Firm reactions to extreme weather and implications for policy design

Maximilian Huppertz*

University of Michigan, Ann Arbor

October 17, 2023

Click here for latest version

Abstract

Climate change and extreme weather events are a global problem but especially affect poor countries, including non-agricultural firms in these countries. Weather shocks may affect these firms through both a demand and supply channel. Depending on which channel dominates, short-run firm adjustments to the shocks differ, and understanding these adjustments can help in designing better policy to reduce the impact of climate change on poor countries. I combine firm-level World Bank data with high-resolution weather data, using year-to-year weather variation to identify the effect of weather shocks. I show that weather shocks are primarily supply shocks, affecting non-agricultural firms across sub-Saharan Africa and South Asia by reducing their productivity. I then show that firms react to these shocks by scaling back their expenditures on productive capability, hiring less machinery, less office space and fewer non-production workers. These reactions further reduce firm productivity. I develop a simple international trade model featuring hired productive capability that rationalizes my reduced form findings. Combining the model with machine learning estimates of the impact of climate change, I highlight the importance of productivity adjustments for policy design: Productivity responses make trade cost reductions 1.7 times more effective at countering welfare losses from climate change, compared to a standard model without these responses.

---

*Contact: mhupp@umich.edu. I am grateful to Lauren Falcao Bergquist, Hoyt Bleakley, Sebastian Sotelo and Dean Yang for their immensely helpful comments, advice and mentorship. I thank Russell Morton and Shwetha Raghuraman for their constant support, suggestions and feedback. I also thank Juan Sebastian Fernandez and Andrei Levchenko for incredibly helpful suggestions.

Climate change and extreme weather events are a global problem but especially affect poor countries. While there is an extensive literature on the effects of extreme weather on agricultural production in poor countries, we know relatively little about the effects of weather shocks on non-agricultural firms in these countries. Specifically, we do know that weather affects non-agricultural firms, with extreme temperatures decreasing their sales, and that this effect can operate both through a demand channel, by reducing local demand, and through a supply channel, by decreasing firm productivity (Adhvaryu, Kala, & Nyshadham, 2019; Costinot, Donaldson, & Smith, 2016; Nath, 2020; Santangelo, 2019; Somanathan, Somanathan, Sudarshan, & Tewari, 2021; Zhang, Dêschenes, Meng, & Zhang, 2018). We do not yet, however, have a good understanding of firm reactions to these shocks. Understanding firm reactions, and how we can leverage these reactions in policy design, could significantly improve our ability to help poor countries cope with the impact of climate change.

In this paper, I show that firms react to weather shocks by endogenously adjusting their productivity and that these reactions are quantitatively important for policy design. I combine World Bank data on firms across Africa and South Asia with global weather data sets and projections of future weather, using well-identified reduced form regressions to quantify the effects of weather shocks on non-agricultural firms, and the firms' reactions. I then develop and estimate a model of international trade capturing key reduced form relationships. Finally, I combine the model with machine learning estimates of the effect of climate change on firms to highlight the importance of firm productivity reactions for policy design.

My argument proceeds in five steps. First, I use World Bank Enterprise Surveys data combined with high-resolution weather data to test whether, across sub-Saharan Africa and South Asia, weather shocks are predominantly supply or demand shocks. (Knowing whether firms are facing supply or demand shocks is a necessary first step for understanding their reaction, since firms would react to both types of shocks differently.) The test I use is based on a basic open economy intuition about exporters: They are somewhat insulated from local demand shocks, but less able to pass on marginal cost increases to their international buyers, and thus more exposed to supply shocks. I regress log sales on a temperature index combining mean temperature, temperature variance the number of days with temperatures exceeding 32°C (89.6°F), using location fixed effects to isolate random year-to-year weather variation, and check whether exporters see a smaller or larger effect on sales. I find that negative weather shocks have a significantly larger relative impact on exporters' total sales than on purely domestically active firms' total sales: An 80$^{\text{th}}$ percentile weather shock decreases non-exporters' sales by 3.9 percent, but exporters' sales by 6.9 percent. That exporters are more affected by extreme weather implies that weather is, on net, a supply shock for these firms, rather than a local demand shock.

Second, I turn to firm reactions to the shock. Since weather shocks are supply shocks, I first test for a basic characteristic of the production structure firms face, assessing whether they operate under constant returns to scale. To do this, I check whether exporters also see a larger impact of weather shocks on domestic sales. I find that this is the case, suggesting that a firm's performance in one market affects its performance in other markets. I argue that this link is due to a key firm reaction to weather shocks: spending on *productive capability*. Productive capability comprises rented or hired equipment, facilities and non-production personnel. For example, it includes rented machinery, rented office space or a sales team. Productive capability improves overall performance across all markets a firm is active in, by increasing labor productivity (through providing workers with sufficient equipment or space) or by making it easier to sell the firm's output — it lowers the cost of providing the firm's products across all markets the firm serves. Faced with higher temperatures, firms scale back expenditures on productive capability, since these kinds of productivity-enhancing expenditures are complementary to firm productivity. I see this reaction in the data: In response to an 80$^{\text{th}}$ percentile weather shock, domestic producers reduce their expenditure on productive capability by 2.9 percent, while exporters reduce their expenditure by 6.7 percent. A mediation analysis shows that controlling for productive capability, fully interacted with exporter status and the temperature index, removes the differential impact on exporters' domestic sales. This provides strong evidence that productive capability is causing the market linkage.

Third, based on these reduced form results and adapting the basic framework of Hyun and Kim (2022), I develop an international trade model that features one simple addition to Melitz (2003) — allowing firms to hire productive capability. The model immediately generates the patterns I observe in the data: In reaction to a negative productivity shock, firms scale back productive capability expenditures, reducing their productivity and therefore their sales across all markets they are active in. The effect is larger for exporters, because exporters may no longer want to incur the cost of trading with some of their export destinations. When they exit those markets, they see a discontinuous fall in total sales and productive capability, which then leads to a discontinuous fall in sales to the domestic market. Adding productive capability makes the model computationally more challenging, since firms' decisions to enter various markets are no longer independent: If a firm enters a new market, the additional demand it can now reach makes it worthwhile to hire additional productive capability, which in turn can make it profitable to enter additional markets. The usual approach to this kind of problem is to find bounds on the set of active markets and check all possible combinations of markets between the bounds (Antràs, Fort, & Tintelnot, 2017; Jia, 2008). I show that in my setting, I can find tighter and computationally more efficient bounds on the set of active markets. I also develop a novel and efficient algorithm for finding the optimal set of markets when the lower and upper bound do not coincide. Instead of searching over all possible combinations of markets between the bounds, which for some firms is a very high dimensional combinatorial problem, I invert the problem. I find bounds on the optimal productive capability the firm hires and do a grid search across productive capability between those bounds. Finally, while the model remains computationally more burdensome than a standard Melitz (2003) model, I show that it can be readily estimated using novel small open economy methods (Bartelme, Lan, & Levchenko, 2023; Demidova, Naito, & Rodríguez-Clare, 2022). I use Zambia as a small open economy, and am able to estimate large parts of the model using reduced form approaches, which reduces the complexity of the estimation considerably without sacrificing key insights from the model. I find that the estimated model matches targeted and non-targeted moments well.

Fourth, I then turn to estimating the causal effect of climate change, based on an array of high-resolution weather projections covering a range of climate change scenarios. I use the generalized random forest (GRF) algorithm (Athey, Tibshirani, & Wager, 2019) to estimate the average firm's change in sales under different climate scenarios. GRFs are especially well suited to this high dimensional, inherently out of sample exercise. The estimation uses a wide range of temperature and precipitation measures, including averages, variances and day counts for temperature and precipitation; an important feature of GRFs is that they perform well even with such a large selection of right hand side variables. I incorporate adaptation to climate change into the estimation by also including long-term means and variances of all weather measures. I take uncertainty about future climate into account, estimating the impact of climate change on firms under three different climate change scenarios and combining predictions from 27 different climate models for each scenario. Under a severe climate change scenario, I estimate that the average firm faces an almost eleven percent decrease in sales by the late 2080s. Even under a mild scenario, I find that the average firm's sales would drop by over six percent.

Finally, I combine these projections with the model to calibrate a counterfactual baseline scenario under climate change, matching the estimated impact on the average firm to a shift of the productivity distribution. I conduct trade policy experiments based on this scenario, comparing my model's results to those of a basic Melitz (2003) model. I show that trade policy is markedly more effective when I take productive capability reactions into account: The welfare impact of a ten percent reduction in variable trade cost is 1.7 times larger due to these productivity responses. This effect is strongest for firms in countries with smaller domestic markets, where purely domestic producers do not find large productive capability expenditures worthwhile, but access to large foreign markets warrants considerable increases in productive capability. My findings are especially relevant as richer countries consider onshoring and consciously shortening their supply chains (Goldberg & Reed, 2023). I highlight that such policies can have especially negative consequences for poor countries and should be well-targeted to avoid collateral damage.

I contribute to the literature on the impact of climate change on poor countries, especially its impact on firms and trade (e.g. Conte, 2022; Costinot et al., 2016; Nath, 2020; Santangelo, 2019; Somanathan et al., 2021; Zhang et al., 2018). I add two important stylized facts to this literature, namely that weather is, on net, a supply rather than a demand shock, and that non-agricultural firms in poor countries react to weather shocks by adjusting their productive capability. I show that this means trade policy can increase firm productivity, because improved access to foreign markets makes it worthwhile for firms to invest in additional productive capability. Since climate change decreases firm productivity, trade policy can directly counteract its effects. Existing studies have conceptualized weather shocks as either demand (Santangelo, 2019) or supply shocks (Nath, 2020; Somanathan et al., 2021; Zhang et al., 2018) for firms, but have not tried to assess which of the two dominates. Knowing that weather is, on net, a supply issue is helpful because firm reactions to either kind of shock can differ. Additionally knowing that firms react by adjusting their productive capability, and thus endogenously changing their productivity, helps model firm responses to climate change. I provide a tractable way of doing this which, in turn, can inform policy design around leveraging firm responses to climate change. For example, this suggests that reducing trade cost raises firm productivity and thus directly counteracts some of the negative effects of climate change.

I further contribute to the broader literature on estimating the effects of weather shocks and climate change (e.g. Auffhammer, Hsiang, Schlenker, & Sobel, 2013; Burke & Emerick, 2016; Burke, Hsiang, & Miguel, 2015; Burke & Tanutama, 2019; Carleton & Hsiang, 2016; Carleton et al., 2022; Dell, Jones, & Olken, 2012; Deschênes & Greenstone, 2007, 2011; Lin, Schmid, & Weisbach, 2019; Nath, 2020; Ortiz-Bobea, 2021; Somanathan et al., 2021; Zhang et al., 2018), highlighting a novel approach to estimation using GRFs (Athey et al., 2019). These are especially well-suited to the problem, since they can easily handle high-dimensional weather data and are optimized for out of sample performance (future climate change is necessarily out of sample). Not having to pick specific weather measures to keep estimation feasible or tractable reduces the risk of results depending on the inclusion of a specific weather variable. The GRF relates current outcomes to current weather data and estimates the causal effect of moving to a new set of weather realizations, for example drawn from a future climate. Results can then easily be summarized as the average change in outcomes under the new climate.

The closest paper to mine is Nath (2020), who shows that under climate change, labor is drawn into agriculture, because countries need to grow sufficient food to feed their population. This labor reallocation is inefficient, because climate change especially reduces agricultural productivity. Improved trade policy allows countries to import more food and to operate closer to an efficient allocation of labor, reducing overall damages from climate change. My paper is different, since I focus on firm-level reactions to extreme weather rather than aggregate reallocations. I also use a different approach to estimating the effects of climate change, turning to causal forests, which I argue are especially well-suited to this task. Nevertheless, my paper is complementary to Nath (2020). His model does not feature the endogenous productivity reactions I find in the data and include in my model. I, on the other hand, abstract away from agricultural production. One could combine his and my model and have both channels operate at the same time, which would lead to an even larger role for trade policy in reducing the impact of climate change. Not only would reduced trade costs allow countries to shift labor into manufacturing and services, as he shows, but they would also allow firms in those sectors to recoup some of the productivity lost to climate change, as I show. This, in turn, would lead to additional labor being drawn into these more efficient sectors, further reinforcing the effect.

The rest of the paper is organized as follows: Section 1 describes the data I use. Section 2 presents motivating evidence, showing that weather acts as a net supply shock, highlighting the importance of productivity responses, and ruling out alternative explanations and informing my modeling choices. Section 3 lays out an international trade model informed by my reduced form results, which can generate the pattern I see in the data using a parsimonious mechanism: firms are able to hire productive capability, such as machinery, office space, or a sales team. Section 4 estimates the causal impact of climate change on firms, taking seriously the challenges involved in this, including the high dimensionality of weather data, the inherently out of sample nature of the problem and firm adaptation to climate change. Section 5 combines the climate change results with the model, presenting counterfactual simulations that show how productivity reactions increase the effectiveness of, for example, trade policy at counteracting some of the negative effects of climate change. Section 6 summarizes my main findings and concludes.

1 Data

1.1 Firm data: World Bank Enterprise Surveys

For data on firm outcomes and characteristics I use the World Bank Enterprise Surveys (ES).[1] The ES data include formal companies with at least five employees in the manufacturing and service sectors. Interviews cover specific fiscal years; I obtained meta data from the World Bank that allow me to fill in fiscal year dates where they are missing in the data. I use the harmonized data set provided by the World Bank, comprised of surveys between 2006 and 2020, and combine this with location data provided to me by the ES unit in the World Bank. For firms that lack location data, I further use information on the city, state and country the firm is located in to geocode the firm's location, using three different web services accessible via `Python` (OpenStreetMap, GeoNames and Google Maps).

Table 1 shows basic summary statistics for firms across sub-Saharan Africa and South Asia; Figure 1 shows the locations of all firms across sub-Saharan Africa and South Asia. I focus on these two regions for two reasons. First, they contain the countries on Earth with the highest fraction of people living in absolute poverty and the largest number of people living in absolute poverty. As climate change stands to be especially damaging to these regions (Costinot et al., 2016), mitigation policy needs to take into account how their economies work. Second, my main argument is most relevant to countries with relatively small domestic markets compared to world markets — relatively poor countries. The ES data for sub-Saharan Africa and South Asia cover many of the poorest countries on Earth.

1.2 Weather data: CHIRPS and Berkeley Earth

I use precipitation data from the Climate Hazards Group InfraRed Precipitation with Stations (CHIRPS) data set (Funk et al., 2015).[2] CHIRPS is a global, daily, high resolution (0.05° grid) precipitation data set going back to 1981. I combine these with daily maximum temperature data from the Berkeley Earth (BKE) data set (Rohde et al., 2013).[3] These data are at a lower resolution

[1] More information on the ES data and how to access them is available at https://www.enterprisesurveys.org/
[2] More information on CHIRPS is available at https://www.chc.ucsb.edu/data/chirps
[3] More information on BKE is available at https://berkeleyearth.org/data/

(1° grid) but cover recent years, which is important since my most recent data points come from 2020. Figure 2 shows daily maximum temperature on April 24, 1991, to illustrate the resolution of the temperature data.

Since the World Bank provides firms locations that are slightly randomly offset from the actual firm location, it happens in some cases that firm locations are over the water, where CHIRPS and BKE do not cover them. For these firms, I interpolate their data using weather data from the closest firm that does not have this problem.

1.3 Climate projections: NEX-GDDP-CMIP6

I obtain projections for future weather (weather drawn from a changed climate) from the NEX-GDDP-CMIP6 data set (Thrasher, Wang, Michaelis, Melton, Lee, & Nemani, 2022; Thrasher, Wang, Michaelis, & Nemani, 2021). These are results of climate model runs that are part of Coupled Model Intercomparison Project Phase 6 (CMIP6), downscaled to a higher resolution and bias corrected by the NASA Center for Climate Simulation. The data contain daily projections for temperature and precipitation, though I do not need them to accurately project temperature on any given day; I just need them to produce reasonable projections of expected weather patterns in future years.

I use projections for three different climate change scenarios; these scenarios are called Shared Socioeconomic Pathways (SSPs). Each SSP describes a different path for future climate change based on different assumptions about greenhouse gas emissions, population and international cooperation. The three different scenarios I consider, which are the most commonly used SSPs, are SSP1/2.6, which is a very optimistic scenario featuring climate change mitigation and sustainable development, SSP2/4.5, which is a middle of the road scenario featuring some mitigation, and SSP5/8.5, which features the most rapid climatic change.[4] Studying results for different SSPs allows me to incorporate deep uncertainty about the broad parameters governning the future path of climate change.

Beyond this deep uncertainty, the NEX-GDDP-CMIP6 data contain results for 27 different climate models for each SSP, reflecting uncertainty about modeling climate even for a given set of broad parameters. I combine all of these data when projecting the causal impact of climate change in Section 4. My results therefore incorporate modeling uncertainty about future weather as well. See Appendix C for more detail on data processing.

Figure 4 shows the trajectory of yearly average daily maximum temperature in the actual data and across SSPs (taking the average across all models within SSP). Starting in 2040, the differences

[4] See O'Neill et al. (2017) and Riahi et al. (2017) for more detail on the SSPs.

between the three scenarios become apparent, with temperature rising fastest in SSP5/8.5, and plateauing (in fact slightly decreasing towards the end of the century) in SSP1/2.6.

2 Motivating reduced-form evidence

This section presents simple reduced form results that make four points. First, firms across sub-Saharan Africa and South Asia are impacted by weather, with hotter years depressing sales. Second, this effect is especially strong for exporters, who see a larger relative decrease in total sales due to negative weather shocks. One implication of this is that weather shocks are, on net, supply shocks — their effect on firms' marginal cost outweighs their effect on local demand, as I explain below. Third, exporters' *domestic* sales also decline more as a result of negative weather shocks, suggesting a link between their international and domestic activities. Fourth, this is not easily explained by common correlates of being an exporter, but is consistent with firms cutting back hired productive capability, such as machinery, office space, or a sales team, in response to productivity shocks.

The functional forms I use in this section are simple, and are not meant to capture the full effects of weather, or the effects of climate change, on firm outcomes. I propose a solution to that challenge in Section 4. Instead, this section provides the key motivation for my modeling choices in Section 3.

2.1 Identification

When regressing firm outcomes on weather, the key challenge to identification is that more or less productive firms could be more likely to be located in hotter or colder places (e.g. Burke & Emerick, 2016). To overcome this, location fixed effects can be used to isolate random year-to-year variation in weather variables. I do not have panel data on firms, so I instead group firms into clusters based on geographic proximity and average weather variables within each cluster-year combination. This ensures that, conditional on cluster fixed effects, there is no correlation between unobserved firm characteristics and weather shocks.

I group firms into clusters using an algorithm called Density-Based Spatial Clustering of Applications with Noise (DBSCAN), which I implement using the `Python` package `scikit-learn` (Pedregosa et al., 2011). This algorithm takes a distance parameter to determine core groups of firms that are located close to each other, and in a second step adds periphery firms to a cluster if they are close enough to a set of core firms. The core trade-off here is that large clusters introduce measurement error due to the cluster averaging of weather variables (the cluster-average shock is different from the shock at the firm's exact location), but small clusters include fewer firms and leave some firms out of any cluster altogether (dropping them from the analysis).

