Bayesian Model for Multisensory Integration and Segregation

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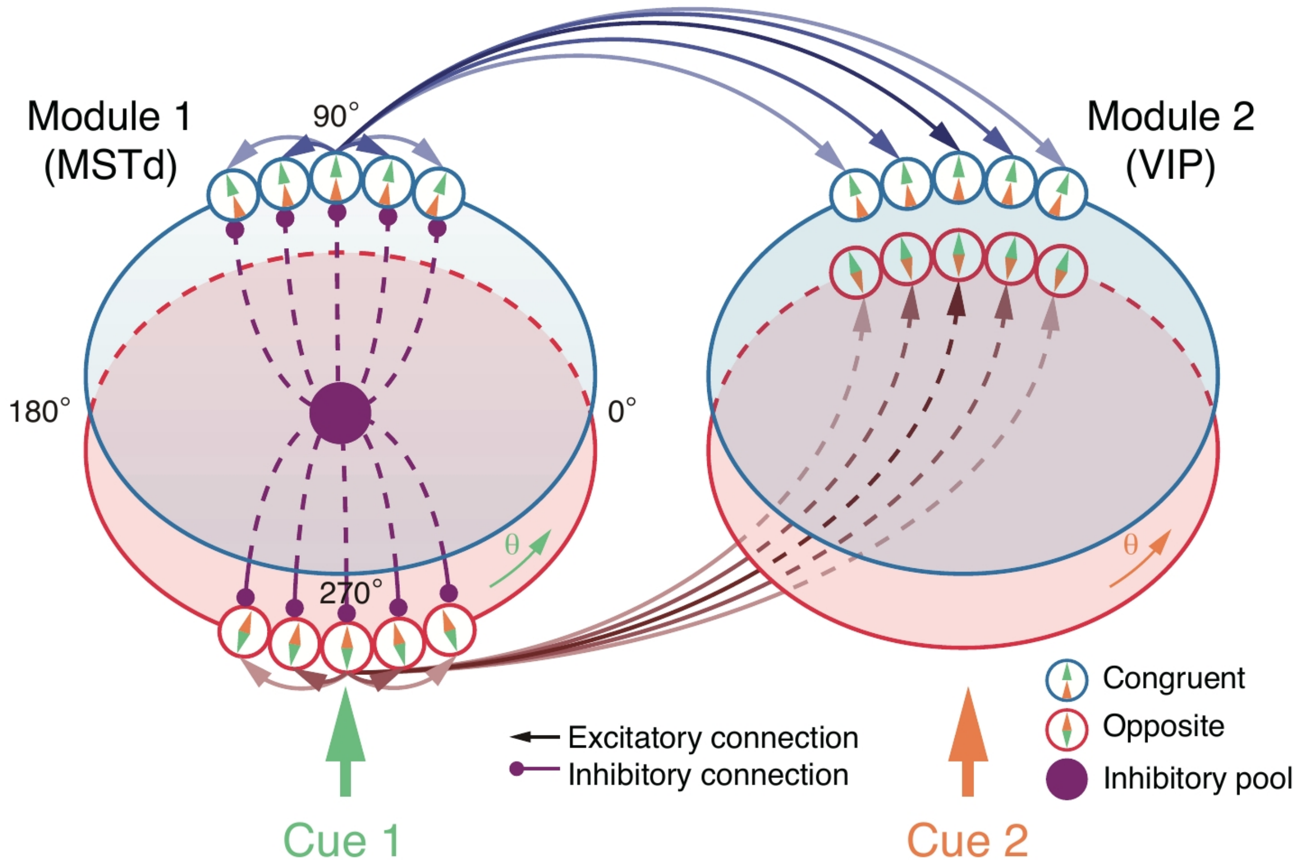
Near Optimal Model



