Fourier Discussion

To simplify this task, we consider $I_1=I_2=I_n$.

Projection

Mean

When $\Delta s=rac{\pi}{2}$ $(\Delta x>rac{\pi}{2})$, project the dynamic equation onto different modes.

$$u_{0} = \frac{\rho(J_{rc} + J_{rp})}{D_{n}} (u_{0}^{2} + \frac{u_{1}^{2}}{2} + \frac{u_{2}^{2}}{2} + \frac{u_{3}^{2}}{2}) + \frac{I_{n}}{2\pi} + I_{b}$$

$$u_{1} = \frac{\rho J_{rc}}{D_{n}} B(1, a_{0}) (2u_{0}u_{1} + u_{1}u_{2}) - \frac{\rho J_{rp}}{D_{n}} B(1, a_{0}) u_{1}u_{3} + \frac{I_{n}B(1, a_{0}/2)}{\pi}$$

$$0 = \frac{\rho J_{rc}}{D_{n}} B(1, a_{0}) u_{1}u_{3} + \frac{\rho J_{rp}}{D_{n}} B(1, a_{0}) (2u_{0}u_{1} + u_{1}u_{2}) - \frac{I_{n}B(1, a_{0}/2)}{\pi} s_{1}$$

$$u_{2} = \frac{\rho(J_{rc} - J_{rp})}{D_{n}} (2u_{0}u_{2} + \frac{u_{1}^{2}}{2}) B(2, a_{0}) + \frac{I_{n}B(2, a_{0}/2)}{\pi}$$

$$u_{3} = \frac{\rho(J_{rc} - J_{rp})}{D_{n}} 2u_{0}u_{3}B(2, a_{0}) - \frac{I_{n}B(2, a_{0}/2)}{\pi} 2s_{1}$$

Since $u_3 \to 0$, we obtain

$$rac{s_1}{u_1}pprox rac{\pi}{I_n B(1,a_0/2)} rac{
ho J_{rp}}{D_n} B(1,a_0) (2u_0+u_2)_{\Delta s=rac{\pi}{2}}$$

Concentration

When $\Delta s=0$ $(\Delta x=0)$, project the dynamic equation onto different modes.

$$u_0 = \frac{\rho(J_{rc} + J_{rp})}{D_n} (u_0^2 + \frac{u_1^2}{2} + \frac{u_2^2}{2} + \frac{u_3^2}{2}) + \frac{I_n}{2\pi} + I_b$$

$$u_1 = \frac{\rho(J_{rc} + J_{rp})}{D_n} B(1, a_0) (2u_0 u_1 + u_1 u_2) + \frac{I_n B(1, a_0/2)}{\pi}$$

$$0 = \frac{\rho(J_{rc} + J_{rp})}{D_n} B(1, a_0) u_1 u_3 - \frac{I_n B(1, a_0/2)}{\pi} s_1$$

$$egin{align} u_2 &= rac{
ho(J_{rc} + J_{rp})}{D_n}(2u_0u_2 + rac{u_1^2}{2})B(2,a_0) + rac{I_nB(2,a_0/2)}{\pi} \ & u_3 &= rac{
ho(J_{rc} + J_{rp})}{D_n}2u_0u_3B(2,a_0) - rac{I_nB(2,a_0/2)}{\pi}2s_1 \ & \end{array}$$

Matrix

$$G_{11} = 1 - \frac{\rho J_{rc}}{D_n} (2u_0 + u_2) B(1, a_0), \ \bar{G}_{11} = 1 - \frac{\rho J_{rc}}{\bar{D}_n} (2\bar{u}_0 + \bar{u}_2) B(1, a_0)$$

$$G_{12} = -\frac{\rho J_{rp}}{D_n} (2u_0 + u_2) B(1, a_0), \ \bar{G}_{12} = \frac{\rho J_{rp}}{\bar{D}_n} (2\bar{u}_0 + \bar{u}_2) B(1, a_0)$$

Temperature

$$\begin{split} T_n &= \frac{F}{2\pi^2 \rho u_1^2} \bigg[\frac{I_n}{2} (1 - B(2, a_0/2)) + \pi I_b \bigg] \\ \bar{T}_n &= \frac{F}{2\pi^2 \rho \bar{u}_1^2} \bigg[\frac{I_n}{2} (1 - B(2, a_0/2)) + \pi I_b \bigg] \end{split}$$

Finally we obtain

$$\sigma_1^2 = rac{G_{11}T_n}{(G_{11}^2 - G_{12}^2) au} \ \kappa_1 = rac{1}{\sigma_1^2} = rac{ au}{T_n}(G_{11} - rac{G_{12}^2}{G_{11}}) pprox rac{ au}{T_n} \ rac{\kappa_1}{ar{\kappa}_1} pprox rac{u_1^2}{ar{u}_1^2}$$

Approximation

When $\Delta s = 0$, divide both sides by u_1

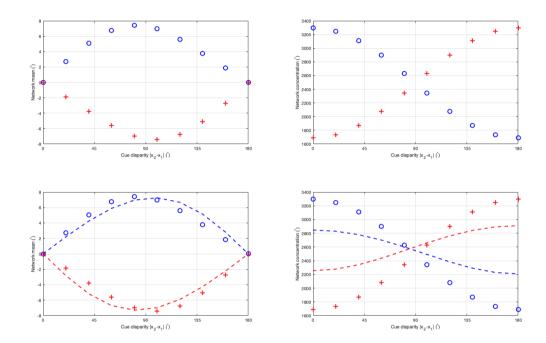
$$1 = \frac{\rho(J_{rc} + J_{rp})}{D_n} B(1, a_0)(2u_0 + u_2)_{\Delta s = 0} + \frac{I_n B(1, a_0/2)}{\pi u_1}$$

Varying Δs , we assume the first term in the right hand side remain unchanged. We replace the first term with $\frac{s_1}{u_1}$.

$$u_{1} = \frac{I_{n}B(1, a_{0}/2)}{\pi} \left[1 + \frac{J_{rc} + J_{rp}}{Jrp} s_{1} \right]$$
$$\bar{u}_{1} = \frac{I_{n}B(1, a_{0}/2)}{\pi} \left[1 + \frac{J_{rc} - J_{rp}}{Jrp} \bar{s}_{1} \right]$$

Assuming $s_1 \approx \bar{s}_1$, we consider the first order Taylor expansion of the ratio.

$$egin{align} rac{u_1^2}{ar{u}_1^2} &pprox \left[rac{1+(rac{J_{rc}}{J_{rp}}+1)s_1}{1+(rac{J_{rc}}{J_{rp}}-1)s_1}
ight]^2 pprox 1+4s_1 \ &rac{\kappa-ar{\kappa}}{\kappa+ar{\kappa}} pprox rac{4s_1}{2+4s_1} pprox 2s_1 \end{aligned}$$



Discussion