An additional econometric challenge is correlation of errors across space, an issue raised for the historical persistence literature in Kelly (2020). Fortunately, I use variation over time rather than just cross-sectional variation across space; by choosing an appropriate clustering distance, I can ensure that errors are correlated within, but not across clusters. As a result, clustered errors are sufficient for correct inference, which helps with inference for the more complicated estimation of the effects of climate change in Section 4. My preferred clustering distance is 10.0 km, since the fraction of firms included in any cluster plateaus at this distance and it is still relatively small. Therefore, the measurement error induced by clustering at this distance should likewise be small. I also fail to reject the null that errors at this clustering distance are uncorrelated across clusters. See Appendix Table 13 for a formal test of error correlation at various distances, as well as the fraction of firms included in the clusters.

With firms grouped into clusters, I estimate regressions of the form

$$y_{jt} = \beta_1 x_{jt} + \gamma_{n(j)} + \delta_t + \varepsilon_{jt}$$

where y_{jt} is an outcome for firm j in cluster n at time t and x_{jt} is a measure of weather over the preceding year. I include location fixed effects $\gamma_{n(j)}$ for identification, as explained above, and year fixed effects δ_t purely to gain precision in my estimates. To explore heterogeneity, I interact weather with firm characteristics $\mathbf{z}_{jt}$,

$$y_{jt} = \beta_1 x_{jt} + \mathbf{z}'_{jt}\boldsymbol{\beta}_2 + x_{jt}\mathbf{z}'_{jt}\boldsymbol{\beta}_3 + \gamma_{n(j)} + \delta_t + \varepsilon_{jt}$$

To provide motivating evidence, I use a weather measure that is parsimonious, easy to understand and captures weather dynamics over the entire year: an index of three commonly used temperature variables. The index combines average temperature over the year, variance of temperature over the year and the number of days with temperatures exceeding 32°C (89.6°F). These three measures provide different lenses on how hot a year is, and each have their own strengths and weaknesses. Combining them into a single index provides a parsimonious combined measures of temperature. To make the index components comparable, I calculate location-specific z-scores for each of the three components x_{jt} as

$$\tilde{x}_{jt} = \frac{x_{jt} - \bar{x}_{jt}}{\sqrt{\widehat{\mathbb{V}}\left(x_{jt}\right)}}$$

where $\bar{x}_{jt}$ is the average of the variable at firm j's location over the last 20 years and $\sqrt{\widehat{\mathbb{V}}\left(x_{jt}\right)}$ is the corresponding standard deviation.[5] The index is then just the average of the three z-scores.

To make the effect size more interpretable, I scale the index by its standard deviation across locations after partialling out cluster fixed effects. (I use the standard deviation after removing fixed effects since that is the identifying variation the regressions use.) A one unit increase in the re-scaled index now corresponds to a one standard deviation weather shock. Figure 3 shows a histogram of the resulting standardized index after partialling out fixed effects; the figure also indicates the 20^{th} and 80^{th} percentiles of the variable. A one standard deviation weather shock in either direction is quite large; most shocks are smaller in absolute magnitude than this. To give a sense of scale, I will convert the one standard deviation effect sizes into 80^{th} percentile weather shocks, or 0.318 standard deviations, in the following discussion.

Since I do not care about the impact of temperature compared to some other weather variable, I do not control for precipitation, for example. The idea is not to isolate the effect of this specific variable. Rather, I am interested in the effect of weather over the year on firm outcomes, and a higher temperature index serves as an indicator of generally unfavorable weather conditions.

2.2 Results

Table 2 shows the effect of weather on firms' log sales. An 80^{th} percentile weather shock leads to a 7.2 percent decline in sales. This is a substantial effect, but note again that I use a very simple functional form here which does not distinguish between small weather variations and truly extreme events — the idea is to highlight that weather affects firms, rather than to exactly quantify its effect, which I turn to in Section 4.

Breaking the weather effect up by exporter status, Table 3 shows that purely domestic firms see a 3.9 percent decline in sales in response to an 80^{th} percentile weather shock, while currently exporting firms see a 6.9 percent decrease, with the difference significant at the one percent level. Appendix A.3 shows that this exporter interaction is not sensitive to using alternative ways of

[5] Another advantage of the location-specific de-meaning is that, since my estimations include cluster fixed effects, I now effectively use deviations of location-specific shocks from a linear growth trend to identify the effects of weather shocks on firms, rather than relying purely on the randomness of weather shocks.

measuring exporter status (for example, using past exporter status instead of current exporter status). The estimate of the base effect for domestic producers is somewhat noisy, but as Section 4 shows, using estimation methods that can fully capture the complexity of weather data, I do estimate a significant overall effect of weather on firm performance.[6]

That exporters see a larger negative effect may seem surprising, given that weather shocks affect local demand (Santangelo, 2019) and that exporters have access to a foreign source of demand that is insulated from the local shock. The differential impact on exporters makes sense, however, because weather shocks also affect firms' marginal cost (Nath, 2020; Somanathan et al., 2021; Zhang et al., 2018). In a basic open economy framework where international demand is more elastic than domestic demand, exporters will respond to a negative productivity shock by reducing their international sales more than their domestic sales. My results therefore imply that, for sub-Saharan Africa and South Asia, the supply effect of weather shocks outweighs their demand effect.

This basic open economy model cannot explain one additional result, however. Table 4 shows that exporters' national sales also see a larger decline in response to negative weather shocks. Purely domestically active firms see a 5.4 percent decline in domestic sales in response to an 80$^{\text{th}}$ percentile weather shock, but exporters see a 7.7 percent decrease. This suggests a link between exporters' international and national sales, which I will formalize in Section 3 by allowing firms to hire productive capability.

To preview the underlying idea, exporters reach a larger total market size than purely domestic producers. This incentivizes them to spend additional money on hired productive capability, in the form of (for example) machinery, office space, or a sales team, which lowers their marginal cost. In turn, when a negative supply shock decreases their productivity, they may no longer be able to profitably serve some foreign markets, because the profit they would make there is smaller than the fixed cost of accessing the market (e.g. keeping a up-to-date export license). In response, they scale back their expenses on rented productive capability, which increases their marginal cost, raises their prices and leads to lower sales even in the markets they can still profitably service. This leads to a larger relative decrease in domestic sales in response to weather shocks compared to non-exporters, explaining the pattern in Table 4. I show formally that the model I build generates these comparative statics in Section 3.

To provide direct support of the hypothesis that expenditures on fixed productive capability are

[6] To highlight that year fixed effects are present purely to increase precision, and do not affect point estimates much, Appendix Table 14 shows an estimation without year fixed effects. Results for the effect on non-exporters are much less precise. The difference in the effect for exporters remains highly significant and similar in magnitude, however.

important in this context, Table 5 shows the effect of weather shocks on productive capability expenditures by exporter status. Productive capability expenditures combine the cost of communications, sales, transportation, and rent for buildings, equipment and land. I see a 2.9 percent decrease in productive capability expenditures for domestically active firms in response to an 80^{th} percentile weather shock, but a 6.7 percent decrease for exporters.[7] I only have detailed cost breakdowns for a sub-sample of firms, which is why this analysis uses fewer observations than the preceding results, but this nevertheless provides direct evidence of productive capability adjustments when faced with negative weather events. Exporters scale back fixed expenditures considerably more than non-exporters in response to negative weather shocks. This is because exporters may no longer be able to profitably serve some markets, making some of their productive capability redundant. When they cut back on this productive capability, their overall productivity decreases.

This is also evident in the second column of Table 5, which shows that sales per employee see a larger relative decline for exporters compared to non-exporters (a 2.6 vs. 1.1 percent decline in response to an 80^{th} percentile weather shock). Further, Table 7 shows a mediation analysis which adds log productive capability expenditures, fully interacted with exporter status and the temperature index, to the national sales regression from Table 4. (I de-mean log productive capability expenditures, so all coefficients shown are evaluated at mean log productive capability, not at zero productive capability.) Since this mediation analysis adds a clearly endogenous regressor, I am careful in over-interpreting these results, but the interaction between the exporter effect and the temperature index flips signs and is longer statistically significantly different from zero, even at the ten percent level. This suggests productive capability 'explains' the larger impact of weather on exporters' domestic sales, though again, since productive capability is an endogenous regressor, I am cautious in interpreting this result. Nevertheless, together with the results from Table 5 that exporters scale back productive capability more than non-exporters and that their sales per employee decline more, hinting at a larger drop in productivity, I take the evidence as very consistent with the theory that productive capability expenditures are driving the market linkage.

[7] It is possible that weather shocks lead firms to reduce the valuation of their productive capability, rather than its physical quantity. Appendix Table 19 shows, however, that weather has no effect on firms' valuation of their stock of machinery, where I should see that same effect at play if it mattered. This suggests the productive capability effect I find is due to a reduction in its quantity, rather than just due to a change in valuation.

2.3 Underlying mechanism

My results show an effect of weather on output, which I interpret as evidence of a net productivity reduction. There is a large literature showing that weather shocks decrease productivity. For example, Adhvaryu et al. (2019) find direct evidence for lower worker productivity on assembly lines during hot days, exacerbated by heat-generating lighting, Somanathan et al. (2021) find lower worker productivity in Indian manufacturing firms on hot days, and Zhang et al. (2018) find reduced total factor productivity in Chinese manufacturing during hotter years. In their wide-ranging literature summary, Carleton and Hsiang (2016) note negative impacts of temperature on labor supply, which are also found to be important in Somanathan et al. (2021) and Santangelo (2019). If workers supply less labor at each wage level, for example because they need to work on their own subsistence farms, that has an effect similar to lower worker productivity — to produce a given quantity, the firm needs to incur higher labor cost. These arguments may pertain more to employees in lower skilled jobs, but as Carleton and Hsiang (2016) note, previous research also finds that extreme temperatures reduce cognitive performance, e.g. lowering math test scores. With imperfect climate control, even workers in an office setting would face lower productivity due to extreme heat. Through this cognitive performance channel, temperature can reduce worker productivity even for firms in the service sector, for example. Ultimately, working in extreme heat makes it hard for anyone to perform their best, is at best unpleasant and at worst outright dangerous.

Table 6 shows evidence that these channels are present in my data, albeit the estimates are all somewhat noisy. For example, I see sales per employee declining by 2.4 percent following an 80$^{\text{th}}$ percentile weather shock, which is a direct indication that labor productivity is reduced. I also see firms' total operating hours increasing by 4.3 percent in response to a 80$^{\text{th}}$ percentile weather shock. Since firms' sales are falling, this suggests lower output per hour, as e.g. found by Adhvaryu et al. (2019). I further find that hotter years lead to more power outages, with an 80$^{\text{th}}$ percentile weather shock increasing the likelihood of an outage by 1.7 percentage points (again, the estimate is somewhat noisy). Outages directly decrease firm productivity by making it impossible to operate machinery or computers.

2.4 Alternative explanations

Of course, exporters are different from other firms in many ways; Table 8 shows a comparison of exporting and non-exporting firms' characteristics. It could be that these differences simply make them more susceptible to weather shocks. To address this concern, I run an extensive battery of robustness checks that regress log total sales on the temperature index interacted with exporter status and additionally interacted with other firm characteristics, plus base effects for those characteristics. If it were true that another firm characteristic is the 'true' reason for the effects I find, I would expect that once I include that characteristic in the regression, the interaction between exporter status and weather shocks loses significance and/or sees a drastically smaller point estimate. I find that neither happens for any of the three alternative hypotheses I describe in the rest of this section. Appendix Table 20 summarizes these robustness checks.

First, exporters tend to be large firms, which could be more reliant on short-term hired labor that can get drawn into agriculture when negative weather shocks hit (Santangelo, 2019); accordingly, I check whether the initial number of employees or the number of employees three years ago can explain the exporter effect. Second, since exporters are more likely to be in large-scale manufacturing, where temperature control can be a problem (Adhvaryu et al., 2019), I control for two- and four-digit ISIC sectors, fully interacted with weather shocks. This also addresses concerns around an effect through input prices — if it were true that exporters simply use a different import structure, I would expect firms in similar sectors to face the same issue. Third, exporters could potentially be using more complex production processes (Costinot, 2009), so I control for measures of complexity: whether a firm has an international quality certification, the firm's ownership structure, and the manager's years of experience. None of these alternative hypotheses alone can explain the different effects for exporters, and I find that weather has a differential impact on exporters even when including them all in the regression at once.[8]

I conclude from this that the effect I find is not due to an obvious correlate of being an exporter, but is instead about the production structure of exporting firms. As I show in Section 3, a simple extension of Melitz (2003) immediately yields the patterns I find. All that is required is that firms can acquire any kind of productive capability — including, but not limited to items such as machinery, office space, or a sales team — and the model generates the comparative statics I find. That firms can hire productive assets is not, in fact, an unrealistic assumption, and provides a parsimonious explanation of the patterns I see in the data, including the greater decrease in productive capability seen in Table 5.

[8] A final concern would be differential measurement error for exporters and non-exporters. To rule out this possibility, I estimate the main regression using only data on firms coming directly from the firms' books. Appendix Table 15 shows the results; point estimates become noisier but remain very similar to my main results.

3 Model

I now develop a simple international trade model that can rationalize the core reduced form results I highlight in the previous section. That is, this model can explain why exporters see a larger effect of weather shocks on total sales and why this carries over to domestic sales as well. The core mechanism is firms' ability to hire productive capability, such as machinery, office space, or a sales team.

3.1 Demand

There are N countries and a mass of goods $\mathcal{G}_n$ is available in each country n. Consumers have CES preferences with elasticity of substitution σ, a budget of X_n and solve

$$\max_{\{q_n(j)\}} \left(\int_{\mathcal{G}_n} q_n(j)^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}j \right)^{\frac{\sigma}{\sigma-1}} \text{ s.t. } X_n = \int_{\mathcal{G}_n} q_n(j) p_n(j) \, \mathrm{d}j \tag{1}$$

This yields quantity demanded as

$$q_n(j) = \underbrace{X_n \mathcal{P}_n^{\sigma-1}}_{\equiv \alpha_n} p_n(j)^{-\sigma} = \alpha_n p_n(j)^{-\sigma} \tag{2}$$

where

$$\mathcal{P}_n = \left(\int_{\mathcal{G}_n} p_n(j)^{1-\sigma} \, \mathrm{d}j \right)^{\frac{1}{1-\sigma}}$$

is the optimal price index and I introduce the shorthand α_n to denote demand factors — from the perspective of firms selling in market n, a higher α_n translates into higher sales in that market at any given price they charge there. The α_n factors will be an important component of solving the model.

3.2 Supply

Firms are located in their home country i and export to other countries n. Each country has an infinite mass of potential entrants. Firm j, producing the proprietary variety j, is characterized by its core productivity $a_j \sim F_i$, where the distribution of productivities is country specific. I adapt the framework of Hyun and Kim (2022), which is an extension of Melitz (2003) allowing firms to choose a common quality level across all markets they are active in. Instead of linking decisions across markets via demand, I let firms purchase or hire productive capability c_j, such as machinery, office space, or a sales team. Additional productive capability makes it cheaper to provide goods in all markets and links choices, including entry decisions, across markets. The cost of acquiring c_j is $f(c_j)$ and measured in units of labor. Firm j, active in a set of markets (countries) $\mathcal{M}_j$, has effective productivity $a_j c_j^{\delta}$ and incurs variable cost

$$v(\{q_n(j)\}) = \sum_{n\in\mathcal{M}_j} d_{ni} q_n(j) \frac{w_i}{a_j c_j^{\delta}}$$

which includes an iceberg trade cost d_{ni} and the wage w_i. (Since Hyun and Kim (2022) focus on entry in various domestic markets within one country instead of international trade, their model does not feature an iceberg cost.) The firm's profit maximization problem is

$$\begin{aligned}
&\max_{\{p_n(j),q_n(j)\},c_j,\mathcal{M}_j} \left[\sum_{n\in\mathcal{M}_j} p_n(j)q_n(j) - d_{ni}q_n(j)\frac{w_i}{a_j c_j^{\delta}} - f_{ni}w_i\right] - f(c_j)\,w_i - f_i w_i \\
\overset{(2)}{\Leftrightarrow} &\max_{\{p_n(j)\},c_j,\mathcal{M}_j} \left[\sum_{n\in\mathcal{M}_j} \alpha_n p_n(j)^{1-\sigma} - d_{ni}\alpha_n p_n(j)^{-\sigma}\frac{w_i}{a_j c_j^{\delta}} - f_{ni}w_i\right] - b\beta c_j^{\frac{1}{\beta}} w_i - f_i w_i \qquad (3)
\end{aligned}$$

where I follow Hyun and Kim (2022) and set $f(c_j) = b\beta c_j^{\frac{1}{\beta}}$. Unlike Hyun and Kim (2022) I explicitly consider market entry and exit, so I include a fixed cost f_{ni}, measured in units of labor, for operating in each market, as in Melitz (2003). I assume $f_{ii} = 0$ and $d_{ii} = 1$ for simplicity, because it makes the model computationally easier to solve and because the data I use cover only active firms. I further include a fixed start-up cost f_i, also measured in units of labor, which entrants have to pay to discover their productivity a_j, again as in Melitz (2003). For a given c_j, optimal prices $p_n(j)$, quantities $q_n(j)$ and sales $S_n(j)$ for firm j in market n, as well as profits across all markets $\pi(j)$, follow from first order conditions as

$$p_n(j) = \underbrace{\frac{\sigma}{\sigma-1}}_{\equiv\mu} d_{ni}\frac{w_i}{a_j c_j^{\delta}} \qquad (4)$$

$$q_n(j) = \alpha_n\left(\mu d_{ni}\frac{w_i}{a_j c_j^{\delta}}\right)^{-\sigma} \qquad (5)$$

$$S_n(j) = \alpha_n\left(\mu d_{ni}\frac{w_i}{a_j c_j^{\delta}}\right)^{1-\sigma} \qquad (6)$$

$$\pi(j) = \frac{1}{\sigma}\left(\mu\frac{w_i}{a_j c_j{}^{\delta}}\right)^{1-\sigma}\left(\sum_{n\in\mathcal{M}_j} d_{ni}^{1-\sigma}\alpha_n\right) - \left(\sum_{n\in\mathcal{M}_j} f_{ni}\right) w_i - b\beta c_j^{\frac{1}{\beta}} w_i - f_i w_i$$

After solving for prices and quantities, the FOC for the optimal distribution network gives[9]

$$c_j = \left[\frac{1}{b}\delta(\mu w_i)^{-\sigma} a_j^{\sigma-1}\left(\sum_{n\in\mathcal{M}_j} {d_{ni}}^{1-\sigma}\alpha_n\right)\right]^{\frac{\beta}{1-\beta(\sigma-1)\delta}} \tag{7}$$

Analogous to Hyun and Kim (2022), the parameter restrictions to ensure an interior solution are $\sigma > 1$, which is a standard CES assumption and ensures that goods are substitutes, $\delta > 0$, which ensures that additional productive capability decreases marginal cost, and $\beta(\sigma-1)\delta < 1$, which ensures that the optimal c_j is finite, because the increase in the cost of acquiring productive capability outpaces the decrease in marginal cost. As I discuss in Section 3.4, I find that these restrictions are fulfilled in the data.

