3 \right) / 3 = J_n^{-2} \frac{1}{12} \operatorname{vol}(P_j) |\mathcal{X}_{\ell}|^2.$$

Then we have, using the above results and applying Assumption 2(a) and Lemma A.7:

$$\int_{\mathcal{X}} (\mathbb{E} \left[ \hat{\mu}(x) \right] - \mu(x))^{2} f(x) dx = \int_{\mathcal{X}} \sum_{j=1}^{J_{n}^{d}} \mathbb{I}_{P_{j}}(x) \left( \mu(\overline{p}_{j}) - \mu(x) \right)^{2} w(x) dx \left( 1 + o(J_{n}^{-d}) \right) \\
= \sum_{j=1}^{J_{n}^{d}} \int_{P_{j}} \left( \sum_{\ell=1}^{d} \frac{\partial \mu(\overline{p}_{j})}{\partial x_{\ell}} (x_{\ell} - \overline{p}_{\ell,j}) + o(J_{n}^{-1}) \right)^{2} w(x) dx \left( 1 + o(J_{n}^{-d}) \right) \\
= \sum_{j=1}^{J_{n}^{d}} \int_{P_{j}} \left( \sum_{\ell=1}^{d} \frac{\partial \mu(\overline{p}_{j})}{\partial x_{\ell}} (x_{\ell} - \overline{p}_{\ell,j}) + o(J_{n}^{-1}) \right)^{2} \\
\times \left( w(\overline{p}_{j}) + (w(x) - w(\overline{p}_{j})) \right) dx \left( 1 + o(J_{n}^{-d}) \right) \\
= \sum_{j=1}^{J_{n}^{d}} w(\overline{p}_{j}) \sum_{\ell=1}^{d} \left( \frac{\partial \mu(\overline{p}_{j})}{\partial x_{\ell}} \right)^{2} \int_{P_{j}} (x_{\ell} - \overline{p}_{\ell,j})^{2} dx + o(J_{n}^{-2}) \\
= J_{n}^{-2} \frac{1}{12} \sum_{j=1}^{J_{n}^{d}} w(\overline{p}_{j}) \sum_{\ell=1}^{d} |\mathcal{X}_{\ell}|^{2} \left( \frac{\partial \mu(\overline{p}_{j})}{\partial x_{\ell}} \right)^{2} vol(P_{j}) + o(J_{n}^{-2}) \\
= J_{n}^{-2} \frac{1}{12} \sum_{\ell=1}^{d} |\mathcal{X}_{\ell}|^{2} \int_{\mathcal{X}} \left( \frac{\partial \mu(x)}{\partial x_{\ell}} \right)^{2} w(x) dx [1 + o(1)] + o(J_{n}^{-2}). \quad (B.6)$$

Adding Eqns. (B.1), (B.5), and (B.6) gives the result.

## C COMPLETE SIMULATION RESULTS

This section contains the results from an exhaustive simulation study. We vastly extend the study contained in Section 5, but keep the aims and set up broadly the same; indeed the results from the main text are a subset of those given here. As such, herein we detail only the extensions considered, referring the reader to Section 5 for the for the general set up, tuning parameter choice, and description of the tables. Further, we do not attempt to interpret the many results presented beyond what is already discussed in the text.

Relative to Section 5, four extensions are presented. First, we consider univariate and trivariate data, in addition to the bivariate study.<sup>1</sup> For d = 1 and d = 3, we also choose appropriate points to examine boundary issues. Second, we also allow for a noisier model, setting  $\sigma^2 = 4$ , in addition to the unit variance. Third, we add the case where  $X_{i,\ell} \sim \text{Beta}(2,2)$  (still truncated). Finally, keeping the tuning parameter  $J_n$  fixed, we place the cell boundaries (and knots) at the appropriate quantiles of the data. This is not technically covered by the theory, but is interesting with an eye toward applications. The regression functions considered are detailed below. Univariate results are presented in Section C.1, followed by complete bivariate results in C.2, and lastly Section C.3. contains results for d = 3.

For d=1, we use the following specifications for  $\mu(x)$ . Models 1.1, 1.2, 1.3, and 1.7 are taken from Fan and Gijbels [1996. Local polynomial modelling and its applications. Chapter 4] and Models 1.4, 1.5, and 1.6 are adapted from Braun and Huang [2005. Kernel Spline Regression. The Canadian Journal of Statistics 33, 259–278]. Model 5 is altered to be discontinuous, not only nondifferentiable, whereas Model 6 has been smoothed. For Models 1.4 and 1.5, we set  $J_n^*=5$  in infeasible estimation. These regression functions are plotted in Figure C.1.

Model 1.1: 
$$\mu(x) = \sin(8x - 4) + 2 \exp\{-256(x - 1/2)^2\}$$
  
Model 1.2:  $\mu(x) = 4x - 2 + 2 \exp\{-256(x - 1/2)^2\}$   
Model 1.3:  $\mu(x) = 8x/5 + 1/5$   
Model 1.4:  $\mu(x) = \mathbb{I}\{|4x - 2| < 0.1\}(1 - 100(4x - 2)^2)^4$   
Model 1.5:  $\mu(x) = \mathbb{I}\{(4x - 2) \in [-2, 1]\}((4x - 2)^7 - 19)/20$   
 $- \mathbb{I}\{(4x - 2) \in (1, 0]\}(4x - 2)^2$   
 $+ \mathbb{I}\{(4x - 2) \in (0, 1/2]\}(4x - 2)^4/2$   
 $+ \mathbb{I}\{(4x - 2) \in (1/2, 1]\}(4x - 2)^5$   
 $+ \mathbb{I}\{(4x - 2) \in (1, 2]\}(2 - (4x - 2)^3)$   
Model 1.6:  $\mu(x) = (1 - (4x - 2)^2)^4$ 

<sup>&</sup>lt;sup>1</sup>For d = 3, we perform 1,000 replications instead of 5,000, due to computational limitations.

Model 1.7: 
$$\mu(x) = 0.3 \exp\{-64(x - 1/4)^2\} + 0.7 \exp\{-256(x - 1/2)^2\}$$
  
Model 1.8:  $\mu(x) = (x - 1/2) + 8(x - 1/2)^2 + 6(x - 1/2)^3 - 30(x - 1/2)^4 - 30(x - 1/2)^5$ 

The first four bivariate specifications are as in Section 5. To these, we add four further models. Model 2.5 is taken from Fan and Gijbels [1996. Local polynomial modelling and its applications. Chapter 7]. All are plotted in Figure C.2.

Model 2.1: 
$$\mu(x_1, x_2) = 0.7 \exp\left\{-3\left((4x_1 - 2 + 0.8)^2 + 8(x_2 - 1/2)^2\right)\right\}$$
  
  $+ \exp\left\{-3\left((4x_1 - 2 - 0.8)^2 + 8(x_2 - 1/2)^2\right)\right\}$   
Model 2.2:  $\mu(x_1, x_2) = \sin(5x_1)\sin(10x_2)$   
Model 2.3:  $\mu(x_1, x_2) = \left((1 - (4x_1 - 2)^2)^2\right)\left(\sin(5x_2)/5\right)$   
Model 2.4:  $\mu(x_1, x_2) = \mathbb{I}\left\{(4x_1 - 2) \in [-2, 1]\right\}\left((4x_1 - 2)^7 - 19\right)/20$   
  $- \mathbb{I}\left\{(4x_1 - 2) \in (-1, 0]\right\}\left(4x_1 - 2\right)^2$   
  $+ \mathbb{I}\left\{(4x_1 - 2) \in (0, 1/2]\right\}\left(4x_1 - 2\right)^4/2$   
  $+ \mathbb{I}\left\{(4x_1 - 2) \in (1/2, 1]\right\}\left(2 - (4x_1 - 2)^3\right)$   
  $+ \mathbb{I}\left\{(4x_1 - 2) \in (1, 2]\right\}\left(2 - (4x_1 - 2)^3\right)$   
  $+ \mathbb{I}\left\{(4x_1 - 2) \in (1, 2)\right\}\left(2 - (4x_1 - 2)^3\right)$   
  $+ \mathbb{I}\left\{(4x_1 - 2) \in (1, 2)\right\}\left(2 - (4x_1 - 2)^3\right)$   
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  $+ \mathbb{I}\left\{(4x_1 - 2) \in (1, 2)\right\}\left($ 

Finally, for d=3, we consider the five specifications below, which are mainly additive and interactive combinations of the above models. For Model 3.5 we set  $J_n^*=3$  in infeasible estimation.

Model 3.1: 
$$\mu(x_1, x_2, x_3) = \text{Model } 1.3(x_1) + \text{Model } 1.3(x_2) + \text{Model } 1.3(x_3)$$
  
Model 3.2:  $\mu(x_1, x_2, x_3) = \text{Model } 2.7(x_1, x_2) \cos(8x_3)$   
Model 3.3:  $\mu(x_1, x_2, x_3) = (\text{Model } 1.8(x_1)) (\text{Model } 1.8(x_2)) (\text{Model } 1.8(x_3))$   
Model 3.4:  $\mu(x_1, x_2, x_3) = (\text{Model } 1.6(x_1)) (4x_2 - 1)(x_3 - 1/2)$   
Model 3.5:  $\mu(x_1, x_2, x_3) = (\text{Model } 1.5(x_1)) \sin(x_2) \cos(8x_3)$ 

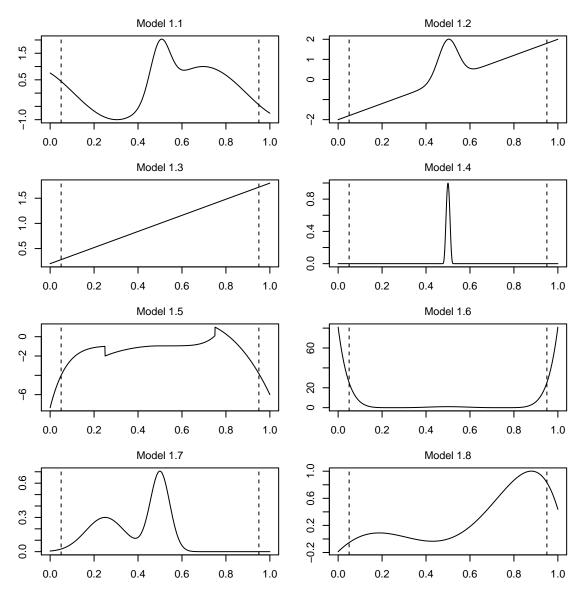


Figure C.1: Regression functions for univariate simulations. The functions are depicted over the domain [0,1], with the interval  $\mathcal{X} = [0.05, 0.95]$  given in dotted lines.

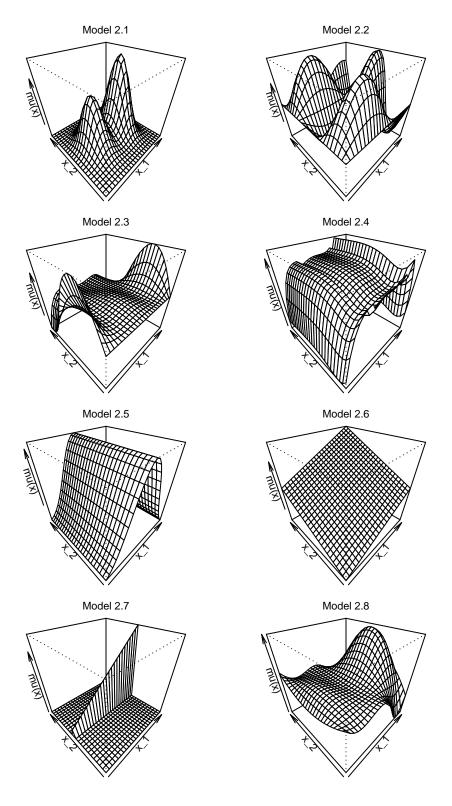


Figure C.2: Regression functions for bivariate simulations. The functions are depicted over the domain  $[0,1] \times [0,1]$ , although  $\mathcal{X} = [0.05, 0.95] \times [0.05, 0.95]$ .

- C.1 Univariate Simulations
- C.1.1 Uniform Cell Boundaries

Table C.1: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 500, \sigma^2 = 1, X_i \sim \beta(0.5, 0.5), \text{ Uniform Cells}$ 

	Tun Parar		Root In		Inget M		Pc (0.		ation RMS	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model						
Infeasible Estimation				Model	1.1					
Local Polynomial	0.05	0.09	0.164	0.178	0.128	0.139	0.305	0.219	0.124	0.169
B-splines	9	5	0.243	0.328	0.178	0.242	0.708	1.002	0.107	0.172
Partitioning	9	5	0.214	0.207	0.162	0.158	0.383	0.250	0.117	0.159
Feasible Estimation										
Local Polynomial	0.12	0.19	0.229	0.216	0.159	0.158	0.804	0.655	0.106	0.164
B-splines Partitioning	4	$\frac{2}{2}$	$0.282 \\ 0.257$	0.376 $0.199$	$0.206 \\ 0.184$	$0.272 \\ 0.146$	0.677 $0.571$	1.184 $0.413$	$0.108 \\ 0.106$	$0.145 \\ 0.112$
1 artitioning	-4	2	0.231			0.140	0.571	0.413	0.100	0.112
I-fill- E-titi				Model	1.2					
Infeasible Estimation Local Polynomial	0.05	0.09	0.164	0.178	0.128	0.139	0.307	0.219	0.123	0.169
B-splines	9	5	0.242	0.328	0.177	0.133	0.708	1.002	0.126	0.170
Partitioning	9	5	0.213	0.207	0.161	0.158	0.383	0.250	0.117	0.159
Feasible Estimation										
Local Polynomial	0.14	0.19	0.258	0.218	0.167	0.159	0.945	0.664	0.107	0.162
B-splines	3	2	0.373	0.363	0.257	0.265	1.226	1.174	0.121	0.120
Partitioning	3	2	0.328	0.192	0.206	0.142	1.032	0.384	0.104	0.118
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.071	0.045	0.072	0.081	0.102
B-splines Partitioning	1 1	$\frac{1}{1}$	$0.063 \\ 0.063$	$0.089 \\ 0.089$	$0.049 \\ 0.049$	$0.069 \\ 0.069$	$0.045 \\ 0.045$	$0.072 \\ 0.072$	$0.078 \\ 0.078$	$0.102 \\ 0.102$
Feasible Estimation	1	1	0.003	0.069	0.049	0.009	0.045	0.072	0.078	0.102
Local Polynomial	0.3	0.3	0.087	0.122	0.066	0.094	0.072	0.102	0.106	0.130
B-splines	2	2	0.074	0.097	0.057	0.075	0.082	0.090	0.089	0.102
Partitioning	2	2	0.085	0.119	0.065	0.090	0.117	0.248	0.092	0.103
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.142	0.166	0.090	0.119	0.929	0.866	0.108	0.156
B-splines	5	5	0.155	0.165	0.103	0.116	0.934	0.909	0.106	0.122
Partitioning	5	5	0.176	0.220	0.125	0.165	0.893	0.767	0.106	0.159
Feasible Estimation	0.20	0.29	0.140	0.162	0.005	0.112	0.042	0.902	0.106	0.129
Local Polynomial B-splines	0.29 2	0.29	$0.140 \\ 0.133$	0.162 $0.146$	$0.085 \\ 0.078$	0.112	0.943 $0.943$	0.902 $0.930$	0.106	0.129 $0.102$
Partitioning	2	2	0.140	0.152	0.084	0.106	0.939	0.789	0.095	0.105
	_	_	0.2.20			0.200	0.000		0.000	0.200
Infeasible Estimation				Model	1.5					
Local Polynomial	0.2	0.2	0.447	0.224	0.332	0.164	0.098	0.120	0.323	0.159
B-splines	5	5	0.369	0.273	0.310	0.213	0.252	0.208	0.250	0.187
Partitioning	5	5	0.273	0.217	0.211	0.164	0.116	0.171	0.262	0.159
Feasible Estimation										
Local Polynomial	0.1	0.23	0.244	0.240	0.181	0.173	0.120	0.119	0.231	0.145
B-splines	5 5	$\frac{2}{2}$	0.371	$0.364 \\ 0.290$	$0.312 \\ 0.212$	0.289	0.253 $0.119$	$0.405 \\ 0.521$	$0.249 \\ 0.261$	$0.173 \\ 0.134$
Partitioning	5	2	0.275			0.223	0.119	0.521	0.201	0.134
				Model	1.6					
Infeasible Estimation	0.02	0.07	0.280	0.207	0.222	0.161	0.297	0.201	0.279	0.185
Local Polynomial B-splines	28	7	0.292	0.207	0.224	0.155	0.351	0.128	0.219	0.165
Partitioning	28	7	0.369	0.243	0.286	0.186	0.577	0.202	0.310	0.184
Feasible Estimation										
Local Polynomial	0.06	0.15	0.754	0.359	0.477	0.273	0.158	0.144	1.900	0.484
B-splines	8	3	1.677	1.601	1.100	1.286	0.293	0.905	3.246	2.541
Partitioning	8	3	1.441	0.769	0.814	0.592	0.304	0.140	2.795	1.182
				Model	1.7					
$In feasible\ Estimation$										
Local Polynomial	0.08	0.12	0.135	0.159	0.106	0.125	0.216	0.179	0.109	0.164
B-splines	6	4	0.132	0.154	0.104	0.122	0.191	0.322	0.107	0.124
Partitioning Feasible Estimation	6	4	0.160	0.180	0.124	0.138	0.246	0.428	0.107	0.142
Local Polynomial	0.25	0.27	0.148	0.155	0.111	0.121	0.439	0.340	0.108	0.132
B-splines	2	2	0.151	0.152	0.116	0.117	0.416	0.406	0.115	0.103
Partitioning	2	2	0.155	0.140	0.116	0.107	0.419	0.318	0.114	0.114
Ŭ				Model						
Infeasible Estimation				Model	1.0					
Local Polynomial	0.12	0.33	0.112	0.121	0.087	0.093	0.109	0.093	0.107	0.125
B-splines	4	1	0.115	0.135	0.091	0.109	0.159	0.144	0.105	0.104
Partitioning	4	1	0.134	0.135	0.104	0.109	0.205	0.144	0.106	0.104
Feasible Estimation		0.22		0.100		0.00=	0.335	0.101	0.100	0.700
Local Polynomial	0.2	0.29	0.107	0.123	0.084	0.095	0.110	0.104	0.109	0.130
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	$0.123 \\ 0.126$	0.113 $0.131$	$0.097 \\ 0.097$	$0.088 \\ 0.102$	$0.141 \\ 0.127$	$0.113 \\ 0.261$	$0.103 \\ 0.104$	$0.103 \\ 0.105$
i areieieiiiig	- 4	- 4	0.120	0.101	0.001	0.102	0.141	0.201	0.104	0.100

Table C.2: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 500, \sigma^2 = 1, X_i \sim \beta(1, 1), \text{ Uniform Cells}$ 

	Tuni Param			tegrated SE	Inget M			oint Estima	ation RMS	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
$In feasible\ Estimation$										
Local Polynomial	0.05	0.09	0.168	0.181	0.132	0.140	0.282	0.199	0.149	0.200
B-splines Partitioning	9 9	5 5	$0.261 \\ 0.220$	0.354 $0.210$	$0.196 \\ 0.169$	$0.271 \\ 0.161$	$0.701 \\ 0.375$	$0.976 \\ 0.235$	0.127 $0.136$	0.199 $0.182$
Feasible Estimation	3	9	0.220	0.210	0.103	0.101	0.373	0.233	0.130	0.102
Local Polynomial	0.11	0.18	0.234	0.222	0.167	0.165	0.745	0.610	0.128	0.191
B-splines	4	3	0.325	0.410	0.243	0.308	0.765	1.182	0.131	0.140
Partitioning	4	3	0.279	0.232	0.204	0.171	0.597	0.494	0.128	0.131
				Model	1.2					
Infeasible Estimation	0.05	0.00	0.100	0.101	0.101	0.140	0.004	0.100	0.140	0.000
Local Polynomial B-splines	0.05 $9$	0.09 5	$0.168 \\ 0.261$	$0.181 \\ 0.354$	$0.131 \\ 0.196$	$0.140 \\ 0.271$	0.284 0.701	$0.199 \\ 0.976$	$0.149 \\ 0.127$	$0.200 \\ 0.197$
Partitioning	9	5	0.219	0.210	0.168	0.161	0.375	0.235	0.136	0.137
Feasible Estimation		-	******	0.2-0	0.200		0.0.0	0.200	0.200	*****
Local Polynomial	0.13	0.19	0.261	0.224	0.177	0.166	0.871	0.619	0.129	0.189
B-splines	4	3	0.342	0.403	0.255	0.305	0.920	1.175	0.143	0.131
Partitioning	4	3	0.308	0.227	0.215	0.167	0.776	0.473	0.123	0.133
				Model 1	1.3					
Infeasible Estimation	0.0	0.9	0.065	0.001	0.051	0.070	0.045	0.069	0.006	0.110
Local Polynomial B-splines	0.9 1	0.9	$0.065 \\ 0.063$	0.091 $0.090$	0.051 $0.049$	0.070	0.045 0.045	$0.068 \\ 0.068$	$0.086 \\ 0.082$	0.119 $0.118$
Partitioning	1	1	0.063	0.090	0.049	0.069	0.045	0.068	0.082	0.118
Feasible Estimation										
Local Polynomial	0.28	0.29	0.089	0.123	0.067	0.093	0.068	0.093	0.126	0.143
B-splines	2	2	0.076	0.098	0.059	0.075	0.079	0.084	0.098	0.120
Partitioning	2	2	0.088	0.120	0.067	0.091	0.114	0.223	0.102	0.121
I C III E C C				Model 1	1.4					
Infeasible Estimation Local Polynomial	0.2	0.2	0.153	0.174	0.094	0.121	0.927	0.864	0.128	0.172
B-splines	5	5	0.165	0.175	0.109	0.121	0.932	0.905	0.127	0.172
Partitioning	5	5	0.185	0.226	0.131	0.170	0.892	0.761	0.127	0.182
Feasible Estimation										
Local Polynomial	0.27	0.28	0.152	0.171	0.091	0.115	0.937	0.897	0.126	0.143
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	$0.145 \\ 0.152$	0.156 $0.160$	0.084 $0.092$	$0.100 \\ 0.111$	0.936 $0.927$	$0.922 \\ 0.757$	$0.102 \\ 0.106$	$0.120 \\ 0.122$
1 at titioning	2	2	0.152			0.111	0.321	0.757	0.100	0.122
Infeasible Estimation				Model :	1.5					
Local Polynomial	0.2	0.2	0.426	0.229	0.307	0.168	0.092	0.107	0.267	0.176
B-splines	5	5	0.359	0.278	0.301	0.216	0.224	0.182	0.195	0.211
Partitioning	5	5	0.265	0.219	0.200	0.166	0.103	0.152	0.232	0.182
Feasible Estimation										
Local Polynomial B-splines	0.1 5	0.21 2	$0.246 \\ 0.368$	0.237 $0.374$	$0.179 \\ 0.306$	$0.172 \\ 0.305$	0.106 0.226	$0.106 \\ 0.364$	0.227 $0.194$	0.168 $0.168$
Partitioning	5	2	0.308 $0.271$	0.291	0.204	0.303	0.220	0.304 $0.475$	0.134	0.142
g				Model						
Infeasible Estimation				Model .	1.0					
Local Polynomial	0.02	0.07	0.268	0.203	0.210	0.155	0.250	0.177	0.320	0.214
B-splines	25	6	0.283	0.276	0.209	0.199	0.184	0.161	0.241	0.438
Partitioning	25	6	0.352	0.234	0.268	0.180	0.238	0.438	0.342	0.199
Feasible Estimation Local Polynomial	0.06	0.14	0.632	0.323	0.370	0.240	0.142	0.127	1.833	0.433
B-splines	8	3	1.404	1.399	0.825	1.047	0.254	0.635	3.034	2.349
Partitioning	8	3	1.209	0.697	0.616	0.510	0.266	0.127	2.692	1.273
				Model	1.7					
Infeasible Estimation										
Local Polynomial	0.08	0.12	0.139	0.162	0.108	0.125	0.200	0.162	0.131	0.192
B-splines	6	4	0.134	0.160	0.107	0.127	0.176	0.309	0.128	0.140
Partitioning Feasible Estimation	6	4	0.162	0.181	0.126	0.139	0.224	0.373	0.128	0.160
Local Polynomial	0.24	0.26	0.157	0.159	0.119	0.124	0.424	0.323	0.130	0.148
B-splines	2	2	0.159	0.162	0.124	0.126	0.406	0.392	0.125	0.123
Partitioning	2	2	0.164	0.141	0.125	0.109	0.396	0.285	0.126	0.129
				Model :	1.8					
$Infeasible\ Estimation$										
Local Polynomial	0.13	0.33	0.110	0.120	0.085	0.091	0.095	0.084	0.128	0.136
B-splines Partitioning	4 4	1 1	0.114 $0.133$	$0.131 \\ 0.131$	0.088 $0.104$	$0.103 \\ 0.103$	0.144 $0.187$	$0.125 \\ 0.125$	$0.123 \\ 0.125$	0.129 $0.129$
Feasible Estimation	4	1	0.133	0.131	0.104	0.103	0.187	0.120	0.120	0.129
Local Polynomial	0.2	0.28	0.107	0.124	0.082	0.094	0.101	0.094	0.129	0.144
B-splines	3	2	0.119	0.110	0.092	0.085	0.127	0.101	0.118	0.123
Partitioning	3	2	0.122	0.130	0.094	0.100	0.114	0.234	0.118	0.123
Notes. Tuning parameter	ore are loc	l polynomial	bandwi	dth and the	number of	colle for	partitioning	etimation	and Be	nlinge ac

Table C.3: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=500, \sigma^2=1, X_i \sim \beta(2,2),$  Uniform Cells

	Tun Parar		Root In			rated AE		oint Estim	ation RM (0	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.173	0.182	0.133	0.136	0.252	0.176	0.231	0.307
B-splines	10	5	0.156	0.390	0.120	0.314	0.215	0.920	0.203	0.259
Partitioning Feasible Estimation	10	5	0.204	0.211	0.156	0.161	0.293	0.216	0.215	0.250
Local Polynomial	0.1	0.17	0.236	0.225	0.174	0.170	0.649	0.536	0.205	0.273
B-splines	5	3	0.446	0.458	0.344	0.365	1.102	1.143	0.204	0.212
Partitioning	5	3	0.327	0.271	0.230	0.207	0.749	0.547	0.196	0.195
				Model	1.2					
$In feasible\ Estimation$										
Local Polynomial	0.05	0.09	0.173	0.182	0.132	0.136	0.253	0.176	0.230	0.307
B-splines Partitioning	10 10	5 5	0.155 $0.203$	$0.390 \\ 0.211$	0.119 $0.156$	0.314 $0.161$	0.215 $0.293$	$0.920 \\ 0.216$	$0.203 \\ 0.215$	$0.256 \\ 0.250$
Feasible Estimation	10	5	0.203	0.211	0.136	0.161	0.293	0.210	0.215	0.230
Local Polynomial	0.11	0.17	0.259	0.227	0.185	0.171	0.744	0.542	0.203	0.270
B-splines	4	3	0.351	0.457	0.283	0.365	0.759	1.141	0.210	0.210
Partitioning	4	3	0.301	0.270	0.231	0.205	0.550	0.542	0.187	0.195
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.068	0.045	0.063	0.098	0.173
B-splines	1	1	0.063	0.090	0.049	0.067	0.045	0.063	0.094	0.170
Partitioning	1	1	0.063	0.090	0.049	0.067	0.045	0.063	0.094	0.170
Feasible Estimation Local Polynomial	0.26	0.28	0.091	0.123	0.066	0.088	0.062	0.082	0.188	0.201
B-splines	2	2	0.078	0.098	0.059	0.073	0.075	0.078	0.126	0.182
Partitioning	2	2	0.091	0.122	0.069	0.091	0.104	0.197	0.135	0.187
_				Model	1.4					
Infeasible Estimation				1110401						
Local Polynomial	0.2	0.2	0.169	0.186	0.101	0.125	0.923	0.859	0.191	0.226
B-splines	5	5	0.180	0.188	0.116	0.126	0.926	0.895	0.191	0.197
Partitioning	5	5	0.198	0.234	0.139	0.175	0.886	0.752	0.196	0.250
Feasible Estimation Local Polynomial	0.24	0.27	0.169	0.184	0.100	0.121	0.926	0.885	0.189	0.204
B-splines	2	2	0.163	0.184 $0.172$	0.100	0.121	0.923	0.883	0.140	0.204
Partitioning	2	2	0.170	0.169	0.105	0.118	0.898	0.685	0.150	0.191
				Model						
Infeasible Estimation				Model	1.0					
Local Polynomial	0.2	0.2	0.367	0.230	0.251	0.164	0.083	0.093	0.224	0.229
B-splines	5	5	0.335	0.272	0.273	0.201	0.180	0.136	0.252	0.260
Partitioning	5	5	0.244	0.219	0.175	0.163	0.090	0.132	0.228	0.250
Feasible Estimation	0.1	0.0	0.045	0.000	0.100	0.104	0.000	0.004	0.050	0.000
Local Polynomial B-splines	0.1 5	$0.2 \\ 2$	0.245 $0.399$	0.229 $0.365$	$0.169 \\ 0.307$	$0.164 \\ 0.297$	$0.090 \\ 0.188$	0.094 $0.278$	$0.256 \\ 0.296$	0.238 $0.202$
Partitioning	5 5	2	0.399 $0.297$	0.365 $0.279$	0.205	0.297	0.143	0.278	0.296 $0.276$	0.202
1 di vivioning		-	0.20.	Model		0.210	0.110	0.001	0.210	0.200
Infeasible Estimation				Model	1.0					
Local Polynomial	0.02	0.08	0.240	0.190	0.181	0.140	0.193	0.144	0.443	0.323
B-splines	21	6	0.256	0.218	0.179	0.144	0.147	0.124	0.720	0.562
Partitioning	21	6	0.323	0.227	0.234	0.170	0.194	0.339	0.823	0.290
Feasible Estimation										
Local Polynomial	0.06	0.14	0.419	0.257	0.223	0.180	0.125	0.111	1.649	0.403
B-splines Partitioning	8 8	3 3	0.934 $0.808$	0.986 $0.536$	$0.445 \\ 0.352$	0.621 $0.354$	$0.200 \\ 0.229$	$0.279 \\ 0.112$	2.501 $2.368$	1.768 $1.303$
rartitioning	0	3	0.808			0.334	0.229	0.112	2.300	1.303
Infeasible Estimation				Model	1.7					
Local Polynomial	0.07	0.11	0.142	0.163	0.108	0.121	0.180	0.143	0.208	0.276
B-splines	6	4	0.137	0.166	0.108	0.133	0.158	0.284	0.199	0.199
Partitioning	6	4	0.163	0.181	0.126	0.137	0.188	0.309	0.202	0.220
Feasible Estimation										
Local Polynomial	0.21	0.24	0.166	0.162	0.128	0.124	0.392	0.289	0.195	0.212
B-splines	2	2	0.175	0.174	0.138	0.140	0.399	0.365	0.151	0.196
Partitioning	2	2	0.178	0.140	0.141	0.107	0.370	0.237	0.166	0.192
Infeasible Estimation				Model	1.8					
Infeasible Estimation Local Polynomial	0.15	0.34	0.108	0.116	0.079	0.084	0.082	0.074	0.206	0.195
B-splines	3	0.54	0.108	0.116	0.079	0.084	0.116	0.074	0.206 $0.173$	0.193
Partitioning	3	1	0.121	0.119	0.093	0.090	0.087	0.098	0.167	0.202
Feasible Estimation	-		_	-			'			
Local Polynomial	0.19	0.27	0.106	0.124	0.079	0.089	0.090	0.084	0.197	0.203
B-splines	3	2	0.109	0.106	0.083	0.079	0.109	0.085	0.159	0.187
Partitioning	3	2	0.114	0.127	0.087	0.096	0.103	0.204	0.156	0.192

Table C.4: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 500, \sigma^2 = 4, X_i \sim \beta(0.5, 0.5),$ Uniform Cells

	Tun Parar		Root In		Inget M		Po (0.		ation RMS	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model						
Infeasible Estimation				Model	1.1					
Local Polynomial	0.07	0.11	0.290	0.330	0.226	0.258	0.494	0.383	0.223	0.333
B-splines	7	4	0.374	0.349	0.284	0.274	0.967	0.843	0.211	0.248
Partitioning	7	4	0.366	0.361	0.279	0.276	0.588	0.890	0.215	0.284
Feasible Estimation										
Local Polynomial	0.13	0.2	0.310	0.325	0.232	0.251	0.885	0.744	0.214	0.315
B-splines Partitioning	4	$\frac{2}{2}$	$0.368 \\ 0.360$	$0.415 \\ 0.304$	$0.275 \\ 0.266$	0.313 $0.231$	0.922 $0.840$	$\frac{1.185}{0.547}$	$0.224 \\ 0.212$	0.229 $0.219$
1 at titioning	-1	2	0.300			0.231	0.040	0.541	0.212	0.213
I-fill- E-titi				Model	1.2					
Infeasible Estimation Local Polynomial	0.07	0.11	0.289	0.330	0.225	0.258	0.497	0.383	0.222	0.333
B-splines	7	4	0.373	0.349	0.283	0.236 $0.274$	0.967	0.843	0.211	0.244
Partitioning	7	4	0.365	0.361	0.279	0.276	0.588	0.890	0.215	0.284
Feasible Estimation										
Local Polynomial	0.16	0.21	0.330	0.326	0.238	0.251	1.034	0.753	0.214	0.312
B-splines	3	2	0.422	0.402	0.304	0.304	1.294	1.183	0.218	0.213
Partitioning	3	2	0.397	0.301	0.279	0.229	1.123	0.550	0.211	0.222
				Model	1.3					
Infeasible Estimation			0.100	0.101		0.1.1		0 1 10	0.100	0.000
Local Polynomial	0.9	0.9	0.129	0.181	0.101	0.141	0.090	0.143	0.163	0.203
B-splines	1 1	$\frac{1}{1}$	0.126	0.178	0.098	0.139	0.090	$0.143 \\ 0.143$	0.156	0.203
Partitioning Feasible Estimation	1	1	0.126	0.178	0.098	0.139	0.090	0.145	0.156	0.203
Local Polynomial	0.24	0.26	0.184	0.249	0.140	0.192	0.161	0.216	0.215	0.268
B-splines	2	2	0.160	0.197	0.124	0.153	0.170	0.187	0.187	0.204
Partitioning	2	2	0.191	0.247	0.146	0.190	0.241	0.528	0.193	0.208
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.214	0.283	0.157	0.217	0.940	0.889	0.215	0.312
B-splines	5	5	0.245	0.274	0.185	0.211	0.947	0.926	0.212	0.242
Partitioning	5	5	0.302	0.410	0.231	0.314	0.913	0.822	0.212	0.318
Feasible Estimation	0.24	0.26	0.914	0.270	0.156	0.206	0.042	0.912	0.215	0.269
Local Polynomial B-splines	$0.24 \\ 2$	0.26	$0.214 \\ 0.195$	$0.270 \\ 0.225$	$0.156 \\ 0.140$	0.206 $0.167$	$0.943 \\ 0.953$	0.912 $0.941$	0.215	0.269 $0.204$
Partitioning	2	2	0.221	0.264	0.160	0.199	0.952	0.892	0.194	0.209
	_	_				0.200			0.20	
Infeasible Estimation				Model	1.0					
Local Polynomial	0.2	0.2	0.474	0.321	0.361	0.249	0.172	0.239	0.374	0.313
B-splines	5	5	0.414	0.349	0.342	0.276	0.295	0.276	0.310	0.281
Partitioning	5	5	0.366	0.408	0.284	0.312	0.229	0.342	0.320	0.318
Feasible Estimation										
Local Polynomial	0.1	0.24	0.326	0.335	0.252	0.259	0.232	0.232	0.299	0.285
B-splines	5 5	$\frac{2}{2}$	$0.475 \\ 0.400$	0.417 $0.383$	$0.386 \\ 0.309$	$0.331 \\ 0.295$	$0.322 \\ 0.308$	$0.437 \\ 0.688$	$0.299 \\ 0.312$	$0.246 \\ 0.227$
Partitioning	5	2	0.400			0.295	0.308	0.000	0.312	0.221
				Model	1.6					
Infeasible Estimation	0.02	0.08	0.486	0.377	0.385	0.295	0.507	0.372	0.484	0.347
Local Polynomial B-splines	21	6	0.512	0.377	0.393	0.298	0.372	0.312	0.464	0.435
Partitioning	21	6	0.640	0.448	0.496	0.343	0.480	1.011	0.628	0.338
Feasible Estimation										
Local Polynomial	0.06	0.15	0.810	0.451	0.559	0.354	0.305	0.283	1.955	0.629
B-splines	8	3	1.695	1.613	1.136	1.292	0.439	0.917	3.252	2.547
Partitioning	8	3	1.475	0.814	0.914	0.641	0.585	0.267	2.804	1.199
				Model	1.7					
Infeasible Estimation										
Local Polynomial	0.11	0.14	0.237	0.299	0.185	0.233	0.332	0.315	0.214	0.316
B-splines	$\frac{4}{4}$	3 3	0.222	0.249 $0.316$	0.175	0.196	0.373	0.449	0.210	0.206
Partitioning Feasible Estimation	4	3	0.265	0.316	0.205	0.242	0.443	0.330	0.211	0.237
Local Polynomial	0.23	0.26	0.218	0.266	0.169	0.208	0.445	0.379	0.216	0.271
B-splines	2	2	0.208	0.229	0.162	0.180	0.449	0.436	0.197	0.204
Partitioning	2	2	0.230	0.257	0.176	0.198	0.460	0.549	0.202	0.214
				Model	1.8					
Infeasible Estimation				1110401	1.0					
Local Polynomial	0.16	0.38	0.202	0.222	0.157	0.171	0.187	0.170	0.218	0.221
B-splines	3	1	0.202	0.205	0.160	0.161	0.192	0.190	0.203	0.204
Partitioning	3	1	0.229	0.205	0.178	0.161	0.183	0.190	0.205	0.204
Feasible Estimation	0.91	0.26	0.107	0.250	0.150	0.102	0.100	0.217	0.015	0.069
Local Polynomial B-splines	0.21 2	$0.26 \\ 2$	0.197 $0.193$	$0.250 \\ 0.201$	0.153 $0.151$	0.193 $0.157$	0.188 $0.221$	0.217 $0.195$	$0.215 \\ 0.199$	$0.268 \\ 0.204$
Partitioning	2	2	0.193 $0.217$	$0.201 \\ 0.251$	$0.151 \\ 0.168$	0.157	0.221 $0.251$	$0.195 \\ 0.532$	0.199 $0.203$	0.204 $0.209$
			0.211	0.201	0.100	0.130	0.201	0.002	0.200	0.203

Table C.5: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=500, \sigma^2=4, X_i \sim \beta(1,1),$ Uniform Cells

	Tun Parar		Root In	tegrated SE		rated AE	Pc (0		ation RM	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation	0.05	0.11	0.005	0.005	0.000	0.050	0.450	0.045	0.000	0.000
Local Polynomial	0.07	0.11	0.295	0.335	0.230	0.259	0.458	0.347	0.268	$0.390 \\ 0.281$
B-splines Partitioning	7 7	$\frac{4}{4}$	0.397 $0.374$	$0.367 \\ 0.363$	$0.308 \\ 0.289$	$0.292 \\ 0.279$	0.955 $0.573$	0.813 $0.780$	$0.255 \\ 0.257$	0.281 $0.319$
Feasible Estimation	,	4	0.374	0.303	0.269	0.219	0.575	0.780	0.237	0.319
Local Polynomial	0.12	0.2	0.321	0.334	0.243	0.257	0.844	0.705	0.257	0.356
B-splines	4	2	0.376	0.443	0.291	0.342	0.814	1.156	0.265	0.251
Partitioning	4	2	0.368	0.318	0.281	0.242	0.734	0.511	0.251	0.257
				Model	1.2					
Infeasible Estimation Local Polynomial	0.07	0.11	0.294	0.335	0.229	0.259	0.460	0.347	0.268	0.390
B-splines	7	4	0.396	0.366	0.307	0.291	0.954	0.813	0.255	0.330
Partitioning	7	4	0.374	0.363	0.288	0.279	0.573	0.780	0.257	0.319
Feasible Estimation										
Local Polynomial	0.15	0.2	0.343	0.335	0.252	0.258	0.977	0.712	0.257	0.353
B-splines	3	$\frac{2}{2}$	0.435	0.433	0.327	0.335	1.166	1.153	0.263	0.246
Partitioning	3	2	0.411	0.315	0.300	0.240	1.024	0.509	0.247	0.258
Infeasible Estimation				Model	1.3					
Local Polynomial	0.9	0.9	0.130	0.183	0.101	0.140	0.090	0.136	0.171	0.238
B-splines	1	1	0.127	0.180	0.099	0.138	0.090	0.136	0.164	0.236
Partitioning	1	1	0.127	0.180	0.099	0.138	0.090	0.136	0.164	0.236
Feasible Estimation			0.400				0.440			
Local Polynomial	0.23	0.26	0.188	0.250	0.141	0.189	0.148	0.194	0.256	0.296
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	0.163 $0.194$	0.199 $0.249$	$0.126 \\ 0.149$	0.153 $0.190$	$0.165 \\ 0.227$	$0.174 \\ 0.473$	$0.209 \\ 0.218$	0.241 $0.243$
1 at titioning	2	2	0.134	Model		0.130	0.221	0.475	0.210	0.240
Infeasible Estimation				Model	1.4					
Local Polynomial	0.2	0.2	0.222	0.288	0.160	0.217	0.937	0.884	0.256	0.344
B-splines	5	5	0.252	0.281	0.190	0.212	0.944	0.921	0.253	0.274
Partitioning	5	5	0.307	0.414	0.236	0.319	0.909	0.804	0.255	0.364
Feasible Estimation Local Polynomial	0.23	0.26	0.224	0.277	0.160	0.206	0.937	0.907	0.256	0.298
B-splines	0.23	2	0.224 $0.205$	0.277	0.146	0.200 $0.171$	0.946	0.934	0.230	0.298
Partitioning	2	2	0.229	0.269	0.166	0.202	0.939	0.839	0.219	0.244
				Model	1.5					
$In feasible\ Estimation$										
Local Polynomial	0.2	0.2	0.455	0.324	0.337	0.249	0.158	0.214	0.347	0.347
B-splines	5	5	0.407	0.355	0.333	0.277	0.265	0.246	0.293	0.318
Partitioning Feasible Estimation	5	5	0.361	0.411	0.278	0.315	0.202	0.303	0.320	0.364
Local Polynomial	0.11	0.23	0.329	0.334	0.251	0.256	0.203	0.210	0.319	0.321
B-splines	5	2	0.486	0.417	0.388	0.336	0.298	0.394	0.287	0.269
Partitioning	5	2	0.404	0.374	0.308	0.292	0.301	0.610	0.308	0.259
				Model	1.6					
$In feasible\ Estimation$										
Local Polynomial	0.02	0.09	0.463	0.370	0.363	0.285	0.433	0.327	0.550	0.407
B-splines Partitioning	19 19	5 5	$0.495 \\ 0.612$	$0.598 \\ 0.430$	$0.368 \\ 0.468$	$0.466 \\ 0.330$	0.316 $0.413$	$0.466 \\ 0.303$	1.113 $1.129$	$\frac{1.088}{0.392}$
Feasible Estimation	13	3	0.012	0.430	0.400	0.550	0.413	0.303	1.123	0.332
Local Polynomial	0.06	0.15	0.700	0.424	0.459	0.324	0.271	0.249	1.916	0.628
B-splines	8	3	1.438	1.412	0.877	1.055	0.390	0.650	3.053	2.359
Partitioning	8	3	1.259	0.748	0.727	0.564	0.502	0.240	2.720	1.295
				Model	1.7					
Infeasible Estimation	0.1	0.10	0.044	0.202	0.100	0.000	0.911	0.004	0.055	0.205
Local Polynomial B-splines	0.1 5	0.13 4	$0.244 \\ 0.256$	$0.303 \\ 0.260$	0.188 $0.203$	0.233 $0.204$	0.311 $0.442$	$0.284 \\ 0.360$	$0.257 \\ 0.254$	$0.365 \\ 0.258$
Partitioning	5	4	0.297	0.250	0.203	0.276	0.344	0.715	0.254 $0.255$	0.238
Feasible Estimation	Ů	-	0.231	0.005	0.201	0.210	0.044	0.110	0.200	0.013
Local Polynomial	0.22	0.25	0.226	0.270	0.175	0.208	0.432	0.363	0.259	0.301
B-splines	2	2	0.217	0.237	0.170	0.186	0.440	0.421	0.222	0.242
Partitioning	2	2	0.238	0.259	0.184	0.200	0.442	0.493	0.229	0.247
Infeasible Estimation				Model	1.8					
Local Polynomial	0.17	0.39	0.200	0.221	0.153	0.167	0.165	0.155	0.260	0.251
B-splines	3	1	0.201	0.204	0.157	0.157	0.175	0.171	0.232	0.242
Partitioning	3	1	0.229	0.204	0.178	0.157	0.166	0.171	0.236	0.242
Feasible Estimation										
Local Polynomial	0.21	0.26	0.199	0.251	0.151	0.189	0.170	0.195	0.257	0.297
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	$0.192 \\ 0.216$	$0.203 \\ 0.251$	$0.149 \\ 0.167$	0.156 $0.193$	$0.202 \\ 0.231$	$0.179 \\ 0.477$	$0.225 \\ 0.232$	$0.242 \\ 0.244$
Votes Tuning paramet							0.231			

Table C.6: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=500, \sigma^2=4, X_i \sim \beta(2,2),$  Uniform Cells

	Tur Parai	ning neter	Root In		Inget M			oint Estim	ation RMS	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
$In feasible\ Estimation$										
Local Polynomial	0.06	0.1	0.303	0.338	0.230	0.251	0.410	0.307	0.422	0.573
B-splines	7	5	0.428	0.447	0.342	0.360	0.919	0.931	0.406	0.425
Partitioning	7	5	0.383	0.406	0.296	0.308	0.546	0.314	0.409	0.499
Feasible Estimation Local Polynomial	0.12	0.19	0.336	0.342	0.255	0.260	0.781	0.640	0.406	0.493
B-splines	4	3	0.405	0.342	0.324	0.384	0.765	1.098	0.391	0.493 $0.407$
Partitioning	4	3	0.384	0.339	0.301	0.259	0.623	0.483	0.377	0.387
_				Model	1.2					
Infeasible Estimation				Model	1.2					
Local Polynomial	0.07	0.1	0.302	0.338	0.230	0.251	0.412	0.307	0.422	0.573
B-splines	7	5	0.427	0.447	0.341	0.360	0.919	0.931	0.406	0.424
Partitioning	7	5	0.382	0.406	0.295	0.308	0.546	0.314	0.409	0.499
Feasible Estimation	0.12	0.19	0.256	0.249	0.266	0.261	0.000	0.644	0.401	0.401
Local Polynomial B-splines	0.13 4	0.19	0.356 $0.443$	$0.342 \\ 0.475$	$0.266 \\ 0.350$	0.261 $0.380$	0.883 $0.974$	0.644 $1.095$	$0.401 \\ 0.391$	0.491 $0.393$
Partitioning	4	3	0.422	0.337	0.322	0.350 $0.257$	0.850	0.479	0.353	0.387
				Model						
Infeasible Estimation				Model	1.5					
Local Polynomial	0.9	0.9	0.129	0.182	0.100	0.136	0.090	0.125	0.197	0.346
B-splines	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Partitioning	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Feasible Estimation										
Local Polynomial	0.22	0.26	0.190	0.250	0.137	0.179	0.133	0.170	0.388	0.411
B-splines	$\frac{2}{2}$	2 2	0.166	0.199	0.126	0.148	0.150	0.158	0.274	0.366
Partitioning	2	2	0.197	0.249	0.150	0.187	0.201	0.407	0.294	0.377
				Model	1.4					
Infeasible Estimation	0.2	0.2	0.234	0.294	0.162	0.212	0.930	0.874	0.382	0.453
Local Polynomial B-splines	5	5	0.262	0.289	0.102	0.212	0.933	0.907	0.382	0.433
Partitioning	5	5	0.315	0.418	0.241	0.319	0.899	0.785	0.392	0.499
Feasible Estimation										
Local Polynomial	0.22	0.25	0.238	0.285	0.164	0.204	0.927	0.893	0.389	0.414
B-splines	2	2	0.220	0.244	0.155	0.175	0.930	0.918	0.282	0.368
Partitioning	2	2	0.244	0.275	0.176	0.204	0.910	0.757	0.301	0.380
				Model	1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.402	0.323	0.283	0.238	0.139	0.185	0.401	0.454
B-splines Partitioning	5 5	5 5	0.386 $0.346$	0.349 $0.410$	$0.305 \\ 0.260$	0.261 $0.309$	$0.217 \\ 0.177$	$0.199 \\ 0.263$	$0.416 \\ 0.411$	0.427 $0.499$
Feasible Estimation	9	3	0.340	0.410	0.200	0.303	0.177	0.203	0.411	0.433
Local Polynomial	0.12	0.21	0.328	0.328	0.239	0.242	0.170	0.184	0.436	0.451
B-splines	4	2	0.495	0.411	0.371	0.328	0.263	0.314	0.450	0.384
Partitioning	4	2	0.417	0.366	0.300	0.284	0.298	0.480	0.448	0.396
				Model	1.6					
$Infeasible\ Estimation$										
Local Polynomial	0.03	0.1	0.418	0.350	0.314	0.259	0.333	0.266	0.756	0.594
B-splines	16	5	0.449	0.483	0.314	0.346	0.385	0.341	0.723	1.186
Partitioning	16	5	0.555	0.416	0.410	0.312	0.588	0.263	1.129	0.557
Feasible Estimation Local Polynomial	0.06	0.15	0.515	0.371	0.318	0.268	0.231	0.215	1.810	0.703
B-splines	7	3	1.105	1.005	0.581	0.635	0.302	0.307	2.442	1.793
Partitioning	7	3	0.979	0.599	0.504	0.418	0.295	0.210	2.430	1.342
o .				Model						
Infeasible Estimation				Model	1.1					
Local Polynomial	0.1	0.13	0.250	0.305	0.187	0.225	0.282	0.252	0.409	0.511
B-splines	5	4	0.266	0.264	0.211	0.202	0.422	0.326	0.382	0.385
Partitioning	5	4	0.301	0.360	0.234	0.272	0.322	0.591	0.392	0.439
Feasible Estimation	0.01	0.04	0.000	0.050	0.101	0.004	0.40	0.000	0.001	0.400
Local Polynomial B-splines	0.21 2	$0.24 \\ 2$	$0.236 \\ 0.230$	$0.273 \\ 0.246$	$0.181 \\ 0.181$	0.204 $0.193$	$0.407 \\ 0.419$	0.333 $0.393$	$0.391 \\ 0.291$	$0.420 \\ 0.374$
Partitioning	2	2	0.251	0.240	0.196	0.198	0.419	0.393 $0.423$	0.231	0.381
- 0.0.0.0	-	-	5.201			0.100	3.400	0.420	0.010	0.001
Infeasible Estimation				Model	1.8					
Local Polynomial	0.19	0.4	0.193	0.215	0.142	0.156	0.142	0.139	0.386	0.386
B-splines	2	1	0.162	0.196	0.142	0.136	0.142	0.139	0.350 $0.254$	0.357
Partitioning	2	1	0.185	0.196	0.143	0.146	0.236	0.146	0.268	0.357
Feasible Estimation										
Local Polynomial	0.2	0.25	0.200	0.250	0.146	0.180	0.152	0.171	0.392	0.412
B-splines	3	2	0.186	0.202	0.142	0.151	0.176	0.161	0.299	0.367
Partitioning	3	2	0.213	0.252	0.162	0.190	0.204	0.411	0.312	0.379

Table C.7: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 1000, \sigma^2 = 1, X_i \sim \beta(0.5, 0.5),$ Uniform Cells

	Tun Parar		Root In	tegrated SE	Inget M		Pc (0		ation RMS	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model						
Infeasible Estimation				Model	1.1					
Local Polynomial	0.05	0.09	0.125	0.132	0.097	0.103	0.240	0.168	0.093	0.118
B-splines	10	5	0.109	0.317	0.086	0.230	0.242	1.000	0.077	0.149
Partitioning	10	5	0.144	0.152	0.111	0.116	0.296	0.222	0.089	0.111
Feasible Estimation										
Local Polynomial	0.1	0.18	0.187	0.178	0.126	0.126	0.686	0.576	0.075	0.127
B-splines Partitioning	5 5	3 3	0.351 $0.255$	$0.375 \\ 0.210$	0.236 $0.159$	0.268 $0.144$	0.766	$\frac{1.244}{0.571}$	$0.074 \\ 0.075$	0.117 $0.083$
1 at titioning	9	3	0.255			0.144	0.700	0.571	0.075	0.000
I-fill- E-titi				Model	1.2					
Infeasible Estimation Local Polynomial	0.05	0.09	0.124	0.132	0.097	0.103	0.242	0.168	0.093	0.118
B-splines	10	5	0.109	0.317	0.085	0.230	0.242	1.000	0.033	0.116
Partitioning	10	5	0.144	0.152	0.111	0.116	0.296	0.222	0.089	0.111
Feasible Estimation										
Local Polynomial	0.12	0.18	0.215	0.180	0.134	0.127	0.822	0.585	0.075	0.125
B-splines	4	3	0.269	0.373	0.196	0.267	0.734	1.241	0.079	0.104
Partitioning	4	3	0.241	0.208	0.164	0.142	0.599	0.563	0.074	0.083
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.065	0.036	0.050	0.032	0.051	0.057	0.072
B-splines	1 1	1 1	0.045	0.063	0.035	0.049	0.032	0.051	0.055	0.071
Partitioning Feasible Estimation	1	1	0.045	0.063	0.035	0.049	0.032	0.051	0.055	0.071
Local Polynomial	0.3	0.3	0.062	0.087	0.047	0.067	0.051	0.072	0.075	0.090
B-splines	2	2	0.053	0.069	0.041	0.054	0.058	0.065	0.063	0.071
Partitioning	2	2	0.061	0.084	0.047	0.064	0.082	0.177	0.065	0.073
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.127	0.138	0.071	0.091	0.927	0.864	0.076	0.109
B-splines	5	5	0.135	0.140	0.081	0.090	0.932	0.905	0.075	0.087
Partitioning	5	5	0.146	0.170	0.096	0.123	0.891	0.760	0.075	0.111
Feasible Estimation	0.00	0.00	0.100	0.107	0.000	0.007	0.020	0.000	0.075	0.092
Local Polynomial B-splines	$0.28 \\ 2$	$0.28 \\ 2$	$0.126 \\ 0.123$	0.137 $0.129$	$0.069 \\ 0.064$	$0.087 \\ 0.075$	0.939 $0.940$	$0.898 \\ 0.925$	$0.075 \\ 0.066$	0.092 $0.072$
Partitioning	2	2	0.123	0.128	0.069	0.073	0.934	0.763	0.068	0.072
1 artifolding	-	-	0.121			0.000	0.564	0.100	0.000	0.014
Infeasible Estimation				Model	1.5					
Local Polynomial	0.2	0.2	0.443	0.205	0.327	0.143	0.079	0.084	0.315	0.114
B-splines	5	5	0.360	0.259	0.305	0.201	0.243	0.194	0.238	0.165
Partitioning	5	5	0.255	0.165	0.196	0.123	0.082	0.121	0.251	0.111
Feasible Estimation										
Local Polynomial	0.09	0.21	0.207	0.210	0.150	0.146	0.091	0.084	0.202	0.112
B-splines	6	$\frac{2}{2}$	0.288	0.353	0.222	0.281	0.168	0.396	0.256	0.158
Partitioning	6	2	0.235	0.271	0.179	0.209	0.153	0.468	0.248	0.114
				Model	1.6					
Infeasible Estimation	0.00	0.07	0.011	0.144	0.160	0.110	0.000	0.140	0.005	0.101
Local Polynomial B-splines	$0.02 \\ 32$	$0.07 \\ 7$	0.211 $0.221$	0.144 $0.177$	0.168 $0.170$	0.113 $0.134$	0.223 $0.268$	0.149 $0.095$	$0.205 \\ 0.168$	$0.131 \\ 0.125$
Partitioning	32	7	0.280	0.176	0.217	0.134	0.424	0.143	0.221	0.123
Feasible Estimation	-	•	0.200	0.2.0	0	0.200				
Local Polynomial	0.05	0.13	0.618	0.228	0.387	0.172	0.121	0.106	1.469	0.120
B-splines	9	4	1.405	1.105	0.873	0.885	0.101	0.819	2.871	1.799
Partitioning	9	4	1.212	0.484	0.659	0.345	0.120	0.264	2.334	0.670
				Model	1.7					
$In feasible\ Estimation$										
Local Polynomial	0.07	0.11	0.103	0.117	0.080	0.092	0.173	0.137	0.079	0.116
B-splines	7	4	0.135	0.130	0.103	0.102	0.340	0.311	0.076	0.096
Partitioning Feasible Estimation	7	4	0.129	0.128	0.099	0.098	0.204	0.317	0.076	0.100
Local Polynomial	0.23	0.25	0.133	0.126	0.097	0.097	0.424	0.318	0.077	0.095
B-splines	2	2	0.133	0.126	0.106	0.102	0.405	0.316	0.084	0.093
Partitioning	2	2	0.140	0.108	0.104	0.083	0.396	0.238	0.082	0.087
Ü				Model						
Infeasible Estimation				Model	1.0					
Local Polynomial	0.11	0.3	0.084	0.091	0.066	0.070	0.082	0.069	0.076	0.097
B-splines	4	2	0.092	0.076	0.072	0.059	0.130	0.072	0.074	0.072
Partitioning	4	2	0.101	0.090	0.079	0.070	0.153	0.203	0.075	0.073
Feasible Estimation	c	0.22		0.000		0.000		0.67:	0.0=0	0.000
Local Polynomial	0.17	0.29	0.082	0.088	0.064	0.068	0.082	0.074	0.078	0.090
B-splines Partitioning	3	$\frac{2}{2}$	$0.110 \\ 0.101$	0.089 $0.100$	$0.090 \\ 0.079$	$0.070 \\ 0.078$	$0.139 \\ 0.092$	$0.092 \\ 0.194$	$0.074 \\ 0.073$	$0.073 \\ 0.074$
1 artificining	J	4	0.101	0.100	0.079	0.010	0.092	0.134	0.073	0.074

Table C.8: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=1,\ n=1000,\ \sigma^2=1,\ X_i\sim\beta(1,1),\ \text{Uniform Cells}$ 

	Tun Parar		Root In		Inget M		Pc (0		ation RMS	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model						
Infeasible Estimation				Model	1.1					
Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.220	0.151	0.111	0.139
B-splines	11	6	0.185	0.207	0.138	0.165	0.514	0.495	0.092	0.137
Partitioning	11	6	0.166	0.157	0.128	0.121	0.273	0.349	0.112	0.136
Feasible Estimation										
Local Polynomial	0.1	0.17	0.190	0.181	0.132	0.131	0.631	0.536	0.090	0.150
B-splines Partitioning	5 5	3 3	$0.407 \\ 0.283$	$0.408 \\ 0.230$	0.287 $0.176$	$0.305 \\ 0.162$	$\frac{1.218}{0.820}$	$\frac{1.216}{0.581}$	0.092 $0.089$	0.111 $0.093$
1 at titioning	9	3	0.263			0.102	0.020	0.561	0.003	0.033
I-fill- E-titi				Model	1.2					
Infeasible Estimation Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.221	0.151	0.111	0.139
B-splines	10	6	0.109	0.207	0.085	0.165	0.227	0.495	0.089	0.138
Partitioning	10	6	0.144	0.157	0.112	0.121	0.273	0.349	0.102	0.136
Feasible Estimation										
Local Polynomial	0.11	0.17	0.216	0.183	0.141	0.132	0.750	0.544	0.089	0.148
B-splines	4	3	0.305	0.408	0.230	0.305	0.773	1.216	0.096	0.102
Partitioning	4	3	0.257	0.229	0.181	0.161	0.578	0.580	0.088	0.093
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.049	0.031	0.047	0.061	0.084
B-splines	1 1	1 1	0.044	0.063	0.035	0.049	0.031 $0.031$	0.047	0.058	0.083
Partitioning Feasible Estimation	1	1	0.044	0.063	0.035	0.049	0.031	0.047	0.058	0.083
Local Polynomial	0.28	0.29	0.062	0.087	0.047	0.065	0.047	0.065	0.088	0.099
B-splines	2	2	0.053	0.069	0.041	0.053	0.055	0.059	0.070	0.084
Partitioning	2	2	0.062	0.085	0.047	0.065	0.079	0.161	0.073	0.084
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.137	0.147	0.075	0.094	0.927	0.863	0.089	0.119
B-splines	5	5	0.145	0.149	0.085	0.094	0.931	0.904	0.088	0.097
Partitioning	5	5	0.155	0.175	0.101	0.128	0.892	0.758	0.089	0.127
Feasible Estimation	0.26	0.00	0.127	0.146	0.073	0.090	0.935	0.894	0.088	0.101
Local Polynomial B-splines	$0.26 \\ 2$	$0.28 \\ 2$	$0.137 \\ 0.134$	$0.146 \\ 0.140$	0.073	0.090	0.935	0.894 $0.921$	0.088	0.101
Partitioning	2	2	0.138	0.135	0.075	0.088	0.925	0.741	0.079	0.086
	_	_	000			0.000		***	0.0.0	
Infeasible Estimation				Model	1.0					
Local Polynomial	0.2	0.2	0.421	0.210	0.301	0.147	0.075	0.076	0.251	0.125
B-splines	5	5	0.351	0.264	0.296	0.205	0.217	0.170	0.169	0.189
Partitioning	5	5	0.247	0.167	0.185	0.124	0.075	0.106	0.212	0.127
Feasible Estimation										
Local Polynomial	0.09	0.2	0.209	0.209	0.147	0.146	0.080	0.077	0.195	0.129
B-splines	6 6	$\frac{2}{2}$	0.291 $0.232$	$0.356 \\ 0.266$	$0.225 \\ 0.172$	0.290	$0.158 \\ 0.129$	$0.343 \\ 0.379$	$0.208 \\ 0.221$	$0.143 \\ 0.115$
Partitioning	Ü	2	0.232			0.207	0.129	0.379	0.221	0.113
				Model	1.6					
Infeasible Estimation	0.02	0.07	0.201	0.141	0.158	0.110	0.184	0.128	0.237	0.151
Local Polynomial B-splines	29	7	0.212	0.141	0.158	0.116	0.134	0.128	0.237	0.156
Partitioning	29	7	0.267	0.174	0.204	0.133	0.175	0.125	0.269	0.152
Feasible Estimation										
Local Polynomial	0.05	0.13	0.517	0.209	0.298	0.155	0.107	0.094	1.430	0.152
B-splines	9	4	1.169	0.939	0.642	0.723	0.099	0.635	2.775	1.738
Partitioning	9	4	1.011	0.411	0.492	0.280	0.109	0.237	2.324	0.691
				Model	1.7					
$Infeasible\ Estimation$										
Local Polynomial	0.07	0.11	0.104	0.118	0.081	0.091	0.160	0.123	0.093	0.135
B-splines	7	4	$0.142 \\ 0.131$	0.136	0.111	0.108	0.336	0.300	0.090	0.109
Partitioning Feasible Estimation	7	4	0.131	0.128	0.102	0.098	0.201	0.281	0.090	0.111
Local Polynomial	0.21	0.24	0.140	0.128	0.103	0.099	0.408	0.300	0.091	0.106
B-splines	2	2	0.149	0.145	0.114	0.112	0.407	0.383	0.092	0.088
Partitioning	2	2	0.149	0.107	0.113	0.083	0.387	0.207	0.092	0.097
				Model	1.8					
Infeasible Estimation				1110401	1.0					
Local Polynomial	0.11	0.31	0.082	0.089	0.063	0.067	0.071	0.061	0.090	0.102
B-splines	4	2	0.089	0.075	0.069	0.057	0.118	0.066	0.086	0.085
Partitioning	4	2	0.098	0.090	0.076	0.069	0.139	0.182	0.087	0.085
Feasible Estimation	0.17	0.00	0.000	0.007	0.000	0.000	0.075	0.007	0.001	0.100
Local Polynomial B-splines	$0.17 \\ 3$	$0.28 \\ 2$	$0.080 \\ 0.105$	$0.087 \\ 0.085$	$0.062 \\ 0.084$	$0.066 \\ 0.065$	$0.075 \\ 0.124$	$0.067 \\ 0.078$	0.091 $0.088$	$0.100 \\ 0.088$
Partitioning	3	2	$0.105 \\ 0.097$	0.085 $0.097$	0.084 $0.075$	0.065 $0.075$	0.124	0.078 $0.173$	0.088	0.088
	3		0.001	0.031	0.010	0.010	0.000	0.110	0.000	0.000

Table C.9: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=1,\ n=1000,\ \sigma^2=1,\ X_i\sim\beta(2,2),\ \text{Uniform Cells}$ 

	Tun Parar		Root In		Inget M		Pc (0.		ation RMS	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model						
Infeasible Estimation				Model	1.1					
Local Polynomial	0.04	0.08	0.132	0.136	0.102	0.101	0.196	0.134	0.169	0.209
B-splines	11	6	0.203	0.225	0.157	0.186	0.498	0.463	0.137	0.195
Partitioning	11	6	0.173	0.159	0.133	0.121	0.264	0.282	0.158	0.194
Feasible Estimation										
Local Polynomial	0.09	0.16	0.191	0.185	0.139	0.138	0.545	0.470	0.140	0.218
B-splines Partitioning	5 5	3 3	0.381 $0.274$	$0.455 \\ 0.256$	0.275 $0.182$	0.363 $0.190$	0.951 $0.653$	$\frac{1.150}{0.566}$	0.133 $0.138$	$0.141 \\ 0.134$
1 at titioning	9	3	0.274			0.130	0.055	0.500	0.136	0.134
I-fill- E-titi				Model	1.2					
Infeasible Estimation Local Polynomial	0.04	0.08	0.131	0.136	0.101	0.101	0.197	0.134	0.169	0.209
B-splines	11	6	0.203	0.225	0.157	0.186	0.498	0.463	0.137	0.196
Partitioning	11	6	0.172	0.159	0.133	0.121	0.264	0.282	0.158	0.194
Feasible Estimation										
Local Polynomial	0.1	0.16	0.212	0.187	0.149	0.139	0.633	0.476	0.139	0.216
B-splines	5	3	0.446	0.454	0.343	0.363	1.145	1.150	0.147	0.142
Partitioning	5	3	0.314	0.256	0.203	0.190	0.783	0.566	0.133	0.134
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.048	0.032	0.044	0.070	0.120
B-splines Partitioning	1 1	1 1	$0.045 \\ 0.045$	$0.064 \\ 0.064$	$0.035 \\ 0.035$	$0.048 \\ 0.048$	$0.032 \\ 0.032$	$0.044 \\ 0.044$	$0.067 \\ 0.067$	0.118 $0.118$
Feasible Estimation	1	1	0.045	0.004	0.033	0.048	0.032	0.044	0.007	0.116
Local Polynomial	0.26	0.28	0.064	0.087	0.046	0.063	0.044	0.057	0.129	0.137
B-splines	2	2	0.055	0.069	0.042	0.052	0.053	0.054	0.088	0.126
Partitioning	2	2	0.065	0.086	0.049	0.065	0.075	0.142	0.094	0.128
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.156	0.162	0.084	0.101	0.923	0.858	0.131	0.156
B-splines	5	5	0.163	0.166	0.095	0.103	0.926	0.895	0.132	0.135
Partitioning	5	5	0.171	0.187	0.111	0.136	0.887	0.753	0.134	0.173
Feasible Estimation	0.22	0.26	0.156	0.162	0.084	0.099	0.002	0.883	0.131	0.140
Local Polynomial B-splines	0.23 2	0.26	0.156 $0.154$	0.162 $0.157$	0.084	0.099	0.923 $0.921$	0.883 $0.907$	0.131 $0.104$	0.140 $0.128$
Partitioning	2	2	0.157	0.146	0.089	0.097	0.903	0.681	0.110	0.130
	_	_	0.20				0.000		0	
Infeasible Estimation				Model	1.0					
Local Polynomial	0.2	0.2	0.361	0.211	0.244	0.146	0.069	0.065	0.162	0.163
B-splines	5	5	0.323	0.259	0.266	0.192	0.173	0.126	0.174	0.222
Partitioning	5	5	0.228	0.168	0.160	0.124	0.064	0.092	0.168	0.173
Feasible Estimation										
Local Polynomial	0.09	0.18	0.210	0.203	0.140	0.140	0.067	0.068	0.203	0.183
B-splines	5 5	3 3	0.317 $0.227$	$0.317 \\ 0.227$	$0.259 \\ 0.159$	$0.256 \\ 0.165$	$0.166 \\ 0.070$	$0.234 \\ 0.145$	$0.181 \\ 0.174$	$0.143 \\ 0.145$
Partitioning	5	3	0.221			0.105	0.070	0.145	0.174	0.145
				Model	1.6					
Infeasible Estimation	0.02	0.08	0.184	0.141	0.138	0.104	0.142	0.104	0.329	0.220
Local Polynomial B-splines	24	6	0.194	0.141	0.135	0.104	0.142	0.104	0.329	0.526
Partitioning	24	6	0.243	0.166	0.178	0.124	0.254	0.251	0.370	0.202
Feasible Estimation										
Local Polynomial	0.05	0.13	0.343	0.169	0.178	0.120	0.094	0.081	1.306	0.244
B-splines	9	4	0.767	0.725	0.331	0.492	0.095	0.387	2.441	1.599
Partitioning	9	4	0.671	0.342	0.274	0.212	0.097	0.189	2.175	0.847
				Model	1.7					
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.107	0.120	0.082	0.090	0.144	0.109	0.144	0.195
B-splines Partitioning	7 7	5 5	0.154 $0.136$	$0.161 \\ 0.144$	$0.124 \\ 0.106$	0.129 $0.110$	0.325 $0.192$	0.334 $0.112$	0.139 $0.140$	$0.144 \\ 0.173$
Feasible Estimation	1	5	0.130	0.144	0.100	0.110	0.192	0.112	0.140	0.173
Local Polynomial	0.18	0.22	0.147	0.130	0.112	0.100	0.373	0.264	0.135	0.151
B-splines	3	2	0.169	0.159	0.131	0.128	0.412	0.361	0.114	0.143
Partitioning	3	2	0.166	0.108	0.130	0.082	0.380	0.164	0.127	0.132
				Model	1.8					
Infeasible Estimation										
Local Polynomial	0.13	0.32	0.080	0.087	0.059	0.062	0.061	0.054	0.137	0.136
B-splines	4	2	0.083	0.074	0.063	0.055	0.100	0.058	0.125	0.128
Partitioning	4	2	0.096	0.090	0.073	0.068	0.118	0.154	0.128	0.130
Feasible Estimation Local Polynomial	0.16	0.27	0.079	0.088	0.059	0.064	0.068	0.059	0.137	0.138
B-splines	3	2	0.079	0.088	0.059 $0.075$	0.054 $0.058$	0.068	0.059	0.137 $0.129$	0.138
Partitioning	3	2	0.091	0.079	0.070	0.038	0.084	0.149	0.129	0.135
	-		2.004		,,		, <b>.</b>			

Table C.10: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 1000, \sigma^2 = 4, X_i \sim \beta(0.5, 0.5),$ Uniform Cells

