Manuscript

Von Mises Analysis

Huge’s Paper

In Huge’s paper, A8:c10 should be:

$$c_{1,0} = \sqrt{\frac{k}{\pi a I_0(2k)}}$$

Project on Width Mode

Note that v_2 is the width mode in Huge’s paper, we use capital V to represent Von Mises function. We give the expression of the convolution.

$\int_{-\pi}^{\pi} v_2(\theta-\theta', k_1) V(\theta', k_2) d\theta'$

$= \frac{I_0(k_1)}{I_0(k_3)} \sqrt{\frac{I_0(2k_3)}{I_0(2k_1)}} \{ \frac{r_0(k_1)}{r_1(k_1)} v_0(\theta, k_3) - \frac{r_0(k_3)}{r_1(k_1) r_0(k_1)} [\sqrt{\frac{k_3}{\pi a_3 I_0(2k_3)}} e^{k_3 cos\theta} (cos\theta - k_3 sin^2\theta)]\}$

$= \frac{2\pi I_0(k_1)}{I_0(k_3)} \sqrt{\frac{I_0(2k_3)}{I_0(2k_1)}} \{ \frac{r_0(k_1)}{r_1(k_1)} \frac{I_0(k_3)}{\sqrt{2\pi I_0(2k_3)}} V(\theta, k_3) - \frac{r_0(k_3)}{r_1(k_1) r_0(k_1)} [\sqrt{\frac{k_3}{\pi a_3 I_0(2k_3)}} I_0(k_3) V(\theta, k_3)(cos\theta - k_3 sin^2\theta)]\}$

$= \sqrt{\frac{2\pi}{I_0(2k_1)}} I_0(k_1) V(\theta, k_3) [\frac{r_0(k_1)}{r_1(k_1)} - \frac{r_0(k_3)\sqrt{2k_3/a_3}}{r_1(k_1) r_0(k_1)} (cos\theta - k_3 sin^2\theta))]$

Here:

$$R_1(k_1) = \frac{r_0(k_1)}{r_1(k_1)} = \sqrt{\frac{k_1 a_1^2}{3k_1 - k_1 a_1^2 - a_1}}$$

$$R_2(k_1, k_3) = \frac{r_0(k_3)\sqrt{2k_3/a_3}}{r_1(k_1) r_0(k_1)} = \sqrt{\frac{4k_3^2}{k_1(3k_1 - k_1 a_1^2 - a_1)}}$$

Where:

$a_1 = A(2k_1)$
$a_3 = A(2k_3)$
$k_3 = A^{-1}(A(k_1) * A(k_2))$
$A(k) = \frac{I_1(k)}{I_0(k)}$

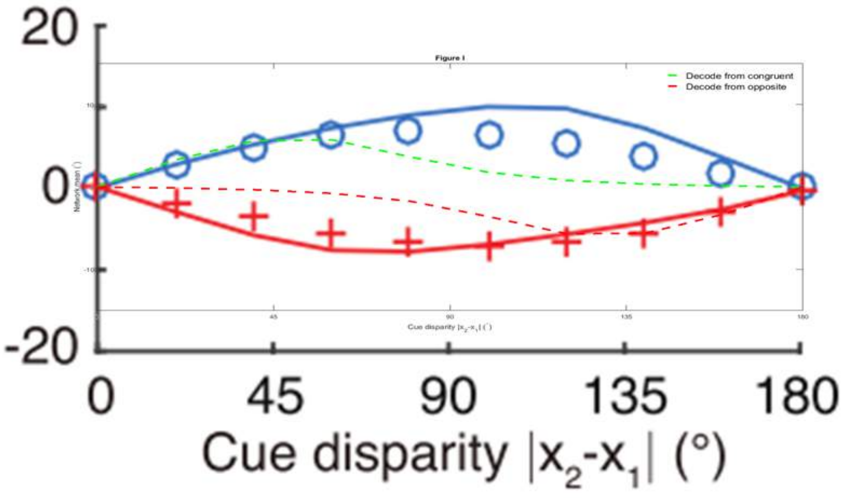
And the steady state:

$g_{11} V(y_1 - s_1, a/2) \approx g_1 2V(y_1 - s_2, a_r) + A_1 V(y_1 - x_1, a/2) + A_b$

Multiply both sides by $v_2(y_1 - s_1, b)$, integrate over y_1:

$$g_{11} V(0, b_u)[R_1(b) - R_2(b, b_u)] \approx g_{12} V(s_2 - s_1, b_r)\{R_1(b) - R_2(b, br)[cos(s_2 - s_1) - b_r sin^2(s_2 - s_1)]\} \\ + A_1 V(x_1 - s_1, b_u)\{R_1(b) - R_2(b, bu)[cos(x_1 - s_1) - b_u sin^2(s_2 - s_1)]\} + A_b R_1(b)$$

But the result was **not good**.