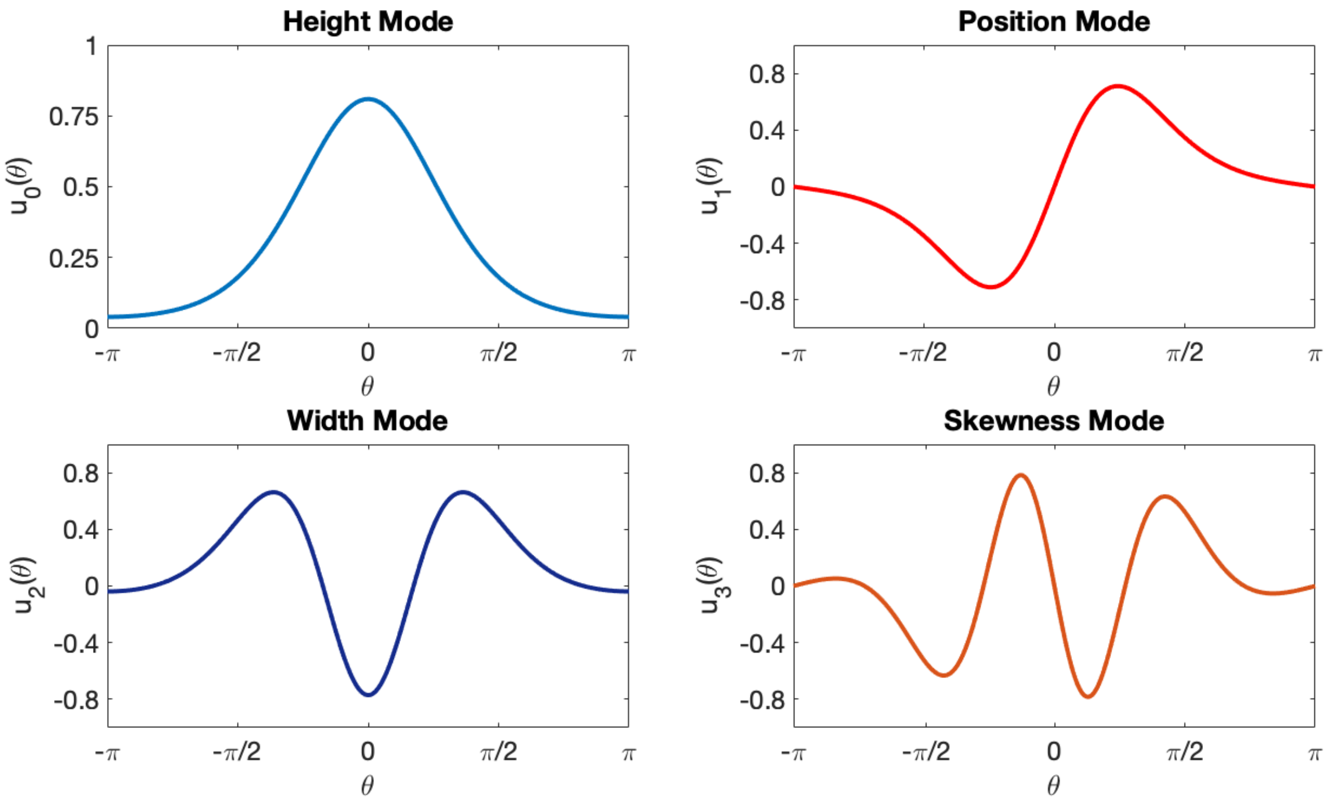
Dynamics of Congruent and Opposite Groups of Neurons

$$\tau \frac{\partial \psi_m(y,t)}{\partial t} = -\psi_m(y,t) + \sum_{y'=-\pi}^{\pi} J_{rc} V(y-y', a_0) R_m(y',t) + \sum_{y'=-\pi}^{\pi} J_{rp} V(y-y', a_0) R_{\bar{m}}(y',t) + I_m^{ext}(y,t)$$

$$\tau \frac{\partial \bar{\psi}_m(y,t)}{\partial t} = -\bar{\psi}_m(y,t) + \sum_{y'=-\pi}^{\pi} J_{rc} V(y-y', a_0) \bar{R}_m(y',t) + \sum_{y'=-\pi}^{\pi} J_{rp} V(y-y'+\pi, a_0) \bar{R}_{\bar{m}}(y',t) + I_m^{ext}(y,t)$$

Basic Functions

When $k = 1.5$



Convolution

We difine two integrals

$$M_{mn}(\theta, k_1, k_2) \equiv \int_{-\pi}^{\pi} \exp[k_1 \cos(\theta - \theta') + k_2 \sin\theta'] \sin^m(\theta - \theta') \sin^n \theta' d\theta'$$

$$T_{mn}(\theta, k_1, k_2) \equiv \int_{-\pi}^{\pi} \exp[k_1 \cos(\theta - \theta') + k_2 \sin\theta'] \sin^m(\theta - \theta') \cos(\theta - \theta') \sin^n \theta' d\theta'$$