	Tur Parai	ning neter	Root In		Inget M		Pc (0		ation RMS	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
$In feasible\ Estimation$										
Local Polynomial	0.06	0.1	0.218	0.243	0.170	0.190	0.394	0.293	0.164	0.234
B-splines	8	5	0.198	0.353	0.156	0.267	0.271	1.008	0.149	0.210
Partitioning Feasible Estimation	8	5	0.257	0.288	0.199	0.221	0.426	0.306	0.156	0.222
Local Polynomial	0.11	0.19	0.247	0.254	0.181	0.193	0.761	0.645	0.150	0.241
B-splines	4	3	0.327	0.395	0.240	0.291	0.852	1.218	0.149	0.174
Partitioning	4	3	0.294	0.262	0.215	0.195	0.670	0.518	0.150	0.159
				Model	1.2					
$In feasible\ Estimation$										
Local Polynomial	0.06	0.1	0.217	0.243	0.170	0.190	0.396	0.293	0.163	0.234
B-splines Partitioning	8 8	5 5	0.197 $0.257$	0.353 $0.288$	0.155 $0.198$	0.267 $0.221$	$0.271 \\ 0.426$	0.306	$0.149 \\ 0.156$	$0.209 \\ 0.222$
Feasible Estimation	0	3	0.237	0.288	0.198	0.221	0.420	0.300	0.150	0.222
Local Polynomial	0.13	0.19	0.271	0.255	0.189	0.194	0.907	0.653	0.151	0.239
B-splines	4	3	0.347	0.387	0.252	0.286	1.017	1.214	0.155	0.160
Partitioning	4	3	0.322	0.259	0.224	0.193	0.881	0.505	0.148	0.160
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.093	0.129	0.072	0.101	0.064	0.102	0.115	0.143
B-splines Partitioning	1 1	1 1	$0.090 \\ 0.090$	$0.127 \\ 0.127$	$0.070 \\ 0.070$	0.099 $0.099$	$0.063 \\ 0.063$	$0.102 \\ 0.102$	$0.109 \\ 0.109$	$0.143 \\ 0.143$
Feasible Estimation	1	1	0.090	0.127	0.070	0.099	0.003	0.102	0.109	0.143
Local Polynomial	0.24	0.27	0.132	0.177	0.101	0.137	0.114	0.151	0.151	0.188
B-splines	2	2	0.114	0.140	0.089	0.110	0.123	0.134	0.132	0.143
Partitioning	2	2	0.136	0.176	0.104	0.135	0.170	0.381	0.136	0.146
				Model	1.4					
$In feasible\ Estimation$										
Local Polynomial	0.2	0.2	0.171	0.214	0.118	0.160	0.933	0.877	0.152	0.218
B-splines	5 5	5 5	0.191	0.209	0.138	0.155 $0.227$	0.939	0.915	0.150	0.173
Partitioning Feasible Estimation	Э	Э	0.227	0.298	0.169	0.227	0.902	0.790	0.150	0.222
Local Polynomial	0.24	0.26	0.171	0.206	0.118	0.152	0.935	0.900	0.151	0.190
B-splines	2	2	0.159	0.178	0.106	0.126	0.945	0.931	0.133	0.143
Partitioning	2	2	0.176	0.199	0.121	0.146	0.937	0.815	0.137	0.147
				Model	1.5					
$Infeasible\ Estimation$										
Local Polynomial	0.2	0.2	0.457	0.261	0.344	0.199	0.128	0.168	0.342	0.221
B-splines	5 5	5 5	0.385	0.302	0.321	0.237	0.267	0.233	0.272	$0.222 \\ 0.222$
Partitioning Feasible Estimation	5	5	0.308	0.295	0.239	0.226	0.160	0.242	0.283	0.222
Local Polynomial	0.09	0.22	0.264	0.272	0.201	0.206	0.176	0.168	0.246	0.212
B-splines	5	2	0.367	0.376	0.302	0.299	0.238	0.410	0.278	0.201
Partitioning	5	2	0.306	0.318	0.237	0.246	0.211	0.543	0.283	0.175
				Model	1.6					
$Infeasible\ Estimation$										
Local Polynomial	0.02	0.08	0.368	0.279	0.293	0.218	0.385	0.276	0.361	0.247
B-splines	$\frac{24}{24}$	6 6	$0.388 \\ 0.486$	0.336 $0.324$	$0.297 \\ 0.378$	$0.259 \\ 0.249$	$0.463 \\ 0.726$	$0.233 \\ 0.708$	$0.250 \\ 0.373$	$0.386 \\ 0.239$
Partitioning Feasible Estimation	24	Ü	0.480	0.324	0.378	0.249	0.720	0.708	0.575	0.239
Local Polynomial	0.05	0.14	0.659	0.294	0.450	0.233	0.233	0.208	1.521	0.229
B-splines	9	3	1.415	1.391	0.904	1.101	0.178	0.898	2.873	2.210
Partitioning	9	3	1.234	0.675	0.733	0.508	0.223	0.394	2.337	0.990
				Model	1.7					
$Infeasible\ Estimation$										
Local Polynomial	0.09	0.13	0.180	0.218	0.141	0.171	0.271	0.239	0.152	0.226
B-splines Partitioning	5	4	0.196	0.195	0.153	0.154	0.434	0.343	0.150	0.164
Feasible Estimation	5	4	0.216	0.254	0.166	0.195	0.323	0.596	0.150	0.199
Local Polynomial	0.22	0.25	0.175	0.199	0.135	0.156	0.424	0.345	0.152	0.193
B-splines	2	2	0.174	0.183	0.134	0.143	0.430	0.415	0.143	0.144
Partitioning	2	2	0.186	0.189	0.142	0.146	0.423	0.404	0.145	0.154
				Model	1.8					
$In feasible\ Estimation$										
Local Polynomial	0.14	0.36	0.152	0.164	0.119	0.127	0.142	0.126	0.154	0.164
B-splines	3 3	1	0.158	0.163	0.127	0.129	0.166	0.161	0.143	0.144
Partitioning Feasible Estimation	3	1	0.169	0.163	0.132	0.129	0.135	0.161	0.145	0.144
Local Polynomial	0.19	0.26	0.146	0.177	0.114	0.137	0.141	0.152	0.152	0.188
B-splines	3	2	0.154	0.147	0.121	0.115	0.188	0.144	0.144	0.143
Partitioning	3	2	0.165	0.180	0.128	0.139	0.188	0.384	0.145	0.147

Table C.11: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=1,\, n=1000,\, \sigma^2=4,\, X_i\sim \beta(1,1),\, \text{Uniform Cells}$ 

	Tun Parar		Root In			rated AE		oint Estim .5)	ation RM (0	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation	0.00	0.1	0.000	0.044	0.170	0.100	0.969	0.000	0.104	0.074
Local Polynomial	0.06	0.1	0.222	0.244	0.173	0.189	0.363	0.263	0.194	0.274
B-splines Partitioning	8 8	5 5	0.199 $0.257$	0.377 $0.289$	0.157 $0.200$	0.293 $0.222$	$0.240 \\ 0.377$	0.984 $0.282$	0.177 $0.181$	$0.242 \\ 0.253$
Feasible Estimation	0	3	0.257	0.289	0.200	0.222	0.377	0.262	0.161	0.200
Local Polynomial	0.11	0.18	0.255	0.260	0.189	0.198	0.723	0.612	0.179	0.278
B-splines	4	3	0.379	0.425	0.283	0.323	0.981	1.197	0.177	0.182
Partitioning	4	3	0.318	0.280	0.233	0.212	0.714	0.533	0.178	0.182
				Model	1.2					
Infeasible Estimation	0.00	0.1	0.001	0.044	0.170	0.100	0.205	0.000	0.100	0.074
Local Polynomial B-splines	0.06	0.1 5	$0.221 \\ 0.198$	$0.244 \\ 0.377$	$0.172 \\ 0.156$	0.189 $0.293$	$0.365 \\ 0.240$	$0.263 \\ 0.984$	$0.193 \\ 0.177$	$0.274 \\ 0.240$
Partitioning	8	5	0.257	0.289	0.200	0.222	0.376	0.282	0.181	0.253
Feasible Estimation	_	-		0.200	0.200			*****	0.202	0.200
Local Polynomial	0.12	0.18	0.277	0.261	0.198	0.199	0.847	0.618	0.179	0.275
B-splines	4	3	0.351	0.420	0.267	0.320	0.884	1.194	0.185	0.176
Partitioning	4	3	0.325	0.277	0.238	0.210	0.753	0.525	0.173	0.183
				Model	1.3					
Infeasible Estimation	0.0	0.0	0.001	0.100	0.071	0.000	0.000	0.004	0.101	0.105
Local Polynomial B-splines	0.9 1	0.9 1	0.091 $0.089$	0.128 $0.126$	0.071 $0.069$	0.099 $0.097$	$0.062 \\ 0.062$	0.094 $0.094$	0.121 $0.116$	$0.167 \\ 0.166$
Partitioning	1	1	0.089	0.126	0.069	0.097	0.062	0.094	0.116	0.166
Feasible Estimation	_	_	0.000	0.220	0.000			0.00-		0.200
Local Polynomial	0.24	0.26	0.131	0.176	0.099	0.133	0.103	0.136	0.178	0.206
B-splines	2	2	0.115	0.140	0.089	0.107	0.115	0.123	0.147	0.169
Partitioning	2	2	0.137	0.176	0.105	0.134	0.159	0.340	0.154	0.170
				Model	1.4					
Infeasible Estimation	0.0	0.0	0.150	0.010	0.100	0.150	0.000	0.055	0.150	0.000
Local Polynomial	0.2	0.2 5	$0.178 \\ 0.197$	0.218 $0.215$	$0.120 \\ 0.141$	0.159 $0.157$	0.933 $0.938$	$0.875 \\ 0.912$	$0.179 \\ 0.177$	0.239 $0.192$
B-splines Partitioning	5 5	5 5	0.197	0.215	0.141	0.137	0.902	0.912 $0.782$	0.177	0.192
Feasible Estimation	0	0	0.202	0.001	0.170	0.200	0.502	0.102	0.177	0.200
Local Polynomial	0.23	0.26	0.179	0.212	0.120	0.153	0.933	0.898	0.179	0.209
B-splines	2	2	0.169	0.185	0.110	0.128	0.942	0.926	0.151	0.169
Partitioning	2	2	0.185	0.204	0.126	0.149	0.932	0.784	0.158	0.170
				Model	1.5					
Infeasible Estimation	0.0	0.0	0.405	0.004	0.015	0.000	0.110	0.151	0.000	0.040
Local Polynomial B-splines	0.2 5	0.2 5	$0.435 \\ 0.375$	0.264 $0.306$	$0.317 \\ 0.312$	$0.200 \\ 0.239$	0.118 $0.239$	$0.151 \\ 0.207$	$0.293 \\ 0.227$	$0.242 \\ 0.252$
Partitioning	5	5	0.302	0.296	0.230	0.227	0.145	0.211	0.260	0.252
Feasible Estimation	0	0	0.002	0.250	0.200	0.221	0.140	0.211	0.200	0.200
Local Polynomial	0.09	0.21	0.266	0.270	0.199	0.203	0.153	0.151	0.252	0.243
B-splines	5	2	0.368	0.379	0.303	0.307	0.224	0.364	0.236	0.204
Partitioning	5	2	0.301	0.314	0.230	0.246	0.168	0.458	0.263	0.191
				Model	1.6					
Infeasible Estimation	0.00	0.00	0.051	0.051	0.070	0.000	0.015	0.005	0.415	0.000
Local Polynomial B-splines	$0.02 \\ 22$	0.08 6	0.351 $0.371$	$0.271 \\ 0.306$	$0.276 \\ 0.276$	$0.209 \\ 0.225$	0.317 $0.391$	$0.235 \\ 0.200$	0.415 $0.400$	$0.286 \\ 0.459$
Partitioning	22	6	0.465	0.320	0.356	0.225	0.607	0.626	0.448	0.439 $0.274$
Feasible Estimation										
Local Polynomial	0.05	0.14	0.568	0.280	0.367	0.217	0.203	0.183	1.509	0.270
B-splines	9	3	1.181	1.244	0.678	0.929	0.166	0.672	2.781	2.140
Partitioning	9	3	1.038	0.627	0.571	0.454	0.197	0.330	2.332	1.103
				Model	1.7					
Infeasible Estimation	0.00	0.10	0.100	0.010	0.140	0.100	0.050	0.015	0.100	0.001
Local Polynomial B-splines	0.09	0.12	0.183	0.219	0.142	0.169	0.253 $0.428$	0.215	0.180	0.261
Partitioning	5 5	4 4	$0.203 \\ 0.218$	0.199 $0.254$	0.159 $0.170$	0.157 $0.195$	0.316	$0.328 \\ 0.522$	$0.178 \\ 0.178$	$0.187 \\ 0.222$
Feasible Estimation	0	-	0.210	0.204	0.170	0.150	0.010	0.022	0.110	0.222
Local Polynomial	0.21	0.25	0.181	0.202	0.140	0.157	0.414	0.331	0.181	0.215
B-splines	2	2	0.183	0.190	0.142	0.149	0.427	0.403	0.163	0.171
Partitioning	2	2	0.195	0.190	0.150	0.146	0.413	0.364	0.168	0.177
				Model	1.8					
Infeasible Estimation		0.22		0.101	6	0.101	6 - 6 - 1	0.110	0.107	0.101
Local Polynomial	0.15	0.36	0.148	0.161	0.113	0.121	0.124	0.112	0.184	0.181
B-splines Partitioning	3	$\frac{1}{1}$	$0.155 \\ 0.167$	0.158 $0.158$	$0.122 \\ 0.130$	0.123 $0.123$	$0.147 \\ 0.122$	0.138 $0.138$	$0.167 \\ 0.167$	$0.174 \\ 0.174$
Feasible Estimation	3	1	0.107	0.100	0.130	0.123	0.122	0.136	0.107	0.174
Local Polynomial	0.19	0.26	0.145	0.176	0.111	0.133	0.128	0.137	0.180	0.207
B-splines	3	2	0.148	0.145	0.115	0.112	0.161	0.129	0.163	0.170
Partitioning	3	2	0.161	0.179	0.124	0.138	0.167	0.346	0.165	0.171

Table C.12: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=1,\, n=1000,\, \sigma^2=4,\, X_i\sim\beta(2,2),\, \text{Uniform Cells}$ 

	Tun Parar		Root In			rated AE	Pc (0		ation RM: (0	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.230	0.249	0.177	0.186	0.325	0.234	0.300	0.401
B-splines	8 8	5	0.204	0.413	0.158	0.333	0.202	0.931	0.279	0.318
Partitioning Feasible Estimation	8	5	0.260	0.292	0.200	0.223	0.324	0.259	0.280	0.346
Local Polynomial	0.1	0.17	0.268	0.270	0.203	0.206	0.666	0.559	0.279	0.382
B-splines	5	3	0.456	0.471	0.356	0.375	1.088	1.141	0.268	0.270
Partitioning	5	3	0.358	0.309	0.265	0.240	0.757	0.547	0.269	0.266
				Model	1.2					
Infeasible Estimation										
Local Polynomial B-splines	0.06	0.1 5	0.229	$0.249 \\ 0.413$	0.176	0.186	0.326	0.234	$0.300 \\ 0.279$	0.401
Partitioning	8	5 5	$0.203 \\ 0.259$	0.413 $0.292$	0.157 $0.200$	0.333 $0.223$	$0.202 \\ 0.324$	$0.931 \\ 0.259$	0.279	0.316 $0.346$
Feasible Estimation	8	3	0.233	0.232	0.200	0.223	0.324	0.203	0.200	0.540
Local Polynomial	0.11	0.18	0.287	0.271	0.213	0.206	0.761	0.563	0.277	0.380
B-splines	4	3	0.387	0.469	0.310	0.374	0.843	1.140	0.275	0.267
Partitioning	4	3	0.341	0.308	0.263	0.239	0.645	0.542	0.257	0.266
				Model	1.3					
Infeasible Estimation			0.000	0.100		0.000		0.000	0 - 10	0.01
Local Polynomial	0.9	0.9	0.092	0.129	0.071	0.096	0.064	0.088	0.140	0.241
B-splines Partitioning	1 1	1 1	$0.090 \\ 0.090$	$0.127 \\ 0.127$	$0.070 \\ 0.070$	$0.095 \\ 0.095$	0.064 $0.064$	$0.088 \\ 0.088$	0.133 $0.133$	$0.236 \\ 0.236$
Feasible Estimation	-	1	0.000	0.121	0.010	0.050	0.004	0.000	0.100	0.200
Local Polynomial	0.23	0.26	0.134	0.177	0.097	0.128	0.092	0.119	0.266	0.281
B-splines	2	2	0.118	0.141	0.090	0.105	0.107	0.111	0.191	0.254
Partitioning	2	2	0.140	0.177	0.106	0.133	0.143	0.290	0.205	0.259
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.193	0.228	0.126	0.161	0.926	0.866	0.263	0.313
B-splines Partitioning	5 5	5 5	0.211 $0.244$	0.228 $0.309$	0.147 $0.181$	$0.161 \\ 0.235$	$0.930 \\ 0.893$	$0.902 \\ 0.771$	$0.263 \\ 0.269$	0.267 $0.346$
Feasible Estimation	5	3	0.244	0.309	0.161	0.233	0.695	0.771	0.209	0.340
Local Polynomial	0.22	0.25	0.195	0.224	0.127	0.156	0.923	0.887	0.267	0.284
B-splines	2	2	0.186	0.199	0.122	0.135	0.924	0.912	0.200	0.255
Partitioning	2	2	0.200	0.213	0.136	0.154	0.909	0.721	0.213	0.260
				Model	1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.379	0.266	0.262	0.195	0.105	0.128	$0.280 \\ 0.287$	0.316
B-splines Partitioning	5 5	5 5	$0.350 \\ 0.287$	$0.302 \\ 0.298$	$0.282 \\ 0.212$	$0.226 \\ 0.225$	$0.193 \\ 0.122$	$0.162 \\ 0.183$	0.288	0.320 $0.347$
Feasible Estimation	0	o	0.201	0.230	0.212	0.220	0.122	0.100	0.200	0.041
Local Polynomial	0.1	0.19	0.269	0.267	0.193	0.195	0.126	0.131	0.317	0.337
B-splines	5	2	0.381	0.369	0.297	0.297	0.196	0.279	0.310	0.264
Partitioning	5	2	0.306	0.305	0.223	0.235	0.159	0.318	0.312	0.269
				Model	1.6					
Infeasible Estimation										
Local Polynomial	0.03	0.09 5	0.319	$0.259 \\ 0.453$	0.240	0.192 $0.326$	0.244	0.193 $0.324$	0.567 $0.953$	0.413 $1.160$
B-splines Partitioning	18 18	5 5	$0.342 \\ 0.423$	0.433 $0.306$	0.237 $0.310$	0.320	$0.292 \\ 0.443$	0.324 $0.183$	1.472	0.417
Feasible Estimation	10	9	0.120	0.000	0.010	0.220	0.110	0.100	1.1.2	0.11.
Local Polynomial	0.06	0.14	0.416	0.261	0.251	0.190	0.174	0.155	1.462	0.428
B-splines	9	3	0.854	0.985	0.412	0.625	0.179	0.319	2.478	1.759
Partitioning	9	3	0.762	0.553	0.376	0.374	0.237	0.182	2.266	1.292
				Model	1.7					
Infeasible Estimation	0.00	0.10	0.100	0.004	0.140	0.100	0.00=	0.100	0.001	0.005
Local Polynomial	0.09	0.12	0.189	0.224	0.143	$0.166 \\ 0.161$	0.227	0.189	0.281	0.365
B-splines Partitioning	6 6	$\frac{4}{4}$	$0.182 \\ 0.225$	$0.205 \\ 0.255$	$0.142 \\ 0.174$	0.101	$0.198 \\ 0.264$	$0.301 \\ 0.433$	$0.273 \\ 0.277$	$0.265 \\ 0.299$
Feasible Estimation	· ·	-	0.220	0.200	0.114	0.154	0.204	0.400	0.211	0.200
Local Polynomial	0.2	0.24	0.192	0.207	0.148	0.158	0.391	0.303	0.270	0.294
B-splines	3	2	0.198	0.202	0.156	0.160	0.408	0.376	0.213	0.262
Partitioning	3	2	0.207	0.192	0.164	0.146	0.389	0.303	0.226	0.261
				Model	1.8					
Infeasible Estimation	0.15	0.07	0.144	0.150	0.100	0.114	0.100	0.101	0.055	0.004
Local Polynomial B-splines	0.17	0.37	$0.144 \\ 0.149$	0.158	0.106	$0.114 \\ 0.112$	$0.106 \\ 0.130$	0.101	0.275	$0.264 \\ 0.259$
Partitioning	3	1 1	0.149 $0.164$	$0.150 \\ 0.150$	$0.116 \\ 0.126$	0.112 $0.112$	0.130	$0.115 \\ 0.115$	$0.231 \\ 0.231$	0.259 $0.259$
Feasible Estimation	Ü	_	5.104	0.100	3.120	V.114	5.110	5.110	0.201	5.203
Local Polynomial	0.19	0.25	0.146	0.178	0.107	0.128	0.114	0.119	0.270	0.282
B-splines	3	2	0.142	0.144	0.109	0.107	0.142	0.115	0.218	0.257
Partitioning	3	2	0.157	0.179	0.120	0.136	0.149	0.292	0.223	0.261

## C.1.2 QUANTILE CELL BOUNDARIES

Table C.13: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 500, \sigma^2 = 1, X_i \sim \beta(0.5, 0.5),$  Quantile Cells

	Tun Parar		Root In			rated AE		oint Estim	ation RM	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation	0.05	0.00	0.104	0.150	0.100	0.100	0.005	0.010	0.104	0.100
Local Polynomial	0.05	0.09	0.164	0.178	0.128	0.139	0.305	0.219	0.124	0.169
B-splines Partitioning	9 9	5 5	$0.270 \\ 0.229$	0.343 $0.218$	$0.195 \\ 0.173$	$0.251 \\ 0.167$	0.787 $0.444$	$\frac{1.065}{0.317}$	0.134 $0.192$	$0.192 \\ 0.167$
Feasible Estimation	9	3	0.229	0.218	0.173	0.107	0.444	0.317	0.192	0.107
Local Polynomial	0.12	0.19	0.229	0.216	0.159	0.158	0.804	0.655	0.106	0.164
B-splines	4	2	0.350	0.380	0.246	0.274	0.946	1.202	0.108	0.145
Partitioning	4	2	0.287	0.225	0.203	0.160	0.641	0.520	0.106	0.115
				Model	1.2					
Infeasible Estimation	0.05	0.00	0.104	0.150	0.100	0.100	0.00=	0.010	0.100	0.100
Local Polynomial B-splines	0.05 9	$0.09 \\ 5$	$0.164 \\ 0.269$	$0.178 \\ 0.343$	$0.128 \\ 0.194$	$0.139 \\ 0.251$	$0.307 \\ 0.787$	$0.219 \\ 1.065$	$0.123 \\ 0.134$	0.169 $0.186$
Partitioning	9	5	0.228	0.218	0.172	0.167	0.444	0.317	0.192	0.167
Feasible Estimation	v	9	0.220	0.210	0.1.2	0.101	0.111	0.011	0.102	0.10.
Local Polynomial	0.14	0.19	0.258	0.218	0.167	0.159	0.945	0.664	0.107	0.162
B-splines	3	2	0.393	0.367	0.272	0.266	1.320	1.192	0.116	0.122
Partitioning	3	2	0.353	0.216	0.227	0.154	1.122	0.487	0.105	0.118
				Model	1.3					
Infeasible Estimation				0.007		0.6=-		0.0=0	0.001	0.100
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.071	0.045	0.072	0.081	0.102
B-splines Partitioning	1 1	$\frac{1}{1}$	$0.063 \\ 0.063$	0.089 $0.089$	$0.049 \\ 0.049$	0.069 $0.069$	$0.045 \\ 0.045$	$0.072 \\ 0.072$	$0.078 \\ 0.078$	$0.102 \\ 0.102$
Feasible Estimation	_	1	0.003	0.003	0.043	0.003	0.045	0.012	0.078	0.102
Local Polynomial	0.3	0.3	0.087	0.122	0.066	0.094	0.072	0.102	0.106	0.130
B-splines	2	2	0.074	0.097	0.057	0.075	0.078	0.090	0.089	0.102
Partitioning	2	2	0.085	0.119	0.065	0.090	0.110	0.192	0.092	0.103
				Model	1.4					
$In feasible\ Estimation$										
Local Polynomial	0.2	0.2	0.142	0.166	0.090	0.119	0.929	0.866	0.108	0.156
B-splines Partitioning	5	5	0.155	0.166	0.103	0.116	0.941	0.915	0.106	0.134
Feasible Estimation	5	5	0.177	0.221	0.127	0.167	0.904	0.788	0.107	0.167
Local Polynomial	0.29	0.29	0.140	0.162	0.085	0.112	0.943	0.902	0.106	0.129
B-splines	2	2	0.134	0.146	0.077	0.094	0.948	0.930	0.092	0.102
Partitioning	2	2	0.139	0.151	0.085	0.105	0.911	0.771	0.096	0.104
				Model	1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.447	0.224	0.332	0.164	0.098	0.120	0.323	0.159
B-splines Partitioning	5 5	5 5	$0.293 \\ 0.262$	$0.282 \\ 0.237$	0.225 $0.199$	0.222 $0.181$	0.187 $0.111$	0.243 $0.163$	$0.273 \\ 0.247$	0.213 $0.167$
Feasible Estimation	5	3	0.202	0.237	0.199	0.161	0.111	0.103	0.241	0.107
Local Polynomial	0.1	0.23	0.244	0.240	0.181	0.173	0.120	0.119	0.231	0.145
B-splines	5	2	0.295	0.363	0.227	0.288	0.188	0.394	0.273	0.173
Partitioning	5	2	0.263	0.292	0.199	0.225	0.113	0.331	0.248	0.135
				Model	1.6					
$In feasible\ Estimation$										
Local Polynomial	0.02	0.07	0.280	0.207	0.222	0.161	0.297	0.201	0.279	0.185
B-splines Partitioning	28 28	7 7	0.257 $0.345$	$0.208 \\ 0.238$	$0.201 \\ 0.268$	0.168 $0.183$	0.234 $0.337$	$0.212 \\ 0.196$	$0.294 \\ 0.368$	$0.204 \\ 0.201$
Feasible Estimation	20	,	0.545	0.236	0.200	0.103	0.331	0.130	0.300	0.201
Local Polynomial	0.06	0.15	0.754	0.359	0.477	0.273	0.158	0.144	1.900	0.484
B-splines	8	3	0.960	1.418	0.553	1.122	0.174	0.677	1.857	2.355
Partitioning	8	3	0.858	0.608	0.491	0.455	0.219	0.151	1.353	0.890
				Model	1.7					
Infeasible Estimation		0.40			0.400					
Local Polynomial B-splines	0.08	0.12	0.135	0.159	0.106	0.125	0.216	0.179	0.109	0.164
Partitioning	6 6	4 4	$0.146 \\ 0.165$	0.158 $0.181$	0.115 $0.129$	0.125 $0.139$	$0.279 \\ 0.220$	$0.349 \\ 0.262$	0.107 $0.113$	0.128 $0.156$
Feasible Estimation	Ü	4	0.105	0.101	0.123	0.133	0.220	0.202	0.110	0.150
Local Polynomial	0.25	0.27	0.148	0.155	0.111	0.121	0.439	0.340	0.108	0.132
B-splines	2	2	0.152	0.152	0.116	0.117	0.430	0.407	0.116	0.103
Partitioning	2	2	0.153	0.141	0.115	0.108	0.394	0.282	0.115	0.113
				Model	1.8					
Infeasible Estimation	0.10	0.00	0.110	0.101	0.00=	0.000	0.100	0.000	0.10=	0.105
Local Polynomial B-splines	0.12	0.33	0.112	0.121	$0.087 \\ 0.087$	0.093 $0.109$	$0.109 \\ 0.126$	$0.093 \\ 0.144$	$0.107 \\ 0.106$	$0.125 \\ 0.104$
Partitioning	4	1 1	$0.111 \\ 0.132$	0.135 $0.135$	0.087	0.109	0.126 $0.171$	0.144 $0.144$	0.106 $0.107$	0.104 $0.104$
Feasible Estimation	-1	1	0.102	0.100	5.102	0.103	0.111	0.144	0.101	0.104
Local Polynomial	0.2	0.29	0.107	0.123	0.084	0.095	0.110	0.104	0.109	0.130
B-splines	2	2	0.128	0.113	0.102	0.088	0.152	0.113	0.104	0.103
Partitioning	2	2	0.126	0.131	0.098	0.102	0.126	0.205	0.105	0.104

Table C.14: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=500, \sigma^2=1, X_i \sim \beta(1,1), \text{ Quantile Cells}$ 

	Tun Parar		Root In			rated AE		oint Estim	ation RM	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
$In feasible\ Estimation$										
Local Polynomial	0.05	0.09	0.168	0.181	0.132	0.140	0.282	0.199	0.149	0.200
B-splines	9	5	0.240	0.349	0.180	0.266	0.606	0.956	0.127	0.199
Partitioning	9	5	0.216	0.208	0.166	0.160	0.337	0.226	0.138	0.182
Feasible Estimation Local Polynomial	0.11	0.18	0.234	0.222	0.167	0.165	0.745	0.610	0.128	0.191
B-splines	4	3	0.331	0.409	0.245	0.307	0.848	1.180	0.129	0.139
Partitioning	4	3	0.268	0.227	0.196	0.167	0.520	0.473	0.128	0.130
_				Model	1.2					
Infeasible Estimation										
Local Polynomial	0.05	0.09	0.168	0.181	0.131	0.140	0.284	0.199	0.149	0.200
B-splines	9	5	0.239	0.349	0.179	0.266	0.606	0.956	0.127	0.196
Partitioning	9	5	0.216	0.208	0.166	0.160	0.337	0.226	0.138	0.182
Feasible Estimation Local Polynomial	0.13	0.19	0.261	0.224	0.177	0.166	0.871	0.619	0.129	0.189
B-splines	4	3	0.350	0.402	0.257	0.100	0.986	1.173	0.129	0.133
Partitioning	4	3	0.300	0.223	0.207	0.164	0.741	0.455	0.123	0.131
1 01 01010111118	•	J	0.000			0.101	0.1.11	0.100	0.120	0.102
Infeasible Estimation				Model	1.3					
Local Polynomial	0.9	0.9	0.065	0.091	0.051	0.070	0.045	0.068	0.086	0.119
B-splines	1	1	0.063	0.090	0.049	0.069	0.045	0.068	0.082	0.118
Partitioning	1	1	0.063	0.090	0.049	0.069	0.045	0.068	0.082	0.118
Feasible Estimation										
Local Polynomial	0.28	0.29	0.089	0.123	0.067	0.093	0.068	0.093	0.126	0.143
B-splines	2	2	0.076	0.098	0.059	0.075	0.076	0.084	0.098	0.120
Partitioning	2	2	0.088	0.120	0.067	0.092	0.107	0.183	0.102	0.121
				Model	1.4					
Infeasible Estimation	0.2	0.2	0.152	0.174	0.094	0.121	0.927	0.864	0.128	0.172
Local Polynomial B-splines	5	5	0.153 $0.164$	$0.174 \\ 0.174$	0.094	0.121	0.927	0.804	0.126	0.172
Partitioning	5	5	0.184	0.224	0.103	0.120	0.879	0.735	0.127	0.182
Feasible Estimation	•	9	0.101	0.221	0.101	0.1.0	0.0.0	0.100	0.12.	0.102
Local Polynomial	0.27	0.28	0.152	0.171	0.091	0.115	0.937	0.897	0.126	0.143
B-splines	2	2	0.145	0.156	0.084	0.100	0.939	0.923	0.102	0.120
Partitioning	2	2	0.150	0.158	0.093	0.110	0.898	0.732	0.107	0.122
				Model	1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.426	0.229	0.307	0.168	0.092	0.107	0.267	0.176
B-splines	5	5	0.365	0.276	0.299	0.214	0.216	0.178	0.197	0.207
Partitioning	5	5	0.264	0.222	0.198	0.168	0.108	0.161	0.229	0.183
Feasible Estimation Local Polynomial	0.1	0.21	0.246	0.237	0.179	0.172	0.106	0.106	0.227	0.168
B-splines	5	2	0.373	0.374	0.304	0.305	0.217	0.358	0.196	0.167
Partitioning	5	2	0.272	0.293	0.202	0.229	0.113	0.322	0.228	0.142
G				Model						
Infeasible Estimation				Model	1.0					
Local Polynomial	0.02	0.07	0.268	0.203	0.210	0.155	0.250	0.177	0.320	0.214
B-splines	25	6	0.301	0.291	0.217	0.207	0.224	0.147	0.449	0.499
Partitioning	25	6	0.359	0.238	0.272	0.182	0.324	0.274	0.513	0.205
Feasible Estimation		0.7.		0.000		0.010	6 - 16	0 - 0 -	1 000	0.10-
Local Polynomial	0.06	0.14	0.632	0.323	0.370	0.240	0.142	0.127	1.833	0.433
B-splines Partitioning	8	3 3	$1.401 \\ 1.221$	1.394 $0.706$	0.810 0.619	$\frac{1.042}{0.513}$	$0.178 \\ 0.215$	$0.629 \\ 0.129$	2.978 $2.675$	$\frac{2.342}{1.297}$
1 artitioning	0	3	1.221			0.515	0.215	0.123	2.015	1.231
Infeasible Estimation				Model	1.7					
Local Polynomial	0.08	0.12	0.139	0.162	0.108	0.125	0.200	0.162	0.131	0.192
B-splines	6	4	0.139	0.160	0.110	0.127	0.220	0.314	0.128	0.140
Partitioning	6	4	0.162	0.181	0.126	0.139	0.197	0.258	0.128	0.159
Feasible Estimation										
Local Polynomial	0.24	0.26	0.157	0.159	0.119	0.124	0.424	0.323	0.130	0.148
B-splines	2	2	0.160	0.162	0.124	0.126	0.414	0.393	0.126	0.123
Partitioning	2	2	0.161	0.142	0.124	0.109	0.373	0.253	0.127	0.129
				Model	1.8					
Infeasible Estimation	0.10	0.00		0.100		0.007		0.001	0.100	0 - 2 - 2 -
Local Polynomial	0.13	0.33	0.110	0.120	0.085	0.091	0.095	0.084	0.128	0.136
B-splines	4	1	0.114	0.131	0.088	0.103	0.123	0.125	0.122	0.129
Partitioning Feasible Estimation	4	1	0.134	0.131	0.104	0.103	0.166	0.125	0.125	0.129
Local Polynomial	0.2	0.28	0.107	0.124	0.082	0.094	0.101	0.094	0.129	0.144
B-splines	3	2	0.119	0.111	0.093	0.085	0.124	0.100	0.118	0.123
Partitioning	3	$\frac{1}{2}$	0.122	0.130	0.094	0.100	0.109	0.194	0.118	0.123

Table C.15: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=500, \sigma^2=1, X_i \sim \beta(2,2),$  Quantile Cells

	Tun Parar		Root In		Inget M.	rated AE		oint Estim	ation RM	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation				0.400	0.400					
Local Polynomial	0.05	0.09	0.173	0.182	0.133	0.136	0.252	0.176	0.231	0.307
B-splines	10	5	0.167	0.340	0.130	0.272	0.235	0.771	0.199	0.213
Partitioning Feasible Estimation	10	5	0.206	0.202	0.160	0.154	0.256	0.173	0.201	0.212
Local Polynomial	0.1	0.17	0.236	0.225	0.174	0.170	0.649	0.536	0.205	0.273
B-splines	5	3	0.364	0.443	0.282	0.353	0.876	1.093	0.183	0.223
Partitioning	5	3	0.241	0.195	0.178	0.147	0.473	0.310	0.196	0.193
				Model	1.2					
Infeasible Estimation	0.05	0.00	0.170	0.100	0.120	0.100	0.052	0.170	0.000	0.205
Local Polynomial B-splines	$0.05 \\ 10$	$0.09 \\ 5$	$0.173 \\ 0.166$	$0.182 \\ 0.340$	$0.132 \\ 0.130$	$0.136 \\ 0.272$	$0.253 \\ 0.235$	$0.176 \\ 0.771$	$0.230 \\ 0.199$	0.307 $0.215$
Partitioning	10	5	0.205	0.202	0.159	0.154	0.256	0.171	0.201	0.212
Feasible Estimation	10	0	0.200	0.202	0.100	0.104	0.200	0.110	0.201	0.212
Local Polynomial	0.11	0.17	0.259	0.227	0.185	0.171	0.744	0.542	0.203	0.270
B-splines	4	3	0.288	0.442	0.223	0.352	0.622	1.092	0.195	0.221
Partitioning	4	3	0.221	0.194	0.165	0.147	0.330	0.307	0.171	0.193
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.065	0.091	0.050	0.068	0.045	0.063	0.098	0.173
B-splines Partitioning	1 1	1 1	$0.063 \\ 0.063$	$0.090 \\ 0.090$	$0.049 \\ 0.049$	$0.067 \\ 0.067$	$0.045 \\ 0.045$	$0.063 \\ 0.063$	$0.094 \\ 0.094$	0.170
Feasible Estimation	1	1	0.003	0.090	0.049	0.007	0.045	0.003	0.094	0.170
Local Polynomial	0.26	0.28	0.091	0.123	0.066	0.088	0.062	0.082	0.188	0.201
B-splines	2	2	0.078	0.098	0.059	0.073	0.073	0.077	0.124	0.182
Partitioning	2	2	0.091	0.122	0.070	0.091	0.105	0.174	0.131	0.187
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.169	0.186	0.101	0.125	0.923	0.859	0.191	0.226
B-splines	5 5	5 5	0.178	0.187	0.117	$0.127 \\ 0.171$	0.900	0.877 $0.644$	$0.174 \\ 0.180$	0.193
Partitioning Feasible Estimation	э	5	0.193	0.226	0.136	0.171	0.832	0.044	0.180	0.212
Local Polynomial	0.24	0.27	0.169	0.184	0.100	0.121	0.926	0.885	0.189	0.204
B-splines	2	2	0.163	0.172	0.096	0.108	0.923	0.910	0.137	0.185
Partitioning	2	2	0.167	0.167	0.105	0.116	0.873	0.678	0.147	0.190
				Model	1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.367	0.230	0.251	0.164	0.083	0.093	0.224	0.229
B-splines	5	5	0.474	0.248	0.346	0.177	0.147	0.097	0.379	0.201
Partitioning	5	5	0.399	0.234	0.273	0.178	0.107	0.161	0.319	0.221
Feasible Estimation Local Polynomial	0.1	0.2	0.245	0.229	0.169	0.164	0.090	0.094	0.256	0.238
B-splines	5	2	0.497	0.368	0.356	0.300	0.148	0.279	0.230	0.204
Partitioning	5	2	0.435	0.282	0.293	0.219	0.138	0.257	0.347	0.203
o o				Model						
Infeasible Estimation				Woder	1.0					
Local Polynomial	0.02	0.08	0.240	0.190	0.181	0.140	0.193	0.144	0.443	0.323
B-splines	21	6	0.649	0.464	0.325	0.321	0.204	0.225	2.148	1.218
Partitioning	21	6	0.609	0.296	0.341	0.217	0.289	0.283	1.888	0.591
Feasible Estimation										
Local Polynomial	0.06	0.14	0.419 $1.531$	0.257	0.223 $0.788$	0.180	$0.125 \\ 0.165$	$0.111 \\ 0.406$	$\frac{1.649}{1.652}$	0.403 $1.720$
B-splines Partitioning	8	3	1.406	1.086 $0.684$	0.643	$0.698 \\ 0.460$	0.103	0.122	1.892	1.534
1 artificining	Ü	o o	1.400	Model		0.400	0.210	0.122	1.002	1.004
Infeasible Estimation				Model	1.7					
Local Polynomial	0.07	0.11	0.142	0.163	0.108	0.121	0.180	0.143	0.208	0.276
B-splines	6	4	0.127	0.157	0.100	0.125	0.156	0.251	0.183	0.195
Partitioning	6	4	0.158	0.180	0.122	0.137	0.195	0.254	0.188	0.200
Feasible Estimation	0.01	0.04	0.100	0.100	0.100	0.104	0.000	0.000	0.105	0.010
Local Polynomial	0.21	0.24	0.166	0.162	0.128	0.124	0.392	$0.289 \\ 0.365$	0.195	0.212
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	0.173 $0.168$	$0.174 \\ 0.141$	0.137 $0.132$	$0.140 \\ 0.107$	$0.392 \\ 0.327$	0.365 $0.215$	$0.147 \\ 0.167$	0.196 $0.192$
1 ar or	2	4	0.100			0.101	0.021	0.210	0.101	0.132
Infeasible Estimation				Model	1.0					
Local Polynomial	0.15	0.34	0.108	0.116	0.079	0.084	0.082	0.074	0.206	0.195
B-splines	3	1	0.108	0.119	0.082	0.090	0.094	0.098	0.162	0.202
Partitioning	3	1	0.119	0.119	0.092	0.090	0.086	0.098	0.160	0.202
Feasible Estimation			_							
Local Polynomial	0.19	0.27	0.106	0.124	0.079	0.089	0.090	0.084	0.197	0.203
B-splines	3	2	0.103	0.106	0.078	0.079	0.093	0.085	0.152	0.187
Partitioning	3	2	0.113	0.127	0.086	0.096	0.099	0.181	0.151	0.19

Table C.16: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 500, \sigma^2 = 4, X_i \sim \beta(0.5, 0.5),$  Quantile Cells

	Tun Parar		Root In			rated AE	Pc (0		ation RM: (0	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation										
Local Polynomial	0.07	0.11	0.290	0.330	0.226	0.258	0.494	0.383	0.223	0.333
B-splines	7 7	$rac{4}{4}$	0.396	0.369	0.299	0.288	1.054	0.938	0.215	0.275
Partitioning Feasible Estimation	1	4	0.381	0.366	0.292	0.282	0.674	0.546	0.256	0.312
Local Polynomial	0.13	0.2	0.310	0.325	0.232	0.251	0.885	0.744	0.214	0.315
B-splines	4	2	0.412	0.418	0.306	0.314	1.097	1.201	0.220	0.230
Partitioning	4	2	0.383	0.318	0.284	0.240	0.895	0.568	0.212	0.225
				Model	1.2					
Infeasible Estimation Local Polynomial	0.07	0.11	0.000	0.220	0.005	0.050	0.407	0.000	0.000	0.000
B-splines	$0.07 \\ 7$	0.11	$0.289 \\ 0.394$	$0.330 \\ 0.368$	$0.225 \\ 0.298$	$0.258 \\ 0.288$	0.497 $1.054$	$0.383 \\ 0.938$	$0.222 \\ 0.215$	0.333 $0.262$
Partitioning	7	4	0.380	0.366	0.291	0.282	0.674	0.546	0.256	0.312
Feasible Estimation	•	_	0.000	0.000	0.202			0.0.0	000	0.0
Local Polynomial	0.16	0.21	0.330	0.326	0.238	0.251	1.034	0.753	0.214	0.312
B-splines	3	2	0.436	0.405	0.315	0.305	1.366	1.199	0.217	0.215
Partitioning	3	2	0.415	0.315	0.295	0.238	1.184	0.567	0.213	0.226
				Model	1.3					
Infeasible Estimation	0.0	0.0	0.100	0.101	0.101	0.141	0.000	0.149	0.169	0.202
Local Polynomial B-splines	0.9 1	0.9 1	0.129 $0.126$	0.181 $0.178$	0.101 0.098	0.141 $0.139$	$0.090 \\ 0.090$	0.143 $0.143$	$0.163 \\ 0.156$	0.203 $0.203$
Partitioning	1	1	0.126	0.178	0.098	0.139	0.090	0.143	0.156	0.203
Feasible Estimation	_	_		0.2.0	0.000	0.200	0.000	***	0.200	000
Local Polynomial	0.24	0.26	0.184	0.249	0.140	0.192	0.161	0.216	0.215	0.268
B-splines	2	2	0.160	0.197	0.124	0.153	0.160	0.186	0.188	0.204
Partitioning	2	2	0.192	0.248	0.146	0.190	0.225	0.411	0.194	0.208
				Model	1.4					
Infeasible Estimation	0.0	0.2	0.014	0.000	0.155	0.017	0.940	0.000	0.015	0.210
Local Polynomial B-splines	0.2 5	5	0.214 $0.245$	0.283 $0.274$	0.157 $0.185$	0.217 $0.210$	0.940 $0.952$	0.889 $0.931$	$0.215 \\ 0.211$	0.312 $0.267$
Partitioning	5	5	0.303	0.411	0.133	0.210	0.932	0.836	0.211	0.335
Feasible Estimation	•	J	0.000	0.111	0.200	0.010	0.021	0.000	0.211	0.000
Local Polynomial	0.24	0.26	0.214	0.270	0.156	0.206	0.943	0.912	0.215	0.269
B-splines	2	2	0.195	0.225	0.140	0.167	0.957	0.942	0.189	0.204
Partitioning	2	2	0.221	0.264	0.161	0.199	0.928	0.824	0.196	0.208
				Model	1.5					
Infeasible Estimation	0.0	0.2	0.474	0.201	0.961	0.040	0.170	0.000	0.074	0.212
Local Polynomial B-splines	0.2 5	5	$0.474 \\ 0.349$	$0.321 \\ 0.357$	$0.361 \\ 0.273$	$0.249 \\ 0.283$	$0.172 \\ 0.237$	$0.239 \\ 0.299$	$0.374 \\ 0.329$	0.313 $0.315$
Partitioning	5	5	0.359	0.420	0.279	0.324	0.216	0.325	0.310	0.335
Feasible Estimation	_			00				****	0.020	0.000
Local Polynomial	0.1	0.24	0.326	0.335	0.252	0.259	0.232	0.232	0.299	0.285
B-splines	5	2	0.390	0.417	0.309	0.330	0.263	0.426	0.323	0.245
Partitioning	5	2	0.361	0.384	0.279	0.297	0.269	0.487	0.311	0.227
				Model	1.6					
Infeasible Estimation Local Polynomial	0.02	0.08	0.486	0.377	0.385	0.295	0.507	0.372	0.484	0.347
B-splines	21	6	0.445	0.304	0.347	0.237	0.407	0.372	0.495	0.333
Partitioning	21	6	0.595	0.440	0.463	0.338	0.568	0.544	0.652	0.354
Feasible Estimation										
Local Polynomial	0.06	0.15	0.810	0.451	0.559	0.354	0.305	0.283	1.955	0.629
B-splines	8	3	0.989	1.430	0.612	1.129	0.305	0.693	1.872	2.362
Partitioning	8	3	0.914	0.665	0.586	0.511	0.424	0.259	1.383	0.916
I f :11 E :: ::				Model	1.7					
Infeasible Estimation Local Polynomial	0.11	0.14	0.237	0.299	0.185	0.233	0.332	0.315	0.214	0.316
B-splines	4	3	0.225	0.249	0.177	0.196	0.401	0.452	0.212	0.207
Partitioning	4	3	0.268	0.320	0.209	0.247	0.396	0.352	0.212	0.255
Feasible Estimation										
Local Polynomial	0.23	0.26	0.218	0.266	0.169	0.208	0.445	0.379	0.216	0.271
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	$0.208 \\ 0.230$	$0.229 \\ 0.258$	$0.163 \\ 0.177$	0.180 $0.199$	0.458 $0.443$	$0.437 \\ 0.444$	0.199 $0.203$	$0.204 \\ 0.213$
r at titioilling	2	∠	0.230			0.199	0.443	0.444	0.203	0.213
Infeasible Estimation				Model	1.8					
Local Polynomial	0.16	0.38	0.202	0.222	0.157	0.171	0.187	0.170	0.218	0.221
B-splines	3	1	0.202	0.205	0.165	0.161	0.205	0.170	0.215	0.204
Partitioning	3	1	0.229	0.205	0.178	0.161	0.177	0.190	0.208	0.204
Feasible Estimation										
Local Polynomial	0.21	0.26	0.197	0.250	0.153	0.193	0.188	0.217	0.215	0.268
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	0.196	0.201	0.154	0.157	0.217	0.193	0.201	0.204
Partitioning			0.217	0.251	0.168	0.193	0.236	0.414	0.205	0.208

Table C.17: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=500, \sigma^2=4, X_i \sim \beta(1,1),$  Quantile Cells

	Tun Paran			tegrated SE	Inget M			oint Estima	ation RMS	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
$In feasible\ Estimation$										
Local Polynomial	0.07	0.11	0.295	0.335	0.230	0.259	0.458	0.347	0.268	0.390
B-splines Partitioning	7 7	4 4	$0.380 \\ 0.371$	$0.370 \\ 0.365$	$0.295 \\ 0.287$	0.294 $0.281$	0.882 $0.532$	0.832 $0.530$	$0.255 \\ 0.258$	0.281 $0.318$
Feasible Estimation	•	4	0.571	0.303	0.201	0.201	0.552	0.550	0.230	0.516
Local Polynomial	0.12	0.2	0.321	0.334	0.243	0.257	0.844	0.705	0.257	0.356
B-splines	4	2	0.385	0.443	0.296	0.342	0.900	1.157	0.265	0.251
Partitioning	4	2	0.361	0.319	0.275	0.243	0.670	0.476	0.251	0.256
				Model	1.2					
Infeasible Estimation	0.07	0.11	0.004	0.005	0.000	0.050	0.400	0.045	0.000	0.000
Local Polynomial B-splines	$0.07 \\ 7$	$0.11 \\ 4$	$0.294 \\ 0.379$	$0.335 \\ 0.370$	$0.229 \\ 0.294$	$0.259 \\ 0.294$	$0.460 \\ 0.882$	0.347 $0.833$	$0.268 \\ 0.255$	$0.390 \\ 0.277$
Partitioning	7	4	0.379	0.365	0.294	0.294	0.532	0.530	0.258	0.211
Feasible Estimation		-		0.000	0.200			0.000	0.200	0.0-0
Local Polynomial	0.15	0.2	0.343	0.335	0.252	0.258	0.977	0.712	0.257	0.353
B-splines	3	2	0.438	0.433	0.328	0.335	1.195	1.155	0.262	0.246
Partitioning	3	2	0.404	0.317	0.294	0.241	0.988	0.474	0.246	0.257
				Model	1.3					
Infeasible Estimation	0.0	0.9	0.120	0.102	0.101	0.140	0.000	0.126	0.171	0.220
Local Polynomial B-splines	0.9 1	0.9	$0.130 \\ 0.127$	0.183 $0.180$	$0.101 \\ 0.099$	$0.140 \\ 0.138$	0.090 0.090	0.136 $0.136$	$0.171 \\ 0.164$	0.238 $0.236$
Partitioning	1	1	0.127 $0.127$	0.180	0.099	0.138	0.090	0.136	0.164	0.236
Feasible Estimation										
Local Polynomial	0.23	0.26	0.188	0.250	0.141	0.189	0.148	0.194	0.256	0.296
B-splines	2	2	0.163	0.199	0.126	0.153	0.158	0.173	0.209	0.241
Partitioning	2	2	0.194	0.249	0.149	0.191	0.218	0.387	0.217	0.243
I f 'II F '' '				Model	1.4					
Infeasible Estimation Local Polynomial	0.2	0.2	0.222	0.288	0.160	0.217	0.937	0.884	0.256	0.344
B-splines	5	5	0.252	0.281	0.190	0.217	0.940	0.918	0.252	0.274
Partitioning	5	5	0.307	0.414	0.236	0.319	0.899	0.786	0.254	0.363
Feasible Estimation										
Local Polynomial	0.23	0.26	0.224	0.277	0.160	0.206	0.937	0.907	0.256	0.298
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	$0.205 \\ 0.228$	0.233 $0.268$	$0.146 \\ 0.167$	$0.171 \\ 0.202$	0.948 0.913	0.934 $0.780$	0.211 $0.219$	$0.241 \\ 0.244$
1 at titioning	2	2	0.220			0.202	0.913	0.780	0.213	0.244
Infeasible Estimation				Model	1.5					
Local Polynomial	0.2	0.2	0.455	0.324	0.337	0.249	0.158	0.214	0.347	0.347
B-splines	5	5	0.412	0.353	0.333	0.276	0.260	0.244	0.295	0.315
Partitioning	5	5	0.361	0.413	0.277	0.317	0.214	0.322	0.317	0.363
Feasible Estimation										
Local Polynomial	0.11	0.23 $2$	0.329	0.334 $0.417$	0.251 $0.385$	0.256	0.203	0.210 $0.388$	0.319 $0.288$	0.321
B-splines Partitioning	5 5	2	$0.485 \\ 0.405$	0.417 $0.375$	0.308	$0.336 \\ 0.293$	$0.280 \\ 0.279$	0.366	0.208	$0.268 \\ 0.258$
		_	0.200	Model		0.200		0.200	0.000	0.200
Infeasible Estimation				Model	1.0					
Local Polynomial	0.02	0.09	0.463	0.370	0.363	0.285	0.433	0.327	0.550	0.407
B-splines	19	5	0.512	0.599	0.375	0.460	0.383	0.459	0.782	1.107
Partitioning	19	5	0.624	0.433	0.475	0.332	0.551	0.322	0.948	0.410
Feasible Estimation Local Polynomial	0.06	0.15	0.700	0.424	0.459	0.324	0.271	0.249	1.916	0.628
B-splines	8	3	1.435	1.407	0.439	1.050	0.311	0.249	2.995	2.353
Partitioning	8	3	1.273	0.756	0.730	0.567	0.416	0.244	2.701	1.318
~				Model						
Infeasible Estimation					-					
Local Polynomial	0.1	0.13	0.244	0.303	0.188	0.233	0.311	0.284	0.257	0.365
B-splines	5	4	0.255	0.261	0.202	0.204	0.433	0.363	0.253	0.258
Partitioning Feasible Estimation	5	4	0.296	0.360	0.231	0.277	0.335	0.503	0.254	0.318
Local Polynomial	0.22	0.25	0.226	0.270	0.175	0.208	0.432	0.363	0.259	0.301
B-splines	2	2	0.217	0.237	0.170	0.186	0.444	0.422	0.223	0.242
Partitioning	2	2	0.237	0.260	0.183	0.200	0.420	0.414	0.230	0.247
				Model	1.8					
$Infeasible\ Estimation$										
Local Polynomial	0.17	0.39	0.200	0.221	0.153	0.167	0.165	0.155	0.260	0.251
B-splines	3	1	0.200	0.204	0.157	0.157	0.175	0.171	0.232	0.242
Partitioning Feasible Estimation	3	1	0.229	0.204	0.178	0.157	0.168	0.171	0.236	0.242
Local Polynomial	0.21	0.26	0.199	0.251	0.151	0.189	0.170	0.195	0.257	0.297
B-splines	2	2	0.192	0.203	0.149	0.156	0.193	0.178	0.225	0.242
Partitioning	2	2	0.217	0.252	0.167	0.193	0.218	0.391	0.232	0.244
Notes Tuning parameter	ore are loc	al polynomial	bandwi	dth and the	number of	colle for	partitioning	etimation	and Be	plines es

Table C.18: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=500, \sigma^2=4, X_i \sim \beta(2,2),$  Quantile Cells

	Tun Parar		Root In	tegrated SE	Inget M		Pc (0		ation RM: (0	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.303	0.338	0.230	0.251	0.410	0.307	0.422	0.573
B-splines	7 7	5 5	0.335 $0.354$	0.404	0.266	0.323	0.614	0.786	0.378	0.395
Partitioning Feasible Estimation	1	Э	0.354	0.401	0.275	0.306	0.386	0.328	0.385	0.425
Local Polynomial	0.12	0.19	0.336	0.342	0.255	0.260	0.781	0.640	0.406	0.493
B-splines	4	3	0.352	0.473	0.279	0.377	0.647	1.068	0.373	0.410
Partitioning	4	3	0.327	0.309	0.253	0.235	0.440	0.354	0.367	0.384
				Model	1.2					
Infeasible Estimation Local Polynomial	0.07	0.1	0.302	0.338	0.230	0.251	0.412	0.307	0.422	0.573
B-splines	7	5	0.334	0.404	0.265	0.323	0.614	0.786	0.377	0.395
Partitioning	7	5	0.353	0.401	0.274	0.306	0.386	0.328	0.385	0.425
Feasible Estimation										
Local Polynomial	0.13	0.19	0.356	0.342	0.266	0.261	0.883	0.644	0.401	0.491
B-splines	4	3	0.405	0.467	0.310	0.374	0.898	1.067	0.375	0.396
Partitioning	4	3	0.354	0.308	0.263	0.234	0.680	0.354	0.331	0.384
				Model	1.3					
Infeasible Estimation Local Polynomial	0.9	0.9	0.129	0.182	0.100	0.136	0.090	0.125	0.197	0.346
B-splines	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Partitioning	1	1	0.127	0.179	0.098	0.134	0.090	0.125	0.187	0.340
Feasible Estimation										
Local Polynomial	0.22	0.26	0.190	0.250	0.137	0.179	0.133	0.170	0.388	0.411
B-splines	2	2	0.165	0.199	0.126	0.148	0.149	0.158	0.265	0.366
Partitioning	2	2	0.196	0.249	0.150	0.187	0.207	0.360	0.281	0.377
T. C. 111 TO 11 11				Model	1.4					
Infeasible Estimation Local Polynomial	0.2	0.2	0.234	0.294	0.162	0.212	0.930	0.874	0.382	0.453
B-splines	5	5	0.234	0.288	0.102	0.212	0.911	0.874	0.347	0.433
Partitioning	5	5	0.311	0.414	0.239	0.317	0.851	0.703	0.359	0.425
Feasible Estimation										
Local Polynomial	0.22	0.25	0.238	0.285	0.164	0.204	0.927	0.893	0.389	0.414
B-splines	2	2	0.219	0.244	0.155	0.175	0.928	0.918	0.272	0.368
Partitioning	2	2	0.241	0.274	0.175	0.204	0.887	0.733	0.289	0.380
I-f:11- F-t:t:				Model	1.5					
Infeasible Estimation Local Polynomial	0.2	0.2	0.402	0.323	0.283	0.238	0.139	0.185	0.401	0.454
B-splines	5	5	0.511	0.331	0.381	0.247	0.205	0.186	0.483	0.389
Partitioning	5	5	0.468	0.418	0.348	0.321	0.214	0.322	0.445	0.429
Feasible Estimation										
Local Polynomial	0.12	0.21	0.328	0.328	0.239	0.242	0.170	0.184	0.436	0.451
B-splines	4	2	0.553	0.413	0.397	0.329	0.231	0.313	0.459	0.385
Partitioning	4	2	0.526	0.367	0.381	0.285	0.302	0.398	0.471	0.394
T. C. 111 TO 11 11				Model	1.6					
Infeasible Estimation Local Polynomial	0.03	0.1	0.418	0.350	0.314	0.259	0.333	0.266	0.756	0.594
B-splines	16	5	0.917	0.700	0.520	0.499	0.371	0.425	2.435	1.584
Partitioning	16	5	0.906	0.490	0.565	0.366	0.540	0.322	2.325	0.892
$Feasible\ Estimation$										
Local Polynomial	0.06	0.15	0.515	0.371	0.318	0.268	0.231	0.215	1.810	0.703
B-splines	7 7	3 3	1.639	1.103	0.885	$0.710 \\ 0.516$	0.273	0.427	1.506	1.746
Partitioning	'	3	1.530	0.734	0.792	0.516	0.309	0.243	1.743	1.566
Infeasible Estimation				Model	1.7					
Local Polynomial	0.1	0.13	0.250	0.305	0.187	0.225	0.282	0.252	0.409	0.511
B-splines	5	4	0.256	0.259	0.203	0.198	0.368	0.304	0.354	0.384
Partitioning	5	4	0.290	0.359	0.226	0.273	0.274	0.504	0.360	0.400
Feasible Estimation										
Local Polynomial	0.21	0.24	0.236	0.273	0.181	0.204	0.407	0.333	0.391	0.420
B-splines Partitioning	$\frac{2}{2}$	$\frac{2}{2}$	$0.228 \\ 0.243$	$0.246 \\ 0.262$	0.180 0.189	0.193 $0.198$	0.413 $0.373$	0.393 $0.381$	$0.280 \\ 0.301$	0.374 $0.380$
r at titioilling	2	∠	0.243			0.190	0.373	0.361	0.301	0.360
Infeasible Estimation				Model	1.8					
Local Polynomial	0.19	0.4	0.193	0.215	0.142	0.156	0.142	0.139	0.386	0.386
B-splines	2	1	0.162	0.196	0.124	0.146	0.162	0.146	0.255	0.357
Partitioning	2	1	0.185	0.196	0.143	0.146	0.233	0.146	0.268	0.357
Feasible Estimation										
Local Polynomial	0.2	0.25	0.200	0.250	0.146	0.180	0.152	0.171	0.392	0.412
B-splines	3	$\frac{2}{2}$	0.183	0.202	$0.139 \\ 0.162$	0.151	$0.167 \\ 0.208$	0.161	0.286	$0.367 \\ 0.379$
Partitioning	3	2 nol polynon	0.212	0.251		0.190		0.364	0.296	

Table C.19: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 1000, \sigma^2 = 1, X_i \sim \beta(0.5, 0.5),$  Quantile Cells

Infassible Estimation		Tun Parar		Root In	tegrated SE		rated AE		oint Estim	ation RM	SE .1)
Infoasible Estimation	Degree:			Linear	Cubic	Linear	Cubic				Cubic
Local Polymonial   0.05	-				Model	1.1					
Besplines											
Partitioning											0.118
Feasible Estimation											0.166
Local Polynomial		10	Э	0.155	0.168	0.120	0.127	0.209	0.309	0.165	0.117
Be-plines		0.1	0.18	0.187	0.178	0.126	0.126	0.686	0.576	0.075	0.127
Partitioning   S											0.119
Infoasible Estimation											0.089
Local Polynomial   0.05   0.09   0.124   0.132   0.097   0.103   0.242   0.168   0.093   0.111   Partitioning   10   5   0.154   0.168   0.119   0.127   0.208   0.309   0.165   Partitioning   10   5   0.154   0.168   0.119   0.127   0.208   0.309   0.165   Partitioning   10   5   0.154   0.168   0.119   0.127   0.208   0.309   0.165   Partitioning   10   1.20   0.270   0.292   0.189   0.170   0.183   0.270   0.393   0.767					Model	1.2					
Besplines											
Partitioning											0.118
Reasible Estimation											$0.160 \\ 0.117$
Local Polynomial   0.12		10	э	0.134	0.108	0.119	0.127	0.208	0.309	0.100	0.117
B-splines		0.12	0.18	0.215	0.180	0.134	0.127	0.822	0.585	0.075	0.125
Partitioning											0.112
Infeasible Estimation	Partitioning	4	3	0.270	0.252	0.189	0.170	0.671	0.726	0.075	0.089
Local Polynomial					Model	1.3					
B-splines											
Partitioning											0.072
Peasible Estimation											0.071
Local Polynomia  0.3		1	1	0.045	0.063	0.035	0.049	0.032	0.051	0.055	0.071
B-splines		0.3	0.3	0.062	0.087	0.047	0.067	0.051	0.072	0.075	0.090
Partitioning   2   2   0.061   0.084   0.047   0.064   0.079   0.143   0.065											0.071
Injeasible Estimation			2	0.061	0.084	0.047	0.064	0.079	0.143	0.065	0.073
Infeasible Estimation					Model	1.4					
B-splines	Infeasible Estimation										
Partitioning   5	Local Polynomial	0.2									0.109
Feasible Estimation											0.096
Local Polynomial   0.28   0.28   0.126   0.137   0.069   0.087   0.939   0.898   0.075		5	5	0.147	0.171	0.097	0.125	0.907	0.790	0.075	0.117
B-splines		0.28	0.28	0.126	0.137	0.069	0.087	0.030	0.808	0.075	0.092
Partitioning   2											0.032
Infeasible Estimation											0.074
Infeasible Estimation					Model	1.5					
B-splines 5 5 5 0.277 0.269 0.211 0.212 0.177 0.236 0.267 Partitioning 5 5 5 0.240 0.193 0.180 0.144 0.079 0.112 0.237 Feasible Estimation  Local Polynomial 0.09 0.21 0.207 0.210 0.150 0.146 0.091 0.084 0.202 B-splines 6 2 0.287 0.353 0.211 0.280 0.131 0.390 0.228 Partitioning 6 2 0.249 0.271 0.178 0.210 0.126 0.311 0.209	Infeasible Estimation				1110401	1.0					
Partitioning	Local Polynomial	0.2	0.2	0.443	0.205	0.327	0.143	0.079	0.084	0.315	0.114
Peasible Estimation											0.194
Local Polynomial   0.09   0.21   0.207   0.210   0.150   0.146   0.091   0.084   0.202		5	5	0.240	0.193	0.180	0.144	0.079	0.112	0.237	0.117
B-splines		0.00	0.21	0.207	0.210	0.150	0.146	0.001	0.084	0.202	0.112
Partitioning   6											0.112
Model 1.6   Infeasible Estimation											0.114
Infeasible Estimation   Cocal Polynomial   Cocal	G										
Local Polynomial         0.02         0.07         0.211         0.144         0.168         0.113         0.223         0.149         0.205           B-splines         32         7         0.192         0.183         0.150         0.152         0.179         0.204         0.207           Partitioning         32         7         0.259         0.169         0.201         0.130         0.257         0.136         0.268           Feasible Estimation           Local Polynomial         0.05         0.13         0.618         0.228         0.387         0.172         0.121         0.106         1.469           B-splines         9         4         0.735         0.881         0.415         0.666         0.156         0.586         1.063           Partitioning         9         4         0.735         0.881         0.415         0.666         0.156         0.586         1.063           Model 1.7           Infeasible Estimation           Local Polynomial         0.07         0.11         0.103         0.117         0.080         0.092         0.173         0.137         0.079           Partitioning         7         4         <	Infeasible Estimation				Model	1.0					
Partitioning   32   7   0.259   0.169   0.201   0.130   0.257   0.136   0.268		0.02	0.07	0.211	0.144	0.168	0.113	0.223	0.149	0.205	0.131
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$											0.172
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		32	7	0.259	0.169	0.201	0.130	0.257	0.136	0.268	0.139
B-splines 9 4 0.735 0.881 0.415 0.666 0.156 0.586 1.063 Partitioning 9 4 0.670 0.345 0.385 0.231 0.127 0.177 0.706 Model 1.7  Infeasible Estimation Local Polynomial 0.07 0.11 0.103 0.117 0.080 0.092 0.173 0.137 0.079 B-splines 7 4 0.142 0.135 0.108 0.105 0.377 0.337 0.078 Partitioning 7 4 0.135 0.129 0.104 0.099 0.238 0.205 0.089 Feasible Estimation Local Polynomial 0.23 0.25 0.133 0.126 0.097 0.097 0.424 0.318 0.077 B-splines 2 2 0.140 0.136 0.106 0.102 0.416 0.397 0.085 Partitioning 2 2 2 0.139 0.108 0.104 0.083 0.383 0.214 0.083  Infeasible Estimation Local Polynomial 0.11 0.3 0.084 0.091 0.066 0.070 0.082 0.069 0.076		0.05	0.12	0.619	0.000	0.207	0.179	0.191	0.106	1 460	0.120
Partitioning         9         4         0.670         0.345         0.385         0.231         0.127         0.177         0.706           Model 1.7           Infeasible Estimation           Local Polynomial         0.07         0.11         0.103         0.117         0.080         0.092         0.173         0.137         0.079           B-splines         7         4         0.142         0.135         0.108         0.105         0.377         0.337         0.078           Partitioning         7         4         0.135         0.129         0.104         0.099         0.238         0.205         0.089           Feasible Estimation           Local Polynomial         0.23         0.25         0.133         0.126         0.097         0.097         0.424         0.318         0.077           B-splines         2         2         0.140         0.136         0.106         0.102         0.416         0.397         0.085           Partitioning         2         2         0.139         0.108         0.104         0.083         0.383         0.214         0.083           Infeasible Estimation           Local P											$0.120 \\ 1.465$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$											0.438
Infeasible Estimation   Local Polynomial   0.07   0.11   0.103   0.117   0.080   0.092   0.173   0.137   0.079   0.189   0.191   0.103   0.117   0.080   0.092   0.173   0.137   0.079   0.078   0.189   0.											
Local Polynomial         0.07         0.11         0.103         0.117         0.080         0.092         0.173         0.137         0.079           B-splines         7         4         0.142         0.135         0.108         0.105         0.377         0.337         0.078           Partitioning         7         4         0.135         0.129         0.104         0.099         0.238         0.205         0.089           Feasible Estimation         Local Polynomial         0.23         0.25         0.133         0.126         0.097         0.097         0.424         0.318         0.077           B-splines         2         2         0.140         0.136         0.106         0.102         0.416         0.397         0.085           Partitioning         2         2         0.139         0.108         0.104         0.083         0.383         0.214         0.083           Model 1.8           Infeasible Estimation           Local Polynomial         0.11         0.3         0.084         0.091         0.066         0.070         0.082         0.069         0.076	Infeasible Estimation				Model						
B-splines         7         4         0.142         0.135         0.108         0.105         0.377         0.337         0.078           Partitioning         7         4         0.135         0.129         0.104         0.099         0.238         0.205         0.089           Feasible Estimation         Local Polynomial         0.23         0.25         0.133         0.126         0.097         0.097         0.424         0.318         0.077           B-splines         2         2         0.140         0.136         0.106         0.102         0.416         0.397         0.085           Partitioning         2         2         0.139         0.108         0.104         0.083         0.383         0.214         0.083           Model 1.8           Infeasible Estimation           Local Polynomial         0.11         0.3         0.084         0.091         0.066         0.070         0.082         0.069         0.076		0.07	0.11	0.103	0.117	0.080	0.092	0.173	0.137	0.079	0.116
Feasible Estimation	B-splines	7	4	0.142	0.135	0.108	0.105	0.377	0.337		0.099
Local Polynomial         0.23         0.25         0.133         0.126         0.097         0.097         0.424         0.318         0.077           B-splines         2         2         0.140         0.136         0.106         0.102         0.416         0.397         0.085           Partitioning         2         2         0.139         0.108         0.104         0.083         0.383         0.214         0.083           Model 1.8           Infeasible Estimation           Local Polynomial         0.11         0.3         0.084         0.091         0.066         0.070         0.082         0.069         0.076		7	4	0.135	0.129	0.104	0.099	0.238	0.205	0.089	0.110
B-splines 2 2 0.140 0.136 0.106 0.102 0.416 0.397 0.085 Partitioning 2 0.139 0.108 0.104 0.083 0.383 0.214 0.083  Model 1.8  Infeasible Estimation Local Polynomial 0.11 0.3 0.084 0.091 0.066 0.070 0.082 0.069 0.076		0.22	0.25	0.122	0.106	0.007	0.007	0.494	0.210	0.077	0.095
Partitioning 2 2 0.139 0.108 0.104 0.083 0.383 0.214 0.083  Model 1.8  Infeasible Estimation Local Polynomial 0.11 0.3 0.084 0.091 0.066 0.070 0.082 0.069 0.076											0.095 $0.073$
Model 1.8  Infeasible Estimation Local Polynomial 0.11 0.3 0.084 0.091 0.066 0.070 0.082 0.069 0.076											0.086
Infeasible Estimation         Local Polynomial         0.11         0.3         0.084         0.091         0.066         0.070         0.082         0.069         0.076	0										
Local Polynomial 0.11 0.3 0.084 0.091 0.066 0.070 0.082 0.069 0.076	Infeasible Estimation				Model	2.0					
		0.11	0.3	0.084	0.091	0.066	0.070	0.082	0.069	0.076	0.097
	B-splines	4	2	0.087	0.076	0.068	0.059	0.107	0.072	0.076	0.072
Partitioning 4 2 0.097 0.090 0.076 0.070 0.131 0.163 0.076		4	2	0.097	0.090	0.076	0.070	0.131	0.163	0.076	0.073
Feasible Estimation		0.17	0.00	0.000	0.000	0.004	0.000	0.000	0.074	0.070	0.000
Local Polynomial 0.17 0.29 0.082 0.088 0.064 0.068 0.082 0.074 0.078 B-splines 3 2 0.118 0.089 0.097 0.070 0.156 0.092 0.074											$0.090 \\ 0.073$
B-splines 3 2 0.118 0.089 0.097 0.070 0.150 0.092 0.074 Partitioning 3 2 0.102 0.100 0.080 0.078 0.101 0.161 0.074											0.073 $0.074$

Table C.20: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=1,\ n=1000,\ \sigma^2=1,\ X_i\sim\beta(1,1),\ {\rm Quantile\ Cells}$ 