Modified Calculation

We rewrite the steady state here:

$$u_1 V(y_1 - s_1, k_{1u}) \approx \rho J_{rc} \frac{u_1^2}{D_1} \frac{I_0(2k_{1u}}{2\pi I_0(k_{1u})^2} V(y_1 - s_1, k_{1r}) + \rho J_{rp} \frac{u_2^2}{D_2} \frac{I_0(2k_{2u}}{2\pi I_0(k_{2u})^2} V(y_1 - s_2, k_{2r}) + I_1 V(y_1 - x_1, a/2) + I_b$$

Let: $y_1 = s_1$

And we assume: $s_1 = x_1$, and $I_b = 0$

$$B_1 = \rho J_{rc} \frac{u_1}{D_1} \frac{I_0(2k_{1u}) e^{k_1 r}}{2\pi I_0(k_{1u}) I_0(k_{1r})}$$
$$B_2 = \frac{I_0(k_{1u})}{u_1} \left(\frac{I_1 e^{a/2}}{I_0(a/2)} + 2\pi I_b \right)$$

We obtain:

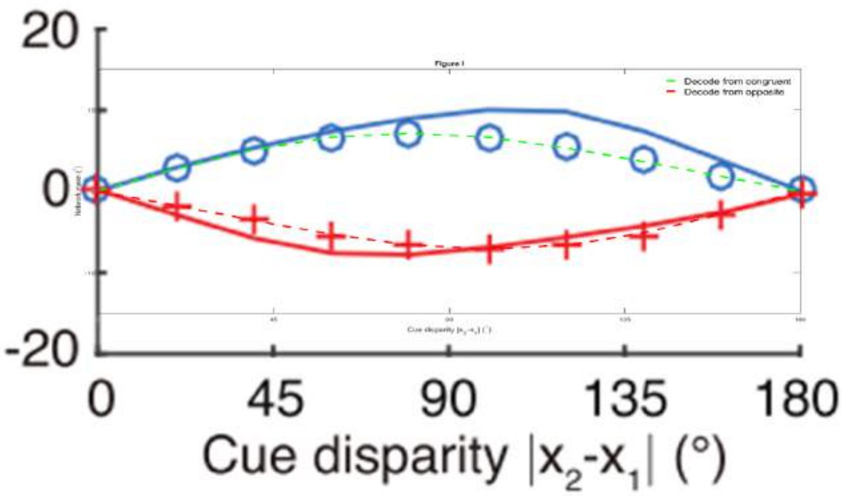
$$k_{1u} = ln(B_1 + B_2) + ln(1 + \frac{B_1}{B_1 + B_2} \frac{J_{rp}}{J_{rc}} e^{k_{1r}(cos\Delta s - 1)})$$
$$k_{1u} = ln(B_1 + B_2) + \frac{B_1}{B_1 + B_2} \frac{J_{rp}}{J_{rc}} e^{k_{1r}(cos\Delta s - 1)}$$

For opposite group, just replace Δs with $\Delta s + \pi$.

Result:

Figure G

mean



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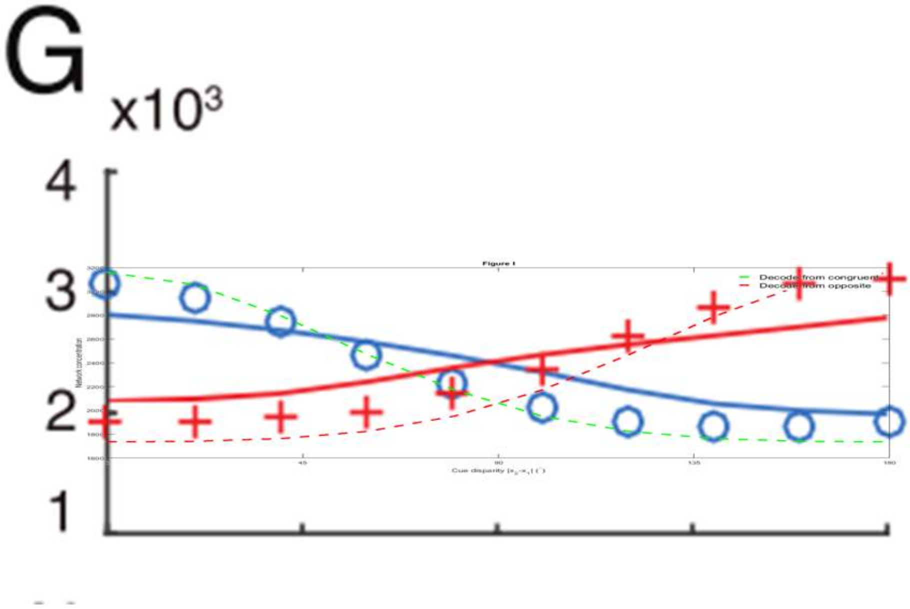
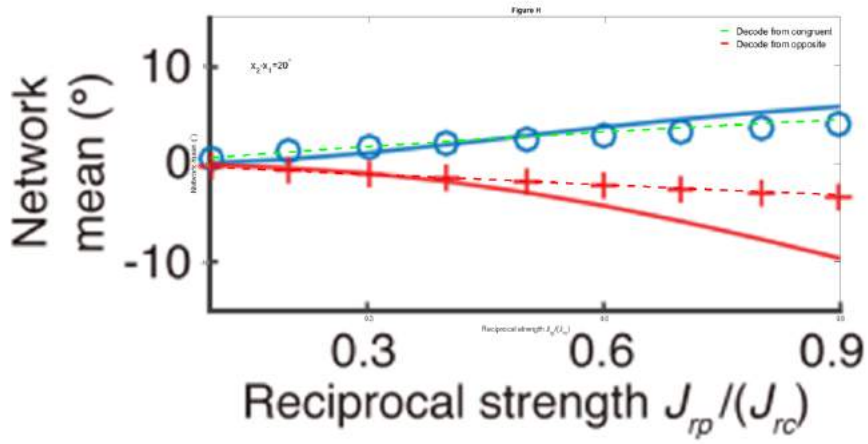


Figure H
mean



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