Derivative

$$\frac{\partial M_{m,n}}{\partial \theta} = mT_{m-1,n} - k_1 M_{m+1,n}$$

$$\frac{\partial T_{m,n}}{\partial \theta} = -k_1 T_{m+1,n} + mM_{m-1,n} - (m+1)M_{m+1,n}$$

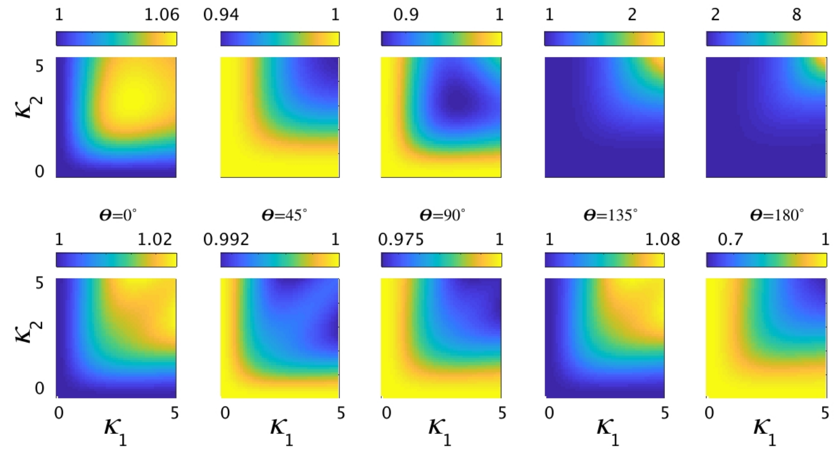
Symmetry

$$M_{m,n}(\theta, k_1, k_2) = M_{n,m}(\theta, k_2, k_1)$$

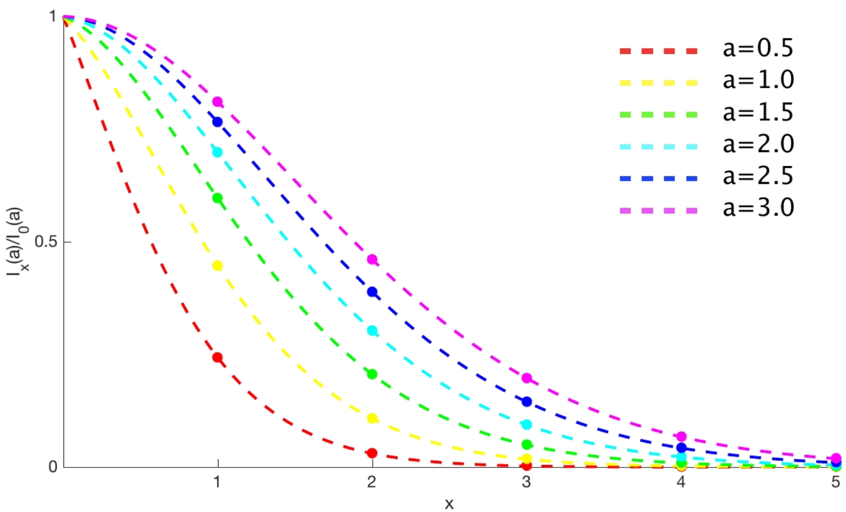
$$T_{m,n}(k_1, k_2) = \cos\theta T_{n,m}(k_2, k_1) + \sin\theta M_{m,n+1}(k_1, k_2)$$