	Tun Parar		Root In			rated AE		oint Estim	ation RM	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation										
Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.220	0.151	0.111	0.139
B-splines	11	6	0.175	0.214	0.131	0.168	0.449	0.521	0.093	0.137
Partitioning Feasible Estimation	11	6	0.164	0.157	0.126	0.121	0.240	0.220	0.115	0.136
Local Polynomial	0.1	0.17	0.190	0.181	0.132	0.131	0.631	0.536	0.090	0.150
B-splines	5	3	0.401	0.407	0.283	0.304	1.199	1.214	0.092	0.110
Partitioning	5	3	0.275	0.224	0.173	0.158	0.788	0.564	0.089	0.093
				Model	1.2					
$In feasible\ Estimation$										
Local Polynomial	0.05	0.08	0.127	0.133	0.099	0.103	0.221	0.151	0.111	0.139
B-splines Partitioning	10 10	6 6	0.134 $0.149$	0.214 $0.157$	$0.102 \\ 0.116$	0.168 $0.121$	$0.232 \\ 0.207$	$0.521 \\ 0.220$	0.090 $0.103$	0.138 $0.136$
Feasible Estimation	10	0	0.149	0.137	0.116	0.121	0.207	0.220	0.103	0.130
Local Polynomial	0.11	0.17	0.216	0.183	0.141	0.132	0.750	0.544	0.089	0.148
B-splines	4	3	0.309	0.407	0.231	0.304	0.831	1.213	0.096	0.102
Partitioning	4	3	0.249	0.224	0.175	0.158	0.516	0.563	0.088	0.093
				Model	1.3					
$Infeasible\ Estimation$										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.049	0.031	0.047	0.061	0.084
B-splines	1	1	0.044	0.063	0.035	0.049	0.031	0.047	0.058	0.083
Partitioning	1	1	0.044	0.063	0.035	0.049	0.031	0.047	0.058	0.083
Feasible Estimation Local Polynomial	0.28	0.29	0.062	0.087	0.047	0.065	0.047	0.065	0.088	0.099
B-splines	2	2	0.053	0.069	0.041	0.053	0.053	0.059	0.069	0.084
Partitioning	2	2	0.062	0.085	0.047	0.065	0.075	0.134	0.072	0.085
				Model	1.4					
Infeasible Estimation				1110401						
Local Polynomial	0.2	0.2	0.137	0.147	0.075	0.094	0.927	0.863	0.089	0.119
B-splines	5	5	0.144	0.149	0.085	0.094	0.929	0.902	0.088	0.097
Partitioning	5	5	0.154	0.174	0.100	0.127	0.886	0.740	0.089	0.126
Feasible Estimation Local Polynomial	0.26	0.28	0.137	0.146	0.073	0.090	0.935	0.894	0.088	0.101
B-splines	2	2	0.134	0.140	0.069	0.090	0.938	0.894 $0.922$	0.088	0.101
Partitioning	2	2	0.136	0.131	0.076	0.086	0.896	0.695	0.079	0.086
o o				Model						
Infeasible Estimation				Model	1.0					
Local Polynomial	0.2	0.2	0.421	0.210	0.301	0.147	0.075	0.076	0.251	0.125
B-splines	5	5	0.354	0.263	0.295	0.203	0.213	0.168	0.171	0.186
Partitioning	5	5	0.243	0.169	0.181	0.125	0.076	0.109	0.211	0.126
Feasible Estimation	0.09	0.2	0.200	0.200	0.147	0.146	0.000	0.077	0.195	0.120
Local Polynomial B-splines	6	2	0.209 $0.300$	$0.209 \\ 0.356$	0.147 $0.230$	0.146 $0.290$	$0.080 \\ 0.148$	$0.077 \\ 0.340$	0.195	0.129 $0.143$
Partitioning	6	2	0.232	0.267	0.172	0.207	0.115	0.269	0.219	0.115
				Model						
Infeasible Estimation				Model	1.0					
Local Polynomial	0.02	0.07	0.201	0.141	0.158	0.110	0.184	0.128	0.237	0.151
B-splines	29	7	0.221	0.168	0.162	0.121	0.169	0.093	0.278	0.195
Partitioning	29	7	0.272	0.174	0.207	0.134	0.240	0.135	0.301	0.150
Feasible Estimation		0.40								
Local Polynomial	0.05	0.13 4	0.517 $1.170$	0.209 $0.936$	0.298	0.155	$0.107 \\ 0.099$	0.094	1.430	0.152 $1.735$
B-splines Partitioning	9 9	4	1.170	0.936 $0.415$	$0.635 \\ 0.493$	0.718 $0.280$	0.099	$0.620 \\ 0.175$	2.740 $2.300$	0.704
1 ar titioning	J	-	1.011	Model		0.200	0.114	0.110	2.000	0.104
Infeasible Estimation				Model	1.7					
Local Polynomial	0.07	0.11	0.104	0.118	0.081	0.091	0.160	0.123	0.093	0.135
B-splines	7	4	0.139	0.136	0.109	0.109	0.322	0.303	0.090	0.109
Partitioning	7	4	0.130	0.128	0.101	0.099	0.188	0.198	0.090	0.111
Feasible Estimation										
Local Polynomial	0.21	0.24	0.140	0.128	0.103	0.099	0.408	0.300	0.091	0.106
B-splines Partitioning	$\frac{2}{2}$	2 2	$0.149 \\ 0.148$	$0.145 \\ 0.108$	0.114	0.112 $0.083$	0.412 $0.369$	0.384 $0.184$	0.092 $0.093$	0.088 $0.096$
r at titioilling	2	4	0.148		0.112	0.083	0.309	0.104	0.093	0.090
Infeasible Estimation				Model	1.8					
Local Polynomial	0.11	0.31	0.082	0.089	0.063	0.067	0.071	0.061	0.090	0.102
B-splines	4	2	0.082	0.075	0.069	0.058	0.102	0.065	0.086	0.102
Partitioning	4	2	0.099	0.090	0.076	0.069	0.124	0.151	0.087	0.085
Feasible Estimation										
Local Polynomial	0.17	0.28	0.080	0.087	0.062	0.066	0.075	0.067	0.091	0.100
B-splines	3	2	0.105	0.085	0.084	0.065	0.121	0.077	0.088	0.088
Partitioning	3	2	0.097	0.097	0.075	0.075	0.082	0.146	0.084	0.088

Table C.21: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=1, n=1000, \sigma^2=1, X_i \sim \beta(2,2), \text{ Quantile Cells}$ 

	Tun Parar		Root In		Inget M		Pc (0.		ation RMS	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model						
Infeasible Estimation				Model	1.1					
Local Polynomial	0.04	0.08	0.132	0.136	0.102	0.101	0.196	0.134	0.169	0.209
B-splines	11	6	0.132	0.166	0.103	0.133	0.244	0.302	0.138	0.149
Partitioning	11	6	0.155	0.156	0.120	0.119	0.169	0.212	0.139	0.155
Feasible Estimation										
Local Polynomial	0.09	0.16	0.191	0.185	0.139	0.138	0.545	0.470	0.140	0.218
B-splines Partitioning	5 5	3 3	0.310 $0.197$	$0.439 \\ 0.166$	0.218 $0.137$	$0.350 \\ 0.123$	0.777 $0.443$	0.314	$0.120 \\ 0.139$	0.154 $0.132$
1 at titioning	9	3	0.137			0.123	0.443	0.514	0.133	0.132
I-fill- E-titi				Model	1.2					
Infeasible Estimation Local Polynomial	0.04	0.08	0.131	0.136	0.101	0.101	0.197	0.134	0.169	0.209
B-splines	11	6	0.131	0.166	0.101	0.133	0.244	0.302	0.138	0.148
Partitioning	11	6	0.154	0.156	0.120	0.119	0.169	0.212	0.139	0.155
Feasible Estimation										
Local Polynomial	0.1	0.16	0.212	0.187	0.149	0.139	0.633	0.476	0.139	0.216
B-splines	5	3	0.365	0.438	0.279	0.350	0.924	1.100	0.152	0.157
Partitioning	5	3	0.219	0.166	0.150	0.123	0.505	0.314	0.124	0.131
				Model	1.3					
Infeasible Estimation										
Local Polynomial	0.9	0.9	0.046	0.064	0.035	0.048	0.032	0.044	0.070	0.120
B-splines	1 1	$\frac{1}{1}$	0.045	0.064	0.035	0.048	0.032	0.044	0.067	0.118
Partitioning Feasible Estimation	1	1	0.045	0.064	0.035	0.048	0.032	0.044	0.067	0.118
Local Polynomial	0.26	0.28	0.064	0.087	0.046	0.063	0.044	0.057	0.129	0.137
B-splines	2	2	0.055	0.069	0.042	0.052	0.052	0.054	0.087	0.126
Partitioning	2	2	0.064	0.086	0.049	0.065	0.074	0.128	0.093	0.128
				Model	1.4					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.156	0.162	0.084	0.101	0.923	0.858	0.131	0.156
B-splines	5	5	0.160	0.164	0.096	0.103	0.902	0.878	0.121	0.131
Partitioning	5	5	0.166	0.178	0.109	0.131	0.840	0.648	0.125	0.145
Feasible Estimation	0.00	0.00	0.150	0.160	0.004	0.000	0.000	0.000	0.101	0.140
Local Polynomial B-splines	0.23 2	$0.26 \\ 2$	$0.156 \\ 0.154$	$0.162 \\ 0.157$	0.084 $0.083$	0.099 $0.091$	0.923 $0.921$	$0.883 \\ 0.907$	$0.131 \\ 0.102$	$0.140 \\ 0.128$
Partitioning	2	2	0.154	0.137	0.090	0.091	0.873	0.643	0.102	0.128
1 ar titioning	-	-	0.104			0.000	0.010	0.040	0.100	0.100
Infeasible Estimation				Model	1.5					
Local Polynomial	0.2	0.2	0.361	0.211	0.244	0.146	0.069	0.065	0.162	0.163
B-splines	5	5	0.471	0.233	0.342	0.162	0.134	0.069	0.343	0.144
Partitioning	5	5	0.391	0.187	0.259	0.139	0.073	0.110	0.272	0.155
$Feasible\ Estimation$										
Local Polynomial	0.09	0.18	0.210	0.203	0.140	0.140	0.067	0.068	0.203	0.183
B-splines	5	3	0.465	0.330	0.337	0.270	0.129	0.249	0.338	0.148
Partitioning	5	3	0.384	0.239	0.253	0.175	0.080	0.125	0.269	0.137
				Model	1.6					
Infeasible Estimation	0.00	0.00	0.104	0.141	0.100	0.104	0.140	0.104	0.000	0.000
Local Polynomial B-splines	$0.02 \\ 24$	0.08	0.184 $0.548$	$0.141 \\ 0.464$	0.138 $0.258$	0.104 $0.323$	$0.142 \\ 0.156$	$0.104 \\ 0.224$	0.329 $1.883$	0.220 $1.220$
Partitioning	24	6	0.512	0.464 $0.254$	0.238	0.323	0.136	0.224	1.553	0.548
Feasible Estimation	2-1	Ü	0.012	0.204	0.210	0.113	0.220	0.211	1.000	0.040
Local Polynomial	0.05	0.13	0.343	0.169	0.178	0.120	0.094	0.081	1.306	0.244
B-splines	9	4	1.436	0.878	0.716	0.607	0.080	0.488	1.826	1.710
Partitioning	9	4	1.305	0.511	0.561	0.329	0.107	0.172	2.073	1.250
				Model	1.7					
$In feasible\ Estimation$										
Local Polynomial	0.06	0.1	0.107	0.120	0.082	0.090	0.144	0.109	0.144	0.195
B-splines	7	5	0.122	0.148	0.097	0.120	0.225	0.289	0.130	0.138
Partitioning Feasible Estimation	7	5	0.125	0.142	0.098	0.109	0.135	0.113	0.133	0.145
Local Polynomial	0.18	0.22	0.147	0.130	0.112	0.100	0.373	0.264	0.135	0.151
B-splines	3	2	0.147	0.159	0.112	0.100	0.399	0.360	0.111	0.131
Partitioning	3	2	0.153	0.106	0.118	0.081	0.325	0.147	0.135	0.132
Ŭ				Model						
Infeasible Estimation				Model	1.0					
Local Polynomial	0.13	0.32	0.080	0.087	0.059	0.062	0.061	0.054	0.137	0.136
B-splines	4	2	0.085	0.074	0.064	0.055	0.089	0.058	0.119	0.128
Partitioning	4	2	0.098	0.090	0.075	0.068	0.119	0.140	0.121	0.130
Feasible Estimation	0.10	0.25		0.000		0.001		0.070	0 - 0 -	0.100
Local Polynomial	0.16	0.27	0.079	0.088	0.059	0.064	0.068	0.059	0.137	0.138
B-splines Partitioning	3	$\frac{2}{2}$	$0.086 \\ 0.090$	$0.079 \\ 0.093$	$0.065 \\ 0.068$	$0.058 \\ 0.070$	$0.086 \\ 0.075$	$0.063 \\ 0.135$	$0.120 \\ 0.113$	$0.133 \\ 0.135$
i ai diddilling	J	- 4	0.090	0.093	0.008	0.070	0.073	0.133	0.113	0.133

Table C.22: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d = 1, n = 1000, \sigma^2 = 4, X_i \sim \beta(0.5, 0.5),$  Quantile Cells

	Tun Parar		Root In	tegrated SE		rated AE		oint Estim	ation RM	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
-				Model	1.1					
Infeasible Estimation										
Local Polynomial	0.06	0.1	0.218	0.243	0.170	0.190	0.394	0.293	0.164	0.234
B-splines	8 8	5 5	0.239	0.370	0.186	0.278	0.448	1.081	0.163	0.233
Partitioning Feasible Estimation	8	Э	0.270	0.297	0.210	0.229	0.333	0.366	0.217	0.235
Local Polynomial	0.11	0.19	0.247	0.254	0.181	0.193	0.761	0.645	0.150	0.241
B-splines	4	3	0.377	0.400	0.273	0.294	1.035	1.243	0.147	0.176
Partitioning	4	3	0.328	0.289	0.238	0.215	0.754	0.638	0.150	0.167
				Model	1.2					
Infeasible Estimation						0.400				
Local Polynomial B-splines	0.06	0.1 5	0.217	0.243 $0.369$	$0.170 \\ 0.184$	$0.190 \\ 0.278$	0.396	0.293	0.163	0.234 $0.229$
Partitioning	8	5 5	$0.238 \\ 0.269$	0.369 $0.297$	0.184	0.278	0.447 $0.331$	0.366	$0.163 \\ 0.217$	0.229
Feasible Estimation	0	0	0.203	0.201	0.200	0.223	0.001	0.000	0.211	0.200
Local Polynomial	0.13	0.19	0.271	0.255	0.189	0.194	0.907	0.653	0.151	0.239
B-splines	4	3	0.371	0.392	0.271	0.288	1.140	1.238	0.153	0.164
Partitioning	4	3	0.348	0.285	0.246	0.211	0.955	0.620	0.149	0.167
				Model	1.3					
Infeasible Estimation				0.400						
Local Polynomial	0.9 1	0.9 1	0.093 $0.090$	0.129 $0.127$	$0.072 \\ 0.070$	0.101 $0.099$	0.064 $0.063$	$0.102 \\ 0.102$	0.115 $0.109$	0.143 $0.143$
B-splines Partitioning	1	1	0.090	0.127 $0.127$	0.070	0.099	0.063	0.102 $0.102$	0.109	0.143
Feasible Estimation	-	1	0.000	0.121	0.010	0.000	0.000	0.102	0.100	0.140
Local Polynomial	0.24	0.27	0.132	0.177	0.101	0.137	0.114	0.151	0.151	0.188
B-splines	2	2	0.114	0.140	0.089	0.110	0.118	0.134	0.133	0.143
Partitioning	2	2	0.137	0.176	0.105	0.135	0.160	0.307	0.136	0.147
				Model	1.4					
Infeasible Estimation						0.400				
Local Polynomial	0.2	0.2	0.171	0.214	0.118	0.160	0.933	0.877	0.152	0.218
B-splines Partitioning	5 5	5 5	0.191 $0.228$	0.209 $0.299$	0.138 $0.171$	0.155 $0.229$	0.948 $0.916$	0.922 $0.814$	$0.149 \\ 0.151$	$0.190 \\ 0.235$
Feasible Estimation	5	3	0.226	0.299	0.171	0.229	0.910	0.014	0.131	0.233
Local Polynomial	0.24	0.26	0.171	0.206	0.118	0.152	0.935	0.900	0.151	0.190
B-splines	2	2	0.160	0.178	0.107	0.126	0.949	0.931	0.134	0.143
Partitioning	2	2	0.175	0.197	0.121	0.146	0.917	0.749	0.138	0.147
				Model	1.5					
Infeasible Estimation										
Local Polynomial	0.2	0.2	0.457	0.261	0.344	0.199	0.128	0.168	0.342	0.221
B-splines Partitioning	5 5	5 5	$0.308 \\ 0.297$	0.310 $0.312$	0.239 $0.229$	0.247 $0.241$	$0.204 \\ 0.148$	$0.267 \\ 0.223$	0.298 $0.272$	0.254 $0.235$
Feasible Estimation	0	0	0.201	0.012	0.225	0.241	0.140	0.220	0.212	0.200
Local Polynomial	0.09	0.22	0.264	0.272	0.201	0.206	0.176	0.168	0.246	0.212
B-splines	5	2	0.312	0.375	0.241	0.297	0.199	0.403	0.286	0.200
Partitioning	5	2	0.301	0.317	0.231	0.245	0.186	0.391	0.263	0.175
				Model	1.6					
Infeasible Estimation	0.00	0.00	0.000	0.050	0.000	0.010	0.005	0.050	0.001	0.045
Local Polynomial B-splines	$0.02 \\ 24$	0.08 6	$0.368 \\ 0.334$	$0.279 \\ 0.228$	0.293 $0.261$	0.218 $0.179$	0.385 $0.311$	$0.276 \\ 0.205$	0.361 $0.390$	0.247 $0.243$
Partitioning	24	6	0.449	0.228	0.349	0.179	0.444	0.203 $0.424$	0.529	0.248
Feasible Estimation										
Local Polynomial	0.05	0.14	0.659	0.294	0.450	0.233	0.233	0.208	1.521	0.229
B-splines	9	3	0.755	1.198	0.459	0.912	0.209	0.664	1.075	1.975
Partitioning	9	3	0.709	0.528	0.454	0.387	0.223	0.276	0.742	0.721
				Model	1.7					
Infeasible Estimation	0.00	0.12	0.100	0.010	0.141	0.171	0.971	0.220	0.159	0.226
Local Polynomial B-splines	$0.09 \\ 5$	0.13 4	$0.180 \\ 0.199$	0.218 $0.198$	0.141 $0.155$	$0.171 \\ 0.156$	$0.271 \\ 0.451$	0.239 $0.364$	$0.152 \\ 0.149$	$0.226 \\ 0.171$
Partitioning	5	4	0.133	0.156	0.173	0.196	0.363	0.389	0.143	0.220
Feasible Estimation										
Local Polynomial	0.22	0.25	0.175	0.199	0.135	0.156	0.424	0.345	0.152	0.193
B-splines	2	2	0.175	0.183	0.135	0.143	0.440	0.416	0.145	0.144
Partitioning	2	2	0.187	0.189	0.143	0.146	0.418	0.341	0.146	0.154
T. C. 11 F				Model	1.8					
Infeasible Estimation Local Polynomial	0.14	0.36	0.152	0.164	0.119	0.127	0.142	0.126	0.154	0.164
B-splines	0.14	0.36	0.152 $0.165$	0.164	0.119 $0.134$	0.127	0.142	0.126 $0.161$	0.154 $0.145$	0.164 $0.144$
Partitioning	3	1	0.170	0.163	0.133	0.129	0.137	0.161	0.147	0.144
Feasible Estimation						-		-		_
Local Polynomial	0.19	0.26	0.146	0.177	0.114	0.137	0.141	0.152	0.152	0.188
B-splines	3	2	0.158	0.147	0.125	0.115	0.187	0.143	0.145	0.143
Partitioning	3	2	0.165	0.180	0.128	0.139	0.179	0.311	0.147	0.147

Table C.23: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=1000, \sigma^2=4, X_i \sim \beta(1,1), \text{ Quantile Cells}$ 

	Tun Parar		Root In			rated AE		oint Estim	ation RM	SE .1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
Infeasible Estimation	0.00	0.1	0.000	0.044	0.150	0.100	0.000	0.000	0.104	0.054
Local Polynomial	0.06	0.1	0.222	0.244	0.173	0.189	0.363	0.263	0.194	0.274 $0.241$
B-splines Partitioning	8 8	5 5	$0.218 \\ 0.261$	$0.374 \\ 0.288$	$0.171 \\ 0.204$	$0.290 \\ 0.222$	0.322 $0.321$	$0.974 \\ 0.277$	0.177 $0.183$	0.241 $0.251$
Feasible Estimation	0	3	0.201	0.288	0.204	0.222	0.321	0.211	0.165	0.231
Local Polynomial	0.11	0.18	0.255	0.260	0.189	0.198	0.723	0.612	0.179	0.278
B-splines	4	3	0.378	0.425	0.281	0.322	0.999	1.195	0.177	0.181
Partitioning	4	3	0.311	0.277	0.229	0.209	0.661	0.516	0.178	0.182
				Model	1.2					
Infeasible Estimation Local Polynomial	0.00	0.1	0.001	0.044	0.170	0.100	0.205	0.000	0.100	0.074
B-splines	0.06	0.1 5	$0.221 \\ 0.217$	$0.244 \\ 0.374$	$0.172 \\ 0.170$	0.189 $0.290$	$0.365 \\ 0.322$	$0.263 \\ 0.974$	$0.193 \\ 0.177$	0.274 $0.239$
Partitioning	8	5	0.261	0.288	0.203	0.222	0.320	0.277	0.183	0.251
Feasible Estimation	_		0.20-	0.200	0.200		0.0_0	v.=		
Local Polynomial	0.12	0.18	0.277	0.261	0.198	0.199	0.847	0.618	0.179	0.275
B-splines	4	3	0.354	0.420	0.268	0.319	0.932	1.192	0.184	0.176
Partitioning	4	3	0.319	0.275	0.233	0.207	0.711	0.509	0.173	0.183
				Model	1.3					
Infeasible Estimation	0.0	0.0	0.001	0.100	0.071	0.000	0.000	0.004	0.101	0.167
Local Polynomial B-splines	0.9 1	0.9 1	0.091 $0.089$	0.128 $0.126$	$0.071 \\ 0.069$	0.099 $0.097$	$0.062 \\ 0.062$	0.094 $0.094$	0.121 $0.116$	$0.167 \\ 0.166$
Partitioning	1	1	0.089	0.126	0.069	0.097	0.062	0.094	0.116	0.166
Feasible Estimation	_	_	0.000	0.220	0.000			0.00-		0.200
Local Polynomial	0.24	0.26	0.131	0.176	0.099	0.133	0.103	0.136	0.178	0.206
B-splines	2	2	0.115	0.140	0.089	0.107	0.111	0.122	0.147	0.169
Partitioning	2	2	0.137	0.176	0.104	0.135	0.153	0.284	0.154	0.170
				Model	1.4					
Infeasible Estimation	0.0	0.0	0.150	0.010	0.100	0.150	0.000	0.055	0.150	0.000
Local Polynomial	0.2	0.2 5	$0.178 \\ 0.197$	0.218 $0.215$	$0.120 \\ 0.140$	0.159 $0.157$	0.933 $0.936$	$0.875 \\ 0.911$	$0.179 \\ 0.177$	0.239 $0.192$
B-splines Partitioning	5 5	5 5	0.197	0.300	0.140 $0.173$	0.137	0.897	0.766	0.177	0.192 $0.251$
Feasible Estimation	0	0	0.202	0.000	0.170	0.200	0.001	0.100	0.170	0.201
Local Polynomial	0.23	0.26	0.179	0.212	0.120	0.153	0.933	0.898	0.179	0.209
B-splines	2	2	0.169	0.185	0.110	0.128	0.944	0.926	0.150	0.169
Partitioning	2	2	0.183	0.201	0.126	0.148	0.906	0.716	0.158	0.171
				Model	1.5					
Infeasible Estimation	0.0	0.0	0.405	0.004	0.015	0.000	0.110	0.151	0.000	0.040
Local Polynomial B-splines	0.2 5	0.2 5	$0.435 \\ 0.378$	$0.264 \\ 0.305$	0.317 $0.311$	$0.200 \\ 0.238$	0.118 $0.236$	$0.151 \\ 0.205$	$0.293 \\ 0.228$	$0.242 \\ 0.250$
Partitioning	5	5	0.378	0.303 $0.297$	0.228	0.238 $0.227$	0.148	0.203	0.259	0.251
Feasible Estimation	0	0	0.230	0.201	0.220	0.221	0.140	0.210	0.200	0.201
Local Polynomial	0.09	0.21	0.266	0.270	0.199	0.203	0.153	0.151	0.252	0.243
B-splines	5	2	0.372	0.378	0.303	0.307	0.222	0.361	0.236	0.204
Partitioning	5	2	0.299	0.314	0.228	0.246	0.163	0.348	0.261	0.191
				Model	1.6					
Infeasible Estimation	0.00	0.00	0.051	0.051	0.070	0.000	0.015	0.005	0.415	0.000
Local Polynomial B-splines	$0.02 \\ 22$	0.08 6	0.351 $0.380$	$0.271 \\ 0.313$	$0.276 \\ 0.280$	$0.209 \\ 0.228$	0.317 $0.301$	0.235 $0.192$	$0.415 \\ 0.561$	0.286 $0.489$
Partitioning	22	6	0.470	0.313	0.359	0.228 $0.247$	0.431	0.192	0.629	0.489 $0.276$
Feasible Estimation										
Local Polynomial	0.05	0.14	0.568	0.280	0.367	0.217	0.203	0.183	1.509	0.270
B-splines	9	3	1.182	1.241	0.673	0.925	0.170	0.663	2.747	2.135
Partitioning	9	3	1.043	0.631	0.572	0.454	0.214	0.254	2.308	1.110
				Model	1.7					
Infeasible Estimation	0.00	0.10	0.100	0.010	0.140	0.100	0.050	0.015	0.100	0.001
Local Polynomial B-splines	0.09	0.12 4	0.183	0.219	0.142	0.169	0.253 $0.423$	0.215	0.180	0.261
Partitioning	5 5	4	$0.202 \\ 0.217$	0.199 $0.254$	0.159 $0.169$	0.157 $0.195$	0.309	$0.331 \\ 0.378$	$0.178 \\ 0.178$	0.187 $0.223$
Feasible Estimation	Ŭ.	*	0.211	0.201	0.100	0.100	0.000	0.010	0.110	0.220
Local Polynomial	0.21	0.25	0.181	0.202	0.140	0.157	0.414	0.331	0.181	0.215
B-splines	2	2	0.183	0.190	0.142	0.149	0.430	0.404	0.163	0.171
Partitioning	2	2	0.193	0.190	0.149	0.146	0.397	0.312	0.168	0.177
				Model	1.8					
Infeasible Estimation	0.15	0.20	0.140	0.161	0.110	0.101	0.104	0.110	0.104	0.107
Local Polynomial B-splines	0.15	$0.36 \\ 1$	$0.148 \\ 0.155$	$0.161 \\ 0.158$	0.113 $0.122$	$0.121 \\ 0.123$	$0.124 \\ 0.147$	$0.112 \\ 0.138$	$0.184 \\ 0.167$	$0.181 \\ 0.174$
Partitioning	3	1	0.155	0.158	0.122	0.123 $0.123$	0.147	0.138	0.167	0.174 $0.174$
Feasible Estimation	Ü	-	5.101	0.100	5.100	0.120	3.122	0.100	0.101	0.114
Local Polynomial	0.19	0.26	0.145	0.176	0.111	0.133	0.128	0.137	0.180	0.207
B-splines	3	2	0.148	0.145	0.115	0.112	0.154	0.128	0.163	0.170
Partitioning	3	2	0.161	0.179	0.124	0.138	0.157	0.288	0.165	0.171

Table C.24: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators

 $d=1, n=1000, \sigma^2=4, X_i \sim \beta(2,2),$  Quantile Cells

	Tun Paran			tegrated SE	Inget M			oint Estima	ation RMS	
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
				Model	1.1					
$In feasible\ Estimation$										
Local Polynomial	0.06	0.1	0.230	0.249	0.177	0.186	0.325	0.234	0.300	0.401
B-splines Partitioning	8	5 5	$0.206 \\ 0.259$	$0.367 \\ 0.285$	$0.162 \\ 0.202$	0.295 $0.218$	0.255 0.336	0.789 $0.230$	$0.267 \\ 0.270$	$0.277 \\ 0.290$
Feasible Estimation	0	0	0.233	0.200	0.202	0.210	0.330	0.230	0.270	0.230
Local Polynomial	0.1	0.17	0.268	0.270	0.203	0.206	0.666	0.559	0.279	0.382
B-splines	5	3	0.379	0.457	0.298	0.364	0.875	1.096	0.247	0.276
Partitioning	5	3	0.280	0.248	0.213	0.190	0.492	0.330	0.260	0.262
				Model	1.2					
Infeasible Estimation Local Polynomial	0.06	0.1	0.229	0.249	0.176	0.186	0.326	0.234	0.300	0.401
B-splines	8	5	0.205	0.367	0.170	0.180	0.255	0.234	0.266	0.401 $0.279$
Partitioning	8	5	0.259	0.285	0.201	0.218	0.336	0.230	0.270	0.290
$Feasible\ Estimation$										
Local Polynomial	0.11	0.18	0.287	0.271	0.213	0.206	0.761	0.563	0.277	0.380
B-splines Partitioning	$\frac{4}{4}$	3 3	$0.323 \\ 0.267$	$0.455 \\ 0.247$	$0.252 \\ 0.201$	$0.363 \\ 0.189$	0.682 $0.424$	$\frac{1.095}{0.328}$	$0.261 \\ 0.239$	$0.275 \\ 0.262$
1 ar titioning	-	Ö	0.201			0.105	0.121	0.020	0.203	0.202
Infoscible Estimation				Model	1.3					
Infeasible Estimation Local Polynomial	0.9	0.9	0.092	0.129	0.071	0.096	0.064	0.088	0.140	0.241
B-splines	1	1	0.090	0.127	0.070	0.095	0.064	0.088	0.133	0.236
Partitioning	1	1	0.090	0.127	0.070	0.095	0.064	0.088	0.133	0.236
Feasible Estimation Local Polynomial	0.23	0.26	0.134	0.177	0.097	0.128	0.092	0.119	0.266	0.281
B-splines	0.23	2	0.134 $0.117$	0.177	0.097	0.128	0.092	0.119	0.286	0.251 $0.254$
Partitioning	2	2	0.139	0.177	0.106	0.134	0.145	0.264	0.197	0.259
J				Model						
Infeasible Estimation				Wodel	1.4					
Local Polynomial	0.2	0.2	0.193	0.228	0.126	0.161	0.926	0.866	0.263	0.313
B-splines	5	5	0.209	0.226	0.148	0.163	0.908	0.885	0.242	0.261
Partitioning	5	5	0.241	0.304	0.180	0.233	0.851	0.678	0.250	0.290
Feasible Estimation Local Polynomial	0.22	0.25	0.195	0.224	0.127	0.156	0.923	0.887	0.267	0.284
B-splines	2	2	0.186	0.199	0.121	0.135	0.922	0.912	0.193	0.255
Partitioning	2	2	0.197	0.209	0.136	0.153	0.880	0.678	0.204	0.260
				Model	1.5					
$In feasible\ Estimation$										
Local Polynomial	0.2	0.2	0.379	0.266	0.262	0.195	0.105	0.128	0.280	0.316
B-splines Partitioning	5 5	5 5	$0.490 \\ 0.428$	$0.280 \\ 0.309$	0.361 $0.309$	$0.205 \\ 0.238$	0.166 $0.144$	0.129 $0.219$	$0.401 \\ 0.348$	$0.268 \\ 0.295$
Feasible Estimation	3	0	0.420	0.309	0.303	0.230	0.144	0.213	0.540	0.235
Local Polynomial	0.1	0.19	0.269	0.267	0.193	0.195	0.126	0.131	0.317	0.337
B-splines	5	2	0.500	0.375	0.366	0.303	0.169	0.285	0.395	0.265
Partitioning	5	2	0.444	0.310	0.316	0.239	0.172	0.275	0.360	0.265
				Model	1.6					
Infeasible Estimation	0.03	0.09	0.210	0.250	0.240	0.192	0.244	0.193	0.567	0.419
Local Polynomial B-splines	18	5	0.319 $0.797$	0.259 $0.693$	$0.240 \\ 0.424$	0.192 $0.496$	0.244 $0.278$	0.193	$\frac{0.367}{2.375}$	0.413 $1.582$
Partitioning	18	5	0.763	0.406	0.447	0.297	0.409	0.220	2.180	0.836
$Feasible\ Estimation$										
Local Polynomial	0.06	0.14	0.416	0.261	0.251	0.190	0.174	0.155	1.462	0.428
B-splines	9 9	3 3	1.493 $1.370$	0.699	$0.777 \\ 0.664$	$0.708 \\ 0.481$	0.184 0.260	$0.444 \\ 0.191$	1.746 $2.002$	1.716 $1.529$
Partitioning	9	3	1.370			0.461	0.200	0.191	2.002	1.529
Infeasible Estimation				Model	1.7					
Local Polynomial	0.09	0.12	0.189	0.224	0.143	0.166	0.227	0.189	0.281	0.365
B-splines	6	4	0.172	0.197	0.134	0.154	0.192	0.269	0.253	0.263
Partitioning	6	4	0.221	0.255	0.172	0.195	0.280	0.383	0.260	0.273
Feasible Estimation	0.0	0.04	0.100	0.00=	0.140	0.150	0.001	0.000	0.050	0.004
Local Polynomial B-splines	0.2	0.24 2	$0.192 \\ 0.195$	$0.207 \\ 0.202$	$0.148 \\ 0.155$	$0.158 \\ 0.160$	0.391 0.399	$0.303 \\ 0.375$	$0.270 \\ 0.205$	$0.294 \\ 0.263$
Partitioning	3	$\frac{2}{2}$	0.193	0.202	0.155	0.146	0.346	0.373	0.203 $0.221$	0.260
	~	•		Model			3.340	. =		
Infeasible Estimation				Model	1.0					
Local Polynomial	0.17	0.37	0.144	0.158	0.106	0.114	0.106	0.101	0.275	0.264
B-splines	3	1	0.141	0.150	0.108	0.112	0.112	0.115	0.218	0.259
Partitioning	3	1	0.162	0.150	0.125	0.112	0.114	0.115	0.221	0.259
Feasible Estimation Local Polynomial	0.19	0.25	0.146	0.178	0.107	0.128	0.114	0.119	0.270	0.282
B-splines	3	2	0.137	0.144	0.107	0.128	0.114	0.115	0.270	0.257
Partitioning	3	2	0.156	0.180	0.120	0.136	0.150	0.267	0.212	0.261
Notes. Tuning parameter	ers are loc	al polynomial	bandwi	dth and the	number of	cells for	partitioning	estimation	and Be	nlines as

- C.2 BIVARIATE SIMULATIONS
- C.2.1 Uniform Cell Boundaries

Table C.25: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=1,\ X_{i,\ell}\sim\beta(0.5,0.5),\ \text{Uniform Cells}$ 

	Tur Parai		Root Integrated MSE		Inget:		Point Estimation RMSE $(0.5,0.5)$ $(0.1,0.5)$ $(0.1,0.1)$						
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	
					Model 2.1								
Infeasible Estimation	0.10	0.05	0.007	0.055	0.150	0.107	0.100	0.004	0.012	0.050	0.040	0.000	
Local Polynomial B-splines	0.19 9	$0.25 \\ 4$	$0.207 \\ 0.214$	0.255 $0.233$	$0.158 \\ 0.167$	0.197 $0.180$	0.199 $0.230$	0.234 $0.271$	0.213 $0.156$	$0.250 \\ 0.233$	0.249 $0.222$	0.288 $0.242$	
Partitioning	9	4	0.243	0.286	0.187	0.130	0.213	1.024	0.202	0.564	0.209	0.242	
Feasible Estimation	Ü	•	0.210	0.200	0.101	0.21.	0.210	1.021	0.202	0.001	0.200	0.2.12	
Local Polynomial	0.3	0.27	0.207	0.217	0.155	0.170	0.088	0.161	0.122	0.145	0.112	0.134	
B-splines	3	1	0.198	0.218	0.150	0.170	0.395	0.261	0.184	0.177	0.181	0.231	
Partitioning	3	1	0.208	0.207	0.158	0.159	0.337	0.321	0.249	0.226	0.165	0.205	
					Model 2.2	2							
$Infeasible\ Estimation$													
Local Polynomial	0.14	0.2	0.262	0.309	0.204	0.239	0.250	0.284	0.261	0.347	0.258	0.411	
B-splines	9	4	0.382	0.233	0.314	0.179	0.481	0.207	0.388	0.229	0.245	0.243	
Partitioning Feasible Estimation	9	4	0.311	0.294	0.246	0.224	0.275	1.131	0.246	0.576	0.221	0.244	
Local Polynomial	0.33	0.24	0.490	0.494	0.392	0.391	0.706	0.757	0.607	0.615	0.287	0.297	
B-splines	4	4	0.355	0.233	0.284	0.180	0.222	0.207	0.206	0.229	0.259	0.243	
Partitioning	4	4	0.386	0.294	0.313	0.224	0.895	1.131	0.351	0.575	0.328	0.244	
<u> </u>					Model 2.3								
Infeasible Estimation					Model 2.c	•							
Local Polynomial	0.17	0.4	0.216	0.204	0.165	0.156	0.178	0.133	0.227	0.176	0.257	0.221	
B-splines	9	4	0.223	0.224	0.175	0.171	0.151	0.207	0.170	0.229	0.223	0.242	
Partitioning	9	4	0.259	0.285	0.202	0.217	0.174	1.047	0.207	0.566	0.215	0.242	
Feasible Estimation													
Local Polynomial	0.34	0.28	0.264	0.276	0.170	0.185	0.118	0.137	0.247	0.259	0.198	0.212	
B-splines	4	2	0.210	0.220	0.160	0.170	0.252	0.183	0.210	0.195	0.183	0.238	
Partitioning	4	2	0.226	0.234	0.174	0.176	0.284	0.623	0.304	0.349	0.160	0.234	
					Model 2.4	Į.							
Infeasible Estimation	0.00	0.00	0.000	0.054	0.400	0.075	0.140	0.00=	0.000	0.000	0.000	0.071	
Local Polynomial	0.33	0.33	0.668	0.354	0.482	0.275	0.149	0.207	0.323	0.206	0.266	0.271	
B-splines Partitioning	9 9	9	$0.731 \\ 0.665$	$0.387 \\ 0.455$	$0.579 \\ 0.524$	0.304 $0.349$	$0.240 \\ 0.196$	$0.307 \\ 0.366$	$0.209 \\ 0.265$	0.243 $0.346$	0.282 $0.249$	$0.270 \\ 0.304$	
Feasible Estimation	9	9	0.005	0.455	0.324	0.349	0.196	0.300	0.265	0.340	0.249	0.304	
Local Polynomial	0.21	0.28	0.638	0.540	0.524	0.443	0.405	0.412	0.668	0.379	0.601	0.907	
B-splines	4	1	0.790	0.568	0.613	0.448	0.681	0.400	0.234	0.246	0.234	0.286	
Partitioning	4	1	0.778	0.556	0.599	0.434	0.753	0.859	0.268	0.425	0.211	0.247	
					Model 2.5	5							
Infeasible Estimation													
Local Polynomial	0.22	0.9	0.186	0.151	0.142	0.120	0.195	0.181	0.192	0.121	0.238	0.145	
B-splines	4	1	0.140	0.209	0.108	0.164	0.247	0.200	0.195	0.170	0.181	0.234	
Partitioning	4	1	0.160	0.178	0.125	0.142	0.303	0.188	0.225	0.146	0.158	0.188	
Feasible Estimation Local Polynomial	0.26	0.29	0.122	0.122	0.096	0.093	0.206	0.104	0.105	0.128	0.105	0.128	
B-splines	4	0.29	0.122	0.122	0.108	0.095 $0.164$	0.200 $0.247$	0.104	0.105 $0.195$	0.128 $0.170$	0.103	0.128	
Partitioning	4	1	0.160	0.182	0.125	0.144	0.303	0.228	0.135	0.156	0.151	0.189	
	_	_	000				0.000	*	00		0.200	0.200	
Infeasible Estimation					Model 2.6	)							
Local Polynomial	0.9	0.9	0.083	0.126	0.064	0.099	0.044	0.091	0.080	0.119	0.110	0.144	
B-splines	1	1	0.089	0.178	0.068	0.136	0.044	0.114	0.077	0.164	0.129	0.230	
Partitioning	1	1	0.076	0.140	0.060	0.109	0.044	0.092	0.076	0.140	0.097	0.185	
Feasible Estimation													
Local Polynomial	0.4	0.31	0.461	0.468	0.405	0.407	0.062	0.110	0.112	0.139	0.647	0.658	
B-splines	1	1	0.092	0.178	0.070	0.136	0.064	0.114	0.086	0.164	0.133	0.230	
Partitioning	1	1	0.082	0.140	0.062	0.109	0.076	0.092	0.088	0.140	0.101	0.185	
					Model 2.7	•							
$Infeasible\ Estimation$													
Local Polynomial	0.33	0.33	0.181	0.254	0.119	0.188	0.960	0.931	0.378	0.384	0.208	0.265	
B-splines	9	9	0.216	0.293	0.153	0.219	0.962	0.947	0.382	0.404	0.222	0.251	
Partitioning Feasible Estimation	9	9	0.261	0.436	0.194	0.330	0.954	0.924	0.396	0.465	0.205	0.298	
Local Polynomial	0.34	0.28	0.148	0.173	0.079	0.110	0.981	0.983	0.369	0.374	0.104	0.128	
B-splines	2	1	0.164	0.217	0.099	0.153	0.962	0.960	0.380	0.387	0.153	0.230	
Partitioning	2	1	0.167	0.190	0.099	0.128	0.968	0.965	0.379	0.380	0.125	0.187	
Ŭ					Model 2.8								
Infeasible Estimation					model 2.8	,							
Local Polynomial	0.33	0.9	0.137	0.130	0.104	0.102	0.086	0.103	0.145	0.119	0.210	0.145	
B-splines	1	1	0.104	0.179	0.079	0.137	0.050	0.116	0.078	0.164	0.130	0.230	
Partitioning	1	1	0.094	0.142	0.072	0.110	0.050	0.105	0.078	0.143	0.097	0.186	
Feasible Estimation													
Local Polynomial	0.34	0.28	0.097	0.132	0.073	0.101	0.062	0.102	0.104	0.128	0.104	0.128	
B-splines	2	1	0.123	0.183	0.093	0.139	0.148	0.123	0.156	0.170	0.161	0.232	
Partitioning	2	1	0.133	0.159	0.100	0.119	0.214	0.299	0.174	0.193	0.136	0.191	

Table C.26: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=1,\ X_{i,\ell}\sim\beta(1,1),\ {\rm Uniform\ Cells}$ 

	Tun Parar		Root Integrated MSE		Ingeti MA		Point Estimation RMSE $(0.5,0.5)$ $(0.1,0.5)$ $(0.1,0.5)$					
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.10	0.05	0.014	0.050	0.100	0.104	0.104	0.011	0.00=	0.004	0.050	0.000
Local Polynomial B-splines	0.18	$0.25 \\ 4$	0.214 $0.226$	0.256 $0.236$	$0.162 \\ 0.177$	$0.194 \\ 0.178$	0.184 $0.221$	0.211 $0.243$	0.237 $0.176$	$0.264 \\ 0.258$	0.376 $0.330$	0.386
Partitioning	9	4	0.226	0.287	0.177	0.178	0.221 $0.192$	0.243 $0.841$	0.176	0.238	0.330 $0.277$	0.398
Feasible Estimation	9	-4	0.240	0.207	0.132	0.210	0.132	0.041	0.213	0.565	0.211	0.572
Local Polynomial	0.29	0.27	0.224	0.231	0.173	0.184	0.072	0.145	0.134	0.159	0.135	0.150
B-splines	3	1	0.205	0.229	0.158	0.178	0.392	0.253	0.241	0.210	0.251	0.359
Partitioning	3	1	0.215	0.221	0.166	0.170	0.348	0.345	0.313	0.285	0.210	0.288
					Model 2.2	!						
Infeasible Estimation												
Local Polynomial	0.14 9	$0.2 \\ 4$	0.262	0.303	0.200	0.229	0.224	0.238	0.288	0.349	0.390	0.55
B-splines Partitioning	9	4	$0.376 \\ 0.310$	0.233 $0.294$	$0.308 \\ 0.245$	$0.175 \\ 0.223$	$0.440 \\ 0.263$	0.186 $0.946$	$0.364 \\ 0.257$	$0.254 \\ 0.605$	0.403 $0.340$	0.40
Feasible Estimation	9	-4	0.310	0.234	0.245	0.223	0.203	0.340	0.201	0.005	0.540	0.57
Local Polynomial	0.3	0.24	0.508	0.513	0.413	0.414	0.671	0.701	0.589	0.586	0.316	0.33
B-splines	4	4	0.337	0.233	0.264	0.175	0.184	0.186	0.242	0.254	0.409	0.40
Partitioning	4	4	0.382	0.294	0.307	0.223	0.779	0.945	0.364	0.604	0.479	0.37
					Model 2.3	;						
Infeasible Estimation												
Local Polynomial	0.18	0.41	0.209	0.194	0.155	0.145	0.146	0.119	0.238	0.185	0.385	0.29
B-splines	9	4	0.217	0.225	0.167	0.168	0.133	0.186	0.191	0.254	0.336	0.39
Partitioning Feasible Estimation	9	4	0.255	0.285	0.197	0.216	0.143	0.835	0.228	0.593	0.288	0.37
Local Polynomial	0.34	0.28	0.217	0.233	0.141	0.155	0.114	0.126	0.233	0.246	0.188	0.20
B-splines	3	2	0.195	0.219	0.144	0.164	0.192	0.166	0.235	0.221	0.244	0.38
Partitioning	3	2	0.208	0.233	0.157	0.173	0.233	0.517	0.296	0.404	0.194	0.33
-					Model 2.4							
Infeasible Estimation						•						
Local Polynomial	0.33	0.33	0.615	0.355	0.429	0.276	0.148	0.186	0.223	0.212	0.336	0.34
B-splines	9	9	0.681	0.392	0.511	0.307	0.155	0.276	0.212	0.253	0.435	0.48
Partitioning	9	9	0.632	0.458	0.479	0.352	0.166	0.289	0.259	0.378	0.346	0.47
Feasible Estimation	0.00	0.00	0.004	0.510	0.405	0.411	0.000	0.005	0.545	0.044	0.704	0.05
Local Polynomial B-splines	$0.22 \\ 4$	0.28	$0.604 \\ 0.724$	0.516 $0.500$	$0.485 \\ 0.524$	0.411 $0.388$	$0.362 \\ 0.521$	$0.365 \\ 0.348$	$0.547 \\ 0.308$	$0.344 \\ 0.267$	$0.734 \\ 0.369$	0.95 0.43
Partitioning	4	1	0.719	0.300 $0.476$	0.524	0.361	0.562	0.348 $0.771$	0.355	0.512	0.334	0.43
1 ar citronning	-	-	0.715	0.410			0.002	0.111	0.000	0.012	0.004	0.01
Infeasible Estimation					Model 2.5	•						
Local Polynomial	0.21	0.9	0.189	0.149	0.143	0.118	0.178	0.163	0.208	0.140	0.345	0.18
B-splines	4	1	0.142	0.208	0.109	0.160	0.233	0.176	0.214	0.197	0.254	0.36
Partitioning	4	1	0.163	0.178	0.127	0.140	0.277	0.170	0.223	0.167	0.204	0.26
Feasible Estimation												
Local Polynomial	0.26	0.29	0.127	0.125	0.102	0.094	0.198	0.095	0.126	0.146	0.126	0.14
B-splines	4	1	0.142	0.210	0.109	0.160	0.233	0.176	0.214	0.203	0.254	0.36
Partitioning	4	1	0.163	0.185	0.127	0.144	0.277	0.230	0.223	0.206	0.204	0.27
T. C. 171 T. C. 11					Model 2.6	i						
Infeasible Estimation Local Polynomial	0.9	0.9	0.083	0.126	0.064	0.097	0.046	0.087	0.088	0.137	0.128	0.18
B-splines	0.9	1	0.090	0.120	0.069	0.037	0.045	0.102	0.084	0.137	0.128	0.16
Partitioning	1	1	0.078	0.142	0.061	0.109	0.045	0.088	0.085	0.153	0.108	0.26
Feasible Estimation												
Local Polynomial	0.4	0.31	0.422	0.430	0.363	0.367	0.058	0.094	0.128	0.153	0.662	0.66
B-splines	1	1	0.094	0.180	0.071	0.134	0.060	0.102	0.100	0.183	0.168	0.35
Partitioning	1	1	0.086	0.142	0.065	0.109	0.085	0.088	0.102	0.153	0.115	0.26
					Model 2.7	•						
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.189	0.255	0.120	0.183	0.959	0.926	0.376	0.391	0.284	0.33
B-splines	9	9	0.225	0.298	0.157	0.218	0.961	0.938	0.383	0.405	0.331	0.44
Partitioning Feasible Estimation	9	9	0.267	0.440	0.199	0.335	0.943	0.898	0.404	0.476	0.275	0.46
Local Polynomial	0.34	0.28	0.159	0.184	0.084	0.113	0.982	0.986	0.367	0.380	0.122	0.14
B-splines	2	1	0.176	0.227	0.107	0.156	0.955	0.954	0.387	0.395	0.210	0.36
Partitioning	2	1	0.181	0.207	0.110	0.138	0.970	0.972	0.374	0.408	0.159	0.27
					Model 2.8	,						
Infeasible Estimation					2.0							
Local Polynomial	0.34	0.9	0.136	0.129	0.100	0.100	0.077	0.093	0.153	0.138	0.283	0.18
B-splines	1	1	0.102	0.180	0.078	0.134	0.052	0.104	0.087	0.183	0.159	0.35
Partitioning	1	1	0.092	0.144	0.071	0.110	0.052	0.095	0.088	0.154	0.110	0.26
Feasible Estimation Local Polynomial	0.04	0.00	0.004	0.100	0.051	0.000	0.000	0.001	0.101	0.140	0.100	0.1.
	0.34	0.28	0.094	0.132	0.071	0.099	0.060	0.091	0.121	0.146	0.122	0.14
B-splines	3	1	0.125	0.186	0.094	0.138	0.134	0.112	0.177	0.194	0.221	0.36

Table C.27: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=500, \sigma^2=1, X_{i,\ell}\sim\beta(2,2),$  Uniform Cells

	Tur Parar		Root Integrated MSE		$\begin{array}{c} \text{Ingetrated} \\ \text{MAE} \end{array}$		(0.5,		Point Estimation RMSE $(0.1,0.5)$ $(0.1,0.1$			
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.17	0.24	0.232	0.249	0.159	0.179	0.151	0.161	0.336	0.335	1.295	1.400
B-splines	9	4	0.232	0.249	0.139	0.179	0.131	0.101	0.330	0.335	0.578	1.157
Partitioning	9	4	0.248	0.287	0.193	0.215	0.164	0.655	0.261	0.706	0.429	0.785
Feasible Estimation												
Local Polynomial	0.27	0.26	0.236	0.237	0.191	0.191	0.058	0.113	0.172	0.204	0.176	0.198
B-splines Partitioning	4	$\frac{2}{2}$	$0.207 \\ 0.218$	0.239 $0.247$	$0.162 \\ 0.170$	0.182 $0.189$	$0.332 \\ 0.377$	0.223 $0.421$	0.353 $0.390$	$0.340 \\ 0.467$	$0.369 \\ 0.261$	0.939 $0.596$
1 artitioning	-4	2	0.210	0.241			0.377	0.421	0.550	0.407	0.201	0.550
Infeasible Estimation					Model 2.2							
Local Polynomial	0.14	0.2	0.259	0.291	0.186	0.206	0.180	0.169	0.379	0.425	4.109	7.040
B-splines	9	4	0.348	0.229	0.278	0.163	0.380	0.144	0.318	0.368	0.771	1.157
Partitioning	9	4	0.299	0.291	0.233	0.217	0.239	0.683	0.275	0.712	0.592	0.790
Feasible Estimation	0.25	0.23	0.516	0.501	0.400	0.495	0.605	0.611	0.516	0.506	0.400	0.427
Local Polynomial B-splines	$0.25 \\ 4$	0.23	0.516 $0.295$	$0.521 \\ 0.229$	$0.422 \\ 0.221$	$0.425 \\ 0.163$	$0.605 \\ 0.200$	0.611 $0.144$	$0.516 \\ 0.372$	$0.526 \\ 0.368$	$0.420 \\ 0.711$	0.437 $1.157$
Partitioning	4	4	0.346	0.291	0.269	0.217	0.621	0.683	0.440	0.712	0.689	0.791
o .					Model 2.3							
Infeasible Estimation					Wiodel 2.0							
Local Polynomial	0.19	0.45	0.189	0.170	0.131	0.122	0.097	0.093	0.295	0.230	0.817	0.440
B-splines	9	4	0.201	0.224	0.148	0.158	0.102	0.145	0.257	0.367	0.582	1.154
Partitioning Feasible Estimation	9	4	0.242	0.285	0.185	0.213	0.106	0.649	0.281	0.708	0.422	0.795
Local Polynomial	0.33	0.27	0.156	0.176	0.107	0.122	0.095	0.100	0.246	0.254	0.207	0.224
B-splines	3	3	0.165	0.215	0.118	0.152	0.146	0.128	0.278	0.339	0.345	1.003
Partitioning	3	3	0.176	0.243	0.130	0.174	0.190	0.484	0.293	0.544	0.239	0.686
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.501	0.342	0.326	0.262	0.137	0.150	0.286	0.263	0.588	0.600
B-splines	9	9	0.561	0.388	0.379	0.297	0.077	0.230	0.409	0.438	0.804	3.013
Partitioning Feasible Estimation	9	9	0.543	0.456	0.381	0.344	0.133	0.196	0.452	0.485	0.612	1.823
Local Polynomial	0.23	0.28	0.529	0.459	0.400	0.350	0.284	0.279	0.345	0.326	0.991	1.024
B-splines	4	2	0.574	0.431	0.363	0.338	0.278	0.282	0.505	0.397	0.612	1.216
Partitioning	4	2	0.578	0.391	0.375	0.300	0.335	0.720	0.515	0.715	0.509	0.835
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.21	0.9	0.188	$0.141 \\ 0.201$	0.136	0.109	0.147	0.122	0.276	0.196	0.688	0.262
B-splines Partitioning	4	1 1	$0.143 \\ 0.162$	0.201	$0.108 \\ 0.126$	$0.146 \\ 0.127$	$0.205 \\ 0.264$	$0.135 \\ 0.126$	0.229 $0.229$	$0.298 \\ 0.246$	$0.368 \\ 0.247$	0.809 $0.426$
Feasible Estimation	-	1	0.102	0.110	0.120	0.121	0.204	0.120	0.223	0.240	0.241	0.420
Local Polynomial	0.25	0.28	0.126	0.122	0.103	0.088	0.172	0.081	0.169	0.193	0.169	0.193
B-splines	4	1	0.143	0.206	0.108	0.149	0.205	0.133	0.229	0.312	0.368	0.879
Partitioning	4	1	0.162	0.195	0.126	0.142	0.264	0.260	0.229	0.348	0.247	0.529
					Model 2.6							
Infeasible Estimation	0.0	0.0	0.001	0.100	0.000	0.001	0.040	0.054	0.00=	0.150	0.100	0.044
Local Polynomial B-splines	0.9 1	0.9 1	0.081 $0.090$	$0.122 \\ 0.178$	$0.062 \\ 0.067$	0.091 $0.126$	0.043 $0.043$	$0.074 \\ 0.085$	0.097 $0.094$	$0.172 \\ 0.250$	0.136 $0.194$	0.244 $0.788$
Partitioning	1	1	0.030	0.141	0.059	0.120	0.043	0.033	0.094	0.187	0.122	0.394
Feasible Estimation												
Local Polynomial	0.39	0.3	0.356	0.366	0.297	0.304	0.052	0.083	0.159	0.207	0.658	0.667
B-splines	1	1	0.098	0.179	0.072	0.126	0.067	0.086	0.127	0.250	0.246	0.789
Partitioning	1	1	0.093	0.143	0.068	0.104	0.086	0.088	0.124	0.191	0.156	0.398
T. C. 111 T. C. 11					Model 2.7							
Infeasible Estimation Local Polynomial	0.33	0.33	0.199	0.250	0.120	0.171	0.951	0.914	0.393	0.414	0.413	0.587
B-splines	9	9	0.234	0.306	0.158	0.214	0.950	0.923	0.400	0.414	0.573	2.513
Partitioning	9	9	0.275	0.445	0.202	0.335	0.934	0.865	0.428	0.573	0.416	1.767
Feasible Estimation												
Local Polynomial	0.33	0.27	0.176	0.198	0.091	0.117	0.969	0.972	0.387	0.407	0.159	0.194
B-splines Partitioning	3	$\frac{2}{2}$	0.195 $0.203$	$0.246 \\ 0.247$	$0.121 \\ 0.131$	$0.165 \\ 0.166$	$0.930 \\ 0.916$	0.934 $0.919$	0.394 $0.373$	$0.454 \\ 0.519$	0.337 $0.230$	0.913 $0.568$
1 areitioning	3	4	0.203	0.441			0.910	0.313	0.373	0.019	0.230	0.008
Infeasible Estimation					Model 2.8							
Local Polynomial	0.36	0.9	0.125	0.124	0.089	0.093	0.060	0.078	0.175	0.172	0.378	0.244
B-splines	1	1	0.098	0.179	0.033	0.033 $0.127$	0.053	0.087	0.101	0.250	0.195	0.788
Partitioning	1	1	0.086	0.142	0.066	0.105	0.052	0.079	0.101	0.187	0.125	0.400
Feasible Estimation												
Local Polynomial	0.33	0.27	0.089	0.127	0.064	0.090	0.055	0.080	0.157	0.194	0.157	0.193
B-splines	3	2	0.127 $0.139$	$0.195 \\ 0.200$	$0.092 \\ 0.104$	0.137 $0.139$	$0.128 \\ 0.179$	$0.105 \\ 0.344$	$0.206 \\ 0.201$	$0.291 \\ 0.408$	0.332	0.911 $0.561$

Table C.28: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=4,\ X_{i,\ell}\sim\beta(0.5,0.5),\ \text{Uniform Cells}$ 

		ning meter	Root In		Ingeti MA		(0.5,		Point Estimation RMSE $(0.1,0.5)$ $(0.1,0.5)$			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.23	0.29	0.345	0.530	0.264	0.405	0.281	0.383	0.363	0.501	0.462	0.668
B-splines	0.23	0.29	0.343	0.350 $0.451$	0.228	0.405	0.281	0.363	0.399	0.301 $0.459$	0.462	0.485
Partitioning	4	4	0.335	0.567	0.260	0.431	0.600	2.040	0.484	1.127	0.322	0.483
Feasible Estimation												
Local Polynomial	0.3	0.26	0.253	0.304	0.196	0.240	0.142	0.244	0.219	0.270	0.214	0.265
B-splines	3	2	0.292	0.409	0.225	0.313	0.522	0.379	0.363	0.392	0.348	0.471
Partitioning	3	2	0.321	0.431	0.247	0.316	0.543	1.273	0.436	0.718	0.306	0.420
T. C. ST. T. C. C.					Model 2.2	2						
Infeasible Estimation Local Polynomial	0.17	0.23	0.437	0.549	0.337	0.423	0.382	0.479	0.463	0.566	0.513	0.673
B-splines	9	4	0.492	0.451	0.396	0.425	0.514	0.414	0.469	0.457	0.455	0.484
Partitioning	9	4	0.509	0.571	0.397	0.435	0.396	2.085	0.426	1.135	0.418	0.484
Feasible Estimation												
Local Polynomial	0.31	0.25	0.510	0.538	0.408	0.426	0.721	0.783	0.632	0.658	0.340	0.369
B-splines	4	4	0.425	0.455	0.337	0.350	0.413	0.426	0.401	0.451	0.404	0.483
Partitioning	4	4	0.471	0.564	0.375	0.431	1.006	1.961	0.536	1.094	0.426	0.483
					Model 2.3	3						
Infeasible Estimation	0.01	0.46	0.969	0.200	0.075	0.075	0.071	0.000	0.200	0.204	0.401	0.207
Local Polynomial B-splines	0.21 4	$0.46 \\ 4$	0.363 $0.311$	$0.360 \\ 0.447$	$0.275 \\ 0.239$	0.275 $0.342$	$0.271 \\ 0.434$	$0.220 \\ 0.414$	$0.390 \\ 0.392$	$0.304 \\ 0.457$	0.481 $0.362$	0.387 $0.484$
Partitioning	4	4	0.351	0.567	0.239	0.431	0.544	2.057	0.392	1.127	0.302	0.483
Feasible Estimation	-	•	0.001	0.001	0.2.0	0.101	0.011	2.00.	0.100	11121	0.010	0.100
Local Polynomial	0.31	0.27	0.300	0.348	0.206	0.251	0.161	0.227	0.306	0.343	0.268	0.310
B-splines	3	2	0.307	0.411	0.233	0.313	0.401	0.335	0.391	0.402	0.349	0.474
Partitioning	3	2	0.339	0.450	0.261	0.328	0.509	1.403	0.480	0.772	0.300	0.440
					Model 2.4	Į.						
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.707	0.529	0.523	0.415	0.205	0.336	0.410	0.406	0.449	0.533
B-splines	9	9	0.793	0.604	0.630	0.471	0.299	0.433	0.339	0.444	0.479	0.512
Partitioning Feasible Estimation	9	9	0.777	0.863	0.619	0.657	0.355	0.731	0.440	0.687	0.435	0.599
Local Polynomial	0.23	0.27	0.685	0.581	0.554	0.469	0.432	0.448	0.671	0.440	0.649	0.939
B-splines	4	2	0.830	0.661	0.640	0.524	0.793	0.493	0.413	0.430	0.390	0.500
Partitioning	4	2	0.835	0.672	0.649	0.532	0.927	1.563	0.472	0.826	0.344	0.441
					Model 2.5	5						
$In feasible\ Estimation$												
Local Polynomial	0.27	0.9	0.314	0.265	0.241	0.209	0.297	0.241	0.323	0.239	0.433	0.288
B-splines	4	1	0.270	0.373	0.207	0.287	0.423	0.281	0.389	0.333	0.361	0.462
Partitioning Feasible Estimation	4	1	0.312	0.302	0.243	0.235	0.548	0.246	0.449	0.284	0.316	0.370
Local Polynomial	0.27	0.27	0.195	0.244	0.152	0.188	0.258	0.210	0.209	0.260	0.209	0.260
B-splines	4	2	0.277	0.398	0.212	0.303	0.436	0.321	0.394	0.383	0.369	0.468
Partitioning	4	2	0.317	0.403	0.247	0.294	0.553	1.136	0.452	0.661	0.325	0.407
					Model 2.6	i						
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.166	0.252	0.129	0.198	0.088	0.183	0.161	0.238	0.221	0.288
B-splines	1	1	0.177	0.357	0.136	0.273	0.087	0.228	0.153	0.329	0.258	0.460
Partitioning	1	1	0.153	0.281	0.119	0.217	0.087	0.183	0.153	0.280	0.193	0.369
Feasible Estimation Local Polynomial	0.34	0.28	0.482	0.513	0.416	0.433	0.126	0.213	0.213	0.263	0.671	0.697
B-splines	2	0.28	0.226	0.365	0.169	0.433 $0.278$	0.120	0.213	0.213	0.203	0.319	0.463
Partitioning	2	1	0.241	0.315	0.176	0.234	0.377	0.582	0.334	0.374	0.262	0.379
o de la companya de l					Model 2.7							
Infeasible Estimation					Model 2.7							
Local Polynomial	0.33	0.33	0.296	0.468	0.217	0.358	0.970	0.970	0.456	0.519	0.416	0.531
B-splines	9	9	0.377	0.549	0.287	0.420	0.978	0.998	0.470	0.553	0.444	0.502
Partitioning	9	9	0.480	0.853	0.370	0.648	1.007	1.130	0.532	0.757	0.410	0.596
Feasible Estimation												
Local Polynomial	0.31	0.27	0.207	0.274	0.140	0.201	0.985	0.997	0.411	0.435	0.208	0.261
B-splines Partitioning	3 3	$\frac{2}{2}$	0.273 $0.298$	$0.405 \\ 0.405$	$0.198 \\ 0.216$	0.304 $0.290$	$\frac{1.001}{1.032}$	0.982 $1.374$	$0.486 \\ 0.523$	$0.521 \\ 0.733$	0.338 $0.289$	$0.466 \\ 0.407$
1 at titioning	3	4	0.298	0.400			1.032	1.014	0.020	0.733	0.209	0.407
Infonsible F-+:					Model 2.8	3						
Infeasible Estimation Local Polynomial	0.42	0.9	0.239	0.254	0.180	0.199	0.129	0.189	0.237	0.238	0.363	0.288
B-splines	0.42	0.9	0.239	0.254 $0.357$	0.180 $0.142$	0.199 $0.273$	0.129	0.189 $0.229$	0.237 $0.154$	0.238 $0.329$	0.363 $0.259$	0.288
Partitioning	1	1	0.162	0.282	0.142	0.218	0.091	0.190	0.154	0.282	0.193	0.370
Feasible Estimation					J							
Local Polynomial	0.31	0.27	0.173	0.249	0.132	0.192	0.129	0.210	0.209	0.261	0.208	0.261
B-splines	3	2	0.251	0.392	0.190	0.297	0.346	0.309	0.349	0.387	0.339	0.468
Partitioning	3	2	0.282	0.408	0.213	0.292	0.472	1.230	0.397	0.699	0.292	0.413

Table C.29: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=4,\ X_{i,\ell}\sim\beta(1,1),\ {\rm Uniform\ Cells}$ 

	Tur Parai		Root Integrated MSE		Ingeti MA		(0.5,		Point Estimation RMSE $(0.1,0.5)$ $(0.1,$			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.22	0.28	0.358	0.540	0.268	0.400	0.264	0.350	0.404	0.528	0.676	0.883
B-splines	4	0.28	0.304	0.340 $0.455$	0.235	0.400	0.204	0.330 $0.402$	0.404 $0.451$	0.528	0.507	0.798
Partitioning	4	4	0.342	0.568	0.266	0.431	0.558	1.666	0.514	1.175	0.411	0.742
Feasible Estimation												
Local Polynomial	0.3	0.26	0.269	0.315	0.211	0.248	0.129	0.222	0.253	0.300	0.257	0.296
B-splines	3	2	0.304	0.415	0.235	0.313	0.489	0.345	0.424	0.431	0.493	0.747
Partitioning	3	2	0.334	0.437	0.259	0.322	0.516	1.036	0.484	0.779	0.393	0.618
T. C. ST. T. C. C.					Model 2.2							
Infeasible Estimation Local Polynomial	0.17	0.23	0.436	0.541	0.329	0.408	0.345	0.408	0.501	0.582	0.762	0.854
B-splines	9	4	0.489	0.453	0.391	0.338	0.466	0.370	0.464	0.508	0.710	0.800
Partitioning	9	4	0.509	0.572	0.398	0.434	0.356	1.726	0.457	1.185	0.592	0.738
Feasible Estimation												
Local Polynomial	0.3	0.25	0.529	0.556	0.429	0.447	0.677	0.717	0.633	0.639	0.376	0.418
B-splines	4	4	0.411	0.457	$0.321 \\ 0.371$	0.342	0.361	0.379	0.447	0.493	0.608	0.795
Partitioning	4	4	0.469	0.565		0.430	0.878	1.655	0.530	1.122	0.600	0.750
I-fill- F-titi					Model 2.3							
Infeasible Estimation Local Polynomial	0.22	0.47	0.351	0.342	0.259	0.256	0.228	0.201	0.404	0.325	0.684	0.517
B-splines	4	1	0.305	0.377	0.231	0.281	0.373	0.231	0.432	0.374	0.509	0.717
Partitioning	4	1	0.345	0.311	0.267	0.236	0.488	0.186	0.485	0.338	0.408	0.545
Feasible Estimation												
Local Polynomial	0.32	0.27	0.262	0.317	0.181	0.227	0.157	0.207	0.312	0.355	0.281	0.328
B-splines Partitioning	3 3	$\frac{2}{2}$	0.297 $0.328$	0.415	$0.223 \\ 0.250$	0.307 $0.330$	0.336	0.306 $1.088$	$0.419 \\ 0.467$	$0.441 \\ 0.844$	0.488 $0.388$	0.754 $0.641$
rartitioning	3	2	0.328	0.455			0.449	1.000	0.407	0.844	0.366	0.041
Infeasible Estimation					Model 2.4							
Local Polynomial	0.33	0.33	0.659	0.523	0.472	0.404	0.197	0.299	0.347	0.420	0.596	0.671
B-splines	9	9	0.749	0.608	0.570	0.466	0.221	0.382	0.357	0.475	0.722	0.918
Partitioning	9	9	0.750	0.866	0.584	0.662	0.293	0.578	0.457	0.747	0.592	0.936
Feasible Estimation												
Local Polynomial	0.23	0.27	0.652	0.560	0.517	0.444	0.392	0.401	0.545	0.428	0.814	0.987
B-splines Partitioning	4	$\frac{2}{2}$	$0.765 \\ 0.775$	$0.636 \\ 0.650$	$0.555 \\ 0.576$	$0.492 \\ 0.502$	$0.620 \\ 0.703$	$0.440 \\ 1.290$	$0.480 \\ 0.525$	0.483 $0.934$	$0.574 \\ 0.491$	$0.798 \\ 0.687$
1 artitioning	4	2	0.775	0.050			0.703	1.230	0.525	0.334	0.431	0.007
Infeasible Estimation					Model 2.5							
Local Polynomial	0.27	0.9	0.320	0.264	0.241	0.206	0.273	0.224	0.353	0.275	0.612	0.361
B-splines	4	1	0.274	0.374	0.209	0.281	0.386	0.250	0.428	0.374	0.508	0.719
Partitioning	4	1	0.316	0.304	0.246	0.234	0.498	0.231	0.446	0.314	0.408	0.527
Feasible Estimation												
Local Polynomial B-splines	0.27	0.27 2	$0.204 \\ 0.279$	0.249	$0.161 \\ 0.212$	0.187 $0.298$	0.252 $0.393$	0.187 $0.286$	0.251 $0.433$	0.294	$0.251 \\ 0.510$	0.294 $0.741$
Partitioning	$\frac{4}{4}$	2	0.279	$0.401 \\ 0.406$	0.212	0.298 $0.295$	0.501	0.2863	$0.455 \\ 0.451$	$0.425 \\ 0.682$	0.310	0.602
1 01 01010111118	•	-	0.020	0.100			0.001	0.000	0.101	0.002	0.110	0.002
Infeasible Estimation					Model 2.6							
Local Polynomial	0.9	0.9	0.166	0.251	0.128	0.194	0.092	0.174	0.177	0.274	0.256	0.361
B-splines	1	1	0.180	0.359	0.137	0.267	0.090	0.204	0.169	0.367	0.318	0.717
Partitioning	1	1	0.156	0.285	0.122	0.217	0.090	0.176	0.169	0.307	0.217	0.526
Feasible Estimation	0.05	0.00	0.440	0.403	0.055	0.000	0.110	0.105	0.040	0.000	0.501	0.711
Local Polynomial B-splines	0.35 $2$	$0.28 \\ 1$	0.446 $0.233$	$0.481 \\ 0.370$	$0.377 \\ 0.172$	$0.399 \\ 0.274$	$0.118 \\ 0.250$	$0.185 \\ 0.221$	0.249 $0.333$	$0.298 \\ 0.388$	$0.701 \\ 0.426$	0.711 $0.724$
Partitioning	2	1	0.233	0.370	0.172	0.244	0.356	0.497	0.342	0.388 $0.471$	0.326	0.724
1 01 01010111118	-	-	0.210	0.020	Model 2.7		0.000	0.10.	0.012	0.111	0.020	0.000
Infeasible Estimation					Model 2.7							
Local Polynomial	0.33	0.33	0.301	0.461	0.216	0.344	0.969	0.956	0.461	0.537	0.568	0.667
B-splines	9	9	0.385	0.553	0.290	0.413	0.976	0.974	0.479	0.571	0.663	0.896
Partitioning	9	9	0.485	0.857	0.376	0.654	0.974	1.031	0.556	0.796	0.550	0.929
Feasible Estimation	0.00	0.07	0.015	0.000	0.145	0.000	0.000	1 000	0.400	0.450	0.045	0.000
Local Polynomial B-splines	0.32	$0.27 \\ 2$	$0.217 \\ 0.286$	$0.283 \\ 0.415$	$0.145 \\ 0.206$	$0.203 \\ 0.306$	$0.990 \\ 0.973$	$\frac{1.003}{0.970}$	$0.420 \\ 0.520$	$0.458 \\ 0.552$	$0.247 \\ 0.479$	0.293 $0.742$
Partitioning	3	2	0.280	0.415 $0.426$	0.229	0.305	1.038	1.290	0.520	0.800	0.479 $0.377$	0.608
		-	3.010		Model 2.8				2.010			2.000
Infeasible Estimation					model 2.8							
Local Polynomial	0.42	0.9	0.235	0.253	0.174	0.196	0.121	0.177	0.251	0.275	0.480	0.361
B-splines	1	1	0.187	0.359	0.142	0.267	0.094	0.204	0.170	0.367	0.318	0.717
Partitioning	1	1	0.163	0.286	0.127	0.218	0.094	0.179	0.171	0.307	0.217	0.530
Feasible Estimation	0.00	0.07	0.175	0.250	0.100	0.100	0.101	0.100	0.047	0.000	0.047	0.000
Local Polynomial B-splines	0.32	0.27 2	$0.175 \\ 0.258$	$0.252 \\ 0.397$	0.132 $0.194$	0.190 $0.293$	$0.121 \\ 0.311$	$0.186 \\ 0.270$	0.247 $0.391$	0.293 $0.433$	0.247 $0.484$	0.293 $0.744$
D-spinies	3	2	0.290	0.397 $0.417$	0.194	0.293 $0.298$	0.311 $0.434$	0.270 $0.975$	0.391 $0.408$	0.433 $0.770$	0.484 $0.378$	0.744

