Fourier Input

Here we suppose the solution to steady state equation is a linear combination of Fourier basic functions

$$\psi_{1c} = u_0 + u_1 cos(y_1 - s_1) + u_2 cos2(y_1 - s_1) + u_3 sin2(y_1 - s_1)$$

And ψ square:

$$\psi_{1c}^2 = (u_0^2 + \frac{u_1^2}{2} + \frac{u_2^2}{2} + \frac{u_3^2}{2}) + (2u_0u_1 + u_1u_2)cos(y_1 - s_1) + u_1u_3 sin(y_1 - s_1) + (2u_0u_2 + \frac{u_1^2}{2})cos2(y_1 - s_1) + \\ 2u_0u_3 sin2(y_1 - s_1) + u_1u_2 cos3(y_1 - s_1) + u_1u_3 sin3(y_1 - s_1) + (\frac{u_2^2}{2} - \frac{u_3^2}{2})cos4(y_1 - s_1) + u_2u_3 sin4(y_1 - s_1)$$

Congruent Group's Equation

$$\psi(y_1) = \frac{\rho J_{rc}}{D_1} \int dy_2 V(y_1 - y_2, a_0)\psi^2(y_2) + \frac{\rho J_{rp}}{D_2} \int dy_2 V(y_1 - y_2, a_0)\bar{\psi}^2(y_2) + I_1 V(y_1 - x_1, a_0/2) + I_{12} V(y_1 - x_2, a_0/2) + I_b \quad (1)$$

Note that opposite groups' inputs from other modules have π phase shift.

Global Inhibition

$$D_n = 1 + \omega \int \rho \left[u_n^2(x,k) + J_{int} u_{\bar{n}}^2(x,k)\right] dx = 1 + \pi\omega\rho \left[2u_0^2 + u_1^2 + u_2^2 + u_3^2 + J_{int}(2\bar{u}_0^2 + \bar{u}_1^2 + \bar{u}_2^2 + \bar{u}_3^2)\right]$$

Let's define:

$$B(n,k) = \frac{I_n(k)}{I_0(k)}$$

The square term:

$$\frac{\rho J_{rc}}{D_1} \int dy_2 V(y_1 - y_2, a_0)\psi^2(y_2) = \frac{\rho J_{rc}}{D_1} \Bigg[(u_0^2 + \frac{u_1^2}{2} + \frac{u_2^2}{2} + \frac{u_3^2}{2}) + (2u_0u_1 + u_1u_2)B(1,a_0)cos(y_1 - s_1) \\ + u_1u_3 B(1,a_0)sin(y_1 - s_1) + (2u_0u_2 + \frac{u_1^2}{2})B(2,a_0)cos2(y_1 - s_1) + 2u_0u_3 B(2,a_0)sin2(y_1 - s_1) + u_1u_2 B(3,a_0)cos3(y_1 - s_1) \\ + u_1u_3 B(3,a_0)sin3(y_1 - s_1) + (\frac{u_2^2}{2} - \frac{u_3^2}{2})B(4,a_0)cos4(y_1 - s_1) + u_2u_3 B(4,a_0)sin4(y_1 - s_1) \Bigg]$$

Projection

Multiply both sides by 1 $cos(y_1 - s_2)$ $sin(y_1 - s_2)$ $cos2(y_1 - s_2)$ $sin2(y_1 - s_2)$ and integrate over y_1

$$u_{10} = \frac{\rho J_{rc}}{D_1}\left[u_{10}^2 + \frac{u_{11}^2}{2} + \frac{u_{12}^2}{2} + \frac{u_{13}^2}{2}\right] + \frac{\rho J_{rp}}{D_2}\left[u_{20}^2 + \frac{u_{21}^2}{2} + \frac{u_{22}^2}{2} + \frac{u_{23}^2}{2}\right] + \frac{I_1}{2\pi} + \frac{I_{12}}{2\pi} + I_b$$