Approximation

A



B



Projection Methods

Bump Profile

Heading direction s_m

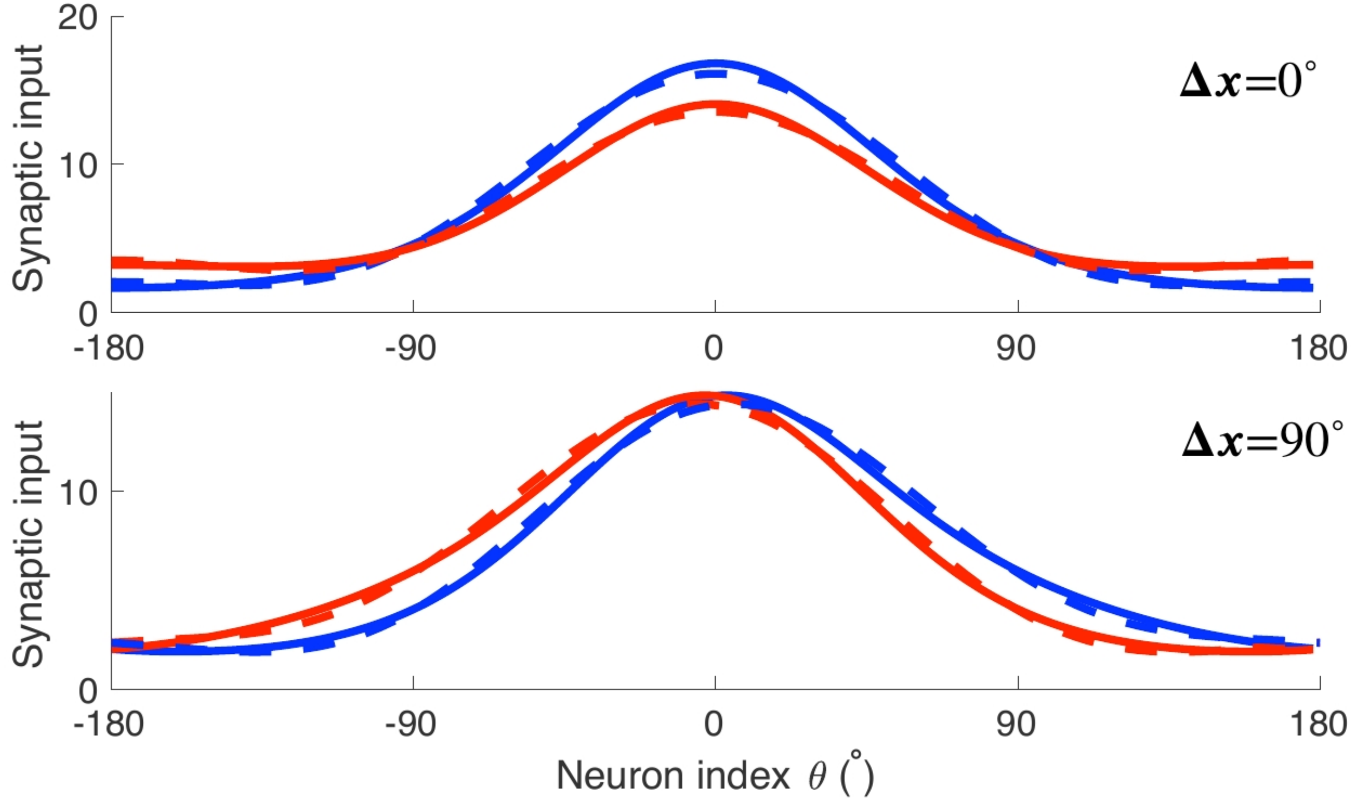
$$\hat{s}_m = \arg(\sum_{y=-\pi}^{\pi} R_m(y,t)e^{jy})$$

Noise Variance

The concentration $\kappa_m = 1/\sigma_m^2$, where

$$\tau\sigma_m^2 = \frac{T_m}{G_{mm} + G_{\bar{m}\bar{m}}} + \frac{T_m G_{\bar{m}\bar{m}}^2 + T_{\bar{m}} G_{m\bar{m}}^2}{(G_{mm}G_{\bar{m}\bar{m}} - G_{m\bar{m}}G_{\bar{m}m})(G_{mm} + G_{\bar{m}\bar{m}})}$$

$$\psi_m = u_{m0} + u_{m1}\cos(y_m - s_m) + u_{m2}\cos 2(y_m - s_m)$$
$$+ u_{m3}\sin 2(y_m - s_m),\ m = 1, 2$$



