Hi Xiangyu,

I want you quickly check the decision probability of choosing a binary decision in a mutual inhibition model presented in Bogacz 2006. Specifically, if the noises received by two populations in Eq. 14 in Bogacz 2006 are not independent, how the decision probability will be? That is, if the Eq. 14 becomes

$$dy_1 = (-ky_1 - wy_2 + I_1)dt + c\left(\sqrt{1-\rho}dW_1 + \sqrt{\rho}dW_0\right)$$
$$dy_2 = (-ky_2 - wy_1 + I_2)dt + c\left(\sqrt{1-\rho}dW_2 + \sqrt{\rho}dW_0\right)$$

where dW_0, dW_1, and dW_2 are three independent standard gaussian white noises, what is the decision probability, i.e., the probability of y_1 and y_2 crossing a fixed boundary, and how this mutual inhibition neural model could be related with parameters in a drift diffusion model (DDM) as presented in Eqs. (5-12)? Mathematically, computing the decision probability is solving the first passage problem in an OU process.
The Bogacz 2006 only presents a simple case where $\rho = 0$, and relate the decision probability in this neural model with DDM (Eqs. 18-26).

The reason I ask this question is I think it probably has the mechanism we want. From the relationship between the DDM and mutual inhibition model (Eqs. 18-30), we see the decision probability in this model has the same mathematical expression as the causal inference, i.e., the exponential term in the denominator in Eq.8 is similar with Bayes factor in causal inference. The intuition is once the drift and fluctuation terms in a DDM (Eq. 5) could be properly set, the decision probability could be consistent with causal inference.

The results presented in Eqs. (24-26) inspire me to consider the possibility of introducing a mutual inhibition circuit driven by the congruent and opposite neurons to implement the causal inference.

1. The drift term in DDM is determined by the difference of the feedforward inputs received by two pools in the network model, $I_1 - I_2$ (Eq. 24). In our network model, I_1 could be the sum of the firing rate of opposite neurons ($B + Bk\cos\Delta x$ in Xiangyu's sigmoid.pdf), and I_2 could be the sum of firing rate of congruent and opposite neurons which is B in sigmoid.pdf.

2. The effective decision boundary, z in Eq. (26), decays with the $I_1 + I_2$, which has similar tendency with the causal inference where (some constant might be missing in below eq.)

$$p(seg|\boldsymbol{x}) = \frac{1}{1 + \exp\left[\kappa_{12s}\cos\Delta x - \ln\left(I_0(\kappa_{12s})\right)\right]}$$

$\ln\left(I_0(\kappa_{12s})\right)$ acts as a flexible decision boundary which is also increases with feedforward input intensity.

3. To match the decision probability in the mutual inhibition model with causal inference, the final terms we need to determine is the noise fluctuation in the mutual inhibition model. If I_1 and I_2 are the sum of firing rate of opposite and congruent neurons, their fluctuations are not independent. That's why I ask you to compute the decision probability in a DDM with correlated noises in the beginning.