I showed in Section 2 that exporters see a larger decline in total sales and domestic sales in response to higher temperatures. This implies that weather is, on net, a negative supply shock for firms, otherwise, I would expect exporters to see a smaller decline in sales in response to these weather shocks. Further, the larger impact on domestic sales can be rationalized if, when they are pushed out of certain markets as a result of the productivity shock, exporters scale back productive capability in response. Matching these reduced form results, exporters see a larger decline in domestic sales if they stop exporting to any markets as a result of a negative productivity shock $a_j' < a_j$. Let ${\mathcal{M}_j}' \subseteq \mathcal{M}_j$ denote the set of markets the firm is active in after the shock. Using primes to denote post-shock quantities, the relative decline in Home sales is

$$\frac{S_i(j)'}{S_i(j)} \overset{(6)}{=} \left(\frac{a_j' {c_j'}^{\delta}}{a_j c_j^{\delta}}\right)^{\sigma-1} \overset{(7)}{=} \left[\left(\frac{a_j'}{a_j}\right)\left(\frac{\sum_{n\in{\mathcal{M}_j}'} {d_{ni}}^{1-\sigma}\alpha_n}{\sum_{n\in\mathcal{M}_j} {d_{ni}}^{1-\sigma}\alpha_n}\right)^{\beta\delta}\right]^{\frac{\sigma-1}{1-\beta(\sigma-1)\delta}}$$

Note that the parameter restrictions ensure the overall exponent is positive. The first term in parentheses depends solely on the change in a_j. The second term in parentheses depends on the change in active markets $\mathcal{M}_j$; it represents an indirect effect of the productivity shock which occurs if leads the firm to exit some markets it was active in. For domestic producers, ${\mathcal{M}_j}' = \mathcal{M}_j$, since they will not exit altogether given that $f_{ii} = 0$. For exporters, if ${\mathcal{M}_j}' \subset \mathcal{M}_j$, the second term in parentheses is smaller than one, exacerbating the effect of the shock and leading to a larger relative decline in sales. This model therefore generates the comparative statics I observe in the data. The standard Melitz (2003) model, in contrast, would only generate this comparative static for total

[9] Detailed derivations for this and all following results which are merely stated can be found in Appendix D.

sales — when an exporter leaves a market following a productivity shock, they see a discontinuous drop in sales, and thus a larger relative decline in *total* sales than a purely domestic producer would. For *Home* sales, however, both firms would see the exact same relative decline.

3.3 Equilibrium

Definition 3.1 *For a given a CES elasticity σ, start-up costs f_n, entry costs f_{ni}, iceberg trade costs d_{ni}, cost parameters b, β and δ, and productivity distributions $F_i(a_j)$, an equilibrium for this model is a set of prices $p_n(j)$, quantities $q_n(j)$, productive capacities c_j, active markets $\mathcal{M}_j$, masses of entrants N_n and active firms n_n, incomes X_n and wages w_n such that, for all firms j and all countries n,*

- *Consumers are maximizing utility* (1)
- *Firms are maximizing profits* (3)
- *Labor supply L_n equals labor demand in all countries*
- *Expected profits prior to entry are zero in all countries*
- *Income equals expenditure in all countries, i.e. trade is balanced*

3.3.1 Optimal choice of active markets

The first step in finding the equilibrium is determining $\mathcal{M}_j$ for a given set of α_n, which in principle is a high-dimensional combinatorial problem. Antràs et al. (2017) confront a similar problem when modeling input sourcing and use the algorithm developed by Jia (2008). This relies on finding an upper bound on $\mathcal{M}_j$ by assuming the firm is active in all N markets and determining whether leaving an individual market n would increase firm profits. If so, as Jia (2008) shows, n cannot be part of the optimal set of markets. A similar procedure yields a lower bound, starting from the firm being active in no markets at all and checking where entry increases profits. To find the optimal $\mathcal{M}_j$, you then only have to calculate profits across all combinations of markets present in both the lower and upper bound.

In my setting, I can use a more computationally efficient algorithm for finding upper and lower bounds. To find an upper bound, for a set of parameter guesses, start by assuming the firm is active in all markets, and set that as the initial $\mathcal{M}'_j$. Then,

1. Calculate c_j if the firm were active in $\mathcal{M}'_j$ and calculate variable profits in each market, that is, sales $S_n(j)$ in each market minus the entry cost f_{ni} (ignoring the cost of acquiring c_j)
2. Drop all markets where the firm would be earning negative variable profits from $\mathcal{M}'_j$, and use those where it makes weakly positive profits as the new $\mathcal{M}'_j$

Iterate until the firm makes weakly positive variable profits in all markets in $\mathcal{M}'_j$; this gives the upper bound $\mathcal{M}_j^{\text{ub}}$ (see Appendix D.3 for a proof).

To find a lower bound, for a set of parameter guesses, start by assuming the firm is active only at Home, and set that as the initial $\mathcal{M}'_j$. (Since I assume $f_{ii} = 0$, firms will always be active in the Home market.) Then,

1. Calculate c_j if the firm were active in $\mathcal{M}'_j$ and calculate variable profits in each market, that is, sales $S_n(j)$ in each market minus the entry cost f_{ni} (ignoring the cost of acquiring c_j)
2. Add all markets where the firm would be earning positive variable profits to $\mathcal{M}'_j$, and use these plus the markets in $\mathcal{M}'_j$ as the new $\mathcal{M}'_j$

Iterate until the firm cannot enter additional markets where it would make weakly positive variable profits when c_j is chosen optimally under $\mathcal{M}'_j$; this gives the lower bound $\mathcal{M}_j^{\text{lb}}$ (see Appendix D.2 for a proof).

In simulations, I find both bounds in many fewer steps than I can find the bounds from Jia (2008). This is largely because finding the bounds from Jia (2008) always requires as many steps as there are markets, whereas my bounding algorithms can often exclude or include multiple markets in one step. I also find that my bounds are usually tighter than those from Jia (2008). Both of these factors speed up computation considerably. The core feature of my model that enables me to use these more efficient bounds is that the link between entering or exiting different markets ultimately comes down to an optimal firm decision on c_j.

Having found the bounds, I know the optimal set of active markets for all firms with $\mathcal{M}_j^{\text{lb}} = \mathcal{M}_j^{\text{ub}}$, which is the case for the majority of firms. For firms where the bounds do not coincide, I could check all possible combinations of markets in between the two bounds. Unfortunately, unlike Antràs et al. (2017), I find that the cardinality of that difference can be considerable: Though most firms only need to decide between a few markets, some have over 100 different markets ($\approx 1.23 \times 10^{30}$ combinations) to choose from. Therefore, I cannot feasibly solve the optimal market problem by checking profit across all combinations of active markets.

Instead of searching over sets of active markets $\mathcal{M}_j$ and determining the optimal profit for each set, I invert the problem. I search across c_j (productive capability) to find optimal profits across possible choices of productive capability. Determining $\mathcal{M}_j$ for a given c_j is easy, since with a known productivity, market entry decisions just boil down to Melitz (2003): Firms enter markets in which they make a variable profit (taking entry costs into account). The lower and upper bounds for markets the firm could be active in, $\mathcal{M}_j^{\text{lb}}$ and $\mathcal{M}_j^{\text{ub}}$, also yield lower and upper bounds on c_j, since as firms increase c_j, they only ever enter additional markets and do not exist markets they are already active in. (Leaving a market the firms is making positive variable profit in cannot increase its total profit for a given c_j.) The c_j the firm would optimally choose if it were active in $\mathcal{M}_j^{\text{lb}}$ thus is a lower bound on the optimal c_j the firm chooses, and similarly the optimal c_j at $\mathcal{M}_j^{\text{ub}}$ yields an upper bound. I then simply conduct a grid search between those two bounds.

3.3.2 Remaining equilibrium objects

To determine α_n, I turn to the free entry condition for country i. It states that expected profits before paying the fixed start-up cost f_i required to discover firm productivity should be zero. Let $\underline{a}_i$ denote the least productive firm that finds it profitable to operate in country i (instead of shutting down after discovering a_j), then the free entry condition is

$$f_i w_i = \int_{\underline{a}_i}^{\infty} \frac{1}{\sigma} \left(\mu \frac{w_i}{a_j {c_j}^{\delta}} \right)^{1-\sigma} \left(\sum_{n \in \mathcal{M}_j} d_{ni}^{1-\sigma} \alpha_n \right) - \left(\sum_{n \in \mathcal{M}_j} f_{ni} \right) w_i - b \beta c_j^{\frac{1}{\beta}} w_i \, \mathrm{d}F_i \left(a_j \right) \tag{8}$$

The integral cannot be solved analytically because it depends on the sets of active markets $\mathcal{M}_j$ (directly but also, non-linearly, through c_j). These sets are a function of a_j and (being sets) do not have an easily computable antiderivative. This condition nevertheless pins down the equilibrium α_n terms, given wages and sets of active markets, in a computationally attractive form: When numerically solving the model (using a sum to simulate the integral), this becomes a system of linear equations in α_n.

The full employment condition for country i yields the mass of entrants

$$N_i = \frac{\mu^{\sigma-1} w_i^{\sigma} L_i}{\int_{\underline{a}_i}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \sum_{n \in \mathcal{M}_j} d_{ni}^{1-\sigma} \alpha_n \, \mathrm{d}F_i \left(a_j \right)}$$

which can be used to find the mass of active firms $n_i = \left[1 - F_i \left(\underline{a}_i \right) \right] N_i$. To derive the gravity equation, I first calculate the price index for country n as

$$\mathcal{P}_n = \mu \left(\sum_{i=1}^{N} n_i (d_{ni} w_i)^{1-\sigma} \int_{\underline{a}_{ni}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \mathrm{d}F_i \left(a_j \right) \right)^{\frac{1}{1-\sigma}}$$

Plugging this into aggregate trade flows from i to n leads to the gravity equation

$$X_{ni} = \frac{n_i (d_{ni} w_i)^{1-\sigma} \int_{\underline{a}_{ni}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \mathrm{d}F_i \left(a_j \right)}{\sum_{k=1}^{N} n_k (d_{nk} w_k)^{1-\sigma} \int_{\underline{a}_{nk}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \mathrm{d}F_k \left(a_j \right)} X_n \tag{9}$$

where $\underline{a}_{ni}$ is the least productive firm from i selling in n. This has the typical gravity structure, but the integrals of effective productivity across producers selling from i to n again cannot be solved analytically. The gravity equation nevertheless pins down wages, closing the model with world GDP as the numeraire.

3.4 Estimation

3.4.1 Reduced form

I follow the common practice of assuming that core productivities come from a Pareto distribution with shape parameter θ, shifted by a country-specific scale parameter $T_i^{\frac{1}{\theta}}$, where T_i captures differences in technology across countries. I estimate both θ and T_i. I simulate firm productivities using draws u_j from a uniform distribution on $(0, 1]$, since

$$u_j = T_i a_j^{-\theta}$$

is uniformly distributed and can be used to back out a_j given the dispersion parameter and technology shifters (Eaton, Kortum, & Kramarz, 2011).

Following Eaton et al. (2011), I estimate σ based on the ratio of firms' sales to their variable costs. Variable costs include the cost of labor, raw materials, fuel, water, electricity, goods for resale, and other costs of production. I first calculate the mark-up for each firm and then use the average across firms to calibrate σ, which yields $\hat{\sigma} = 3.016$ with a standard error of 0.043. Table 9 shows estimates for all parameters of the structural model.

Next, I turn to the parameters governing firms' optimal productive capability, β and δ. Log sales in firm j's home market i can be written as

$$\log\left(S_i(j)\right) = I + \log\left(\alpha_i\right) + (\sigma - 1)\log\left(a_j\right) - (\sigma + \beta\delta - 1)\log\left(w_i\right) + (\sigma - 1)\,\beta\delta\log\left(\mathcal{S}(j)\right) \tag{10}$$

with I a constant. This shows that the elasticity of home market sales with respect to total sales identifies $\beta\delta$ for a known σ. I make the simplifying assumption that $\beta = \delta$ to ease the computational burden during the structural estimation described below. β and δ both discipline firms' choices of productive capability c_j, and equilibrium wages and welfare depend only on the product of both parameters, though they have slightly different implications for price distributions. Normalizing the two is therefore without great loss of generality.

I can then directly estimate δ and β by regressing log home market sales on log total sales, country-year fixed effects (to deal with α_i and w_i) and proxies for core productivity a_j. I proxy for core productivity using country-sector-year fixed effects (obviating the need for country-year fixed effects), the manager's years of experience, log initial number of employees, log number of employees three years ago, whether the firm uses an international quality certification, whether the firm experienced power outages, whether the firm competes against the informal sector, whether the firms introduced a new product or service during the last three years, whether the firm introduced a new process during the last three years, whether the firm had any R&D expenditures, whether the firm is part of a larger firm, whether the firm was formal when it was founded, whether the firm applied for an electricity connection over the last three years, whether the firm applied for a water connection over the last three years, the firm age, and legal status indicators. To allow for more flexibility in the estimation, I add all pairwise interactions as well as fourth-degree polynomials of all continuous variables.

I estimate (10) using only data on exporters, since purely domestically active firms mechanically yield an elasticity of one (their home sales and total sales are identical); see Appendix Table 21 for a summary of the results. Since the estimate for $\beta\delta$ depends on $\hat{\sigma}$, I obtain its standard error via a pairs bootstrap, estimating σ and $\beta\delta$ for 999 bootstrap samples. I find $\widehat{\delta\beta} = 0.388$ with a standard error of 0.016, which results in $\hat{\delta} = \hat{\beta} \approx 0.623$. Regardless of the assumption that $\beta = \delta$, these parameter estimates satisfy the crucial restriction that $\beta(\sigma - 1)\delta < 1$, ensuring an internal solution for firms' productive capability c_j.

Finally, I need to fix the productive capability cost shifter b. This is not separately identified from the technology shifters T_i, because making productive capability overall much cheaper has the same effect as shifting overall productivity. I assume that $b = 1$ to resolve this set identification problem. The interpretation of this assumption is simply that workers can do production work and administrative work equally well.

3.4.2 Structural estimation

All remaining parameters — technology shifters T_i, the technology scale parameter θ, start-up costs f_i, iceberg costs d_{ni} and entry costs f_{ni} — are estimated via the method of simulated moments (MSM). Estimating the model for the entire set of economies I have in my data is computationally infeasible, since finding a single equilibrium of the model for many countries and with a large number of simulated firms takes considerable time. Instead, I estimate the model for a small open economy Home (H), building on the theoretical work by Demidova et al. (2022) and the estimation strategy in Bartelme et al. (2023). Specifically, I solve (8) only for the Home price index α_H, taking all other countries' α_n as given. I take the labor force size L_H from World Bank (2023).

I recover $\alpha_n = X_n \mathcal{P}_n^{\sigma-1}$ for all other countries from the ITPD data on total expenditures X_n and a gravity estimation using data on all countries but Home, similar to Bartelme et al. (2023). I model all other countries' economies as following the model in Melitz (2003), which means their price indices are

$$\mathcal{P}_n = \frac{\sigma}{\sigma-1}\left(\frac{\theta_F}{\theta_F-\sigma+1}\right)^{-\frac{1}{\theta_F}}\left(\frac{\sigma}{X_n}\right)^{\frac{\theta_F-\sigma+1}{\theta_F(\sigma-1)}}\left(\sum_{i=1}^{N} T_i n_i (d_{ni}w_i)^{-\theta_F}(f_{ni}w_i)^{\frac{\theta_F-\sigma+1}{1-\sigma}}\right)^{-\frac{1}{\theta_F}}$$

where θ_F is the dispersion parameter for other economies' core productivity distributions, which I take from Melitz and Redding (2015) as $\theta_F = 4.25$.[10] Everything else in this expression is either data or a parameter I can estimate via the reduced form approaches above, while the final term in parentheses can be recovered from a gravity estimation. Specifically, the model in Melitz (2003) yields the gravity equation

$$X_{ni} = \frac{T_i n_i (d_{ni}w_i)^{-\theta_F}(f_{ni}w_i)^{\frac{\theta_F-\sigma+1}{1-\sigma}}}{\sum_{l=1}^{N} T_l n_l (d_{nl}w_l)^{-\theta_F}(f_{nl}w_l)^{\frac{\theta_F-\sigma+1}{1-\sigma}}} X_n \tag{11}$$

for trade flows from i to n, which allows me to recover $\sum_{i=1}^{N} T_i n_i (d_{ni}w_i)^{-\theta_F}(f_{ni}w_i)^{\frac{\theta_F-\sigma+1}{1-\sigma}}$ from the

[10] The technology scale parameters θ and θ_F capture the dispersion of core productivities, one for Home, the other for all other countries. I take θ_F from the literature since good estimates of this parameter exist, but allow $\theta \neq \theta_F$ because those existing estimates are for the dispersion in firm productivity when a_j completely captures firm productivity. In my model, total firm productivity is $a_J c_j^{\delta}$, which has a different distribution than core productivity a_j by itself. I therefore allow for a different dispersion for core productivity.

importer fixed effect of country n in a standard gravity estimation. I estimate this gravity equation using pseudo-Poisson maximum likelihood estimation (Santos Silva & Tenreyro, 2006), based on data for all countries except Home. Following Bartelme et al. (2023), I use distance and an indicator for contiguity to approximate the bilateral term

$$\tau_{ni} \equiv d_{ni}^{-\theta_F} f_{ni}^{\frac{\theta_F-\sigma+1}{1-\sigma}} \tag{12}$$

To minimize measurement error in the trade data, I calculate average real flows across all years from 2000 to 2019 (the most recent year in the data). Appendix Table 22 shows estimation results for the gravity equation.

The results from this gravity estimation also allow me to ease estimation of d_{ni} and f_{ni}. The estimates for τ_{ni} combine iceberg trade cost and entry cost. I parameterize iceberg cost d_{ni} as a function of the same variables I include in the gravity equation, distance and the contiguity indicator,

$$d_{ni} = 1 + \exp\{\mathbf{X}_{ni}\boldsymbol{\gamma}\}$$

where $\mathbf{X}_{ni}$ also contains a constant term. For a guess of $\boldsymbol{\gamma}$, I can then recover f_{ni} from (12). I maintain the assumptions that $d_{ii} = 1$ and $f_{ii} = 0$. Parameterizing d_{ni} in this way is an exact analogue to estimating parameters via a gravity equation beforehand and feeding the results into the structural algorithm, as done for example in Antràs et al. (2017).

The parameters T_H, f_H and $\boldsymbol{\gamma}$ are estimated via MSM. The targeted moments are Home's share of exporters, exports from Home to each other country,[11] the ratio of Home's trade with itself to its total exports and firms' average productive capability. In simulated data, these moments are exactly sufficient to identify all model parameters. To minimize measurement error in the trade data, I again use average real flows across all years from 2000 to 2019 (the most recent year in the data), as I did when estimating (11). In the firm data, productive capability combines the cost of communications, sales, transportation, and rent for buildings, equipment and land. In the model, it is simply the value of firms' productive capability, $\beta c_j^{\frac{1}{\beta}} w_H$.

While all parameters are identified by all moments, the share of exporters and ratio of Home's trade with itself to total exports are especially helpful for identifying f_H and T_H, the export flows

[11] In order not to overweight a few large export destinations, I take the log of exports and add the fraction of countries Home does not export to as an additional moment.

are especially useful for identifying γ and the average productive capability is needed to identify θ. Table 9 also shows which variation in the data is especially important for identifying which parameters. I simulate the model using one million firms. I also estimate a version of Melitz (2003) in exactly the same way, targeting the same moments except the average productive capability, which that model does not require. For this model, I simply use the dispersion parameter $\theta_F = 4.25$ from Melitz and Redding (2015).

3.5 Estimation results

I estimate the model using Zambia as the small open economy, Home, because I have three rounds worth of Enterprise Surveys data (2007, 2013 and 2019) and because it could reasonably be described as a small open economy. It has a ratio of total exports to total domestic trade (trade with itself) of ≈ 58 percent, so trade makes up a large fraction of its economy, and it exhibits roughly balanced trade: Its trade imbalance (exports minus imports) as a fraction of its total trade (exports plus imports) is only six percent.

