Bayesian Model for Multisensory Integration and Segregation

Xiangyu Ma[1], He Wang[1], Min Yan[1], Wen-Hao Zhang[2], and K. Y. Michael Wong[1].
[1]Hong Kong University of Science and Technology, [2]University of Pittsburgh.

Abstract

Multisensory integration and segregation are distinctly important for the survivorship of higher animals. The experimental data indicate that the brain processes information in a Bayesian way. The model proposed by Wenhao has demonstrated two different neuron groups are related to the multisensory integration and segregation simultaneously. However, the details of the dynamics of this neuron system remain unknown. In this paper, we will discuss the conditions of this neuron network using Bayesian inference to process complementary information.

Keywords: Bayesian inference; multisensory integration and segregation; continuous attractor neural network

Introduction

In Zhang et. al., the neural circuit consists of two modules, each of them contains two groups of excitatory neurons, congruent and opposite neurons. The neuronal dynamics of congruent groups is given by

$$\tau\frac{\partial\psi_m(y,t)}{\partial t} = -\psi_m(y,t) + \frac{J_{rc}}{D_m}\sum_{-\pi}^{\pi}V(y-y',a_0)\psi_m^2(y') + \frac{J_{rp}}{D_{\bar{m}}}\sum_{-\pi}^{\pi}V(y-y',a_0)\psi_{\bar{m}}^2(y') + I_1V(y-x,a_0/2) + I_b \qquad (1)$$

where $\bar{m}$ denotes the complementary index. For opposite groups

$$\tau\frac{\partial\bar{\psi}_m(y,t)}{\partial t} = -\bar{\psi}_m(y,t) + \frac{J_{rc}}{\bar{D}_m}\sum_{-\pi}^{\pi}V(y-y',a_0)\bar{\psi}_m^2(y') + \frac{J_{rp}}{\bar{D}_{\bar{m}}}\sum_{-\pi}^{\pi}V(y-y'+\pi,a_0)\bar{\psi}_{\bar{m}}^2(y') + I_1V(y-x,a_0/2) + I_b \qquad (2)$$

However, this network is near optimal but lacking a theoretical understanding of the dynamics. In this paper, we are going to modify this neural network for a generic Bayesian model and discuss the conditions of such a Bayesian approach.

Projection Methods

Suppose the solution to steady state equations is a linear combination of Fourier series

$$\psi_1 = u_0 + u_1\cos(y_1-s_1) + u_2\cos 2(y_1-s_1) + u_3\sin 2(y_1-s_1)$$

For congruent groups, the steady state equation is given by

$$\psi(y_1) = \frac{\rho J_{rc}}{D_1}\int dy_2 V(y_1-y_2,a_0)\psi^2(y_2) + \frac{\rho J_{rp}}{D_2}\int dy_2 V(y_1-y_2,a_0)\bar{\psi}^2(y_2) + I_1V(y_1-x_1,\frac{a_0}{2}) + I_b$$

Furthermore we define $B_n(k) = \frac{I_n(k)}{I_0(k)}$, where I_n is the modified Bessel function of the first kind. Referring to inhibition pool, D_m could be replace by integration $D_m = 1 + \pi\omega\rho[2u_0^2 + u_1^2 + u_2^2 + u_3^2 + J_{int}(2\bar{u}_0^2 + \bar{u}_1^2 + \bar{u}_2^2 + \bar{u}_3^2)]$. Multiply the steady state equation's both sides by $1\ \cos(y_1-s_1)\ \sin(y_1-s_1)\ \cos 2(y_1-s_1)\ \sin 2(y_1-s_1)$ and integrate over y_1, we obtain different steady state equations of each mode respectively.