Table C.30: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=500, \sigma^2=4, X_{i,\ell}\sim\beta(2,2),$  Uniform Cells

	Tur Parar		Root Int MS		Inget:		(0.5,		oint Estima (0.1,		SE (0.1	,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.21	0.27	0.366	0.453	0.261	0.324	0.220	0.268	0.554	0.587	1.384	1.542
B-splines	4	4	0.309	0.453	0.236	0.324 $0.322$	0.220 $0.427$	0.208	0.534	0.738	0.738	2.308
Partitioning	4	4	0.344	0.568	0.268	0.425	0.538	1.285	0.558	1.407	0.500	1.558
Feasible Estimation	_	_	0.0	0.000	0.200	0	0.000		0.000		0.000	
Local Polynomial	0.29	0.26	0.279	0.320	0.223	0.247	0.112	0.183	0.332	0.398	0.334	0.392
B-splines	4	3	0.310	0.434	0.237	0.313	0.413	0.305	0.518	0.673	0.724	2.065
Partitioning	4	3	0.341	0.496	0.265	0.362	0.522	1.012	0.536	1.115	0.496	1.364
					Model 2.2	!						
$Infeasible\ Estimation$												
Local Polynomial	0.17	0.23	0.423	0.511	0.303	0.365	0.277	0.290	0.654	0.702	6.256	5.066
B-splines	9	4	0.464	0.449	0.360	0.319	0.403	0.288	0.473	0.734	1.265	2.308
Partitioning Feasible Estimation	9	4	0.499	0.570	0.385	0.426	0.297	1.295	0.524	1.409	0.933	1.552
Local Polynomial	0.27	0.24	0.536	0.563	0.436	0.453	0.612	0.628	0.589	0.617	0.506	0.558
B-splines	4	4	0.371	0.450	0.430	0.320	0.312	0.294	0.574	0.723	0.930	2.241
Partitioning	4	4	0.442	0.566	0.341	0.423	0.754	1.266	0.619	1.378	0.809	1.549
3												
Infeasible Estimation					Model 2.3	•						
Local Polynomial	0.25	0.51	0.317	0.304	0.221	0.219	0.156	0.161	0.485	0.415	1.092	0.750
B-splines	4	1	0.288	0.368	0.212	0.213	0.304	0.185	0.484	0.520	0.740	1.574
Partitioning	4	1	0.327	0.298	0.252	0.218	0.426	0.157	0.501	0.415	0.502	0.791
Feasible Estimation												
Local Polynomial	0.31	0.26	0.214	0.277	0.151	0.197	0.132	0.173	0.373	0.424	0.348	0.406
B-splines	3	3	0.281	0.424	0.205	0.299	0.286	0.255	0.470	0.684	0.709	2.067
Partitioning	3	3	0.312	0.497	0.236	0.356	0.394	1.025	0.482	1.163	0.480	1.395
					Model 2.4							
$Infeasible\ Estimation$												
Local Polynomial	0.33	0.33	0.549	0.492	0.370	0.368	0.169	0.229	0.430	0.520	0.931	1.182
B-splines	9	9	0.640	0.606	0.450	0.443	0.148	0.306	0.537	0.785	1.290	5.271
Partitioning	9	9	0.674	0.865	0.502	0.649	0.220	0.392	0.634	0.959	0.950	3.574
Feasible Estimation	0.05	0.27	0.570	0.500	0.491	0.207	0.202	0.311	0.200	0.475	1 105	1.075
Local Polynomial B-splines	$0.25 \\ 4$	3	$0.573 \\ 0.621$	$0.506 \\ 0.581$	$0.431 \\ 0.410$	$0.387 \\ 0.436$	0.303 $0.384$	0.311 $0.352$	$0.398 \\ 0.635$	$0.475 \\ 0.737$	$\frac{1.125}{0.874}$	2.262
Partitioning	4	3	0.638	0.615	0.410	0.463	0.505	1.205	0.640	1.262	0.656	1.540
1 artifolding	-	o	0.000	0.010			0.000	1.200	0.040	1.202	0.000	1.040
Infeasible Estimation					Model 2.5	•						
Local Polynomial	0.26	0.9	0.314	0.254	0.228	0.192	0.225	0.175	0.454	0.357	1.003	0.498
B-splines	4	1	0.273	0.369	0.203	0.263	0.330	0.179	0.454 $0.457$	0.527	0.736	1.579
Partitioning	4	1	0.314	0.298	0.244	0.220	0.456	0.177	0.456	0.409	0.494	0.805
Feasible Estimation												
Local Polynomial	0.27	0.26	0.201	0.248	0.156	0.177	0.214	0.163	0.333	0.390	0.333	0.390
B-splines	4	2	0.275	0.414	0.205	0.292	0.333	0.245	0.461	0.649	0.737	2.018
Partitioning	4	2	0.315	0.463	0.245	0.328	0.457	0.917	0.459	1.004	0.498	1.308
					Model 2.6	;						
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.161	0.243	0.123	0.183	0.087	0.147	0.193	0.343	0.273	0.487
B-splines	1	1	0.179	0.357	0.133	0.252	0.087	0.171	0.189	0.499	0.389	1.577
Partitioning	1	1	0.153	0.283	0.118	0.207	0.086	0.147	0.189	0.374	0.244	0.789
Feasible Estimation Local Polynomial	0.22	0.27	0.294	0.494	0.216	0.241	0.105	0.169	0.224	0.205	0.710	0.741
B-splines	0.33	0.27 2	$0.384 \\ 0.247$	0.424 $0.388$	$0.316 \\ 0.179$	$0.341 \\ 0.272$	$0.105 \\ 0.245$	$0.163 \\ 0.208$	$0.324 \\ 0.408$	$0.395 \\ 0.578$	$0.710 \\ 0.667$	0.741 $1.830$
Partitioning	3	2	0.247	0.398	0.200	0.272	0.348	0.686	0.396	0.788	0.443	1.134
1 artifolding	Ö	-	0.210	0.000			0.040	0.000	0.000	0.100	0.440	1.104
I f the Etc.					Model 2.7							
Infeasible Estimation Local Polynomial	0.33	0.33	0.300	0.432	0.206	0.308	0.954	0.929	0.512	0.613	0.825	1.174
B-splines	9	9	0.387	0.557	0.282	0.396	0.956	0.943	0.536	0.816	1.146	5.026
Partitioning	9	9	0.485	0.859	0.372	0.646	0.949	0.929	0.624	1.011	0.831	3.534
Feasible Estimation		-	0.200	0.000		0.0.0	0.0 -0	0.0_0	0.0		0.00-	
Local Polynomial	0.31	0.26	0.228	0.292	0.147	0.200	0.972	0.982	0.483	0.534	0.324	0.390
B-splines	3	3	0.300	0.443	0.213	0.313	0.949	0.945	0.553	0.748	0.705	2.066
Partitioning	3	3	0.328	0.499	0.242	0.356	0.959	1.242	0.532	1.147	0.478	1.355
					Model 2.8	;						
Infeasible Estimation												
Local Polynomial	0.45	0.9	0.216	0.244	0.156	0.184	0.100	0.150	0.280	0.343	0.573	0.488
B-splines	1	1	0.183	0.357	0.136	0.253	0.092	0.171	0.191	0.499	0.388	1.576
Partitioning	1	1	0.158	0.283	0.122	0.208	0.091	0.150	0.191	0.374	0.245	0.792
Feasible Estimation		0.00	0	0.070		0.150	0.700	0.100	0.000	0.000	0.000	0.00-
Local Polynomial	0.31	0.26	0.171	0.250	0.123	0.178	0.108	0.163	0.323	0.390	0.323	0.389
B-splines	3	3 3	$0.261 \\ 0.294$	$0.416 \\ 0.477$	$0.191 \\ 0.224$	$0.292 \\ 0.338$	$0.276 \\ 0.397$	$0.245 \\ 0.973$	$0.438 \\ 0.431$	$0.655 \\ 1.068$	$0.705 \\ 0.474$	$\frac{2.048}{1.340}$

Table C.31: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=1000,\ \sigma^2=1,\ X_{i,\ell}\sim\beta(0.5,0.5),\ \text{Uniform Cells}$ 

	Tur Parai		Root Int		Ingeti MA		(0.5		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.17	0.23	0.161	0.191	0.124	0.148	0.167	0.184	0.157	0.193	0.177	0.220
B-splines	9	0.23	0.175	0.191	0.124	0.148	0.167 $0.215$	0.184	0.137	0.193 $0.167$	0.160	0.220
Partitioning	9	4	0.180	0.206	0.139	0.154	0.181	0.654	0.140	0.385	0.151	0.169
Feasible Estimation												
Local Polynomial	0.28	0.27	0.199	0.201	0.146	0.155	0.084	0.149	0.096	0.113	0.084	0.096
B-splines	4	1	0.166	0.177	0.124	0.138	0.419	0.243	0.167	0.125	0.141	0.162
Partitioning	4	1	0.177	0.175	0.133	0.134	0.328	0.235	0.248	0.176	0.138	0.159
T. C. ST. T. C. C.					Model 2.2	!						
Infeasible Estimation Local Polynomial	0.12	0.18	0.206	0.233	0.161	0.180	0.202	0.213	0.196	0.273	0.182	0.323
B-splines	16	4	0.210	0.173	0.166	0.133	0.408	0.148	0.267	0.157	0.175	0.168
Partitioning	16	4	0.253	0.218	0.196	0.166	0.555	0.830	0.377	0.396	0.167	0.172
$Feasible\ Estimation$												
Local Polynomial	0.32	0.23	0.487	0.489	0.390	0.387	0.703	0.747	0.602	0.604	0.277	0.285
B-splines Partitioning	$\frac{4}{4}$	$\frac{4}{4}$	$0.343 \\ 0.373$	$0.173 \\ 0.218$	$0.275 \\ 0.305$	0.133 $0.166$	$0.169 \\ 0.884$	$0.148 \\ 0.830$	$0.149 \\ 0.290$	0.157 $0.396$	$0.228 \\ 0.311$	0.168 $0.172$
raititioning	4	4	0.373	0.216			0.884	0.630	0.290	0.390	0.311	0.172
I-fill- F-titi					Model 2.3	<b>;</b>						
Infeasible Estimation Local Polynomial	0.15	0.37	0.168	0.156	0.129	0.120	0.141	0.106	0.168	0.133	0.186	0.174
B-splines	16	4	0.177	0.160	0.123	0.120	0.279	0.148	0.223	0.157	0.174	0.167
Partitioning	16	$\overline{4}$	0.233	0.205	0.181	0.156	0.425	0.667	0.322	0.386	0.177	0.171
Feasible Estimation						0.455						
Local Polynomial	0.34	0.28	0.259 $0.187$	$0.265 \\ 0.167$	$0.164 \\ 0.145$	$0.172 \\ 0.129$	0.114 $0.222$	$0.121 \\ 0.148$	0.232 $0.145$	$0.240 \\ 0.146$	$0.179 \\ 0.140$	0.189 $0.165$
B-splines Partitioning	4	3 3	0.187	0.167	0.145	0.129 $0.149$	0.222 $0.213$	0.148 $0.533$	$0.145 \\ 0.248$	0.146 $0.320$	0.140 $0.127$	0.180
rartitioning	4	3	0.200	0.190			0.213	0.555	0.246	0.320	0.127	0.160
Infeasible Estimation					Model 2.4							
Local Polynomial	0.33	0.33	0.666	0.318	0.479	0.242	0.140	0.175	0.306	0.151	0.197	0.197
B-splines	9	9	0.726	0.342	0.575	0.269	0.223	0.276	0.168	0.179	0.194	0.190
Partitioning	9	9	0.652	0.348	0.512	0.267	0.148	0.233	0.203	0.239	0.175	0.213
Feasible Estimation												
Local Polynomial	$0.19 \\ 4$	0.27 1	$0.610 \\ 0.733$	$0.530 \\ 0.410$	$0.509 \\ 0.579$	0.438	0.394	$0.394 \\ 0.392$	$0.667 \\ 0.166$	$0.363 \\ 0.189$	$0.588 \\ 0.192$	0.902 $0.186$
B-splines Partitioning	4	1	0.733	0.410	0.521	$0.322 \\ 0.274$	$0.303 \\ 0.277$	0.392 $0.757$	0.100	0.189	0.192 $0.173$	0.175
1 artifolding	-	-	0.001	0.000			0.211	0.101	0.200	0.001	0.110	0.110
Infeasible Estimation					Model 2.5	•						
Local Polynomial	0.19	0.9	0.145	0.123	0.112	0.099	0.156	0.170	0.143	0.085	0.174	0.108
B-splines	4	1	0.105	0.169	0.081	0.135	0.204	0.186	0.140	0.119	0.136	0.166
Partitioning	4	1	0.118	0.150	0.092	0.122	0.244	0.179	0.152	0.103	0.122	0.145
Feasible Estimation	0.00	0.00	0.007	0.000	0.077	0.000	0.170	0.070	0.075	0.000	0.075	0.000
Local Polynomial B-splines	0.23	0.29 1	0.097 $0.105$	0.088 $0.169$	$0.077 \\ 0.081$	$0.068 \\ 0.134$	0.173 $0.204$	$0.078 \\ 0.185$	0.075	0.089 $0.119$	0.075 $0.136$	0.089 $0.166$
Partitioning	4	1	0.118	0.153	0.092	0.123	0.244	0.204	0.152	0.119	0.122	0.146
Ü					Model 2.6							
Infeasible Estimation					Model 2.0							
Local Polynomial	0.9	0.9	0.060	0.090	0.046	0.071	0.033	0.066	0.057	0.084	0.083	0.107
B-splines	1	1	0.063	0.127	0.048	0.097	0.033	0.081	0.055	0.117	0.094	0.162
Partitioning	1	1	0.055	0.101	0.043	0.078	0.033	0.066	0.055	0.099	0.072	0.142
Feasible Estimation Local Polynomial	0.41	0.31	0.459	0.462	0.404	0.405	0.045	0.075	0.078	0.096	0.646	0.652
B-splines	0.41	0.31	0.459	0.462 $0.127$	0.050	0.405 $0.097$	0.043	0.075	0.078	0.096 $0.117$	0.046	0.052
Partitioning	1	1	0.060	0.101	0.045	0.078	0.053	0.066	0.066	0.099	0.074	0.142
Ü					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.155	0.198	0.091	0.139	0.960	0.924	0.356	0.353	0.151	0.188
B-splines	9	9	0.176	0.224	0.115	0.162	0.963	0.938	0.359	0.360	0.160	0.171
Partitioning Feasible Estimation	9	9	0.204	0.319	0.144	0.239	0.937	0.883	0.356	0.384	0.148	0.199
Local Polynomial	0.34	0.28	0.137	0.152	0.064	0.085	0.984	0.986	0.360	0.362	0.074	0.090
B-splines	2	1	0.137	0.132	0.004	0.085	0.967	0.959	0.364	0.352	0.074	0.090
Partitioning	2	1	0.147	0.161	0.077	0.099	0.966	0.959	0.361	0.356	0.095	0.144
-					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.29	0.9	0.104	0.096	0.079	0.075	0.071	0.080	0.111	0.086	0.159	0.108
B-splines	4	1	0.097	0.128	0.075	0.098	0.143	0.084	0.139	0.117	0.135	0.162
Partitioning	4	1	0.114	0.103	0.089	0.080	0.205	0.082	0.151	0.101	0.122	0.145
Feasible Estimation Local Polynomial	0.34	0.28	0.079	0.102	0.059	0.078	0.047	0.072	0.073	0.090	0.073	0.090
B-splines	3	0.28	0.079	0.102	0.039	0.078	0.047	0.072	0.073	0.090	0.073	0.090
Partitioning	3	1	0.102	0.113	0.077	0.085	0.164	0.168	0.112	0.129	0.106	0.146

Table C.32: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=1, X_{i,\ell}\sim\beta(1,1),$  Uniform Cells

	Tur Parai		Root In		Ingeti MA		(0.5		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.16	0.23	0.166	0.190	0.126	0.144	0.150	0.161	0.178	0.196	0.253	0.268
B-splines	9	0.23	0.186	0.190 $0.174$	0.126	0.144	0.130	0.101	0.178	0.196	0.233	0.240
Partitioning	9	4	0.183	0.205	0.142	0.156	0.164	0.573	0.152	0.393	0.184	0.232
Feasible Estimation												
Local Polynomial	0.27	0.27	0.215	0.213	0.165	0.170	0.059	0.126	0.101	0.115	0.098	0.105
B-splines	4	1	0.172	0.187	0.132	0.146	0.393	0.236	0.209	0.153	0.169	0.227
Partitioning	4	1	0.181	0.184	0.138	0.143	0.341	0.258	0.288	0.200	0.155	0.199
1.6 21 E.C. C.					Model 2.2	2						
Infeasible Estimation Local Polynomial	0.12	0.18	0.203	0.227	0.156	0.171	0.177	0.174	0.210	0.262	0.259	0.401
B-splines	16	4	0.205	0.227	0.159	0.171	0.353	0.174	0.210	0.182	0.239	0.242
Partitioning	16	4	0.251	0.216	0.195	0.164	0.510	0.696	0.411	0.404	0.217	0.233
Feasible Estimation												
Local Polynomial	0.29	0.23	0.505	0.508	0.411	0.410	0.675	0.704	0.571	0.572	0.313	0.320
B-splines	4	4	0.327	0.170	0.257	0.128	0.165	0.128	0.176	0.182	0.334	0.242
Partitioning	4	4	0.362	0.216	0.292	0.164	0.752	0.696	0.317	0.404	0.429	0.233
1 f 21 F 22					Model 2.3	3						
Infeasible Estimation Local Polynomial	0.16	0.39	0.160	0.148	0.120	0.110	0.109	0.091	0.177	0.138	0.258	0.203
B-splines	9	0.39	0.176	0.148	0.120	0.110	0.109 $0.117$	0.129	0.148	0.138	0.238	0.203
Partitioning	9	4	0.194	0.203	0.150	0.154	0.101	0.582	0.161	0.397	0.192	0.235
Feasible Estimation												
Local Polynomial	0.34	0.28	0.210	0.217	0.133	0.140	0.104	0.103	0.214	0.214	0.164	0.166
B-splines	4	3	0.172	0.164	0.128	0.124	0.172	0.130	0.166	0.171	0.169	0.236
Partitioning	4	3	0.182	0.190	0.137	0.142	0.179	0.459	0.239	0.328	0.137	0.230
					Model 2.4	Į.						
Infeasible Estimation	0.22	0.22	0.612	0.222	0.495	0.246	0.120	0.150	0.100	0.155	0.222	0.004
Local Polynomial B-splines	0.33 9	0.33 9	0.613 $0.677$	0.322 $0.346$	$0.425 \\ 0.507$	$0.246 \\ 0.274$	0.138 0.136	$0.158 \\ 0.254$	$0.190 \\ 0.154$	$0.155 \\ 0.192$	0.223 $0.297$	$0.224 \\ 0.275$
Partitioning	9	9	0.618	0.349	0.464	0.269	0.133	0.188	0.187	0.132	0.235	0.278
Feasible Estimation	Ü	Ü	0.010	0.010	0.101	0.200	0.100	0.100	0.101	0.200	0.200	0.2.0
Local Polynomial	0.19	0.27	0.580	0.506	0.471	0.403	0.345	0.344	0.571	0.325	0.690	0.945
B-splines	4	2	0.688	0.399	0.510	0.323	0.267	0.346	0.185	0.206	0.290	0.257
Partitioning	4	2	0.642	0.335	0.475	0.261	0.285	0.707	0.213	0.412	0.239	0.238
					Model 2.5	5						
Infeasible Estimation	0.10	0.0	0.145	0.100	0.110	0.007	0.100	0.140	0.150	0.000	0.005	0.105
Local Polynomial B-splines	$0.19 \\ 4$	$0.9 \\ 1$	$0.145 \\ 0.105$	$0.120 \\ 0.166$	$0.110 \\ 0.081$	0.097 $0.130$	0.139 $0.193$	$0.149 \\ 0.162$	$0.158 \\ 0.151$	$0.098 \\ 0.146$	$0.235 \\ 0.168$	$0.125 \\ 0.235$
Partitioning	4	1	0.118	0.147	0.092	0.130	0.225	0.156	0.154	0.146	0.136	0.182
Feasible Estimation												
Local Polynomial	0.23	0.29	0.099	0.087	0.079	0.066	0.161	0.067	0.088	0.096	0.088	0.096
B-splines	4	1	0.105	0.166	0.081	0.130	0.193	0.162	0.151	0.149	0.168	0.234
Partitioning	4	1	0.118	0.151	0.092	0.120	0.225	0.187	0.154	0.139	0.136	0.186
					Model 2.6	5						
Infeasible Estimation	0.0	0.9	0.050	0.000	0.046	0.069	0.022	0.061	0.060	0.004	0.000	0.122
Local Polynomial B-splines	$0.9 \\ 1$	0.9	$0.059 \\ 0.064$	0.089 $0.127$	$0.046 \\ 0.049$	0.009	0.033 $0.033$	$0.061 \\ 0.072$	$0.060 \\ 0.058$	0.094 $0.129$	0.089 $0.109$	0.122
Partitioning	1	1	0.055	0.100	0.043	0.034	0.033	0.060	0.057	0.123	0.103	0.171
Feasible Estimation												
Local Polynomial	0.4	0.31	0.419	0.423	0.362	0.364	0.042	0.065	0.086	0.103	0.646	0.651
B-splines	1	1	0.066	0.127	0.050	0.094	0.043	0.072	0.067	0.129	0.115	0.226
Partitioning	1	1	0.060	0.100	0.046	0.076	0.054	0.060	0.068	0.107	0.083	0.171
					Model 2.7	•						
Infeasible Estimation	0.22	0.22	0.164	0.202	0.002	0.127	0.057	0.010	0.250	0.255	0.102	0.015
Local Polynomial B-splines	0.33 9	0.33 9	0.164 $0.185$	$0.202 \\ 0.230$	$0.093 \\ 0.118$	$0.137 \\ 0.161$	0.957 $0.960$	0.918 $0.930$	$0.359 \\ 0.364$	$0.355 \\ 0.371$	0.183 $0.209$	0.215 $0.254$
Partitioning	9	9	0.211	0.323	0.148	0.243	0.937	0.858	0.369	0.390	0.179	0.267
Feasible Estimation												
Local Polynomial	0.34	0.28	0.149	0.162	0.068	0.087	0.978	0.979	0.361	0.363	0.084	0.097
B-splines	2	1	0.157	0.186	0.085	0.118	0.954	0.952	0.368	0.369	0.145	0.226
Partitioning	2	1	0.160	0.174	0.087	0.105	0.956	0.946	0.357	0.369	0.112	0.179
					Model 2.8	3						
Infeasible Estimation			0.105	0.001		0.050		0.0=0	0.110	0.001	0 - 0 -	0.10-
Local Polynomial B-splines	0.3	0.9	0.103	0.094	0.076	0.072	0.061	0.070	0.119	0.094	0.197	0.122
B-splines Partitioning	4	1 1	0.096 $0.113$	0.127 $0.102$	$0.073 \\ 0.088$	$0.095 \\ 0.078$	$0.125 \\ 0.177$	$0.074 \\ 0.072$	0.151 $0.154$	$0.129 \\ 0.109$	0.168 $0.136$	$0.226 \\ 0.176$
Feasible Estimation	4	1	0.113	0.102	0.000	0.010	0.177	0.012	0.134	0.109	0.130	0.170
Local Polynomial	0.34	0.28	0.074	0.097	0.055	0.073	0.043	0.063	0.083	0.096	0.083	0.097
B-splines	3	1	0.091	0.131	0.069	0.097	0.099	0.079	0.126	0.138	0.150	0.225
Partitioning	3	1	0.101	0.118	0.076	0.086	0.145	0.178	0.131	0.156	0.119	0.183

Table C.33: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=1, X_{i,\ell}\sim\beta(2,2),$  Uniform Cells

	Tur Parai		Root Int		Ingeti MA		(0.5,		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.15	0.22	0.174	0.187	0.125	0.134	0.127	0.130	0.237	0.229	0.739	0.554
B-splines	0.15	0.22	0.174	0.187 $0.175$	0.125	0.134 $0.128$	0.127 $0.202$	0.130 $0.159$	0.237 $0.192$	0.229	0.739 $0.391$	0.633
Partitioning	9	4	0.188	0.206	0.147	0.154	0.150	0.456	0.176	0.464	0.305	0.496
Feasible Estimation												
Local Polynomial	0.25	0.26	0.227	0.221	0.183	0.180	0.043	0.098	0.124	0.144	0.136	0.145
B-splines	4	2	0.180	0.194	0.140	0.150	0.326	0.212	0.318	0.241	0.256	0.559
Partitioning	4	2	0.186	0.199	0.144	0.154	0.350	0.322	0.379	0.324	0.194	0.389
					Model 2.2							
Infeasible Estimation Local Polynomial	0.12	0.18	0.201	0.216	0.145	0.154	0.146	0.132	0.263	0.293	0.637	1.093
B-splines	16	4	0.195	0.216	0.145	0.134	0.140	0.103	0.203	0.233	0.539	0.632
Partitioning	16	4	0.249	0.212	0.190	0.158	0.426	0.498	0.459	0.481	0.438	0.509
Feasible Estimation												
Local Polynomial	0.22	0.22	0.513	0.516	0.420	0.422	0.606	0.613	0.510	0.514	0.392	0.400
B-splines	8	4	0.323	0.166	0.257	0.118	0.355	0.103	0.271	0.231	0.567	0.632
Partitioning	8	4	0.267	0.212	0.209	0.158	0.313	0.498	0.229	0.481	0.515	0.509
					Model 2.3							
Infeasible Estimation	0.17	0.42	0.147	0.120	0.102	0.002	0.070	0.072	0.211	0.162	0.527	0.244
Local Polynomial B-splines	0.17 9	$0.42 \\ 4$	0.147 $0.158$	$0.130 \\ 0.158$	$0.102 \\ 0.116$	0.092 $0.111$	$0.079 \\ 0.089$	$0.073 \\ 0.104$	0.211 $0.205$	$0.162 \\ 0.230$	0.537 $0.401$	0.344 $0.633$
Partitioning	9	4	0.180	0.202	0.136	0.111	0.039	0.104	0.198	0.472	0.295	0.506
Feasible Estimation												
Local Polynomial	0.34	0.28	0.146	0.155	0.098	0.104	0.089	0.083	0.207	0.197	0.164	0.164
B-splines	3	3	0.140	0.157	0.098	0.111	0.114	0.102	0.228	0.222	0.243	0.597
Partitioning	3	3	0.147	0.184	0.107	0.134	0.147	0.365	0.245	0.399	0.186	0.462
					Model 2.4							
Infeasible Estimation	0.00	0.00	0.400	0.010	0.000	0.000	0.101	0.100	0.00=	0.150	0.450	0.000
Local Polynomial	0.33 9	0.33 9	0.496	0.313	0.320	0.239	0.131	0.133	0.227	0.172	0.456	0.380
B-splines Partitioning	9	9	0.555 $0.527$	0.344 $0.348$	$0.370 \\ 0.359$	0.269 $0.262$	0.054 $0.113$	$0.217 \\ 0.137$	$0.360 \\ 0.375$	$0.249 \\ 0.295$	0.593 $0.486$	1.193 0.791
Feasible Estimation	5	9	0.021	0.040	0.005	0.202	0.110	0.101	0.010	0.230	0.400	0.731
Local Polynomial	0.21	0.27	0.512	0.450	0.390	0.342	0.273	0.263	0.359	0.271	0.927	1.024
B-splines	4	4	0.567	0.398	0.360	0.320	0.221	0.275	0.448	0.242	0.542	0.690
Partitioning	4	4	0.559	0.327	0.360	0.256	0.251	0.568	0.469	0.478	0.473	0.519
					Model 2.5							
Infeasible Estimation	0.10	0.0	0.140	0.110	0.100	0.000	0.100	0.114	0.100	0.145	0.400	0.104
Local Polynomial B-splines	$0.19 \\ 4$	$0.9 \\ 1$	$0.146 \\ 0.106$	$0.112 \\ 0.157$	$0.106 \\ 0.081$	$0.088 \\ 0.117$	$0.120 \\ 0.181$	0.114 $0.122$	$0.199 \\ 0.156$	$0.147 \\ 0.223$	$0.498 \\ 0.252$	$0.194 \\ 0.528$
Partitioning	4	1	0.119	0.137	0.093	0.105	0.216	0.119	0.164	0.197	0.182	0.313
Feasible Estimation	•	-	0.110	0.10.	0.000	0.100	0.210	0.110	0.101	0.101	0.102	0.010
Local Polynomial	0.22	0.29	0.100	0.087	0.082	0.062	0.145	0.060	0.123	0.135	0.123	0.135
B-splines	4	1	0.106	0.158	0.081	0.117	0.181	0.121	0.156	0.226	0.252	0.547
Partitioning	4	1	0.119	0.147	0.093	0.111	0.216	0.187	0.164	0.224	0.182	0.335
					Model 2.6	;						
Infeasible Estimation	0.0	0.0	0.050	0.000	0.045	0.004	0.000	0.050	0.051	0.100	0.101	0.150
Local Polynomial B-splines	0.9 1	0.9 1	$0.058 \\ 0.063$	0.086 $0.126$	$0.045 \\ 0.047$	0.064 $0.088$	0.033 $0.033$	0.052 $0.060$	$0.071 \\ 0.067$	$0.122 \\ 0.164$	$0.101 \\ 0.137$	0.179 $0.494$
Partitioning	1	1	0.055	0.120	0.047	0.033	0.033	0.052	0.067	0.130	0.137	0.494
Feasible Estimation	-	-	0.000	0.000	0.010	0.0.2	0.000	0.002	0.001	0.100	0.000	0.2.0
Local Polynomial	0.4	0.3	0.353	0.358	0.296	0.299	0.038	0.058	0.115	0.140	0.652	0.658
B-splines	1	1	0.070	0.126	0.051	0.088	0.051	0.060	0.088	0.164	0.172	0.496
Partitioning	1	1	0.068	0.099	0.050	0.073	0.069	0.055	0.092	0.130	0.111	0.279
					Model 2.7	•						
Infeasible Estimation												
Local Polynomial B-splines	0.33	0.33	0.177	0.205	0.097	0.131	0.950	0.910	0.363	0.355	0.294	0.371
Partitioning	9 9	9	0.198 $0.222$	0.241 $0.330$	$0.123 \\ 0.154$	$0.161 \\ 0.245$	$0.950 \\ 0.931$	$0.920 \\ 0.855$	$0.369 \\ 0.372$	$0.397 \\ 0.412$	0.388 $0.290$	0.946 $0.777$
Feasible Estimation	9	3	0.222	0.550	0.134	0.245	0.331	0.000	0.372	0.412	0.230	0.111
Local Polynomial	0.33	0.27	0.166	0.178	0.078	0.094	0.969	0.969	0.362	0.370	0.116	0.137
B-splines	3	2	0.174	0.204	0.099	0.127	0.926	0.930	0.363	0.386	0.232	0.551
Partitioning	3	2	0.179	0.199	0.104	0.125	0.917	0.890	0.340	0.393	0.170	0.368
					Model 2.8							
$In feasible\ Estimation$												
Local Polynomial	0.32	0.9	0.096	0.089	0.067	0.067	0.048	0.058	0.134	0.123	0.309	0.179
B-splines	4	1	0.095	0.126	0.071	0.089	0.106	0.062	0.155	0.165	0.252	0.494
Partitioning Feasible Estimation	4	1	0.111	0.101	0.086	0.074	0.154	0.058	0.163	0.130	0.182	0.288
Local Polynomial	0.34	0.28	0.067	0.092	0.049	0.066	0.042	0.058	0.113	0.135	0.114	0.136
B-splines	3	2	0.091	0.136	0.067	0.095	0.092	0.074	0.135	0.184	0.232	0.545
Partitioning	3	2	0.101	0.135	0.076	0.093	0.135	0.214	0.145	0.236	0.166	0.356

Table C.34: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=1000,\ \sigma^2=4,\ X_{i,\ell}\sim\beta(0.5,0.5),\ \text{Uniform Cells}$ 

	Tun Paran		Root Int		Inget:		(0.5,		oint Estim			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.01	0.07	0.007	0.245	0.005	0.000	0.040	0.205	0.071	0.207	0.000	0.070
Local Polynomial B-splines	0.21 4	$0.27 \\ 4$	0.267 $0.228$	0.345 $0.325$	$0.205 \\ 0.177$	$0.266 \\ 0.250$	$0.240 \\ 0.504$	$0.305 \\ 0.342$	$0.271 \\ 0.300$	0.327 $0.319$	0.339 $0.271$	$0.370 \\ 0.333$
Partitioning	4	4	0.256	0.405	0.177	0.308	0.472	1.301	0.376	0.766	0.251	0.339
Feasible Estimation	_	_	0.200	0.200	0.200	0.000						0.000
Local Polynomial	0.29	0.26	0.225	0.252	0.171	0.198	0.119	0.200	0.159	0.193	0.153	0.184
B-splines	3 3	2 2	0.233	0.299	0.180	0.230	0.463	0.303	0.285	0.265	0.270	$0.326 \\ 0.308$
Partitioning	3	2	0.255	0.310	0.196	0.231	0.446	0.725	0.349	0.461	0.247	0.308
T. C. ST. T. C. C.					Model 2.2	2						
Infeasible Estimation Local Polynomial	0.15	0.21	0.343	0.412	0.266	0.319	0.314	0.361	0.349	0.440	0.361	0.516
B-splines	9	4	0.425	0.325	0.348	0.249	0.489	0.293	0.428	0.313	0.336	0.334
Partitioning	9	4	0.392	0.411	0.309	0.313	0.332	1.402	0.322	0.772	0.309	0.340
Feasible Estimation												
Local Polynomial	0.31	0.24	0.498	0.512	0.399	0.407	0.709	0.757	0.616	0.626	0.304	0.323
B-splines Partitioning	$\frac{4}{4}$	$\frac{4}{4}$	$0.382 \\ 0.420$	$0.326 \\ 0.411$	$0.305 \\ 0.338$	$0.250 \\ 0.313$	$0.300 \\ 0.941$	0.295 $1.399$	0.284 $0.384$	0.313 $0.767$	$0.327 \\ 0.376$	0.334 $0.341$
1 ar troioining	-	-	0.420	0.411			0.541	1.000	0.004	0.101	0.010	0.041
Infeasible Estimation					Model 2.3	•						
Local Polynomial	0.19	0.43	0.279	0.271	0.213	0.208	0.219	0.176	0.286	0.223	0.352	0.300
B-splines	9	4	0.288	0.318	0.225	0.244	0.187	0.294	0.227	0.314	0.321	0.333
Partitioning	9	4	0.351	0.404	0.273	0.307	0.238	1.308	0.278	0.766	0.304	0.339
Feasible Estimation Local Polynomial	0.32	0.27	0.279	0.305	0.186	0.214	0.142	0.178	0.263	0.288	0.218	0.246
B-splines	4	3	0.279	0.303	0.186	0.214	0.142	0.178	0.286	0.280	0.218 $0.267$	0.246
Partitioning	4	3	0.274	0.344	0.212	0.255	0.368	0.960	0.359	0.566	0.240	0.332
					Model 2.4							
Infeasible Estimation					Model 2.4							
Local Polynomial	0.33	0.33	0.686	0.422	0.501	0.332	0.175	0.259	0.351	0.295	0.325	0.382
B-splines	9	9	0.759	0.475	0.603	0.373	0.255	0.350	0.254	0.307	0.337	0.351
Partitioning	9	9	0.712	0.626	0.568	0.478	0.251	0.466	0.311	0.472	0.310	0.406
Feasible Estimation Local Polynomial	0.2	0.27	0.648	0.554	0.532	0.452	0.417	0.423	0.669	0.393	0.613	0.921
B-splines	4	2	0.808	0.562	0.625	0.443	0.704	0.428	0.293	0.333	0.297	0.343
Partitioning	4	2	0.801	0.560	0.620	0.438	0.782	1.101	0.340	0.627	0.272	0.328
					Model 2.5	5						
$In feasible\ Estimation$												
Local Polynomial	0.24	0.9	0.243	0.199	0.187	0.158	0.243	0.207	0.246	0.168	0.325	0.215
B-splines Partitioning	$\frac{4}{4}$	1 1	$0.197 \\ 0.225$	$0.278 \\ 0.230$	$0.151 \\ 0.175$	0.215 $0.181$	0.319 $0.408$	0.234 $0.214$	0.279 $0.303$	0.234 $0.200$	$0.271 \\ 0.244$	$0.326 \\ 0.285$
Feasible Estimation	4	1	0.225	0.230	0.175	0.161	0.408	0.214	0.303	0.200	0.244	0.265
Local Polynomial	0.26	0.27	0.152	0.176	0.120	0.136	0.223	0.151	0.148	0.181	0.148	0.181
B-splines	4	2	0.197	0.291	0.151	0.223	0.319	0.255	0.279	0.258	0.271	0.326
Partitioning	4	2	0.225	0.290	0.175	0.216	0.408	0.644	0.303	0.413	0.244	0.302
					Model 2.6	5						
Infeasible Estimation	0.0	0.0	0.110	0.101	0.000	0.141	0.000	0.100	0.110	0.100	0.105	0.015
Local Polynomial B-splines	0.9 1	0.9 1	0.119 $0.127$	0.181 $0.255$	$0.092 \\ 0.097$	$0.141 \\ 0.194$	0.066 $0.066$	$0.132 \\ 0.161$	0.113 $0.110$	0.168 $0.233$	$0.165 \\ 0.188$	0.215 $0.324$
Partitioning	1	1	0.110	0.202	0.086	0.154	0.066	0.133	0.110	0.199	0.144	0.324 $0.284$
Feasible Estimation	_	_	0.220	0.202		0.200	0.000		0		******	0.20
Local Polynomial	0.35	0.28	0.470	0.486	0.410	0.418	0.093	0.148	0.149	0.181	0.660	0.671
B-splines	2	1	0.163	0.260	0.122	0.197	0.196	0.171	0.221	0.237	0.234	0.324
Partitioning	2	1	0.175	0.223	0.127	0.167	0.273	0.323	0.230	0.259	0.198	0.288
					Model 2.7	•						
Infeasible Estimation Local Polynomial	0.33	0.33	0.227	0.341	0.160	0.259	0.967	0.944	0.394	0.431	0.301	0.376
B-splines	9	9	0.283	0.399	0.210	0.303	0.974	0.964	0.404	0.435	0.320	0.342
Partitioning	9	9	0.352	0.611	0.268	0.463	0.956	0.977	0.423	0.558	0.296	0.398
Feasible Estimation												
Local Polynomial	0.32	0.27	0.172	0.215	0.107	0.150	0.989	0.997	0.381	0.393	0.147	0.181
B-splines Partitioning	3 3	$\frac{2}{2}$	0.213	0.299	0.147	$0.220 \\ 0.208$	0.981	0.969 $1.100$	$0.415 \\ 0.419$	0.426	0.251	0.326 $0.301$
rarmoning	3	2	0.230	0.296	0.159		0.989	1.100	0.419	0.512	0.221	0.301
Infossible F-ti					Model 2.8	3						
Infeasible Estimation Local Polynomial	0.37	0.9	0.183	0.183	0.138	0.144	0.109	0.139	0.189	0.169	0.288	0.215
B-splines	1	1	0.138	0.255	0.106	0.194	0.069	0.162	0.112	0.233	0.188	0.324
Partitioning	1	1	0.123	0.203	0.096	0.157	0.069	0.141	0.112	0.200	0.144	0.285
Feasible Estimation												
Local Polynomial	0.32	0.27	0.130	0.183	0.100	0.142	0.095	0.148	0.147	0.181	0.147	0.182
B-splines	3	$\frac{2}{2}$	0.183 $0.206$	$0.278 \\ 0.285$	$0.139 \\ 0.156$	$0.210 \\ 0.205$	$0.247 \\ 0.342$	$0.212 \\ 0.707$	$0.256 \\ 0.270$	$0.264 \\ 0.435$	$0.257 \\ 0.229$	$0.326 \\ 0.305$

Table C.35: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=4, X_{i,\ell}\sim\beta(1,1),$  Uniform Cells

	Tur Parar		Root Int		Ingeti MA		(0.5,		oint Estim (0.1	ation RM, $0.5$ )		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.9	0.26	0.275	0.249	0.207	0.050	0.917	0.065	0.207	0.220	0.460	0.450
Local Polynomial B-splines	0.2	$0.26 \\ 4$	$0.275 \\ 0.236$	0.342 $0.324$	$0.207 \\ 0.183$	$0.258 \\ 0.244$	0.217 $0.453$	$0.265 \\ 0.303$	0.307 $0.335$	0.339 $0.364$	$0.460 \\ 0.336$	$0.450 \\ 0.480$
Partitioning	4	4	0.260	0.324 $0.402$	0.183	0.244 $0.305$	0.455 $0.451$	1.135	0.395	0.364 $0.783$	0.330	0.460
Feasible Estimation	4	4	0.200	0.402	0.202	0.303	0.431	1.135	0.555	0.765	0.201	0.402
Local Polynomial	0.29	0.26	0.239	0.261	0.187	0.208	0.094	0.170	0.178	0.204	0.177	0.200
B-splines	4	2	0.240	0.304	0.187	0.232	0.430	0.274	0.321	0.312	0.330	0.463
Partitioning	4	2	0.261	0.318	0.202	0.238	0.431	0.643	0.373	0.516	0.279	0.394
					Model 2.2							
$In feasible\ Estimation$												
Local Polynomial	0.15	0.21	0.337	0.402	0.256	0.303	0.272	0.299	0.376	0.431	0.510	0.620
B-splines Partitioning	9	$\frac{4}{4}$	0.419 $0.389$	$0.322 \\ 0.408$	$0.340 \\ 0.305$	0.241 $0.309$	0.447 $0.295$	0.255 $1.202$	0.399 $0.331$	$0.363 \\ 0.788$	0.459 $0.398$	0.482 $0.461$
Feasible Estimation	9	4	0.369	0.408	0.303	0.309	0.293	1.202	0.331	0.766	0.398	0.401
Local Polynomial	0.3	0.24	0.516	0.529	0.419	0.427	0.677	0.715	0.592	0.600	0.343	0.360
B-splines	4	4	0.365	0.323	0.285	0.241	0.252	0.256	0.321	0.363	0.446	0.482
Partitioning	4	4	0.414	0.408	0.330	0.309	0.825	1.198	0.428	0.787	0.498	0.463
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.2	0.44	0.268	0.255	0.198	0.191	0.172	0.153	0.305	0.238	0.464	0.350
B-splines	9	4	0.281	0.316	0.216	0.236	0.162	0.255	0.252	0.363	0.419	0.480
Partitioning	9	4	0.345	0.402	0.268	0.304	0.194	1.143	0.308	0.786	0.365	0.464
Feasible Estimation Local Polynomial	0.32	0.27	0.233	0.264	0.155	0.183	0.127	0.152	0.257	0.272	0.218	0.236
B-splines	4	3	0.235	0.300	0.177	0.103	0.263	0.132	0.309	0.321	0.330	0.470
Partitioning	4	3	0.258	0.336	0.198	0.246	0.325	0.780	0.355	0.602	0.267	0.427
3					Model 2.4							
Infeasible Estimation					Moder 2.4							
Local Polynomial	0.33	0.33	0.635	0.420	0.448	0.327	0.166	0.225	0.266	0.302	0.390	0.436
B-splines	9	9	0.711	0.476	0.538	0.370	0.173	0.313	0.255	0.345	0.473	0.520
Partitioning	9	9	0.681	0.625	0.526	0.478	0.212	0.377	0.324	0.499	0.392	0.540
Feasible Estimation												
Local Polynomial	0.21	0.27	0.618 $0.745$	0.531	0.495	0.421	0.365	0.367	$0.567 \\ 0.361$	0.364	0.732	0.967
B-splines Partitioning	4	$\frac{2}{2}$	$0.745 \\ 0.745$	$0.536 \\ 0.529$	$0.540 \\ 0.547$	$0.418 \\ 0.407$	$0.539 \\ 0.608$	0.388 $1.034$	0.361 $0.397$	$0.364 \\ 0.693$	0.388 $0.350$	0.493 $0.451$
1 artitioning	4	2	0.743	0.523			0.000	1.054	0.551	0.033	0.330	0.401
I-fill- E-titi					Model 2.5							
Infeasible Estimation Local Polynomial	0.24	0.9	0.244	0.195	0.184	0.153	0.216	0.184	0.271	0.189	0.419	0.245
B-splines	4	1	0.196	0.275	0.149	0.208	0.288	0.206	0.302	0.267	0.335	0.456
Partitioning	4	1	0.224	0.228	0.175	0.176	0.364	0.189	0.307	0.225	0.272	0.348
Feasible Estimation												
Local Polynomial	0.25	0.27	0.152	0.173	0.121	0.131	0.205	0.129	0.172	0.195	0.172	0.195
B-splines	4	2	0.196	0.288	0.149	0.216	0.288	0.216	0.302	0.302	0.335	0.462
Partitioning	4	2	0.224	0.288	0.175	0.211	0.364	0.529	0.307	0.453	0.272	0.377
					Model 2.6	;						
Infeasible Estimation	0.0	0.0	0.110	0.170	0.001	0.107	0.000	0.101	0.100	0.107	0.170	0.049
Local Polynomial B-splines	0.9 1	0.9 1	0.118 $0.127$	$0.178 \\ 0.254$	0.091 $0.097$	0.137 $0.188$	$0.066 \\ 0.065$	0.121 $0.143$	$0.120 \\ 0.115$	0.187 $0.258$	$0.179 \\ 0.218$	0.243 $0.453$
Partitioning	1	1	0.110	0.201	0.086	0.153	0.065	0.143 $0.121$	0.115	0.238 $0.214$	0.218	0.433
Feasible Estimation	_	_	0.220	0.202	0.000	0.200	0.000	*****			0.202	0.0.0
Local Polynomial	0.34	0.28	0.431	0.449	0.368	0.379	0.084	0.127	0.170	0.197	0.663	0.673
B-splines	2	1	0.164	0.261	0.122	0.193	0.181	0.155	0.233	0.271	0.292	0.451
Partitioning	2	1	0.176	0.230	0.129	0.168	0.247	0.350	0.241	0.303	0.225	0.356
					Model 2.7	•						
$Infeasible\ Estimation$												
Local Polynomial	0.33	0.33	0.232	0.337	0.159	0.249	0.963	0.931	0.404	0.437	0.366	0.431
B-splines	9	9	0.287	0.400	0.210	0.296	0.969	0.947	0.418	0.467	0.419	0.507
Partitioning Feasible Estimation	9	9	0.354	0.611	0.271	0.465	0.951	0.912	0.454	0.577	0.357	0.533
Local Polynomial	0.32	0.27	0.180	0.221	0.109	0.149	0.981	0.985	0.389	0.399	0.169	0.195
B-splines	3	2	0.222	0.305	0.153	0.220	0.955	0.959	0.363 $0.442$	0.461	0.313	0.462
Partitioning	3	2	0.239	0.306	0.168	0.214	0.973	1.037	0.427	0.571	0.253	0.376
-					Model 2.8							
Infeasible Estimation					Model 2.c							
Local Polynomial	0.38	0.9	0.177	0.180	0.130	0.139	0.095	0.126	0.194	0.187	0.336	0.243
B-splines	1	1	0.136	0.254	0.104	0.188	0.070	0.144	0.119	0.258	0.219	0.453
Partitioning	1	1	0.121	0.202	0.094	0.154	0.070	0.126	0.118	0.215	0.155	0.346
Feasible Estimation	0.00	0.07	0.105	0.170	0.005	0.105	0.000	0.100	0.100	0.105	0.100	0.100
Local Polynomial	0.32	$0.27 \\ 2$	0.125 $0.182$	$0.179 \\ 0.277$	$0.095 \\ 0.137$	0.135 $0.204$	$0.086 \\ 0.220$	$0.128 \\ 0.178$	$0.169 \\ 0.278$	$0.195 \\ 0.305$	$0.169 \\ 0.315$	$0.196 \\ 0.465$
B-splines												

Table C.36: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=4, X_{i,\ell}\sim\beta(2,2),$  Uniform Cells

		ning meter	Root Int		Ingeti MA		(0.5		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.19	0.26	0.285	0.334	0.204	0.238	0.186	0.217	0.397	0.397	0.996	0.883
Local Polynomial B-splines	9	0.20	0.288	0.334 $0.325$	0.232	0.230	0.180	0.217	0.394	0.397	0.996	1.265
Partitioning	9	4	0.340	0.403	0.262	0.300	0.196	0.877	0.348	0.925	0.587	0.985
Feasible Estimation												
Local Polynomial	0.28	0.26	0.252	0.268	0.203	0.214	0.083	0.141	0.239	0.277	0.248	0.279
B-splines	4	3	0.245	0.316	0.190	0.230	0.369	0.247	0.405	0.422	0.506	1.160
Partitioning	4	3	0.265	0.356	0.207	0.262	0.433	0.673	0.466	0.718	0.369	0.848
Infeasible Estimation					Model 2.2	}						
Local Polynomial	0.16	0.21	0.332	0.383	0.238	0.272	0.227	0.228	0.467	0.481	1.373	1.300
B-splines	9	4	0.395	0.320	0.312	0.225	0.391	0.205	0.371	0.460	0.880	1.265
Partitioning	9	4	0.380	0.407	0.295	0.303	0.264	0.894	0.365	0.936	0.705	0.988
Feasible Estimation					0.400							
Local Polynomial	0.25	0.23	0.524	0.537	0.428	0.438	0.610	0.624	0.557	0.570	0.435	0.460
B-splines Partitioning	4	$rac{4}{4}$	$0.324 \\ 0.386$	$0.320 \\ 0.407$	$0.242 \\ 0.299$	$0.225 \\ 0.303$	$0.235 \\ 0.668$	$0.207 \\ 0.894$	$0.433 \\ 0.492$	$0.460 \\ 0.934$	$0.744 \\ 0.734$	1.260 $0.990$
1 01 01010111118	-	-	0.000	0.10.			0.000	0.001	0.102	0.001	0.101	0.000
Infeasible Estimation					Model 2.3	•						
Local Polynomial	0.22	0.48	0.245	0.226	0.169	0.162	0.124	0.123	0.348	0.290	0.842	0.584
B-splines	4	4	0.216	0.316	0.158	0.221	0.218	0.206	0.346	0.460	0.508	1.265
Partitioning	4	4	0.242	0.402	0.185	0.299	0.302	0.876	0.369	0.930	0.373	0.991
Feasible Estimation	0.91	0.07	0.170	0.016	0.104	0.151	0.111	0.101	0.000	0.200	0.054	0.000
Local Polynomial B-splines	0.31	0.27 3	0.178 $0.211$	0.216 $0.302$	0.124 $0.153$	$0.151 \\ 0.211$	0.111 $0.205$	0.131 $0.189$	0.282 $0.342$	$0.306 \\ 0.421$	0.254 $0.486$	0.286 $1.192$
Partitioning	3	3	0.233	0.355	0.175	0.254	0.288	0.716	0.358	0.768	0.357	0.889
		_	0.200	0.000	Model 2.4		0.200					
Infeasible Estimation					Model 2.4	:						
Local Polynomial	0.33	0.33	0.521	0.399	0.345	0.303	0.149	0.183	0.319	0.338	0.690	0.748
B-splines	9	9	0.597	0.476	0.410	0.355	0.103	0.259	0.429	0.456	0.901	2.033
Partitioning	9	9	0.598	0.625	0.433	0.468	0.167	0.275	0.482	0.577	0.704	1.562
Feasible Estimation Local Polynomial	0.23	0.27	0.545	0.477	0.412	0.363	0.292	0.284	0.351	0.355	1.036	1.057
B-splines	4	3	0.596	0.477	0.382	0.303 $0.377$	0.322	0.234	0.535	0.333 $0.478$	0.671	1.286
Partitioning	4	3	0.604	0.485	0.403	0.369	0.394	0.904	0.578	0.889	0.557	0.971
					Model 2.5							
Infeasible Estimation					1110401 210							
Local Polynomial	0.23	0.9	0.243	0.186	0.175	0.141	0.183	0.145	0.329	0.256	0.789	0.363
B-splines	4	1	0.195	0.268	0.146	0.192	0.258	0.160	0.310	0.361	0.504	1.006
Partitioning	4	1	0.223	0.219	0.173	0.163	0.341	0.149	0.326	0.298	0.363	0.573
Feasible Estimation Local Polynomial	0.25	0.27	0.154	0.174	0.121	0.125	0.183	0.118	0.240	0.272	0.240	0.272
B-splines	4	2	0.195	0.295	0.146	0.208	0.258	0.181	0.310	0.412	0.504	1.139
Partitioning	4	2	0.223	0.325	0.173	0.231	0.341	0.589	0.326	0.652	0.363	0.815
					Model 2.6	;						
$In feasible\ Estimation$												
Local Polynomial	0.9	0.9	0.116	0.172	0.089	0.129	0.065	0.104	0.142	0.244	0.202	0.357
B-splines	1 1	1 1	0.126	0.251	0.094	0.177	0.065	0.119	0.134	0.327	0.274	0.988
Partitioning Feasible Estimation	1	1	0.110	0.198	0.086	0.145	0.065	0.103	0.134	0.259	0.175	0.557
Local Polynomial	0.34	0.28	0.367	0.388	0.305	0.319	0.077	0.117	0.230	0.273	0.687	0.703
B-splines	3	2	0.172	0.270	0.125	0.188	0.173	0.146	0.264	0.369	0.455	1.089
Partitioning	3	2	0.188	0.267	0.139	0.184	0.255	0.428	0.275	0.470	0.320	0.704
					Model 2.7							
$In feasible\ Estimation$												
Local Polynomial	0.33	0.33	0.238	0.323	0.156	0.226	0.952	0.916	0.424	0.455	0.588	0.743
B-splines Partitioning	9 9	9	0.295 $0.360$	$0.408 \\ 0.615$	$0.210 \\ 0.273$	0.287 $0.461$	0.954 $0.938$	0.929 $0.886$	$0.440 \\ 0.474$	$0.551 \\ 0.640$	$0.775 \\ 0.581$	1.889 1.554
Feasible Estimation	9	9	0.300	0.013	0.273	0.401	0.936	0.000	0.414	0.040	0.561	1.554
Local Polynomial	0.31	0.26	0.196	0.234	0.116	0.151	0.970	0.971	0.411	0.435	0.234	0.273
B-splines	3	3	0.235	0.328	0.160	0.227	0.927	0.928	0.440	0.522	0.490	1.156
Partitioning	3	3	0.254	0.359	0.180	0.252	0.928	1.002	0.425	0.734	0.353	0.840
					Model 2.8	,						
Infeasible Estimation				0.4		0.4						
Local Polynomial	0.4	0.9	0.165	0.173	0.118	0.130	0.078	0.107	0.221	0.245	0.475	0.357
B-splines Partitioning	1 1	1 1	$0.132 \\ 0.117$	0.252 $0.199$	0.099 $0.091$	$0.177 \\ 0.145$	$0.072 \\ 0.072$	$0.120 \\ 0.107$	0.138 $0.139$	$0.328 \\ 0.259$	$0.276 \\ 0.177$	0.988 $0.562$
Feasible Estimation	1	1	0.117	0.199	0.091	0.140	0.072	0.107	0.139	0.209	0.177	0.502
Local Polynomial	0.31	0.27	0.123	0.177	0.090	0.127	0.080	0.118	0.231	0.273	0.232	0.273
B-splines	3	3	0.183	0.291	0.134	0.202	0.195	0.170	0.289	0.403	0.486	1.153
Partitioning	3	3	0.207	0.333	0.157	0.233	0.283	0.629	0.305	0.692	0.348	0.840

## C.2.2 QUANTILE CELL BOUNDARIES

Table C.37: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=1,\ X_{i,\ell}\sim\beta(0.5,0.5),\ \text{Quantile Cells}$ 

	Tur Parai		Root Int		Ingeti MA		(0.5,		oint Estim (0.1			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.10	0.05	0.007	0.055	0.150	0.107	0.100	0.004	0.012	0.050	0.040	0.000
Local Polynomial B-splines	0.19 9	$0.25 \\ 4$	$0.207 \\ 0.216$	0.255 $0.233$	0.158 $0.169$	0.197 $0.180$	0.199 $0.198$	0.234 $0.271$	0.213 $0.152$	$0.250 \\ 0.231$	0.249 $0.232$	0.288 $0.242$
Partitioning	9	4	0.216 $0.245$	0.233	0.109	0.180	0.198	0.782	0.132 $0.187$	0.231 $0.461$	0.232 $0.214$	0.242
Feasible Estimation	3	4	0.243	0.201	0.130	0.210	0.207	0.162	0.101	0.401	0.214	0.242
Local Polynomial	0.3	0.27	0.207	0.217	0.155	0.170	0.088	0.161	0.122	0.145	0.112	0.134
B-splines	3	1	0.198	0.218	0.150	0.170	0.354	0.261	0.172	0.177	0.180	0.231
Partitioning	3	1	0.208	0.207	0.158	0.159	0.311	0.279	0.248	0.203	0.167	0.204
					Model 2.2							
$In feasible\ Estimation$												
Local Polynomial	0.14	0.2	0.262	0.309	0.204	0.239	0.250	0.284	0.261	0.347	0.258	0.411
B-splines Partitioning	9	4	$0.402 \\ 0.317$	0.234 $0.295$	$0.329 \\ 0.250$	$0.180 \\ 0.225$	0.522 $0.333$	$0.206 \\ 0.880$	$0.423 \\ 0.275$	$0.226 \\ 0.467$	$0.246 \\ 0.218$	0.243 $0.245$
Feasible Estimation	9	4	0.317	0.293	0.230	0.225	0.555	0.000	0.275	0.407	0.218	0.245
Local Polynomial	0.33	0.24	0.490	0.494	0.392	0.391	0.706	0.757	0.607	0.615	0.287	0.297
B-splines	4	4	0.356	0.234	0.285	0.180	0.252	0.206	0.212	0.227	0.262	0.243
Partitioning	4	4	0.386	0.295	0.312	0.225	0.853	0.880	0.313	0.466	0.323	0.244
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.17	0.4	0.216	0.204	0.165	0.156	0.178	0.133	0.227	0.176	0.257	0.221
B-splines	9	4	0.217	0.224	0.169	0.171	0.142	0.204	0.171	0.226	0.233	0.242
Partitioning	9	4	0.255	0.286	0.198	0.217	0.145	0.774	0.194	0.462	0.223	0.243
Feasible Estimation Local Polynomial	0.34	0.28	0.264	0.276	0.170	0.185	0.118	0.137	0.247	0.259	0.198	0.212
B-splines	4	2	0.209	0.220	0.160	0.170	0.223	0.182	0.200	0.194	0.183	0.212
Partitioning	4	2	0.226	0.234	0.174	0.176	0.256	0.433	0.294	0.306	0.159	0.234
3					Model 2.4							
Infeasible Estimation					Moder 2.4							
Local Polynomial	0.33	0.33	0.668	0.354	0.482	0.275	0.149	0.207	0.323	0.206	0.266	0.271
B-splines	9	9	0.693	0.379	0.555	0.296	0.184	0.272	0.238	0.230	0.290	0.266
Partitioning	9	9	0.590	0.444	0.455	0.341	0.188	0.300	0.277	0.323	0.260	0.337
Feasible Estimation												
Local Polynomial	$0.21 \\ 4$	0.28 1	$0.638 \\ 0.784$	$0.540 \\ 0.567$	$0.524 \\ 0.609$	$0.443 \\ 0.447$	0.405	0.412	$0.668 \\ 0.219$	0.379	$0.601 \\ 0.234$	0.907
B-splines Partitioning	4	1	0.769	0.556	0.590	0.447	0.584 $0.661$	$0.395 \\ 0.589$	0.219	0.245 $0.333$	0.234	0.286 $0.247$
1 ai titioning	-	-	0.703	0.000			0.001	0.000	0.200	0.000	0.210	0.241
Infeasible Estimation					Model 2.5							
Local Polynomial	0.22	0.9	0.186	0.151	0.142	0.120	0.195	0.181	0.192	0.121	0.238	0.145
B-splines	4	1	0.143	0.209	0.111	0.164	0.201	0.200	0.185	0.170	0.182	0.234
Partitioning	4	1	0.161	0.178	0.126	0.142	0.272	0.188	0.214	0.146	0.159	0.188
Feasible Estimation												
Local Polynomial	0.26	0.29	0.122	0.122	0.096	0.093	0.206	0.104	0.105	0.128	0.105	0.128
B-splines	4	1	0.143	0.210	0.111	0.164	0.201	0.200	0.185	0.170	0.182	0.235
Partitioning	4	1	0.161	0.182	0.126	0.144	0.272	0.205	0.214	0.152	0.159	0.189
					Model 2.6	i						
Infeasible Estimation Local Polynomial	0.9	0.9	0.083	0.126	0.064	0.099	0.044	0.091	0.080	0.119	0.110	0.144
B-splines	0.9	0.9	0.089	0.126	0.064	0.099	0.044	0.091 $0.114$	0.080	0.119 $0.164$	0.110	0.144
Partitioning	1	1	0.076	0.140	0.060	0.109	0.044	0.092	0.076	0.140	0.097	0.185
Feasible Estimation												
Local Polynomial	0.4	0.31	0.461	0.468	0.405	0.407	0.062	0.110	0.112	0.139	0.647	0.658
B-splines	1	1	0.092	0.178	0.070	0.136	0.060	0.114	0.085	0.164	0.134	0.230
Partitioning	1	1	0.082	0.140	0.062	0.109	0.064	0.092	0.088	0.140	0.101	0.185
					Model 2.7	•						
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.181	0.254	0.119	0.188	0.960	0.931	0.378	0.384	0.208	0.265
B-splines Partitioning	9	9	$0.216 \\ 0.261$	0.293 $0.437$	0.153	0.218 $0.334$	$0.965 \\ 0.952$	0.949 $0.932$	0.382 $0.393$	$0.403 \\ 0.447$	$0.232 \\ 0.212$	0.254 $0.336$
Feasible Estimation	9	9	0.201	0.437	0.195	0.334	0.952	0.932	0.393	0.447	0.212	0.550
Local Polynomial	0.34	0.28	0.148	0.173	0.079	0.110	0.981	0.983	0.369	0.374	0.104	0.128
B-splines	2	1	0.164	0.217	0.099	0.153	0.964	0.960	0.378	0.387	0.153	0.230
Partitioning	2	1	0.166	0.190	0.099	0.128	0.950	0.958	0.370	0.377	0.126	0.187
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.33	0.9	0.137	0.130	0.104	0.102	0.086	0.103	0.145	0.119	0.210	0.145
B-splines	1	1	0.104	0.179	0.079	0.137	0.050	0.116	0.078	0.164	0.130	0.230
Partitioning	1	1	0.094	0.142	0.072	0.110	0.050	0.105	0.078	0.143	0.097	0.186
Feasible Estimation	0.24	0.00	0.007	0.120	0.072	0.101	0.062	0.100	0.104	0.100	0.104	0.100
Local Polynomial B-splines	0.34	0.28 1	0.097 $0.123$	0.132 $0.183$	$0.073 \\ 0.093$	$0.101 \\ 0.140$	$0.062 \\ 0.133$	$0.102 \\ 0.123$	$0.104 \\ 0.149$	$0.128 \\ 0.170$	$0.104 \\ 0.162$	0.128 $0.232$
	_	1	0.123	0.100	0.093	0.140	0.100	0.140	0.149	0.110	0.102	0.434

Table C.38: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=500, \sigma^2=1, X_{i,\ell}\sim\beta(1,1),$  Quantile Cells

		ning neter	Root In		Ingeti MA		(0.5		oint Estim (0.1			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.10	0.05	0.014	0.050	0.160	0.104	0.104	0.011	0.007	0.064	0.076	0.000
Local Polynomial B-splines	0.18 9	$0.25 \\ 4$	0.214 $0.225$	$0.256 \\ 0.236$	$0.162 \\ 0.177$	0.194 $0.179$	0.184 $0.222$	0.211 $0.243$	0.237 $0.177$	$0.264 \\ 0.256$	$0.376 \\ 0.329$	0.386 $0.398$
Partitioning	9	4	0.225	0.286	0.177	0.179	0.222	0.640	0.177	0.256 $0.525$	0.329 $0.277$	0.367
Feasible Estimation	9	4	0.247	0.280	0.132	0.210	0.134	0.040	0.221	0.525	0.211	0.307
Local Polynomial	0.29	0.27	0.224	0.231	0.173	0.184	0.072	0.145	0.134	0.159	0.135	0.150
B-splines	3	1	0.205	0.229	0.158	0.178	0.355	0.253	0.227	0.209	0.252	0.359
Partitioning	3	1	0.215	0.221	0.166	0.170	0.336	0.306	0.312	0.286	0.210	0.288
					Model 2.2	2						
$Infeasible\ Estimation$												
Local Polynomial	0.14	0.2	0.262	0.303	0.200	0.229	0.224	0.238	0.288	0.349	0.390	0.555
B-splines	9	4	0.375	0.234	0.306	0.175	0.436	0.184	0.360	0.253	0.404	0.400
Partitioning	9	4	0.312	0.294	0.246	0.223	0.260	0.717	0.256	0.535	0.342	0.365
Feasible Estimation Local Polynomial	0.3	0.24	0.508	0.513	0.413	0.414	0.671	0.701	0.589	0.586	0.316	0.335
B-splines	4	4	0.339	0.234	0.265	0.175	0.196	0.184	0.221	0.253	0.407	0.400
Partitioning	4	4	0.381	0.294	0.306	0.224	0.755	0.717	0.342	0.535	0.474	0.365
3					Model 2.3							
Infeasible Estimation					Model 2.5	•						
Local Polynomial	0.18	0.41	0.209	0.194	0.155	0.145	0.146	0.119	0.238	0.185	0.385	0.294
B-splines	9	4	0.217	0.225	0.167	0.167	0.133	0.183	0.192	0.252	0.335	0.399
Partitioning	9	4	0.255	0.285	0.198	0.217	0.148	0.631	0.230	0.530	0.287	0.371
Feasible Estimation												
Local Polynomial	0.34	0.28	0.217	0.233	0.141	0.155	0.114	0.126	0.233	0.246	0.188	0.204
B-splines	3	2	0.195	0.218	0.144	0.164	0.177	0.165	0.229	0.220	0.244	0.381
Partitioning	3	2	0.208	0.233	0.157	0.173	0.228	0.407	0.278	0.375	0.194	0.333
					Model 2.4	Į.						
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.615	0.355	0.429	0.276	0.148	0.186	0.223	0.212	0.336	0.340
B-splines	9	9	0.679	0.392	0.509	0.308	0.157	0.276	0.214	0.254	0.435	0.483
Partitioning Feasible Estimation	9	9	0.630	0.458	0.477	0.352	0.171	0.294	0.263	0.386	0.345	0.476
Local Polynomial	0.22	0.28	0.604	0.516	0.485	0.411	0.362	0.365	0.547	0.344	0.734	0.953
B-splines	4	1	0.724	0.499	0.523	0.388	0.451	0.344	0.284	0.266	0.369	0.435
Partitioning	4	1	0.717	0.477	0.521	0.361	0.512	0.587	0.345	0.467	0.336	0.370
					Model 2.5							
Infeasible Estimation					Model 2.c	,						
Local Polynomial	0.21	0.9	0.189	0.149	0.143	0.118	0.178	0.163	0.208	0.140	0.345	0.183
B-splines	4	1	0.145	0.208	0.112	0.160	0.190	0.176	0.204	0.197	0.255	0.366
Partitioning	4	1	0.163	0.178	0.127	0.140	0.263	0.170	0.214	0.167	0.205	0.269
Feasible Estimation										0.440		
Local Polynomial	0.26	0.29	0.127	0.125	0.102	0.094	0.198	0.095	0.126	0.146	0.126	0.146
B-splines Partitioning	4	1 1	$0.145 \\ 0.163$	$0.210 \\ 0.185$	$0.112 \\ 0.127$	$0.160 \\ 0.144$	$0.190 \\ 0.263$	$0.176 \\ 0.203$	$0.204 \\ 0.214$	$0.203 \\ 0.200$	$0.255 \\ 0.205$	$0.367 \\ 0.272$
rantitioning	4	1	0.103	0.165			0.203	0.203	0.214	0.200	0.203	0.212
					Model 2.6	5						
Infeasible Estimation	0.0	0.0	0.000	0.100	0.064	0.007	0.046	0.007	0.000	0.107	0.100	0.101
Local Polynomial B-splines	$0.9 \\ 1$	0.9 1	0.083 0.090	0.126 $0.180$	$0.064 \\ 0.069$	0.097 $0.134$	$0.046 \\ 0.045$	$0.087 \\ 0.102$	0.088 $0.084$	0.137 $0.183$	0.128 $0.159$	0.181 $0.359$
Partitioning	1	1	0.078	0.142	0.061	0.134	0.045	0.102	0.084	0.153	0.108	0.359
Feasible Estimation	1	1	0.010	0.142	0.001	0.103	0.040	0.000	0.000	0.100	0.100	0.200
Local Polynomial	0.4	0.31	0.422	0.430	0.363	0.367	0.058	0.094	0.128	0.153	0.662	0.662
B-splines	1	1	0.094	0.180	0.071	0.134	0.057	0.102	0.098	0.183	0.168	0.359
Partitioning	1	1	0.086	0.142	0.065	0.109	0.083	0.088	0.102	0.153	0.115	0.263
					Model 2.7	,						
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.189	0.255	0.120	0.183	0.959	0.926	0.376	0.391	0.284	0.333
B-splines	9	9	0.224	0.298	0.157	0.218	0.960	0.938	0.383	0.406	0.331	0.446
Partitioning	9	9	0.267	0.440	0.199	0.335	0.944	0.891	0.402	0.480	0.275	0.466
Feasible Estimation	0.04	0.00	0.150	0.104	0.004	0.110	0.000	0.000	0.00=	0.000	0.100	0.140
Local Polynomial	0.34	0.28	0.159	0.184	0.084	0.113	0.982	0.986	0.367	0.380	0.122	0.146
B-splines Partitioning	2 2	1 1	$0.176 \\ 0.179$	$0.227 \\ 0.207$	$0.107 \\ 0.110$	0.156 $0.138$	$0.958 \\ 0.944$	$0.954 \\ 0.953$	0.384 $0.359$	$0.395 \\ 0.402$	$0.210 \\ 0.158$	$0.361 \\ 0.274$
1 areitioning	4	1	0.119	0.201			0.344	0.900	0.339	0.402	0.136	0.214
T. C					Model 2.8	3						
Infeasible Estimation	0.24	0.0	0.126	0.100	0.100	0.100	0.077	0.002	0.159	0.120	0.000	0.101
Local Polynomial B-splines	0.34 1	0.9 1	$0.136 \\ 0.102$	0.129 $0.180$	$0.100 \\ 0.078$	$0.100 \\ 0.134$	$0.077 \\ 0.052$	$0.093 \\ 0.104$	$0.153 \\ 0.087$	$0.138 \\ 0.183$	0.283 $0.159$	$0.181 \\ 0.358$
Partitioning	1	1	0.102	0.180 $0.144$	0.071	0.134	0.052 $0.052$	0.104 $0.095$	0.087	0.183 $0.154$	0.110	0.338
Feasible Estimation	1	1	5.032	0.144	0.011	0.110	0.002	0.030	0.000	0.104	0.110	0.209
Local Polynomial	0.34	0.28	0.094	0.132	0.071	0.099	0.060	0.091	0.121	0.146	0.122	0.146
B-splines	3	1	0.125	0.186	0.094	0.138	0.122	0.112	0.169	0.194	0.221	0.361
Partitioning	3	1	0.135	0.165	0.101	0.122	0.190	0.204	0.175	0.234	0.169	0.281