$$u_{11} = \frac{\rho J_{rc}}{D_1}(2u_{10}u_{11} + u_{11}u_{12})B(1,a_0) + \frac{I_1 B(1,a_0/2)}{\pi}cos(x_1 - s_1) + \frac{I_{12}B(1,a_0/2)}{\pi}cos(x_2 - s_1) + \\ \frac{\rho J_{rp}}{D_2}[(2u_{20}u_{21} + u_{21}u_{22})B(1,a_0)cos(s_2 - s_1) - u_{21}u_{23}B(1,a_0)sin(s_2 - s_1)]$$

$$0 = \frac{\rho J_{rc}}{D_1}u_{11}u_{13}B(1,a_0) + \frac{I_1 B(1,a_0/2)}{\pi}sin(x_1 - s_1) + \frac{I_{12}B(1,a_0/2)}{\pi}sin(x_2 - s_1) + \\ \frac{\rho J_{rp}}{D_2}[(2u_{20}u_{21} + u_{21}u_{22})B(1,a_0)sin(s_2 - s_1) + u_{21}u_{23}B(1,a_0)cos(s_2 - s_1)]$$

$$u_{12}=\frac{\rho J_{rc}}{D_1}(2u_{10}u_{12}+\frac{u_{11}^2}{2})B(2,a_0)+\frac{I_1B(2,a_0/2)}{\pi}cos2(x_1-s_1)+\frac{I_{12}B(2,a_0/2)}{\pi}cos2(x_2-s_1)+$$
$$\frac{\rho J_{rp}}{D_2}\left[(2u_{20}u_{22}+\frac{u_{21}^2}{2})B(2,a_0)cos2(s_2-s_1)-2u_{20}u_{23}B(2,a_0)sin2(s_2-s_1)\right]$$

$$u_{13}=\frac{\rho J_{rc}}{D_1}2u_{10}u_{13}B(2,a_0)+\frac{I_1B(2,a_0/2)}{\pi}sin2(x_1-s_1)+\frac{I_{12}B(2,a_0/2)}{\pi}sin2(x_2-s_1)+$$
$$\frac{\rho J_{rp}}{D_2}\left[(2u_{20}u_{22}+\frac{u_{21}^2}{2})B(2,a_0)sin2(s_2-s_1)+2u_{20}u_{23}B(2,a_0)cos2(s_2-s_1)\right]$$

Mean

Firing rate $R_i(s_1,u_0,u_1,u_2,u_3)=\frac{\psi_i^2}{D_i}$, then $\hat{s}$ will be

$$\hat{s}_i=arg(\sum_{-\pi}^{\pi}R_ie^{j\theta})$$

$Real\ Part=\pi(2u_0u_1+u_1u_2)cos(s)-\pi u_1u_3sin(s)$
$Imaginary\ Part=\pi(2u_0u_1+u_1u_2)sin(s)+\pi u_1u_3cos(s)$

So

$$\hat{s}=atan2[(2u_0u_1+u_1u_2)sin(s)+u_1u_3cos(s),(2u_0u_1+u_1u_2)cos(s)-u_1u_3sin(s)]$$

Concentration

Next we need to take noise into consideration.

$$\tau\frac{\partial}{\partial t}\psi(y_1)=-\psi(y_1)+\frac{\rho J_{rc}}{D_1}\int dy_2V(y_1-y_2,a_0)\psi^2(y_2)+\frac{\rho J_{rp}}{D_2}\int dy_2V(y_1-y_2,a_0)\bar{\psi}^2(y_2)+I_1V(y_1-x_1,a_0/2)+I_b$$
$$+\sqrt{FI_1V(y_1-x_1,a_0/2)}\xi_1+\sqrt{FI_{12}V(y_1-x_2,a_0/2)}\xi_{12}+\sqrt{FI_b}\epsilon_1$$