Table 9 shows the parameter estimates. I estimate a productivity dispersion parameter $\theta = 3.590$, which is somewhat lower than the estimate for the standard Melitz (2003) model I take from the Melitz and Redding (2015). Comparing total productivity distributions, however, Figure 9 shows that my model actually generates a higher dispersion for total productivity among Zambian firms than Melitz (2003) estimated on the same data. This figure shows the percentiles of log a_j for the Melitz (2003) model and log $a_j c_j^{\delta}$ for my model. My model results may look like an extreme productivity distribution, but poor countries often exhibit a relatively large mass of very small firms (Hsieh & Olken, 2014), leading to a stronger productivity dispersion. For example, in the Enterprise Surveys data for Zambia, the ratio of the 95^{th} to the 5^{th} percentile (95/5 ratio) of real sales is ≈ 681, which is even larger than my model's estimate of the 95/5 ratio of ≈ 365. For comparison, the Melitz (2003) model generates a 95/5 ratio of ≈ 8. This highlights that my model's implications for productivity dispersion are not implausible in the Zambian context.

Figure 7 shows a comparison of Zambian log exports and model results. The model matches the data well — the correlation coefficient is 0.63. In addition, figure 8 shows Zambian log imports, which are not a targeted moment in the MSM estimation, compared to the model simulation. The correlation here is even stronger, at 0.79. This is encouraging, since imports depend on the iceberg cost parameters from the MSM estimation and other countries' α_n, which I estimate outside the model. That imports are well approximated suggests that the theoretical model captures key relationships in the data, that the estimates of that model reproduce those relationships for untargeted data moments, and that the MSM estimation and estimations outside the model combine well. Comparisons of the data and estimated values for the other three targeted moments are shown in Table 10. The model produces a share of exporters of 15 percent, which is identical to the share in the data, a ratio of own trade to total exports of 1.73, which is also identical to the data moment, and an average productive capability expenditure of $\approx$ USD $145,000$, which matches the data moment as well.

4 Estimating the impact of climate change

4.1 Setup

For counterfactual simulations that take climate change into account, I require a realistic estimate of the impact of climate change on firms. Three key challenges arise in this context. First, weather is a very high-dimensional object, and I thus need to flexibly estimate how weather affects firm outcomes. Second, projecting the causal impact of climate change is inherently an out of sample exercise and I need to estimate firms' responses to climate change in a way that takes this seriously. Third, firms adapt to climate change and my estimation needs to allow for this, or otherwise take it into account.

To formalize the problem, I observe firm outcomes $\mathbf{y}$ and weather data from the current climate, $\mathbf{X}^{\text{now}} \sim F_{\text{now}}$; those depend on each other as

$$\mathbf{y} = g\left(\mathbf{X}^{\text{now}}\right) + \boldsymbol{\varepsilon}$$

Rather than trying to isolate the effect of a specific x_{ij} on $\mathbb{E}\left[y_i|\mathbf{X}_i\right]$, I am interested in the combined effect that moving to a new set of weather realizations $\mathbf{X}^{\text{future}} \sim F_{\text{future}}$ has on the expected $\mathbf{y}$, but I do not see outcomes for these new weather realizations. Doing inference on the difference in expected outcomes thus requires an estimate of the new outcomes.

One solution is to pick a set of weather variables an estimate a relationship using regression, perhaps including splines or other flexible function forms. However, the choice of weather variables is not obvious. Instead, I use the generalized random forests (GRFs) developed by Athey et al. (2019). GRFs are designed to incorporate high-dimensional data and can be tuned to protect against overfitting and improve out of sample performance. GRFs are generally used to estimate heterogeneous treatment effects. However, I thank Stefan Wager and Erik Sverdrup for pointing out to me that they can also be used to estimate and do inference on unobserved means. Let $D_i = 1$ for data with observed outcomes (the 'treatment' group) and $D_i = 0$ for data without (the 'control' group). Keep $y_i = y_i(1) = y_i$ for observed data and set $y_i = y_i(0) = 0$ for unobserved outcomes. Then, the conditional average treatment effect for the control group is

$$\mathbb{E}\left[y_i(1) - y_i(0)|D_i = 0\right] = \mathbb{E}\left[y_i - 0|D_i = 0\right] = \mathbb{E}\left[y_i|D_i = 0\right]$$

which is the expected outcome among data points with unobserved outcomes. GRFs provide efficient cluster-robust confidence intervals for this expected outcome and can easily be estimated using the `grf` package in `R` (Tibshirani et al., 2023). They provide a solution to the first two challenges: they are able to incorporate high-dimensional weather data and flexibly relating them to firm outcomes, and they perform well out of sample, which is highly desirable when estimating the impacts of climate change.

The third challenge, incorporating adaptation, can be addressed by including long-term moments of weather in the estimation. Adaptation means reacting differently to an identical weather shock depending on the climatic environment. For example, a firm that is used to an average yearly temperature of 28°C may be severely affected by a year that averages 30°C. If over time, the average temperature rises to 30°C, the same firm may adapt to the changed climatic environment, for example by installing climate control measures (Adhvaryu et al., 2019). It may then be less affected by a 30°C year.[12] To capture this, I include longer-term moments of contemporary weather variables in the estimation, akin to Carleton et al. (2022). Specifically, I include the mean and variance over the preceding 20 years.

4.2 Results

I use de-meaned log sales as my outcome of interest. By construction, this has an average of zero for data with observed outcomes. The estimated mean for each SSP can therefore be interpreted as the relative change in sales the average firm sees due to climate change. The weather measures I use include yearly averages, yearly averages of daily values raised to the second, third, fourth, fifth and sixth power (that is, the second to sixth non-centered moments of each variable), as used in

[12] It is also conceivable that rising average temperatures could make firms more vulnerable to weather shocks, for example if they negatively affect local labor markets (Santangelo, 2019). My solution here can take either effect into account.

Carleton et al. (2022), the corresponding centered moments, and counts for days in specific intervals and above certain thresholds. I also include long-term means and variances to capture adaptation, as described above. I partial out cluster fixed effects from all variables, including the outcome, before conducting the estimations. The GRF flexibly estimates the response of firm outcomes to all of these weather measures, including their interactions.

My main reference period for which I estimate the effects of climate change is 2086–2090. That is, for each SSP, I include projections from all 34 climate models and for each year in the 2086–2090 range and predict the causal effect on average sales. I choose this period because at that time, differences between the three SSPs are clearly visible in the data.[13] Table 11 shows the estimated change in sales under the three different SSPs as well as 90 percent confidence intervals. I consistently estimate negative effects, with larger magnitudes under more extreme climate change scenarios. The impacts range from a 6.6 percent decrease in sales for the average firm under SPP1/2.6 to an 8.3 percent decrease under SSP2/4.5 to a 10.8 percent decrease under SSP5/8.5. The latter two are significant at the ten percent level, while the change under SSP1/2.6 is not (though the confidence interval only barely includes zero). I want to highlight here that the confidence intervals I present incorporate two sources of uncertainty. First, they of course reflect uncertainty in fitting the model (statistical uncertainty). Second, since I combine data for many different model projections of future weather under each scenario, the intervals are also affected by scientific uncertainty regarding the path of future weather.

5 Counterfactuals

I now combine the model estimates from Section 3 with the estimated impacts of climate change from Section 4 to show that trade policy is especially suited to help poor countries mitigate the effects of climate change. I first calibrate a counterfactual that changes the technology parameter T_H to match the estimated impact of climate change under SPP5/8.5 from Section 4. Specifically, I calibrate T_H so the average firm experiences a real sales decline of 0.108 log points.[14] That is, I find the shift in the distribution of core productivities a_j would lead to the estimated impact on firms' sales. I call this the *climate change baseline* counterfactual. To be able to compare my model results to conclusions I would draw if I ignored productive capability, I also estimate the model of

[13] Appendix A.7 shows results for additional periods.

[14] To calculate real sales, I calculate prices, quantities and total sales in the estimated model as well as quantities sold in the counterfactual. I then evaluate counterfactual quantities at the actual prices to get real sales.

Melitz (2003). I estimate this in exactly the same way as my model, using the same moments, and calibrate the climate change baseline counterfactual in the same way.

I then calculate the change in welfare resulting from this shift in technology. As the first row Table 12 shows, I find that in my model, welfare would decline by 8.7 percent. Using Melitz (2003) instead, I would have concluded that welfare decreases by 12.5 percent. Note that these welfare changes can also be thought of as changes in real GDP, using the optimal consumer price index to convert nominal to real values.

I focus more on the relative welfare change stemming from different policy interventions compared to this baseline, rather than comparing the level changes. Whatever the exact path of future climate change is, we want to know what policies are most effective at counteracting its effects. To briefly highlight why the level results are different, it is important to note that both models predict that larger firms see a smaller relative decline in real sales. This is highlighted in Figures 10 and 11, which show the change in real sales distributions under the climate change counterfactual, for my model and the Melitz (2003) model, respectively. Both figures show the change in the CDF of log sales across log sales; positive values indicate that the CDF is shifting to the left, and entirely positive values mean the new distribution first order stochastically dominates the old one. In both models, larger firms see a smaller relative reduction in sales.[15] The reason is that smaller firms are the least competitive, and their relative competitiveness decreases even further when the productivity distribution shifts left. In my model, however, this is exacerbated: larger firms, especially exporters, can retain some of their productive capability, while smaller firms cannot afford to do so. Figure 12 shows the change in the distribution of productive capability c_j, highlighting that smaller firms lose productive capability compared to larger firms. This means larger firms stay relatively more competitive in my model.

I then conduct policy experiments under the climate change baseline. I reduce trade costs and assess the change in average sales compared to current levels. The first policy change is a reduction in iceberg trade cost from Home to all other countries, d_{nH}, by 10 percent across the board. (This is an asymmetric reduction — I keep trade costs from all other countries to Home, d_{Hn}, fixed.) The results are presented in row two of Table 12. Compared to current welfare, under my model, Zambia now experiences only a 5.9 percent decline, which means trade policy reduces the impact of climate change by 2.8 percentage points, or 32 percent ($\approx 2.8/8.7$). Ignoring productivity responses, Melitz

[15] The Melitz (2003) model has a small area with a negative shift, because in general equilibrium, wage changes allow some firms to access a market they were previously unable to profitably export to. This is purely a reallocation of sales within this group of firms and firms right below that cutoff, however.

(2003) would instead yield a 10.2 percent decline, a 2.3 percentage point or 18 percent ($\approx 2.3/12.5$) improvement.

Thus, I find that almost half of the welfare impact of trade policy, or 44 percent ($\approx (32-18)/32$), is due to productivity responses, making trade policy 1.7 times ($\approx 32/18$) more effective. This highlights that trade policy is especially important for small open economies, such as Zambia, where access to foreign markets allows firms to invest in operations they could not support based solely on domestic market demand.

The second policy change is a reduction in entry cost f_{nH} by 10 percent across the board. In my model, the effects are negligible. The Melitz (2003) model predicts some improvements, though these are much smaller than the predicted effects for variable trade cost reductions. The reason is that only relatively unproductive firms benefit from the entry cost reductions, whereas variable trade cost reductions benefit already exporting firms, which are more productive. In my model this is further exacerbated by that fact that variable trade cost reductions allow large firms to expand productive capability, making them even more productive.

6 Conclusion

I show that weather shocks are especially detrimental for exporting firms in poor countries, which implies that weather shocks are a net supply shock for these firms (rather than a demand shock). I further show that even exporters' domestic sales are more severely affected, which means firm operations across markets are linked. This is because of firms' expenditures on productive capability, such as machinery, office space, or a sales team. Productive capability lowers the cost of providing their products across all markets they serve. In response to a negative productivity shock, firms reduce productive capability, which in turn makes them less productive across all markets they are active in, reducing sales there as well. I support this by showing that exporters scale back productive capability more than non-exporters in response to negative weather shocks, and via a mediation analysis showing that once I control for productive capability spending, the differential effect on exporters' domestic sales disappears. I rule out competing explanations based on well-known differences between exporters and purely domestic producers, such as their size, sector or complexity of their production structure.

I then develop an international trade model featuring hired productive capability and show that it yields the same comparative statics. I estimate it using a small open economy framework, which allows me to discipline this otherwise computationally complex model. I combine the model with causal forest estimates of the impact of climate change, taking the large dimensionality of weather as well as firm adaptation into account. Policy simulations under this climate change scenario show that trade policy is considerably more effective at mitigating the impact of climate change when I take productive capability responses into account. I find that the welfare effect of a ten percent reduction in trade costs is 1.7 times larger due to these responses.

This is important as we consider what policies both rich and poor countries can adapt so poor countries can better mitigate the impact of climate change. The endogenous productivity response I highlight means trade policy is more effective than we might have assumed, is especially well-suited to counteracting the negative productivity effects of climate change, and should play a significant role as we work on coping with climate change. My results are especially important for poor countries, since the key mechanism is especially pronounced when the domestic market is small relative to foreign markets. My results also highlight that as rich countries consider onshoring and shortening supply chains (Goldberg & Reed, 2023), they should ensure those policies do not impose unnecessary collateral damage on poor countries. Ill-targeted, such policies could in fact greatly reduce poor countries' ability to deal with climate change.

Of course, none of this detracts from the fact that trade policy can also play a key role in preventing or reducing the severity of climate change (Farrokhi & Lashkaripour, 2021). More generally, there are good reasons to think that prevention is preferable to accepting climate change and resolving ourselves to dealing with it when it happens. Nevertheless, some amount of climate change is already occurring and will continue to occur over the near and medium term, and poor countries will be affected by it. Understanding what policies are especially suited for allowing them to deal with that reality is a crucial task for contemporary social science.

References

Adhvaryu, A., Kala, N., & Nyshadham, A. (2019). The light and the heat: Productivity co-benefits of energy-saving technology. *The Review of Economic Studies*, 1–36. https://doi.org/10.1162/rest_a_00886

Antràs, P., Fort, T. C., & Tintelnot, F. (2017). The margins of global sourcing: Theory and evidence from US firms. *American Economic Review*, *107*(9), 2514–2564. https://doi.org/10.1257/aer.20141685

Athey, S., Tibshirani, J., & Wager, S. (2019). Generalized random forests. *Annals of Statistics*, *47*(2), 1148–1178. https://doi.org/10.1214/18-AOS1709

Auffhammer, M., Hsiang, S. M., Schlenker, W., & Sobel, A. (2013). Using weather data and climate model output in economic analyses of climate change. *Review of Environmental Economics and Policy*, *7*(2), 181–198. https://doi.org/10.1093/reep/ret016

Bartelme, D., Lan, T., & Levchenko, A. A. (2023). *Specialization, market access and real income*. Working paper. https://alevchenko.com/Bartelme_Lan_Levchenko.pdf

Burke, M., & Emerick, K. (2016). Adaptation to climate change: Evidence from US agriculture. *American Economic Journal: Economic Policy*, *8*(3), 106–40. https://doi.org/10.1257/pol.20130025

Burke, M., Hsiang, S., & Miguel, E. (2015). Global non-linear effect of temperature on economic production. *Nature*, *527*, 235–239. https://doi.org/10.1038/nature15725

Burke, M., & Tanutama, V. (2019). *Climatic constraints on aggregate economic output*. NBER working paper 25779. https://www.nber.org/papers/w25779

Carleton, T. A., & Hsiang, S. M. (2016). Social and economic impacts of climate. *Science*, *353*(6304). https://doi.org/10.1126/science.aad9837

Carleton, T. A., Jina, A., Delgado, M., Greenstone, M., Houser, T., Hsiang, S., Hultgren, A., Kopp, R. E., McCusker, K. E., Nath, I., Rising, J., Rode, A., Seo, H. K., Viaene, A., Yuan, J., & Zhang, A. T. (2022). Valuing the Global Mortality Consequences of Climate Change Accounting for Adaptation Costs and Benefits. *The Quarterly Journal of Economics*, *137*(4), 2037–2105. https://doi.org/10.1093/qje/qjac020

Conte, B. (2022). *Climate change and migration: The case of Africa*. CESifo Working Paper No. 9948. https://www.cesifo.org/en/publications/2022/working-paper/climate-change-and-migration-case-africa

Costinot, A. (2009). On the origins of comparative advantage. *Journal of International Economics*, *77*(2), 255–264. https://doi.org/10.1016/j.jinteco.2009.01.007

Costinot, A., Donaldson, D., & Smith, C. (2016). Evolving comparative advantage and the impact of climate change in agricultural markets: Evidence from 1.7 million fields around the world. *Journal of Political Economy*, *124*(1), 205–248. https://doi.org/10.1086/684719

Dell, M., Jones, B. F., & Olken, B. A. (2012). Temperature shocks and economic growth: Evidence from the last half century. *American Economic Journal: Macroeconomics*, *4*(3), 66–95. https://doi.org/10.1257/mac.4.3.66

Demidova, S., Naito, T., & Rodríguez-Clare, A. (2022). *The small open economy in a generalized gravity model*. NBER Working Paper 30394. https://www.nber.org/papers/w30394

Deschênes, O., & Greenstone, M. (2007). The economic impacts of climate change: Evidence from agricultural output and random fluctuations in weather. *American Economic Review*, *97*(1), 354–385. https://doi.org/10.1257/aer.97.1.354

Deschênes, O., & Greenstone, M. (2011). Climate change, mortality, and adaptation: Evidence from annual fluctuations in weather in the US. *American Economic Journal: Applied Economics*, *3*(4), 152–185. https://doi.org/10.1257/app.3.4.152

Eaton, J., Kortum, S., & Kramarz, F. (2011). An anatomy of international trade: Evidence from french firms. *Econometrica*, *79*(5), 1453–1498. https://doi.org/https://doi.org/10.3982/ECTA8318

Farrokhi, F., & Lashkaripour, A. (2021). Can trade policy mitigate climate change? https://alashkar.pages.iu.edu/FL2021_Climate_Policy.pdf

Funk, C., Peterson, P., Landsfeld, M., Pedreros, D., Verdin, J., Shukla, S., Husak, G., Rowland, J., Harrison, L., Hoell, A., & Michaelsen, J. (2015). The climate hazards infrared precipitation with stations — A new environmental record for monitoring extremes. *Scientific Data*, *2*(150066). https://doi.org/10.1038/sdata.2015.66

Goldberg, P. K., & Reed, T. (2023). *Is the global economy deglobalizing? and if so, why? and what is next?* NBER Working Paper 31115. http://www.nber.org/papers/w31115

Hsieh, C.-T., & Olken, B. A. (2014). The missing "missing middle". *Journal of Economic Perspectives*, *28*(3), 89–108. https://doi.org/10.1257/jep.28.3.89

Hyun, J., & Kim, R. (2022). *Spillovers through multimarket firms: The uniform product replacement channel*. Working paper. https://www.dropbox.com/s/jly2dztl0nmu58w/draft.pdf

Jia, P. (2008). Whan happens when Wal-Mart comes to town: An empirical analysis of the discount retailing industry. *Econometrica*, *76*(6), 1263–1316. https://doi.org/10.3982/ECTA6649

Kelly, M. (2020). *Understanding persistence*. CEPR Press Discussion Paper No. 15246. https://cepr.org/publications/dp15246

Lin, C., Schmid, T., & Weisbach, M. S. (2019). *Climate change, operating flexibility and corporate investment decisions*. NBER Working Paper 26441. https://www.nber.org/papers/w26441

Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, *71*(6), 1695–1725. https://doi.org/10.1111/1468-0262.00467

Melitz, M. J., & Redding, S. J. (2015). New trade models, new welfare implications. *American Economic Review*, *105*(3), 1105–1146. https://doi.org/10.1257/aer.20130351

Nath, I. B. (2020). *The food problem and the aggregate productivity consequences of climate change*. NBER Working Paper 27297. https://www.nber.org/papers/w27297

