Add Width Mode

Refer to Basic Functions

We use v_i to represent basic functions. Here we suppose the solution to steady state equation is a linear combination of basic functions

$$\psi_{1c} = u_0 v_0(y_1 - s_1, k) + u_2 v_2(y_1 - s_1, k)$$

Steady State

First we consider equal stimulus strength case.

Congruent Group's Equation

$$\psi(y_1) = \frac{\rho J_{rc}}{D_1}\int dy_2 V(y_1 - y_2, a_0)\psi^2(y_2) + \frac{\rho J_{rp}}{D_2}\int dy_2 V(y_1 - y_2, a_0)\bar{\psi}^2(y_2) + I_1 V(y_1 - x_1, a_0/2) + I_b \quad (1)$$

Global Inhibition

$$D_n = 1 + \omega\int\rho\left[u_n^2(x,k) + J_{int}u_{\bar{n}}^2(x,k)\right]dx = 1 + \omega\rho\left[u_0^2 + u_2^2 + J_{int}(\bar{u}_0^2 + \bar{u}_2^2)\right]$$

Firing Rate

$$\psi^2(y_2) = e^{2kcos\theta}[u_2^2 c_2^2 sin^4(y_2 - s_1) + 2u_2c_2(u_0c_0 - u_2\frac{c_2N_2}{N_0})sin^2(y_2 - s_1) + (u_0c_0 - u_2\frac{c_2N_2}{N_0})^2]$$

$$\begin{aligned}
\int dy_2 e^{a_0cos(y_1-y_2)}\psi^2(y_2) &= u_2^2c_2^2M_{0,4}(y_1, a_0, 2k) + 2u_2c_2(u_0c_0 - u_2\frac{c_2N_2}{N_0})M_{0,2}(y_1, a_0, 2k) + (u_0c_0 - u_2\frac{c_2N_2}{N_0})^2M_{0,0}(y_1, a_0, 2k) \\
&= e^{k_3cos(y_1-s_1)}\{(u_0c_0 - u_2\frac{c_2N_2}{N_0})^2F + u_2c_2(u_0c_0 - u_2\frac{c_2N_2}{N_0})\frac{F'(2k)}{k} + \frac{u_2^2c_2^2}{16k^4}[-12F'(2k)k + 12Fk^2 + 3Fk_3^2 - 6Fkk_3\frac{\partial k_3}{\partial k}] \\
&\quad + \left[u_2c_2(u_0c_0 - u_2\frac{c_2N_2}{N_0})\frac{Fk_3^2}{2k^2} - \frac{3k_3u_2^2c_2^2}{16k^4}[3Fk_3 - 4F'(2k)kk_3 - 6Fk\frac{\partial k_3}{\partial k}]\right]sin^2(y_1 - s_1) \\
&\quad + \frac{u_2^2c_2^2Fk_3^4}{16k^4}sin^4(y_1 - s_1) \\
&\quad + \left[u_2c_2(u_0c_0 - u_2\frac{c_2N_2}{N_0})\frac{F}{2k}(\frac{\partial k_3}{\partial k} - \frac{k_3}{k}) + \frac{u_2^2c_2^2}{16k^4}(12Fk_3 - 12F'(2k)kk_3 - 12Fk\frac{\partial k_3}{\partial k})\right]cos(y_1 - s_1) \\
&\quad + \frac{6u_2^2c_2^2Fk_3^2}{16k^4}[-k_3 + k\frac{\partial k_3}{\partial k}]sin^2(y_1 - s_1)cos(y_1 - s_1)\} \\
&= e^{k_3cos(y_1-s_1)}[R_0 + R_2sin^2(y_1 - s_1) + R_4sin^4(y_1 - s_1) + R_{01}cos(y_1 - s_1) + R_{21}sin^2(y_1 - s_1)cos(y_1 - s_1)]
\end{aligned}$$

Note $k_3 = A^{-1}[A(a_0)A(2k)]$.

- Steady State

$$\begin{gathered}
u_0v_0(y_1 - s_1, k) + u_2v_2(y_1 - s_1, k) = \\
\frac{\rho J_{rc}e^{k_3cos(y_1-s_1)}}{2\pi I_0(a_0)D_n}\left[R_0 + R_2sin^2(y_1 - s_1) + R_4sin^4(y_1 - s_1) + R_{01}cos(y_1 - s_1) + R_{21}sin^2(y_1 - s_1)cos(y_1 - s_1)\right] + \\
\frac{\rho J_{rp}e^{k_3cos(y_1-s_2)}}{2\pi I_0(a_0)D_n}\left[R_0 + R_2sin^2(y_1 - s_2) + R_4sin^4(y_1 - s_2) + R_{01}cos(y_1 - s_2) + R_{21}sin^2(y_1 - s_2)cos(y_1 - s_2)\right] + \\
I_1V(y_1 - x_1, a_0/2) + I_b
\end{gathered}$$

Projection

We project steady state equation to height mode, position mode and width mode.