Next we need take noise into consideration. Gaussian noise $\sqrt{F_0 I_1 V(y-x,a_0/2)}\xi_1 + \sqrt{F_0 I_b}\varepsilon_1$ will be added to Eq. (1) and Eq. (2), where F_0 is the Fano factor, $\xi_m(\theta,t)$ and $\varepsilon_m(\theta,t)$ is the Gaussian white noise of zero mean and variance satisfying $\langle\xi_m(\theta,t),\xi_{m'}(\theta',t')\rangle = \delta_{mm'}\delta(\theta-\theta')\delta(t-t')$, and $\langle\varepsilon_m(\theta,t),\varepsilon_{m'}(\theta',t')\rangle = \delta_{mm'}\delta(\theta-\theta')\delta(t-t')$.Consider the dynamics of displacement mode and multiply both sides by $sin(y_1-s_1)$, integrate over y_1

$$\tau\frac{\partial}{\partial t}\delta s_1 = -\delta s_1 + \frac{\rho J_{rc}}{D_1u_{11}}B_1(a_0)\left[2u_{10}u_{11} + u_{11}u_{12}\right]\delta s_1 + \frac{\rho J_{rp}}{D_2u_{11}}B_1(a_0)u_{21}\left[(2u_{20}+u_{22})cos\Delta s - u_{23}sin\Delta s\right]\delta s_2 + \frac{\sqrt{F_0}}{\pi u_{11}}\int\left[\sqrt{I_1V(y-x,a_0/2)}\xi_1 + \sqrt{I_b}\right]sin(y_1-s_1)\varepsilon_1 dy_1$$

The equivalent noise temperature is given by

$$T_1 = \frac{F_0}{2\pi^2\rho u_{11}^2}\int\left[I_1V(y_1-x_1,a_0/2)+I_b\right]\sin^2(y_1-s_1)dy_1 = \frac{F_0}{2\pi^2\rho u_{11}^2}\left[(\frac{I_1}{2}+\pi I_b) - \frac{I_1}{2}B(2,a_0/2)\cos 2(x_1-s_1)\right]$$

The concentration $\kappa_1 = \frac{1}{\sigma_1^2}$ for the matrix G_{ij}, where

$$\sigma_1^2 = \frac{T_1}{(G_{11}+G_{22})\tau} + \frac{T_1G_{22}^2 + T_2G_{12}^2}{(G_{11}G_{22}-G_{12}G_{21})(G_{11}+G_{22})\tau}$$

Bayesian Inference

We focus on the case that the stimulus strengths are equal, that is $I_1 = I_2 = I$. We consider competition between congruent and opposite group in module 1. The solution is symmetric with respect to the perception displacement. That is $s_1 - x_1 = x_2 - s_2$. We have $G_{11} = G_{22},\ G_{12} = G_{21},\ \bar{G}_{11} = \bar{G}_{22},\ \bar{G}_{12} = \bar{G}_{21}$ because of symmetry. The concentration could be approximate by

$$\kappa_1 = \frac{1}{\sigma_1^2} = \frac{\tau}{T}(G_{11} - \frac{G_{12}^2}{G_{11}}) \approx \frac{\tau}{T}$$

Hence $\frac{\kappa_1}{\bar{\kappa}_1} \approx \frac{u_1^2}{\bar{u}_1^2}$. The information of position and variance is decoded from the second and the third equations.

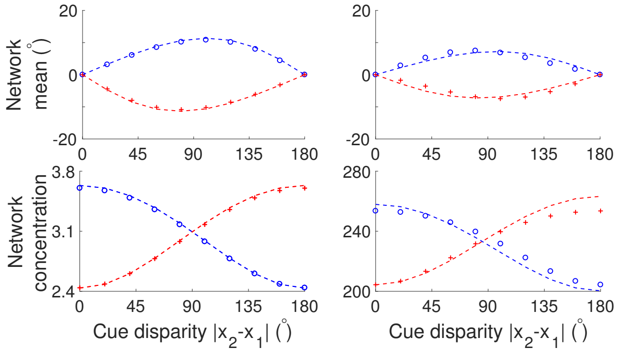

Figure 1: Weak input (left column) and strong input (right column). Symbols: network results; dash lines: Bayesian prediction. The blue and red colors represent congruent and opposite groups in module 1 respectively.