Consider the dynamics of the displacement mode and multiply both sides by $sin(y_1-s_1)$, integrate over y_1

$$\tau\frac{\partial}{\partial t}\delta s_1=-\delta s_1+\frac{\rho J_{rc}}{D_1u_{11}}(2u_{10}u_{11}+u_{11}u_{12})B(1,a_0)\delta s_1+$$
$$\frac{\rho J_{rp}}{D_2u_{11}}[(2u_{20}u_{21}+u_{21}u_{22})B(1,a_0)cos(s_2-s_1)-u_{21}u_{23}B(1,a_0)sin(s_2-s_1)]\,\delta s_2+$$
$$\frac{\sqrt{FI_1}}{\pi u_{11}}\int\sqrt{V(y_1-x_1,a_0/2)}sin(y_1-s_1)\xi_1dy_1+\frac{\sqrt{FI_{12}}}{\pi u_{11}}\int\sqrt{V(y_1-x_2,a_0/2)}sin(y_1-s_1)\xi_{12}dy_1+\frac{\sqrt{FI_b}}{\pi u_{11}}\int sin(y_1-s_1)\epsilon_1dy_1$$

Noise Temperature

$$T_1=\frac{F}{2\pi^2\rho u_{11}^2}\left[I_1\int V(y_1-x_1,a_0/2)sin^2(y_1-s_1)dy_1+I_{12}\int V(y_1-x_2,a_0/2)sin^2(y_1-s_1)dy_1+I_b\int sin^2(y_1-s_1)dy_1\right]$$
$$=\frac{F}{2\pi^2\rho u_{11}^2}\left[(\frac{I_1}{2}+\frac{I_{12}}{2}+\pi I_b)-\frac{I_1}{2}B(2,a_0/2)cos2(x_1-s_1)-\frac{I_{12}}{2}B(2,a_0/2)cos2(x_2-s_1)\right]$$

And T_2

$$T_2=\frac{F}{2\pi^2\rho u_{21}^2}\left[(\frac{I_2}{2}+\frac{I_{21}}{2}+\pi I_b)-\frac{I_2}{2}B(2,a_0/2)cos2(x_2-s_2)-\frac{I_{21}}{2}B(2,a_0/2)cos2(x_1-s_2)\right]$$

Matrix

$$G_{11}=1-\frac{\rho J_{rc}}{D_1u_{11}}(2u_{10}u_{11}+u_{11}u_{12})B(1,a_0)$$
$$G_{12}=-\frac{\rho J_{rp}}{D_2u_{11}}[(2u_{20}u_{21}+u_{21}u_{22})B(1,a_0)cos(s_2-s_1)-u_{21}u_{23}B(1,a_0)sin(s_2-s_1)]$$
$$G_{21}=-\frac{\rho J_{rp}}{D_1u_{21}}[(2u_{10}u_{11}+u_{11}u_{12})B(1,a_0)cos(s_1-s_2)-u_{11}u_{13}B(1,a_0)sin(s_1-s_2)]$$
$$G_{22}=1-\frac{\rho J_{rc}}{D_2u_{21}}(2u_{20}u_{21}+u_{21}u_{22})B(1,a_0)$$