Table C.39: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=500, \sigma^2=1, X_{i,\ell}\sim\beta(2,2),$  Quantile Cells

	Tur Parai		Root Int		Ingeti MA		(0.5,		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.17	0.24	0.232	0.249	0.150	0.179	0.151	0.161	0.336	0.335	1 205	1.400
Local Polynomial B-splines	9.17	0.24	0.232 $0.234$	0.249	$0.159 \\ 0.184$	0.179 $0.170$	$0.151 \\ 0.231$	$0.161 \\ 0.187$	0.336	0.335 $0.375$	$\frac{1.295}{0.517}$	1.400
Partitioning	9	4	0.248	0.286	0.193	0.214	0.166	0.556	0.301	0.672	0.385	0.792
Feasible Estimation	Ü	-	0.210	0.200	0.100	0.211	0.100	0.000	0.001	0.012	0.000	0.102
Local Polynomial	0.27	0.26	0.236	0.237	0.191	0.191	0.058	0.113	0.172	0.204	0.176	0.198
B-splines	4	2	0.207	0.239	0.162	0.182	0.316	0.223	0.340	0.340	0.369	0.937
Partitioning	4	2	0.217	0.247	0.169	0.189	0.354	0.383	0.396	0.448	0.263	0.601
					Model 2.2							
$Infeasible\ Estimation$												
Local Polynomial	0.14	0.2	0.259	0.291	0.186	0.206	0.180	0.169	0.379	0.425	4.109	7.040
B-splines	9	4	0.311	0.229	0.243	0.163	0.304	0.144	0.257	0.366	0.753	1.157
Partitioning Feasible Estimation	9	4	0.301	0.290	0.234	0.217	0.189	0.586	0.292	0.680	0.662	0.799
Local Polynomial	0.25	0.23	0.516	0.521	0.422	0.425	0.605	0.611	0.516	0.526	0.420	0.437
B-splines	4	4	0.292	0.229	0.422	0.423	0.172	0.144	0.341	0.366	0.708	1.157
Partitioning	4	4	0.347	0.290	0.269	0.217	0.580	0.586	0.424	0.680	0.697	0.801
3												
Infeasible Estimation					Model 2.3							
Local Polynomial	0.19	0.45	0.189	0.170	0.131	0.122	0.097	0.093	0.295	0.230	0.817	0.440
B-splines	9	4	0.202	0.224	0.150	0.158	0.106	0.144	0.267	0.366	0.520	1.156
Partitioning	9	4	0.245	0.284	0.189	0.212	0.144	0.539	0.318	0.677	0.366	0.801
Feasible Estimation												
Local Polynomial	0.33	0.27	0.156	0.176	0.107	0.122	0.095	0.100	0.246	0.254	0.207	0.224
B-splines	3	3	0.165	0.215	0.118	0.152	0.138	0.127	0.275	0.338	0.344	1.003
Partitioning	3	3	0.176	0.242	0.129	0.174	0.196	0.415	0.286	0.526	0.239	0.686
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.501	0.342	0.326	0.262	0.137	0.150	0.286	0.263	0.588	0.600
B-splines	9	9	0.569	0.399	0.379	0.309	0.098	0.245	0.431	0.475	0.736	2.468
Partitioning Feasible Estimation	9	9	0.571	0.463	0.401	0.354	0.152	0.294	0.475	0.591	0.550	1.355
Local Polynomial	0.23	0.28	0.529	0.459	0.400	0.350	0.284	0.279	0.345	0.326	0.991	1.024
B-splines	4	2	0.575	0.430	0.363	0.337	0.252	0.280	0.487	0.396	0.604	1.221
Partitioning	4	2	0.578	0.390	0.376	0.300	0.311	0.580	0.524	0.703	0.505	0.843
_					Model 2.5							
Infeasible Estimation					Woder 2.5							
Local Polynomial	0.21	0.9	0.188	0.141	0.136	0.109	0.147	0.122	0.276	0.196	0.688	0.262
B-splines	4	1	0.144	0.201	0.109	0.146	0.181	0.135	0.223	0.298	0.372	0.809
Partitioning	4	1	0.162	0.170	0.126	0.127	0.249	0.126	0.226	0.246	0.250	0.426
Feasible Estimation												
Local Polynomial	0.25	0.28	0.126	0.122	0.103	0.088	0.172	0.081	0.169	0.193	0.169	0.193
B-splines	4	1	0.144	0.206	0.109	0.149	0.181	0.133	0.223	0.312	0.372	0.876
Partitioning	4	1	0.162	0.195	0.126	0.142	0.249	0.239	0.226	0.343	0.250	0.525
					Model 2.6	;						
Infeasible Estimation	0.0	0.0	0.001	0.100	0.000	0.001	0.040	0.074	0.00=	0.150	0.100	0.044
Local Polynomial	0.9	0.9	0.081	0.122	0.062	0.091	0.043	0.074	0.097	0.172	0.136	0.244
B-splines Partitioning	$\frac{1}{1}$	1 1	$0.090 \\ 0.077$	$0.178 \\ 0.141$	$0.067 \\ 0.059$	$0.126 \\ 0.104$	0.043 $0.043$	$0.085 \\ 0.073$	$0.094 \\ 0.094$	$0.250 \\ 0.187$	$0.194 \\ 0.122$	0.788 $0.394$
Feasible Estimation	1	1	0.011	0.141	0.059	0.104	0.043	0.073	0.034	0.101	0.122	0.334
Local Polynomial	0.39	0.3	0.356	0.366	0.297	0.304	0.052	0.083	0.159	0.207	0.658	0.667
B-splines	1	1	0.099	0.179	0.072	0.126	0.064	0.086	0.124	0.250	0.246	0.789
Partitioning	1	1	0.093	0.143	0.068	0.104	0.095	0.079	0.124	0.193	0.157	0.398
					Model 2.7							
Infeasible Estimation					1110401 211							
Local Polynomial	0.33	0.33	0.199	0.250	0.120	0.171	0.951	0.914	0.393	0.414	0.413	0.587
B-splines	9	9	0.233	0.306	0.160	0.216	0.943	0.918	0.401	0.504	0.512	2.002
Partitioning	9	9	0.273	0.442	0.203	0.335	0.918	0.827	0.443	0.656	0.361	1.323
Feasible Estimation												
Local Polynomial	0.33	0.27	0.176	0.198	0.091	0.117	0.969	0.972	0.387	0.407	0.159	0.194
B-splines Partitioning	3	2	0.195	0.246	0.121	0.165	0.932	0.934	0.391	0.454	0.337	0.913
rarmoning	3	2	0.201	0.246	0.130	0.166	0.901	0.910	0.357	0.511	0.230	0.567
					Model 2.8							
Infeasible Estimation	0.00	0.0	0.105	0.104	0.000	0.000	0.000	0.050	0.155	0.170	0.050	0.041
Local Polynomial B-splines	0.36	0.9	$0.125 \\ 0.098$	$0.124 \\ 0.179$	$0.089 \\ 0.073$	$0.093 \\ 0.127$	$0.060 \\ 0.053$	$0.078 \\ 0.087$	$0.175 \\ 0.101$	0.172	0.378	0.244 $0.788$
Partitioning	1 1	1 1	0.098	0.179 $0.142$	0.073	0.127	0.053 $0.052$	0.087	0.101	$0.250 \\ 0.187$	$0.195 \\ 0.125$	0.788
Feasible Estimation	1	1	0.000	0.142	0.000	0.100	0.052	0.079	0.101	0.107	0.120	0.400
Local Polynomial	0.33	0.27	0.089	0.127	0.064	0.090	0.055	0.080	0.157	0.194	0.157	0.193
B-splines	3	2	0.127	0.195	0.093	0.137	0.120	0.104	0.199	0.292	0.332	0.911
Partitioning	3	2	0.139	0.200	0.104	0.139	0.181	0.320	0.201	0.390	0.223	0.560

Table C.40: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=4,\ X_{i,\ell}\sim\beta(0.5,0.5),\ {\rm Quantile\ Cells}$ 

		ning neter	Root Int		Ingeti MA		(0.5,		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.23	0.29	0.345	0.530	0.264	0.405	0.281	0.383	0.363	0.501	0.462	0.668
B-splines	0.23	0.29	0.345 $0.294$	0.330 $0.451$	0.204	0.405 $0.346$	0.281	0.363 $0.442$	0.373	0.301 $0.454$	0.462	0.484
Partitioning	4	4	0.334	0.568	0.260	0.432	0.551	1.551	0.467	0.918	0.324	0.484
Feasible Estimation												
Local Polynomial	0.3	0.26	0.253	0.304	0.196	0.240	0.142	0.244	0.219	0.270	0.214	0.265
B-splines	3	2	0.292	0.409	0.225	0.313	0.467	0.377	0.341	0.390	0.348	0.471
Partitioning	3	2	0.322	0.432	0.248	0.316	0.503	0.931	0.424	0.571	0.309	0.419
T. C. ST. T. C. C.					Model 2.2							
Infeasible Estimation Local Polynomial	0.17	0.23	0.437	0.549	0.337	0.423	0.382	0.479	0.463	0.566	0.513	0.673
B-splines	9	4	0.507	0.452	0.409	0.425	0.547	0.409	0.495	0.452	0.470	0.484
Partitioning	9	4	0.511	0.573	0.400	0.436	0.408	1.609	0.422	0.922	0.427	0.485
Feasible Estimation												
Local Polynomial	0.31	0.25	0.510	0.538	0.408	0.426	0.721	0.783	0.632	0.658	0.340	0.369
B-splines	$\frac{4}{4}$	4	0.426	0.456	$0.338 \\ 0.374$	0.351	0.400	0.422	0.388	0.446	$0.408 \\ 0.422$	0.483
Partitioning	4	4	0.470	0.565		0.432	0.947	1.535	0.491	0.865	0.422	0.484
I-f:LI- F-t:t:					Model 2.3							
Infeasible Estimation Local Polynomial	0.21	0.46	0.363	0.360	0.275	0.275	0.271	0.220	0.390	0.304	0.481	0.387
B-splines	4	4	0.310	0.447	0.238	0.342	0.383	0.408	0.369	0.452	0.362	0.483
Partitioning	4	4	0.350	0.568	0.273	0.432	0.498	1.539	0.474	0.918	0.319	0.484
$Feasible\ Estimation$												
Local Polynomial	0.31	0.27	0.300	0.348	0.206	0.251	0.161	0.227	0.306	0.343	0.268	0.310
B-splines Partitioning	3 3	$\frac{2}{2}$	$0.306 \\ 0.339$	0.411 $0.451$	0.233 $0.261$	0.313 $0.329$	0.355	0.332 $0.973$	$0.370 \\ 0.461$	$0.400 \\ 0.636$	$0.350 \\ 0.301$	$0.474 \\ 0.440$
rartitioning	3	2	0.559	0.451			0.460	0.973	0.401	0.030	0.301	0.440
Infeasible Estimation					Model 2.4							
Local Polynomial	0.33	0.33	0.707	0.529	0.523	0.415	0.205	0.336	0.410	0.406	0.449	0.533
B-splines	9	9	0.758	0.599	0.605	0.464	0.246	0.399	0.355	0.434	0.496	0.515
Partitioning	9	9	0.713	0.858	0.559	0.657	0.303	0.599	0.427	0.640	0.450	0.671
Feasible Estimation												
Local Polynomial	0.23	0.27	0.685	0.581	0.554	0.469	0.432	0.448	0.671	0.440	0.649	0.939
B-splines Partitioning	4 4	$\frac{2}{2}$	$0.830 \\ 0.834$	$0.661 \\ 0.672$	$0.639 \\ 0.648$	0.523 $0.533$	$0.687 \\ 0.812$	0.487 $1.075$	0.387 $0.450$	$0.426 \\ 0.654$	0.389 $0.345$	0.499 $0.440$
1 artitioning	4	2	0.034	0.012			0.012	1.075	0.450	0.054	0.545	0.440
Infeasible Estimation					Model 2.5							
Local Polynomial	0.27	0.9	0.314	0.265	0.241	0.209	0.297	0.241	0.323	0.239	0.433	0.288
B-splines	4	1	0.272	0.373	0.209	0.287	0.365	0.281	0.367	0.333	0.362	0.462
Partitioning	4	1	0.312	0.302	0.244	0.235	0.500	0.246	0.426	0.284	0.318	0.370
Feasible Estimation	0.05	0.07	0.105	0.044	0.150	0.100	0.050	0.010	0.000	0.000	0.000	0.000
Local Polynomial B-splines	0.27 $4$	0.27 2	0.195 $0.279$	0.244 $0.398$	$0.152 \\ 0.214$	0.188 $0.303$	0.258 $0.381$	0.210 $0.319$	0.209 $0.373$	$0.260 \\ 0.381$	$0.209 \\ 0.369$	$0.260 \\ 0.468$
Partitioning	4	2	0.279	0.398	0.214	0.303 $0.294$	0.509	0.805	0.373	0.523	0.326	0.408
					Model 2.6							
Infeasible Estimation					Model 2.0							
Local Polynomial	0.9	0.9	0.166	0.252	0.129	0.198	0.088	0.183	0.161	0.238	0.221	0.288
B-splines	1	1	0.177	0.357	0.136	0.273	0.087	0.228	0.153	0.329	0.258	0.460
Partitioning	1	1	0.153	0.281	0.119	0.217	0.087	0.183	0.153	0.280	0.193	0.369
Feasible Estimation	0.24	0.00	0.400	0.519	0.416	0.422	0.100	0.010	0.010	0.000	0.671	0.007
Local Polynomial B-splines	0.34 2	0.28	$0.482 \\ 0.226$	0.513 $0.365$	$0.416 \\ 0.169$	$0.433 \\ 0.278$	$0.126 \\ 0.242$	$0.213 \\ 0.240$	0.213 $0.284$	$0.263 \\ 0.340$	$0.671 \\ 0.320$	$0.697 \\ 0.463$
Partitioning	2	1	0.241	0.315	0.176	0.235	0.335	0.426	0.319	0.337	0.263	0.378
G G					Model 2.7							
Infeasible Estimation					Woder 2.7							
Local Polynomial	0.33	0.33	0.296	0.468	0.217	0.358	0.970	0.970	0.456	0.519	0.416	0.531
B-splines	9	9	0.377	0.549	0.286	0.418	0.978	0.996	0.467	0.549	0.463	0.509
Partitioning	9	9	0.478	0.854	0.371	0.654	0.982	1.082	0.514	0.711	0.425	0.673
Feasible Estimation Local Polynomial	0.31	0.27	0.207	0.274	0.140	0.201	0.985	0.997	0.411	0.435	0.208	0.261
B-splines	0.31	0.27	0.207 $0.274$	0.274	0.140	0.201 $0.304$	0.985	0.997 $0.982$	$0.411 \\ 0.474$	$0.435 \\ 0.520$	0.208	0.466
Partitioning	3	2	0.298	0.406	0.216	0.290	0.992	1.194	0.489	0.620	0.291	0.406
0	-				Model 2.8			-			- '	
Infeasible Estimation					model 2.c							
Local Polynomial	0.42	0.9	0.239	0.254	0.180	0.199	0.129	0.189	0.237	0.238	0.363	0.288
B-splines	1	1	0.185	0.357	0.142	0.273	0.091	0.229	0.154	0.329	0.259	0.460
Partitioning	1	1	0.162	0.282	0.126	0.218	0.091	0.190	0.154	0.282	0.193	0.370
Feasible Estimation Local Polynomial	0.31	0.27	0.173	0.249	0.132	0.192	0.129	0.210	0.209	0.261	0.208	0.261
B-splines	0.31	0.27	0.173 $0.251$	0.249	0.132	0.192 $0.297$	0.129	0.210	0.209	$0.261 \\ 0.385$	0.208	0.261
Partitioning	3	2	0.282	0.408	0.213	0.293	0.424	0.859	0.379	0.551	0.333	0.411

Table C.41: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=4,\ X_{i,\ell}\sim\beta(1,1),\ {\rm Quantile\ Cells}$ 

	Tur Parai	ning meter	Root Int		Ingeti MA		(0.5,		oint Estima (0.1,		SE (0.1	,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubi
					Model 2.1							
Infeasible Estimation	0.22	0.28	0.358	0.540	0.268	0.400	0.264	0.350	0.404	0.528	0.676	0.88
Local Polynomial B-splines	0.22	0.28	0.304	0.340 $0.455$	0.235	0.400	0.264 $0.474$	0.399	0.404 $0.427$	0.528	0.506	0.88
Partitioning	4	4	0.340	0.568	0.265	0.431	0.539	1.264	0.507	1.047	0.411	0.73
Feasible Estimation	-	_	0.0.0	0.000	0.200		0.000				*****	
Local Polynomial	0.3	0.26	0.269	0.315	0.211	0.248	0.129	0.222	0.253	0.300	0.257	0.29
B-splines	3	2	0.304	0.415	0.236	0.313	0.442	0.344	0.402	0.430	0.493	0.74
Partitioning	3	2	0.332	0.437	0.258	0.322	0.506	0.784	0.477	0.718	0.393	0.61
					Model 2.2							
Infeasible Estimation	0.15	0.00	0.400	0.543	0.000	0.400	0.045	0.400	0.501	0.500	0.700	0.05
Local Polynomial B-splines	0.17 9	$0.23 \\ 4$	$0.436 \\ 0.488$	$0.541 \\ 0.454$	$0.329 \\ 0.389$	$0.408 \\ 0.338$	$0.345 \\ 0.464$	$0.408 \\ 0.366$	$0.501 \\ 0.461$	$0.582 \\ 0.505$	$0.762 \\ 0.710$	0.85
Partitioning	9	4	0.511	0.571	0.399	0.434	0.359	1.310	0.459	1.054	0.593	0.72
Feasible Estimation	,	4	0.011	0.011	0.000	0.404	0.000	1.010	0.400	1.004	0.000	0.12
Local Polynomial	0.3	0.25	0.529	0.556	0.429	0.447	0.677	0.717	0.633	0.639	0.376	0.41
B-splines	4	4	0.413	0.457	0.322	0.342	0.345	0.375	0.419	0.491	0.606	0.79
Partitioning	4	4	0.467	0.565	0.370	0.431	0.849	1.230	0.498	0.993	0.597	0.74
					Model 2.3							
Infeasible Estimation												
Local Polynomial	0.22	0.47	0.351	0.342	0.259	0.256	0.228	0.201	0.404	0.325	0.684	0.51
B-splines	4	1	0.305	0.377	0.231	0.281	0.340	0.231	0.413	0.374	0.509	0.71
Partitioning Feasible Estimation	4	1	0.343	0.311	0.266	0.236	0.472	0.186	0.455	0.338	0.408	0.54
Local Polynomial	0.32	0.27	0.262	0.317	0.181	0.227	0.157	0.207	0.312	0.355	0.281	0.32
B-splines	3	2	0.297	0.415	0.223	0.307	0.307	0.303	0.402	0.440	0.488	0.7
Partitioning	3	2	0.327	0.454	0.249	0.329	0.439	0.867	0.441	0.787	0.388	0.63
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.659	0.523	0.472	0.404	0.197	0.299	0.347	0.420	0.596	0.67
B-splines	9	9	0.747	0.609	0.568	0.467	0.225	0.383	0.358	0.478	0.721	0.9
Partitioning	9	9	0.749	0.867	0.583	0.663	0.303	0.588	0.462	0.763	0.590	0.93
Feasible Estimation	0.00	0.07	0.050	0.500	0.515	0.444	0.000	0.401	0 5 4 5	0.400	0.014	0.0
Local Polynomial B-splines	0.23 $4$	$0.27 \\ 2$	$0.652 \\ 0.765$	$0.560 \\ 0.635$	0.517 $0.555$	$0.444 \\ 0.491$	$0.392 \\ 0.540$	$0.401 \\ 0.436$	$0.545 \\ 0.449$	$0.428 \\ 0.481$	0.814 $0.575$	0.98
Partitioning	4	2	0.774	0.649	0.575	0.502	0.654	0.994	0.516	0.451	0.491	0.67
Infeasible Estimation					Model 2.5							
Local Polynomial	0.27	0.9	0.320	0.264	0.241	0.206	0.273	0.224	0.353	0.275	0.612	0.36
B-splines	4	1	0.276	0.374	0.210	0.281	0.336	0.250	0.406	0.374	0.508	0.7
Partitioning	4	1	0.315	0.304	0.245	0.234	0.479	0.231	0.427	0.314	0.408	0.52
Feasible Estimation												
Local Polynomial	0.27	0.27	0.204	0.249	0.161	0.187	0.252	0.187	0.251	0.294	0.251	0.29
B-splines Partitioning	4	$\frac{2}{2}$	0.281 $0.319$	$0.400 \\ 0.406$	$0.214 \\ 0.248$	$0.298 \\ 0.295$	0.343 $0.484$	0.284 $0.698$	$0.413 \\ 0.432$	$0.423 \\ 0.663$	$0.510 \\ 0.413$	0.74 $0.59$
1 artitioning	4	2	0.313	0.400			0.404	0.030	0.432	0.003	0.415	0.5.
r f - 21 - Fi - C					Model 2.6							
Infeasible Estimation Local Polynomial	0.9	0.9	0.166	0.251	0.128	0.194	0.092	0.174	0.177	0.274	0.256	0.30
B-splines	1	1	0.180	0.359	0.123	0.154	0.092	0.204	0.169	0.367	0.230	0.7
Partitioning	1	1	0.156	0.285	0.122	0.217	0.090	0.176	0.169	0.307	0.217	0.52
Feasible Estimation												
Local Polynomial	0.35	0.28	0.446	0.481	0.377	0.399	0.118	0.185	0.249	0.298	0.701	0.7
B-splines	2	1	0.233	0.370	0.172	0.274	0.228	0.221	0.317	0.388	0.426	0.72
Partitioning	2	1	0.248	0.328	0.182	0.241	0.347	0.400	0.330	0.464	0.324	0.5
					Model 2.7							
Infeasible Estimation	0.00	0.00	0.001	0.401	0.010	0.944	0.000	0.050	0.401	0.505	0.500	
Local Polynomial	0.33	0.33	0.301	0.461	0.216	0.344	0.969	0.956	0.461	0.537	0.568	0.60
B-splines Partitioning	9 9	9	0.384 $0.485$	0.553 $0.858$	$0.290 \\ 0.376$	0.413 $0.654$	0.974 $0.980$	0.974 $1.028$	$0.479 \\ 0.555$	$0.572 \\ 0.810$	$0.661 \\ 0.550$	0.9
Feasible Estimation	9	3	0.400	0.000	0.570	0.054	0.560	1.020	0.555	0.010	0.550	0.5
Local Polynomial	0.32	0.27	0.217	0.283	0.145	0.203	0.990	1.003	0.420	0.458	0.247	0.2
B-splines	3	2	0.286	0.415	0.206	0.306	0.973	0.970	0.507	0.551	0.478	0.74
Partitioning	3	2	0.311	0.426	0.228	0.305	0.993	1.137	0.479	0.748	0.376	0.6
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.42	0.9	0.235	0.253	0.174	0.196	0.121	0.177	0.251	0.275	0.480	0.3
B-splines	1	1	0.187	0.359	0.142	0.267	0.094	0.204	0.170	0.367	0.318	0.7
Partitioning	1	1	0.163	0.286	0.127	0.218	0.094	0.179	0.171	0.307	0.217	0.5
Feasible Estimation Local Polynomial	0.32	0.27	0.175	0.252	0.132	0.190	0.121	0.186	0.247	0.293	0.247	0.2
B-splines	3	2	0.175	0.232	0.132	0.190	0.121	0.186	0.372	0.293 $0.431$	0.483	0.2
Partitioning	3	2	0.289	0.417	0.219	0.298	0.423	0.749	0.387	0.704	0.378	0.6

Table C.42: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2,\ n=500,\ \sigma^2=4,\ X_{i,\ell}\sim\beta(2,2),\ {\rm Quantile\ Cells}$ 

	Tur Parai	ning neter	Root Int MS		Ingeti MA		(0.5,		oint Estima (0.1,			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubi
					Model 2.1							
Infeasible Estimation	0.01	0.05	0.000	0.450	0.001	0.004	0.000	0.000	0.554	0.505	1.004	
Local Polynomial	0.21	0.27	0.366	0.453	0.261	0.324	0.220	0.268	0.554	0.587	1.384	1.54
B-splines Partitioning	4 4	$rac{4}{4}$	$0.309 \\ 0.342$	$0.453 \\ 0.567$	$0.236 \\ 0.266$	0.323 $0.424$	$0.405 \\ 0.514$	0.311 $1.086$	$0.515 \\ 0.562$	0.736 $1.342$	$0.738 \\ 0.503$	2.31 1.57
Feasible Estimation	4	4	0.342	0.567	0.200	0.424	0.514	1.080	0.362	1.342	0.505	1.57
Local Polynomial	0.29	0.26	0.279	0.320	0.223	0.247	0.112	0.183	0.332	0.398	0.334	0.39
B-splines	4	3	0.310	0.434	0.223	0.313	0.392	0.304	0.499	0.672	0.724	2.06
Partitioning	4	3	0.340	0.495	0.264	0.362	0.502	0.859	0.542	1.094	0.500	1.35
Infeasible Estimation					Model 2.2							
Local Polynomial	0.17	0.23	0.423	0.511	0.303	0.365	0.277	0.290	0.654	0.702	6.256	5.06
B-splines	9	4	0.438	0.450	0.336	0.319	0.339	0.286	0.441	0.731	1.171	2.31
Partitioning	9	4	0.501	0.569	0.389	0.425	0.309	1.103	0.576	1.346	0.908	1.57
Feasible Estimation	_	_	0.00-	0.000	0.000	0	0.000		0.0.0		0.000	
Local Polynomial	0.27	0.24	0.536	0.563	0.436	0.453	0.612	0.628	0.589	0.617	0.506	0.55
B-splines	4	4	0.372	0.450	0.279	0.320	0.287	0.292	0.542	0.721	0.937	2.23
Partitioning	4	4	0.441	0.564	0.340	0.422	0.714	1.080	0.595	1.307	0.807	1.56
					Model 2.3							
Infeasible Estimation					Model 2.c							
Local Polynomial	0.25	0.51	0.317	0.304	0.221	0.219	0.156	0.161	0.485	0.415	1.092	0.75
B-splines	4	1	0.288	0.368	0.212	0.261	0.285	0.185	0.471	0.520	0.740	1.57
Partitioning	4	1	0.325	0.298	0.251	0.218	0.420	0.157	0.486	0.415	0.504	0.79
Feasible Estimation												
Local Polynomial	0.31	0.26	0.214	0.277	0.151	0.197	0.132	0.173	0.373	0.424	0.348	0.40
B-splines	3	3	0.281	0.424	0.205	0.299	0.268	0.254	0.458	0.682	0.710	2.06
Partitioning	3	3	0.311	0.495	0.235	0.355	0.396	0.886	0.466	1.114	0.479	1.40
					Model 2.4							
Infeasible Estimation					1110401 211							
Local Polynomial	0.33	0.33	0.549	0.492	0.370	0.368	0.169	0.229	0.430	0.520	0.931	1.18
B-splines	9	9	0.647	0.613	0.454	0.454	0.176	0.327	0.557	0.820	1.163	4.24
Partitioning	9	9	0.697	0.868	0.523	0.660	0.290	0.588	0.678	1.179	0.832	2.67
Feasible Estimation												
Local Polynomial	0.25	0.27	0.573	0.506	0.431	0.387	0.303	0.311	0.398	0.475	1.125	1.07
B-splines	4	3	0.621	0.581	0.410	0.436	0.354	0.349	0.613	0.735	0.875	2.26
Partitioning	4	3	0.638	0.613	0.442	0.462	0.480	1.010	0.650	1.245	0.660	1.54
					Model 2.5	i						
Infeasible Estimation												
Local Polynomial	0.26	0.9	0.314	0.254	0.228	0.192	0.225	0.175	0.454	0.357	1.003	0.49
B-splines	4	1	0.274	0.369	0.204	0.263	0.304	0.199	0.443	0.527	0.740	1.57
Partitioning	4	1	0.312	0.298	0.243	0.220	0.440	0.177	0.451	0.409	0.497	0.80
Feasible Estimation												
Local Polynomial	0.27	0.26	0.201	0.248	0.156	0.177	0.214	0.163	0.333	0.390	0.333	0.39
B-splines	4	2	0.276	0.414	0.206	0.292	0.307	0.244	0.447	0.648	0.740	2.0
Partitioning	4	2	0.313	0.461	0.244	0.327	0.442	0.784	0.454	0.995	0.502	1.30
					Model 2.6	;						
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.161	0.243	0.123	0.183	0.087	0.147	0.193	0.343	0.273	0.48
B-splines	1	1	0.179	0.357	0.133	0.252	0.087	0.171	0.189	0.499	0.389	1.57
Partitioning	1	1	0.153	0.283	0.118	0.207	0.086	0.147	0.189	0.374	0.244	0.78
Feasible Estimation	0.00	0.07	0.004	0.404	0.010	0.041	0.105	0.100	0.004	0.005	0.710	c =
Local Polynomial	0.33	0.27	0.384	0.424	0.316	0.341	0.105	0.163	0.324	0.395	0.710	0.74
B-splines	3	2	0.247	0.388	0.179	0.272	0.231	0.207	0.395	0.578	0.667	1.82
Partitioning	3	2	0.269	0.397	0.200	0.275	0.349	0.632	0.392	0.766	0.441	1.13
					Model 2.7	•						
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.300	0.432	0.206	0.308	0.954	0.929	0.512	0.613	0.825	1.1'
B-splines	9	9	0.386	0.556	0.286	0.400	0.952	0.941	0.542	0.833	1.023	4.00
Partitioning	9	9	0.485	0.858	0.375	0.650	0.955	0.965	0.672	1.216	0.722	2.6
Feasible Estimation	0.91	0.00	0.000	0.000	0.147	0.000	0.070	0.000	0.400	0.504	0.204	0.00
Local Polynomial B-splines	0.31	$0.26 \\ 3$	$0.228 \\ 0.300$	$0.292 \\ 0.443$	$0.147 \\ 0.213$	$0.200 \\ 0.313$	$0.972 \\ 0.947$	0.982 $0.945$	$0.483 \\ 0.543$	$0.534 \\ 0.747$	$0.324 \\ 0.705$	0.39 2.05
Partitioning	3	3	0.300 $0.326$	0.443 $0.498$	0.213 $0.241$	0.313 $0.355$	0.947	0.945 $1.135$	0.543 $0.512$	$\frac{0.747}{1.117}$	$0.705 \\ 0.477$	1.35
1 ai titioning	3	3	0.320	0.430			0.901	1.130	0.012	1.111	0.411	1.36
					Model 2.8	;						
Infeasible Estimation												_
Local Polynomial	0.45	0.9	0.216	0.244	0.156	0.184	0.100	0.150	0.280	0.343	0.573	0.48
B-splines	1	1	0.183	0.357	0.136	0.253	0.092	0.171	0.191	0.499	0.388	1.5
Partitioning	1	1	0.158	0.283	0.122	0.208	0.091	0.150	0.191	0.374	0.245	0.79
Feasible Estimation	0.91	0.00	0.171	0.050	0.100	0.170	0.100	0.100	0.202	0.200	0.000	0.0
Local Polynomial	0.31	0.26	0.171	0.250	0.123	0.178	0.108	0.163	0.323	0.390	0.323	0.38
B-splines	3 3	3	0.261	0.416	0.191	0.292	0.258	0.244	0.424	0.654 $1.059$	0.705	2.04 1.33

Table C.43: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=1, X_{i,\ell}\sim\beta(0.5,0.5),$  Quantile Cells

	Tur Parai		Root Int		Ingeti MA		(0.5,		oint Estim (0.1			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation Local Polynomial	0.17	0.23	0.161	0.191	0.124	0.148	0.167	0.184	0.157	0.193	0.177	0.220
B-splines	9	0.23	0.101 $0.177$	0.191	0.124	0.148 $0.134$	0.183	0.184 $0.232$	0.109	0.193	0.165	0.220
Partitioning	9	4	0.184	0.205	0.142	0.156	0.184	0.532	0.127	0.327	0.154	0.168
Feasible Estimation												
Local Polynomial	0.28	0.27	0.199	0.201	0.146	0.155	0.084	0.149	0.096	0.113	0.084	0.096
B-splines	4	1	0.166	0.177	0.124	0.138	0.384	0.243	0.158	0.125	0.141	0.162
Partitioning	4	1	0.177	0.175	0.133	0.134	0.306	0.213	0.248	0.168	0.138	0.159
					Model 2.2							
Infeasible Estimation Local Polynomial	0.12	0.18	0.206	0.233	0.161	0.180	0.202	0.213	0.196	0.273	0.182	0.323
B-splines	16	4	0.195	0.233 $0.174$	0.151	0.134	0.202	0.213	0.190	0.157	0.182	0.169
Partitioning	16	4	0.253	0.218	0.196	0.166	0.542	0.658	0.318	0.333	0.179	0.170
Feasible Estimation												
Local Polynomial	0.32	0.23	0.487	0.489	0.390	0.387	0.703	0.747	0.602	0.604	0.277	0.285
B-splines	4	4	0.344	0.174	0.276	0.134	0.202	0.148	0.159	0.157	0.228	0.169
Partitioning	4	4	0.373	0.218	0.304	0.166	0.850	0.658	0.278	0.333	0.309	0.170
					Model 2.3							
Infeasible Estimation	0.15	0.27	0.169	0.156	0.120	0.120	0.141	0.106	0.169	0.199	0.196	0.174
Local Polynomial B-splines	0.15 16	$0.37 \\ 4$	$0.168 \\ 0.169$	0.156 $0.160$	0.129 $0.130$	$0.120 \\ 0.122$	$0.141 \\ 0.210$	$0.106 \\ 0.147$	0.168 $0.204$	0.133 $0.157$	0.186 $0.179$	0.174 $0.167$
Partitioning	16	4	0.229	0.205	0.178	0.122	0.330	0.532	0.277	0.327	0.173	0.169
Feasible Estimation												
Local Polynomial	0.34	0.28	0.259	0.265	0.164	0.172	0.114	0.121	0.232	0.240	0.179	0.189
B-splines	4	3	0.187	0.167	0.145	0.129	0.205	0.148	0.140	0.146	0.140	0.165
Partitioning	4	3	0.200	0.196	0.155	0.149	0.199	0.427	0.243	0.280	0.128	0.179
					Model 2.4							
Infeasible Estimation	0.00	0.00	0.000	0.010	0.450	0.040	0.140	0.155	0.000	0.151	0.105	0.10
Local Polynomial B-splines	0.33 9	0.33 9	0.666	0.318	0.479	0.242	0.140	0.175	0.306	0.151	0.197	0.197
Partitioning	9	9	$0.690 \\ 0.577$	0.332 $0.332$	0.553 $0.445$	$0.258 \\ 0.253$	0.167 $0.163$	$0.240 \\ 0.189$	$0.207 \\ 0.231$	$0.163 \\ 0.232$	0.199 $0.187$	0.182 $0.223$
Feasible Estimation	9	J	0.011	0.002	0.440	0.200	0.100	0.100	0.201	0.202	0.101	0.220
Local Polynomial	0.19	0.27	0.610	0.530	0.509	0.438	0.394	0.394	0.667	0.363	0.588	0.902
B-splines	4	1	0.699	0.409	0.559	0.322	0.247	0.387	0.199	0.188	0.196	0.185
Partitioning	4	1	0.599	0.360	0.460	0.274	0.269	0.582	0.225	0.342	0.183	0.173
					Model 2.5							
Infeasible Estimation												
Local Polynomial B-splines	$0.19 \\ 4$	$0.9 \\ 1$	$0.145 \\ 0.107$	0.123 $0.169$	$0.112 \\ 0.084$	0.099 $0.135$	$0.156 \\ 0.168$	$0.170 \\ 0.186$	$0.143 \\ 0.135$	$0.085 \\ 0.119$	$0.174 \\ 0.137$	$0.108 \\ 0.166$
Partitioning	4	1	0.119	0.150	0.084	0.133 $0.122$	0.108	0.179	0.156	0.113	0.137	0.145
Feasible Estimation	-	-	0.113	0.100	0.030	0.122	0.210	0.175	0.100	0.100	0.122	0.140
Local Polynomial	0.23	0.29	0.097	0.088	0.077	0.068	0.173	0.078	0.075	0.089	0.075	0.089
B-splines	4	1	0.107	0.169	0.084	0.134	0.168	0.185	0.135	0.119	0.137	0.166
Partitioning	4	1	0.119	0.153	0.093	0.123	0.218	0.204	0.156	0.115	0.122	0.147
					Model 2.6	;						
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.060	0.090	0.046	0.071	0.033	0.066	0.057	0.084	0.083	0.107
B-splines Partitioning	1 1	1 1	$0.063 \\ 0.055$	$0.127 \\ 0.101$	$0.048 \\ 0.043$	$0.097 \\ 0.078$	0.033 $0.033$	$0.081 \\ 0.066$	$0.055 \\ 0.055$	0.117 $0.099$	$0.094 \\ 0.072$	$0.162 \\ 0.142$
Feasible Estimation	-	-	0.000	0.101	0.040	0.010	0.000	0.000	0.000	0.055	0.012	0.142
Local Polynomial	0.41	0.31	0.459	0.462	0.404	0.405	0.045	0.075	0.078	0.096	0.646	0.652
B-splines	1	1	0.066	0.127	0.050	0.097	0.042	0.081	0.065	0.117	0.097	0.162
Partitioning	1	1	0.060	0.101	0.045	0.078	0.056	0.066	0.065	0.099	0.075	0.142
					Model 2.7	•						
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.155	0.198	0.091	0.139	0.960	0.924	0.356	0.353	0.151	0.188
B-splines Partitioning	9	9	$0.177 \\ 0.204$	0.225	0.115	$0.161 \\ 0.241$	$0.967 \\ 0.949$	0.941 $0.891$	0.359 $0.356$	$0.360 \\ 0.384$	$0.165 \\ 0.153$	$0.172 \\ 0.218$
Feasible Estimation	9	9	0.204	0.320	0.145	0.241	0.949	0.891	0.330	0.364	0.155	0.218
Local Polynomial	0.34	0.28	0.137	0.152	0.064	0.085	0.984	0.986	0.360	0.362	0.074	0.090
B-splines	2	1	0.145	0.177	0.077	0.115	0.968	0.959	0.364	0.359	0.114	0.162
Partitioning	2	1	0.146	0.161	0.077	0.099	0.953	0.958	0.358	0.354	0.095	0.144
					Model 2.8							
Infeasible Estimation												
Local Polynomial	0.29	0.9	0.104	0.096	0.079	0.075	0.071	0.080	0.111	0.086	0.159	0.108
B-splines	4	1	0.097	0.128	0.075	0.098	0.132	0.084	0.134	0.117	0.135	0.162
Partitioning	4	1	0.114	0.103	0.089	0.080	0.192	0.082	0.155	0.101	0.122	0.145
Feasible Estimation Local Polynomial	0.24	0.28	0.070	0.100	0.050	0.079	0.047	0.079	0.072	0.000	0.072	0.000
	0.34	0.28	0.079 $0.093$	$0.102 \\ 0.130$	$0.059 \\ 0.070$	0.078 $0.099$	$0.047 \\ 0.101$	$0.072 \\ 0.088$	0.073 $0.108$	$0.090 \\ 0.118$	0.073 $0.121$	$0.090 \\ 0.162$
B-splines									0.100	0.110		

Table C.44: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=1, X_{i,\ell}\sim\beta(1,1),$  Quantile Cells

	Tur Parai		Root Int		Inget:		(0.5,		oint Estim			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.10	0.00	0.100	0.100	0.100	0.144	0.150	0.101	0.150	0.100	0.050	0.000
Local Polynomial B-splines	0.16 9	$0.23 \\ 4$	0.166 $0.186$	$0.190 \\ 0.175$	$0.126 \\ 0.146$	0.144 $0.134$	$0.150 \\ 0.210$	$0.161 \\ 0.206$	0.178 $0.133$	$0.196 \\ 0.184$	0.253 $0.209$	0.268 $0.240$
Partitioning	9	4	0.183	0.175	0.146	0.154 $0.156$	0.210	0.206 $0.475$	0.155 $0.151$	0.164 $0.353$	0.209	0.240
Feasible Estimation	3	4	0.103	0.205	0.143	0.150	0.105	0.415	0.101	0.000	0.100	0.231
Local Polynomial	0.27	0.27	0.215	0.213	0.165	0.170	0.059	0.126	0.101	0.115	0.098	0.105
B-splines	4	1	0.173	0.187	0.132	0.146	0.368	0.236	0.199	0.153	0.169	0.227
Partitioning	4	1	0.180	0.184	0.138	0.143	0.324	0.235	0.280	0.200	0.154	0.199
					Model 2.2	2						
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.203	0.227	0.156	0.171	0.177	0.174	0.210	0.262	0.259	0.401
B-splines	16	4	0.207	0.171	0.160	0.129	0.277	0.128	0.255	0.182	0.245	0.242
Partitioning	16	4	0.251	0.216	0.195	0.164	0.433	0.567	0.381	0.361	0.215	0.231
Feasible Estimation												
Local Polynomial	0.29	0.23	0.505	0.508	0.411	0.410	0.675	0.704	0.571	0.572	0.313	0.320
B-splines	$\frac{4}{4}$	4	0.328	0.171	0.258	0.129	0.173	0.128	0.164	0.182	0.335	0.242
Partitioning	4	4	0.362	0.216	0.292	0.164	0.721	0.567	0.301	0.361	0.427	0.231
					Model 2.3	3						
Infeasible Estimation												
Local Polynomial	0.16	0.39	0.160	0.148	0.120	0.110	0.109	0.091	0.177	0.138	0.258	0.203
B-splines	9	4	0.176	0.159	0.136	0.118	0.117	0.128	0.148	0.181	0.212	0.240
Partitioning Feasible Estimation	9	4	0.195	0.203	0.150	0.154	0.104	0.487	0.159	0.356	0.192	0.235
Local Polynomial	0.34	0.28	0.210	0.217	0.133	0.140	0.104	0.103	0.214	0.214	0.164	0.166
B-splines	4	3	0.172	0.164	0.133	0.124	0.162	0.130	0.164	0.171	0.169	0.100
Partitioning	4	3	0.182	0.190	0.137	0.142	0.171	0.388	0.241	0.301	0.136	0.230
1 drononing	-	J	0.102	0.100			0.1.1	0.000	0.211	0.001	0.100	0.200
Infeasible Estimation					Model 2.4	Ŀ						
Local Polynomial	0.33	0.33	0.613	0.322	0.425	0.246	0.138	0.158	0.190	0.155	0.223	0.224
B-splines	9	9	0.676	0.347	0.506	0.240	0.137	0.155	0.155	0.193	0.226	0.224
Partitioning	9	9	0.619	0.349	0.464	0.269	0.134	0.194	0.185	0.252	0.234	0.280
Feasible Estimation	Ü		0.010	0.010	0.101	0.200	0.101	0.101	0.100	0.202	0.201	0.200
Local Polynomial	0.19	0.27	0.580	0.506	0.471	0.403	0.345	0.344	0.571	0.325	0.690	0.945
B-splines	4	2	0.686	0.399	0.510	0.322	0.245	0.343	0.179	0.205	0.289	0.257
Partitioning	4	2	0.641	0.335	0.474	0.261	0.271	0.570	0.205	0.367	0.238	0.237
					Model 2.5	i						
Infeasible Estimation												
Local Polynomial	0.19	0.9	0.145	0.120	0.110	0.097	0.139	0.149	0.158	0.098	0.235	0.125
B-splines	4	1	0.107	0.166	0.082	0.130	0.163	0.162	0.146	0.146	0.168	0.235
Partitioning	4	1	0.118	0.147	0.092	0.118	0.211	0.156	0.149	0.126	0.136	0.182
Feasible Estimation												
Local Polynomial	0.23	0.29	0.099	0.087	0.079	0.066	0.161	0.067	0.088	0.096	0.088	0.096
B-splines	4	1 1	$0.107 \\ 0.118$	$0.166 \\ 0.151$	$0.082 \\ 0.092$	$0.130 \\ 0.120$	$0.163 \\ 0.211$	$0.162 \\ 0.183$	$0.146 \\ 0.149$	$0.149 \\ 0.137$	$0.168 \\ 0.136$	0.234 $0.186$
Partitioning	4	1	0.116	0.131			0.211	0.165	0.149	0.137	0.130	0.180
					Model 2.6	5						
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.059	0.089	0.046	0.069	0.033	0.061	0.060	0.094	0.089	0.122
B-splines Partitioning	1 1	1 1	$0.064 \\ 0.055$	$0.127 \\ 0.100$	$0.049 \\ 0.043$	$0.094 \\ 0.076$	0.033 $0.033$	$0.072 \\ 0.060$	$0.058 \\ 0.057$	$0.129 \\ 0.107$	$0.109 \\ 0.077$	$0.226 \\ 0.171$
Feasible Estimation	1	1	0.055	0.100	0.045	0.070	0.033	0.000	0.007	0.107	0.077	0.171
Local Polynomial	0.4	0.31	0.419	0.423	0.362	0.364	0.042	0.065	0.086	0.103	0.646	0.651
B-splines	1	1	0.066	0.127	0.050	0.094	0.041	0.072	0.067	0.129	0.115	0.226
Partitioning	1	1	0.060	0.100	0.046	0.076	0.050	0.060	0.067	0.107	0.083	0.171
3					Model 2.7							
Infeasible Estimation					Model 2.1							
Local Polynomial	0.33	0.33	0.164	0.202	0.093	0.137	0.957	0.918	0.359	0.355	0.183	0.215
B-splines	9	9	0.185	0.230	0.118	0.161	0.959	0.930	0.364	0.371	0.209	0.254
Partitioning	9	9	0.211	0.323	0.148	0.243	0.935	0.856	0.367	0.387	0.179	0.269
Feasible Estimation												
Local Polynomial	0.34	0.28	0.149	0.162	0.068	0.087	0.978	0.979	0.361	0.363	0.084	0.097
B-splines	2	1	0.157	0.186	0.085	0.118	0.955	0.952	0.367	0.369	0.145	0.226
Partitioning	2	1	0.158	0.173	0.087	0.105	0.937	0.940	0.348	0.370	0.112	0.178
					Model 2.8	3						
Infeasible Estimation												
Local Polynomial	0.3	0.9	0.103	0.094	0.076	0.072	0.061	0.070	0.119	0.094	0.197	0.122
B-splines	4	1	0.096	0.127	0.073	0.095	0.117	0.074	0.146	0.129	0.167	0.226
Partitioning	4	1	0.112	0.102	0.087	0.078	0.171	0.072	0.149	0.109	0.135	0.176
Feasible Estimation												
Local Polynomial	0.34	0.28	0.074	0.097	0.055	0.073	0.043	0.063	0.083	0.096	0.083	0.097
B-splines	3	1	0.091	0.131	0.069	0.097	0.093	0.079	0.121	0.138	0.150	0.225
Partitioning	3	1	0.100	0.117	0.076	0.086	0.144	0.159	0.124	0.157	0.118	0.183

Table C.45: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=1, X_{i,\ell}\sim\beta(2,2),$  Quantile Cells

	Tur Parai	ning meter	Root In		Ingeti MA		(0.5,		oint Estima (0.1,		SE (0.1	,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubi
					Model 2.1							
Infeasible Estimation	0.15	0.22	0.174	0.187	0.125	0.134	0.127	0.130	0.237	0.229	0.739	0.55
Local Polynomial B-splines	0.15	0.22	0.174 $0.199$	0.187	0.125	0.134	0.127	0.130 $0.159$	0.237	0.229 $0.235$	0.739	0.63
Partitioning	9	4	0.187	0.206	0.146	0.123	0.133	0.133	0.208	0.452	0.333	0.49
Feasible Estimation	Ü	-	0.101	0.200	0.110	0.101	0.100	0.120	0.200	0.102	0.201	0.10
Local Polynomial	0.25	0.26	0.227	0.221	0.183	0.180	0.043	0.098	0.124	0.144	0.136	0.14
B-splines	4	2	0.180	0.194	0.141	0.150	0.314	0.212	0.309	0.240	0.257	0.56
Partitioning	4	2	0.186	0.199	0.144	0.154	0.335	0.318	0.375	0.324	0.195	0.39
					Model 2.2							
Infeasible Estimation												
Local Polynomial	0.12	0.18	0.201	0.216	0.145	0.154	0.146	0.132	0.263	0.293	0.637	1.09
B-splines	16	4	0.218	0.166	0.166	0.118	0.261	0.102	0.329	0.230	0.507	0.63
Partitioning Feasible Estimation	16	4	0.254	0.212	0.197	0.158	0.356	0.468	0.422	0.468	0.441	0.50
Local Polynomial	0.22	0.22	0.513	0.516	0.420	0.422	0.606	0.613	0.510	0.514	0.392	0.40
B-splines	8	4	0.289	0.166	0.224	0.422	0.284	0.102	0.204	0.230	0.589	0.40
Partitioning	8	4	0.269	0.212	0.208	0.158	0.247	0.468	0.216	0.468	0.608	0.50
Infeasible Estimation					Model 2.3							
Local Polynomial	0.17	0.42	0.147	0.130	0.102	0.092	0.079	0.073	0.211	0.162	0.537	0.34
B-splines	9	4	0.160	0.158	0.119	0.111	0.090	0.103	0.211	0.229	0.365	0.63
Partitioning	9	4	0.184	0.202	0.141	0.150	0.100	0.419	0.215	0.460	0.258	0.50
Feasible Estimation												
Local Polynomial	0.34	0.28	0.146	0.155	0.098	0.104	0.089	0.083	0.207	0.197	0.164	0.16
B-splines	3	3	0.140	0.157	0.098	0.111	0.110	0.102	0.227	0.221	0.243	0.59
Partitioning	3	3	0.147	0.184	0.106	0.134	0.142	0.349	0.251	0.384	0.187	0.46
					Model 2.4							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.496	0.313	0.320	0.239	0.131	0.133	0.227	0.172	0.456	0.38
B-splines	9	9	0.564	0.359	0.369	0.284	0.075	0.235	0.384	0.275	0.557	1.1
Partitioning Feasible Estimation	9	9	0.557	0.358	0.379	0.273	0.110	0.198	0.403	0.342	0.457	0.67
Local Polynomial	0.21	0.27	0.512	0.450	0.390	0.342	0.273	0.263	0.359	0.271	0.927	1.02
B-splines	4	4	0.570	0.398	0.360	0.320	0.206	0.274	0.444	0.241	0.529	0.69
Partitioning	4	4	0.568	0.327	0.366	0.256	0.236	0.509	0.469	0.459	0.461	0.52
					Model 2.5							
Infeasible Estimation					Woder 2.5							
Local Polynomial	0.19	0.9	0.146	0.112	0.106	0.088	0.120	0.114	0.199	0.147	0.498	0.19
B-splines	4	1	0.107	0.157	0.082	0.117	0.160	0.122	0.153	0.223	0.253	0.52
Partitioning	4	1	0.119	0.137	0.093	0.105	0.206	0.119	0.166	0.197	0.183	0.3
Feasible Estimation												
Local Polynomial	0.22	0.29	0.100	0.087	0.082	0.062	0.145	0.060	0.123	0.135	0.123	0.13
B-splines	4	1	0.107	0.158	0.082	0.117	0.160	0.121	0.153	0.226	0.253	0.54
Partitioning	4	1	0.119	0.147	0.093	0.111	0.206	0.185	0.166	0.233	0.183	0.33
					Model 2.6							
Infeasible Estimation	0.0	0.0	0.050	0.000	0.045	0.004	0.000	0.050	0.051	0.100	0.101	0.11
Local Polynomial	0.9	0.9	0.058	0.086	0.045	0.064	0.033	0.052	0.071	0.122	0.101	0.17
B-splines Partitioning	1	1 1	$0.063 \\ 0.055$	$0.126 \\ 0.099$	$0.047 \\ 0.043$	$0.088 \\ 0.072$	0.033 $0.033$	$0.060 \\ 0.052$	$0.067 \\ 0.067$	$0.164 \\ 0.130$	$0.137 \\ 0.088$	0.49 0.27
Feasible Estimation	-	1	0.055	0.033	0.043	0.072	0.033	0.052	0.007	0.130	0.000	0.2
Local Polynomial	0.4	0.3	0.353	0.358	0.296	0.299	0.038	0.058	0.115	0.140	0.652	0.6
B-splines	1	1	0.070	0.126	0.051	0.088	0.050	0.060	0.087	0.164	0.172	0.49
Partitioning	1	1	0.068	0.099	0.050	0.073	0.071	0.060	0.092	0.130	0.112	0.27
					Model 2.7							
Infeasible Estimation					1110401 211							
Local Polynomial	0.33	0.33	0.177	0.205	0.097	0.131	0.950	0.910	0.363	0.355	0.294	0.3'
B-splines	9	9	0.198	0.241	0.125	0.163	0.943	0.915	0.368	0.399	0.349	0.8
Partitioning	9	9	0.220	0.327	0.154	0.245	0.910	0.806	0.370	0.436	0.254	0.66
Feasible Estimation				0.4				0.6				
Local Polynomial	0.33	0.27	0.166	0.178	0.078	0.094	0.969	0.969	0.362	0.370	0.116	0.13
B-splines	3	2	0.174	0.204	0.099	0.127	0.928	0.930	0.362	0.386	0.232	0.55
Partitioning	3	2	0.177	0.199	0.105	0.125	0.898	0.877	0.332	0.388	0.171	0.30
					Model 2.8							
Infeasible Estimation	0.00	0.0	0.000	0.000	0.00=	0.00=	0.040	0.050	0.104	0.100	0.000	0 -
Local Polynomial	0.32	0.9	0.096	0.089	0.067	0.067	0.048	0.058	0.134	0.123	0.309	0.1
B-splines	4	1	0.095	0.126	0.071	0.089	0.101	0.062	0.151	0.165	0.252	0.49
Partitioning Feasible Estimation	4	1	0.111	0.101	0.086	0.074	0.150	0.058	0.164	0.130	0.183	0.28
Local Polynomial	0.34	0.28	0.067	0.092	0.049	0.066	0.042	0.058	0.113	0.135	0.114	0.13
B-splines	3	2	0.091	0.136	0.043	0.095	0.042	0.074	0.113	0.184	0.233	0.5
Partitioning	3	2	0.101	0.135	0.076	0.093	0.131	0.207	0.140	0.232	0.166	0.3

Table C.46: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=4, X_{i,\ell}\sim\beta(0.5,0.5),$  Quantile Cells

	Tur Parai		Root Int MS		Inget: MA		(0.5,		oint Estima (0.1,			,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubi
					Model 2.1							
Infeasible Estimation	0.01	0.05	0.00=	0.045	0.005	0.000	0.040	0.005	0.051	0.00=	0.000	0.05
Local Polynomial	0.21	0.27	0.267	0.345	0.205	0.266	0.240	0.305	0.271	0.327	0.339	0.37
B-splines Partitioning	4 4	$\frac{4}{4}$	$0.229 \\ 0.256$	$0.325 \\ 0.404$	$0.177 \\ 0.198$	$0.250 \\ 0.307$	$0.462 \\ 0.438$	0.342 $1.055$	0.287 $0.381$	0.318 $0.651$	$0.270 \\ 0.251$	0.33
Feasible Estimation	4	4	0.230	0.404	0.198	0.307	0.438	1.055	0.361	0.031	0.231	0.55
Local Polynomial	0.29	0.26	0.225	0.252	0.171	0.198	0.119	0.200	0.159	0.193	0.153	0.18
B-splines	3	2	0.233	0.299	0.171	0.130	0.426	0.303	0.133	0.153	0.153	0.13
Partitioning	3	2	0.255	0.310	0.196	0.231	0.411	0.598	0.353	0.402	0.247	0.30
	_	_	0.200	0.020				0.000				
Infeasible Estimation					Model 2.2	:						
Local Polynomial	0.15	0.21	0.343	0.412	0.266	0.319	0.314	0.361	0.349	0.440	0.361	0.51
B-splines	9	4	0.444	0.326	0.362	0.250	0.530	0.292	0.460	0.313	0.340	0.33
Partitioning	9	4	0.396	0.410	0.312	0.312	0.365	1.128	0.337	0.654	0.310	0.33
Feasible Estimation	3	-	0.050	0.410	0.012	0.012	0.000	1.120	0.001	0.004	0.010	0.00
Local Polynomial	0.31	0.24	0.498	0.512	0.399	0.407	0.709	0.757	0.616	0.626	0.304	0.32
B-splines	4	4	0.383	0.327	0.306	0.251	0.304	0.294	0.280	0.312	0.327	0.33
Partitioning	4	4	0.419	0.411	0.337	0.313	0.899	1.126	0.384	0.652	0.375	0.33
_					Model 2.3							
infeasible Estimation					Model 2.3	•						
Local Polynomial	0.19	0.43	0.279	0.271	0.213	0.208	0.219	0.176	0.286	0.223	0.352	0.30
B-splines	9	4	0.219	0.318	0.213	0.244	0.219	0.170	0.222	0.223	0.332	0.33
Partitioning	9	4	0.348	0.403	0.220	0.307	0.173	1.051	0.253	0.651	0.331	0.33
Feasible Estimation	,	-	2.010		J.2.1		2.200		J. <b>2</b> 00	2.001	0.010	5.50
Local Polynomial	0.32	0.27	0.279	0.305	0.186	0.214	0.142	0.178	0.263	0.288	0.218	0.24
B-splines	4	3	0.250	0.304	0.192	0.232	0.296	0.256	0.276	0.280	0.267	0.32
Partitioning	4	3	0.274	0.343	0.212	0.255	0.352	0.771	0.358	0.490	0.240	0.33
3					Model 2.4							
Infeasible Estimation					Model 2.4							
Local Polynomial	0.33	0.33	0.686	0.422	0.501	0.332	0.175	0.259	0.351	0.295	0.325	0.38
B-splines	9	9	0.724	0.468	0.580	0.365	0.201	0.316	0.331 $0.277$	0.294	0.348	0.34
Partitioning	9	9	0.644	0.617	0.505	0.471	0.234	0.377	0.315	0.459	0.323	0.44
Feasible Estimation	3	9	0.044	0.011	0.000	0.471	0.204	0.011	0.010	0.405	0.020	0.4-
Local Polynomial	0.2	0.27	0.648	0.554	0.532	0.452	0.417	0.423	0.669	0.393	0.613	0.92
B-splines	4	2	0.803	0.562	0.621	0.443	0.629	0.424	0.281	0.311	0.298	0.34
Partitioning	4	2	0.793	0.559	0.611	0.437	0.699	0.920	0.336	0.553	0.272	0.32
					Model 2.5	:						
Infeasible Estimation					Woder 2.0	•						
Local Polynomial	0.24	0.9	0.243	0.199	0.187	0.158	0.243	0.207	0.246	0.168	0.325	0.21
B-splines	4	1	0.198	0.278	0.153	0.215	0.283	0.234	0.268	0.234	0.271	0.32
Partitioning	4	1	0.226	0.230	0.176	0.181	0.375	0.214	0.311	0.200	0.244	0.28
Feasible Estimation												
Local Polynomial	0.26	0.27	0.152	0.176	0.120	0.136	0.223	0.151	0.148	0.181	0.148	0.18
B-splines	4	2	0.198	0.291	0.153	0.223	0.283	0.254	0.268	0.258	0.271	0.32
Partitioning	4	2	0.226	0.290	0.176	0.216	0.375	0.555	0.311	0.368	0.244	0.30
					Model 2.6							
Infeasible Estimation					Model 2.0	•						
Local Polynomial	0.9	0.9	0.119	0.181	0.092	0.141	0.066	0.132	0.113	0.168	0.165	0.2
B-splines	1	1	0.127	0.255	0.097	0.194	0.066	0.161	0.110	0.233	0.188	0.32
Partitioning	1	1	0.110	0.202	0.086	0.154	0.066	0.133	0.110	0.199	0.144	0.32
Feasible Estimation	-	-			3.000	0.200	2.000	5.100		5.100		5.20
Local Polynomial	0.35	0.28	0.470	0.486	0.410	0.418	0.093	0.148	0.149	0.181	0.660	0.67
B-splines	2	1	0.163	0.260	0.122	0.197	0.182	0.171	0.213	0.237	0.234	0.32
Partitioning	2	1	0.175	0.223	0.127	0.167	0.262	0.290	0.241	0.241	0.198	0.28
3					Model 2.7							
Infospible Fetimation					Model 2.1							
Infeasible Estimation Local Polynomial	0.33	0.33	0.227	0.341	0.160	0.259	0.967	0.944	0.394	0.431	0.301	0.37
B-splines	9	9	0.283	0.399	0.209	0.302	0.976	0.964	0.401	0.433	0.330	0.34
Partitioning	9	9	0.351	0.611	0.270	0.466	0.964	0.950	0.412	0.549	0.306	0.43
Feasible Estimation	9	9	0.331	0.011	0.270	0.400	0.904	0.950	0.412	0.549	0.300	0.4.
Local Polynomial	0.32	0.27	0.172	0.215	0.107	0.150	0.989	0.997	0.381	0.393	0.147	0.18
B-splines	3	2	0.214	0.219	0.147	0.130	0.981	0.969	0.409	0.426	0.250	0.32
Partitioning	3	2	0.229	0.295	0.159	0.208	0.972	1.035	0.415	0.479	0.220	0.30
	9	-					2.0.2	500			J. <b></b> J	0.0
. f . 21 . E . C . C					Model 2.8	•						
Infeasible Estimation	0.97	0.0	0.100	0.100	0.100	0.144	0.100	0.100	0.100	0.100	0.000	0.0
Local Polynomial	0.37	0.9	0.183	0.183	0.138	0.144	0.109	0.139	0.189	0.169	0.288	0.2
B-splines	1	1	0.138	0.255	0.106	0.194	0.069	0.162	0.112	0.233	0.188	0.32
Partitioning	1	1	0.123	0.203	0.096	0.157	0.069	0.141	0.112	0.200	0.144	0.28
Feasible Estimation	0.22	0.27	0.120	0.192	0.100	0.149	0.005	0.140	0.147	0.101	0.147	0.74
Local Polynomial	0.32	0.27	0.130	0.183	0.100	0.142	0.095	0.148	0.147	0.181	0.147	0.18
B-splines	3 3	2 2	0.183 $0.206$	$0.278 \\ 0.285$	$0.139 \\ 0.156$	$0.210 \\ 0.205$	$0.229 \\ 0.321$	$0.211 \\ 0.579$	$0.245 \\ 0.280$	$0.263 \\ 0.386$	$0.256 \\ 0.229$	0.32

Table C.47: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=4, X_{i,\ell}\sim\beta(1,1),$  Quantile Cells

	Tur Parai		Root Int		Ingeti MA		(0.5,		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.2	0.26	0.275	0.342	0.207	0.258	0.017	0.265	0.307	0.339	0.460	0.450
Local Polynomial B-splines	4	4	0.275	0.344	0.183	0.238 $0.244$	0.217 $0.424$	0.203	0.322	0.363	0.335	0.480
Partitioning	4	4	0.259	0.402	0.201	0.305	0.431	0.943	0.379	0.704	0.279	0.460
Feasible Estimation												
Local Polynomial	0.29	0.26	0.239	0.261	0.187	0.208	0.094	0.170	0.178	0.204	0.177	0.200
B-splines	4	2	0.240	0.304	0.187	0.232	0.403	0.274	0.308	0.311	0.329	0.463
Partitioning	4	2	0.259	0.318	0.201	0.238	0.414	0.574	0.365	0.481	0.277	0.394
					Model 2.2							
Infeasible Estimation Local Polynomial	0.15	0.21	0.337	0.402	0.256	0.303	0.272	0.299	0.376	0.431	0.510	0.620
B-splines	9	4	0.418	0.322	0.339	0.303 $0.241$	0.444	0.254	0.397	0.362	0.457	0.481
Partitioning	9	4	0.390	0.408	0.306	0.309	0.299	0.993	0.327	0.707	0.398	0.459
Feasible Estimation												
Local Polynomial	0.3	0.24	0.516	0.529	0.419	0.427	0.677	0.715	0.592	0.600	0.343	0.360
B-splines	4	4	0.365	0.323	0.286	0.242	0.246	0.255	0.305	0.362	0.447	0.482
Partitioning	4	4	0.413	0.408	0.330	0.309	0.791	0.992	0.408	0.705	0.496	0.460
					Model 2.3							
Infeasible Estimation	0.2	0.44	0.269	0.255	0.109	0.101	0.179	0.152	0.205	0.228	0.464	0.250
Local Polynomial B-splines	0.2 9	$0.44 \\ 4$	$0.268 \\ 0.281$	0.255 $0.316$	0.198 $0.215$	$0.191 \\ 0.236$	$0.172 \\ 0.163$	$0.153 \\ 0.254$	$0.305 \\ 0.253$	$0.238 \\ 0.362$	$0.464 \\ 0.418$	$0.350 \\ 0.480$
Partitioning	9	4	0.345	0.401	0.268	0.304	0.199	0.955	0.304	0.706	0.365	0.463
Feasible Estimation												
Local Polynomial	0.32	0.27	0.233	0.264	0.155	0.183	0.127	0.152	0.257	0.272	0.218	0.236
B-splines	4	3	0.235	0.300	0.176	0.224	0.246	0.219	0.301	0.321	0.329	0.470
Partitioning	4	3	0.257	0.336	0.197	0.246	0.316	0.684	0.353	0.548	0.266	0.427
					Model 2.4							
Infeasible Estimation	0.00	0.00	0.005	0.400	0.440	0.00=	0.100	0.005	0.000	0.000	0.000	0.400
Local Polynomial B-splines	0.33 9	0.33 9	0.635	$0.420 \\ 0.477$	0.448	0.327	0.166	0.225	0.266	0.302	0.390	0.436
Partitioning	9	9	0.711 $0.681$	0.477	0.538 $0.526$	$0.370 \\ 0.479$	$0.174 \\ 0.216$	0.314 $0.387$	0.257 $0.320$	$0.346 \\ 0.494$	$0.470 \\ 0.391$	$0.520 \\ 0.544$
Feasible Estimation	5	J	0.001	0.020	0.020	0.410	0.210	0.001	0.020	0.404	0.001	0.011
Local Polynomial	0.21	0.27	0.618	0.531	0.495	0.421	0.365	0.367	0.567	0.364	0.732	0.967
B-splines	4	2	0.745	0.536	0.540	0.418	0.490	0.386	0.343	0.363	0.388	0.492
Partitioning	4	2	0.745	0.529	0.547	0.407	0.573	0.857	0.385	0.626	0.350	0.451
					Model 2.5							
Infeasible Estimation					0.404							
Local Polynomial B-splines	$0.24 \\ 4$	$0.9 \\ 1$	$0.244 \\ 0.196$	$0.195 \\ 0.275$	$0.184 \\ 0.150$	$0.153 \\ 0.208$	$0.216 \\ 0.259$	0.184 $0.206$	$0.271 \\ 0.291$	$0.189 \\ 0.267$	0.419 $0.335$	$0.245 \\ 0.456$
Partitioning	4	1	0.190	0.273	0.174	0.208	0.350	0.189	0.291	0.225	0.333	0.430
Feasible Estimation	*	-	0.220	0.220	0.114	0.170	0.000	0.105	0.201	0.220	0.210	0.040
Local Polynomial	0.25	0.27	0.152	0.173	0.121	0.131	0.205	0.129	0.172	0.195	0.172	0.195
B-splines	4	2	0.196	0.288	0.150	0.216	0.259	0.215	0.291	0.301	0.335	0.462
Partitioning	4	2	0.223	0.287	0.174	0.211	0.350	0.475	0.297	0.419	0.270	0.377
					Model 2.6	i						
Infeasible Estimation												
Local Polynomial	0.9	0.9	0.118	0.178	0.091	0.137	0.066	0.121	0.120	0.187	0.179	0.243
B-splines Partitioning	1	1 1	$0.127 \\ 0.110$	$0.254 \\ 0.201$	$0.097 \\ 0.086$	0.188 $0.153$	$0.065 \\ 0.065$	$0.143 \\ 0.121$	$0.115 \\ 0.115$	$0.258 \\ 0.214$	$0.218 \\ 0.154$	0.453 $0.343$
Feasible Estimation	-	-	0.110	0.201	0.000	0.100	0.000	0.121	0.110	0.211	0.101	0.010
Local Polynomial	0.34	0.28	0.431	0.449	0.368	0.379	0.084	0.127	0.170	0.197	0.663	0.673
B-splines	2	1	0.164	0.261	0.122	0.193	0.169	0.155	0.225	0.271	0.292	0.451
Partitioning	2	1	0.175	0.230	0.128	0.168	0.244	0.303	0.226	0.313	0.224	0.356
					Model 2.7							
Infeasible Estimation												
Local Polynomial	0.33	0.33	0.232	0.337	0.159	0.249	0.963	0.931	0.404	0.437	0.366	0.431
B-splines Partitioning	9 9	9	0.287 $0.355$	$0.400 \\ 0.611$	0.210	$0.296 \\ 0.465$	0.969 $0.949$	0.947 $0.913$	0.419	$0.468 \\ 0.572$	0.417 $0.358$	0.507 $0.537$
Feasible Estimation	9	9	0.555	0.011	0.271	0.405	0.949	0.913	0.449	0.572	0.558	0.557
Local Polynomial	0.32	0.27	0.180	0.221	0.109	0.149	0.981	0.985	0.389	0.399	0.169	0.195
B-splines	3	2	0.222	0.305	0.153	0.220	0.955	0.959	0.437	0.461	0.312	0.462
Partitioning	3	2	0.236	0.306	0.167	0.214	0.937	0.996	0.414	0.541	0.250	0.376
					Model 2.8							
$In feasible\ Estimation$												
Local Polynomial	0.38	0.9	0.177	0.180	0.130	0.139	0.095	0.126	0.194	0.187	0.336	0.243
B-splines	1	1	0.136	0.254	0.104	0.188	0.070	0.144	0.119	0.258	0.219	0.453
Partitioning	1	1	0.121	0.202	0.094	0.154	0.070	0.126	0.118	0.215	0.155	0.346
Feasible Estimation Local Polynomial	0.32	0.27	0.125	0.179	0.095	0.135	0.086	0.128	0.169	0.195	0.169	0.196
B-splines	3	2	0.123	0.179	0.095	0.133 $0.204$	0.086	0.128	0.169 $0.267$	0.195 $0.304$	0.109	0.196
Partitioning	3	2	0.204	0.285	0.155	0.203	0.298	0.530	0.276	0.448	0.252	0.383