Incompleteness of Old Model

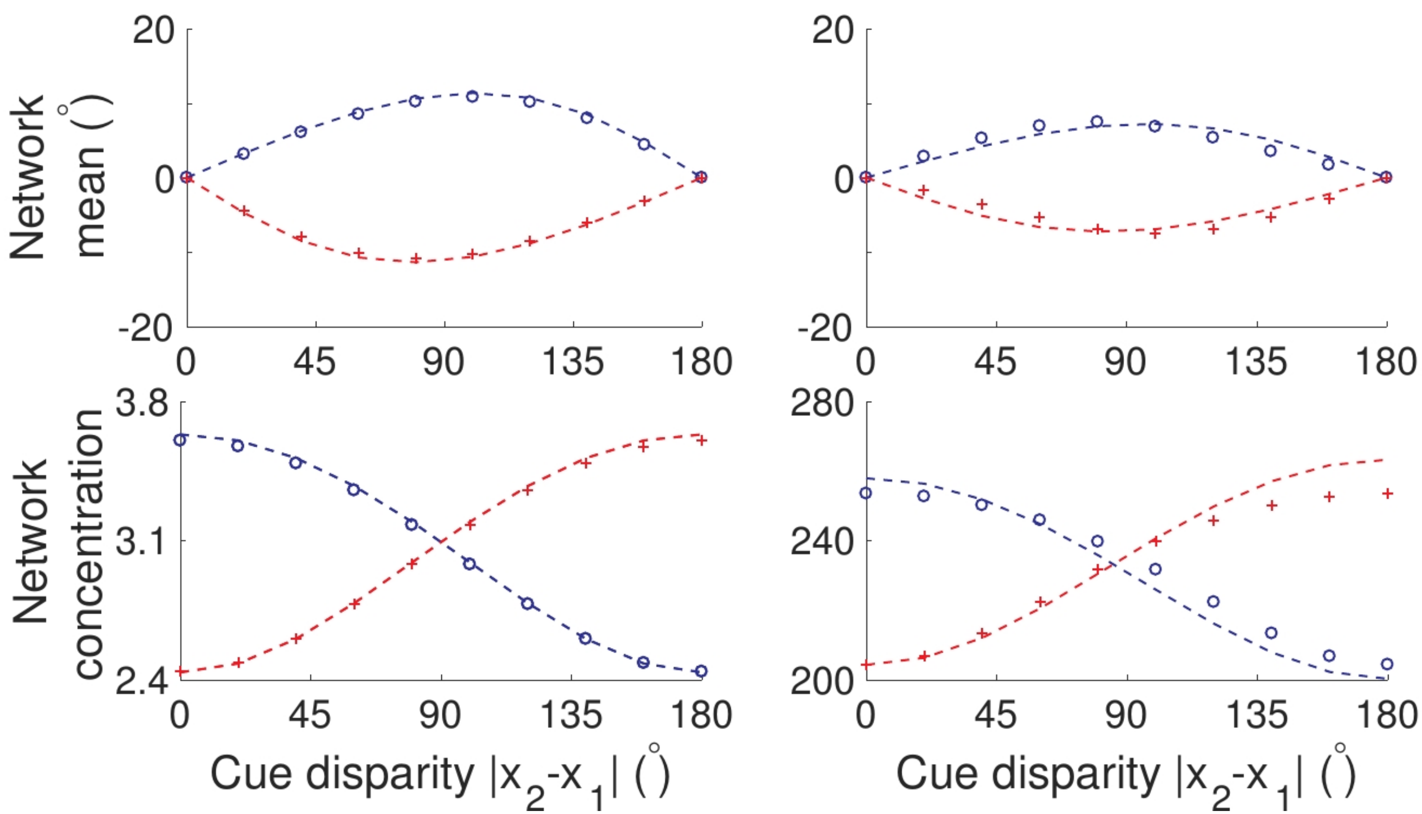
$$\left(\frac{\hat{\kappa}_1 - \hat{\bar{\kappa}}_1}{\hat{\kappa}_1 + \hat{\bar{\kappa}}_1}\right)_{\Delta x=0} = \frac{\left(\frac{u_1^2}{\bar{u}_1^2}\right)_{\Delta x=0} - 1}{\left(\frac{u_1^2}{\bar{u}_1^2}\right)_{\Delta x=0} + 1} \approx 2(s_1 - x_1)_{\Delta s=\frac{\pi}{2}}$$