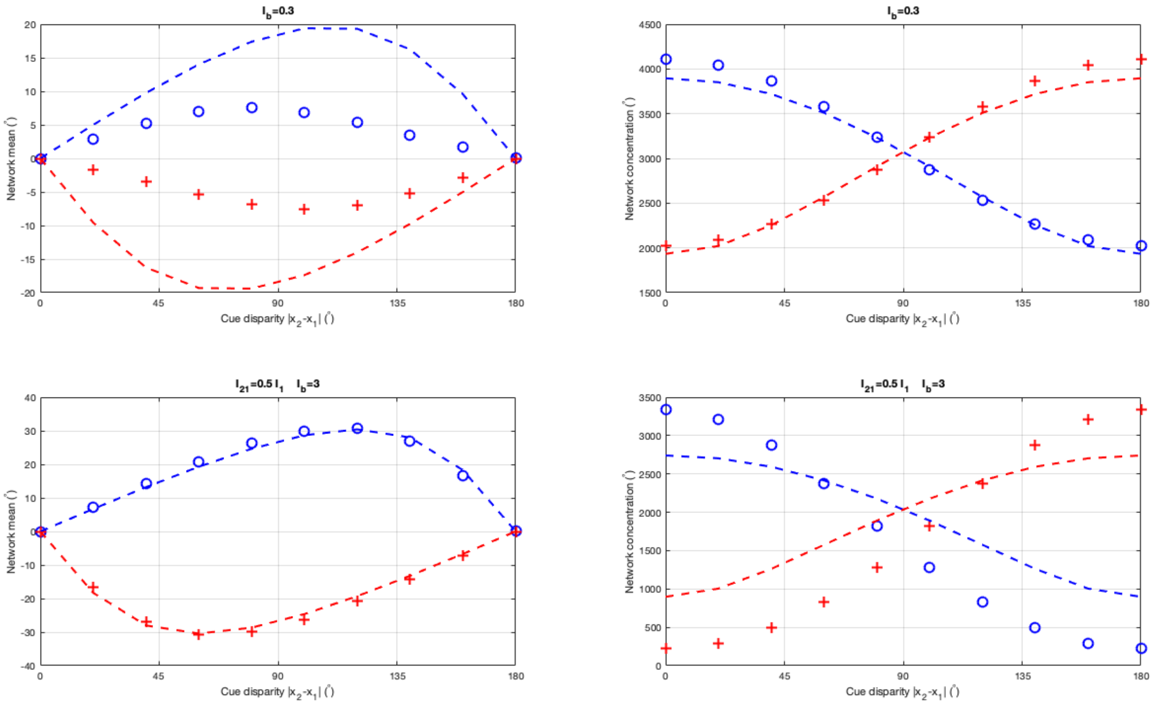
Variance

$$\sigma_1^2 = \frac{T_1}{(G_{11} + G_{22})\tau} + \frac{T_1 G_{22}^2 + T_2 G_{12}^2}{(G_{11}G_{22} - G_{12}G_{21})(G_{11} + G_{22})\tau}$$