O'Neill, B. C., Kriegler, E., Ebi, K. L., Kemp-Benedict, E., Riahi, K., Rothman, D. S., van Ruijven, B. J., van Vuuren, D. P., Birkmann, J., Kok, K., Levy, M., & Solecki, W. (2017). The roads ahead: Narratives for shared socioeconomic pathways describing world futures in the 21st century. *Global Environmental Change*, *42*, 169–180. https://doi.org/https://doi.org/10.1016/j.gloenvcha.2015.01.004

Ortiz-Bobea, A. (2021). Chapter 76 - the empirical analysis of climate change impacts and adaptation in agriculture. In C. B. Barrett & D. R. Just (Eds.), *Handbook of agricultural economics* (pp. 3981–4073). Elsevier. https://doi.org/10.1016/bs.hesagr.2021.10.002

Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., Blondel, M., Prettenhofer, P., Weiss, R., Dubourg, V., Vanderplas, J., Passos, A., Cournapeau, D., Brucher, M., Perrot, M., & Duchesnay, E. (2011). Scikit-learn: Machine learning in Python. *Journal of Machine Learning Research*, *12*, 2825–2830. https://jmlr.csail.mit.edu/papers/v12/pedregosa11a.html

Riahi, K., van Vuuren, D. P., Kriegler, E., Edmonds, J., O'Neill, B. C., Fujimori, S., Bauer, N., Calvin, K., Dellink, R., Fricko, O., Lutz, W., Popp, A., Cuaresma, J. C., KC, S., Leimbach, M., Jiang, L., Kram, T., Rao, S., Emmerling, J., … Tavoni, M. (2017). The shared socioeconomic pathways and their energy, land use, and greenhouse gas emissions implications: An overview. *Global Environmental Change*, *42*, 153–168. https://doi.org/https://doi.org/10.1016/j.gloenvcha.2016.05.009

Rohde, R., Muller, R. A., Jacobsen, R., Muller, E., Perlmutter, S., Rosenfeld, A., Wurtele, J., Groom, D., & Wickham, C. (2013). A new estimate of the average earth surface land temperature spanning 1753 to 2011. *Geoinformatics & Geostatistics: An Overview*, *1*(1). https://doi.org/10.4172/2327-4581.1000101

Santangelo, G. (2019). *Firms and farms: The local effects of farm income on firms' demand*. Working paper. https://gabriellasantangelo.files.wordpress.com/2019/03/gabriella_santangelo_-_jmp_-_latest.pdf

Santos Silva, J. M. C., & Tenreyro, S. (2006). The log of gravity. *The Review of Economics and Statistics*, *88*(4), 641–658. https://doi.org/10.1162/rest.88.4.641

Somanathan, E., Somanathan, R., Sudarshan, A., & Tewari, M. (2021). The impact of temperature on productivity and labor supply: Evidence from indian manufacturing. *Journal of Political Economy*, *129*(6), 1797–1827. https://doi.org/10.1086/713733

Thrasher, B., Wang, W., Michaelis, A., Melton, F., Lee, T., & Nemani, R. (2022). NASA global daily downscaled projections, CMIP6. *Scientific Data*, *9*(262 (2022)). https://doi.org/10.1038/s41597-022-01393-4

Thrasher, B., Wang, W., Michaelis, A., & Nemani, R. (2021). *NEX-GDDP-CMIP6*. NASA Center for Climate Simulation. https://doi.org/10.7917/OFSG3345

Tibshirani, J., Athey, S., Friedberg, R., Hadad, V., Hirshberg, D., Miner, L., Sverdrup, E., Wager, S., & Wright, M. (2023). *grf* (Version 2.2.1). https://grf-labs.github.io/grf/index.html

World Bank. (2023). *World development indicators*. The World Bank Group. https://datatopics.worldbank.org/world-development-indicators/

Zhang, P., Dêschenes, O., Meng, K., & Zhang, J. (2018). Temperature effects on productivity and factor reallocation: Evidence from half a million Chinese manufacturing plants. *Journal of Environmental Economics and Management*, *88*, 1–17. https://doi.org/10.1016/j.jeem.2017.11.001

Tables

Table 1: Firm summary statistics

Variable	Count	Mean	P25	Median	P75
Sales (real 2009 USD)	37,100	44,451,183.54	27,648.78	104,665.36	500,123.04
Cost (real 2009 USD)	38,358	2,309,691.81	8,393.78	30,651.68	134,594.01
Labor cost (real 2009 USD)	36,664	221,888.62	4,747.38	14,736.47	56,391.69
Number of employees	46,347	40.21	6.00	10.00	23.00
Initial number of employees	39,455	24.36	4.00	6.00	12.00
Exporter	45,884	0.12	0.00	0.00	0.00
Manufacturing	46,694	0.32	0.00	0.00	1.00
Internat. quality cert.	45,302	0.13	0.00	0.00	0.00
Manager experience (years)	45,949	14.17	7.00	12.00	20.00
Yearly mean temperature (°C)	26,309	29.68	26.72	30.56	33.15
Yearly total precipitation (1,000 mm)	26,309	1.03	0.56	0.92	1.28

Note: The lower observation counts for weather variables stem from the fact that I can only match firm and weather data for firms that have non-missing location information.

Table 2: ATE of weather shocks

Variable	Log sales
Temperature index	-0.223^{*}
	[0.091]
Year FE	Yes
Cluster FE	Yes
Clusters	587
Observations	18,273

Note: *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80^{th} percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95^{th} percentile for each survey round. p-values in brackets.

Table 3: Effect of weather shocks by exporter status

Variable	Log sales
Temperature index	-0.125
	[0.296]
Temperature index × Current exporter	-0.094^{***}
	[0.006]
Current exporter	1.603^{***}
	[0.000]
Year FE	Yes
Cluster FE	Yes
Clusters	376
Observations	17,024

Note: *Current exporters* are firms that export in the current fiscal year. *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80^{th} percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95^{th} percentile for each survey round. p-values in brackets.

Table 4: Effect of weather shocks on domestic sales

Variable	Log domestic sales
Temperature index	-0.186
	[0.118]
Temperature index × Current exporter	-0.071^{**}
	[0.013]
Current exporter	0.886^{***}
	[0.000]
Year FE	Yes
Cluster FE	Yes
Clusters	362
Observations	16,196

Note: *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80^{th} percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95^{th} percentile for each survey round. p-values in brackets.

Table 5: Effect of weather shocks on productive capability and productivity

Variable	Log productive capability	Log sales/employee
Temperature index	-0.049	-0.036
	[0.753]	[0.722]
Temperature index $\times$ Current exporter	-0.117^{***}	-0.048^{**}
	[0.001]	[0.019]
Current exporter	1.233^{***}	0.631^{***}
	[0.000]	[0.000]
Year FE	Yes	Yes
Cluster FE	Yes	Yes
Clusters	201	375
Observations	7,510	16,920

Note: *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80^{th} percentile weather shock. Productive capability expenditures combine cost of communications, sales, transportation, and rent for buildings, equipment and land. Standard errors clustered by firm cluster. Outcomes winsorized at the 95^{th} percentile for each survey round. p-values in brackets.

Table 6: Indications of productivity impact

Variable	Log sales/employee	Log weekly hours	Outage
Temperature index	-0.074	0.142^{*}	0.065^{*}
	[0.491]	[0.088]	[0.075]
Year FE	Yes	Yes	Yes
Cluster FE	Yes	Yes	Yes
Clusters	587	522	595
Observations	18,133	11,837	22,327

Note: *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80^{th} percentile weather shock. Each column shows results for a different outcome. *Log weekly hours* is the log of the firm's total operating hours per week. *Log female employment* is the log of the firm's number of female employees. *Outage* is an indicator for whether the firm experienced power outages. Outcomes winsorized at the 95^{th} percentile, except indicators. p-values in brackets.

Table 7: Effect of weather shocks on domestic sales, mediation via productive capability

Variable	Log domestic sales
Temperature index	−0.182 [0.139]
Temperature index × Current exporter	0.041 [0.438]
Current exporter	0.179 [0.112]
Log productive capability controls	Yes
Year FE	Yes
Cluster FE	Yes
Clusters	375
Observations	7,447

Note: *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80$^{\text{th}}$ percentile weather shock. *Log productive capability controls* comprises log productive capability expenditures fully interacted with exporter status and the temperature index. I do not show the coefficients on these endogenous regressors. *(DM)* indicates the variable is de-meaned to center interaction terms. Productive capability expenditures combine cost of communications, sales, transportation, and rent for buildings, equipment and land. Standard errors clustered by firm cluster. Outcomes winsorized at the 95$^{\text{th}}$ percentile for each survey round. p-values in brackets.

Table 8: Exporters compared to non-exporters

Variable	Mean non-exporter	Mean exporter	p-value
Sales (real 2009 USD)	870,913.66	2,646,253.68	0.000***
Cost (real 2009 USD)	267,630.22	1,093,091.57	0.000***
Labor cost (real 2009 USD)	74,375.21	232,004.51	0.000***
Number of employees	23.14	66.31	0.000***
Initial number of employees	11.68	23.97	0.000***
Manufacturing	0.29	0.49	0.000***
Internat. quality cert.	0.10	0.28	0.000***
Manager experience (years)	13.65	15.45	0.006***
Yearly mean temperature (°C)	29.61	29.69	0.593
Yearly total precipitation (1,000 mm)	0.98	0.98	0.561

Table 9: Parameter estimates for structural model

Parameter	Source/identifying variation	Estimate
Panel A: Reduced form and data		
σ	Sales, variable cost	3.016 (0.043)
$\beta\delta$	Sales regression (10)	0.384 (0.016)
Panel B: Structural estimation		
θ	Average productive capability	9.047
$T_H w_H$	Ratio of Foreign to Home sales	0.000
$f_H w_H$	Fraction of exporters	0.031
γ_0		-2.251
γ_{dist}	Export flows	0.893
γ_{contig}		-1.622

Note: Standard errors in parentheses where available. I present the minimum productivity T_H and start-up cost f_H times the estimated Home wage w_H to convert them into an easier to interpret unit, millions of USD, rather than presenting them in units of labor. The three components of $\boldsymbol{\gamma}$ are the intercept γ_0, the coefficient on log distance γ_{dist} and the coefficient on the contiguity indicator γ_{contig}.

Table 10: Moment comparisons for structural model

Moment	Data	Model
Fraction exporting	0.152	0.152
Ratio own trade/total exports	1.727	1.727
Mean fixed cost	0.145	0.145

Note: Mean productive capability is in millions of USD. In the data, productive capability expenditures combine cost of communications, sales, transportation, and rent for buildings, equipment and land. In the model, productive capability expenditures are equal to the value of firms' productive capability. The other set of targeted moments, log exports and the fraction of firms with zero exports, is shown in Figure 7.

Table 11: Causal forest estimates for 2086–2090

Scenario	Change in log sales
SSP1/2.6	-0.066 $(-0.131, 0.000)$
SSP2/4.5	-0.083 $(-0.160, -0.006)$
SSP5/8.5	-0.108 $(-0.188, -0.028)$

Note: Standard errors clustered by firm cluster. 90 percent confidence intervals in parentheses.

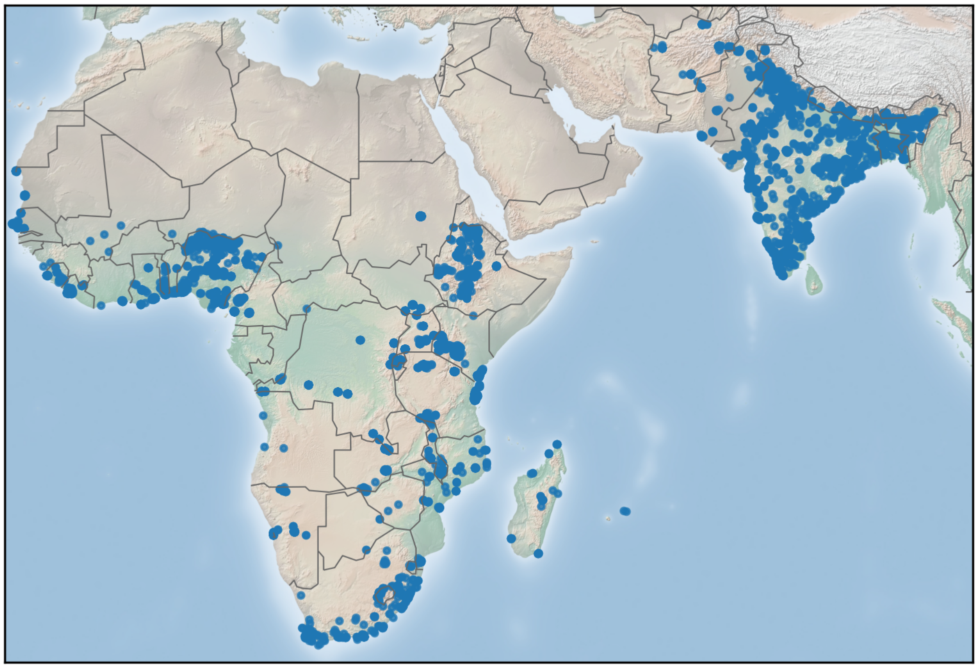
Table 12: Counterfactual change in welfare

Scenario	Full model	Melitz (2003)
Climate change baseline	-0.087	-0.125
Iceberg cost reduction	-0.059	-0.102
Entry cost reduction	-0.087	-0.119

Note: Each row presents the relative change in welfare under a different counterfactual scenario compared to the baseline estimates for my model. These welfare changes can also be thought of as changes in real GDP, using the optimal consumer price index to convert nominal to real values. *Climate change baseline* uses the technology parameter T_H to match the estimated impact of climate change on the Zambian economy. *Iceberg cost reduction* reduces variable trade costs from Zambia to all other markets by ten percent, while *entry cost reduction* reduces entry cost for Zambian firms to all other markets by ten percent. *Full model* shows estimated welfare changes from my model, *Melitz (2003)* shows estimates based on the model by Melitz (2003).

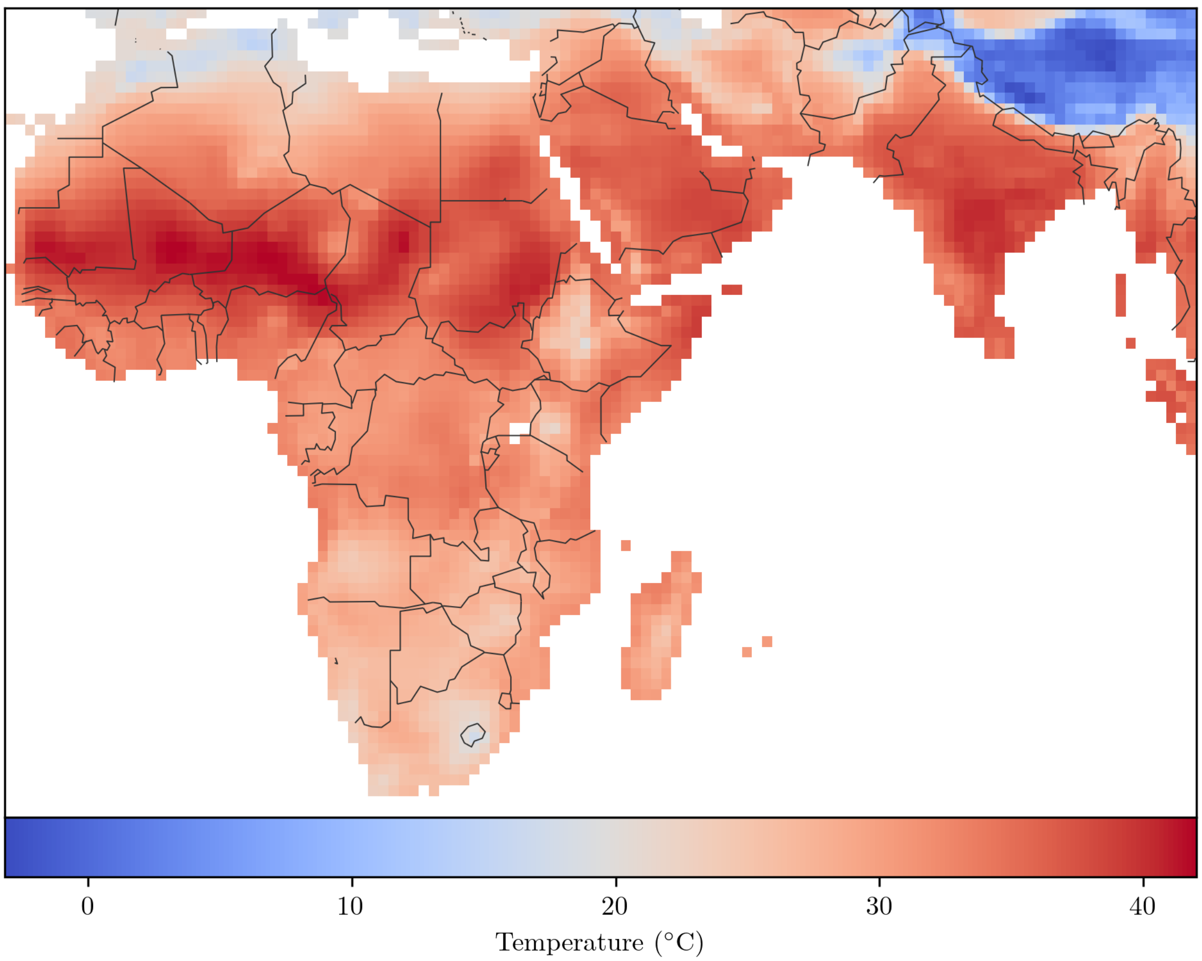
Figures

Figure 1: Locations of firms and firm clusters across Africa and South Asia



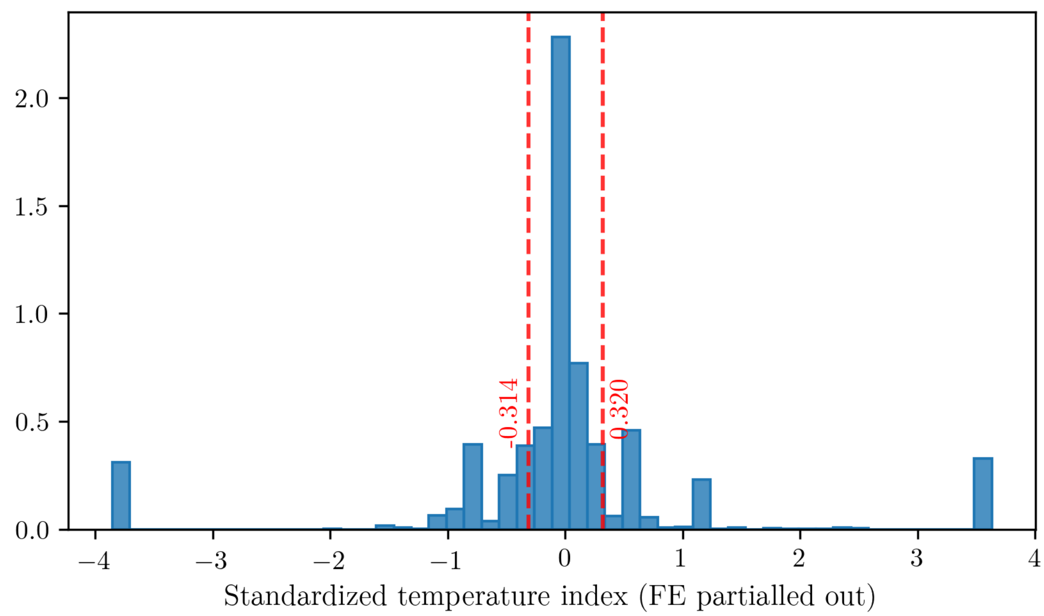
Note: Each dot is either the location of a single firm or the location of a firm cluster. Clusters appear if several firms were recorded as having the same location in the Enterprise Surveys data or if I was able to determine the firms' location via geolocation methods based on the city firms are located in.