Output-Dependent Noise

White noise $\sqrt{F_0 \hat{A}}\epsilon_m$, where

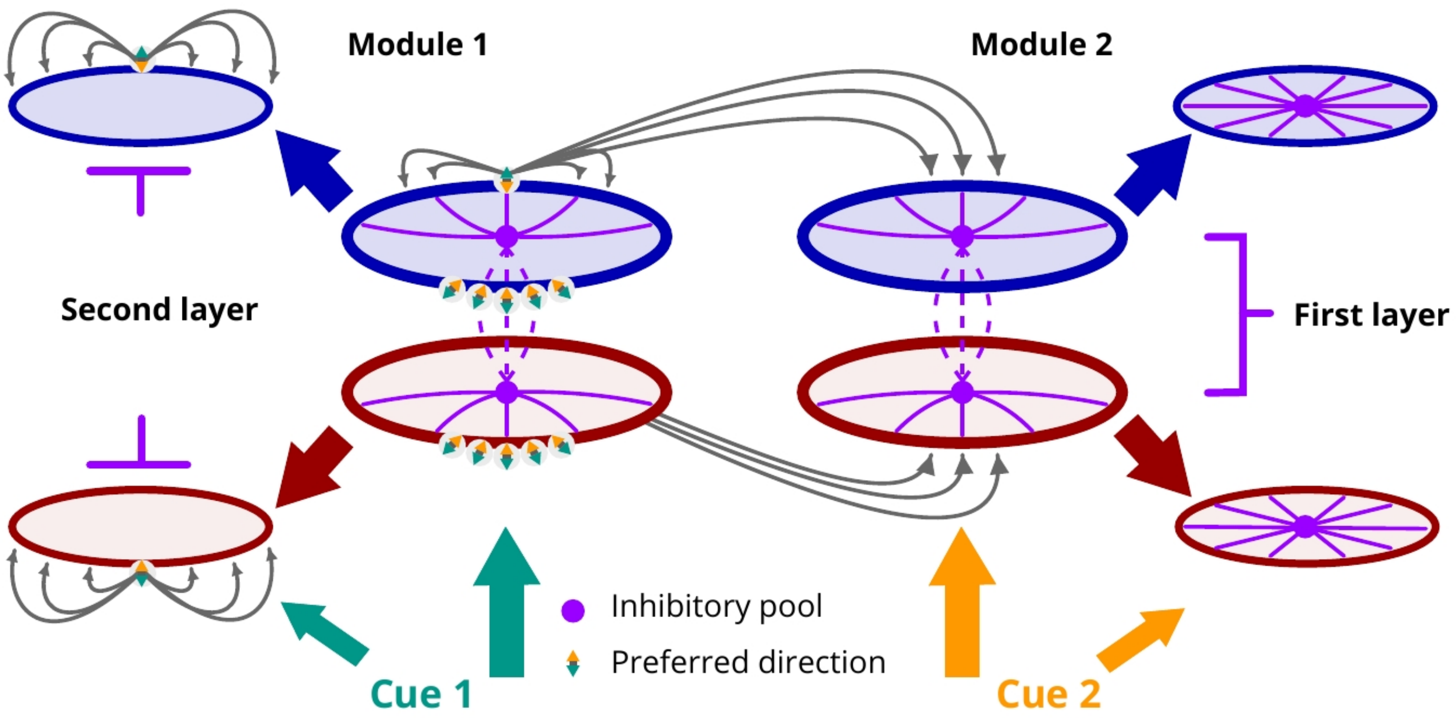
$$\hat{A} \equiv \mathrm{mod}(\frac{1}{N}\sum_{y=-\pi}^{\pi} R_m(y)e^{jy})$$

Results



Network mean (°)
20
0
-20
0 45 90 135 180
20
0
-20
0 45 90 135 180
Network concentration
3.8
3.1
2.4
0 45 90 135 180
280
240
200
0 45 90 135 180
Cue disparity |x2-x1| (°)
Cue disparity |x2-x1| (°)

New Model


Module 1
Module 2
Second layer
First layer
Inhibitory pool
Preferred direction
Cue 1
Cue 2

Prior with an Indepentent Component

Consider the prior $p(s_1, s_2)$ for congruent groups of neurons

$$p(s_1, s_2) = \frac{p_0}{2\pi} V(s_1 - s_2, \kappa_s) + \frac{1 - p_0}{(2\pi)^2}$$