Table C.48: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=2, n=1000, \sigma^2=4, X_{i,\ell}\sim\beta(2,2),$  Quantile Cells

		ning meter	Root Int		Ingeti MA		(0.5,		oint Estim (0.1	ation RM ,0.5)		,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
					Model 2.1							
Infeasible Estimation	0.19	0.26	0.285	0.334	0.204	0.238	0.186	0.217	0.397	0.397	0.996	0.883
Local Polynomial B-splines	9	0.20	0.285	0.334 $0.325$	0.230	0.238	0.186	0.217	0.340	0.397	0.996	1.266
Partitioning	9	4	0.340	0.404	0.264	0.301	0.215	0.832	0.379	0.900	0.521	0.987
Feasible Estimation												
Local Polynomial	0.28	0.26	0.252	0.268	0.203	0.214	0.083	0.141	0.239	0.277	0.248	0.279
B-splines	4	3	0.245	0.316	0.190	0.230	0.354	0.247	0.394	0.420	0.507	1.161
Partitioning	4	3	0.265	0.356	0.207	0.262	0.415	0.645	0.460	0.693	0.370	0.856
					Model 2.2							
Infeasible Estimation Local Polynomial	0.16	0.21	0.332	0.383	0.238	0.272	0.227	0.228	0.467	0.481	1.373	1.300
B-splines	9	4	0.364	0.320	0.282	0.272	0.321	0.204	0.321	0.451	0.842	1.267
Partitioning	9	4	0.382	0.407	0.297	0.303	0.237	0.854	0.363	0.911	0.742	0.989
Feasible Estimation												
Local Polynomial	0.25	0.23	0.524	0.537	0.428	0.438	0.610	0.624	0.557	0.570	0.435	0.460
B-splines	4	4	0.322	0.320	0.241	0.225	0.214	0.206	0.406	0.458	0.746	1.262
Partitioning	4	4	0.386	0.407	0.299	0.303	0.633	0.853	0.480	0.909	0.742	0.991
					Model 2.3							
Infeasible Estimation	0.00	0.40	0.045	0.000	0.160	0.160	0.104	0.100	0.040	0.000	0.040	0.504
Local Polynomial B-splines	0.22	0.48	$0.245 \\ 0.216$	0.226 $0.316$	$0.169 \\ 0.158$	$0.162 \\ 0.221$	$0.124 \\ 0.209$	0.123 $0.205$	0.348 $0.339$	0.290	0.842 $0.509$	0.584 $1.267$
Partitioning	4	$\frac{4}{4}$	0.210	0.310	0.185	0.221 $0.299$	0.209	0.203	0.339 $0.379$	$0.458 \\ 0.906$	0.309 $0.375$	0.994
Feasible Estimation	*	4	0.242	0.402	0.100	0.233	0.250	0.001	0.015	0.500	0.010	0.554
Local Polynomial	0.31	0.27	0.178	0.216	0.124	0.151	0.111	0.131	0.282	0.306	0.254	0.286
B-splines	3	3	0.211	0.302	0.153	0.211	0.197	0.188	0.336	0.419	0.487	1.194
Partitioning	3	3	0.233	0.355	0.175	0.254	0.279	0.687	0.362	0.727	0.357	0.894
					Model 2.4							
$In feasible\ Estimation$												
Local Polynomial	0.33	0.33	0.521	0.399	0.345	0.303	0.149	0.183	0.319	0.338	0.690	0.748
B-splines	9	9	0.605	0.487	0.412	0.368	0.126	0.280	0.452	0.481	0.823	1.852
Partitioning	9	9	0.625	0.631	0.454	0.479	0.202	0.395	0.514	0.677	0.638	1.330
Feasible Estimation Local Polynomial	0.23	0.27	0.545	0.477	0.412	0.363	0.292	0.284	0.351	0.355	1.036	1.057
B-splines	4	3	0.596	0.492	0.382	0.377	0.301	0.311	0.521	0.476	0.670	1.288
Partitioning	4	3	0.604	0.486	0.403	0.369	0.381	0.837	0.567	0.859	0.558	0.975
					Model 2.5							
Infeasible Estimation												
Local Polynomial	0.23	0.9	0.243	0.186	0.175	0.141	0.183	0.145	0.329	0.256	0.789	0.363
B-splines	4	1	0.195	0.268	0.146	0.192	0.237	0.160	0.303	0.361	0.504	1.006
Partitioning	4	1	0.223	0.219	0.173	0.163	0.331	0.149	0.329	0.298	0.365	0.573
Feasible Estimation Local Polynomial	0.25	0.27	0.154	0.174	0.121	0.125	0.183	0.118	0.240	0.272	0.240	0.272
B-splines	4	2	0.195	0.174	0.121	0.123	0.183	0.118	0.303	0.411	0.504	1.141
Partitioning	4	2	0.223	0.325	0.173	0.231	0.331	0.576	0.329	0.632	0.365	0.822
o de la companya de l					Model 2.6							
Infeasible Estimation					Model 2.0							
Local Polynomial	0.9	0.9	0.116	0.172	0.089	0.129	0.065	0.104	0.142	0.244	0.202	0.357
B-splines	1	1	0.126	0.251	0.094	0.177	0.065	0.119	0.134	0.327	0.274	0.988
Partitioning	1	1	0.110	0.198	0.086	0.145	0.065	0.103	0.134	0.259	0.175	0.557
Feasible Estimation												
Local Polynomial	0.34	0.28	0.367	0.388	0.305	0.319	0.077	0.117	0.230	0.273	0.687	0.703
B-splines Partitioning	3 3	$\frac{2}{2}$	0.172 $0.189$	$0.270 \\ 0.267$	0.125 $0.139$	0.188 $0.184$	$0.166 \\ 0.244$	$0.145 \\ 0.416$	$0.258 \\ 0.271$	$0.368 \\ 0.467$	$0.456 \\ 0.321$	1.090 0.708
rartitioning	3	2	0.169	0.207			0.244	0.410	0.211	0.407	0.321	0.708
T.C. III T.C. C.					Model 2.7							
Infeasible Estimation Local Polynomial	0.33	0.33	0.238	0.323	0.156	0.226	0.952	0.916	0.424	0.455	0.588	0.743
B-splines	9	9	0.295	0.323	0.130	0.220	0.932	0.916	0.424	0.455	0.698	1.693
Partitioning	9	9	0.359	0.614	0.274	0.463	0.924	0.871	0.483	0.726	0.509	1.325
Feasible Estimation												
Local Polynomial	0.31	0.26	0.196	0.234	0.116	0.151	0.970	0.971	0.411	0.435	0.234	0.273
B-splines	3	3	0.235	0.328	0.160	0.227	0.929	0.929	0.436	0.521	0.491	1.158
Partitioning	3	3	0.252	0.358	0.180	0.252	0.903	0.963	0.413	0.720	0.355	0.846
					Model 2.8							
Infeasible Estimation			0.46-									
Local Polynomial	0.4	0.9	0.165	0.173	0.118	0.130	0.078	0.107	0.221	0.245	0.475	0.357
B-splines Partitioning	1 1	1 1	0.132	0.252	0.099	$0.177 \\ 0.145$	0.072	$0.120 \\ 0.107$	0.138 $0.139$	0.328	0.276	0.988 $0.562$
Feasible Estimation	1	1	0.117	0.199	0.091	0.145	0.072	0.107	0.139	0.259	0.177	0.562
Local Polynomial	0.31	0.27	0.123	0.177	0.090	0.127	0.080	0.118	0.231	0.273	0.232	0.273
B-splines	3	3	0.183	0.291	0.134	0.202	0.186	0.169	0.282	0.401	0.487	1.155
Partitioning	3	3	0.207	0.333	0.157	0.233	0.277	0.615	0.299	0.668	0.349	0.846

- C.3 Trivariate Simulations
- C.3.1 Uniform Cell Boundaries

Table C.49: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 500, \sigma^2 = 1, X_{i,\ell} \sim \beta(0.5, 0.5), \text{Uniform Cells}$ 

	1 ming Parameter	ıng aeter	Root Integrated MSE	egrateu 3E	MAE	\E	(0.5, 0.5, 0.5)	5,0.5)	(0.1,0.5,0.5)	5,0.5)	(0.5,0.5) $(0.1,0.1,0.5)$	1,0.5)	(0.1, 0.1, 0.1)	1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
: : :						Model 3.1								
Injeasible Estimation Local Polymomial	0	0	0.469	0.468	707	0.406	0.051	0000	0000	0.133	0.131	0 167	0.55	0.663
B-splines	1.5	9	0.463	0.483	0.405	0.413	0.050	0.124	0.085	0.181	0.152	0.261	0.663	0.694
Partitioning	1	1	0.090	0.199	0.070	0.153	0.045	0.103	0.075	0.173	0.100	0.237	0.121	0.336
Feasible Estimation														
Local Polynomial	0.45	0.31	0.648	0.653	0.529	0.533	0.062	0.115	0.118	0.155	0.651	0.659	1.285	1.289
B-splines	1	_	0.463	0.483	0.405	0.413	0.050	0.124	0.085	0.181	0.152	0.261	0.663	0.694
Partitioning	ī	-	0.090	0.199	0.070	0.153	0.045	0.103	0.075	0.173	0.100	0.237	0.121	0.336
:						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.17	0.24	0.401	0.426	0.314	0.333	0.470	0.508	0.426	0.482	0.416	0.475	0.344	0.371
B-splines	27	00	0.398	0.413	0.314	0.321	0.407	0.509	0.361	0.456	0.429	0.422	0.306	0.328
Partitioning	27	œ	0.493	0.576	0.383	0.444	0.585	4.445	1.113	2.740	0.481	1.557	0.444	0.746
Feasible Estimation	0	0	0	0	0	0	0	0	0	0	0	1	i	0
Local Polynomial	0.38	0.28	0.373	0.383	0.286	0.296	0.300	0.301	0.293	0.308	0.308	0.315	0.275	0.230
B-splines		٠,	0.376	0.398	0.289	0.312	0.375	0.413	0.271	0.366	0.359	0.425	0.262	0.311
Fartitioning	7	Т	0.377	0.398	0.290	0.312	0.395	0.400	0.299	0.553	0.346	0.335	0.270	0.354
						Model 3.3								
Infeasible Estimation	0	0		0	0		0	0	1	0	0	0	0	0
Local Folynomial	0.25	0.32	0.188	0.253	0.138	0.191	0.120	0.166	0.178	0.221	0.231	0.287	0.231	0.287
B-splines	<b>x</b> 0 0	٠,	0.167	0.204	0.122	0.152	0.207	0.116	0.201	0.166	0.193	0.236	0.193	0.236
Fartitioning Feasible Estimation	o	<b>-</b>	0.258	0.207	0.201	0.159	0.495	0.100	0.405	0.170	0.547	0.241	0.280	0.357
Local Polynomial	0.39	0.28	0.142	0.169	0.087	0.117	0.061	0.100	0.101	0.125	0.101	0.125	0.101	0.125
B-sulines		-	0.140	0.204	0.094	0.152	0.058	0.116	0.077	0.166	0.137	0.236	0.137	0.236
Partitioning	-		0.142	0.207	0.097	0.159	0.077	0.106	0.081	0.175	0.120	0.241	0.163	0.357
)						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.07	6.0	1.791	2.044	0.814	0.847	0.644	0.180	0.513	0.357	1.378	0.678	4.044	3.830
B-splines	216	27	1.955	1.903	0.905	0.880	0.633	0.193	0.602	0.465	1.614	1.346	4.138	4.021
Partitioning	216	27	1.443	1.113	0.598	0.554	0.243	0.024	0.576	0.523	1.278	1.796	3.388	4.360
Feasible Estimation														
Local Polynomial	0.29	0.26	2.077	2.061	0.820	0.839	0.070	0.107	0.475	0.336	0.475	0.336	3.792	3.787
B-splines	œ	1	2.061	2.005	0.882	0.880	0.282	0.157	0.426	0.446	0.838	1.344	3.864	4.025
Partitioning	œ	п	1.598	1.743	1.015	1.018	2.690	1.147	0.641	0.523	1.529	0.801	1.456	2.308
						Model 3.5								
Infeasible Estimation						ļ	,			:	,			
Local Polynomial	0.33	0.33	0.819	0.598	0.639	0.477	1.138	1.281	1.305	1.141	0.617	0.792	0.642	0.509
B-splines	27	27	0.878	0.617	0.699	0.493	0.843	1.428	1.158	1.313	0.794	0.684	0.510	0.606
Partitioning	27	27	0.764	1.276	0.604	0.991	0.936	0.383	1.394	1.109	0.560	3.010	0.533	3.962
Feasible Estimation	0	o c	1	11	, i	0 0 0	0	0	0 10 10 10 10 10 10 10 10 10 10 10 10 10	14 14 7	7	100	000	403
Local Folynomial	77:0	0.20	007.0	0.0.0	0.092	0.460	1.205	1.219	1.559	1.100	0.470	0.001	0.690	0.400
B-splines	<b>x</b> 0 0	٠,	0.930	0.822	0.734	0.661	1.0562	0.716	1.175	1.166	0.754	0.787	0.499	0.545
Partitioning	ø	- 	0.901	0.801	007.00	0.641	1.016	U.87U	1.449	1.322	0.814	0.588	0.632	U.bU4

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimated by averaging over the design points in each simulated data set.

Table C.50: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=500,\,\sigma^2=1,\,X_{i,\ell}\sim\beta(1,1),$  Uniform Cells

Degree: Linear Infeasible Estimation 0.9 B-splines 1 Fasible Estimation 0.44 B-splines 1 B-splines 1 Infeasible Estimation 0.17 B-splines 27 B-splines 8 B-splines 10.38 Feasible Estimation 0.28 B-splines 8 B-splines 10.38			INDE	TATA	MAE	(0.5, 0.5, 0.5)	5,0.5)	$(0.1, 0.5, 0.5) \qquad (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, $	5,0.5)	(0.1, 0.1, 0.5)	(2.0,1	(0.1, 0.1, 0.1)	(1.0,1)
0 0 0 0	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0					Model 3.1								
	1			1	1	1		1	,	1	,	1	1
	0.0	0.421	0.429	0.362	0.366	0.047	0.094	0.090	0.142	0.130	0.191	0.654	0.667
e		0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
e e	0.3	0.591	0.596	0.482	0.486	0.057	0.100	0.126	0.164	0.655	0.663	1.289	1.292
g g	1	0.423	0.446	0.363	0.375	0.046	0.112	0.085	0.196	0.162	0.364	0.659	0.712
e e	1	0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
e . e .					Model 3.2								
. 8	0.24	0.407	0.430	0.317	0.335	0.435	0.462	0.395	0.420	0.462	0.487	0.472	0.487
. 8	œ ·	0.403	0.421	0.317	0.327	0.401	0.463	0.346	0.415	0.437	0.483	0.406	0.491
. 8	œ	0.491	0.575	0.383	0.442	0.341	3.287	0.430	2.566	0.556	1.861	0.630	1.296
	0.07	0.374	28.0	986 0	906 0	0 977	088	0.319	0 207	0 300	308	7000	008
		0.014	0.004	0.280	0.230	# C86 C	0.330	0.012	0.027	0.300	0.308	40.50 40.50	0.009
		0.379	0.404	0.291	0.315	0.400	0.411	0.338	0.481	0.330	0.368	0.345	0.484
					Model 3.3								
_	0.34	0.168	0.231	0.120	0.171	0.088	0.132	0.164	0.197	0.282	0.323	0.282	0.323
_	1	0.156	0.195	0.114	0.142	0.182	0.105	0.203	0.178	0.236	0.331	0.236	0.331
_	1	0.258	0.206	0.201	0.155	0.443	0.105	0.396	0.184	0.367	0.284	0.351	0.477
	0.27	0.123	0.156	0.077	0.109	0.056	0.093	0.114	0.142	0.114	0.142	0.114	0.142
B-splines 1	1	0.126	0.195	0.085	0.142	0.060	0.105	0.087	0.178	0.153	0.331	0.153	0.331
Partitioning 1	1	0.131	0.206	0.090	0.155	0.106	0.105	0.110	0.184	0.130	0.284	0.174	0.477
					Model 3.4								
2	(	1	0	i i	0	0			0	1	0	0	0
Local Polynomial 0.08	0.9	1.785	1.320	0.508	0.496	0.331	0.148	0.413	0.353	7.493	9.005	8.533	3.831
'nœ	200	1.272	1.207	0.475	0.543	0.00 4.00 4.00 4.00 4.00 4.00 4.00 4.00	1318	0.01	200.5	2.090	7.569	3.754	0 5 5 X
ntion	!	! !					)	1	)			)	
Local Polynomial 0.29	0.26	1.345	1.327	0.469	0.489	0.063	0.096	0.431	0.325	0.431	0.325	3.805	3.800
B-splines 8	2	1.333	1.271	0.524	0.533	0.233	0.145	0.375	0.408	0.800	1.625	3.872	4.147
ng	2	1.114	1.112	0.646	0.600	1.499	1.894	0.543	1.036	1.176	1.011	2.217	2.803
					Model 3.5								
u	0	1	(	0	0	1	1	1	0	i		0	0
ynomial 0	0.33	0.789	0.613	0.616	0.493	1.130	1.205	1.151	1.052	0.716	0.734	0.622	0.641
Despines 21	2 6	0.850	1.264	0.009	010:0	0.925	1.544 1.359	1.002	9 200	0.893	7 469	0.541	0.73
ntion	ī	-	1				1000	5	1	50.0	100		
Local Polynomial 0.28	0.28	0.750	0.593	0.587	0.477	1.216	1.225	1.262	1.173	0.580	0.669	0.598	0.514
	1	0.890	0.805	0.688	0.642	0.681	0.814	0.969	0.980	0.825	0.944	0.488	0.596
Partitioning 8	1	0.853	0.786	0.656	0.623	1.023	0.912	1.168	1.066	0.939	0.767	0.786	0.824

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.51: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3, n=500, \sigma^2=1, X_{i,\ell}\sim \beta(2,2),$  Uniform Cells

Degree: Infeasible Estimation Local Polynomial B-splines	raran	Parameter	MSE	3E	M	MAE	(0.5,0.5,0.5)	5,0.5)	$(0.1,0.5,0.5) \qquad (0.1,0.1,0.1,0.1)$	5,0.5)	(0.1, 0.1, 0.5)	(3.0,	(0.1,0.1,0.1)	(1.0,1)
Infeasible Estimation Local Polynomial B-splines	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
Infeasible Estimation Local Polynomial B-splines Partitioning						Model 3.1								
Local Polynomial B-splines Partitioning	1		1	1	1			1		1	1	1	1	1
Partitioning	6.0	0.0 -	0.356	0.366	0.298	0.304	0.048	0.083	0.104	0.190	0.151	0.277	0.655	1.044
Similaria			0.000	0.200	0.070	0.147	0.044	0.087	0.000	0.219	0.127	0.426	0.152	0.705
Feasible Estimation														
Local Polynomial	0.43	0.29	0.497	0.504	0.404	0.409	0.053	0.087	0.164	0.226	0.662	0.679	1.291	1.300
B-splines	1	1	0.359	0.387	0.300	0.317	0.047	0.093	0.097	0.256	0.206	0.818	0.671	1.044
Partitioning	1	1	0.090	0.200	0.070	0.147	0.044	0.087	0.090	0.219	0.127	0.426	0.152	0.702
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.18	0.25	0.401	0.419	0.305	0.320	0.365	0.369	0.428	0.413	1.556	1.224	1.557	1.240
B-splines	27	00	0.398	0.416	0.308	0.317	0.368	0.369	0.358	0.452	0.645	1.348	0.685	1.329
Partitioning 7.1	2.5	xo	0.489	0.571	0.375	0.435	0.245	2.043	0.432	2.504	0.832	3.185	1.519	3.821
reasible Estimation	96 0	90 0	096.0	0000	0 0 0 0	000	0 901	900	0.04	096 0	0060	000	0000	0 0
Benlinge	5.0	0.50	0.309	0.360	0.20	0.230	0.331	0.363	0.040	0.00	0.383	1.001	0.026 73.75	1 000
Partitioning	4	<b>1</b> 4	0.380	0.479	0.291	0.361	0.485	1.379	0.459	1.620	0.514	2.010	0.729	2.589
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.34	0.36	0.138	0.195	0.095	0.138	0.060	0.097	0.182	0.237	0.399	0.549	0.399	0.549
B-splines	1	1	0.106	0.186	0.074	0.131	0.046	0.087	0.091	0.240	0.197	0.754	0.197	0.754
Partitioning	П	1	0.107	0.203	0.077	0.149	0.046	0.088	0.091	0.221	0.134	0.426	0.169	0.745
Local Polynomial	0.37	0.27	0.100	0.137	0.066	0.095	0.051	0.081	0.157	0.203	0.157	0.203	0.157	0.203
B-splines	2	1	0.117	0.187	0.081	0.131	0.083	0.088	0.139	0.243	0.253	0.770	0.253	0.770
Partitioning	2	1	0.151	0.213	0.101	0.153	0.184	0.217	0.204	0.338	0.231	0.543	0.260	0.804
						Model 3.4								
Infeasible Estimation	,		1	1	1	1	1	1		1		1	1	1
Local Polynomial	0.11	0.0	0.588	0.551	0.256	0.208	0.166	0.095	0.473	0.339	4.660	0.507	5.969	3.837
D-spilles Partitioning	4 6	27.0	0.034	0.4.0	0.202	0.203	0.909	0.120	1 916	6 145	2 477	19.806	4.255	3 994
Feasible Estimation		i				0.110			2	0				
Local Polynomial	0.32	0.26	0.561	0.544	0.184	0.202	0.053	0.082	0.361	0.415	0.361	0.415	3.818	3.821
B-splines	7	က	0.561	0.514	0.222	0.242	0.158	0.114	0.281	0.397	0.671	2.137	3.862	4.322
Partitioning	7	က	0.543	0.524	0.298	0.310	0.589	1.075	0.421	1.317	0.661	1.870	3.250	3.776
						Model 3.5								
Infeasible Estimation					1		,	,			,	1		1
Local Polynomial	0.33	0.33	0.720	0.611	0.560	0.493	1.090	1.058	0.831	0.952	1.026	0.858	0.623	0.958
D-spines	2 6	27.	0770	1 103	0.038	0.512	0.995	1.173 0.535	0.717	6.429	1.229	10.892	1.636	0.042
Feasible Estimation	1	ī	9	2				0000				1000	200-1	1
Local Polynomial	0.28	0.27	0.713	0.616	0.558	0.495	1.199	1.194	0.984	1.178	0.882	0.697	0.362	0.534
B-splines	œ	4	0.775	0.705	0.592	0.560	0.785	1.056	0.630	0.830	0.976	1.696	0.527	1.390
Partitioning	œ	4	0.725	0.669	0.543	0.517	1.024	1.780	0.839	1.844	1.188	2.477	1.078	3.162

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.52: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 500, \sigma^2 = 4, X_{i,\ell} \sim \beta(0.5, 0.5), \text{ Uniform Cells}$ 

Doggo.	Parameter	arameter	MSE	E	M	MAE	(0.5, 0.5, 0.5)	5,0.5)	$(0.1, 0.5, 0.5) \qquad (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, $	5,0.5)	(0.1, 0.1, 0.5)	(3.0,	(0.1, 0.1, 0.1)	,0.1)
Degree.	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation	1	1		1	1			1	1				1	1
Local Polynomial	0.0	6.0	0.484	0.516	0.417	0.434	0.094	0.185	0.165	0.246	0.242	0.304	0.693	0.715
D-spines Partitioning	-	- ۱	0.179	0.399	0.141	0.307	060.0	0.200	0.151	0.345	0.231	0.450	0.242	0.671
Feasible Estimation	+	4	1.0		1				101.0		2			5
Local Polynomial	0.38	0.28	0.662	0.686	0.540	0.558	0.122	0.213	0.213	0.270	0.676	0.695	1.299	1.309
B-splines	1	1	0.489	0.573	0.420	0.470	0.106	0.235	0.169	0.340	0.284	0.486	0.710	0.809
Partitioning	П	1	0.190	0.399	0.145	0.307	0.161	0.206	0.185	0.345	0.218	0.475	0.253	0.671
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.497	0.583	0.391	0.456	0.505	0.590	0.531	0.616	0.589	0.643	0.535	0.563
B-splines	œ	œ	0.447	0.565	0.350	0.441	0.612	0.621	0.566	0.619	0.544	0.602	0.427	0.540
Partitioning	œ	œ	0.594	1.136	0.466	0.874	1.040	8.320	1.028	5.138	0.816	3.056	0.633	1.467
Feasible Estimation	ç	9	000	0 0 0	000	6	0	2.0	0.70	1	1 2 2	000	0	7000
Local Folynomial	40.0	0.20	0.000	0.4.0	0.503	0.045	0.57.0	0.403	0.542	0.570	0.557	0.000	0.527	0.004
b-spines Partitioning	4 4		0.425	0.504	0.331	0.397	0.486	0.460	0.435	0.408	0.488	0.592	0.387	0.513
0	ı	ı				Model 3.3								
Infeasible Estimation														
Local Polynomial	0.3	0.37	0.303	0.426	0.227	0.326	0.192	0.287	0.316	0.372	0.450	0.472	0.450	0.472
B-splines	П	1	0.208	0.370	0.154	0.281	0.097	0.232	0.151	0.331	0.271	0.473	0.271	0.473
Partitioning	-1	1	0.209	0.403	0.158	0.310	0.097	0.207	0.151	0.346	0.206	0.477	0.261	0.682
reasivie Estimation Local Polymomial	0 7 8	96 0	0 100	0.973	0 143	0.907	0 194	806.0	7060	0.259	0.904	0.050	0.907	0.950
B-splines		-	0.240	0.370	0.145	182.0	0.246	0.232	0.266	0.233	0.204	0.203	0.319	0.473
Partitioning	က	-	0.346	0.407	0.239	0.311	0.564	0.394	0.496	0.472	0.459	0.531	0.409	0.683
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.09	0.0	1.872	2.055	1.007	0.891	0.771	0.234	0.735	0.413	1.504	0.726	4.087	3.835
B-splines	125	27	2.039	1.959	1.032	0.999	0.361	0.362	0.560	0.598	1.620	1.427	4.138	4.047
Fartitioning	125	7.7.	1.510	1.547	0.944	0.892	0.660	0.048	0.941	1.063	1.786	3.500	5.96.6	7.999
reasible Estimation Local Polynomial	0.3	0.26	2.083	2.070	0.846	883	0.135	0.211	0.506	0.408	0.506	0.408	3.795	3.793
B-splines	, oo	; ;	2.074	2.029	0.925	0.946	0.460	0.274	0.552	0.535	0.897	1.407	3.872	4.041
Partitioning	œ	1	1.657	1.772	1.081	1.068	2.807	2.929	0.950	1.219	1.623	1.117	1.538	2.407
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.852	0.715	0.666	0.569	1.145	1.303	1.331	1.202	0.720	0.924	0.739	0.686
B-splines	27.	2.7.	0.930	0.773	0.741	0.613	0.859	1.452	1.192	1.374	0.895	0.826	0.644	0.760
Fartitioning Feasible Fetimation	7	7	1.109	T.003	0.010	1.001	1.340	0.999	4.024	1.409	0.307	4.019	0.922	061.1
Local Polynomial	0.28	0.26	0.783	0.605	0.614	0.486	1.200	1.214	1.364	1.184	0.514	0.709	0.706	0.547
B-splines	∞	1	0.969	0.878	0.759	0.703	0.709	0.740	1.215	1.202	0.847	0.885	0.593	0.678
Partitioning	œ	1	1.004	0.872	0.790	969.0	1.284	0.887	1.595	1.354	1.033	0.720	0.788	0.834

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.53: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=500,\,\sigma^2=4,\,X_{i,\ell}\sim\beta(1,1),$  Uniform Cells

Degree: Li  Infeasible Estimation Local Polynomial B-splines Partitioning Partitioning Partitioning B-splines P-splines Partitioning Infeasible Estimation Local Polynomial B-splines Partitioning Partitioning	Linear 0.9 0.38 0.38 0.21 8 8 0.34 5 5	Cubic 0.9 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	Linear 0.443 0.174 0.174 0.606 0.450 0.194 0.501 0.501 0.506	Cubic 0.480 0.542	Linear 0 375	Cubic Model 3.1	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
	0.9 0.38 0.21 8 8 0.34	0.9 0.27 0.28 0.28 0.25 0.25	0.443 0.174 0.174 0.606 0.450 0.194 0.501 0.506 0.596	0.480	0.375	Model 3.1								
	0.9 0.38 0.21 0.34 0.34	0.9 0.27 0.28 0.28 0.25 0.25	0.443 0.174 0.174 0.606 0.450 0.194 0.501 0.506 0.596	0.480	0.375									
	0.3 0.38 0.34 0.34 0.34	0.29 0.27 0.28 0.25 0.25 1	0.443 0.174 0.174 0.606 0.450 0.194 0.501 0.506 0.596	0.542	375	0	0	1	1	1	(	0	0	1
	0.38 0.38 0.34 0.34 0.34	0.27 0.28 0.28 0.25 0.25	0.450 0.174 0.606 0.450 0.194 0.501 0.596	0.042	0.373	0.398	0.089	0.179	0.170	0.267	0.244	0.361	0.688	0.735
	0.38 88 88 88 5 5 5 5 5 5 5 5 5 5 5 5 5 5	0.27 0.28 0.25 0.25 1.1	0.606 0.450 0.194 0.501 0.450 0.596	מממ	0.579	0.437	0.000	0.212	0.100	0.367	0.505	0.002	0.708	0.033
	0.38 0.21 0.34 0.34	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	0.606 0.450 0.194 0.501 0.506 0.596	66.0	0.150	100.0	0.000	107:0	0.100	0.00	0.0	500.0	0.5	50.0
	0.21 8 8 8 0.34 7	0 0.28 0 0.25 0 0.25	0.450 0.194 0.501 0.596 0.596	0.633	0.494	0.514	0.112	0.192	0.239	0.297	0.687	0.710	1.306	1.319
	0.21 8 8 8 0.34 5	0.28 8 8 8 0.25 1	0.194 0.501 0.450 0.596	0.542	0.380	0.437	0.105	0.212	0.170	0.367	0.310	0.682	0.710	0.899
	0.21 8 8 0.34 5	$\begin{array}{ccc} 0.28 & & & \\ 8 & 8 & & \\ 0.25 & & & \\ 1 & & & \end{array}$	0.501 0.450 0.596	0.399	0.144	0.301	0.210	0.204	0.222	0.361	0.245	0.567	0.278	0.874
	0.21 8 8 0.34 5	0.28 8 8 0.25 1	0.501 0.450 0.596			Model 3.2								
	0.21 8 8 8 8 5 7	0.28 8 0.25 1	0.501 0.450 0.596 0.399											
B-splines Partitioning	8 8 8 8 E C C C C	0.25 1	0.450 0.596 0.399	0.582	0.389	0.450	0.466	0.533	0.502	0.542	0.705	0.719	0.724	0.729
Partitioning	0.34 5	8 0.25 1	0.596	0.571	0.350	0.441	0.568	0.561	0.511	0.586	0.566	0.813	0.537	0.839
	0.34 5 5	0.25 $1$	0 399	1.135	0.467	0.873	0.957	6.199	0.955	4.840	0.855	3.580	0.858	2.477
	0.34 52	0.25	55%	0	0	0	0		1		0	0	0	0
ynomial	ကေ	-	3 0	0.439	0.309	0.343	0.387	0.416	0.373	0.410	0.363	0.396	0.358	0.396
B-splines	o	-	0.432	0.510	0.335	0.398	0.495	0.456	0.436	0.470	0.511	1.059	0.487	0.732
Fartitioning		7	0.519	0.571	0.398	0.434	0.799	1.403	0.714	0.889	0.084	1.002	0.079	0.982
						Model 3.3								
Infeasible Estimation	2	0	1	1	500	900	0	0	0	0.0	, 1	li Li	i c	i.
Local Folynomial	U.34	0.50	0.277	0.395	0.201	0.230	0.140	0.232	0.294	0.540	0.032	0.000	70000	0.000
D-spines	٦.	٠.	0.195	0.365	0.145	0.270	0.000	0.209	0.159	0.555	0.230	0.007	0.298	0.007
r artifolisiig Feasible Estimation	4	-	0.130	0.407	0.140	U.304	0.030	0.70	0.1.00	0.00	0.213	0.00	0.403	0.033
	0.34	0.25	0.185	0.264	0.133	0.197	0.116	0.192	0.234	0.285	0.234	0.285	0.234	0.285
	4	-	0.245	0.367	0.179	0.271	0.262	0.215	0.318	0.362	0.408	0.665	0.408	0.665
Partitioning	4	1	0.390	0.428	0.274	0.314	0.622	0.714	0.550	0.659	0.546	0.766	0.558	0.957
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.1	6.0	1.209	1.337	0.676	0.550	0.517	0.215	0.653	0.420	2.118	0.693	4.419	3.844
B-splines	125	27	1.338	1.264	0.707	0.676	0.291	0.305	0.554	0.601	2.127	2.164	4.424	4.442
Partitioning	125	27	1.528	1.569	0.914	0.830	1.193	2.635	1.729	4.020	2.222	15.117	4.131	15.210
Feasible Estimation														
Local Polynomial	O.3	0.25	1.353	1.342	0.500	0.539	0.123	0.193	0.475	0.413	0.475	0.413	3.809	3.807
B-splines	χo	no	1.353	1.310	0.576	0.616	1.650	0.270	0.517	0.528	1 224	1.758 1.955	3.899	2.221
rateteloning	0	2	1.130	617:1	0.140	0.1.0	T.000	0.4.0	0.00	404.4	T-00-T	1.000	0.40.4	9.000
Infoamily Detion ation						Model 3.5								
	000	000	000	0 700	0.641	7 14 14	1 190	0000	1 1 7 1	1 006	0.00	010	0 100	042
	0.00	0.00	0.022	0.787	0.041	0.07.0	0.043	1.220	1.1.1	1.090	1 0 2 2	1.085	0.750	1 196
Partitioning	27	27	1.091	1.613	0.857	1.285	0.930	2.643	1.041	4.209	1.168	14.812	1.331	15.577
Feasible Estimation														
ynomial	0.29	0.26	0.774	0.624	0.605	0.501	1.212	1.220	1.265	1.205	0.629	0.708	0.619	0.575
B-splines	œ	1	0.922	0.860	0.711	0.682	0.786	0.864	0.998	1.023	0.933	1.081	0.641	0.839
Partitioning	œ	1	0.960	0.869	0.744	0.684	1.264	1.371	1.320	1.277	1.150	1.103	0.997	1.210

Table C.54: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=500,\,\sigma^2=4,\,X_{i,\ell}\sim\beta(2,2),$  Uniform Cells

threst         Cubic         Linear         Linear         Linear         Linear         Linear         Linear <th< th=""><th></th><th>Tuning Parameter</th><th>Tuning arameter</th><th>Root Integrated MSE</th><th>egrated</th><th>Ingetrated</th><th>rateu 1E</th><th>(0.5,0.5,0.5)</th><th>5,0.5)</th><th>Foint E <math>(0.1,0.5,0.5)</math></th><th>(0.5,0.5) <math>(0.1,0.1)</math></th><th>(0.1,0.1,0.5)</th><th>1,0.5)</th><th>(0.1,0.1,0.1)</th><th>(1.0.1)</th></th<>		Tuning Parameter	Tuning arameter	Root Integrated MSE	egrated	Ingetrated	rateu 1E	(0.5,0.5,0.5)	5,0.5)	Foint E $(0.1,0.5,0.5)$	(0.5,0.5) $(0.1,0.1)$	(0.1,0.1,0.5)	1,0.5)	(0.1,0.1,0.1)	(1.0.1)
Nodel 3.1   Nodel 3.2   Node	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.9         0.38         0.423         0.346         0.344         0.091         0.156         0.148         0.092         0.177         0.185         0.438         0.259         0.777         0.185         0.438         0.259         0.777         0.185         0.2498         0.077         0.185         0.039         0.177         0.185         0.2498         0.078         0.177         0.185         0.2498         0.089         0.177         0.185         0.039         0.177         0.185         0.0498         0.177         0.185         0.0498         0.177         0.185         0.0498         0.177         0.185         0.0498         0.185         0.0499         0.177         0.185         0.0499         0.177         0.185         0.0499         0.177         0.185         0.0499         0.177         0.0498         0.0499         0.0740         0.0499         0.0740         0.0499         0.0776         0.0740         0.0499         0.0740         0.0499         0.0740         0.0499         0.0740         0.0499         0.0740         0.0499         0.0740         0.0499         0.0740         0.0499         0.0740         0.0490         0.0740         0.0490         0.0740         0.0490         0.0740							Model 3.1								
1,	nfeasible Estimation	d	c	0000	400	2100	0	100	, ,	0 101	696 0	000	000	0000	000
1	Local Folynomial	e:-	S	0.303	0.423	0.510	0.344	0.091	0.159	0.197	0.303	0.230	1 550	0.039	0.020 1.020 1.020
0.37         0.27         0.516         0.547         0.418         0.440         0.105         0.176         0.272         0.427         0.426         0.78         0.185         0.176         0.272         0.427         0.407         0.679         0.427         0.679         0.427         0.679         0.427         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.681         1.077         0.681         0.682         0.878         0.678         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.679         0.678         0.681         0.681         0.681         0.682         0.687         0.681         0.678         0.679         0.679         0.679         0.679         0.679         0.678         0.681         0.681         0.682         0.687         0.	D-spinies Partitioning	-	-	0.180	0.401	0.139	0.294	0.080	0.175	0.180	0.438	0.253	0.851	0.303	1.409
0.37         0.27         0.27         0.56         0.75         0.76         0.78         1.22           2         1         0.41         0.42         0.345         0.136         0.176         0.427         0.769         1.671         0.78         1.287           2         1         0.0274         0.420         0.185         0.392         0.179         0.78         1.671         0.481         0.678         0.784         1.671         0.788         1.672         1.287           8         8         0.447         0.584         0.371         0.441         0.441         0.441         0.442         0.524         0.788         0.734         0.768         1.385         1.671         0.481           8         8         0.585         1.133         0.446         0.863         0.449         0.441         0.442         0.442         0.442         0.442         0.442         0.442         0.442         0.442	easible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.37	0.27	0.516	0.547	0.418	0.440	0.103	0.166	0.327	0.422	0.719	0.768	1.322	1.349
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	2	1	0.401	0.497	0.328	0.389	0.155	0.179	0.272	0.497	0.496	1.579	0.811	1.719
0.21         0.28         0.492         0.594         0.371         0.441         0.391         0.416         0.594         0.635         1.385         1.672         1.387           8         0.447         0.588         0.447         0.426         0.841         0.492         0.583         1.417         0.793         0.758         1.287         1.790           8         8         0.447         0.588         0.447         0.426         0.441         0.526         0.150         0.426         0.747         0.790         0.748         0.774         1.700         6.217         1.790           0.33         0.25         0.396         0.445         0.526         0.030         0.742         0.437         0.606         0.714         0.706         6.714         0.706         6.217         0.710           0.41         0.42	Partitioning	2	1	0.274	0.420	0.185	0.302	0.345	0.428	0.397	0.679	0.424	1.071	0.482	1.534
0.21         0.28         0.447         0.594         0.371         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.441         0.426         0.437         0.426         0.437         0.441         0.426         0.437         0.441         0.442         0.442         0.443         0.444         0.456         0.441         0.552         0.437         0.441         0.441         0.552         0.442         0.549         0.549         0.744         0.746         0.447         0.441         0.447         0.441         0.441         0.444         0.441         0							Model 3.2								
0.21         0.28         0.442         0.594         0.371         0.441         0.391         0.446         0.594         0.574         0.655         1.752         1.385         0.7734         2.577         1.079         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.734         0.747         0.747         0.704         0.747         0.704         0.774         0.707         0.734         0.735         0.747         0.706         0.774	nfeasible Estimation														
8         0.247         0.568         0.341         0.442         0.441         0.442         0.524         0.524         0.754         2.517         0.799           0.33         0.25         0.244         0.568         0.441         0.426         0.504         4.797         1.098         0.437         0.669         0.744         0.706         2.277         0.706           7         6         0.560         1.008         0.435         0.435         0.437         0.817         3.994         0.598         0.577         0.500         0.437         0.706         2.279         0.776           1         0.1         0.560         1.008         0.426         0.248         0.107         0.835         0.998         0.934         5.375         1.215           0.41         0.1         0.260         0.144         0.295         0.089         0.174         0.181         0.440         0.505         0.175         0.181         0.440         0.560         0.527         0.560         0.589         0.589         0.074         0.686         0.625         0.648         0.666         0.674         0.667         0.181         0.480         0.780         0.417         0.181         0.480	Local Polynomial	0.21	0.28	0.492	0.594	0.371	0.441	0.391	0.416	0.594	0.635	1.385	1.672	1.387	1.681
8         0.588         1.133         0.456         0.863         0.876         3.964         0.901         4.797         1.009         6.217         1.308           0.33         0.25         0.399         0.437         0.392         0.450         0.450         0.519         0.450         0.500         0.719         0.500         0.776           7         6         0.560         1.008         0.432         0.740         0.817         3.397         0.852         3.998         0.934         5.375         1.215           0.41         0.41         0.520         1.008         0.442         0.826         0.740         0.887         0.989         0.174         0.805         0.747         0.880         0.392         1.215         1.215           0.41         0.41         0.223         0.740         0.887         0.276         0.105         0.174         0.181         0.499         0.880         0.934         5.755         1.219         1.219           0.33         0.25         0.145         0.286         0.286         0.050         0.714         0.706         0.417         0.050         0.714         0.707         0.881         0.392         0.417         0.181	B-splines	œ i	oo ·	0.447	0.568	0.341	0.426	0.441	0.442	0.524	0.758	0.734	2.517	0.790	2.492
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	00	x	0.585	1.133	0.456	0.863	0.876	3.964	0.901	4.797	1.009	6.217	1.308	7.213
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	66.0	C C	9060	707	0000	7000	0000	007	24	0 1	400	000	707	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Benlines	7.00	0.4.0	0.330	0.4.0 0.4.0	0.302	0.004	0.032	0.409	0.400 707	0.019	0.425	9 270	0.457	9 965
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	- 1-	ာဖ	0.560	1.008	0.432	0.740	0.817	3.397	0.832	3.998	0.934	5.375	1.215	6.184
0.41         0.41         0.231         0.346         0.165         0.248         0.105         0.174         0.305         0.442         0.625         0.966         0.625         0.966         0.625         0.969         0.174         0.181         0.480         0.392         1.509         0.392         1.509         0.392           1         1         0.189         0.364         0.126         0.089         0.174         0.181         0.480         0.392         1.509         0.392           0.33         0.25         0.173         0.255         0.123         0.182         0.069         0.175         0.181         0.480         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.489         0.417         0.489         0.417         0.489         0.417         0.489         0.417         0.329         0.417         0.489         0.417	)						Model 3.3								
0.41         0.431         0.346         0.155         0.248         0.105         0.174         0.305         0.442         0.625         0.956         0.0492         0.174         0.181         0.480         0.256         0.059         0.174         0.181         0.480         0.257         0.058         0.174         0.181         0.480         0.257         0.059         0.177         0.181         0.480         0.257         0.182         0.182         0.182         0.182         0.059         0.177         0.181         0.480         0.257         0.059         0.177         0.181         0.480         0.317         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.329         0.417         0.418         0.044         0.056         0.277         0.267         0.166         0.245         0.277         0.267         0.166         0.245         0.428         0.412         0.418         0.418         0.418         0.411         0.418         0.412 <th< td=""><td>nfeasible Estimation</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></th<>	nfeasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.41	0.41	0.231	0.346	0.165	0.248	0.105	0.177	0.305	0.442	0.625	0.956	0.625	0.956
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	н	1	0.188	0.361	0.138	0.256	0.089	0.174	0.181	0.480	0.392	1.509	0.392	1.509
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	П	П	0.189	0.402	0.144	0.295	0.089	0.175	0.181	0.439	0.257	0.851	0.313	1.431
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.25	0.173	0.255	0.123	0.182	0.105	0.166	0.329	0.417	0.329	0.417	0.329	0.417
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	9	1	0.260	0.413	0.190	0.291	0.261	0.247	0.425	0.625	0.648	2.135	0.648	2.135
0.13 0.9 0.632 0.591 0.387 0.277 0.267 0.166 0.746 0.453 4.138 0.672 5.652 0.591 0.387 0.277 0.456 0.236 0.872 0.779 2.471 6.213 4.448 0.39 0.39 0.542 0.776 1.819 0.961 3.752 12.288 4.721 39.529 5.277 0.456 0.236 0.872 0.779 0.457 0.457 0.456 0.236 0.872 0.779 2.471 6.213 4.448 0.31 0.25 0.656 0.627 0.224 0.267 0.106 0.167 0.481 0.723 0.890 3.024 3.908 0.696 0.607 0.629 0.726 0.865 3.380 0.786 4.128 0.880 3.024 3.908 0.308 0.691 1.019 0.459 0.726 0.865 3.380 0.786 4.128 0.980 5.223 3.319 0.33 0.755 0.705 0.585 0.560 1.005 1.002 1.189 0.799 1.152 1.572 6.088 1.259 0.390 0.643 0.615 1.002 1.189 0.790 1.2471 1.687 39.465 2.983 0.299 0.790 0.790 0.643 0.615 1.200 0.802 0.914 0.782 0.455 0.790 0.775 0.829 0.770 0.809 0.775 0.809 0.777 0.809 0.77	Partitioning	9	ıΩ	0.459	0.880	0.339	0.604	0.662	2.917	0.696	3.451	0.760	4.610	0.815	5.435
0.13         0.9         0.632         0.591         0.387         0.277         0.267         0.166         0.453         4.138         0.675         5.652           64         27         0.658         0.666         0.396         0.427         0.456         0.236         0.872         0.779         2.471         6.213         4.448           64         27         1.297         1.321         0.942         0.776         1.819         0.961         3.752         12.288         4.721         39.529         5.277           8         6         0.679         0.582         0.276         0.206         0.267         0.167         0.459         0.541         3.752         12.288         4.721         39.529         5.277           8         6         0.606         0.627         0.267         0.166         0.167         0.459         0.563         0.481         0.723         0.890         5.223         3.319           9         0.609         1.019         0.459         0.756         0.266         3.380         0.786         4.128         0.980         5.223         3.319           0.53         0.73         0.643         0.560         1.002         1.089							Model 3.4								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	sfeasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.13	6.0	0.632	0.591	0.387	0.277	0.267	0.166	0.746	0.453	4.138	0.672	5.652	3.864
0.31 0.25 0.579 0.582 0.224 0.267 0.106 0.167 0.459 0.544 0.459 0.544 3.828 0.606 0.627 0.294 0.370 0.296 0.263 0.481 0.723 0.890 3.024 3.908 0.606 0.627 0.294 0.370 0.296 0.263 0.481 0.723 0.890 3.024 3.908 0.691 1.019 0.459 0.726 0.865 3.380 0.786 4.128 0.980 5.223 3.319 0.33 0.33 0.755 0.705 0.585 0.560 1.095 1.071 0.892 1.043 1.250 1.368 0.944 0.785 0.790 0.643 0.615 1.002 1.189 0.799 1.152 1.572 6.088 1.259 0.790 0.643 0.615 1.002 1.186 1.011 1.236 0.944 0.782 0.455 0.645 0.657 0.809 1.200 0.802 0.992 0.992 0.992 0.992 0.994 0.782 0.455 0.645 0.677 0.612 0.873 1.186 1.011 1.236 0.944 0.782 0.455 0.645 0.677 0.612 0.873 1.152 0.741 1.674 1.152 0.779 0.779 0.778 0.677 0.612 0.872 0.741 1.074 1.152 0.782 0.455 0.779 0.779 0.778 0.677 0.612 0.872 0.741 1.074 1.152 0.888 0.779 0.779 0.778 0.677 0.612 0.872 0.741 1.074 1.152 0.638 0.779 0.779 0.778 0.778 0.677 0.612 0.872 0.741 1.074 1.152 0.638 0.779 0.779 0.778 0.677 0.612 0.677 0.612 0.779 0.771 0.774 1.152 0.638 0.779 0.779 0.778 0.677 0.612 0.677 0.612 0.771 0.774 1.152 0.638 0.779 0.779 0.779 0.778 0.779 0.779 0.778 0.779 0.779 0.779 0.779 0.778 0.779 0.779 0.779 0.779 0.779 0.779 0.778 0.779 0.77	B-splines	64	27	0.658	0.666	0.396	0.427	0.456	0.236	0.872	0.779	2.471	6.213	4.448	7.154
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	64	27	1.297	1.321	0.942	0.776	1.819	0.961	3.752	12.288	4.721	39.529	5.277	4.588
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	5	Ç	11	0	200	0	001	0	7	о 2	2 7	, 1	000	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial Renlines	0.31	0.75	0.579	0.582	0.224	0.267	0.106	0.167	0.459	0.544	0.459	3.024	3.008	3.835
Model 3.5 $ \begin{array}{ccccccccccccccccccccccccccccccccccc$	Partitioning	° 00	9	0.691	1.019	0.459	0.726	0.865	3.380	0.786	4.128	0.980	5.223	3.319	6.607
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	)						Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	sfeasible Estimation														
27         27         0.829         0.730         0.643         0.615         1.002         1.189         0.739         1.152         1.572         6.088         1.259           27         27         1.051         1.515         0.809         1.200         0.802         0.992         0.950         12.471         1.687         39.465         2.983           .         0.29         0.25         0.733         0.646         0.572         0.517         1.197         1.1186         1.011         1.236         0.944         0.782         0.455           8         6         0.809         0.778         0.617         0.612         0.632         1.152         0.741         1.074         1.152         2.638         0.779	Local Polynomial	0.33	0.33	0.755	0.705	0.585	0.560	1.095	1.071	0.892	1.043	1.250	1.368	0.944	1.438
27         27         1.051         1.515         0.809         1.200         0.802         0.992         0.950         12.471         1.687         39.465         2.983           0.29         0.25         0.733         0.646         0.572         0.517         1.197         1.186         1.011         1.236         0.944         0.782         0.455           8         6         0.809         0.778         0.617         0.612         0.832         1.152         0.741         1.074         1.152         2.638         0.779	B-splines	27	27	0.829	0.790	0.643	0.615	1.002	1.189	0.799	1.152	1.572	880.9	1.259	5.874
0.29 0.25 0.733 0.646 0.572 0.517 1.197 1.186 1.011 1.236 0.944 0.782 0.455 8 6 0.809 0.778 0.617 0.612 0.832 1.152 0.741 1.074 1.152 2.638 0.799	Partitioning	27	27	1.051	1.515	0.809	1.200	0.802	0.992	0.950	12.471	1.687	39.465	2.983	3.067
8 6 0.809 0.617 0.612 0.612 0.832 1.152 0.741 1.074 1.152 2.639 0.799	easible Estimation	06.0	C C	0 400	979	0 110	1	101	100	1 011	1 096	7700	0 100	2 7 11	0.40
8 0 0.809 0.178 0.017 0.012 0.832 1.152 0.741 1.1074 1.152 1.038 0.799	Local Folynomial	0.29	0.20	0.733	0.040	0.072	0.517	1.197	1.180	1.011	1.230	444	0.782	0.455	0.048
	B-splines	<b>x</b> 0 (	9	0.809	0.778	0.617	0.612	0.832	1.152	0.74I	1.074	1.152	2.638	0.799	2.463

Table C.55: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 1000, \sigma^2 = 1, X_{i,\ell} \sim \beta(0.5, 0.5), \text{Uniform Cells}$ 

nation omial ution omial omial	Linear 0.9 1 1 0.46	Cubic					(0:0,0:0,0:0)	, - , -						(=:::::::::::::::::::::::::::::::::::::
Infeasible Estimation Local Polynomial B-splines Partitioning Partitioning Local Polynomial B-splines Partitioning Partitioning Partitioning Partitioning Partitioning Partitioning Second Polynomial B-splines	0.9 1 1 0.46		Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
njeasible Estimation Local Polynomial B-splines Partitioning Partitioning B-splines Fartitioning P-splines Partitioning Partitioning Partitioning Partitioning Partitioning Partitioning Partitioning P-splines	0.9 1 1 0.46					Model 3.1								
Decal Polynomial B-splines Partitioning Partitioning Local Polynomial B-splines Partitioning Partitioning Local Polynomial B-splines Local Polynomial	0.9 1 1 0.46 1													
Partitioning Partitioning Partitioning Partitioning Partitioning Partitioning Partitioning Partitioning Local Polynomial Local Polynomial B-splines	0.46	6.0	0.458	0.462	0.403	0.404	0.034	0.073	0.062	0.092	0.091	0.116	0.648	0.656
easible Estimation Local Polynomial B-splines Partitioning  ryleasible Estimation Local Polynomial	0.46 1 1	٦.	0.459	0.403	0.404	0.407	4000	0.030	0.060	0.124	0.103	0.177	0.000	00.00
Local Polynomial B-splines Partitioning  nfeasible Estimation Local Polynomial B-splines	0.46	1	0.007	0.141	0.049	0.103	0.030	0.07	0.034	0.113	0.07	0.100	0.00	0.220
B-splines Partitioning nfeasible Estimation Local Polynomial B-splines		0.31	0.645	0.648	0.526	0.528	0.043	0.081	0.080	0.106	0.648	0.653	1.285	1.288
Partitioning nfeasible Estimation Local Polynomial B-splines	п	;	0.459	0.469	0.404	0.407	0.034	0.093	090.0	0.124	0.103	0.177	0.650	0.667
nfeasible Estimation Local Polynomial B-splines		1	0.062	0.141	0.049	0.109	0.030	0.077	0.054	0.119	0.071	0.168	0.085	0.225
nfeasible Estimation Local Polynomial B-splines						Model 3.2								
Local Polynomial B-splines														
B-splines	0.16	0.23	0.382	0.396	0.297	0.307	0.468	0.497	0.392	0.432	0.370	0.402	0.287	0.319
	27	œ	0.380	0.386	0.296	0.297	0.403	0.495	0.335	0.406	0.384	0.377	0.257	0.268
Partitioning	27	œ	0.374	0.421	0.292	0.322	0.321	2.568	0.327	1.655	0.294	0.903	0.301	0.417
Feasible Estimation	0	o c	0	0	6	1000	1100	0	000	000	000	000	900	0
Local Polynomial	0.38	0.28	0.309	0.374	0.281	0.287	0.357	0.353	0.284	0.293	0.298	0.302	0.205	0.274
b-spinnes Partitioning			0.370	0.375	0.284	0.294	0.388	0.408	0.271	0.534	0.335	0.280	0.246	0.249
)						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.22	0.3	0.156	0.202	0.112	0.151	0.095	0.127	0.131	0.170	0.166	0.216	0.166	0.216
B-splines	œ	œ	0.138	0.186	960.0	0.138	0.151	0.153	0.142	0.159	0.135	0.166	0.135	0.166
Partitioning	œ	œ	0.187	0.400	0.145	0.305	0.356	2.133	0.278	1.308	0.250	0.832	0.198	0.410
easible Estimation	0	0	0	07.	0	700	0.00	0	0	500	0	100	0	100
Local Folynomial	0.09	0.28	0.130	0.140	0.074	0.094	0.040	0.07	0.072	0.091	0.07	0.091	0.072	0.091
D-spines		٦.	0.126	0.102	0.080	0.110	0.043	0.000	0.00	0.114	0.030	0.101	0.030	0.101
t di dicioning	-	4	0	0.10	100.0	0.111	1000	0000	0000	0.17	0.100	7.7.	7	0.400
Infeasible Estimation						Model 5.4								
Local Polynomial	0.07	6.0	1.844	2.068	0.813	0.823	0.361	0.133	0.275	0.237	0.945	0.472	3.936	3.837
B-splines	343	64	2.009	1.964	0.876	0.859	0.179	0.225	0.268	0.317	1.137	0.825	4.003	3.904
Partitioning	343	64	1.162	1.130	0.543	0.505	0.115	0.000	0.466	1.182	8.935	8.965	2.670	3.624
Feasible Estimation														
Local Polynomial	0.29	0.26	2.084	2.076	0.803	0.816	0.049	0.077	0.317	0.226	0.317	0.226	3.817	3.800
B-splines	œ ·	7	2.076	2.047	0.846	0.848	0.202	0.119	0.307	0.310	0.606	0.970	3.865	3.951
Partitioning	œ	7	1.618	1.740	1.022	0.984	2.474	1.815	0.450	0.501	1.492	0.562	1.313	2.095
:						Model 3.5								
Infeasible Estimation	0	0	0	0	0	1	,	1	0	1	1	!	0	
Local Polynomial	0.33	0.33	0.818	0.582	0.639	0.465	1.141	1.285	1.306	1.119	0.578	0.747	0.619	0.468
D-splines	1 -	7 10	0.870	0.595	0.030	0.470	0.845	1.455	1.159	1.290	0.749	0.020	0.408	0.073
Fartitioning Feasible Estimation	7	7	071.0	0.740	0.575	0.501	0.011	7.000	0.071	2.100	0.341	1.030	0.030	0.050
Local Polynomial	0.25	0.28	0.725	0.567	0.570	0.455	1.212	1.213	1.385	1.156	0.438	0.667	0.712	0.487
B-splines	00	-	0.936	0.820	0.735	0.661	223	0.711	1.156	1.166	0.716	0.724	0.473	0.501
Dartitioning	oo	٠ -	868.0	797.0	0.704	0.001	0.000	0.862	1.380	1 321	0.747	0.534	0.590	0.532

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.56: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=1000,\,\sigma^2=1,\,X_{i,\ell}\sim\beta(1,1),$  Uniform Cells

	Parameter	arameter	MSE	<b>E</b>	MAE	1E	(0.5, 0.5, 0.5)	5,0.5)	(0.1,0.5,0.5)	5,0.5)	(0.5,0.5) $(0.1,0.1,0.5)$	1,0.5)	(0.1, 0.1, 0.1)	1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
:						Model 3.1								
Infeasible Estimation		¢		0	0	0	0	0	0	0	0	1	0	0
Local Polynomial R-splines	0.0 -	o.o	0.419	0.423	0.362	0.364	0.036	0.065	0.067	0.104	0.095	0.135	0.646	0.649
Partitioning		٠.	0.065	0.142	0.051	0.107	0.033	0.069	0.059	0.127	0.077	0.197	0.090	0.288
Feasible Estimation														
Local Polynomial	0.45	0.31	0.590	0.592	0.481	0.484	0.042	0.073	0.094	0.119	0.648	0.653	1.285	1.287
B-splines	1	1	0.420	0.432	0.362	0.368	0.035	0.076	0.064	0.136	0.117	0.245	0.651	0.677
Partitioning	1	1	0.065	0.142	0.051	0.107	0.033	0.069	0.059	0.127	0.077	0.197	0.090	0.288
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.390	0.403	0.303	0.312	0.424	0.442	0.368	0.391	0.386	0.412	0.366	0.386
B-splines	27	00	0.386	0.394	0.300	0.305	0.389	0.438	0.329	0.379	0.374	0.383	0.330	0.357
Partitioning	27	00	0.373	0.419	0.292	0.320	0.266	1.835	0.292	1.465	0.342	1.006	0.396	0.711
reastote Estimation Local Polynomial	0.37	0.27	0.372	0.377	0.283	886.0	0.370	0.373	0.301	0.309	0.289	0.294	0.283	0.291
B-splines		-	0.373	385	0.285	862.0	0.380	0.393	0.304	0.333	0.314	0.381	0.287	0.342
Partitioning	1 61		0.373	0.383	0.285	0.296	0.401	0.402	0.357	0.469	0.320	0.302	0.325	0.346
						Model 3.3								
Infeasible Estimation		6	9	0	1	6	6	6	0	1	6	6	0	6
Local Polynomial	0.25	0.32	0.139	0.183	0.097	0.133	0.069	0.098	0.130	0.155	0.208	0.236	0.208	0.236
B-splines	<b>x</b> 0 0		0.126	0.151	0.088	0.107	0.124	0.071	0.144	0.126	0.165	0.229	0.165	0.229
Fartitioning Feasible Estimation	x	Т	0.186	0.150	0.145	0.113	0.318	0.072	0.275	0.131	0.240	0.199	0.225	0.335
Local Polynomial	0.38	0.28	0.112	0.130	0.065	0.085	0.043	0.067	0.085	0.103	0.085	0.103	0.085	0.103
B-splines	1	1	0.111	0.151	0.072	0.107	0.046	0.071	0.069	0.126	0.113	0.229	0.113	0.229
Partitioning	1	1	0.115	0.150	0.075	0.113	0.092	0.072	0.075	0.131	0.105	0.199	0.142	0.335
						Model 3.4								
Infeasible Estimation		1	,			1	1	1	1		,			
Local Polynomial	0.08	0.0 E	1.284	1.342	0.491	0.472	0.235	0.096	0.265	0.254	1.381	0.447	4.003	3.809
B-spines	210	7 7 7	1.292	1.263	0.529	0.511	0.345	0.114	0.409	1.060	1.545	1.350	4.065	3.990
Fartitioning	210	77	1.034	0.749	0.079	0.584	2.080	0.002	2.250	1.000	1.771	2.178	3.008	7.42/
Local Polynomial	0.29	0.26	1.355	1.346	0.454	0.468	0.046	0.069	0.316	0.237	0.316	0.237	3.802	3.790
B-splines	, oo	က	1.349	1.313	0.493	0.501	0.161	0.104	0.268	0.310	0.586	1.199	3.820	3.942
Partitioning	œ	က	1.127	1.083	0.640	0.558	1.390	1.710	0.437	0.694	1.082	0.744	2.128	2.509
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.789	0.601	0.616	0.485	1.125	1.200	1.147	1.047	0.663	0.680	0.570	0.567
B-splines	27	27	0.849	0.615	0.669	0.496	0.917	1.340	0.994	1.206	0.837	0.616	0.446	0.659
Fartitioning	7.7	7.7	0.701	0.748	0.545	0.583	0.780	0.001	0.720	1.00 t	0.444	2.000	0.469	2.010
Local Polynomial	0.25	0.28	0.720	0.589	0.566	0.474	1.225	1.225	1.286	1.166	0.540	0.662	0.616	0.499
B-splines	œ	-	0.890	0.802	0.688	0.641	0.648	0.808	0.961	0.968	0.791	0.854	0.431	0.475
Partitioning	oo	-	0.848	0.780	0.651	0.619	0.947	0.907	1.150	1.062	0.877	0.729	0.704	0.665

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.57: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3, n=1000, \sigma^2=1, X_{i,\ell}\sim \beta(2,2),$  Uniform Cells

Degree         Linear         Cubic         Linear		Tuning Parameter	Tuning Parameter	Root Integrated MSE	egrated E	Ingetrated MAE	ated	(0.5,0.5,0.5)	5,0.5)	Point E (0.1,0.5,0.5)	Point Estimation RMSE 0.5,0.5) (0.1,0.1,0	ation RMSE $(0.1,0.1,0.5)$	$_{1,0.5)}^{ m SE}$	(0.1, 0.1, 0.1)	1,0.1)
Model 3.1   Model 3.2   Mode	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.9         0.35         0.358         0.296         0.296         0.038         0.068         0.071         0.139         0.010         0.188         0.648           1         1         0.034         0.354         0.366         0.366         0.037         0.008         0.176         0.039         0.049         0.009         0.039         0.049         0.009         0.030         0.016         0.068         0.176         0.049         0.009         0.017         0.068         0.176         0.049         0.099         0.009         0.013         0.156         0.059         0.019         0.009         0.013         0.156         0.059         0.009         0.014         0.008         0.116         0.059         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.009         0.014         0.014							Model 3.1								
1.1   1.2	njeasible Estimation	0	0	0 273	с п о	906 0	0000	0.033	77	0.071	0 134	101	0 1 8 8	878	0 880
1	Benlines	9.5	e -	0.354	368	0.230	0.305	0.033	0.033	0.068	0.176	0.101	0.188	0.048	0.00
0.43         0.3         0.499         0.409         0.409         0.409         0.409         0.409         0.409         0.409         0.409         0.609         0.105         0.015         0.055         0.	Partitioning			0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
0.43         0.49         0.403         0.406         0.115         0.155         0.165         0.126         0.159         0.159         0.105         0.159         0.159         0.159         0.159         0.159         0.159         0.159         0.159         0.159         0.159         0.159         0.159         0.054         0.046         0.102         0.048         0.102         0.048         0.102         0.054         0.059         0.151         0.065         0.159         0.159         0.053         0.054         0.150         0.072         0.150         0.102           27         8         0.379         0.389         0.292         0.299         0.357         0.359         0.371         0.273         0.349         0.574         0.278         0.379         0.379         0.379         0.389         0.371         0.273         0.379         0.374         0.274         0.279         0.378         0.379         0.374         0.274         0.279         0.378         0.379         0.374         0.279         0.378         0.379         0.374         0.439         0.349         0.374         0.449         0.349         0.374         0.449         0.349         0.374         0.449         0.349	Peasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.43	0.3	0.495	0.499	0.403	0.406	0.037	0.060	0.113	0.155	0.655	0.669	1.290	1.301
1   1   0.062   0.140   0.048   0.102   0.051   0.065   0.151   0.085   0.280   0.105	B-splines	1	1	0.354	0.368	0.296	0.305	0.033	0.064	0.068	0.176	0.143	0.490	0.659	0.814
Model 3.2   Model 3.3   Mode	Partitioning	1	1	0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
0.16         0.23         0.384         0.389         0.299         0.350         0.350         0.350         0.350         0.379         0.369         0.299         0.384         0.270         0.278         0.389         0.389         0.384         0.277         0.279         0.389         0.389         0.384         0.277         0.279         0.378         0.399         0.389         0.384         0.277         0.279         0.378         0.378         0.371         0.378         0.379         0.378         0.378         0.379         0.378         0							Model 3.2								
0.16         0.23         0.384         0.399         0.299         0.381         0.381         0.384         0.462         0.472         0.553         0.773           27         8         0.389         0.299         0.299         0.329         0.381         0.384         0.714         0.299         0.392         0.394         1.434         0.529         0.714         0.729         0.037         0.279         0.286         0.371         0.274         0.289         0.388         0.297         0.378         0.399         0.384         0.277         0.299         0.378         0.399         0.384         0.277         0.299         0.388         0.299         0.389         0.389         0.384         0.277         0.299         0.389	nfeasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.16	0.23	0.384	0.393	0.292	0.299	0.352	0.351	0.363	0.350	0.722	0.553	0.743	0.590
27         8         0.369         0.413         0.285         0.310         0.218         1.289         0.304         1.434         0.524         1.720         0.775           0.36         0.371         0.274         0.299         0.376         0.378         0.390         0.392         0.370         0.439         0.036         0.374         0.274         0.299         0.376         0.386         0.377         0.274         0.290         0.386         0.377         0.046         0.077         0.439         1.061         0.448         0.175         0.439         0.370         0.448         0.370         0.448         0.370         0.448         0.370         0.448         0.177         0.449         0.370         0.478         0.439         0.370         0.448         0.439         0.074         0.106         0.074         0.106         0.074         0.106         0.074         0.107         0.149         0.106         0.074         0.107         0.149         0.106         0.074         0.107         0.149         0.106         0.074         0.107         0.149         0.106         0.074         0.103         0.042         0.143         0.140         0.106         0.074         0.143         0.173	B-splines	27	œ	0.379	0.389	0.291	0.296	0.362	0.350	0.319	0.364	0.462	0.714	0.529	0.744
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	27	œ	0.369	0.413	0.285	0.310	0.218	1.289	0.304	1.434	0.524	1.720	0.775	2.121
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	0		0	1	1	1	1	1	0	0	0	0	0	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.36	0.27	0.366	0.371	0.274	0.279	0.376	0.378	0.320	0.328	0.293	0.310	0.302	0.311
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	4 -	4.4	0.369	0.384	0.277	0.292	0.360	0.353	0.301	0.352	0.320	0.644	0.374	0.681
0.31 0.34 0.110 0.150 0.074 0.105 0.046 0.074 0.143 0.175 0.319 0.350 0.319  8 1 0.140 0.150 0.077 0.105 0.094 0.105 0.061 0.163 0.173 0.240 0.470 0.240  8 1 0.182 0.145 0.141 0.005 0.271 0.062 0.153 0.153 0.241 0.305  2 1 0.083 0.106 0.061 0.094 0.062 0.115 0.142 0.115 0.140 0.470 0.240  8 1 0.0107 0.136 0.070 0.007 0.007 0.007 0.015 0.153 0.173 0.240 0.470 0.105  2 1 0.0091 0.136 0.061 0.094 0.056 0.001 0.193 0.173 0.165 0.183  8 0.182 0.192 0.145 0.007 0.0094 0.005 0.116 0.194 0.155 0.183  9 0.192 0.192 0.193 0.106 0.001 0.0094 0.005 0.116 0.1094 0.105 0.116 0.116  9 0.194 0.195 0.195 0.1094 0.005 0.134 0.107 0.105 0.109 0.116  9 0.194 0.195 0.195 0.193 0.137 0.005 0.134 0.157 0.155 0.281 0.183  9 0.194 0.195 0.196 0.198 0.198 0.198 0.199 0.19	Fartitioning	4	4	0.360	0.397	0.274	0.300	0.453	0.975	0.439	1.061	0.469	1.297	607.0	1.743
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.3								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation	Ċ	i d		1	1	1	0	1		1	0	i c	0	0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.31	0.34	0.110	0.150	0.074	0.105	0.046	0.074	0.143	0.175	0.319	0.350	0.319	0.350
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	b-spines	<b>x</b> 0 0	<b>⊣</b> -	0.107	0.136	0.076	0.094	0.109	0.061	0.103	0.173	0.240	0.470	0.240	0.470
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	rartitioning easible Estimation	o	1	0.102	0.143	0.141	0.103	0.271	0.007	0.213	0.104	0.739	0.201	0.909	0.020
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.37	0.27	0.083	0.106	0.051	0.070	0.037	0.057	0.115	0.142	0.115	0.142	0.115	0.142
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	2	-	0.091	0.136	0.061	0.094	0.056	0.061	0.093	0.173	0.165	0.470	0.165	0.470
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	73	1	0.112	0.145	0.074	0.105	0.134	0.062	0.136	0.154	0.155	0.281	0.183	0.525
$\begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.4								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	ifeasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.1	6.0	0.485	0.554	0.222	0.183	0.137	0.065	0.331	0.248	2.492	0.368	4.498	3.820
125   27   0.741   0.748   0.491   0.536   0.260   0.269   0.866   0.791   1.790   6.145   3.850   3	B-splines	125	27	0.528	0.491	0.237	0.226	0.081	0.089	0.284	0.344	1.975	2.346	4.276	4.467
0.31         0.26         0.560         0.551         0.168         0.182         0.038         0.0565         0.265         0.297         0.265         0.297         0.263         0.265         0.278         0.265         0.297         0.265         0.297         0.263         0.297         0.766         0.297         0.766         0.766         0.766         0.766         0.766         0.767         0.769         0.766         0.767         0.1020         0.766         0.766         0.766         0.766         0.767         0.766         0.841         0.716         0.823         0.767         0.823         0.767         0.823         0.767         0.823         0.841         0.716         0.823         0.767         0.104         0.104         0.104         0.104         0.104         0.104         0.104 <th< td=""><td>Partitioning</td><td>125</td><td>27</td><td>0.741</td><td>0.748</td><td>0.491</td><td>0.536</td><td>0.260</td><td>0.269</td><td>0.866</td><td>0.791</td><td>1.790</td><td>6.145</td><td>3.850</td><td>397.445</td></th<>	Partitioning	125	27	0.741	0.748	0.491	0.536	0.260	0.269	0.866	0.791	1.790	6.145	3.850	397.445
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation														
8 3 0.560 0.525 0.483 0.265 0.467 0.891 0.300 0.498 1.636 3.833    Model 3.5	Local Polynomial	0.31	0.26	0.560	0.551	0.168	0.182	0.038	0.058	0.265	0.297	0.265	0.297	3.799	3.798
8 3 0.525 0.483 0.256 0.467 0.891 0.326 0.726 0.570 1.086 3.154  Model 3.5  0.33 0.719 0.602 0.559 0.487 1.087 1.049 0.812 0.924 0.896 0.657 0.498  27 27 0.767 0.622 0.596 0.501 0.990 1.169 0.692 0.960 1.020 1.510 0.566  27 27 0.766 0.514 0.584 0.756 0.357 0.485 0.841 0.715 6.931 0.885 31  0.26 0.27 0.696 0.612 0.548 0.492 1.207 1.192 1.014 1.157 0.823 0.685 0.361  8 7 0.773 0.664 0.591 0.533 0.757 1.160 0.596 0.936 0.913 1.027 0.414	B-splines	œ	က	0.560	0.525	0.195	0.212	0.121	0.084	0.219	0.300	0.498	1.636	3.833	4.146
	Partitioning	œ	ന	0.525	0.483	0.263	0.256	0.467	0.891	0.326	0.726	0.570	1.086	3.154	3.149
0.33         0.33         0.719         0.602         0.559         0.487         1.087         1.049         0.812         0.924         0.896         0.657         0.498           27         27         0.767         0.622         0.596         0.501         0.990         1.169         0.692         0.960         1.020         1.510         0.566           27         27         0.629         0.776         0.756         0.357         0.485         0.841         0.715         6.931         0.885         31           0.26         0.27         0.696         0.612         0.548         0.492         1.207         1.192         1.014         1.157         0.823         0.685         0.361           8         7         0.773         0.664         0.591         0.533         0.757         1.160         0.596         0.936         0.913         1.027         0.414							Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation					1									
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.719	0.602	0.559	0.487	1.087	1.049	0.812	0.924	0.896	0.657	0.498	0.752
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	27	27	0.767	0.622	0.596	0.501	0.990	1.169	0.692	0.960	1.020	1.510	0.566	1.330
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	27	27	0.629	0.766	0.474	0.584	0.756	0.357	0.485	0.841	0.715	6.931	0.885	306.734
omia 0.20 0.21 0.090 0.012 0.548 0.492 1.20 1.192 1.014 1.151 0.523 0.001 0.501 0.501 0.773 0.664 0.591 0.533 0.777 1.160 0.596 0.936 0.913 1.027 0.414	easible Estimation	0	100	0	0	7	9	100	00		1	0	i c	0	001
8 7 0.773 0.664 0.591 0.533 0.757 1.160 0.596 0.513 1.027 0.414	Local Polynomial	0.26	0.27	0.090	0.012	0.548	0.492	1.207	1.192	1.014	1.157	0.823	0.085	0.361	0.498
	B-splines	00	7	0.773	0.664	0.591	0.533	0.757	1.160	0.596	0.936	0.913	1.027	0.414	0.926