Figure 2: Maximum temperature on April 24, 1991



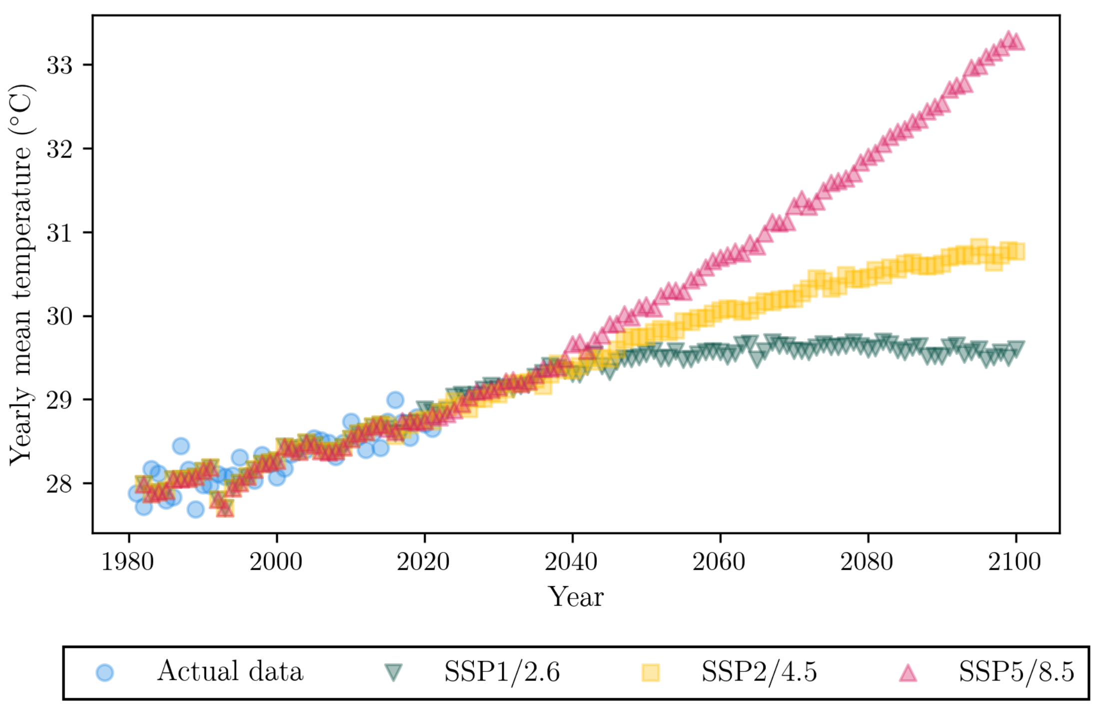
Note: The figure shows temperature from the Berkeley Earth dataset.

Figure 3: Histogram of temperature index after partialling out cluster FE



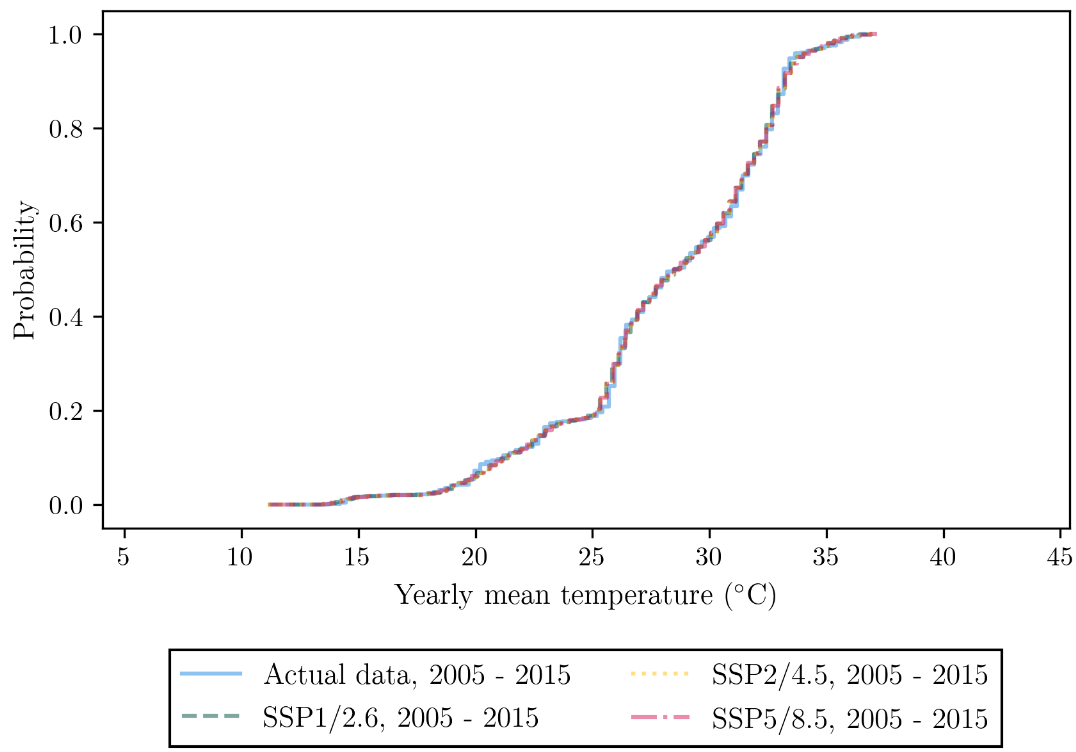
Note: The figure shows the temperature index after partialling out cluster fixed effects. This is the same identifying variation used in the regressions I estimate — the variation remaining in the standardized temperature index after cluster fixed effects are taken into account. Dashed lines indicate the 20th and 80th percentile. Observations without variation after partialling out FE not shown.

Figure 4: Yearly average daily maximum temperature across climate change scenarios



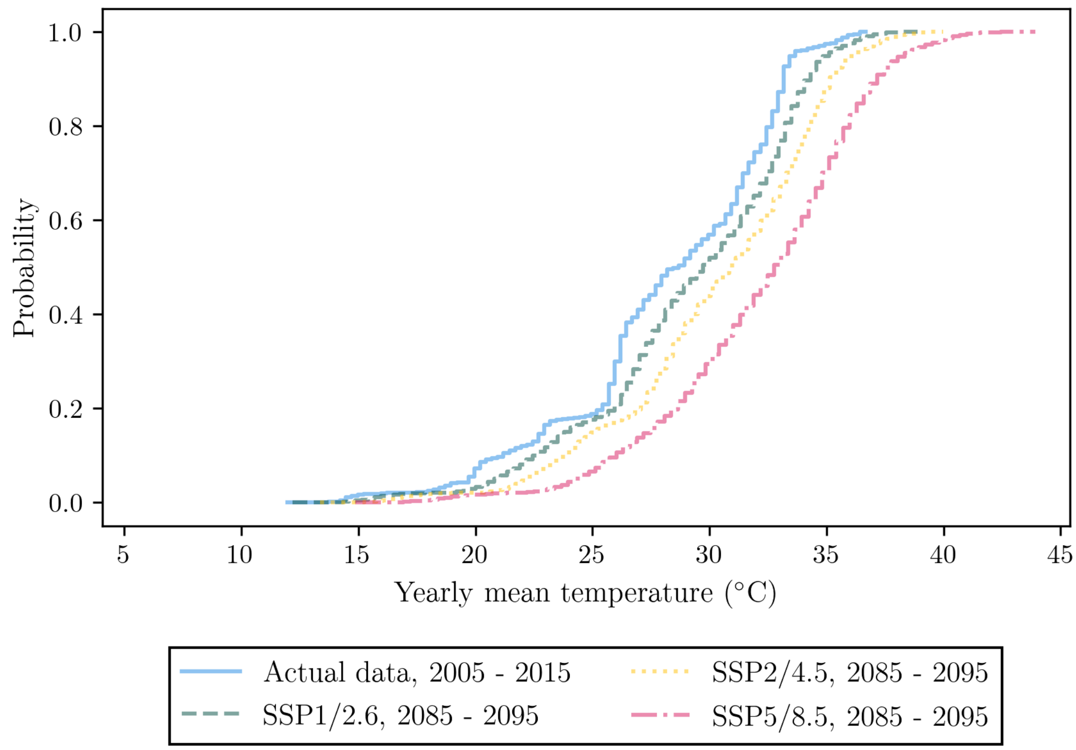
Note: The figure shows yearly averages of daily maximum temperatures. The climate change projections are adjusted for differences in baseline temperature means for each day of the year, as described in Appendix C.

Figure 5: Distribution of yearly average daily maximum temperature, 2005–2015, for actual data and climate change projections



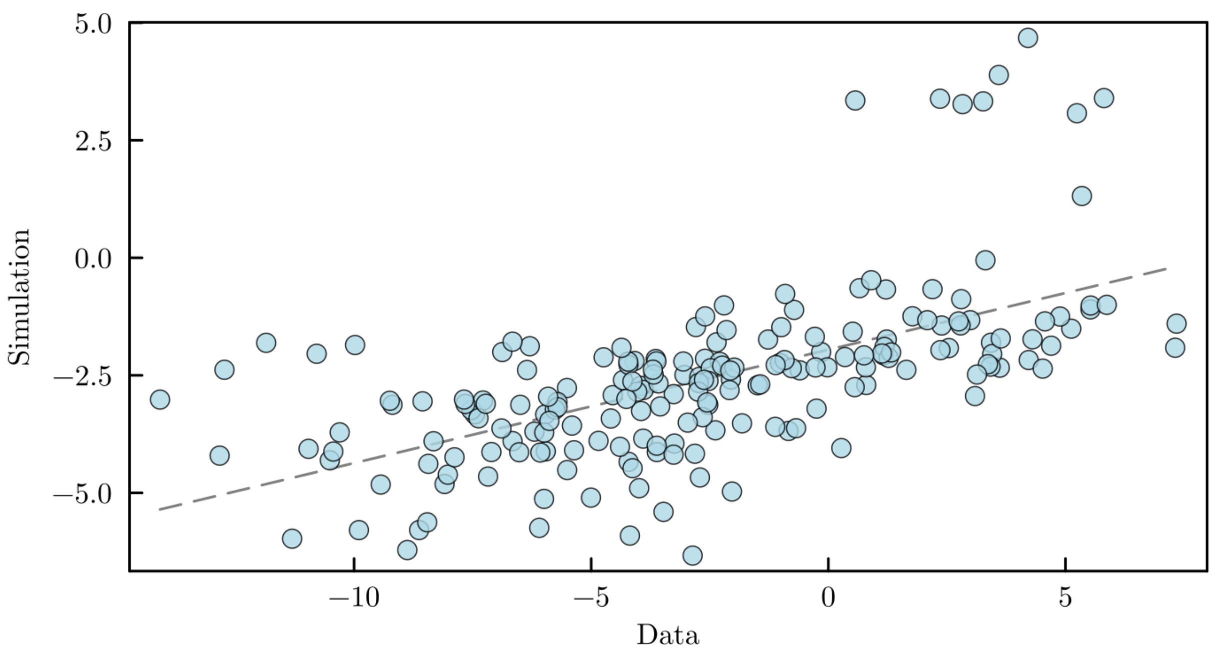
Note: The figure shows the empirical CDF of the yearly average of daily maximum temperatures across firms. The climate change projections are adjusted for differences in baseline temperature means for each day of the year, as described in Appendix C.

Figure 6: Distribution of yearly average daily maximum temperature, 2005–2015 for actual data and 2085–2095 for climate change projections



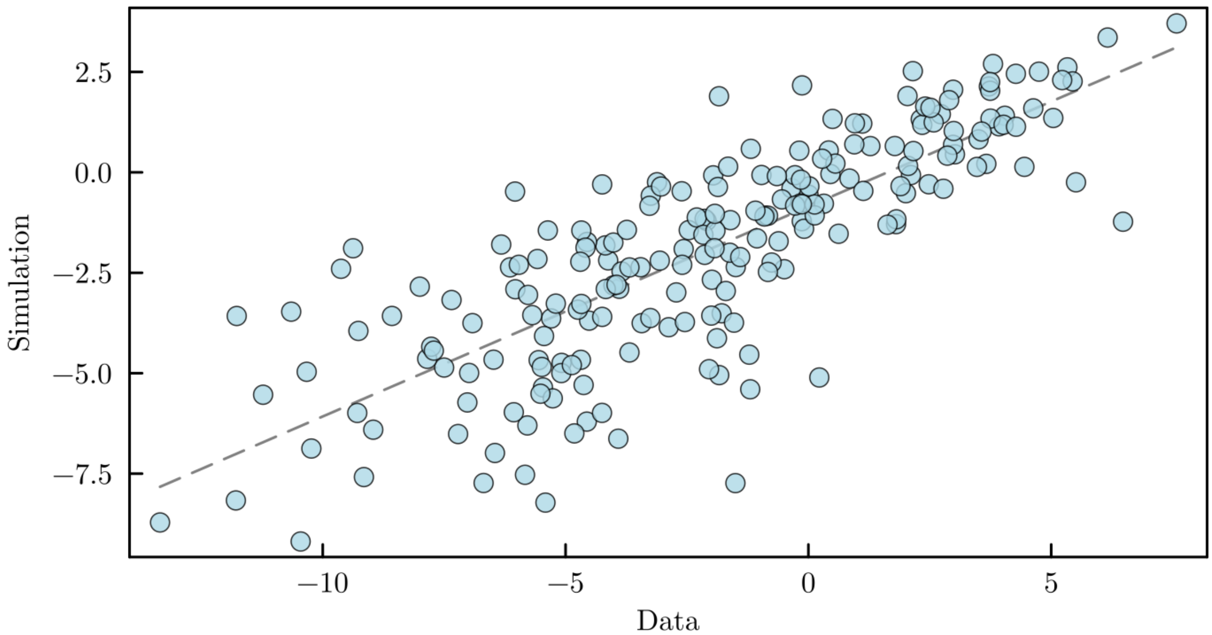
Note: The figure shows the empirical CDF of the yearly average of daily maximum temperatures across firms. The climate change projections are adjusted for differences in baseline temperature means for each day of the year, as described in Appendix C.

Figure 7: Zambian log exports vs. model simulation



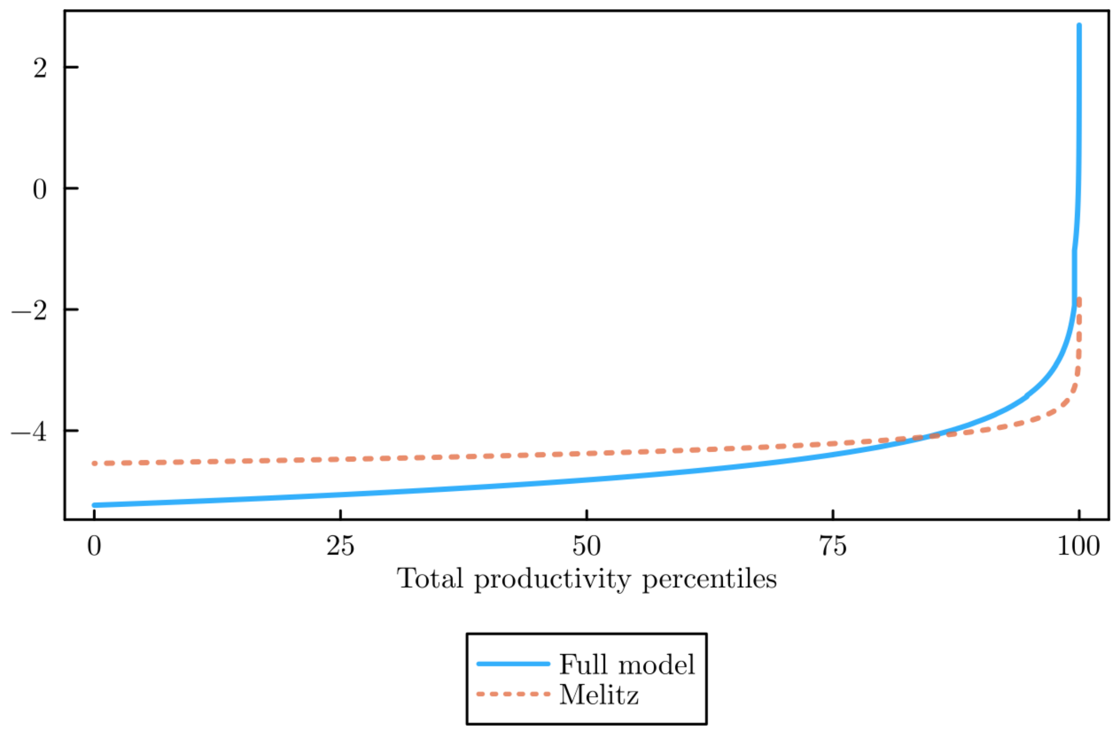
Note: Log exports are a targeted moment, together with the fraction of countries with zero exports.

Figure 8: Zambian log imports vs. model simulation



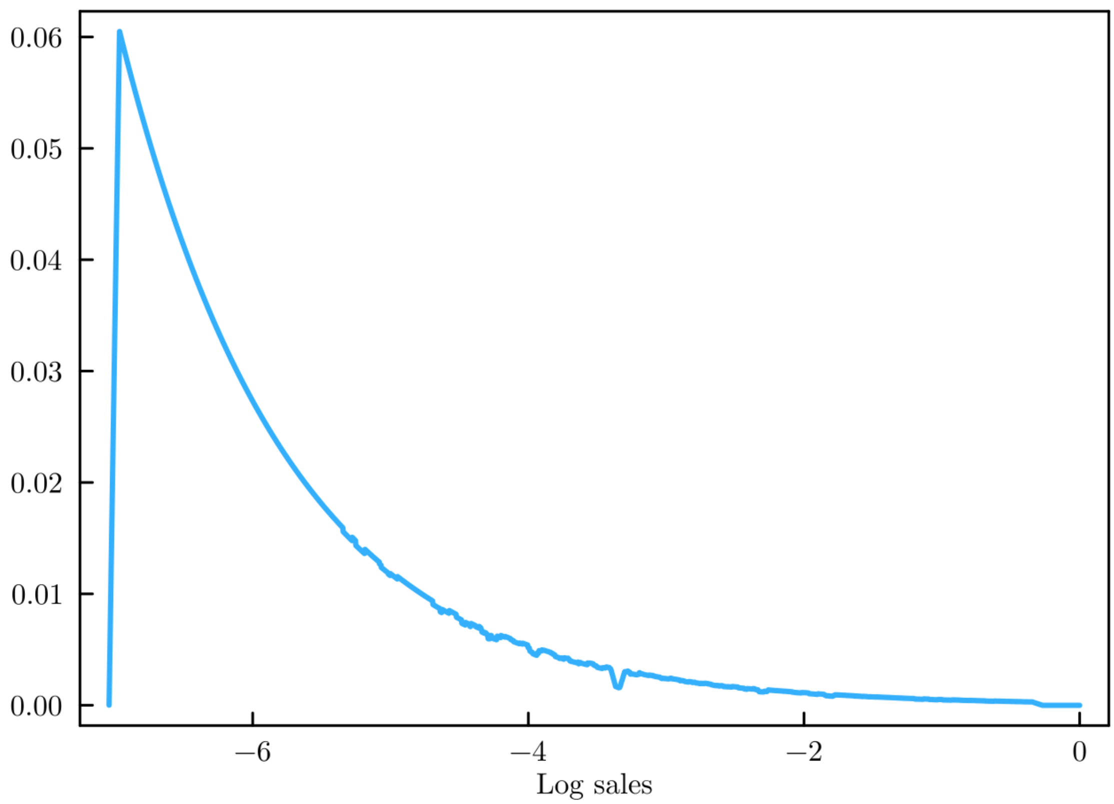
Note: Log imports are an untargeted moment.

