Fourier Discussion

To simplify this task, we consider $I_1 = I_2 = I_n$.

Projection

Mean

When $\Delta s = \frac{\pi}{2}$ $(\Delta x > \frac{\pi}{2})$, project the dynamic equation onto different modes.

$$u_0 = \frac{\rho(J_{rc} + J_{rp})}{D_n}(u_0^2 + \frac{u_1^2}{2} + \frac{u_2^2}{2} + \frac{u_3^2}{2}) + \frac{I_n}{2\pi} + I_b$$

$$u_1 = \frac{\rho J_{rc}}{D_n} B(1, a_0)(2u_0u_1 + u_1u_2) - \frac{\rho J_{rp}}{D_n} B(1, a_0) u_1 u_3 + \frac{I_n B(1, a_0/2)}{\pi}$$

$$0 = \frac{\rho J_{rc}}{D_n} B(1, a_0) u_1 u_3 + \frac{\rho J_{rp}}{D_n} B(1, a_0)(2u_0u_1 + u_1u_2) - \frac{I_n B(1, a_0/2)}{\pi} s_1$$

$$u_2 = \frac{\rho(J_{rc} - J_{rp})}{D_n}(2u_0u_2 + \frac{u_1^2}{2}) B(2, a_0) + \frac{I_n B(2, a_0/2)}{\pi}$$

$$u_3 = \frac{\rho(J_{rc} - J_{rp})}{D_n} 2u_0u_3 B(2, a_0) - \frac{I_n B(2, a_0/2)}{\pi} 2s_1$$

Since $u_3 \to 0$, we obtain

$$\frac{s_1}{u_1} \approx \frac{\pi}{I_n B(1, a_0/2)} \frac{\rho J_{rp}}{D_n} B(1, a_0)(2u_0 + u_2)_{\Delta s = \frac{\pi}{2}}$$

Concentration

When $\Delta s = 0$ $(\Delta x = 0)$, project the dynamic equation onto different modes.

$$u_0 = \frac{\rho(J_{rc} + J_{rp})}{D_n}(u_0^2 + \frac{u_1^2}{2} + \frac{u_2^2}{2} + \frac{u_3^2}{2}) + \frac{I_n}{2\pi} + I_b$$

$$u_1 = \frac{\rho(J_{rc} + J_{rp})}{D_n} B(1, a_0)(2u_0u_1 + u_1u_2) + \frac{I_n B(1, a_0/2)}{\pi}$$

$$0 = \frac{\rho(J_{rc} + J_{rp})}{D_n} B(1, a_0) u_1 u_3 - \frac{I_n B(1, a_0/2)}{\pi} s_1$$

$$u_2 = \frac{\rho(J_{rc}+J_{rp})}{D_n}(2u_0u_2 + \frac{u_1^2}{2})B(2,a_0) + \frac{I_nB(2,a_0/2)}{\pi}$$

$$u_3 = \frac{\rho(J_{rc}+J_{rp})}{D_n}2u_0u_3B(2,a_0) - \frac{I_nB(2,a_0/2)}{\pi}2s_1$$

Matrix

$$G_{11} = 1 - \frac{\rho J_{rc}}{D_n}(2u_0+u_2)B(1,a_0),\ \bar{G}_{11} = 1 - \frac{\rho J_{rc}}{\bar{D}_n}(2\bar{u}_0+\bar{u}_2)B(1,a_0)$$

$$G_{12} = -\frac{\rho J_{rp}}{D_n}(2u_0+u_2)B(1,a_0),\ \bar{G}_{12} = \frac{\rho J_{rp}}{\bar{D}_n}(2\bar{u}_0+\bar{u}_2)B(1,a_0)$$

Temperature

$$T_n = \frac{F}{2\pi^2\rho u_1^2}\left[\frac{I_n}{2}(1-B(2,a_0/2)) + \pi I_b\right]$$

$$\bar{T}_n = \frac{F}{2\pi^2\rho \bar{u}_1^2}\left[\frac{I_n}{2}(1-B(2,a_0/2)) + \pi I_b\right]$$

