Modified Calculation

In Huge's Paper

The basic functions are given by:

$$c_{0,0} = \frac{1}{\sqrt{2\pi I_0(2k)}}$$
$$c_{1,0} = \sqrt{\frac{k}{\pi a I_0(2k)}}$$
$$\begin{pmatrix} c_{2,0} \\ c_{2,1} \\ c_{2,2} \end{pmatrix} = \sqrt{\frac{k}{2\pi I_0(2k)(3k-a^2k-a)}} \begin{pmatrix} k+a \\ -2 \\ -k \end{pmatrix}$$

Where $a = A(2k)$. The coefficients multiply Von Mises functions respectively, which are given by:

$$e^{kcos\theta} \quad e^{kcos\theta} sin\theta \quad e^{kcos\theta} \begin{pmatrix} 1 & cos\theta & cos2\theta \end{pmatrix}$$

Analysis

We use $v_0,\ v_1,\ v_2$ to represent the first three order basic functions respectively.

Projection Method

We suppose $u_{1,c}(y_1 - s_1, k) = u_0 v_0(y_1 - s_1, k) + u_1 v_1(y_1 - s_1, k) + u_2 v_2(y_1 - s_1, k)$, with the width fixed. We set the width of the basic functions and synaptic input to be $a/2$. We may adjust the width of the basic functions to see whether the result converge faster. The coefficients of basic functions remain unknown. We project them on the height modes, position modes and width modes then we'll obtain three equations. The last equation is the self-consistent equation, which is given by:

$$z(t) = \frac{\int_{-\infty}^{\infty} dx U(x,t) x}{\int_{-\infty}^{\infty} dx U(x,t)}$$

And circular self-consistent equation:

$$s = arg(\sum_{-\pi}^{\pi} U(y,k)e^{iy}) = arg(\int_{-\pi}^{\pi} \rho U(y,k) e^{iy} dy) = arg(\int_{-\pi}^{\pi} U(y,k) e^{iy} dy) \tag{1}$$

So we have four equations to decide four variables.

Steady State

- Self-Consistent Equation

$$0 = arg\left(\int_{-\pi}^{\pi} dy \left[u_0 v_0(y,k) + u_1 v_1(y,k) + u_2 v_2(y,k)\right] e^{iy}\right)$$

- Equal Stimulus Strength

Steady state:

$$\psi(y_1) = \frac{\rho J_{rc}}{D_1} \int dy_2 V(y_1 - y_2, a) \psi^2(y_2) + \frac{\rho J_{rp}}{D_2} \int dy_2 V(y_1 - y_2, a) \bar{\psi}^2(y_2) + I_1 V(y_1 - x_1, a/2) + I_b$$

Global inhibition D_n:

$$D_n = 1 + \omega \int \rho \left[u_n^2(x,k) + J_{int} u_{\bar{n}}^2(x,k) \right] dx = 1 + \omega\rho \left[(u_0^2 + u_1^2 + u_2^2) + J_{int}(\bar{u}_0^2 + \bar{u}_1^2 + \bar{u}_2^2) \right]$$

Here we define:

$$M_{j,k}^i(\theta, k_1, k_2, k_2) = \int_{-\pi}^{\pi} v_i(\theta - \theta', k_1) v_j(\theta', k_2) v_k(\theta', k_3) d\theta' \tag{2}$$

Using the recursive relationship and integration by parts, we obtain:

$$\begin{aligned} M_{j,k+1}^i(\theta, k_1, k_2, k_3) &= \int_{-\pi}^{\pi} v_i(\theta - \theta', k_1) v_j(\theta', k_2) v_{k+1}(\theta', k_3) d\theta' \\ &= \int_{-\pi}^{\pi} d\theta' v_i(\theta - \theta', k_1) v_j(\theta', k_2) \left[\frac{r_{k-1} v_{k-1} - v_k'}{r_k} \right] \end{aligned}$$

$$r_{k-1}(k_3) M_{j,k-1}^i - r_k(k_3) M_{j,k+1}^i + r_{j-1}(k_2) M_{j-1,k}^i - r_j(k_2) M_{j+1,k}^i = r_{i-1}(k_1) M_{j,k}^{i-1} - r_i(k_1) M_{j,k}^{i+1} \tag{2.1}$$

And:

$$M_{j,k}^{i+1} = \frac{r_{i-1}(k_1)}{r_i(k_1)} M_{j,k}^{i-1} - \frac{1}{r_i(k_1)} \partial_\theta M_{j,k}^i \tag{2.2}$$

The first few terms are given below:

$$M_{0,0}^0(\theta, k_1, k_2, k_2) = \frac{I_0(k_1)}{I_0(k_3)} \sqrt{\frac{I_0(2k_3)}{I_0(2k_1)}} v_0(\theta, k_3) \quad where\ k_3 = A^{-1}\left[A(k_1) A(2k_2) \right]$$

$$M_{j,k}^1 = -\frac{1}{r_0(k_1)} \partial_\theta M_{j,k}^0$$

$$M_{0,1}^0(\theta, k_1, k_2, k_2) = \frac{r_0(k_1)}{2r_0(k_2)} M_{0,0}^1$$

$$M_{1,1}^0(\theta, k_1, k_2, k_2) =$$

$$M_{0,2}^0(\theta, k_1, k_2, k_2) =$$

$$M_{1,2}^0(\theta, k_1, k_2, k_2) =$$

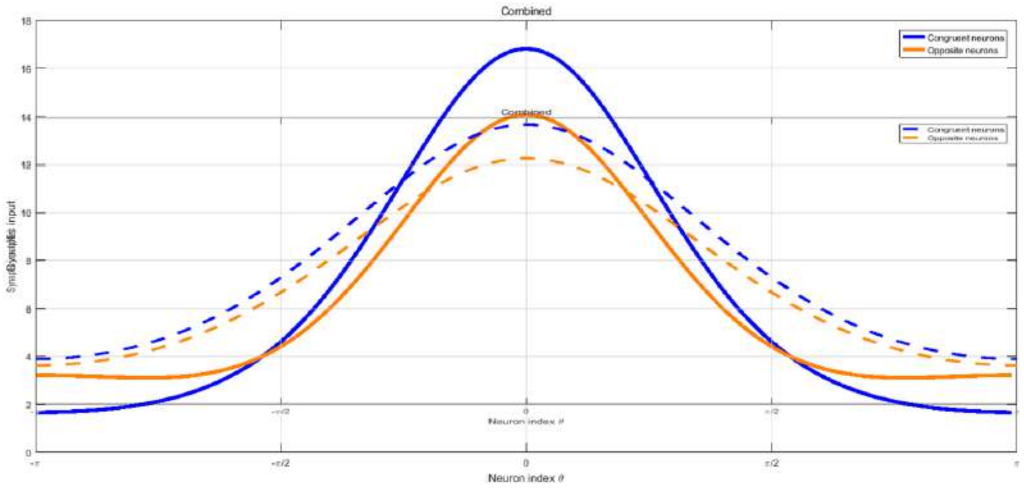
$$M_{2,2}^0(\theta, k_1, k_2, k_2) =$$

Consider the square term:

$\int dy_2 V(y_1 - y_2, a) \psi^2(y_2) = \frac{\sqrt{2\pi I_0(2a)}}{2\pi I_0(a)} \sum_{i,j=0}^{2} M_{i,j}^0(y_1, a, k, k)$

Where:

Bump Profile



Combined
Congruent neurons
Opposite neurons
Synaptic input
Neuron index θ
-π
-π/2
0
π/2
π