$$1 = HJ_{rc} + HJ_{rp}\cos\Delta s + \frac{F}{u_1}\cos(x_1 - s_1) \tag{3}$$

$$0 = HJ_{rp}\sin\Delta s + \frac{F}{u_1}\sin(x_1 - s_1) \tag{4}$$

where $H = \frac{\rho}{D}(2u_0 + u_2)B_1(a_0)$, $F = \frac{IB_1(a_0/2)}{\pi}$, under the weak input limit H can be treated as a constant. Since HJ_{rp} is small, the ratio of u_1 to $\bar{u}_1$ can be approximated by

$$\left(\frac{u_1}{\bar{u}_1}\right)_{\Delta x=0} = \frac{1 - HJ_{rc} + HJ_{rp}}{1 - HJ_{rc} - HJ_{rp}} \approx 1 + 2(s_1 - x_1)_{\Delta s=\frac{\pi}{2}}$$

$$\left(\frac{\kappa_1 - \bar{\kappa}_1}{\kappa_1 + \bar{\kappa}_1}\right)_{\Delta x=0} = \frac{\left(\frac{u_1^2}{\bar{u}_1^2}\right)_{\Delta x=0} - 1}{\left(\frac{u_1^2}{\bar{u}_1^2}\right)_{\Delta x=0} + 1} \approx 2(s_1 - x_1)_{\Delta s=\frac{\pi}{2}}$$

That is, under weak input limit, if we are going to fit the variance in this network, the angle decoded from population vector will be twice as large as the real angle. This ratio is always greater than 2 when input strength is in normal range.

Multisensory Integration

Here we define the length of population vector $\hat{A} = mod(\frac{1}{N}\sum_{-\pi}^{\pi} R_i e^{j\theta}) = \frac{\rho}{ND}u_1\sqrt{(2u_0 + u_2)^2 + u_3^2}$, $\hat{A}$ is used to rescale the noise variance. We set $J_{int} = 1$ and rewrite the noise term $\sqrt{F_0 I_b \hat{A}}\varepsilon_1$. The noise temperature

$$T = \frac{\sqrt{(2u_0 + u_2)^2 + u_3^2}}{2\pi N D u_1} F_0 I_b$$

$$\bar{T} = \frac{\sqrt{(2\bar{u}_0 + \bar{u}_2)^2 + \bar{u}_3^2}}{2\pi N \bar{D} \bar{u}_1} F_0 I_b$$

Consider weak input limit, $u_0 \approx \bar{u}_0$

$$\kappa \approx \frac{\pi\tau N D}{F_0 I_b u_0} u_1 = c_0 u_1$$

$$\bar{\kappa} \approx \frac{\pi\tau N \bar{D}}{F_0 I_b \bar{u}_0} \bar{u}_1 = c_0 \bar{u}_1$$

c_0 denotes the coefficient of u_1 and $\bar{u}_1$, the concentration is proportional to the coefficient of the height mode. From (3) and (4) we have

$$\left(\frac{F}{u_1}\right)^2 = (1 - HJ_{rc})^2 + (HJ_{rp})^2 - 2(1 - HJ_{rc})HJ_{rp}cos\Delta s$$

$$u_1^2 = \frac{F^2}{(1 - HJ_{rc})^2 + (HJ_{rp})^2 - 2(1 - HJ_{rc})HJ_{rp}cos\Delta s}$$

Actually HJ_{rp} is small, and $cos\Delta s = cos[\Delta x - 2(s_1 - x_1)] \approx cos\Delta x$. We expand u_1^2 and $tan(s_1 - x_1)$ then get

$$tan(s_1 - x_1) \approx \frac{HJ_{rp}\sin\Delta x}{1 - HJ_{rc} + HJ_{rp}\cos\Delta x}$$

$$\kappa^2 \approx \frac{c_0^2 F^2}{(1 - HJ_{rc})^2}\left[1 + \frac{2HJ_{rp}}{1 - HJ_{rc}}\cos\Delta x + (\frac{HJ_{rp}}{1 - HJ_{rc}})^2\right]$$