Dynamics of the Second Layer

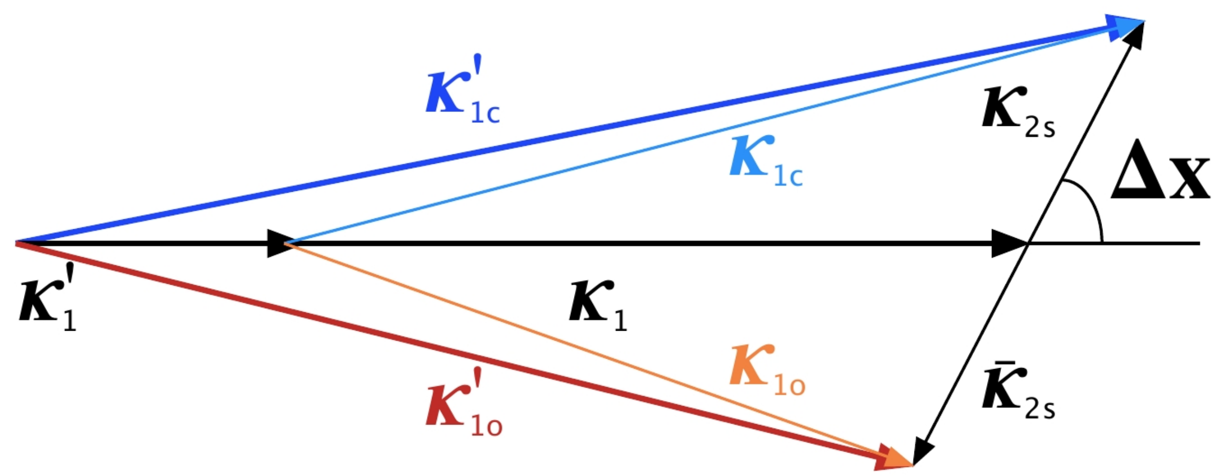
$$\tau \frac{\partial \psi_{2m}(y,t)}{\partial t} = -\psi_{2m}(y,t) + \sum_{y'=-\pi}^{\pi} J_{rc} V(y - y', a_0) R_{2m}(y', t)$$
$$+ p_0 \sum_{y'=-\pi}^{\pi} c_k \cos(y - y') R_m(y', t) + I_m^{ext}(y, t)$$

Information Segregation

For opposite groups of neurons

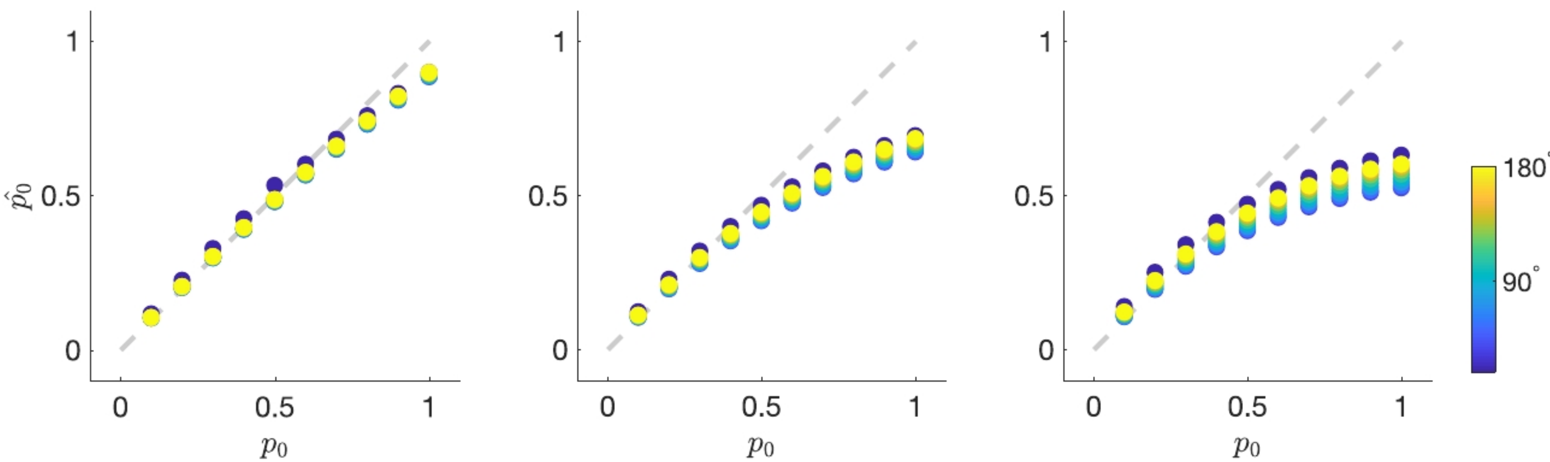
$$p(s_1, s_2) = \frac{p_0}{2\pi} V(s_1 - s_2, -\kappa_s) + \frac{1-p_0}{(2\pi)^2}$$