Table C.58: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 1000, \sigma^2 = 4, X_{i,\ell} \sim \beta(0.5, 0.5), \text{Uniform Cells}$ 

$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		Tuning Parameter	ing neter	Root Integrated MSE	egrated E	Ingetrated MAE	rated \E	(0.5, 0.5, 0.5)	5,0.5)	Point Es $(0.1, 0.5, 0.5)$	nt Estima 5,0.5)	Point Estimation RMSE (0.1,0.5,0.5)	E 1,0.5)	(0.1, 0.1, 0.1)	1,0.1)
0.39 0.9 0.469 0.487 0.409 0.418 0.063 0.136 0.117 0.172 1 1 0.125 0.653 0.249 0.246 0.246 0.248 0.248 0.248 0.249 0.248 0.241 0.258 0.249 0.247 0.060 0.154 0.103 0.229 0.241 0.245 0.242 0.065 0.154 0.108 0.239 0.241 0.283 0.241 0.247 0.060 0.154 0.107 0.229 0.249 0.248 0.249 0.247 0.067 0.154 0.107 0.232 0.249 0.247 0.247 0.249 0.247 0.247 0.242 0.248 0.249 0.247 0.242 0.248 0.249 0.247 0.242 0.248 0.249 0.247 0.248	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.469         0.487         0.409         0.418         0.063         0.136         0.117         0.117         0.117         0.117         0.117         0.117         0.117         0.117         0.117         0.117         0.117         0.117         0.111         0.222           1         1         0.125         0.655         0.652         0.646         0.048         0.154         0.101         0.117         0.117         0.128           1         1         0.122         0.658         0.641         0.416         0.124         0.126         0.117         0.117         0.128           2         1         0.122         0.626         0.641         0.246         0.124         0.124         0.124         0.124         0.117         0.117         0.127         0.117         0.124         0.117         0.124         0.117         0.124         0.117         0.124         0.117         0.124         0.117         0.124         0.118         0.124         0.118         0.124         0.118         0.124         0.118         0.125         0.124         0.118         0.124         0.118         0.125         0.124         0.124         0.124         0.124         0.124	:						Model 3.1								
0.39         0.45         0.450         0.458         0.419         0.418         0.003         0.117         0.118         0.239           0.39         0.28         0.652         0.652         0.652         0.041         0.041         0.117         0.117         0.118         0.118         0.023           27         8         0.652         0.641         0.247         0.049         0.747         0.150         0.154         0.117         0.148         0.039           27         8         0.643         0.443         0.443         0.249         0.345         0.452         0.447         0.545         0.447         0.148         0.328         0.458         0.447         0.542         0.548         0.447         0.548         0.547         0.548         0.548         0.544         0.147         0.148         0.328         0.649         0.641         0.641         0.642<	Infeasible Estimation	c c	Ó			0		0	0	1		i	(	0	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial R-splines	0.9	0.0 -	0.469	0.487	0.409	0.418	0.063	0.136	0.117	0.172	0.171	0.218	0.665	0.685
0.39         0.28         0.652         0.665         0.632         0.641         0.043         0.113         0.114         0.119           1         1         0.472         0.518         0.411         0.436         0.073         0.174         0.117         0.232           0.19         0.28         0.411         0.436         0.078         0.174         0.117         0.239           27         8         0.439         0.474         0.348         0.386         0.449         0.477         0.529         0.619         0.619         0.626         0.381         0.486         0.499         0.370         0.427         0.569         0.381         0.486         0.499         0.370         0.427         0.569         0.381         0.486         0.499         0.370         0.449         0.330         0.619         0.437         0.486         0.487         0.486         0.487         0.489         0.384         0.469         0.384         0.469         0.384         0.469         0.384         0.469         0.384         0.477         0.589         0.289         0.289         0.384         0.619         0.449         0.619         0.619         0.619         0.619         0.619         0.619 </td <td>Partitioning</td> <td>-</td> <td>-</td> <td>0.125</td> <td>0.283</td> <td>0.098</td> <td>0.217</td> <td>090:0</td> <td>0.154</td> <td>0.108</td> <td>0.239</td> <td>0.143</td> <td>0.337</td> <td>0.170</td> <td>0.451</td>	Partitioning	-	-	0.125	0.283	0.098	0.217	090:0	0.154	0.108	0.239	0.143	0.337	0.170	0.451
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Feasible Estimation														
1         1         0.472         0.518         0.411         0.436         0.073         0.174         0.117         0.239           1         1         0.132         0.283         0.101         0.217         0.120         0.154         0.122         0.239           27         8         0.439         0.444         0.349         0.370         0.497         0.552         0.381         0.486           27         8         0.682         0.811         0.529         0.619         0.528         0.497         0.529         0.619         0.588         0.486         0.486         0.497         0.560         0.381         0.486         0.497         0.560         0.381         0.486         0.487         0.560         0.381         0.486         0.489         0.382         0.619         0.529         0.487         0.619         0.589         0.449         0.332         0.245         0.453         0.645         0.417         0.529         0.645         0.457         0.458         0.459         0.645         0.459         0.458         0.459         0.449         0.382         0.220         0.645         0.645         0.645         0.645         0.645         0.645         0.645	Local Polynomial	0.39	0.28	0.652	0.665	0.532	0.542	0.084	0.153	0.148	0.192	0.661	0.672	1.293	1.299
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	1	1	0.472	0.518	0.411	0.436	0.073	0.174	0.117	0.232	0.196	0.330	0.669	0.725
0.19         0.26         0.443         0.493         0.348         0.386         0.491         0.548         0.447         0.512           27         8         0.443         0.493         0.348         0.386         0.491         0.552         0.381         0.486           27         8         0.439         0.474         0.529         0.519         0.552         0.581         0.486           4         1         0.395         0.449         0.529         0.314         0.362         0.377         0.389         0.381           0.25         0.26         0.381         0.449         0.325         0.345         0.447         0.389         0.487         0.389         0.387           0.27         0.346         0.449         0.325         0.325         0.325         0.367         0.419         0.581         0.667           0.27         0.34         0.225         0.449         0.325         0.225         0.435         0.449         0.325           0.35         0.26         0.162         0.289         0.272         0.645         0.157         0.529         0.289           0.35         0.26         0.162         0.283         0.225	Partitioning	1	1	0.132	0.283	0.101	0.217	0.120	0.154	0.122	0.239	0.153	0.337	0.173	0.451
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.2								
0.19         0.26         0.443         0.493         0.348         0.386         0.491         0.548         0.447         0.512           27         8         0.682         0.871         0.619         0.538         4.500         0.538         2.836           27         8         0.682         0.871         0.529         0.619         0.538         4.500         0.538         2.836           0.35         0.26         0.345         0.345         0.437         0.339         0.334         0.607         0.439         0.387           4         1         0.329         0.449         0.325         0.367         0.419         0.581         0.607         0.419         0.538         0.607         0.419         0.538         0.607         0.419         0.538         0.607         0.419         0.538         0.222         0.607         0.419         0.528         0.222         0.607         0.419         0.528         0.222         0.645         0.157         0.228         0.222         0.645         0.157         0.228         0.228         0.222         0.645         0.157         0.228         0.242         0.445         0.528         0.240           0.35	Infeasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.19	0.26	0.443	0.493	0.348	0.386	0.491	0.548	0.447	0.512	0.477	0.509	0.409	0.444
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	27	<b>x</b> 0 (	0.439	0.474	0.349	0.370	0.427	0.562	0.381	0.486	0.475	0.476	0.376	0.390
0.35         0.26         0.381         0.403         0.294         0.314         0.365         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.455         0.607         0.434         0.581         0.581         0.581         0.605           0.27         0.34         0.235         0.325         0.162         0.205         0.301         0.170         0.528         0.288           0.37         0.26         0.283         0.225         0.205         0.170         0.528         0.228           0.35         0.26         0.283         0.225         0.045         0.157         0.556         0.240           0.35         0.26         0.283         0.205         0.157         0.157         0.284         0.156         0.284         0.157         0.156         0.289         0.284         0.156         0.284         0.156         0.284         0.157         0.156         0.284         0.157         0.156         0.284         0.156         0.156         0.284         0.156         0.156         0.284         0.156         0.156         0.	Fartitioning	7.7	x	0.682	0.811	0.529	0.619	0.538	4.500	0.598	2.836	0.580	T.708	0.591	0.822
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	reassure Estimation	0.35	98.0	0.381	0.403	0 294	0.314	0.362	0.378	0.309	0.334	0.325	0.342	0 292	0.318
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	4	-	0.395	0.438	0.306	0.345	0.453	0.437	0.380	0.387	0.416	0.474	0.312	0.380
0.27         0.34         0.235         0.325         0.175         0.248         0.152         0.299         0.284         0.228           8         1         0.215         0.272         0.162         0.205         0.301         0.170         0.284         0.228           8         1         0.215         0.289         0.283         0.222         0.645         0.170         0.284         0.228           9.35         0.26         0.163         0.210         0.110         0.155         0.0645         0.157         0.556         0.240           9.35         0.26         0.163         0.272         0.174         0.222         0.055         0.157         0.556         0.240           9.3         0.26         0.164         0.272         0.174         0.222         0.055         0.015         0.156         0.240           9.3         1         0.250         0.289         0.174         0.262         0.157         0.157         0.186         0.186           0.08         0.9         0.196         0.289         0.174         0.956         0.855         0.187         0.228         0.208           2.16         2.0         0.240 <t< td=""><td>Partitioning</td><td>4</td><td>1</td><td>0.429</td><td>0.449</td><td>0.332</td><td>0.353</td><td>0.607</td><td>0.419</td><td>0.581</td><td>0.605</td><td>0.487</td><td>0.411</td><td>0.377</td><td>0.463</td></t<>	Partitioning	4	1	0.429	0.449	0.332	0.353	0.607	0.419	0.581	0.605	0.487	0.411	0.377	0.463
$\begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.3								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation	0	0	0	0	1	0	( ) 1		0	0	0	ì	0	i C
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.27	0.34	0.235	0.325	0.175	0.248	0.152	0.219	0.229	0.283	0.309	0.355	0.309	0.355
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	b-spines	<b>x</b> 00		0.215	0.272	0.162	0.205	0.301	0.170	0.284	0.228	0.270	0.323	0.270	0.323
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	raturoning Feasible Estimation	0	4	0.505	0.209	0.203	0.522	0.0	0.10	0.000	0.54	000.0	0.00	0.09	0.40
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.35	0.26	0.163	0.210	0.110	0.155	0.088	0.152	0.146	0.186	0.146	0.186	0.146	0.186
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	8	1	0.184	0.272	0.133	0.205	0.191	0.170	0.195	0.228	0.224	0.323	0.224	0.323
0.08 0.9 1.957 2.074 0.956 0.852 0.175 0.457 0.278 216 27 2.060 2.024 0.979 0.935 0.831 0.265 0.609 0.406 216 27 1.444 1.457 0.961 1.120 2.111 5.544 2.865 3.849 0.3 0.25 2.087 2.081 0.819 0.844 0.095 0.152 0.376 8 2 2.083 2.059 0.873 0.892 0.331 0.200 0.397 0.376 8 2 1.647 1.767 1.053 1.023 2.524 2.178 0.661 0.899  0.3 0.33 0.835 0.645 0.652 0.515 1.147 1.302 1.314 1.145 27 27 0.903 0.680 0.720 0.543 0.855 1.453 1.171 1.316 27 27 27 0.904 1.452 0.732 1.126 0.920 5.644 1.010 4.058 0.26 0.26 0.26 0.746 0.581 0.585 0.467 1.211 1.204 1.374 1.169 8 1 0.650 0.849 0.746 0.681 0.682 1.463 1.171 1.204 1.374 1.169 8 1 0.650 0.849 0.746 0.681 0.682 1.463 1.474 1.465 1.468 1.469	Partitioning	က	1	0.250	0.289	0.174	0.222	0.365	0.157	0.328	0.240	0.311	0.338	0.286	0.465
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.4								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation	0	Ċ	1	1	0	0	1	1	1		1	100	0	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.08	0.0	1.957	2.074	0.956	0.852	0.552	0.175	0.457	0.278	1.047	0.507	3.965	3.837
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	D-spines	012	2 6	2.060	1 4 5 7 7	0.979	1.30	0.001	0.263 F F 4.4	0.003	9.6406	1.194 9.979	0.970	9.013	0.820
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	ratutuoning Jeasible Estimation	017	4	# # # #	10#:T	0.301	1.120	7.1.1	5.0.5	7.000	0.040	0.0.0	6.0.0	4.003	1.000
8 2 2.083 2.059 0.873 0.892 0.331 0.200 0.397 0.376 8	Local Polynomial	0.3	0.25	2.087	2.081	0.819	0.844	0.095	0.152	0.340	0.281	0.340	0.281	3.816	3.805
8 2 1.647 1.767 1.053 1.023 2.524 2.178 0.661 0.899  Model 3.5  0.33 0.33 0.835 0.645 0.652 0.515 1.147 1.302 1.314 1.145  27 27 27 0.903 0.680 0.720 0.543 0.855 1.453 1.171 1.316  0.26 0.26 0.746 0.581 0.585 0.467 1.211 1.204 1.374 1.169  8 1 0.650 0.849 0.746 0.681 0.682 1.77 0.688 1.179	B-splines	00	7	2.083	2.059	0.873	0.892	0.331	0.200	0.397	0.376	0.653	1.006	3.874	3.958
Model 3.5  0.33  0.33  0.33  0.835  0.645  0.652  0.515  1.147  1.302  1.314  1.145  27  27  27  27  0.903  0.680  0.720  0.543  0.855  1.453  1.771  1.316  27  27  27  0.924  1.452  0.732  1.126  0.920  5.644  1.010  4.058  0.26  0.26  0.26  0.27  1.17  1.199  8  1.199  9.746  0.849  0.746  0.881  0.681  0.681  0.681  0.682  1.77  1.71  1.179  1.179	Partitioning	∞	61	1.647	1.767	1.053	1.023	2.524	2.178	0.661	0.899	1.561	0.788	1.348	2.148
0.33         0.33         0.835         0.645         0.652         0.515         1.147         1.302         1.314         1.145           27         27         27         0.903         0.680         0.720         0.543         0.855         1.453         1.771         1.316           27         27         0.924         1.452         0.732         1.126         0.920         5.644         1.010         4.058           0.26         0.26         0.746         0.581         0.585         0.467         1.211         1.204         1.374         1.169           8         1         0.650         0.849         0.746         0.681         0.682         0.775         0.777         1.468         1.179							Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Infeasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.835	0.645	0.652	0.515	1.147	1.302	1.314	1.145	0.636	0.812	0.668	0.565
27 27 0.924 1.452 0.732 1.126 0.920 5.044 1.010 4.058 0.26 0.26 0.746 0.581 0.585 0.467 1.211 1.204 1.374 1.169 8 1 0.951 0.849 0.746 0.881 0.682 0.732 1.178 1.179 8 1.170 0.510 0.524 0.727 1.660 1.345	B-splines	27	27	0.903	0.680	0.720	0.543	0.855	1.453	1.171	1.316	0.800	0.695	0.541	0.644
0.26 0.26 0.746 0.581 0.585 0.467 1.211 1.204 1.374 1.169 8 1 0.951 0.849 0.746 0.681 0.602 0.732 1.179 8 1 0.050 0.334 0.745 0.689 1.779 0.486 1.179	Fartitioning	7.7	7.7.	0.924	1.452	0.732	1.126	0.920	5.644	1.010	4.058	0.602	2.047	0.646	1.299
8 1 0.951 0.849 0.746 0.681 0.602 0.732 1.178 1.179 8 1 0.050 0.834 0.747 0.688 1.077 0.877 1.480 1.948	Local Polynomial	0.26	0.26	0.746	0.581	0.585	0.467	1.211	1.204	1.374	1.169	0.471	0.684	0.708	0.514
0 1 0 0 5 0 0 2 1 0 0 5 0 0 1 0 7 0 5 0 0 1 0 7 0 0 5 0 1 0 7 0 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0	B-splines	000	-	0.951	0.849	0.746	0.681	0.602	0.732	1.178	1.179	0.752	0.777	0.528	0.571
040.1 604.1 1/0.0 1/0.1 600.0 141.0 460.0 008.0	Partitioning	oc	-	0.950	0.834	0.747	0.668	1.077	0.877	1.459	1.346	0.865	0.612	0.678	0.657

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.59: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=1000,\,\sigma^2=4,\,X_{i,\ell}\sim\beta(1,1),$  Uniform Cells

Dogectes:         Linear         Cubic         Linear <t< th=""><th></th><th>Tuning</th><th>nng neter</th><th>Root Integrated MSE</th><th>egrated</th><th><math>M^{\prime}</math></th><th>MAE</th><th>(0.5, 0.5, 0.5)</th><th>5,0.5)</th><th>(0.1, 0.5, 0.5)</th><th>5,0.5)</th><th>(0.5,0.5) <math>(0.1,0.1,0.5)</math></th><th>E 1,0.5)</th><th>(0.1, 0.1, 0.1)</th><th>.,0.1)</th></t<>		Tuning	nng neter	Root Integrated MSE	egrated	$M^{\prime}$	MAE	(0.5, 0.5, 0.5)	5,0.5)	(0.1, 0.5, 0.5)	5,0.5)	(0.5,0.5) $(0.1,0.1,0.5)$	E 1,0.5)	(0.1, 0.1, 0.1)	.,0.1)
Model 3.1	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.09         0.432         0.4454         0.380         0.380         0.007         0.122         0.126         0.136         0.157         0.126         0	:						Model 3.1								
0.38         0.26         0.35         0.445         0.35         0.445         0.145         0.445         0.145         0.445         0.145         0.445         0.145         0.445         0.145         0.445         0.144         0.1	Infeasible Estimation	4	1		1	1	1	1	,	1	1	1	1	1	
1	Local Polynomial R-splines	0.9	0.0 -	0.432	0.451	0.369	0.380	0.067	0.122	0.126	0.195	0.180	0.254	0.663	0.682
0.38         0.28         0.598         0.612         0.488         0.498         0.083         0.113         0.178         0.218         0.689         0.684         0.772         0.411         0.078         0.114         0.124         0.284         0.077         0.144         0.124         0.284         0.077         0.014         0.017         0.014         0.017         0.014         0.017         0.017         0.015         0.017         0.015         0.026         0.045         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.018         0.017         0.019         0.025         0.044         0.050         0.035         0.037         0.040         0.044         0.040         0.035         0.040         0.047         0.041         0.040         0.044         0.040         0.057         0.042         0.042         0.058         0.044         0.040         0.044         0.040         0.044         0.040         0.044         0.040         0.041         0.040         0.041         0.040         0.041         0.040         0.041         0.041         0.040         0.041         0	Partitioning			0.129	0.284	0.101	0.215	0.066	0.137	0.118	0.254	0.155	0.394	0.180	0.575
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Feasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.38	0.28	0.598	0.612	0.488	0.498	0.083	0.139	0.178	0.218	0.669	0.682	1.297	1.304
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	П	1	0.436	0.484	0.372	0.401	0.078	0.144	0.133	0.257	0.226	0.465	0.681	0.781
Model 3.2   Model 3.3   Mode	Partitioning	1	П	0.144	0.284	0.107	0.215	0.158	0.137	0.150	0.254	0.185	0.394	0.190	0.575
0.19         0.26         0.440         0.486         0.381         0.386         0.480         0.480         0.480         0.382         0.480         0.480         0.582         0.481         0.485         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.882         0.883         0.882         0.883         0.882         0.883         0.882         0.883         0.867         0.883         0.883         0.883         0.884         0.883         0.867         0.883         0.884         0.883         0.884         0.883         0.884         0.883         0.884         0.883         0.884         0.883         0.884         0.883         0.884         0.883         0.884         0.883         0.884         0.883         0.884         0							Model 3.2								
0.19         0.26         0.450         0.458         0.480         0.480         0.331         0.386         0.480         0.480         0.480         0.480         0.480         0.480         0.480         0.480         0.480         0.687         0.480         0.687         0.687         0.480         0.687         0.687         0.680         0.687         0.687         0.687         0.687         0.687         0.687         0.687         0.687         0.687         0.689         0.687         0.687         0.689         0.687         0.687         0.689         0.687         0.687         0.689         0.687         0.687         0.689         0.688         0.688         0.687         0.687         0.689         0.688         0.688         0.689         0.687         0.687         0.689         0.689         0.688         0.689         0.687         0.689         0.689         0.688         0.689         0.687         0.689         0.689         0.689         0.689         0.689         0.687         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0.689         0	Infeasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.19	0.26	0.450	0.496	0.351	0.386	0.438	0.480	0.440	0.474	0.553	0.558	0.541	0.544
27         8         0.680         0.809         0.531         0.617         0.410         3.201         0.535         2.547         0.672         1.865           0.34         0.26         0.385         0.404         0.311         0.548         0.344         0.316         0.377         0.393         0.334         0.329         0.548           5         1         0.442         0.401         0.343         0.360         0.597         0.522         0.629         0.384         0.329         0.548           0.3         0.26         0.241         0.343         0.360         0.597         0.522         0.629         0.583         0.511         0.507           1         0.442         0.311         0.360         0.597         0.519         0.528         0.501           1         0.442         0.301         0.146         0.137         0.141         0.329         0.583         0.511         0.507         0.419         0.507         0.419         0.507         0.419         0.507         0.419         0.507         0.419         0.507         0.419         0.507         0.419         0.507         0.419         0.507         0.419         0.507         0.419         0	B-splines	27	œ ·	0.444	0.480	0.352	0.373	0.401	0.485	0.382	0.484	0.523	0.567	0.497	0.555
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	27	œ	0.680	0.809	0.531	0.617	0.410	3.201	0.535	2.547	0.672	1.865	0.784	1.322
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	reasible Estimation	ć	90	0	07	0	910	1	000	C C	2	000	0 0 0	0	000
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Folynomial	4.0.0	0.20	0.300	0.407	0.297	0.516	0.07	0.090	0.555	4000	0.529	0.040	0.010	0.000
0.3 0.36 0.218 0.301 0.158 0.224 0.113 0.171 0.233 0.267 0.401 0.405 1 0.157 0.266 0.114 0.197 0.071 0.141 0.199 0.252 0.216 0.459 0.19 0.157 0.268 0.114 0.197 0.071 0.141 0.199 0.252 0.216 0.395 0.19 0.157 0.288 0.197 0.197 0.071 0.141 0.289 0.197 0.252 0.166 0.395 0.197 0.280 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.197 0.289 0.253 0.289 0.417 0.289 0.197 0.289 0.197 0.289 0.299 0.197 0.289 0.299 0.2	D-spinnes Partitioning	o ro		0.403	0.444	0.343	0.360	0.437	0.522	0.629	0.030	0.511	0.507	0.532	0.520
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0	ı	ı				Model 3.3								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Infereible Estimation						2000								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.3	0.36	0.218	0.301	0.158	0.224	0.113	0.171	0.233	0.267	0.401	0.405	0.401	0.405
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	П	П	0.157	0.266	0.114	0.197	0.071	0.141	0.119	0.252	0.216	0.459	0.216	0.459
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	1	1	0.157	0.288	0.116	0.218	0.071	0.139	0.119	0.257	0.166	0.395	0.209	0.604
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	0	(	1	0	1	1	0	0	1	0	1	0	1	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.34	0.26	0.151	0.201	0.105	0.147	0.085	0.138	0.174	0.208	0.174 0.200	0.208	0.174	0.208
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	4.	н,	0.187	0.266	0.136	0.197	0.185	0.141	0.230	0.253	0.283	0.459	0.283	0.459
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Fartitioning	4	7	0.280	0.292	0.198	0.219	0.435	0.197	0.405	0.264	0.358	0.417	0.349	0.607
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	:						Model 3.4								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation	0	c c	ì	ì		0	0	0	ì	0		0	0	1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial Benline	0.09 198	0.9	1.255	1.351	0.615	0.508	0.379	0.139	0.451	0.302	1.481	1.430	4.046	3.817
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	D-spines Partitioning	10.5	- 12	1.043	1 476	1 082	1140	1.916	1 203	1 740	0.401	1.436	7 310	3 341	1.021
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Pasible Estimation		i	101	2	1000	2	2		0	1	2	9		
8 3 1.359 1.334 0.557 0.555 0.271 0.182 0.369 0.391 0.655 1.257	Local Polynomial	0.3	0.26	1.359	1.354	0.475	0.501	0.090	0.139	0.348	0.300	0.348	0.300	3.801	3.790
8 3 1.169 1.171 0.692 0.648 1.480 2.050 0.652 1.153 1.161 1.099  Model 3.5  0.33 0.33 0.806 0.659 0.629 0.530 1.126 1.208 1.159 1.073 0.777  27 27 27 0.877 0.698 0.691 0.558 0.920 1.348 1.010 1.234 0.908 0.748  27 27 27 0.977 0.698 0.691 0.558 0.920 1.348 1.010 1.234 0.908 0.748  28 0.740 0.602 0.581 0.484 1.223 1.214 1.274 1.180 0.582 0.684  8 1 0.906 0.828 0.700 0.659 0.659 0.659 0.659 0.659 0.659  9 1 0.907 0.828 0.700 0.659 0.659 0.659 0.659 0.659 0.659 0.659	B-splines	∞	က	1.359	1.334	0.527	0.555	0.271	0.182	0.369	0.391	0.655	1.257	3.832	3.968
Model 3.5  0.33  0.33  0.806  0.659  0.629  0.629  0.630  1.126  1.126  1.126  1.1298  1.1159  1.1073  0.773  0.777  27  27  27  27  27  27  27  27  27	Partitioning	œ	က	1.169	1.171	0.692	0.648	1.480	2.050	0.652	1.153	1.161	1.099	2.173	2.602
0.33         0.33         0.806         0.659         0.629         0.530         1.126         1.208         1.159         1.073         0.733         0.777           27         27         0.877         0.698         0.691         0.558         0.920         1.348         1.010         1.234         0.908         0.748           27         27         0.903         1.475         0.710         1.149         0.836         1.242         0.843         2.094         0.729         4.199           0.26         0.26         0.740         0.602         0.581         0.484         1.223         1.214         1.274         1.180         0.582         0.684           8         1         0.906         0.828         0.700         0.659         0.679							Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.806	0.659	0.629	0.530	1.126	1.208	1.159	1.073	0.733	0.777	0.657	0.686
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	27	27	0.877	0.698	0.691	0.558	0.920	1.348	1.010	1.234	0.908	0.748	0.576	0.788
0.26         0.26         0.26         0.581         0.484         1.223         1.214         1.274         1.180         0.582         0.684           8         1         0.906         0.828         0.700         0.659         0.678         0.846         0.991         0.997         0.840         0.927           9         1         0.000         0.693         0.678         0.678         0.741         1.154         0.664         0.997	Partitioning	.57	.57	0.903	1.475	0.710	1.149	0.836	1.242	0.843	2.094	0.729	4.199	0.823	3.981
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	reasible Estimation	96.0	96 0	0.740	0.609	27.07.0	787 0	1 993	1 917	1 274	1 80	о С	080	0.614	0 530
0 1 0.000 0.020 0.100 0.000 0.010 0.030 0.030 0.030 0.031 0.034 0.032 0.034 0.032 0.034 0.032 0.034 0.	Designed	9.0	04:	0.000	0000	0.001	0.464	0.10	1.214	1.60.0	0.007	0.00	0.034	0.0	0000
	D-spines	0 0	٦.	0.906	0.020	0.700	0.039	0.070	1 103	1.931	1.33	0.040	0.927	0.010	0.020

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.60: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 1000, \sigma^2 = 4, X_{i,\ell} \sim \beta(2, 2),$ Uniform Cells

	Tur Parar	Tuning Parameter	Root Integ MSE	Root Integrated MSE	$rac{Ingetrated}{MAE}$	ated E	(0.5,0.5,0.5)	5,0.5)	Point E (0.1,0.5,0.5)	oint Estin 5,0.5)	Point Estimation RMSE (0.5,0.5)	SE 1,0.5)	(0.1,0	(0.1,0.1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Local Polynomial	6.0	6.0	0.366	0.387	0.305	0.319	0.063	0.106	0.137	0.262	0.194	0.360	0.670	0.736
B-splines Partitioning			0.370 $0.124$	0.428	0.307	0.344	0.063	0.123	0.132 $0.131$	0.344 $0.302$	$0.276 \\ 0.170$	0.946 $0.561$	$0.705 \\ 0.211$	1.153
Feasible Estimation	0 84	0 94	л п	162	0.410	0.455	0.073	0 117	0.939	006.0	0 803	0.791	1 213	1 223
B-splines	2.2	0.21	0.374	0.428	0.310	0.344	0.105	0.123	0.180	0.344	0.316	0.946	0.722	1.153
Partitioning	2	-	0.179	0.281	0.122	0.204	0.209	0.121	0.252	0.302	0.274	0.561	0.301	0.935
7. 6						Model 3.2								
yeasible Estimation Local Polynomial	0.19	0.26	0.441	0.479	0.334	0.363	0.368	0.380	0.486	0.476	0.972	0.850	0.985	0.870
B-splines	27	oc o	0.438	0.475	0.340	0.359	0.369	0.392	0.402	0.550	0.805	1.317	0.845	1.335
Feasible Estimation	1	0	0.00	0.00	6.0	0000	0.50	7	0.00	100.7	0.930	0.1.0	010.1	0.0
Local Polynomial	0.33	0.25	0.379	0.400	0.287	0.305	0.380	0.388	0.379	0.405	0.365	0.405	0.364	0.392
B-splines Partitioning		99	0.401 $0.453$	0.463 $0.705$	0.305	0.350 $0.515$	0.388	0.380	0.393	0.516 $2.195$	0.510 $0.731$	1.219 $2.550$	0.569 $1.014$	1.241 $3.195$
						Model 3.3								
Infeasible Estimation			1	1		,	1	,		1	1		1	
Local Polynomial Replines	0.38	0.39	0.178	0.257	0.124	0.183	0.078	0.133	0.243	0.326	0.504	0.629	0.504	0.629
Partitioning			0.137	0.283	0.103	0.205	0.064	0.122	0.131	0.304	0.175	0.561	0.222	0.968
Feasible Estimation														
Local Polynomial	0.33	0.25	0.131	0.185	0.091	0.131	0.074	0.117	0.240	0.289	0.240	0.289	0.240	0.285
p-spillies Partitioning	9	ာက	0.318	0.469	0.233	0.303	0.192 $0.461$	1.202	0.482	1.312	0.518	1.507	0.548	2.059
						Model 3.4								
Infeasible Estimation	,	,	1	i				;	1			!		
Local Polynomial	0.12	0.0	0.565	0.574	0.318	0.228	0.224	0.112	0.555	0.334	2.406	0.476	4.487	3.832
D-splines Partitioning	64	2.7.	0.603	1.412	0.770	1.050	1.222	0.173	1.662	0.550	3.031	12.237	7.126	840.145
Feasible Estimation		i		1	)					)			)    -  -	
Local Polynomial	0.31	0.25	0.569	0.569	0.194	0.223	0.075	0.117	0.335	0.382	0.335	0.382	3.799	3.799
B-splines	œ.	ທີ	0.582	0.580	0.241	0.289	0.223	0.186	0.359	0.497	0.647	1.972	3.853	4.277
Partitioning	00	ro	0.607	0.728	0.365	0.482	0.634	1.866	0.578	2.031	0.760	2.396	3.209	3.842
						Model 3.5								
Infeasible Estimation Local Polynomial	0.33	0.33	0.736	0.651	0.572	0.523	1.086	1.052	0.839	0.965	1.023	0.901	0.701	0.971
B-splines	27	27	0.798	0.703	0.619	0.557	0.991	1.174	0.731	1.033	1.211	2.233	0.864	2.122
Partitioning	27	27	0.848	1.421	0.651	1.092	0.774	0.587	0.681	1.606	1.105	12.939	1.568	747.807
Feasible Estimation	0 0	C C	0 41	700	n n	0	1 201	1 1 1 2 0	1 000	1 176	000	200	006.0	о Н
Local Polynomial Bealines	0.2.0	0.75	0.790	0.624	0.559	0.501	1.201	1.179 1.179	1.002	1.176	1.005	1 500	0.590	0.000
D-spinnes Partitioning	o oo	- 1-	0.779	0.830	0.586	0.631	1.046	2.306	0.910	2.476	1.238	2.838	1.138	3.512
0							7	, , , , ,			/ 1	,		

## C.3.2 Quantile Cell Boundaries

Table C.61: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 500, \sigma^2 = 1, X_{i,\ell} \sim \beta(0.5, 0.5),$  Quantile Cells

		Tuning Parameter	ing neter	Root In	Root Integrated MSE	$rac{ ext{Inget}}{ ext{M}}$	$\begin{array}{c} {\rm Ingetrated} \\ {\rm MAE} \end{array}$	(0.5, 0.5, 0.5)	5,0.5)	Point Es (0.1,0.5,0.5)	int Estime 5,0.5)	Point Estimation RMSE (0.1,0.5,0.5)	E 1,0.5)	(0.1, 0.1, 0.1)	1,0.1)
Model 3.1   Model 3.1   Model 3.1   Model 3.1   Model 3.1	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.0         0.0         0.462         0.468         0.406         0.406         0.001         0.001         0.131         0.161         0.056         0.132         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.056         0.131         0.045         0.131         0.045         0.131         0.045         0.131         0.045         0.131         0.045         0.132         0.045         0.132         0.045         0.132	:						Model 3.1								
0.3         0.45         0.15	Infeasible Estimation	Ó	c c	0	0	C T	0	0	0	0	0	0	0	0	0
1	Local Folynomial R-splines	9.0	e	0.462	0.468	0.405	0.406	0.051	0.099	0.090	0.155	0.151	0.164	0.663	0.003
0.45         0.48         0.648         0.673         0.529         0.533         0.045         0.115         0.118         0.156         0.656         0.158         0.156         0.656         0.158         0.156         0.656         0.158         0.156         0.656         0.158         0.158         0.656         0.158         0.056         0.158         0.158         0.059         0.070         0.043         0.045         0.114         0.056         0.1124         0.056         0.118         0.156         0.157         0.177         0.179         0.177         0.179         0.079         0.075         0.075         0.017	Partitioning	- ·	٠.	060 0	0.199	0.070	0.153	0.030	0.103	0.020	0.173	0.100	0.237	0.121	0.336
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Feasible Estimation	4	•		9	)	9		)	)		9	1	1	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.45	0.31	0.648	0.653	0.529	0.533	0.062	0.115	0.118	0.155	0.651	0.659	1.285	1.289
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	П	1	0.463	0.483	0.405	0.413	0.050	0.124	0.085	0.181	0.152	0.261	0.663	0.694
0.17         0.24         0.401         0.426         0.314         0.333         0.470         0.508         0.426         0.445         0.445         0.334         0.334         0.336         0.475         0.437         0.447         0.316         0.426         0.445         0.445         0.344         0.334         0.344         0.334         0.347         0.347         0.348         0.346         0.447         0.348         0.347         0.447         0.348         0.346         0.447         0.348         0.346         0.447         0.348         0.346         0.446         0.346         0.347         0.348         0	Partitioning	1	1	0.090	0.199	0.070	0.153	0.045	0.103	0.075	0.173	0.100	0.237	0.121	0.336
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.2								
0.17         0.24         0.401         0.426         0.346         0.325         0.445         0.426         0.347         0.507         0.426         0.426         0.347         0.321         0.436         0.445         0.446         0.321         0.359         0.427         0.349         0.341         0.347         0.348         0.048         0.246         0.349         0.349         0.341         0.347         0.349         0.348         0.286         0.286         0.346         0.346         0.349         0.348         0.289         0.347         0.440         0.299         0.346         0.340         0.348         0.349         0.341         0.240         0.340         0.240         0.346         0	nfeasible Estimation														
27         8         0.399         0.4413         0.341         0.321         0.345         0.455         0.445         0.422         0.316           9.7         8         0.399         0.431         0.391         0.350         0.365         0.445         0.365         0.445         0.495         0.316           1.3         0.28         0.286         0.286         0.346         0.366         0.356         0.366         0.356         0.316         0.275         0.027         0.366         0.356         0.326         0.326         0.376         0.346         0.366         0.356         0.326         0.326         0.346         0.346         0.326         0.346         0.366         0.356         0.366         0.356         0.346         0.336         0.366         0.366         0.366         0.366         0.366         0.366         0.366         0.366         0.366         0.366         0.366         0.376         0.366         0.376         0.366         0.376         0.386         0.366         0.386         0.366         0.386         0.366         0.386         0.366         0.386         0.366         0.386         0.386         0.386         0.386         0.386         0.386 <td< td=""><td>Local Polynomial</td><td>0.17</td><td>0.24</td><td>0.401</td><td>0.426</td><td>0.314</td><td>0.333</td><td>0.470</td><td>0.508</td><td>0.426</td><td>0.482</td><td>0.416</td><td>0.475</td><td>0.344</td><td>0.371</td></td<>	Local Polynomial	0.17	0.24	0.401	0.426	0.314	0.333	0.470	0.508	0.426	0.482	0.416	0.475	0.344	0.371
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	27	œ	0.399	0.413	0.314	0.321	0.398	0.507	0.352	0.455	0.426	0.422	0.316	0.328
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	27	œ	0.495	0.576	0.387	0.444	0.391	3.315	0.427	2.301	0.453	1.313	0.501	0.760
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	0	0	0	0	0	0	0	0	0	0	0	0	ì	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.38	0.78	0.373	0.383	0.286	0.296	0.360	0.361	0.293	0.308	0.308	0.315	0.275	0.230
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines		п,	0.376	0.398	0.289	0.312	0.375	0.413	0.270	0.366	0.359	0.425	0.262	0.311
0.25         0.32         0.188         0.253         0.138         0.191         0.120         0.166         0.178         0.221         0.231         0.287         0.231           8         1         0.167         0.204         0.129         0.159         0.166         0.178         0.193         0.236         0.193           8         1         0.167         0.204         0.129         0.189         0.116         0.191         0.166         0.183         0.193         0.286         0.193         0.228         0.193         0.284         0.193         0.286         0.116         0.191         0.166         0.189         0.189         0.193         0.241         0.189         0.241         0.189         0.116         0.191         0.166         0.189         0.189         0.189         0.189         0.116         0.191         0.166         0.189         0.	Fartitioning	7	Т	0.377	0.398	0.230	0.312	0.381	0.400	0.299	0.553	0.346	0.335	0.269	0.354
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.3								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation	0	0	0	0	0		0		1		0	0		0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.25	0.32	0.188	0.253	0.138	0.191	0.120	0.166	0.178	0.221	0.231	0.287	0.231	0.287
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	<b>x</b> 0 0	٦.	0.167	0.204	0.122	0.152	0.186	0.116	0.191	0.166	0.193	0.236	0.193	0.236
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Fartitioning Fasible Estimation	o	ī	0.258	0.207	0.201	601.0	0.432	0.100	0.384	0.170	0.340	0.241	0.284	0.337
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.39	0.28	0.142	0.169	0.087	0.117	0.061	0.100	0.101	0.125	0.101	0.125	0.101	0.125
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines			0.140	0.204	0.094	0.152	0.058	0.116	0.077	0.166	0.137	0.236	0.137	0.236
Model 3.4  0.07 0.99 1.791 2.044 0.814 0.834 0.180 0.513 0.357 1.378 0.678 4.044  216 27 1.902 1.890 0.873 0.873 0.394 0.183 0.493 0.448 1.565 1.326 4.118  216 27 1.902 1.890 0.873 0.873 0.373 1.572 1.693 0.448 1.565 1.326 4.118  216 27 1.902 1.890 0.873 0.873 0.394 0.183 0.493 0.448 1.565 1.326 4.118  217 2.061 2.077 2.061 0.820 0.880 0.244 0.156 0.390 0.445 0.838 1.343 3.864  228 1 1.599 1.743 1.015 1.018 2.269 1.146 0.594 0.490 1.548 0.751 1.458  229 0.33 0.33 0.819 0.598 0.639 0.477 1.138 1.281 1.305 1.141 0.617 0.792 0.642  27 27 0.845 0.612 0.676 0.489 0.875 1.383 1.185 1.282 0.761 0.712 0.548  28 0.27 27 0.845 0.612 0.655 1.093 0.946 1.727 1.040 2.383 0.518 3.710 0.594  29 0.28 0.756 0.573 0.594 0.661 1.727 1.040 2.383 0.518 3.710 0.594  20 0.29 0.20 0.20 0.20 0.20 0.20 0.20 0.	Partitioning	1	1	0.142	0.207	0.097	0.159	0.069	0.106	0.079	0.175	0.117	0.241	0.163	0.357
0.07         0.9         1.791         2.044         0.814         0.644         0.180         0.513         0.357         1.378         0.678         4.044           216         27         1.902         1.890         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.873         0.889         0.167         0.185         0.448         1.565         1.326         4.118           0.29         0.26         2.077         2.061         0.820         0.880         0.070         0.107         0.475         0.836         0.475         0.836         0.475         0.836         0.475         0.838         1.343         3.864           8         1         1.599         1.743         1.015         1.018         2.269         1.146         0.594         0.490         1.548         0.751         1.458           0.33         0.819         0.598         0.676         0.489         0.875         1.146         0.594         0.761         0.771         0.752         0.642           27         27         0.709         1.395         0.676 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td>Model 3.4</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>							Model 3.4								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.07	6.0	1.791	2.044	0.814	0.847	0.644	0.180	0.513	0.357	1.378	0.678	4.044	3.830
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	216	27	1.902	1.890	0.873	0.873	0.394	0.183	0.493	0.448	1.565	1.326	4.118	4.010
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	216	27	1.926	1.762	0.757	0.758	1.572	1.693	0.673	2.185	0.784	2.899	3.748	4.395
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.29	0.26	2.077	2.061	0.820	0.839	0.070	0.107	0.475	0.336	0.475	0.336	3.792	3.787
8 1 1.599 1.743 1.015 1.018 2.269 1.146 0.594 0.490 1.548 0.751 1.458  0.33 0.33 0.819 0.598 0.639 0.477 1.138 1.281 1.305 1.141 0.617 0.792 0.642  27 27 0.845 0.612 0.676 0.489 0.875 1.383 1.185 1.282 0.761 0.712 0.548  27 0.709 1.395 0.555 1.093 0.946 1.727 1.040 2.383 0.518 3.710 0.599  0.27 0.28 0.756 0.573 0.592 0.460 1.205 1.219 1.359 1.155 0.476 0.681 0.690  8 1 0.936 0.822 0.744 0.661 0.653 0.747 1.318 1.329 0.705 0.582 0.497  8 1 0.900 0.801 0.705 0.643 0.653 0.705 0.705 0.705 0.705	B-splines	oo ·	1	2.061	2.005	0.882	0.880	0.244	0.156	0.390	0.445	0.838	1.343	3.864	4.024
Model 3.5  0.33  0.33  0.33  0.34  0.35  0.819  0.598  0.676  0.477  1.138  1.281  1.305  1.141  0.617  0.792  0.642  0.648  2.7  27  27  27  27  27  27  27  27  2	Partitioning	œ	1	1.599	1.743	1.015	1.018	2.269	1.146	0.594	0.490	1.548	0.751	1.458	2.308
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.819	0.598	0.639	0.477	1.138	1.281	1.305	1.141	0.617	0.792	0.642	0.509
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	7 7 7	0.845	1.305	0.070	1.003	0.875	1.383	1.185	1.282	0.761	0.712	0.548	0.580
0.27 0.28 0.756 0.573 0.592 0.460 1.205 1.219 1.359 1.155 0.476 0.681 0.690 8 1 0.936 0.822 0.734 0.661 0.622 0.716 1.175 1.166 0.756 0.782 0.497 8 1 0.900 0.801 0.755 0.641 0.953 0.870 1.318 1.322 0.795 0.588 0.638	Fartitioning	7	7	607.0	1.090	0.000	1.090	0.940	1.121	1.040	7.000	0.010	9.710	0.088	0.400
8 1 0.936 0.822 0.734 0.601 0.622 0.716 1.15 1.166 0.756 0.782 0.497 8 1 0.900 0.811 0.755 0.784 0.651 0.852 0.870 1.318 1.329 0.705 0.588 0.628	Local Polynomial	0.27	86.0	0.756	0.573	0.592	0.460	1 205	1 219	1 359	155	0.476	0.681	0.690	0.493
8 1 0.000 0.	B-enlines	ox	-	0 936	0.00	0.334	0.661	0.630	0.716	1 175	1 166	0.756	0.001	0.000	0.545
	Dontitioning	0	٠.	00000	1080	00	0.641	0.0	0.10	1 210	1 223	100	0 0	0690	0.040

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.62: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=500,\,\sigma^2=1,\,X_{i,\ell}\sim\beta(1,1),$  Quantile Cells

	Tuning Parameter	ing neter	Root In M	Root Integrated MSE	Inget M	Ingetrated MAE	(0.5, 0.5, 0.5)	5,0.5)	Point Es $(0.1, 0.5, 0.5)$	int Estime 5,0.5)	Point Estimation RMSE (0.1,0.5,0.5)	iE 1,0.5)	(0.1, 0.1, 0.1)	1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
:						Model 3.1								
Infeasible Estimation	d	d	0	0	0	000	5	200	0	6	0	0	, F	0
Local Polynomial B-splines	9.0 T	0.9 1	0.423	0.446	0.363	0.375	0.047	0.034	0.085	0.196	0.150	0.364	0.659	0.007
Partitioning	1	1	0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
Feasible Estimation														
Local Polynomial	0.44	0.3	0.591	0.596	0.482	0.486	0.057	0.100	0.126	0.164	0.655	0.663	1.289	1.292
B-splines			0.423	0.446	0.363	0.375	0.046	0.112	0.085	0.196	0.162	0.364	0.659	0.712
Fartitioning	Т	Т	0.087	0.200	0.068	0.151	0.043	0.102	0.079	0.180	0.103	0.284	0.125	0.437
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.17	0.24	0.407	0.430	0.317	0.335	0.435	0.462	0.395	0.420	0.462	0.487	0.472	0.487
B-splines	27	00	0.403	0.421	0.317	0.327	0.401	0.463	0.347	0.414	0.437	0.483	0.407	0.490
Partitioning	27	œ	0.491	0.575	0.383	0.443	0.345	2.468	0.440	2.176	0.539	1.702	0.621	1.288
reasible Estimation	0 04	0.07	0 0 7 7	7000	900 0	906.0	0.077	000	010	0 997	0060	0060	7000	0060
Dealine	6.0	0.5	0.074	0.384	0.200	0.230	1000	0.300	0.512	0.027	0.300	0.300	0.594	0.303
D-spines Partitioning	1 01		0.379	0.404	0.291	0.315	0.401	0.411	0.337	0.481	0.337	0.368	0.343	0.484
)						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.28	0.34	0.168	0.231	0.120	0.171	0.088	0.132	0.164	0.197	0.282	0.323	0.282	0.323
B-splines	œ	1	0.156	0.195	0.114	0.142	0.165	0.105	0.192	0.178	0.234	0.331	0.234	0.331
Partitioning	∞	1	0.257	0.206	0.201	0.155	0.424	0.105	0.373	0.184	0.358	0.284	0.348	0.477
reasible Estimation	000	0 0	0 1 0 0	- C	0 044	0010	0 0	600	711	0 140	7	0.45	-	0.145
B-enlines	0.00	77.0	0.125	0.136	0.00	0.109	0.000	0.035	1000	0.142	0.114	0.142	0.114	0.142
D-spines Partitioning			0.131	0.206	0.090	0.155	0.104	0.105	0.104	0.184	0.129	0.284	0.173	0.477
0						Model 2.4								
Infeasible Estimation						Fro lanomi								
Local Polynomial	0.08	0.9	1.785	1.320	0.508	0.496	0.331	0.148	0.413	0.353	7.493	0.619	8.533	3.831
B-splines	216	27	1.224	1.175	0.570	0.539	0.389	0.162	0.520	0.459	2.179	1.985	4.443	4.338
Partitioning	216	27	1.281	1.214	0.470	0.536	2.240	1.419	0.948	3.605	1.165	4.904	3.797	8.345
Feasible Estimation														
Local Polynomial	0.29	0.26	1.345	1.327	0.469	0.489	0.063	0.096	0.431	0.325	0.431	0.325	3.805	3.800
D-spunes Partitioning	o oc	N 6	1.555	1.271	0.524	0.033	1.328	1.213	0.550	0.400	1 123	0.974	2.0.0	2.807
0	)	1	•	1		100 - E		1		-	1		1	i
Infeasible Estimation						Model 5.5								
Local Polynomial	0.33	0.33	0.789	0.613	0.616	0.493	1.130	1.205	1.151	1.052	0.716	0.734	0.622	0.641
B-splines	27	27	0.848	0.635	0.668	0.510	0.928	1.341	1.003	1.207	0.893	0.787	0.541	0.799
Partitioning	27	27	0.738	1.279	0.576	1.013	0.790	1.478	0.789	4.460	0.658	5.563	0.762	7.308
Feasible Estimation	0	000	7	0 100	0 1	77	1 016	1 005	1 263	1 170	000	0990	000	о 2
Dealine	0.1.0	0.40	000.0	0.00	9890	0.410	1.610	0.44.0	1.202	080	0.000	0.003	0.00 0.00 0.00 0.00	0.014
D-spines Doutitioning	0 0		0.690	0.603	0.000	0.042	0.440	0.014	1 110	1.071	0.026	0.769	0.400	0.030
Farmoung	5	7	0.000	001.0	0.000	0.020	0.30	0.310	1.110	11011	0.000	0.105	0.110	0.044

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.63: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=500,\,\sigma^2=1,\,X_{i,\ell}\sim\beta(2,2),$  Quantile Cells

Degree         Jinoz         Cubic         Linoz         Cubic         <		rumng Parameter	Tuning arameter	Root Integrated MSE	SE	MAE	\E	(0.5,0.5,0.5)	5,0.5)	(0.1,0)	(0.1,0.5,0.5) $(0.1,0.1,0.1)$	(0.1,0.1,0.5)	1,0.5)	(0.1,0.1,0.1)	1,0.1)
Nodel 3.1   Nodel 3.2   Node	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.9         0.9         0.356         0.386         0.304         0.048         0.083         0.104         0.109         0.11							Model 3.1								
1	njeasible Estimation Local Polymomial	0	60	0.356	0.366	998	0.304	0.048	0.083	0 104	0 100	121	0.977	0.65	0.697
1	B-splines		; <del>-</del>	0.359	0.387	0.300	0.317	0.047	0.093	0.097	0.256	0.206	0.818	0.671	1.044
0.43         0.29         0.497         0.5044         0.404         0.404         0.404         0.404         0.404         0.404         0.044         0.053         0.057         0.056         0.626         0.626         0.626         0.676         0.671         1.201           1         1         0.0590         0.237         0.307         0.177         0.044         0.097         0.055         0.266         0.818         0.671         0.127         0.046         0.097         0.026         0.266         0.818         0.017         0.017         0.017         0.018         0.008         0.018         0.018         0.008         0.026	Partitioning	1		0.000	0.200	0.070	0.147	0.044	0.087	0.090	0.219	0.127	0.426	0.152	0.705
0.43         0.29         0.447         0.504         0.404         0.403         0.087         0.104         0.226         0.206         0.675         1.21           1         1         0.090         0.239         0.407         0.147         0.044         0.087         0.007         0.219         0.127         0.426         0.067         0.121           1.1         1         0.090         0.200         0.070         0.147         0.044         0.087         0.000         0.219         0.127         0.426         0.067         0.126         0.067         0.128         0.236         0.144         0.571         2.271         0.127         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0.128         0	easible Estimation														
1   1   0.339   0.387   0.300   0.317   0.047   0.093   0.007   0.226   0.226   0.248   0.057   0.007   0.047   0.094   0.087   0.026   0.048   0.041   0.049   0.027   0.049   0.048   0.041   0.049   0.027   0.048   0.041   0.049   0.048   0.048   0.041   0.049   0.028   0.048   0.041   0.049   0.028   0.04	Local Polynomial	0.43	0.29	0.497	0.504	0.404	0.409	0.053	0.087	0.164	0.226	0.662	0.679	1.291	1.300
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	1	1	0.359	0.387	0.300	0.317	0.047	0.093	0.097	0.256	0.206	0.818	0.671	1.044
0.18         0.25         0.401         0.305         0.365         0.369         0.428         0.413         1.556         1.224         1.557           2.7         8         0.381         0.416         0.387         0.337         0.365         0.369         0.428         0.413         1.506         0.367         0.369         0.355         0.416         0.387         0.317         0.385         0.386         0.414         0.389         0.414         0.389         0.714         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.717         0.389         0.311         0.387         0.386         0.414         1.500         0.338         0.717         0.718         0.744         1.500         0.328         0.718         0.744         1.500         0.328         0.718         0.749         0.749         0.749         0.749         0.749         0.744         1.500         0.328         0.749         0.749         0.749         0.744         1.500         0.749	Partitioning	1	1	0.090	0.200	0.070	0.147	0.044	0.087	0.090	0.219	0.127	0.426	0.152	0.705
0.18         0.25         0.401         0.419         0.305         0.385         0.389         0.428         0.413         1.556         1.254         1.557           27         8         0.491         0.435         0.380         0.389         0.450         0.589         0.459         0.459         0.459         0.589							Model 3.2								
0.18         0.25         0.401         0.419         0.305         0.326         0.389         0.428         0.418         1.254         1.257         1.224         1.557           27         8         0.391         0.435         0.386         0.389         0.449         0.571         0.385         0.436         0.438         0.378         0.389         0.441         0.571         0.589         2.714         0.980           36         0.26         0.389         0.436         0.389         0.378         0.386         0.344         0.389         2.714         0.980           4         4         0.376         0.479         0.281         0.386         0.344         0.389         0.349         0.371         0.389         0.344         0.389         0.345         0.371         0.388         0.345         0.444         1.500         0.388         0.345         0.449         1.778         0.435         0.449         0.444         1.500         0.345         0.445         0.445         0.445         0.445         0.445         0.445         0.445         0.446         0.444         1.500         0.564         0.446         0.446         0.446         0.446         0.446         0.446 <td>feasible Estimation</td> <td></td>	feasible Estimation														
27         8         0.388         0.416         0.347         0.345         0.346         0.355         0.346         0.355         0.346         0.355         0.346         0.355         0.346         0.355         0.346         0.355         0.346         0.355         0.347         0.375         0.348         0.356         0.349         0.355         0.346         0.356         0.346         0.356         0.344         0.356         0.349         0.356         0.346         0.378         0.344         0.360         0.378         0.346         0.366         0.367         0.344         1.500         0.502         1.778         0.725           0.34         0.36         0.446         0.386         0.387         0.367         0.369         0.349         0.356         0.348         0.388         0.376         0.369         0.376         0.369         0.376         0.369         0.376         0.369         0.376         0.369         0.348         0.369         0.348         0.369         0.348         0.369         0.348         0.369         0.348         0.369         0.348         0.369         0.348         0.369         0.348         0.349         0.348         0.349         0.348         0.349	Local Polynomial	0.18	0.25	0.401	0.419	0.305	0.320	0.365	0.369	0.428	0.413	1.556	1.224	1.557	1.240
27         8         0.491         0.571         0.382         0.435         0.435         0.435         0.714         0.589         2.714         0.580           0.36         0.28         0.28         0.381         0.385         0.385         0.345         0.345         0.349         0.378           4         4         0.380         0.475         0.286         0.387         0.344         1.500         0.502         1.778         0.378           0.34         0.36         0.479         0.281         0.366         0.087         0.444         1.500         0.502         1.778         0.725           1         1         0.106         0.188         0.095         0.138         0.060         0.097         0.187         0.240         0.197         0.744         1.197         0.740         0.187         0.741         0.197         0.049         0.087         0.091         0.240         0.197         0.744         0.051         0.049         0.087         0.091         0.240         0.197         0.049         0.081         0.147         0.197         0.049         0.081         0.157         0.240         0.197         0.187         0.240         0.197         0.187         <	B-splines	27	œ	0.398	0.416	0.307	0.317	0.365	0.369	0.355	0.450	0.584	1.350	0.637	1.330
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	27	œ	0.491	0.571	0.382	0.435	0.309	1.914	0.571	2.271	0.839	2.714	0.980	3.852
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	0	000	0	0	0	000	000	0	2	0	0	000	900	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Folynomial	0.00	0.20	0.309	0.380	0.4.0	0.200	0.301	0.505	0.540	0.309	0.503	0.000	0.320	1 000
0.34         0.36         0.138         0.195         0.095         0.138         0.060         0.097         0.182         0.237         0.399         0.549         0.399           1         1         0.106         0.186         0.066         0.087         0.091         0.220         0.197         0.774         0.197         0.077         0.146         0.087         0.091         0.220         0.197         0.754         0.197           0.37         0.106         0.186         0.077         0.149         0.046         0.087         0.091         0.220         0.197         0.159           0.37         0.27         0.187         0.095         0.051         0.088         0.135         0.025         0.167         0.189         0.157         0.203         0.157         0.203         0.157         0.203         0.157         0.203         0.157         0.203         0.157         0.203         0.157         0.189         0.157         0.189         0.157         0.189         0.157         0.189         0.157         0.189         0.157         0.189         0.157         0.189         0.157         0.189         0.157         0.189         0.157         0.189         0.189         0	D-spilles Partitioning	4 4	4 4	0.380	0.403	0.293	0.361	0.373	1.308	0.320	1.500	0.502	1.032	0.455	2.676
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0		ı		)	1	Model 3.3							) 	i
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	foreible Hetimotion						or concern								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Jeasible Estimation Local Polynomial	0.34	0.36	0.138	0.195	0.095	0.138	090.0	0.097	0.182	0.237	0.399	0.549	0.399	0.549
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	1	1	0.106	0.186	0.074	0.131	0.046	0.087	0.091	0.240	0.197	0.754	0.197	0.754
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	1	1	0.107	0.203	0.077	0.149	0.046	0.088	0.091	0.221	0.134	0.426	0.169	0.745
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	asible Estimation	1								1		1		,	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.37	0.27	0.100	0.137	0.066	0.095	0.051	0.081	0.157	0.203	0.157	0.203	0.157	0.203
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	6		0.117	0.187	0.081	0.131	0.079	0.088	0.135	0.243	0.253	0.770	0.253	0.770
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	77	-	0.152	0.213	0.102	0.153	0.157	0.267	0.187	0.383	0.227	0.507	0.257	0.820
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	:						Model 3.4								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	jeasible Estimation		0	0	1	0	0	1	0		0	0	1	0	0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.11	0.0	0.588	0.551	0.256	0.208	0.166	0.095	0.473	0.339	4.660	0.507	5.969	3.837
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	D-spines	40	1 -	0.000	0.400	0.273	0.703	0.233	0.170	0.400	10.430	1.000	17.023	4.000	0.147
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	rartitioning geible Retimation	04	7	0.799	0.721	0.001	0.550	1.040	7.100	1.741	10.099	7.037	17.040	9.090	34.300
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.32	0.26	0.561	0.544	0.184	0.202	0.053	0.082	0.361	0.415	0.361	0.415	818	3.821
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	1	e e e	0.561	0.514	0.222	0.242	0.146	0.113	0.268	0.396	0.672	2.145	3.864	4.330
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	7	8	0.543	0.524	0.298	0.310	0.538	1.083	0.426	1.224	0.656	1.560	3.255	3.847
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	feasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.720	0.611	0.560	0.493	1.090	1.058	0.831	0.952	1.026	0.858	0.623	0.958
27 27 0.699 1.127 0.536 0.914 0.606 2.796 0.698 9.955 0.937 17.105 1.039 3 0.28 0.27 0.713 0.616 0.558 0.495 1.199 1.194 0.984 1.178 0.882 0.697 0.362 8 4 0.775 0.705 0.590 0.560 0.805 1.055 0.637 0.830 0.977 1.698 0.526	B-splines	27	27	0.775	0.646	0.598	0.517	0.961	1.175	0.695	0.930	1.149	3.559	0.714	3.221
0.28 0.27 0.713 0.616 0.558 0.495 1.199 1.194 0.984 1.178 0.882 0.697 0.362 8 4 0.775 0.705 0.592 0.560 0.805 1.055 0.637 0.830 0.977 1.698 0.526	Partitioning	27	27	0.699	1.127	0.536	0.914	909.0	2.796	0.698	9.955	0.937	17.105	1.039	37.494
omiai 0.28 0.27 0.775 0.705 0.592 0.560 0.805 1.055 0.634 0.305 0.977 0.809 0.592 0.500 0.500 0.650 0.	easible Estimation	o o	1	1	0	i i	2	000		9	7	0	0	0	, ,
8 4 0.175 0.109 0.109 0.800 1.050 0.850 0.850 0.175 0.	Local Polynomial	0.28	0.27	0.713	0.010	0.558	0.495	1.199	1.194	0.984	1.178	0.882	1.697	0.362	0.534
	B-splines	<b>x</b> 0 0	4.	0.775	0.705	0.592	0.560	0.805	1.055	0.637	0.830	0.977	1.698	0.526	1.391

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.64: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 500, \sigma^2 = 4, X_{i,\ell} \sim \beta(0.5, 0.5),$  Quantile Cells

	Tuning Parameter	ing neter	Root Integrated MSE	tegrated SE	$rac{Ingetrated}{MAE}$	getrated MAE	(0.5, 0.5, 0.5)	5,0.5)	Point E; (0.1,0.5,0.5)	int Estima 5,0.5)	Point Estimation RMSE $(0.5,0.5)$	E 1,0.5)	(0.1, 0.1, 0.1)	(1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
:						Model 3.1								
nfeasible Estimation	d	d	6		1	0.7	200	0	9	0.00	0.70	6	0	1
Local Folynomial B-splines	0.0 -	 -	0.484	0.516	0.417	0.434	0.094	0.185	0.105	0.240	0.242	0.304	0.093	0.715
Partitioning			0.179	0.399	0.141	0.307	0.090	0.206	0.151	0.345	0.199	0.475	0.242	0.671
Feasible Estimation														
Local Polynomial	0.38	0.28	0.662	0.686	0.540	0.558	0.122	0.213	0.213	0.270	0.676	0.695	1.299	1.309
B-splines	1	1	0.489	0.573	0.420	0.470	0.103	0.235	0.167	0.340	0.284	0.486	0.710	0.809
Partitioning	1	1	0.190	0.399	0.145	0.307	0.148	0.206	0.191	0.345	0.210	0.475	0.250	0.671
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.21	0.28	0.497	0.583	0.391	0.456	0.505	0.590	0.531	0.616	0.589	0.643	0.535	0.563
B-splines	<b>x</b> 0 (	<b>x</b> 0 (	0.447	0.566	0.350	0.441	0.573	0.617	0.544	0.615	0.544	0.601	0.427	0.539
Partitioning	x	x	0.594	1.136	0.467	0.875	0.940	6.140	0.940	4.325	0.782	2.587	0.630	1.496
reasible Estimation	760	96 0	0000	0 4 9 0	006 0	0.049	0.979	700	0.00	0 0 0	0 0 11	0000	0 007	1960
Benlines	, c	0.50	0.000	20.4.0	0.303	0.397	0.0.0	0.400	0.042	0.0	0.00	0.000	0.00	0.00
Partitioning	4 4		0.495	0.532	0.380	0.418	0.694	0.800	0.677	0.699	0.601	0.545	0.462	0.685
)						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.3	0.37	0.303	0.426	0.227	0.326	0.192	0.287	0.316	0.372	0.450	0.472	0.450	0.472
B-splines	1	1	0.208	0.370	0.154	0.281	0.097	0.232	0.151	0.331	0.271	0.473	0.271	0.473
Partitioning	П	-	0.209	0.403	0.158	0.310	0.097	0.207	0.151	0.346	0.206	0.477	0.261	0.682
Local Polynomial	0.35	0.26	0.199	0.273	0.143	0.207	0.124	0.208	0.204	0.259	0.204	0.259	0.204	0.259
B-splines	, e	-	0.240	0.370	0.177	0.281	0.225	0.232	0.255	0.333	0.318	0.473	0.318	0.473
Partitioning	3	1	0.348	0.407	0.240	0.311	0.497	0.348	0.475	0.442	0.442	0.491	0.400	0.683
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.09	0.9	1.872	2.055	1.007	0.891	0.771	0.234	0.735	0.413	1.504	0.726	4.087	3.835
B-splines	125	2.7	2.004	1.947	1.006	0.880	0.315	0.344	0.551	0.580	1.690	1.412	4.168	4.037
Fartitioning	125	7.5	1.878	2.010	1.076	1.024	1.971	3.386	2.111	4.385	4.015	5.901	3.903	6.906
Local Polynomial	0	96 0	2 083	0.000	0.846	0.880	0 135	0.911	0.506	0.408	508	0.408	3 795	3 703
B-splines	000		2,074	2.028	0.925	0.946	0.408	0.272	0.513	0.534	868.0	1.407	3.872	4.041
Partitioning	œ	1	1.658	1.772	1.080	1.069	2.360	2.135	0.897	1.257	1.664	0.951	1.539	2.408
						Model 3.5								
Infeasible Estimation	0	0	0	1	0	0	1	0	0	0	1	0	1	0
Local Polynomial	0.33	0.33	0.852	0.715	0.666	0.569	1.145	1.303	1.331	1.202	0.720	0.924	0.739	0.686
B-splines	72.0	72.0	0.900	1.007	0.719	0.609	0.888	1.405	1.216	1.342	0.869	0.852	0.683	0.741
Fartitioning	7	7	1.072	1.697	0.840	1.359	1.075	5.397	1.230	4.503	0.918	0.327	1.043	0.370
Local Polynomial	0.28	0.26	0.783	0,605	0.614	0.486	1.200	1.214	1.364	1.184	0.514	0.709	0.706	0.547
B-splines	œ	-	0.969	278	0.759	0.703	0.739	0.740	1.210	1.202	0.849	200	0.591	0.678
Partitioning	ooc		1,003	0.872	0.789	0.696	1.183	0.887	1.435	1.354	1,009	0.720	0.783	0.834

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.65: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=500,\,\sigma^2=4,\,X_{i,\ell}\sim\beta(1,1),$  Quantile Cells