Figure 9: Log total productivity distribution compared to Melitz (2003)



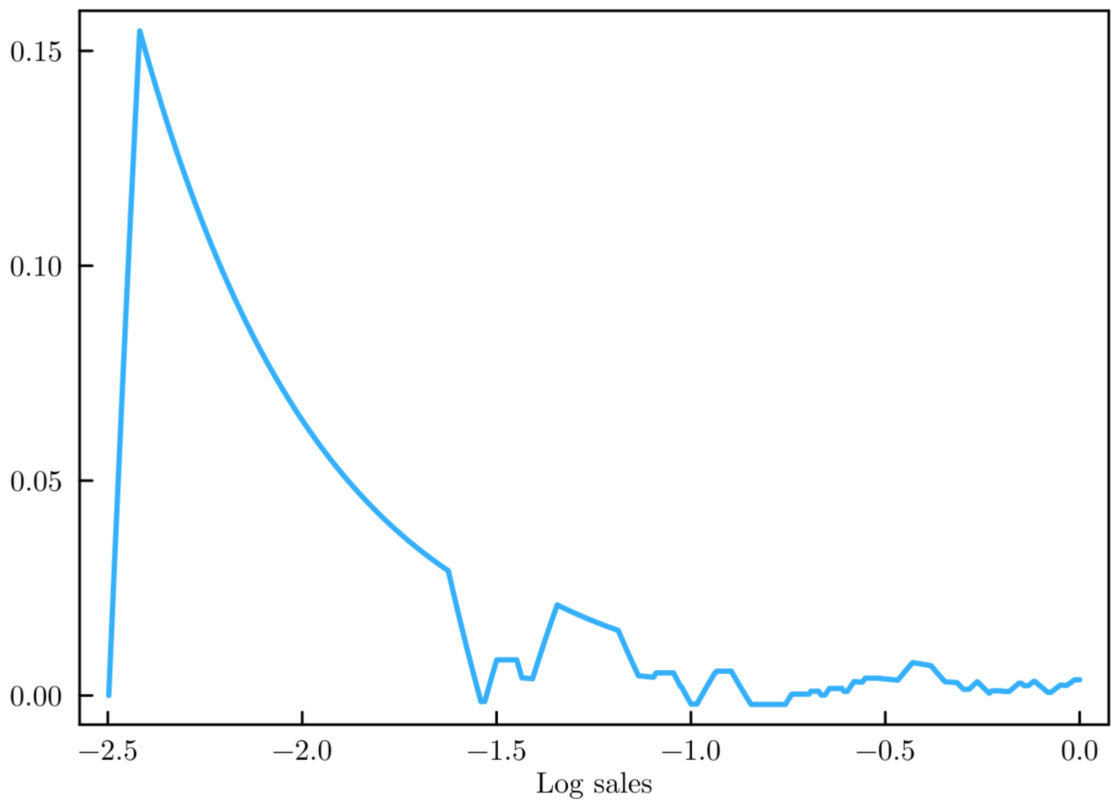
Note: The figure shows the first through 99$^{\text{th}}$ percentiles of log total productivity for my model and the model of Melitz (2003) estimated on the same data. For my model, total productivity is the product of core productivity a_j and productive capability c_j^{δ}, whereas for Melitz (2003) it is simply a_j.

Figure 10: Change in real sales distribution under climate change baseline counterfactual, full model



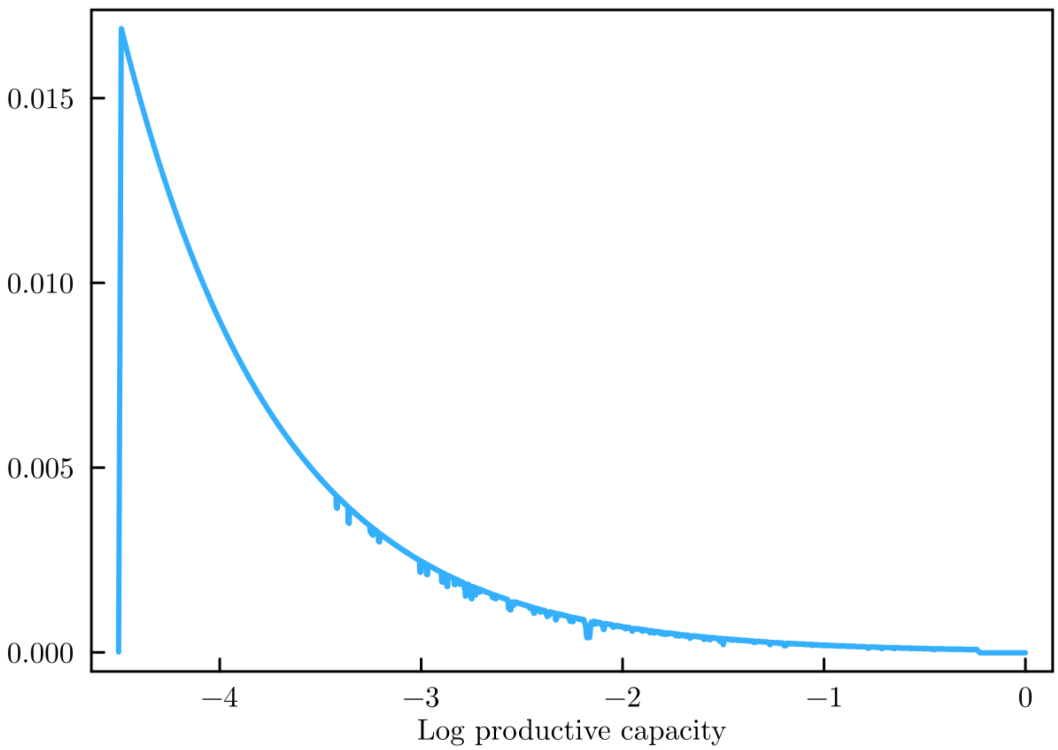
Note: The figure shows the change in the CDF of log sales across log sales. Positive values indicate that the CDF is shifting to the left, and entirely positive values mean the new distribution first order stochastically dominates the old one.

Figure 11: Change in real sales distribution under climate change baseline counterfactual, Melitz (2003) model



Note: The figure shows the change in the CDF of log sales across log sales. Positive values indicate that the CDF is shifting to the left, and entirely positive values mean the new distribution first order stochastically dominates the old one.

Figure 12: Change in productive capability under climate change baseline counterfactual



Note: The figure shows the change in the CDF of log productive capability c_j across log productive capability. Positive values indicate that the CDF is shifting to the left, and entirely positive values mean the new distribution first order stochastically dominates the old one.

Appendix A Additional tables

A.1 Clustering distance choice

Table 13: Moran test for spatial correlation

Clustering distance	p-value	Adjusted p-value	Fraction included
0.25 km	0.116	1.000	0.110
0.5 km	0.686	1.000	0.215
1.0 km	0.535	1.000	0.325
2.5 km	0.147	1.000	0.399
5.0 km	0.582	1.000	0.406
10.0 km	0.130	1.000	0.414
15.0 km	0.161	1.000	0.416
20.0 km	0.422	1.000	0.417
25.0 km	0.440	1.000	0.423
50.0 km	0.678	1.000	0.430
100.0 km	0.605	1.000	0.432
200.0 km	0.757	1.000	0.440
500.0 km	0.500	1.000	0.509

Note: The Moran test is for the null that residuals from a regression of log sales on a temperature index are not correlated across clusters. The index combines mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). Standard errors clustered by firm cluster. *Adjusted p-values* are adjusted for multiple hypothesis testing using the Holm-Bonferroni correction.

A.2 Alternative specifications for exporter effect

Table 14: Effect of weather shocks by exporter status, no year FE

Variable	Log sales
Temperature index	−0.004 [0.938]
Temperature index × Current exporter	−0.094*** [0.006]
Current exporter	1.630*** [0.000]
Cluster FE	Yes
Clusters	586
Observations	17,975

Note: *Current exporters* are firms that export in the current fiscal year. *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80th percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95th percentile for each survey round. p-values in brackets.

Table 15: Effect of weather shocks by exporter status, most reliable numbers only

Variable	Log sales
Temperature index	−0.120 [0.308]
Temperature index × Current exporter	−0.069 [0.216]
Current exporter	1.473*** [0.000]
Year FE	Yes
Cluster FE	Yes
Clusters	244
Observations	5,746

Note: This estimation uses only data that came directly from firm records, as opposed to being estimates, for example. *Current exporters* are firms that export in the current fiscal year. *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80th percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95th percentile for each survey round. p-values in brackets.

A.3 Alternative indicators for exporter status

Table 16: Exporter effect using past exporter status

Variable	Log sales
Temperature index	0.119 [0.358]
Temperature index × Past exporter	-0.074^{**} [0.036]
Past exporter	1.208^{***} [0.000]
Year FE	Yes
Cluster FE	Yes
Clusters	587
Observations	18,273

Note: *Past exporter* is an indicator for firms reporting a past year as their first year of exporting. *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80$^{\text{th}}$ percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95$^{\text{th}}$ percentile for each survey round. p-values in brackets.

Table 17: Exporter effect using ever exporter status

Variable	Log sales
Temperature index	0.141 [0.269]
Temperature index × Ever exporter	-0.079^{**} [0.022]
Ever exporter	1.384^{***} [0.000]
Year FE	Yes
Cluster FE	Yes
Clusters	586
Observations	17,975

Note: *Ever exporter* is an indicator for firms which exported in the past and/or report international sales this year. *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80$^{\text{th}}$ percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95$^{\text{th}}$ percentile for each survey round. p-values in brackets.

Table 18: Effect on continuing, discontinuing, and first-time exporters

Variable	Log sales
Temperature index	-0.143 [0.178]
Temperature index × Continuing exporter	-0.097^{***} [0.005]
Temperature index × Discontinuing exporter	-1.451^{***} [0.002]
Temperature index × First-time exporter	0.092 [0.573]
Year FE	Yes
Exporter status FE	Yes
Cluster FE	Yes
Clusters	586
Observations	17,975

Note: *Continuing exporters* are firms that exported in the past and do so in the observed year. *Discontinuing past exporters* are firms that exported in the past and are not doing so in the observed year. *First-time exporters* did not export in the past, but are doing so now. *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific *z*-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80$^{\text{th}}$ percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95$^{\text{th}}$ percentile for each survey round. *p*-values in brackets.

A.4 Additional regressions

Table 19: Effect of weather shocks on cost of repurchasing machinery

Variable	Log value of re-purchasing machinery
Temperature index	0.121
	[0.656]
Temperature index × Current exporter	0.035
	[0.465]
Current exporter	1.635***
	[0.000]
Year FE	Yes
Cluster FE	Yes
Clusters	215
Observations	6,312

Note: *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80th percentile weather shock. Outcomes winsorized at the 95th percentile for each survey round. p-values in brackets.

Table 20: Checks for alternative explanations

Variable	Log sales	Log sales	Log sales	Log sales	Log sales	Log sales	Log sales	Log sales
Temperature index	-0.269^{**} [0.023]	-0.049 [0.671]	-0.113 [0.240]	-0.077 [0.378]	-0.105 [0.303]	-0.050 [0.611]	-0.101 [0.357]	0.064 [0.312]
Temperature index × Current exporter	-0.088^{***} [0.006]	-0.063^{**} [0.025]	-0.089^{**} [0.015]	-0.101^{***} [0.000]	-0.079^{**} [0.014]	-0.099^{***} [0.000]	-0.098^{***} [0.003]	-0.049^{***} [0.002]
Temperature index × Initial no. of employees > median	-0.017 [0.516]							-0.014 [0.444]
Initial no. of employees > median	1.208^{***} [0.000]							0.434^{***} [0.000]
Temperature index × No. of employees 3 yrs. ago > median		-0.036 [0.146]						-0.042^{*} [0.094]
No. of employees 3 yrs. ago > median		1.772^{***} [0.000]						1.321^{***} [0.000]
Temperature index × International certification					-0.026 [0.646]			0.008 [0.789]
International certification					1.251^{***} [0.000]			0.597^{***} [0.000]
Temperature index × Manager's years of experience > median							-0.060^{***} [0.000]	-0.014 [0.394]
Manager's years of experience > median							0.563^{***} [0.000]	0.241^{***} [0.000]
ISIC2 FE			Yes					
ISIC2 FE × weather variables			Yes					
ISIC4 FE				Yes				Yes
ISIC4 FE × weather variables				Yes				Yes
Legal status FE						Yes		Yes
Legal status FE × weather variables						Yes		Yes
Current exporter	1.277^{***} [0.000]	1.050^{***} [0.000]	1.494^{***} [0.000]	1.419^{***} [0.000]	1.365^{***} [0.000]	1.286^{***} [0.000]	1.568^{***} [0.000]	0.733^{***} [0.000]
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Cluster FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Clusters	584	582	584	583	586	586	586	573
Observations	16,169	16,554	16,446	16,221	17,537	17,903	17,774	13,417

Note: *ISIC2 FE* and *ISIC4 FE* are indicators for two-digit and four-digit ISIC sectors, respectively. *International certification* is an indicator for firms reporting that they have an international quality certification. *Legal status FE* are indicators for the legal status of the firm. The omitted status are sole proprietorships. *Temperature index* is an index combining mean temperature, temperature variance and the number of days with temperatures exceeding 32°C (89.6°F). The index is an average of location-specific z-scores for each variable (using the 20-year mean and standard deviation of reach variable). An 0.318 increase in the index is an 80$^{\text{th}}$ percentile weather shock. Standard errors clustered by firm cluster. Outcomes winsorized at the 95$^{\text{th}}$ percentile for each survey round. p-values in brackets.

A.5 Reduced form parameter estimation for structural model

Table 21: Sales regression

Variable	Trade flows
Log sales	0.816^{***} [0.000]
Manager's years of experience	0.037^{***} [0.000]
Log initial no. of employees	0.333^{***} [0.000]
Log no. of employees 3 yrs. ago	-0.668^{***} [0.000]
International certification	0.124^{***} [0.000]
Had power outage	0.060^{***} [0.000]
Competes against informal sector	-0.103^{***} [0.000]
Introduced new product	0.238^{***} [0.000]
Introduced new process	0.174^{***} [0.000]
Had RD expenditure	-0.131^{***} [0.000]
Part of larger firm	0.133^{***} [0.000]
Formal when founded	0.183^{***} [0.000]
Applied for grid connection	-0.085^{***} [0.000]
Applied for water connection	-0.103^{***} [0.000]
Firm age	0.021^{***} [0.000]
4^{th} degree polynomials	Yes
Pairwise interactions	Yes
Country-sector (ISIC4)-year FE	Yes
Observations	4,160

Note: *Fourth degree polynomials* are included for all continuous variables besides log sales. *Pairwise interactions* include only level variables, not variables raised to a power as part of the polynomials. Standard errors are robust. p-values in brackets.

Table 22: Gravity estimation

Variable	Trade flows
Log distance	-1.171
	[0.000]
Contiguous	0.649
	[0.000]
Importer FE	Yes
Exporter FE	Yes

Note: Standard errors are robust. p-values in brackets.

A.6 Moment comparisons for Melitz (2003) estimation

Table 23: Moment comparisons for structural estimation of Melitz (2003)

Moment	Data	Model
Fraction exporting	0.152	0.152
Ratio own trade/total exports	1.819	1.819

Note: The other set of targeted moments, log exports and the fraction of firms with zero exports, is shown in Figure 13.

A.7 Additional causal forest results

Table 24: Causal forest estimates for 2091–2095

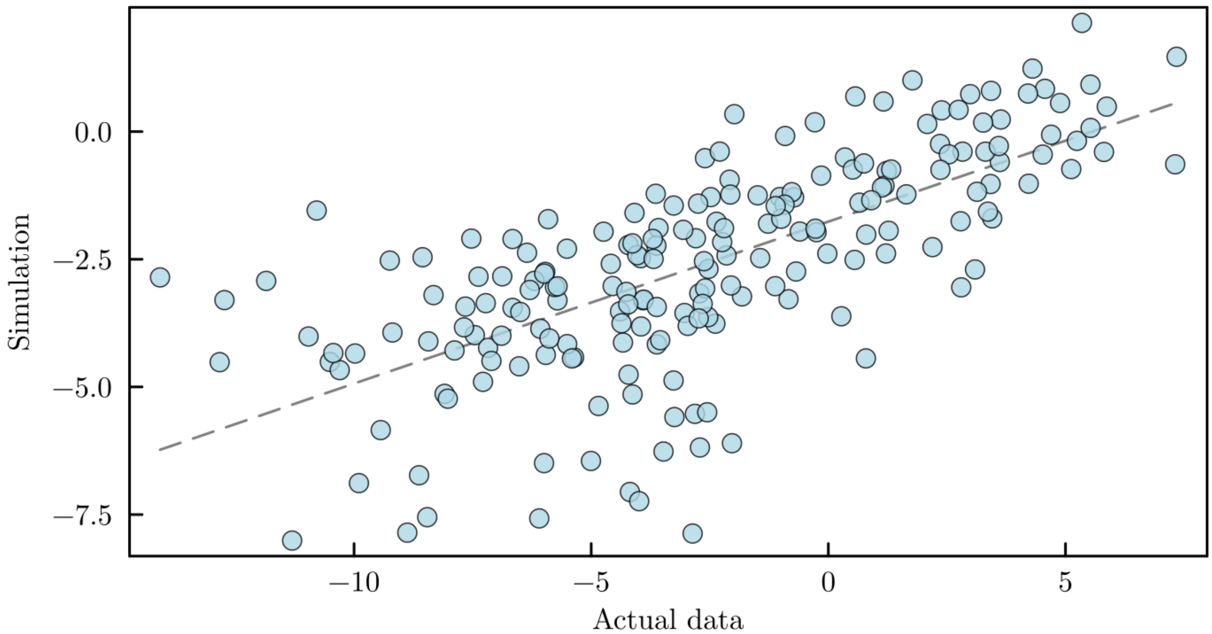
Scenario	Change in log sales
SSP1/2.6	-0.048 $(-0.121, 0.026)$
SSP2/4.5	-0.059 $(-0.133, 0.015)$
SSP5/8.5	-0.101 $(-0.176, -0.026)$

Note: Standard errors clustered by firm cluster. 90 percent confidence intervals in parentheses.