Vector Space

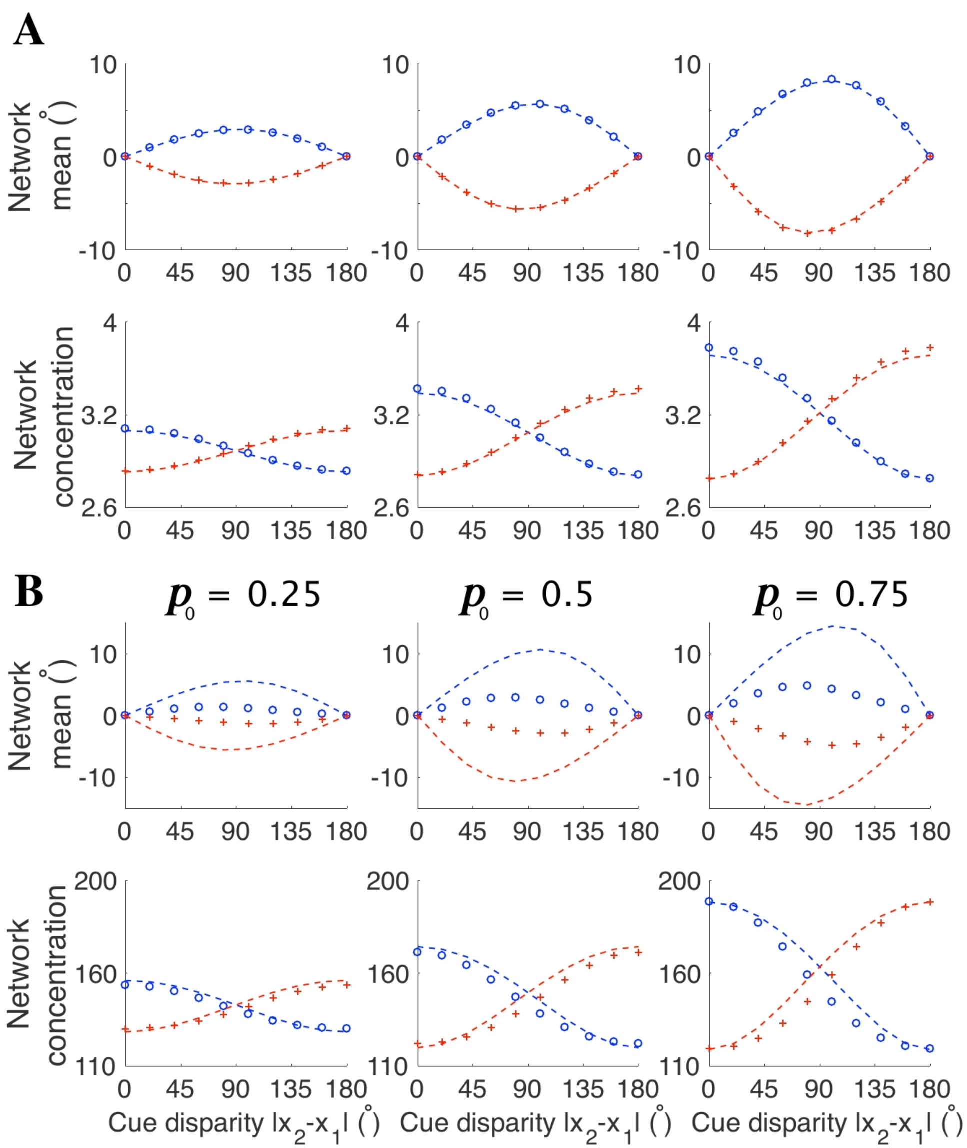


Weak Input Limit

If we let $c_k = \frac{J_{rc}+J_{rp}}{1-\sqrt{1-4\rho(J_{rc}+J_{rp})}}$, then the probability of correlation $\hat{p}_0 = 1 - \frac{|\kappa_{1c}'|\cos s_1' - |\kappa_{1c}|\cos s_1}{\lambda|\kappa_{1o}'|\cos \bar{s}_1' + |\kappa_{1c}'|\cos s_1'}(\lambda+1)$, where $\lambda = -\frac{|\kappa_{1c}'|\sin s_1'}{|\kappa_{1o}'|\sin \bar{s}_1'}$



Outputs from the Second Layer



Future Work

Bayesian Model

It is possible to generalize the model to prior distributions with more than two components

Casual Inference

The probabilistic model $p(x, s, C) = p(x|s)p(s|C)p(C)$

$$p(x, s, C) \propto \exp[-\frac{(s_1 - x_1)^2}{2\sigma_1^2} - \frac{(s_2 - x_2)^2}{2\sigma_2^2} - C\frac{(s_2 - s_1)^2}{2\sigma_3^2}]$$

Thank you for your attention!