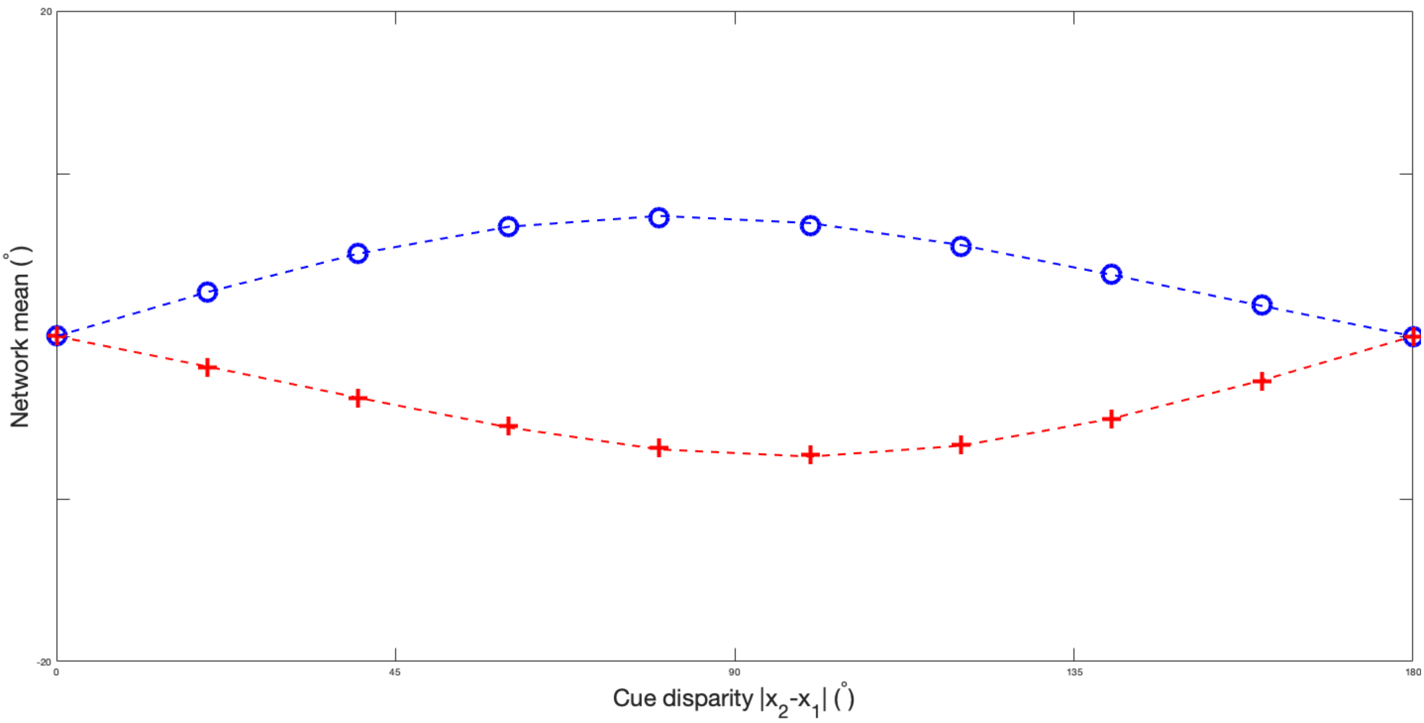
Concentration

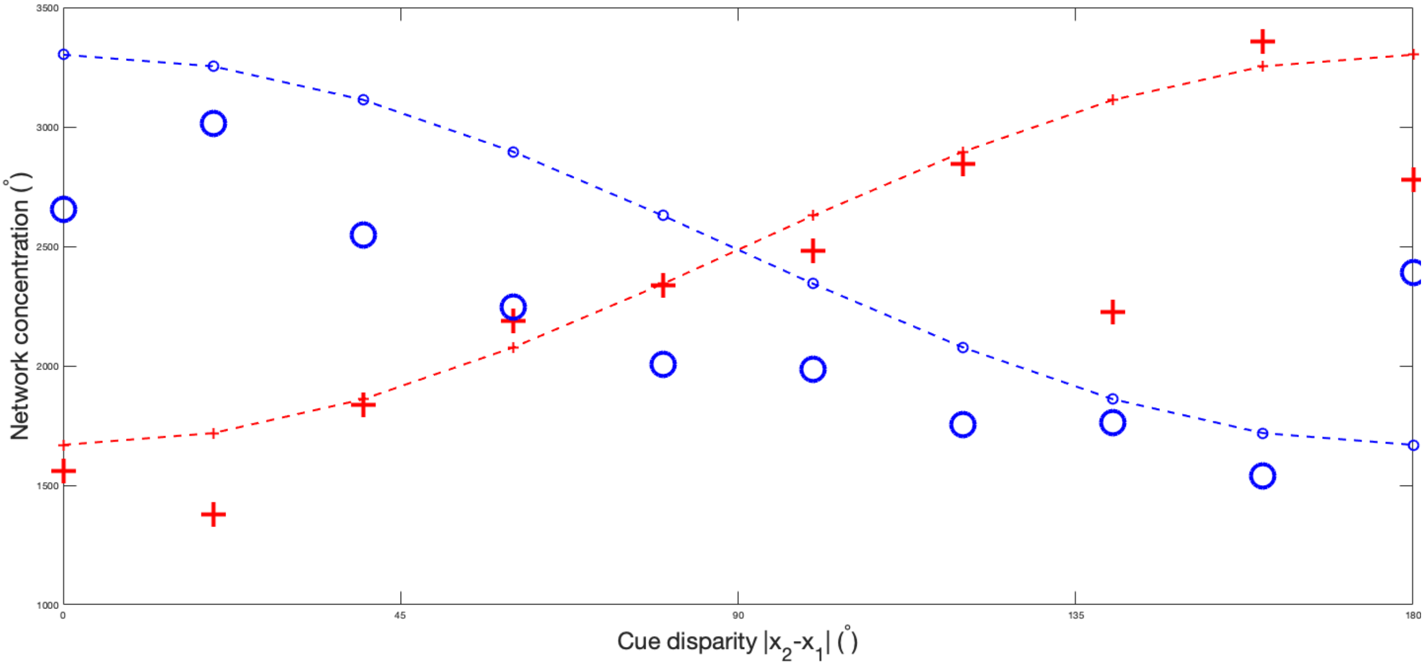
$$\kappa_1 = \frac{1}{\sigma_1^2}$$

Results



Wenhao Simulation




3500
3000
2500
2000
1500
1000
0
45
90
135
180
Network concentration (°)
Cue disparity $|x_2-x_1|$ (°)