Height Mode

Multiply both side by $e^{bcos(y_1-s_1)}$ and integrate over y_1. Note $F_\kappa = F(b,\kappa)$ and $b_\kappa = A^{-1}[A(b)A(\kappa)]$.

$LHS = (u_0 c_0 - u_2 c_2 \frac{N_2}{N_0}) N_0(k+b) + u_2 c_2 N_2(k+b)$

Input

$$RHS(Input) = \frac{I_1}{2\pi I_0(a_0/2)} M_{0,0}(s_1 - x_1; a_0/2, b) + I_b N_0(b)$$

Coupling

$$RHS(RC) = \frac{\rho J_{rc} e^{b_{k_3}}}{2\pi I_0(a_0) D_n} [R_0 N_0(k_3+b) + R_2 N_2(k_3+b) + R_4 N_4(k_3+b) + R_{0,1} N_0'(k_3+b) + R_{2,1} N_2'(k_3+b)]$$

$$RHS(RP) = \frac{\rho J_{rp} e^{b_{k_3} cos(s_1-s_2)}}{2\pi I_0(a_0) D_n} [R_0 M_{0,0}(s_1-s_2; k_3, b) + R_2 M_{2,0}(s_1-s_2; k_3, b) + R_4 M_{4,0}(s_1-s_2; k_3, b)$$
$$+ R_{01} T_{0,1}(s_1-s_2; k_3, b) + R_{21} T_{2,1}(s_1-s_2; k_3, b)]$$

Position Mode

Multiply both side by $e^{bcos(y_1-s_1)} sin(y_1 - s_1)$ and integrate over y_1. Note $F_\kappa = F(b,\kappa)$ and $b_\kappa = A^{-1}[A(b)A(\kappa)]$.

$LHS = 0$

Input

$$RHS(Input) = \frac{I_1}{2\pi I_0(a_0/2)} M_{0,1}(s_1 - x_1; a_0/2, b)$$

Coupling

$$RHS(RC) = 0$$

$$RHS(RP) = \frac{\rho J_{rp} e^{b_{k_3} cos(s_1-s_2)}}{2\pi I_0(a_0) D_n} [R_0 M_{0,1}(s_1-s_2; k_3, b) + R_2 M_{2,1}(s_1-s_2; k_3, b) + R_4 M_{4,1}(s_1-s_2; k_3, b)$$
$$+ R_{01} T_{0,1,1}(s_1-s_2; k_3, b) + R_{21} T_{2,1,1}(s_1-s_2; k_3, b)]$$

Width Mode

Multiply both side by $e^{bcos(y_1-s_1)} [sin^2(y_1 - s_1) - N_b]$ and integrate over y_1. Note $N_b = \frac{N_2(2b)}{N_0(2b)}$, $F_\kappa = F(b,\kappa)$ and $b_\kappa = A^{-1}[A(b)A(\kappa)]$.

$LHS = -N_b(u_0 c_0 - \frac{N_2}{N_0}) N_0(k+b) + (u_0 c_0 - \frac{N_2}{N_0} - N_b u_2 c_2) N_2(k+b) + u_2 c_2 N_4(k+b)$

$RHS(Input) = \frac{I_1}{2\pi I_0(a_0/2)} [M_{0,2}(s_1 - x_1; a_0/2, b) - N_b M_{0,0}(s_1 - x_1; a_0/2, b)] + I_b[N_2(b) - N_b N_0(b)]$

Coupling

$$RHS(RC) = \frac{\rho J_{rc} e^{b_{k_3}}}{2\pi I_0(a_0) D_n} \{R_0[N_2(k_3+b) - N_b N_0(k_3+b)] + R_2[N_4(k_3+b) - N_b N_2(k_3+b)]$$
$$+ R_4[N_6(k_3+b) - N_b N_4(k_3+b)] + R_{0,1}[N_2'(k_3+b) - N_b N_0'(k_3+b)] + R_{2,1}[N_4'(k_3+b) - N_b N_2'(k_3+b)]\}$$

$$RHS(RP) = \frac{\rho J_{rp} e^{b_{k_3} cos(s_1-s_2)}}{2\pi I_0(a_0) D_n} \{R_0[M_{0,2}(s_1-s_2; k_3, b) - N_b M_{0,0}(s_1-s_2; k_3, b)] + R_2[M_{2,2}(s_1-s_2; k_3, b) - N_b M_{2,0}(s_1-s_2; k_3, b)]$$
$$+ R_4[M_{4,2}(s_1-s_2; k_3, b) - N_b M_{4,0}(s_1-s_2; k_3, b)] + R_{01}[T_{0,1,2}(s_1-s_2; k_3, b) - N_b T_{0,1,0}(s_1-s_2; k_3, b)]$$
$$+ R_{21}[T_{2,1,2}(s_1-s_2; k_3, b) - N_b T_{2,1,0}(s_1-s_2; k_3, b)]\}$$