Finally we obtain

$$\sigma_1^2 = \frac{G_{11}T_n}{(G_{11}^2 - G_{12}^2)\tau}$$

$$\kappa_1 = \frac{1}{\sigma_1^2} = \frac{\tau}{T_n}(G_{11} - \frac{G_{12}^2}{G_{11}}) \approx \frac{\tau}{T_n}$$

$$\frac{\kappa_1}{\bar{\kappa}_1} \approx \frac{u_1^2}{\bar{u}_1^2}$$

Approximation

When $\Delta s = 0$, divide both sides by u_1

$$1 = \frac{\rho(J_{rc}+J_{rp})}{D_n}B(1,a_0)(2u_0+u_2)_{\Delta s=0} + \frac{I_nB(1,a_0/2)}{\pi u_1}$$

Varying Δs, we assume the first term in the right hand side remain unchanged. We replace the first term with $\frac{s_1}{u_1}$.

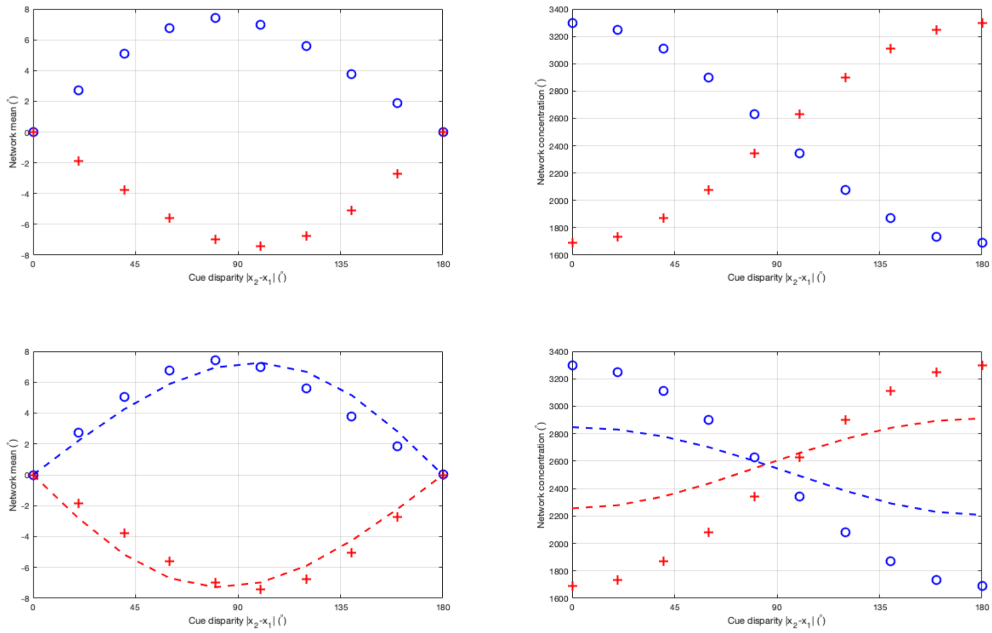
$$u_1 = \frac{I_nB(1,a_0/2)}{\pi}\left[1 + \frac{J_{rc}+J_{rp}}{Jrp}s_1\right]$$

$$\bar{u}_1 = \frac{I_nB(1,a_0/2)}{\pi}\left[1 + \frac{J_{rc}-J_{rp}}{Jrp}\bar{s}_1\right]$$

Assuming $s_1 \approx \bar{s}_1$, we consider the first order Taylor expansion of the ratio.

$$\frac{u_1^2}{\bar{u}_1^2} \approx \left[\frac{1 + (\frac{J_{rc}}{J_{rp}} + 1)s_1}{1 + (\frac{J_{rc}}{J_{rp}} - 1)s_1} \right]^2 \approx 1 + 4s_1$$

$$\frac{\kappa - \bar{\kappa}}{\kappa + \bar{\kappa}} \approx \frac{4s_1}{2 + 4s_1} \approx 2s_1$$



Discussion