		Tuning Parameter	ing neter	Root Integrated MSE	egrated E	Ingetrated MAE	ated	(0.5,0.5,0.5)	5,0.5)	Point E $(0.1,0.5,0.5)$	oint Estim 5,0.5)	Point Estimation RMSE 0.5,0.5) (0.1,0.1,0.5)	iE 1,0.5)	(0.1, 0.1, 0.1)	(1,0.1)
Model 31   Model 32	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.9         0.445         0.450         0.375         0.386         0.179         0.170         0.297         0.244         0.387         0.480         0.487         0.179         0.179         0.179         0.179         0.088         0.179         0.179         0.087         0.087         0.087         0.087         0.087         0.087         0.087         0.087         0.087         0.088         0.179         0.179         0.088         0.179         0.019         0.089         0.019         0.019         0.0204         0.019         0.0204         0.019         0.0204         0.019         0.0204         0.037         0.019         0.0204         0.037         0.019         0.0204         0.037         0.010         0.0204         0.037         0.0204         0.037         0.040	:						Model 3.1								
0.38         0.470         0.487	nfeasible Estimation	0	0	0.443	0.480	0 27	308	080 0	0 1 70	0.170	796.0	0.044	0.361	889	737
1	B-enlines	6.0		0.449	0.430	0.379	0.330	0.003	0.212	0.170	0.367	0.305	0.00	0.000	00%
0.38         0.27         0.606         0.633         0.494         0.514         0.115         0.192         0.239         0.297         0.701         0.193         0.297         0.687         0.710         1.306           1         1         0.450         0.542         0.347         0.101         0.212         0.199         0.397         0.701         0.770         0.701         0.203         0.687         0.701         0.702         0.702         0.703         0.687         0.701         0.702         0.703         0.704         0.702         0.703         0.703         0.704         0.702         0.703         0.703         0.704         0.704         0.704         0.704	Partitioning			0.174	0.399	0.136	0.301	0.086	0.204	0.158	0.361	0.207	0.567	0.249	0.874
0.38         0.27         0.66         0.633         0.544         0.514         0.112         0.129         0.290         0.297         0.297         0.710         1.306           1         1         0.406         0.542         0.344         0.501         0.144         0.301         0.175         0.206         0.381         0.290         0.710         0.175         0.206         0.381         0.290         0.711         0.710         0.711         0.710         0.720         0.710         0.711	easible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.38	0.27	0.606	0.633	0.494	0.514	0.112	0.192	0.239	0.297	0.687	0.710	1.306	1.319
1   1   0.194   0.399   0.144   0.301   0.175   0.204   0.206   0.361   0.250   0.567   0.276     8	B-splines	1	1	0.450	0.542	0.380	0.437	0.101	0.212	0.169	0.367	0.310	0.682	0.710	0.899
0.21         0.28         0.562         0.582         0.456         0.533         0.502         0.779         0.779         0.774           8         8         0.456         0.456         0.533         0.502         0.779         0.779         0.774           8         8         0.456         0.456         0.446         0.546         4.056         0.882         0.779         0.779         0.774           8         8         0.456         0.477         0.877         0.441         0.548         0.589         0.571         3.286         0.410         0.589         0.571         3.296         0.882         0.514         0.589         0.547         0.877         0.410         0.589         0.589         0.771         1.084         0.589         0.588         0.771         1.086         0.479         0.466         0.479         0.479         0.466         0.479	Partitioning	1	1	0.194	0.399	0.144	0.301	0.175	0.204	0.206	0.361	0.250	0.567	0.276	0.874
0.21         0.28         0.501         0.582         0.389         0.450         0.466         0.583         0.502         0.570         0.705         0.719         0.724           8         8         0.540         0.572         0.440         0.573         0.440         0.583         0.459         0.579         0.708         0.719         0.524           9         0.549         0.549         0.470         0.689         0.441         0.589         0.440         0.589         0.440         0.589         0.440         0.589         0.440         0.589         0.440         0.589         0.440         0.589         0.440         0.589         0.446         0.589 <t< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td>Model 3.2</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></t<>							Model 3.2								
0.21         0.50         0.58         0.58         0.41         0.54         0.563         0.562         0.542         0.74         0.73           8         8         0.450         0.582         0.359         0.441         0.543         0.563         0.562         0.882         0.411         0.543         0.583         0.589         0.818         0.589         0.882         0.411         0.588         0.589         0.489         0.888         0.414         0.589         0.889	nfeasible Estimation														
8         0.450         0.572         0.431         0.543         0.450         0.572         0.543         0.544         0.563         0.543         0.543         0.544         0.563         0.544         0.563         0.544         0.563         0.544         0.563         0.543         0.543         0.544         0.563         0.543         0.543         0.655         0.943         0.567         0.589         0.657         0.589         0.675         0.543         0.574         0.486         0.574         0.676         0.590         0.156         0.675         0.523         0.234         0.552         0.524         0.156         0.676         0.580         0.156         0.675         0.528         0.675         0.523         0.675         0.675         0.675         0.675         0.674         0.675         0.675         0.674         0.675         0.675         0.674         0.6	Local Polynomial	0.21	0.28	0.501	0.582	0.389	0.450	0.466	0.533	0.502	0.542	0.705	0.719	0.724	0.729
0.34         0.25         1.130         0.407         0.873         0.910         4.009         0.888         4.141         0.941         3.209         0.538           0.34         0.25         0.399         0.430         0.334         0.384         0.478         0.416         0.373         0.040         0.5363         0.396         0.388         0.378         0.441         0.057         0.980         0.724         0.479         0.655         0.441         0.508         0.042         0.479         0.880         0.657         0.980         0.724         0.479         0.880         0.652         0.943         0.679         0.880         0.724         0.470         0.880         0.724         0.470         0.880         0.724         0.470         0.880         0.675         0.980         0.724         0.770         0.880         0.675         0.980         0.724         0.770         0.880         0.724         0.780         0.675         0.980         0.675         0.980         0.675         0.980         0.675         0.980         0.675         0.980         0.675         0.990         0.676         0.676         0.680         0.675         0.990         0.675         0.940         0.676         0.880<	B-splines	<b>x</b> 0 c	<b>x</b> 0 0	0.450	0.572	0.350	0.441	0.543	0.559	0.491	0.583	0.563	0.812	0.534	0.837
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Fartitioning	0	ю	0.595	1.130	0.467	0.873	0.916	4.605	0.888	4.141	0.841	3.209	0.852	7.457
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.34	0.25	0.399	0.439	0.309	0.343	0.387	0.416	0.373	0.410	0.363	0.396	0.358	0.396
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	10		0.432	0.510	0,335	0.398	0.479	0.456	0.422	0.470	0,509	0.724	0.486	0.732
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	rO	1	0.518	0.571	0.397	0.435	0.711	1.039	0.655	0.943	0.679	0.880	0.677	0.967
0.34         0.38         0.277         0.395         0.201         0.296         0.146         0.235         0.294         0.346         0.355         0.298         0.655         0.298         0.655         0.298         0.655         0.298         0.655         0.298         0.655         0.298         0.655         0.298         0.665         0.298         0.665         0.298         0.665         0.298         0.665         0.298         0.665         0.298         0.665         0.298         0.667         0.298         0.667         0.298         0.667         0.298         0.205         0.158         0.208         0.298         0.666         0.405         0.205         0.158         0.208         0.666         0.405         0.206         0.158         0.208         0.666         0.405         0.206         0.158         0.234         0.285         0.234         0.285         0.234         0.285         0.215         0.218         0.236         0.405         0.216         0.218         0.236         0.234         0.286         0.428         0.211         0.218         0.286         0.284         0.218         0.284         0.211         0.286         0.234         0.218         0.286         0.284         0							Model 3.3								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	feasible Estimation				1	1	1	,				1	1	1	1
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.34	0.38	0.277	0.395	0.201	0.296	0.146	0.232	0.294	0.346	0.532	0.555	0.532	0.555
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines			0.195	0.365	0.143	0.270	0.090	0.209	0.159	0.355	0.298	0.662	0.298	0.662
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning easible Estimation	-	Т	0.196	0.402	0.148	0.304	0.090	0.205	0.158	0.363	0.213	0.567	0.269	0.899
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.34	0.25	0.185	0.264	0.133	0.197	0.116	0.192	0.234	0.285	0.234	0.285	0.234	0.285
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	4		0.245	0.367	0.179	0.271	0.238	0.215	0.302	0.362	0.405	0.666	0.405	0.666
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	4	1	0.389	0.428	0.274	0.314	0.582	0.547	0.518	0.680	0.543	0.711	0.553	0.941
0.1         0.9         1.209         1.337         0.676         0.550         0.517         0.215         0.653         0.420         2.118         0.693         4.420           125         27         1.337         1.263         0.708         0.676         0.309         0.307         0.653         0.603         2.095         2.143         4.420           125         27         1.531         1.559         0.910         0.805         1.702         2.838         1.778         7.185         1.861         9.825         4.003         1.9           0.3         0.25         1.353         1.342         0.500         0.539         0.123         0.486         0.528         0.891         1.758         3.899           8         3         1.198         1.219         0.745         0.735         1.499         2.596         0.824         1.992         1.773         1.663         2.326           0.33         0.33         0.822         0.775         0.735         1.139         1.228         1.171         1.096         0.843         0.918         0.783           0.34         0.35         0.787         0.711         0.622         0.944         1.367         1.071 <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td>Model 3.4</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td>							Model 3.4								
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	feasible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.1	6.0	1.209	1.337	0.676	0.550	0.517	0.215	0.653	0.420	2.118	0.693	4.419	3.844
125 27 1.531 1.559 0.910 0.805 1.702 2.838 1.778 7.185 1.861 9.825 4.003 1 1 6.0.3	B-splines	125	27	1.337	1.263	0.708	0.676	0.309	0.307	0.563	0.603	2.095	2.143	4.420	4.439
0.3 0.25 1.353 1.342 0.500 0.539 0.123 0.193 0.475 0.413 0.475 0.413 3.809 8 3 1.1353 1.342 0.576 0.616 0.350 0.268 0.486 0.528 0.891 1.758 3.899 8 3 1.136 1.219 0.745 0.735 1.499 2.596 0.824 1.992 1.273 1.663 2.326  Model 3.5  0.33 0.33 0.33 0.822 0.723 0.641 0.575 1.139 1.228 1.171 1.096 0.843 0.918 0.783  27 27 27 0.903 0.787 0.711 0.622 0.994 1.367 1.031 1.262 1.023 1.087 0.760  0.29 0.26 0.774 0.624 0.605 0.711 0.682 0.884 1.220 1.265 1.220 0.708 0.619  8 1 0.922 0.860 0.711 0.682 0.884 1.70 1.70 1.70 1.70 1.70 1.70 1.70 1.70	Partitioning	125	27	1.531	1.559	0.910	0.805	1.702	2.838	1.778	7.185	1.861	9.825	4.003	15.041
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	c	1	0	0	1	0	00	0	1	0	i	0	0	0
8 3 1.138 1.219 0.745 0.735 1.499 2.596 0.824 1.992 1.273 1.663 2.326 0.83 0.83 0.83 0.83 0.83 0.83 0.745 0.711 0.622 0.944 1.367 1.099 7.927 1.109 0.843 0.918 0.783 0.78 0.711 0.622 0.944 1.367 1.099 7.927 1.116 10.23 1.087 0.760 0.82 0.82 0.83 0.93 0.83 0.83 0.83 0.83 0.83 0.83 0.83 0.8	Local Polynomial R-enlines	ç.⊙ ⊗	0.2:0	1.353	1.342	0.500	0.559	0.123	0.195	0.473	0.415	0.475	1 758	3.800	0.00.0
0.33 0.33 0.822 0.723 0.641 0.575 1.139 1.228 1.171 1.096 0.843 0.918 0.783   27 27 0.903 0.787 0.711 0.622 0.944 1.367 1.031 1.262 1.023 1.087 0.760   27 27 27 1.089 1.610 0.855 1.282 0.952 2.873 1.069 7.927 1.116 10.236 1.325 1  0.29 0.26 0.774 0.624 0.605 0.501 1.212 1.220 1.265 1.265 0.629 0.708 0.619   8 1 0.922 0.860 0.711 0.682 0.812 0.864 1.023 1.90 0.686 0.688   8 1 0.688 0.860 0.711 0.682 0.711 0.682 0.814 1.023 0.933 1.080 0.688   9 1 0.688 0.860 0.711 0.682 0.817 1.70 1.70 1.70 1.71 1.71 1.71 1.71 1.	Partitioning	oo	ာက	1.198	1.219	0.745	0.735	1.499	2.596	0.824	1.992	1.273	1.663	2.326	3.068
0.33         0.33         0.822         0.723         0.641         0.575         1.139         1.228         1.171         1.096         0.843         0.918         0.783           27         27         0.903         0.787         0.711         0.622         0.944         1.367         1.031         1.262         1.023         1.087         0.760           27         27         1.089         1.610         0.855         1.282         0.932         2.873         1.069         7.927         1.116         10.236         1.325         1           0.29         0.26         0.774         0.654         0.605         0.501         1.212         1.226         1.265         1.265         0.629         0.708         0.619           8         1         0.922         0.860         0.771         0.682         0.771         0.682         0.864         1.79         1.79         1.79         1.79         1.79         1.77         1.79         1.79         1.79         1.79         1.79         1.75         1.66         0.686							Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	sfeasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.822	0.723	0.641	0.575	1.139	1.228	1.171	1.096	0.843	0.918	0.783	0.872
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	27	27	0.903	0.787	0.711	0.622	0.944	1.367	1.031	1.262	1.023	1.087	0.760	1.127
0.29 0.26 0.774 0.624 0.605 0.501 1.212 1.220 1.265 1.205 0.629 0.708 0.619 8 1 0.922 0.860 0.711 0.682 0.812 0.864 0.994 1.023 0.933 1.080 0.636 8 1 0.958 0.860 0.743 0.84 1.179 1.319 1.959 1.04 1.137 1.058 0.838	Partitioning	27	27	1.089	1.610	0.855	1.282	0.932	2.873	1.069	7.927	1.116	10.236	1.325	14.255
6 1 0.922 0.860 0.711 0.682 0.812 0.864 1.023 0.023 1.080 0.636 8 1 0.958 0.711 0.682 1.717 1.17 1.10 1.257 1.17 1.058 0.858	easible Estimation	06.0	96 0	777	0.634	0 GOR	0 501	1 919	1 250	1 26.	1 205	0890	0 406	0.810	0 71 71
0 1 0.592 0.501 0.111 0.005 0.011 0.000 0.111 0.000 0.000 0.000 0.000 0.000 0.111 0.005 0.000 0.	Deal Polynomia	67.0	0.50	# 660	#70.0 0.00	0.000	100.0	41414	1.000	1.200	007.1	0.023	1 080	0.019	0.00
	D-spines	00	٦.	0.922	0.900	0.711	700.0	0.012	1 210	0.094 4.050	1.023	0.855	1.000	0.000	1.000

Table C.66: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3,\,n=500,\,\sigma^2=4,\,X_{i,\ell}\sim\beta(2,2),$ Quantile Cells

Linear         Cubic         Linear         Cubic         Linear         Linear         Cubic         Linear         Linear         Cubic         Linear         Linear         Cubic         Linear		Tuning Parameter	ing neter	Root In	Root Integrated MSE	Ingetrated MAE	rated AE	(0.5,0.5,0.5)	5,0.5)	P. (0.1,0)	Point Estim (0.1,0.5,0.5)	Point Estimation RMSE ,0.5,0.5) (0.1,0.1,0.5)	E 1,0.5)	(0.1,0.1,0.1)	1,0.1)
No.	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.383         0.426         0.384         0.091         0.159         0.197         0.185         0.489         0.290           1         1         0.180         0.486         0.090         0.177         0.185         0.489         0.290           2         1         0.180         0.446         0.239         0.248         0.090         0.177         0.185         0.489           2         1         0.180         0.447         0.438         0.389         0.197         0.176         0.489         0.293           2         1         0.401         0.447         0.185         0.389         0.179         0.176         0.489         0.499           8         0.447         0.428         0.389         0.446         0.894         0.176         0.754         0.449         0.496           8         0.447         0.466         0.826         0.446         0.426         0.484         0.761         0.741         0.751         0.773           9.33         0.25         0.447         0.466         0.324         0.446         0.449         0.394         0.496         0.449         0.751         0.773         0.449         0.774         0.							Model 3.1								
0.37         0.27         0.23         0.443         0.234         0.034         0.159         0.159         0.253         0.43           1         1         0.180         0.443         0.231         0.234         0.049         0.177         0.180         0.435         0.249         0.049         0.177         0.180         0.435         0.249         0.049         0.180         0.440         0.180         0.249         0.049         0.177         0.180         0.477         0.180         0.477         0.480         0.499         0.177         0.180         0.447         0.48         0.440         0.185         0.380         0.199         0.177         0.180         0.442         0.492         0.749         0.749         0.492         0.749         0.447         0.48         0.447         0.441	Infeasible Estimation	0	¢	0	0	0	0	0	1	1	0	0	0	0	0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	D. C	o.o	0.383	0.423	0.316	0.344	0.091	0.159	0.197 0.198	0.363	0.230	1 550	0.699	1.828
0.37         0.27         0.516         0.447         0.418         0.440         0.103         0.166         0.327         0.426         0.436         0.436         0.133         0.166         0.327         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.437         0.448         0.436         0.437         0.449         0.179         0.276         0.436         0.447         0.447         0.426         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.436         0.442         0.436         0.436         0.443         0.444         0.556         0.436         0.436         0.442         0.442         0.442         0.442         0.444         0.556         0.436         0.443         0.444         0.556         0.436         0.436         0.436         0.444         0.436         0.436         0.444         0.436         0.436         0.444         0.436         0.436         0.444         0.434         0.431         0.444         0.436         0	Partitioning		-	0.180	0.401	0.139	0.294	0.080	0.175	0.180	0.438	0.253	0.851	0.303	1.409
0.37         0.27         0.516         0.547         0.418         0.440         0.103         0.106         0.327         0.422         0.754           2         1         0.275         0.421         0.185         0.302         0.292         0.527         0.427         0.449         0.834         0.449         0.275         0.427         0.449         0.834         0.759         0.527         0.446         0.447         0.849         0.441	Feasible Estimation	•	•		1		1					1	)		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.37	0.27	0.516	0.547	0.418	0.440	0.103	0.166	0.327	0.422	0.719	0.768	1.322	1.349
2         1         0.275         0.421         0.185         0.302         0.529         0.520         0.520         0.520         0.520         0.450         0.441         0.441         0.441         0.441         0.441         0.441         0.441         0.542         0.753         0.753         0.753         0.753         0.753         0.754         0.754         0.753         0.753         0.754         0.751         0.751         0.751         0.751         0.751         0.751         0.751         0.751         0.751         0.751         0.752         0.752 </td <td>B-splines</td> <td>2</td> <td>П</td> <td>0.401</td> <td>0.497</td> <td>0.328</td> <td>0.389</td> <td>0.149</td> <td>0.179</td> <td>0.267</td> <td>0.496</td> <td>0.496</td> <td>1.579</td> <td>0.811</td> <td>1.719</td>	B-splines	2	П	0.401	0.497	0.328	0.389	0.149	0.179	0.267	0.496	0.496	1.579	0.811	1.719
0.21         0.28         0.492         0.594         0.371         0.441         0.391         0.416         0.594         0.635         1.385           8         8         0.447         0.568         0.426         0.433         0.442         0.512         0.755         0.773           9.33         0.286         1.133         0.445         0.563         0.834         3.674         0.6712         0.755         0.731           0.33         0.25         0.441         0.562         0.337         0.415         0.492         0.549         0.740         0.894         3.671         0.703           1         6         0.441         0.552         0.337         0.415         0.751         3.217         0.449         0.713         0.703           1         0.41         0.561         1.008         0.435         0.435         0.444         0.712         0.741         0.892         0.744         0.751         0.744         0.712         0.744         0.755         0.744         0.755         0.744         0.755         0.744         0.755         0.744         0.755         0.744         0.755         0.744         0.755         0.744         0.755         0.744 <t< td=""><td>Partitioning</td><td>2</td><td>1</td><td>0.275</td><td>0.421</td><td>0.185</td><td>0.302</td><td>0.292</td><td>0.520</td><td>0.361</td><td>0.754</td><td>0.421</td><td>1.007</td><td>0.497</td><td>1.568</td></t<>	Partitioning	2	1	0.275	0.421	0.185	0.302	0.292	0.520	0.361	0.754	0.421	1.007	0.497	1.568
0.21         0.28         0.492         0.584         0.371         0.441         0.391         0.446         0.552         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.755         0.751         0.755         0							Model 3.2								
0.21         0.28         0.492         0.594         0.371         0.441         0.331         0.416         0.594         0.655         1.385           8         8         0.447         0.568         0.623         0.449         0.658         0.755         0.755         0.735         0.749         0.657         0.775         0.735         0.735         0.731         0.749         0.657         0.775         0.713         0.703	nfeasible Estimation														
8         8         0.447         0.568         0.426         0.434         0.442         0.512         0.755         0.731           8         8         0.447         0.568         0.863         0.863         0.644         0.671         0.712         0.734         0.712         0.741         0.712         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.713         0.714         0.712         0.714         0.712         0.714         0.712         0.714         0.714         0.712         0.724         0.175         0.181         0.442         0.712         0.723           0.41         0.189         0.046         0.165         0.248         0.175         0.181         0.442         0.724         0.742         0.752           0.41         0.189         0.046         0.184         0.295         0.089         0.175         0.181         0.442         0.753         0.742         0.754           0.42         0.42         0.144         0.295         0.144         0.295         0.174         0.181	Local Polynomial	0.21	0.28	0.492	0.594	0.371	0.441	0.391	0.416	0.594	0.635	1.385	1.672	1.387	1.681
8 8 8 0.586 1.133 0.456 0.863 0.834 3.674 0.874 4.349 1.010 0.33 0.25 0.396 0.437 0.332 0.334 0.332 0.409 0.450 0.450 0.519 0.423 0.411 0.552 0.337 0.415 0.421 0.431 0.494 0.712 0.703 0.441 0.552 0.337 0.415 0.421 0.431 0.494 0.712 0.703 0.441 0.518 0.441 0.521 0.435 0.421 0.421 0.421 0.421 0.431 0.432 0.442 0.529 0.441 0.411 0.231 0.346 0.165 0.248 0.105 0.177 0.305 0.442 0.625 0.392 0.417 0.188 0.361 0.438 0.256 0.089 0.177 0.305 0.442 0.625 0.392 0.417 0.393 0.257 0.400 0.494 0.772 0.400 0.291 0.494 0.291 0.494 0.772 0.291 0.417 0.395 0.442 0.494 0.297 0.291 0.494 0.291 0.494 0.291 0.494 0.291 0.494 0.291 0.494 0.494 0.494 0.292 0.494 0.494 0.292 0.494 0	B-splines	œ	∞	0.447	0.568	0.342	0.426	0.433	0.442	0.512	0.755	0.731	2.521	0.789	2.495
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	œ	00	0.586	1.133	0.456	0.863	0.834	3.674	0.874	4.349	1.010	5.289	1.297	7.269
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation	0	0	0	1	0	0	000	0	1	1	007	1	1	ì
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.25	0.396	0.437	0.302	0.334	0.392	0.409	0.450	0.519	0.423	0.500	0.437	0.508
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-spines	1 ~	o w	0.441	1.008	0.337	0.415	0.421	0.431	0.494	0.712	0.703	7.280	1 304	2.205
Model 3.3  0.41	a di titolilling	-	>	0.001	7.000	70.0	0,000	00	1 1 1	0.01	5	0.0	500	101	0.4.00
0.41         0.431         0.231         0.346         0.165         0.248         0.105         0.174         0.385         0.442         0.625           1         1         0.188         0.361         0.138         0.256         0.089         0.174         0.181         0.480         0.357           0.33         0.25         0.173         0.255         0.144         0.295         0.089         0.175         0.181         0.480         0.357           0.33         0.25         0.173         0.255         0.123         0.182         0.105         0.166         0.329         0.417         0.439         0.257           0.46         0.881         0.190         0.291         0.266         0.246         0.411         0.623         0.645           0.13         0.9         0.660         0.881         0.394         0.605         2.794         0.688         3.117         0.764           0.13         0.25         0.591         0.387         0.277         0.267         0.166         0.453         0.543           0.31         0.25         0.572         0.460         0.572         0.574         0.576         0.459         0.724         0.576         0.	:						Model 3.3								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	teasible Estimation	0.41	0.41	0.931	0.346	187	876.0	101	0.177	308	0.449	7690	0 0 25	0.695	0 976
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines		1.	0.188	0.361	0.138	0.256	0.089	0.174	0.181	0.480	0.392	1.509	0.392	1.509
0.33         0.25         0.173         0.255         0.123         0.182         0.105         0.106         0.266         0.246         0.411         0.623         0.645           6         5         0.261         0.413         0.291         0.246         0.246         0.411         0.623         0.645           6         5         0.460         0.881         0.99         0.694         0.665         2.794         0.688         3.117         0.764           0.13         0.9         0.660         0.881         0.387         0.277         0.267         0.166         0.746         0.453         4.138           64         27         0.674         0.671         0.412         0.434         0.267         0.166         0.746         0.453         4.138           64         27         0.674         0.671         0.412         0.434         0.267         0.166         0.146         0.453         4.138           6         0.666         0.657         0.295         0.278         0.267         0.464         0.785         3.659         0.980           8         6         0.666         0.657         0.705         0.585         0.560         0.97	Partitioning	1		0.189	0.402	0.144	0.295	0.089	0.175	0.181	0.439	0.257	0.851	0.313	1.431
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation														
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.25	0.173	0.255	0.123	0.182	0.105	0.166	0.329	0.417	0.329	0.417	0.329	0.417
6 5 0.460 0.881 0.340 0.604 0.605 2.794 0.688 3.117 0.764  Model 3.4  0.13 0.9 0.632 0.591 0.387 0.277 0.267 0.166 0.746 0.453 4.138 64 27 0.674 0.671 0.412 0.434 0.594 0.251 0.831 0.801 1.853 64 27 0.674 0.671 0.412 0.434 0.594 0.251 0.831 0.801 1.853 65 0.606 0.627 0.295 0.297 0.278 0.262 0.464 0.459 8 6 0.602 0.627 0.295 0.370 0.278 0.262 0.464 0.459 8 6 0.602 0.795 0.795 0.585 0.560 1.095 1.071 0.892 1.043 1.250  0.33 0.33 0.755 0.705 0.585 0.560 0.971 1.193 0.780 1.124 1.443  27 27 0.834 0.795 0.644 0.620 0.971 1.193 0.780 1.124 1.443  28 6 0.613 0.646 0.572 0.517 1.186 1.011 1.236 0.944  8 6 0.830 0.778 0.617 0.617 0.617 0.617 0.781 1.268 1.196 0.703 1.151	B-splines	9	ro	0.261	0.413	0.190	0.291	0.246	0.246	0.411	0.623	0.645	2.143	0.645	2.143
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Partitioning	9	ю	0.460	0.881	0.340	0.604	0.605	2.794	0.688	3.117	0.764	3.925	0.803	5.428
0.13         0.9         0.632         0.591         0.387         0.277         0.267         0.166         0.746         0.453         4.138           64         27         0.674         0.671         0.412         0.434         0.564         0.251         0.831         0.801         1.853           64         27         0.674         0.671         0.412         0.434         0.564         0.251         0.831         0.801         1.853           6         0.674         0.677         0.627         0.224         0.267         0.106         0.167         0.459         0.544         0.459           8         6         0.666         0.627         0.295         0.370         0.278         0.662         0.449         0.727           9         0.33         0.755         0.705         0.585         0.560         0.971         1.193         0.780         1.124         1.443           27         27         1.066         1.486         0.581         0.560         0.971         1.193         0.780         1.124         1.443           8         6         0.775         0.775         0.517         0.517         0.741         1.150         0.74	1						Model 3.4								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation		0	0	1	0	1	0	1	i	1		0	1	0
0.31 0.25 0.579 0.224 0.267 0.262 0.283 0.262 0.464 0.289 0.289 0.393 0.755 0.705 0.284 0.287 0.287 0.288 0.689 0.582 0.284 0.788 0.289 0.289 0.389 0.389 0.785 0.689 0.787 0.891 0.789 0.289 0.980 0.389 0.389 0.785 0.689 0.787 0.891 0.789 0.289 0.980 0.389 0.789 0.	Local Polynomial	0.13	0.0	0.632	0.591	0.387	0.277	0.267	0.166	0.746	0.453	4.138	0.672	5.652	3.864
0.31 0.25 0.579 0.582 0.224 0.267 0.106 0.167 0.459 0.544 0.459 0.370 0.370 0.278 0.262 0.464 0.720 0.980 0.33 0.33 0.755 0.705 0.585 0.595 0.971 1.193 0.780 1.124 1.443 0.29 0.25 0.733 0.575 0.575 0.575 0.575 0.571 1.193 0.780 1.194 1.443 0.29 0.25 0.733 0.464 0.572 0.517 1.197 1.186 1.011 1.236 0.944 0.572 0.517 1.197 1.186 1.011 1.236 0.944 0.572 0.517 1.197 1.186 1.011 1.236 0.944 0.572 0.517 1.197 1.186 1.011 1.236 0.944 0.572 0.517 1.197 1.186 1.011 1.236 0.944 0.572 0.517 0.781 0.781 0.778 0.570 0.570 0.778 0.570 0.778 0.570 0.778 0.570 0.778 0.570 0.778 0.577 0.577 0.577 0.577 0.577 0.577 0.577 0.577 0.577 0.577 0.577 0.570 0.779 0.778 0.577 0.	Dartitioning	4	. 10	1.390	1 207	1.057	0.434	3 697	5.566	3 478	21.390	255.4	35 146	5 091	68 472
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	easible Estimation		1	000		-	1			5	000		2	50.0	1
8 6 0.606 0.627 0.295 0.370 0.278 0.262 0.464 0.720 0.890  8 8 6 0.6092 1.019 0.459 0.727 0.801 3.201 0.785 3.659 0.980  Model 3.5  0.33 0.33 0.755 0.705 0.585 0.560 1.095 1.071 0.892 1.043 1.250  27 27 27 0.834 0.795 0.644 0.620 0.971 1.193 0.780 1.124 1.443  27 27 27 0.834 0.795 0.644 0.620 0.971 1.193 0.780 1.124 1.443  0.29 0.25 0.733 0.646 0.572 0.517 1.197 1.186 1.011 1.236 0.944  8 6 0.810 0.778 0.617 0.612 0.847 1.150 0.740 1.073 1.151	Local Polynomial	0.31	0.25	0.579	0.582	0.224	0.267	0.106	0.167	0.459	0.544	0.459	0.544	3.828	3.835
8 6 0.692 1.019 0.459 0.727 0.801 3.201 0.785 3.659 0.980  Model 3.5  0.33 0.35 0.755 0.705 0.585 0.560 1.095 1.071 0.892 1.043 1.250  27 27 27 0.834 0.795 0.644 0.620 0.971 1.193 0.780 1.124 1.443  27 27 27 1.066 1.486 0.830 1.180 0.781 5.568 1.196 20.552 1.588  0.29 0.25 0.733 0.646 0.572 0.517 1.197 1.186 1.011 1.236 0.944  8 6 0.810 0.778 0.617 0.612 0.847 1.150 0.740 1.073 1.151	B-splines	œ	9	0.606	0.627	0.295	0.370	0.278	0.262	0.464	0.720	0.890	3.030	3.910	4.752
Model 3.5  0.33  0.35  0.36  0.37  0.40  0.585  0.560  0.644  0.620  0.971  0.892  1.043  1.250  1.250  2.7  2.7  2.7  1.066  1.486  0.830  1.180  0.781  1.193  0.781  1.195  0.781  1.195  0.782  1.196  1.196  1.196  0.972  1.197  1.197  1.197  1.197  1.197  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191  1.191	Partitioning	œ	9	0.692	1.019	0.459	0.727	0.801	3.201	0.785	3.659	0.980	4.489	3.325	6.612
0.33         0.33         0.755         0.705         0.585         0.560         1.095         1.071         0.892         1.043         1.250           27         27         0.834         0.795         0.644         0.620         0.971         1.193         0.780         1.124         1.443           27         27         1.066         1.486         0.830         1.180         0.781         5.568         1.196         20.552         1.588           0.29         0.25         0.733         0.646         0.572         0.517         1.197         1.186         1.011         1.236         0.944           8         6         0.810         0.778         0.617         0.612         0.847         1.150         0.740         1.073         1.151							Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	nfeasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.755	0.705	0.585	0.560	1.095	1.071	0.892	1.043	1.250	1.368	0.944	1.438
27     27     1.066     1.486     0.830     1.180     0.781     5.568     1.196     20.552     1.588       0.29     0.25     0.733     0.646     0.572     0.517     1.197     1.186     1.011     1.236     0.944       8     6     0.810     0.778     0.617     0.612     0.847     1.150     0.740     1.073     1.151	B-splines	27	27	0.834	0.795	0.644	0.620	0.971	1.193	0.780	1.124	1.443	5.097	1.123	4.857
0.29 0.25 0.733 0.646 0.572 0.517 1.197 1.186 1.011 1.236 0.944 8 6 0.810 0.778 0.617 0.612 0.847 1.150 0.740 1.073 1.151	Partitioning	27	27	1.066	1.486	0.830	1.180	0.781	5.568	1.196	20.552	1.588	34.372	1.812	70.198
8 6 0.810 0.778 0.617 0.612 0.847 1.150 0.740 1.073 1.151	easible Estimation	000	C C	0 420	0.646	0	1 1 1	101	1 100	1 011	1 096	6	200	2 7 7	0.40
161.1 (2).1 (1).0 (61.1 ).50.0 71.0 (1).0 (	Local Folynomial	67.0	0.23	0.733	0.040	0.012	0.517	1.197	1.100	1.011	1.230	0.344	0.107	0.400	0.040
000 000	B-spines	<b>x</b> 0 0	9	U.81U	0.7.0	0.617	0.612	U.841	0000	0.740	1.073	1.151	2.037	1.97	2.461

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.67: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 1000, \sigma^2 = 1, X_{i,\ell} \sim \beta(0.5, 0.5),$  Quantile Cells

	Tuning Parameter	ning neter	Root In M	Root Integrated MSE	$rac{Inget}{M_{ u}}$	$rac{Ingetrated}{MAE}$	(0.5,0.5,0.5)	5,0.5)	Point Es (0.1,0.5,0.5)	Point Estimation RMSE $(0.1,0.5,0.5)$	tion RMSE $(0.1,0.1,0.5)$	E 1,0.5)	(0.1,0.1,0.1)	(1.0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation	d	d		0.7	0 4 0	0	0	1	000	0	000	9	9	0
Local Polynomial	o	0.0 F	0.458	0.462	0.403	0.404	0.034	0.073	0.060	0.092	0.091	0.116	0.648	0.050
Partitioning			0.062	0.141	0.049	0.109	0.034	0.077	0.054	0.119	0.071	0.168	0.030	0.225
Feasible Estimation														
Local Polynomial	0.46	0.31	0.645	0.648	0.526	0.528	0.043	0.081	0.080	0.106	0.648	0.653	1.285	1.288
B-splines	1	П	0.459	0.469	0.404	0.407	0.034	0.093	0.060	0.124	0.103	0.177	0.650	0.667
Partitioning	1	1	0.062	0.141	0.049	0.109	0.030	0.077	0.054	0.119	0.071	0.168	0.085	0.225
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.382	0.396	0.297	0.307	0.468	0.497	0.392	0.432	0.370	0.402	0.287	0.319
B-splines	27	<b>x</b> 0	0.380	0.386	0.296	0.297	0.394	0.494	0.327	0.405	0.383	0.377	0.263	0.268
Partitioning	27	00	0.378	0.421	0.296	0.322	0.309	1.913	0.305	1.287	0.285	0.774	0.332	0.420
reastote Estimation	080	86 0	0 360	0.877	196.0	0.087	0 20 71	272	786	0 903	806 0	0 303	786	0.077
Benlines	5.5	5.	0.309	0.380	0.281	0.287	0.327	0.00	0.234	0.233	0.238	0.381	0.200	0.961
Partitioning			0.370	0.375	0.284	0.291	0.387	0.394	0.300	0.534	0.332	0.280	0.263	0.249
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.22	0.3	0.156	0.202	0.112	0.151	0.095	0.127	0.131	0.170	0.166	0.216	0.166	0.216
B-splines	<b>∞</b>	<b>∞</b>	0.138	0.186	0.096	0.138	0.139	0.151	0.136	0.158	0.135	0.166	0.135	0.166
Partitioning Feasible Estimation	<b>x</b> 0	x	0.187	0.400	0.146	0.305	0.342	1.529	0.252	1.064	0.241	0.719	0.198	0.411
Local Polynomial	0.39	0.28	0.130	0.146	0.074	0.094	0.046	0.072	0.072	0.091	0.072	0.091	0.072	0.091
B-splines	1	П	0.125	0.162	0.080	0.116	0.049	0.085	0.057	0.114	0.096	0.161	0.096	0.161
Partitioning	1	1	0.126	0.153	0.081	0.117	0.061	0.083	0.059	0.122	0.099	0.171	0.142	0.255
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.02	6.0	1.844	2.068	0.813	0.823	0.361	0.133	0.275	0.237	0.945	0.472	3.936	3.837
B-splines	343	64	1.972	1.948	0.852	0.849	0.159	0.202	0.259	0.297	0.972	0.782	3.939	3.888
Partitioning	343	64	1.795	1.982	0.743	0.771	0.599	49.860	0.497	7.261	0.687	4.008	3.591	3.773
reasible Estimation	000	96 0	200.0	2 076	6000	2100	070	0.077	0 017	966.0	0 917	9660	0 017	0000
Local Fotymoninal	5 ×	0.20	2.084	2.070	0.803	0.010	0.043	0.07	2880	0.220	0.605	0.420	3 864	3 951
Partitioning	000	1 (3)	1.618	1.740	1.022	0.984	2.244	1.327	0.415	0.474	1.442	0.550	1.318	2.094
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.818	0.582	0.639	0.465	1.141	1.285	1.306	1.119	0.578	0.747	0.619	0.468
B-splines	27	27	0.845	0.589	0.677	0.471	0.871	1.391	1.187	1.264	0.716	0.658	0.505	0.544
Partitioning	27	27	0.665	0.743	0.520	0.579	0.933	0.677	1.002	908.0	0.328	0.779	0.423	1.042
Feasible Estimation Local Polynomial	20.0	86.0	0 795	0 567	0.570	777	1 919	1 913	385	1 7 8	0.438	0.667	0.719	787
B-enlines	) ()	-	9200	0000	0.0.0	0.460	1 C	711	1.000	1 166	717	0.00	0.770	0.101
D-spines Dartitioning	0 00	-	00000	0.820	0.133	0.001	0000	0.862	1.136	1 391	0.738	0.124	2.4.0 0.00 0.00	0.501
0			1						1	1			1	

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.68: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 1000, \sigma^2 = 1, X_{i,\ell} \sim \beta(1,1),$  Quantile Cells

	Tuning Parameter	ing neter	Root In	Root Integrated MSE	Inget MA	Ingetrated MAE	(0.5, 0.5, 0.5)	5,0.5)	Point Es $(0.1, 0.5, 0.5)$	int Estima 5,0.5)	Point Estimation RMSE (0.1,0.5,0.5)	E 1,0.5)	(0.1, 0.1, 0.1)	1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
:						Model 3.1								
Infeasible Estimation	d	d	0	070	0	200	0	9	1000	0	0	5	0.00	9
Local Folynomial B-splines	e.e.	0.9 1	0.420	0.432	0.362	0.368	0.035	0.076	0.064	0.104	0.095	0.135	0.651	0.677
Partitioning	1	1	0.065	0.142	0.051	0.107	0.033	0.069	0.059	0.127	0.077	0.197	0.090	0.288
Feasible Estimation														
Local Polynomial	0.45	0.31	0.590	0.592	0.481	0.484	0.042	0.073	0.094	0.119	0.648	0.653	1.285	1.287
B-splines		٦.	0.420	0.432	0.362	0.368	0.035	0.076	0.064	0.136	0.117	0.245	0.651	0.677
raruma	4	4	0.003	0.142	0.031	0.101	0.033	0.003	0.03	0.121	0.00	0.137	0.030	0.700
						Model 3.2								
Infeasible Estimation	,	0	0	9	0	0	0		0	0	0		0	0
Local Polynomial	0.16	0.23	0.390	0.403	0.303	0.312	0.424	0.442	0.368	0.391	0.386	0.412	0.366	0.386
D-spilles Dortitioning	2 6	0 o	0.380	0.394	0.300	0.505	0.580	1 700	0.550	1 304	0.074	0.383	0.523	0.007
Feasible Estimation	1	0				1		7.7	000				005.0	5
Local Polynomial	0.37	0.27	0.372	0.377	0.283	0.288	0.370	0.373	0.301	0.309	0.289	0.294	0.283	0.291
B-splines	2	1	0.373	0.385	0.285	0.298	0.379	0.393	0.303	0.333	0.314	0.381	0.287	0.342
Partitioning	2	1	0.373	0.383	0.285	0.296	0.397	0.402	0.333	0.469	0.316	0.302	0.325	0.346
						Model 3.3								
Infeasible Estimation														
Local Polynomial	0.25	0.32	0.139	0.183	0.097	0.133	0.069	0.098	0.130	0.155	0.208	0.236	0.208	0.236
b-spines	<b>x</b> 0 0	٦.	0.126	0.151	0.088	0.107	0.11.0	0.071	0.139	0.126	0.165	0.229	0.100	0.229
Fartitioning Feasible Estimation	0	-	0.100	0.1.00	0.143	0.113	0.530	0.012	0.271	0.191	0.243	0.133	0.22.0	0.000
Local Polynomial	0.38	0.28	0.112	0.130	0.065	0.085	0.043	0.067	0.085	0.103	0.085	0.103	0.085	0.103
B-splines	1	1	0.111	0.151	0.072	0.107	0.045	0.071	0.068	0.126	0.114	0.229	0.114	0.229
Partitioning	1	1	0.115	0.150	0.075	0.113	0.073	0.072	0.076	0.131	0.106	0.199	0.142	0.335
						Model 3.4								
Infeasible Estimation	0	Ó		(	0		0	0	(	1			0	0
Local Polynomial	91.08	9.0	1.284	1.342	0.491	0.472	0.235	0.096	0.265	0.254	1.381	0.447	4.003	3.809
Partitioning	216	2 5 5 7 5 7	1.040	0.749	0.582	0.584	2.265	0.582	2.102	0.920	1.354	1.951	3.390	2.491
Feasible Estimation														
Local Polynomial	0.29	0.26	1.355	1.346	0.454	0.468	0.046	0.069	0.316	0.237	0.316	0.237	3.802	3.790
B-splines	oo i	က	1.349	1.313	0.493	0.501	0.146	0.104	0.254	0.310	0.587	1.198	3.820	3.942
Partitioning	00	n	1.127	1.083	0.640	0.557	1.295	1.460	0.420	0.634	1.052	0.741	2.136	2.513
						Model 3.5								
Infeasible Estimation	000	000	1	1000	2120	0 0		000	- 7	17	0000	000	1	0 100
B-enlines	27.0	27.0	0.103	0.001	0.010	0.483	0.919	1.339	0.995	1.047	0.003	0.080	0.570	0.00
Partitioning	27	27.	0.700	0.747	0.544	0.584	0.773	0.628	0.720	0.937	0.446	1.831	0.479	2.136
Feasible Estimation														
Local Polynomial	0.25	0.28	0.720	0.589	0.566	0.474	1.225	1.225	1.286	1.166	0.540	0.662	0.616	0.499
B-splines	œ	П	0.890	0.802	0.688	0.641	0.683	0.808	0.967	0.968	0.793	0.854	0.430	0.475
Partitioning	∞	1	0.847	0.780	0.650	0.619	0.914	0.923	1.066	1.068	098.0	0.729	0.704	0.665

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.69: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3, n=1000, \sigma^2=1, X_{i,\ell}\sim \beta(2,2),$  Quantile Cells

	Tuning Paramete	Tuning Parameter	Root In M	Root Integrated MSE	Ingetrated MAE	getrated MAE	(0.5,0.5,0.5)	5,0.5)	Point E: (0.1,0.5,0.5)	oint Estim 5,0.5)	Point Estimation RMSE ,0.5,0.5) (0.1,0.1,0.5)	3E 1,0.5)	(0.1,0.1,0.1)	1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation	c	c c	0	0	0	0	0	ì	i	0	0	0	9	0
Local Polynomial B-splines	0.0 -	0.0 -	0.353	0.358	0.296	0.299	0.033	0.055	0.071	0.134	0.101	0.188	0.659	0.009
Partitioning			0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
Feasible Estimation														
Local Polynomial	0.43	0.3	0.495	0.499	0.403	0.406	0.037	0.060	0.113	0.155	0.655	0.669	1.290	1.301
B-splines	П	1	0.354	0.368	0.296	0.305	0.033	0.064	0.068	0.176	0.143	0.490	0.659	0.814
Partitioning	1	1	0.062	0.140	0.048	0.102	0.031	0.061	0.065	0.151	0.085	0.280	0.105	0.468
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.16	0.23	0.384	0.393	0.292	0.299	0.352	0.351	0.363	0.350	0.722	0.553	0.743	0.590
B-splines	27	œ	0.379	0.389	0.290	0.296	0.357	0.350	0.317	0.364	0.424	0.714	0.500	0.745
Partitioning	27	œ	0.372	0.413	0.290	0.311	0.210	1.118	0.361	1.408	0.559	1.598	0.671	2.076
Feasible Estimation	0	0	9900	6	0	0	0	0 0	0	000	000	6	000	
Local Folynomia	0.30	0.27	0.300	0.971	0.274	0.279	0.570	0.570	0.520	0.020	0.290	0.510	0.302	0.500
b-spines Partitioning	4 4	4.4	0.360	0.384	0.277	0.292	0.360	0.353	0.300	1.060	0.320	1 203	0.374	1 715
0	1	1			! !	Model 3.3								
Infeasible Estimation														
Local Polynomial	0.31	0.34	0.110	0.150	0.074	0.105	0.046	0.074	0.143	0.175	0.319	0.320	0.319	0.350
B-splines	œ	1	0.108	0.136	0.076	0.094	0.104	0.061	0.160	0.173	0.239	0.470	0.239	0.470
Partitioning	∞	П	0.181	0.145	0.140	0.105	0.258	0.062	0.277	0.154	0.278	0.281	0.305	0.525
Feasible Estimation	0	1	0	0	0	1	0	1	1		1		1	
Local Polynomial	0.37	0.27	0.083	0.106	0.051	0.070	0.037	0.057	0.115	0.142	0.115	0.142	0.115	0.142
B-splines	21 (		0.091	0.136	0.061	0.094	0.054	0.061	0.091	0.173	0.165	0.470	0.165	0.470
Fartitioning	N	Т	0.112	0.145	0.074	0.105	0.124	0.062	0.132	0.154	0.149	0.281	0.180	0.525
:						Model 3.4								
Infeasible Estimation	(	Ġ	0	ì	0	0	i	9	0	0	0	0	9	0
Local Polynomial	U.T	9.0	0.485	0.554	0.222	0.183	0.137	0.065	0.331	0.248	2.492	0.368	4.498	3.820
D-spinies	120	2 0	0.000	0.40	0.243 7.70	0.230	0.107	0.030	0.200	0.040	1.442	2.203	210.4	45.094
Faithmilli	071	4	0.143	0.1.0	0.040	t. 0.0	00	0.010	1.002	7.730	# T	4.040	0.000	10.200
Local Polynomial	0.31	0.26	0.560	0.551	0.168	0.182	0.038	0.058	0.265	0.997	0.265	0.297	3 799	3.798
B-splines	oc	er.	0.560	0.525	0.195	0.212	0.114	0.084	0.212	0.299	0.498	1.634	3.833	4.145
Partitioning	00	8	0.525	0.483	0.263	0.256	0.470	0.739	0.328	0.792	0.564	1.105	3.153	3.158
						Model 3.5								
Infeasible Estimation														
Local Polynomial	0.33	0.33	0.719	0.602	0.559	0.487	1.087	1.049	0.812	0.924	0.896	0.657	0.498	0.752
B-splines	27.	72.0	0.773	0.629	0.596	0.507	0.954	1.172	0.669	0.914	0.990	1.450	0.517	10.000
Partitioning	.57	27	0.652	0.748	0.489	0.584	0.561	0.625	0.456	2.292	0.666	4.910	0.674	10.386
Feasible Estimation Local Polynomial	96.0	0.97	969 0	0.619	24.8	0.492	1 907	1 199	1 014	1 157	0.893	685	0.361	0.498
Benlings	) 1	1	0.000	0.012	0.540	0.430	1010	1 160	1.0.1	0.937	0.020	1.036	0.001	0.000
D-spilles Partitioning	0 00	- 1-	0.715	0.512	0.532	0.392	0.930	1.270	0.759	1.384	1.119	1.502	0.991	1.949
											:			

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimated by averaging over the design points in each simulated data set.

Table C.70: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 1000, \sigma^2 = 4, X_{i,\ell} \sim \beta(0.5, 0.5),$  Quantile Cells

Degree:	* ***	Parameter	MSE	ਸ਼੍ਰ	MŁ	MAE	(0.5, 0.5, 0.5)	5,0.5)	$(0.1, 0.5, 0.5) \qquad (0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, $	5,0.5)	(0.1, 0.1, 0.5)	(2.0,1	(0.1,0.1,0.1)	(1.0.1)
	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
						Model 3.1								
Infeasible Estimation	c c	(	0	1	0		0	0	1	1	i	(	0	0
Local Polynomial R-splines	6.0	6.0	0.469	0.487	0.409	0.418	0.063	0.136	0.117	0.172	0.171	0.218	0.665	0.685
Partitioning			0.125	0.283	0.098	0.217	090:0	0.154	0.108	0.239	0.143	0.337	0.170	0.451
Feasible Estimation														
Local Polynomial	0.39	0.28	0.652	0.665	0.532	0.542	0.084	0.153	0.148	0.192	0.661	0.672	1.293	1.299
B-splines	1	1	0.472	0.518	0.411	0.436	0.071	0.174	0.116	0.232	0.195	0.330	0.669	0.725
Partitioning	1	1	0.132	0.283	0.101	0.217	0.110	0.154	0.124	0.239	0.155	0.337	0.173	0.451
						Model 3.2								
Infeasible Estimation														
Local Polynomial	0.19	0.26	0.443	0.493	0.348	0.386	0.491	0.548	0.447	0.512	0.477	0.509	0.409	0.444
B-splines	27	oo ·	0.439	0.474	0.349	0.370	0.414	0.559	0.371	0.485	0.479	0.476	0.386	0.390
Partitioning	27	∞	0.684	0.810	0.535	0.619	0.428	3.265	0.503	2.244	0.552	1.468	0.657	0.825
reasible Estimation Local Polynomial	0 7 7	96.0	0.381	0.403	0.994	0.314	0.369	0.378	0.300	0 334	0 395	0 349	0.999	0.318
B-splines	4	-	0.395	0.438	0.306	0.345	0.442	0.437	0.373	0.387	0.416	0.474	0.311	0.380
Partitioning	4		0.429	0.449	0.332	0.353	0.577	0.420	0.558	0.614	0.471	0.408	0.377	0.463
						Model 3.3								
Infeasible Estimation	0	· ·	0	0	1	0	1		0	0	0	i i	0	i i
Local Folynomial	0.27	0.34	0.235	0.325	0.175	0.248	0.152	0.219	0.229	0.283	0.308	0.355	0.308	0.355
D-spinies Dartitioning	o о	٠.	0.213	0.27	0.162	0.203	7190	0.170	0.27.0	0.220	0.263	0.323	0.263	0.025
Feasible Estimation		•				1								
Local Polynomial	0.35	0.26	0.163	0.210	0.110	0.155	0.088	0.152	0.146	0.186	0.146	0.186	0.146	0.186
B-splines	9	1	0.184	0.272	0.133	0.205	0.177	0.170	0.189	0.228	0.223	0.323	0.223	0.323
Partitioning	3	П	0.251	0.289	0.174	0.222	0.377	0.157	0.306	0.240	0.309	0.338	0.288	0.465
						Model 3.4								
Infeasible Estimation	0	o o	1	i	0	0	i i	i i	1	i	1	i i	1	0
Local Polynomial Bealines	0.08	0.9	1.957	2.074	0.956	0.852	0.552	0.175	0.457	0.278	1.047	0.507	3.965	3.837
Partitioning	216	22	1.780	1.479	1.105	1.152	4.550	1.170	5.118	1.460	2.295	1.531	3.443	2.271
Feasible Estimation														
Local Polynomial	0.3	0.25	2.087	2.081	0.819	0.844	0.095	0.152	0.340	0.281	0.340	0.281	3.816	3.805
B-splines	œ	2	2.083	2.059	0.873	0.892	0.301	0.201	0.376	0.375	0.652	1.006	3.873	3.958
Partitioning	œ	5	1.647	1.767	1.053	1.023	2.302	1.373	0.597	0.731	1.502	0.722	1.353	2.147
:						Model 3.5								
Infeasible Estimation	0	0	0	1	0	1		0		1	0	0	0	ì
Local Polynomial	0.33	0.33	0.835	0.645	0.652	0.515	1.147	1.302	1.314	1.145	0.636	0.812	0.668	0.565
D-spinies Partitioning	27.	27.	0.876	1.473	0.689	1.147	0.976	1.219	1.079	1.497	0.573	1.534	0.711	2.081
Feasible Estimation	i	i		)				1			)	i ) )		1
Local Polynomial	0.26	0.26	0.746	0.581	0.585	0.467	1.211	1.204	1.374	1.169	0.471	0.684	0.708	0.514
B-splines	œ	1	0.951	0.849	0.746	0.681	0.638	0.732	1.179	1.179	0.753	0.777	0.527	0.571
Partitioning	8	1	0.950	0.834	0.747	0.668	1.024	0.877	1.382	1.350	0.855	0.609	0.677	0.658

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.

Table C.71: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d = 3, n = 1000, \sigma^2 = 4, X_{i,\ell} \sim \beta(1,1),$  Quantile Cells

	Tuning Parameter	ing neter	Root Integrated MSE	egrated E	$rac{Ingetrated}{MAE}$	getrated MAE	(0.5, 0.5, 0.5)	5,0.5)	Point E: $(0.1, 0.5, 0.5)$	Foint Estimation KMSE ,0.5,0.5) (0.1,0.1,0	tion KMSE $(0.1,0.1,0.5)$	E 1,0.5)	(0.1, 0.1, 0.1)	1,0.1)
Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1						Model 3.1								
Tocal Polymomial	0	0	0.439	177	0 369	0 380	0.067	0.199	0.196	105	0.180	0.954	0.663	0.89
B-splines	S:-	5.0	0.435	0.484	0.371	0.401	0.067	0.144	0.120	0.257	0.221	0.465	0.680	0.781
Partitioning	П	-	0.129	0.284	0.101	0.215	0.066	0.137	0.118	0.254	0.155	0.394	0.180	0.575
Feasible Estimation														
Local Polynomial	0.38	0.28	0.598	0.612	0.488	0.498	0.083	0.139	0.178	0.218	0.669	0.682	1.297	1.304
B-splines			0.436	0.484	0.372	0.401	0.077	0.144	0.131	0.257	0.226	0.465	0.681	0.781
Fartitioning	7	7	0.144	0.284	0.107	0.215	0.131	0.137	0.144	0.254	0.182	0.394	0.193	0.575
						Model 3.2								
Infeasible Estimation	0	0	0	0	1	0	0			į	1	1	1	1
Local Polynomial	0.19	0.26	0.450	0.496	0.351	0.386	0.438	0.480	0.440	0.474	0.553	0.558	0.541	0.544
D-spines	7 6	00	0.444	0.480	0.352	0.573	0.402	0.484	0.000	404.0	0.022	1.756	0.495	1 200
rattioning Feasible Estimation	1	0	0.001	0.010	0.002	0.017	0.414	4.304	0.040	7.00	0.0.0	T. / 00	0.130	1.300
Local Polynomial	0.34	0.26	0.385	0.407	0.297	0.316	0.377	0.393	0.335	0.354	0.329	0.348	0.318	0.338
B-splines	ro	1	0.403	0.444	0.311	0.348	0.429	0.411	0.389	0.398	0.425	0.548	0.383	0.525
Partitioning	5	1	0.442	0.461	0.343	0.360	0.593	0.525	0.593	0.594	0.514	0.499	0.534	0.624
						Model 3.3								
Infeasible Estimation	c c	0	0	0	1	0		i	0	100	0	1	0	2
Local Polynomial	0.3	0.36	0.218	0.301	0.158	0.224	0.113	0.171	0.233	0.267	0.401	0.405	0.401	0.405
b-spiines			0.157	0.200	0.114	0.197	0.071	0.141	0.119	0.252	0.216	0.409	0.210	0.459
rarutuoning Feasible Estimation	<b>-</b>	-	0.137	0.700	0.110	0.210	0.071	0.109	0.119	0.20	0.100	0.030	0.203	0.004
Local Polynomial	0.34	0.26	0.151	0.201	0.105	0.147	0.085	0.138	0.174	0.208	0.174	0.208	0.174	0.208
B-splines	4	1	0.187	0.266	0.136	0.197	0.173	0.141	0.223	0.253	0.283	0.459	0.283	0.459
Partitioning	4	1	0.280	0.293	0.198	0.219	0.398	0.270	0.391	0.322	0.367	0.417	0.353	0.612
						Model 3.4								
Infeasible Estimation														
Local Polynomial	0.09	0.0	1.255	1.351	0.615	0.508	0.379	0.139	0.451	0.302	1.481	0.496	4.046	3.817
b-spines	125	7 7 7	1.348	1.305	0.018	0.001	0.213	0.218	0.401	0.431	1.494	1.429	4.056	4.020
Fartitioning	125	7.5	1.438	1.477	1.091	1.150	1.635	1.163	1.799	1.833	2.576	3.885	3.095	4.473
Local Polynomial	0.3	0.26	1.359	1.354	0.475	0.501	060.0	0.139	0.348	0.300	0.348	0.300	3.801	3.790
B-splines	000	n n	1.359	1.334	0.527	0.555	0.250	0.181	0.353	0.390	0.656	1.257	3.831	3.967
Partitioning	œ	8	1.169	1.171	0.692	0.648	1.382	1.786	0.633	1.030	1.139	1.071	2.182	2.607
						Model 3.5								
Infeasible Estimation	0	0	0	1	0	0	7	0	, 1	1	0	1	1 0	0
Local Polynomial	0.33	0.33	0.806	0.659	0.629	0.530	1.126	1.208	1.159	1.073	0.733	0.777	0.657	0.686
B-spines	7 7 7	7 5	0.876	0.098	0.690	0.558	0.923	1.347	1.011	1.231	0.907	0.748	0.576	0.787
Fartitioning	7	77	0.903	1.470	0.709	061.1	0.831	1.189	0.850	1.833	0.732	3.702	0.837	4.304
Local Polynomial	0.26	0.26	0.740	0.602	0.581	0.484	1.223	1.214	1.274	1.180	0.582	0.684	0.614	0.530
B-splines	ox	-	0.906	8080	0.700	0.659	0 707	0.846	0 005	0 997	0.842	0.997	0.513	0.690
D-spinies Partitioning	o oc	٠.	0.900	0.828	0.696	0.650	1.021	1.082	1.152	1.169	0.963	288	0.809	0.020
0				111111111111111111111111111111111111111					1	1	1	1	1	1

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimated by averaging over the design points in each simulated data set.

Table C.72: Error Comparisons for Local Polynomials, B-Splines, and Partitioning Estimators  $d=3, n=1000, \sigma^2=4, X_{i,\ell}\sim \beta(2,2),$  Quantile Cells

Degrees         Linear Cubic         Cubic         Linear Cubic         Cubic         Linear Cubic         Cubic         Linear Cubic         Cubic         1.12         0.36         0.38         0.38         0.34         0.06         0.13         0.32         0.37         0.34         0.05         0.13         0.32         0.37         0.34         0.06         0.13         0.33         0.37         0.34         0.06         0.13         0.33         0.37         0.34         0.06         0.13         0.32         0.37         0.36         0.37         0.34         0.06         0.32         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06         0.34         0.06		Tuning Parameter	ing neter	Root Integrated MSE	segrated	Ingetrated MAE	getrated MAE	(0.5,0.5,0.5)	5,0.5)	Point E (0.1,0.5,0.5)	int Estime 5,0.5)	Point Estimation RMSE ,0.5,0.5) (0.1,0.1,0.5)	E ,,0.5)	(0.1, 0.1, 0.1)	.,0.1)
Model 3.1   Model 3.1   Model 3.1   Model 3.1   Model 3.1   Model 3.1   Model 3.2   Mode	Degree:	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic	Linear	Cubic
0.9         0.9         0.9         0.346         0.387         0.349         0.083         0.106         0.132         0.132         0.134         0.249         0.775         0.444         0.096         0.244         0.093         0.123         0.132         0.344         0.023         0.123         0.344         0.023         0.132         0.349         0.274         0.344         0.023         0.132         0.344         0.132         0.132         0.344         0.132         0.134         0.344         0.012         0.134         0.344         0.17         0.344         0.344         0.040         0.122         0.117         0.344         0.344         0.040         0.122         0.147         0.344         0.344         0.044         0.142         0.144         0.142         0.144         0.144         0.444         0.445         0.344         0.348         0.380         0.348<	;						Model 3.1								
0.9         0.9 <td>Infeasible Estimation</td> <td>1</td> <td></td> <td>1</td> <td>1</td> <td>i</td>	Infeasible Estimation	1	1	1	1	1	1	1	1	1	1		1	1	i
1	Local Polynomial	0.0 -	0.0 1	0.366	0.387	0.305	0.319	0.063	0.106	0.137	0.262	0.194	0.360	0.670	0.736
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	D-spinnes Dartitioning	٦.		0.370	0.420	0.50	0.944	0.063	0.123	0.132	0.344	0.270	0.940	0.703	0.100
0.37         0.27         0.505         0.5251         0.442         0.073         0.117         0.226         0.229         0.721         0.372         0.727         0.728         0.727         0.727         0.728         0.727         0.728         0.727         0.728         0.727         0.728         0.728         0.728         0.728         0.728         0.728         0.728	Feasible Estimation	+	+	F 7 1.0	0.50	0000	500	0000	77.0	101.0	0000	0.1.0	0.00	7.0	0.000
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.37	0.27	0.505	0.521	0.410	0.422	0.073	0.117	0.232	0.290	0.692	0.721	1.313	1.332
2         1         0.180         0.281         0.122         0.204         0.205         0.302         0.262         0.561         0.299           0.19         0.26         0.447         0.478         0.344         0.363         0.380         0.440         0.549         0.729         0.561         0.299           27         8         0.4437         0.475         0.384         0.386         0.340         0.549         0.729         1.380         0.773           27         8         0.680         0.807         0.386         0.386         0.390         0.404         0.549         0.729         1.380         0.773           27         8         0.680         0.807         0.386         0.390         0.406         0.723         1.188         0.390         0.516         0.492         0.493         0.773         1.188         0.493         0.516         0.493         0.718         0.493         0.718         0.494         0.418         0.789         0.493         0.718         0.789         0.494         0.718         0.790         0.494         0.718         0.790         0.496         0.718         0.790         0.790         0.496         0.719         0.790 <td< td=""><td>B-splines</td><td>2</td><td>П</td><td>0.374</td><td>0.428</td><td>0.310</td><td>0.344</td><td>0.102</td><td>0.123</td><td>0.177</td><td>0.344</td><td>0.317</td><td>0.946</td><td>0.722</td><td>1.153</td></td<>	B-splines	2	П	0.374	0.428	0.310	0.344	0.102	0.123	0.177	0.344	0.317	0.946	0.722	1.153
Model 3.2   Model 3.3   Mode	Partitioning	2	1	0.180	0.281	0.122	0.204	0.206	0.121	0.240	0.302	0.262	0.561	0.299	0.935
0.19         0.26         0.441         0.479         0.334         0.363         0.368         0.369         0.486         0.446         0.549         0.572         0.580         0.585           27         8         0.6437         0.475         0.339         0.369         0.367         0.391         0.446         0.549         0.579         0.579         0.773           27         8         0.680         0.687         0.360         0.367         0.391         0.446         0.549         0.702         0.329         0.705         0.702         0.702         0.390         0.706         0.703         0.706         0.703         0.706         0.707         0.396         0.705         0.707         0.705         0.705         0.705         0.705         0.705         0.705         0.705         0.705         0.706         0.707         0.708         0.708         0.712         0.707         0.709         0.707         0.709 <td< td=""><td></td><td></td><td></td><td></td><td></td><td></td><td>Model 3.2</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>							Model 3.2								
0.19         0.26         0.441         0.479         0.334         0.353         0.368         0.386         0.386         0.386         0.386         0.386         0.386         0.386         0.386         0.386         0.0729         1.220         0.773         1.286         0.0789         0.773         2.856         0.992         2.923         1.158         0.773         0.779         0.778         0.779 <th< td=""><td>Infeasible Estimation</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></th<>	Infeasible Estimation														
27         8         0.437         0.437         0.339         0.359         0.367         0.404         0.407         0.475         0.475         0.475         0.475         0.475         0.406         0.367         0.407         0.401         0.475         0.475         0.475         0.406         0.475         0.406         0.387         0.366         0.437         0.406         0.287         0.366         0.378         0.379         0.406         0.287         0.366         0.378         0.379         0.406         0.403         0.705         0.386         0.386         0.378         0.406         0.717         0.569         0.1219         0.406         0.569         0.1178         0.664         0.126         0.1178         0.664         0.126         0.1178         0.664         0.127         0.107         0.1179         0.064         0.127         0.127         0.108         0.108         0.108         0.1174         0.127         0.1179         0.064         0.127         0.137         0.274         0.108         0.109         0.1174         0.127         0.117         0.064         0.122         0.131         0.138         0.139         0.137         0.131         0.138         0.132         0.064 <t< td=""><td>Local Polynomial</td><td>0.19</td><td>0.26</td><td>0.441</td><td>0.479</td><td>0.334</td><td>0.363</td><td>0.368</td><td>0.380</td><td>0.486</td><td>0.476</td><td>0.972</td><td>0.850</td><td>0.985</td><td>0.870</td></t<>	Local Polynomial	0.19	0.26	0.441	0.479	0.334	0.363	0.368	0.380	0.486	0.476	0.972	0.850	0.985	0.870
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	27	<b>x</b> 0 0	0.437	0.475	0.339	0.359	0.367	0.391	0.404	0.549	0.729	1.320	0.773	1.338
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Fartitioning	7	ю	0.680	0.807	0.529	0.000	0.367	2.030	0.713	2.585	0.992	2.923	1.158	3.043
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.25	0.379	0.400	0.287	0.305	0.380	0.388	0.379	0.405	0.365	0.405	0.364	0.392
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	2	9	0.401	0.463	0.305	0.350	0.385	0.379	0.390	0.515	0.509	1.219	0.568	1.243
0.38         0.39         0.178         0.257         0.124         0.183         0.078         0.133         0.243         0.346         0.504         0.629         0.504           1         1         0.137         0.257         0.124         0.183         0.078         0.131         0.345         0.573         0.941         0.273           1         1         0.137         0.257         0.109         0.179         0.064         0.122         0.131         0.345         0.273         0.991           0.33         0.25         0.131         0.183         0.091         0.133         0.109         0.183         0.144         0.181         0.074         0.131         0.346         0.274         0.139         0.274         0.139         0.149         0.649         0.149         0.649         0.149         0.649         0.149         0.649 <td>Partitioning</td> <td>- 1-</td> <td>9</td> <td>0.453</td> <td>0.705</td> <td>0.350</td> <td>0.516</td> <td>0.614</td> <td>1.664</td> <td>0.649</td> <td>2.150</td> <td>0.707</td> <td>2.384</td> <td>1.008</td> <td>3.120</td>	Partitioning	- 1-	9	0.453	0.705	0.350	0.516	0.614	1.664	0.649	2.150	0.707	2.384	1.008	3.120
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$							Model 3.3								
0.38         0.39         0.178         0.257         0.124         0.183         0.078         0.133         0.243         0.345         0.054         0.050         0.054         0.057         0.124         0.183         0.078         0.134         0.345         0.054         0.054         0.054         0.054         0.054         0.054         0.054         0.055         0.054         0.057         0.049         0.057         0.064         0.122         0.131         0.034         0.078         0.034         0.034         0.056         0.056         0.056         0.054         0.054         0.054         0.057         0.054         0.054         0.078         0.034         0.034         0.049         0.056         0.064         0.057         0.013         0.054         0.059         0.056         0.049         0.056         0.057         0.056         0.056         0.056         0	Infeasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.38	0.39	0.178	0.257	0.124	0.183	0.078	0.133	0.243	0.326	0.504	0.629	0.504	0.629
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Dartitioning	٠.		0.137	0.237	0.100	0.173	0.064	0.122	0.131	0.345	0.273	0.541	0.273	0.941
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Feasible Estimation	1	4						1			-			
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.25	0.131	0.185	0.091	0.131	0.074	0.117	0.240	0.289	0.240	0.289	0.240	0.289
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	9	3	0.183	0.274	0.133	0.190	0.183	0.154	0.283	0.373	0.439	1.048	0.439	1.048
0.12 0.9 0.565 0.574 0.318 0.228 0.224 0.112 0.555 0.334 2.406 0.476 4.487 0.318 0.228 0.325 0.399 0.187 0.583 0.541 1.381 2.687 4.054 0.308 0.31 0.25 0.569 0.569 0.242 0.283 0.317 0.322 2.228 4.593 2.056 9.670 3.085 2.08 0.31 0.25 0.582 0.580 0.242 0.289 0.075 0.117 0.335 0.382 0.335 0.382 3.799 0.31 0.25 0.607 0.729 0.365 0.482 0.212 0.385 0.381 0.496 0.647 1.969 3.854 0.308 0.31 0.25 0.607 0.729 0.365 0.482 0.212 0.385 0.381 0.496 0.647 1.969 3.854 0.308	Partitioning	9	က	0.318	0.469	0.232	0.302	0.437	0.961	0.481	1.279	0.489	1.430	0.545	1.991
0.12         0.9         0.565         0.574         0.318         0.224         0.112         0.555         0.334         2.406         0.476         4.487           64         27         0.616         0.595         0.325         0.336         0.399         0.187         0.583         0.541         1.381         2.687         4.054           64         27         1.063         1.471         0.817         1.143         2.389         0.187         0.583         0.541         1.381         2.687         4.054           0.31         0.25         0.569         0.569         0.194         0.223         0.075         0.117         0.335         0.382         0.356         0.389         3.799           8         5         0.607         0.729         0.242         0.289         0.212         0.185         0.381         0.496         0.647         1.969         3.854           8         5         0.607         0.729         0.365         0.165         0.636         1.052         0.881         0.744         2.288         3.206           9.33         0.03         0.651         0.522         0.581         1.074         2.288         3.074         3.854							Model 3.4								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	njeasible Estimation	0	d	9	7	010	0000	7000	0110	n n	7000	2010	0.476	7	0000
64 27 1.063 1.471 0.817 1.143 2.389 1.232 2.228 4.593 2.056 9.670 3.085 2.088	Local Folynomial B-splines	0.12	9.3	0.563	0.595	0.325	0.336	0.399	0.187	0.00	0.541	1.381	2.687	4.467	4.659
0.31         0.25         0.569         0.569         0.194         0.223         0.075         0.117         0.335         0.385         0.385         0.385         3.799           8         5         0.582         0.589         0.212         0.185         0.351         0.496         0.647         1.969         3.854           8         5         0.607         0.729         0.242         0.289         0.212         0.185         0.351         0.496         0.647         1.969         3.854           8         5         0.607         0.729         0.365         0.482         0.636         1.632         0.581         0.744         2.258         3.206           9         0.61         0.572         0.523         1.086         1.052         0.839         0.965         1.153         2.078         0.781           27         27         0.803         0.709         0.619         0.562         0.955         1.178         0.714         4.568         1.063         9.675         1.154         2.           27         27         0.806         1.476         0.669         1.148         0.632         1.235         0.764         4.568         1.063	Partitioning	64	27	1.063	1.471	0.817	1.143	2.389	1.232	2.228	4.593	2.056	9.670	3.085	20.072
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Feasible Estimation														
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.31	$0.25_{-}$	0.569	0.569	0.194	0.223	0.075	0.117	0.335	0.382	0.335	0.382	3.799	3.799
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	B-splines	x 0	oн	0.582	0.580	0.242	0.289	0.212	0.185	0.351	9.496	0.647	1.969	3.854	9.276
Model 3.5  0.33  0.33  0.33  0.736  0.651  0.572  0.523  1.086  1.052  0.955  1.178  0.713  0.965  1.023  0.901  0.701  0.701  27  27  27  27  28  0.713  0.955  1.154  27  27  0.784  0.659  1.148  0.632  1.235  0.764  4.568  1.063  0.771  0.781  0.781  1.154  1.154  2.781  1.154  2.781  1.154  2.781  1.154  2.781  1.154  2.781  1.154  2.781  1.154  2.781  1.154  2.781  1.157  1	a di ci ci ci ci ci ci	0	0	00:0	641.0	0000	70.407	0.000	T.00.T	0.00	7.00.7	# # -:-	004:4	0.500	# 0.0
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Infeasible Estimation						Model 3.5								
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Local Polynomial	0.33	0.33	0.736	0.651	0.572	0.523	1.086	1.052	0.839	0.965	1.023	0.901	0.701	0.971
27 27 0.866 1.476 0.669 1.148 0.632 1.235 0.764 4.568 1.063 9.675 1.154 2 0.27 0.25 0.712 0.624 0.559 0.501 1.201 1.179 1.002 1.176 0.881 0.736 0.390 8 7 0.790 0.719 0.603 0.572 0.787 1.155 0.662 1.006 1.005 1.501 0.582 8 7 0.779 0.830 0.575 0.673 1.013 2.026 0.888 2.453 1.217 2.440 1.135	B-splines	27	27	0.803	0.709	0.619	0.562	0.955	1.178	0.713	0.995	1.153	2.078	0.781	1.934
0.27 0.25 0.712 0.624 0.559 0.501 1.201 1.179 1.002 1.176 0.881 0.736 0.390 8 7 0.790 0.719 0.603 0.572 0.787 1.155 0.662 1.006 1.005 1.501 0.582 8 7 0.779 0.830 0.585 0.633 1.013 2.026 0.888 2.453 1.217 2.640 1.135	Partitioning	27	27	0.866	1.476	0.669	1.148	0.632	1.235	0.764	4.568	1.063	9.675	1.154	20.204
0.21 0.20 0.112 0.024 0.303 0.301 1.201 1.119 1.002 1.110 0.881 0.130 0.380 8 7 0.719 0.603 0.572 0.787 1.155 0.662 1.006 1.005 1.501 0.582 8 7 0.779 0.830 0.855 0.633 1.013 2.026 0.888 2.453 1.317 2.640 1.135	Feasible Estimation	0.07	5	0.71	200	о н С	2	1 201	140	1 000	1 176	1000	202.0	0060	л п
0 1 0779 0.830 0.585 0.632 1.013 2.026 0.026 1.002 1.002 1.002 1.003	Benlines	ο 1	2 1-	0.700	0.024	0.00	0.001	1.201	1.1.	0.002	1.170	1001	1 501	0.000	1 433
	Dertitioning	o ox	- 1-	0.130	0.830	00.0 00.0 00.0 00.0	0.012	1.013	9 00 6	200.0	2 473	1 217	2,640	1 1 2 2	2 / 25

Notes. Tuning parameters are local polynomial bandwidth and the number of cells for partitioning estimation and B-splines, as described in the text. Feasible tuning parameters reported are the (rounded) mean of all estimated values. Integrated MSE and MAE are estimated by averaging over the design points in each simulated data set.