Appendix B Additional figures

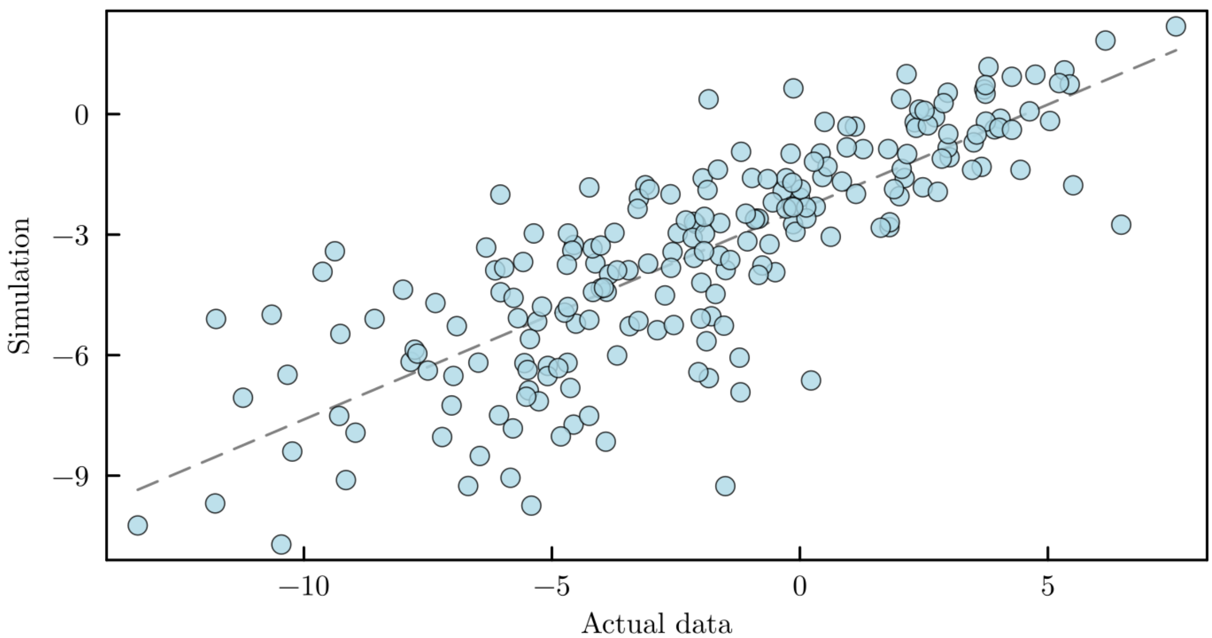
B.1 Moment comparisons for Melitz (2003) estimation

Figure 13: Zambian log exports vs. model simulation from Melitz (2003) estimation



Note: Log exports are a targeted moment, together with the fraction of countries with zero exports.

Figure 14: Zambian log imports vs. model simulation from Melitz (2003) estimation



Note: Log imports are an untargeted moment.

Appendix C Climate data processing

For firms that, due to their offset locations, ended up with interpolated data from CHIRPS or BKE, I also interpolate projection data in the same way to ensure that observed changes in weather are due to differences in weather over time at the same location, rather than weather data and projections coming from different locations. Since I combine weather data (CHIRPS and BKE) with the NEX-GDDP-CMIP6 projections, I need to take care to remove underlying differences in average weather at baseline, to isolate the effect of changes in weather patterns over time (Auffhammer et al., 2013). To this end, I also download historical runs of each model for the period from 1980–2014. This gives me an overlapping period of 34 years to assess existing biases across models and correct for them. For both temperature and precipitation, I calculate the average value for each day of the year (e.g. January 1) across this overlapping period and subtract the difference from projection data, as recommended by Auffhammer et al. (2013). For one of the climate models, TaiESM1, temperature jumps significantly between the historical run and climate change projection, making it impossible to adjust for bias and making me question the validity of the projection. I thus exclude the TaiESM1 projections for both temperature and precipitation from my analyses. No other model has this issue.

Appendix D Proofs and derivations

D.1 Optimal c

The FOC for the optimal distribution network gives

$$
\begin{aligned}
0 &= \delta \frac{w_i}{a_j} c_j^{-\delta-1} \left(\sum_{n \in \mathcal{M}_j} d_{ni} \alpha_n p_n(j)^{-\sigma} \right) - b c_j^{\frac{1}{\beta}-1} w_i \\
\Leftrightarrow c_j^{\frac{1}{\beta}+\delta} &= \frac{1}{b} \delta \frac{1}{a_j} \left(\sum_{n \in \mathcal{M}_j} d_{ni} \alpha_n p_n(j)^{-\sigma} \right) \\
\overset{(4)}{\Leftrightarrow} c_j^{\frac{1}{\beta}+\delta} &= \frac{1}{b} \delta (\mu w_i)^{-\sigma} a_j^{\sigma-1} \left(\sum_{n \in \mathcal{M}_j} {d_{ni}}^{1-\sigma} \alpha_n \right) c_j^{\delta\sigma} \qquad (13) \\
\Leftrightarrow c_j &= \left[\frac{1}{b} \delta (\mu w_i)^{-\sigma} a_j^{\sigma-1} \left(\sum_{n \in \mathcal{M}_j} {d_{ni}}^{1-\sigma} \alpha_n \right) \right]^{\frac{\beta}{1-\beta(\sigma-1)\delta}}
\end{aligned}
$$

D.2 Proof that $\mathcal{M}_j^{\text{lb}}$ is a lower bound

I need to show that there is no set of markets $\mathcal{M}_j^{\text{cand}}$ included in $\mathcal{M}_j^{\text{lb}}$ such that exiting these markets would increase firm profits. I first show that there is no single market included in the lower bound that the firm could profitably exit, and then show that this also implies there is no set of markets included in the lower bound that the firm could profitably exit.

Suppose firm j is active in markets $\mathcal{M}_j \supseteq \mathcal{M}_j^{\text{lb}}$ with c_j optimally chosen and considers whether exiting a market $n \in \mathcal{M}_j^{\text{lb}}$ could increase its profit. Let $\widetilde{\mathcal{M}}_j = \mathcal{M}_j \setminus \{n\}$ and let $\tilde{c}_j$ be the corresponding optimal productive capability. Since c_j is increasing in $|\mathcal{M}_j|$, I know that $\tilde{c}_j < c_j$. I also know that at the iteration of the algorithm during which n was added to $\mathcal{M}_j^{\text{lb}}$ the firm was active in a set of markets $\mathcal{M}'_j$ not including n and would have made a variable profit in n at the optimal c'_j. Since the algorithm only ever adds markets at each iteration and $\mathcal{M}_j \supseteq \mathcal{M}_j^{\text{lb}}$ I know that every market in $\mathcal{M}'_j$ must also be in $\widetilde{\mathcal{M}}_j = \mathcal{M}_j \setminus \{n\}$. I therefore have $\mathcal{M}'_j \subseteq \widetilde{\mathcal{M}}_j$ which implies $c'_j \leq \tilde{c}_j$. That is, the firm made a variable profit in n at some $c'_j \leq \tilde{c}_j < c_j$. Therefore, it will certainly make a variable profit in n at c_j or $\tilde{c}_j$, and variable losses incurred in n cannot be the reason to exit it (since there are none). Exiting n could still increase the firm's total profit because at $\widetilde{\mathcal{M}}_j$, the fact that $\tilde{c}_j < c_j$ decreases its cost of acquiring productive capability. But there is nothing stopping the firm from choosing $\tilde{c}_j$ at $\mathcal{M}_j$ while still incurring a variable profit in n. Since at $\mathcal{M}_j$, the firm instead optimally chooses c_j, deviating to $\tilde{c}_j$ cannot increase profits. Therefore, the firm would never want to exit any market $n \in \mathcal{M}_j^{\text{lb}}$.

This argument extends to exiting a set of markets $\mathcal{M}_j^{\text{cand}}$ that the algorithm included in $\mathcal{M}_j^{\text{lb}}$, because c_j depends only on the total effective demand of all markets in $\mathcal{M}_j^{\text{cand}}$ (not on their indices, for example). The firm would make a profit in any market $n \in \mathcal{M}^{\text{cand}}$, so direct losses cannot be the reason to exit. For the firm's choice of c_j, exiting several markets at once is just like exiting one large market, because c_j depends only on the sum of α_n across all markets the firm is active in. For the same reason as above, choosing a different c_j cannot increase profits.

D.3 Proof that $\mathcal{M}_j^{\text{ub}}$ is an upper bound

I need to show that there is no set of markets $\mathcal{M}_j^{\text{cand}}$ excluded from $\mathcal{M}_j^{\text{ub}}$ such that entering these markets would increase firm profits. I first show that there is no single market excluded from the upper bound that the firm could profitably enter, and then show that this also implies there is no set of markets excluded from the upper bound that the firm could profitably enter.

Suppose firm j is active in markets $\mathcal{M}_j \subseteq \mathcal{M}_j^{\text{ub}}$ with c_j optimally chosen and considers whether entering a market $n \notin \mathcal{M}_j^{\text{ub}}$ could increase its profit. Let $\widetilde{\mathcal{M}}_j = \mathcal{M}_j \cup \{n\}$ and let $\tilde{c}_j$ be the corresponding optimal productive capability. Since c_j is increasing in $|\mathcal{M}_j|$, I know that $\tilde{c}_j > c_j$. I also know that at the iteration of the algorithm during which n was dropped from $\mathcal{M}_j^{\text{ub}}$ the firm was active in a set of markets $\mathcal{M}'_j$ including n and did not make a variable profit in n at the optimal c'_j. Since the algorithm only ever drops markets at each iteration and $\mathcal{M}_j \subseteq \mathcal{M}_j^{\text{ub}}$, I know that every market in $\widetilde{\mathcal{M}}_j = \mathcal{M}_j \cup \{n\}$ must also be in $\mathcal{M}'_j$. I therefore have $\mathcal{M}'_j \supseteq \widetilde{\mathcal{M}}_j$ which implies $c'_j \geq \tilde{c}_j$. That is, the firm made a variable loss in n at some $c'_j \geq \tilde{c}_j > c_j$. Therefore, it will certainly make a variable loss in n at c_j or $\tilde{c}_j$ and variable profits made in n cannot be the reason to enter it (since there are none). Entering n could still increase the firm's total profit because at $\widetilde{\mathcal{M}}_j$, the fact that $\tilde{c}_j > c_j$ increases its profit in other markets. But there is nothing stopping the firm from choosing $\tilde{c}_j$ at $\mathcal{M}_j$ without incurring a variable loss in n. Since at $\mathcal{M}_j$, the firm instead optimally chooses c_j, deviating to $\tilde{c}_j$ cannot increase total profits. Therefore, the firm would never want to enter any market $n \notin \mathcal{M}_j^{\text{ub}}$.

This argument extends to entering a set of markets $\mathcal{M}_j^{\text{cand}}$ that the algorithm excluded from $\mathcal{M}_j^{\text{ub}}$, because c_j depends only on the total effective demand of all markets in $\mathcal{M}_j^{\text{cand}}$ (not on their indices, for example). The firm would make a variable loss in any market $n \in \mathcal{M}^{\text{cand}}$, so variable profits cannot be the reason to enter. For the firm's choice of c_j, entering several markets at once is just like entering one large market, because c_j depends only on the sum of α_n across all markets the firm is active in. For the same reason as above, choosing a different c_j cannot increase profits.

D.4 Mass of entrants

Letting Ω_i denote the set of entrants in i, so $N_i = |\Omega_i|$, country i's full employment condition is that

$$
\begin{aligned}
L_i &= \int_{\Omega_i} \mathbb{1}\left[a_j \geq \underline{a}_i\right] \left\{ \left[\sum_{n \in \mathcal{M}_j} d_{ni} q_n(j) \frac{1}{a_j c_j^{\delta}} + f_{ni} \right] + b\beta c_j^{\frac{1}{\beta}} \right\} + f_i \ \mathrm{d}j \\
&= N_i \left(\int_{\underline{a}_i}^{\infty} \left[\sum_{n \in \mathcal{M}_j} d_{ni} q_n(j) \frac{1}{a_j c_j^{\delta}} + f_{ni} \right] + b\beta c_j^{\frac{1}{\beta}} \ \mathrm{d}F_i\left(a_j\right) + f_i \right) \\
\Leftrightarrow w_i L_i &= N_i \left(\int_{\underline{a}_i}^{\infty} \left[\sum_{n \in \mathcal{M}_j} d_{ni} q_n(j) \frac{w_i}{a_j c_j^{\delta}} + f_{ni} w_i \right] + b\beta c_j^{\frac{1}{\beta}} w_i \ \mathrm{d}F_i\left(a_j\right) + f_i w_i \right) \\
&\overset{(5)}{=} N_i \left(\int_{\underline{a}_i}^{\infty} \left[\sum_{n \in \mathcal{M}_j} \mu^{-\sigma} \left(d_{ni} \frac{w_i}{a_j c_j^{\delta}} \right)^{1-\sigma} \alpha_n + f_{ni} w_i \right] + b\beta c_j^{\frac{1}{\beta}} w_i \ \mathrm{d}F_i\left(a_j\right) + f_i w_i \right) \\
&\overset{(8)}{=} N_i \int_{\underline{a}_i}^{\infty} \sum_{n \in \mathcal{M}_j} \left(\frac{1}{\mu} + \frac{1}{\sigma} \right) \left(\mu d_{ni} \frac{w_i}{a_j c_j^{\delta}} \right)^{1-\sigma} \alpha_n \ \mathrm{d}F_i\left(a_j\right) \\
\Leftrightarrow N_i &= \frac{\mu^{\sigma-1} w_i^{\sigma} L_i}{\int_{\underline{a}_i}^{\infty} \left(a_j c_j^{\delta}\right)^{\sigma-1} \sum_{n \in \mathcal{M}_j} d_{ni}^{1-\sigma} \alpha_n \ \mathrm{d}F_i\left(a_j\right)}
\end{aligned}
$$

D.5 Price index

The price index of country n is

$$
\mathcal{P}_n = \left(\int_{\mathcal{G}_n} p_n(j)^{1-\sigma} \ \mathrm{d}j \right)^{\frac{1}{1-\sigma}}
$$

Letting $\mathcal{E}_{ni}$ denote the set of goods produced in i and exported to n,

$$
= \left(\sum_{i=1}^{N} \int_{\mathcal{E}_{ni}} p_n(j)^{1-\sigma} \ \mathrm{d}j \right)^{\frac{1}{1-\sigma}}
$$

Dealing with $\mathcal{E}_{ni}$ directly is cumbersome, because it involves conditional probabilities. Instead, let $\mathcal{O}_i$ denote the set of goods produced in i, regardless of where they're shipped to. Then,

$$
= \left(\sum_{i=1}^{N} \int_{\mathcal{O}_i} \mathbb{1}\left[j \in \mathcal{E}_{ni}\right] p_n(j)^{1-\sigma} \ \mathrm{d}j \right)^{\frac{1}{1-\sigma}}
$$

which, using that $j \in \mathcal{E}_{ni} \Leftrightarrow n \in \mathcal{M}_j$, switching to integrating over the CDF of productivities and remembering that there is a mass n_i of firms active in country i,

$$= \left(\sum_{i=1}^{N} n_i \int_{\underline{a}_i}^{\infty} \mathbb{1}\left[n \in \mathcal{M}_j\right] p_n(j)^{1-\sigma} \; \mathrm{d}F_i\left(a_j\right) \right)^{\frac{1}{1-\sigma}}$$

$$\stackrel{(4)}{=} \left(\sum_{i=1}^{N} n_i \int_{\underline{a}_i}^{\infty} \mathbb{1}\left[n \in \mathcal{M}_j\right] \left(\mu d_{ni} \frac{w_i}{a_j c_j^{\delta}} \right)^{1-\sigma} \; \mathrm{d}F_i\left(a_j\right) \right)^{\frac{1}{1-\sigma}}$$

Since there will also be a cutoff $\underline{a}_{ni}$ such that all firms in i with $a_j \geq \underline{a}_{ni}$ will sell in n, and all other firms in i will not,

$$= \mu \left(\sum_{i=1}^{N} n_i (d_{ni} w_i)^{1-\sigma} \int_{\underline{a}_{ni}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \; \mathrm{d}F_i\left(a_j\right) \right)^{\frac{1}{1-\sigma}} \tag{14}$$

D.6 Gravity equation

Sales from firms in i to n are

$$\begin{aligned}
X_{ni} &= \int_{\mathcal{E}_{ni}} S_n(j) \; \mathrm{d}j \\
&= n_i \int_{\underline{a}_{ni}}^{\infty} S_n(j) \; \mathrm{d}F_i\left(a_j\right) \\
&\stackrel{(6)}{=} n_i \int_{\underline{a}_{ni}}^{\infty} \alpha_n \left(\mu d_{ni} \frac{w_i}{a_j c_j^{\delta}} \right)^{1-\sigma} \; \mathrm{d}F_i\left(a_j\right) \\
&= n_i (\mu d_{ni} w_i)^{1-\sigma} \alpha_n \int_{\underline{a}_{ni}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \; \mathrm{d}F_i\left(a_j\right)
\end{aligned}$$

which, by definition of α_n,

$$\begin{aligned}
&= n_i (\mu d_{ni} w_i)^{1-\sigma} X_n \mathcal{P}_n^{\sigma-1} \int_{\underline{a}_{ni}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \; \mathrm{d}F_i\left(a_j\right) \\
&\stackrel{(14)}{=} \frac{n_i (d_{ni} w_i)^{1-\sigma} \int_{\underline{a}_{ni}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \; \mathrm{d}F_i\left(a_j\right)}{\sum_{k=1}^{N} n_k (d_{nk} w_k)^{1-\sigma} \int_{\underline{a}_{nk}}^{\infty} \left(a_j c_j^{\delta} \right)^{\sigma-1} \; \mathrm{d}F_k\left(a_j\right)} X_n
\end{aligned}$$

D.7 Home sales as a log-linear function of total sales

From (13), the optimal c can be written as

$$c_j^{\frac{1}{\beta}+\delta} = \frac{1}{b} \delta (\mu w_i)^{-\sigma} a_j^{\sigma-1} \left(\sum_{n \in \mathcal{M}_j} {d_{ni}}^{1-\sigma} \alpha_n \right) c_j^{\delta\sigma}$$

$$
\begin{aligned}
&\Leftrightarrow c_j^{\frac{1}{\beta}} = \frac{1}{b}\delta(\mu w_i)^{-\sigma} a_j^{\sigma-1}\left(\sum_{n\in\mathcal{M}_j} {d_{ni}}^{1-\sigma}\alpha_n\right) c_j^{\delta(\sigma-1)} \\
&\Leftrightarrow c_j = \left[\frac{1}{b}\frac{\delta}{\mu w_i}\left(\mu\frac{w_i}{a_j c_j^{\delta}}\right)^{1-\sigma}\left(\sum_{n\in\mathcal{M}_j} {d_{ni}}^{1-\sigma}\alpha_n\right)\right]^{\beta} \\
&\overset{(6)}{=} \left[\frac{1}{b}\frac{\delta}{\mu w_i}\underbrace{\left(\sum_{n\in\mathcal{M}_j} S_n(j)\right)}_{\equiv\mathcal{S}(j)}\right]^{\beta}
\end{aligned} \tag{15}
$$

Plugging (15) into sales to the Home market (6) and remembering that $d_{ii} = 1$ by assumption,

$$
\begin{aligned}
S_i(j) &= \alpha_i\left(\mu\frac{w_i}{a_j}\left[\frac{1}{b}\frac{\delta}{\mu w_i}\mathcal{S}(j)\right]^{-\beta\delta}\right)^{1-\sigma} \\
\Leftrightarrow \log\left(S_i(j)\right) &= \log\left(\alpha_i\right) + (\sigma-1)\left[\log\left(\frac{1}{\mu}\frac{a_j}{w_i}\right) + \beta\delta\log\left(\frac{1}{b}\frac{\delta}{\mu w_i}\mathcal{S}(j)\right)\right] \\
\Leftrightarrow \log\left(S_i(j)\right) &= I + \log\left(\alpha_i\right) + (\sigma-1)\log\left(a_j\right) - (\sigma+\beta\delta-1)\log\left(w_i\right) + (\sigma-1)\,\beta\delta\log\left(\mathcal{S}(j)\right)
\end{aligned}
$$

with

$$
I \equiv (\sigma-1)\left[\log\left(\frac{1}{\mu}\right) + \beta\delta\log\left(\frac{1}{b}\frac{\delta}{\mu}\right)\right]
$$