When $I_1 = I$ and $I_2 = 0$, this network is asymmetric. The bump in module 2 is much weaker. The concentration decoded from module 1 is $\kappa_1 = \frac{c_0 F}{(1 - HJ_{rc})}$. When $I_2 = I$ and $I_1 = 0$, the concentration decoded from module 1 is $\kappa_{12} = c_0 u_{11} = \frac{c_0 F HJ_{rp}}{(1 - HJ_{rc})^2}$. Compare the results, we have

$$\kappa e^{js_1} = \kappa_1 e^{jx_1} + \kappa_{12} e^{jx_2}$$

where j is the imaginary unit. Similarly, for opposite groups we also have

$$\bar{\kappa} e^{j\bar{s}_1} = \bar{\kappa}_1 e^{jx_1} + \bar{\kappa}_{12} e^{j(x_2 + \pi)}$$

The parameters are listed. Each network consist of $N = 180$ congruent and opposite neurons respectively. The connection width $a_0 = 3$ and the time step is 0.01τ using Euler method where τ is rescaled to 1. The strength of background input is $I_b = 1$. The strength of divisive normalization $\omega = 3 \times 10^{-4}$. F_0 in the noise term is the Fano factor which is set to 0.5.

In simulation we fix $x_1 = 0$, that is, $\Delta x = x_2 - x_1 = x_2$. J_c is the minimal recurrent strength and we choose $J_{rc} = 0.3J_c$ and $J_{rp} = 0.5J_{rc}$. J_c can be found by solving the dynamic equations, which is given by

$$J_c = \sqrt{\frac{8\pi I_0(a_0/2)^2\omega(1 + J_{int})}{I_0(a_0)\rho}}$$

U_0 is the rescaled input strength $U_0 = \frac{J_c e^{a_0/2}}{2\pi\omega(1 + J_{int})I_0(a_0/2)}$. We compare the case of weak input $I = 0.01U_0$ (left column) and strong input $I = 0.7U_0$ (right coulmn) in module 1. The results are derived by solving dynamic equations. Figure 1 shows this network could implement Bayesian inference under weak input limit. When external inputs are strong, the prediction from this network is close to the Bayesian way.

Causal Inference

The prior, which gave in Wenhao's paper, is $p(s_1,s_2) \propto M(s_1-s_2,\kappa_s)$. We rewrite the prior of congruent and opposite groups

$$p(s_1,s_2) = \frac{p_0}{2\pi}M(s_1,s_2|\pm\kappa_s) + \frac{1-p_0}{(2\pi)^2}$$

Actually the posterior can be calculated by marginalizing $p(s_1,s_2|x_1,x_2)$. We integrate over s_2 yielding

$$\begin{aligned} p(s_1|x_1,x_2) &= (2\pi)^2 p(x_1|s_1)\int_{-\pi}^{\pi} p(x_2|s_2)p(s_1,s_2)ds_2 \\ &\approx p_0 CM(s_1|x_1,\kappa_1)M(s_1|x_2,\pm\kappa_s) + (1-p_0)M(s_1|x_1,\kappa_1) \end{aligned}$$

Where C is norm constant, normalization pool $D' = 1 + \omega\sum_{-\pi}^{\pi}[\psi]_+^2$. By adding another module, we can implement the second component as an independent part. The dynamics of congruent and opposite groups is given by

$$\begin{aligned} &\tau\frac{\partial\psi(y,t)}{\partial t} = -\psi(y,t) + \frac{J_{rc}}{D'}\sum_{-\pi}^{\pi}V(y-y',a_0)\psi^2(y') \\ &+ p_0\frac{c_k}{D}\sum_{-\pi}^{\pi}\cos(y-y')\psi^2(y') + (1-p_0)IV(y-x,a_0/2) + I_b \end{aligned} \tag{5}$$

Where $\frac{\psi^2(y')}{D}$ is the firing rate with kernel $c_k\cos(y-y')$ from lower layer, that is, the module introduced by Wenhao's paper. In Figure 2 A, we use synaptic inputs as the feedforward inputs in the upper layer. Without reciprocal connections from other modules, read out the concentration $\kappa'_m(\bar{\kappa}'_m)$ from population vectors in these modules. The final output will be the weighted sum of two. That is, for the case of combined cues, the final output of congruent group in module 1 will become

$$p_0\kappa e^{js_1} + (1-p_0)\kappa'_1 e^{jx_1}$$

The first component is a complex number since it contains position information. Meanwhile, κ'_1 doesn't change when this network only receive input 1, the output then will be

$$p_0\kappa_1 e^{jx_1} + (1-p_0)\kappa'_1 e^{jx_1}$$

Only cue 2, the final output from congruent group in module 1 is $p_0\kappa_{12}e^{jx_2}$. From previous discussion, we know this network still behave in a Bayesian way.

Dynamics

Similarly, we project Eq. (5) onto height and position modes. Consider the dynamics of the congruent group in module 1 under weak input limit, we know the solution is

$$\begin{aligned} u'_1 =& (1-p_0)\frac{F cos(x_1-s'_1)}{1-H'J_{rc}} + p_0\frac{c_k H u_1 cos(s_1-s'_1)}{J_{rc}B_1(a_0)(1-H'J_{rc})} \\ 0 =& (1-p_0)\frac{F sin(x_1-s'_1)}{1-H'J_{rc}} + p_0\frac{c_k H u_1 sin(s_1-s'_1)}{J_{rc}B_1(a_0)(1-H'J_{rc})} \end{aligned}$$

where $H' = \frac{\rho J_{rc}}{D'}(2u'_0+u'_2)B_1(a_0),\ F = \frac{IB_1(a_0/2)}{\pi}$.

That is, for combined case, we have $u'_1 e^{js'_1} = (1-p_0)\frac{F}{1-H'J_{rc}}e^{jx_1} + p_0\frac{c_k H u_1}{J_{rc}B(1,a_0)(1-H'J_{rc})}e^{js_1}$. When cue 1 is applied, this expression doesn't change except the second term u_1, which refers to the single cue case (only cue 1). On the other hand, when cue 2 is applied, the first term vanishes, the second term u_1 refers to the single cue case (only cue 2). Under this circumstance we won't have Bayesian inference unless $\kappa \propto u_1$ is still held.

Here we still consider using $\hat{A}$ to rescale the noise variance. The noise temperature doesn't change since they have the same expression. The noise variance can be calculate

$$\sigma^2 = \frac{T_1}{[1-\frac{\rho J_{rc}}{D'}(2u'_0+u'_2)B_1(a_0)]\tau}$$

Hence we get concentration

$$\kappa'_1 = \frac{1}{\sigma^2} \approx \frac{\tau}{T_1} \approx c_0 u'_1$$

This approximation is under weak input limit, c_0 is defined in previous section. We consider weak inputs and strong inputs, that is, $I = 0.01U_0$ and $I = 0.7U_0$, and compare the results with Bayesian prediction in Figure 4. So far the outputs from the second layer will behave in Bayesian way under weak input limit. Although the prior is the sum of two von Mises functions, it won't get two peaks since the position disparity between inputs and feedforward inputs is actually small. Figure 3 shows in vector space, causal inference can be achieved by taking advantage of the properties of this network.

Information Segregation

In Zhang et. al., the definition of the disparity information is given by

$$p_d(s_1|x_1,x_2) \propto p(s_1|x_1)/p(s_1|x_2)$$

However, $p_d \propto \frac{M(s_1-x_1,\kappa_1}{p_0 CM(s_1-x_1,\kappa_{2s})+(1-p_0)}$ is different from that in Wenhao's paper. In order to implement information segregation, we need to modify our model.

Although p_d is hard to obtain from this neuron network, however, this ratio is equivalent to p_d^{-1}, which contains the same amount of information of cue segregation. p_d^{-1} can be written by

$$\begin{aligned} p_d^{-1} =& p_0 CM(s_1-x_2,\kappa_{2s})M(s_1-x_1+\pi,\kappa_1) \\ &+ (1-p_0)M(s_1-x_1+\pi,\kappa_1) \end{aligned}$$

where C is norm constant. That is, input's direction in module 1 will be shifted by π for the opposite group. Actually, it is relatively easy to confirm that the modified model has the same structure as that we define in the previous section. The only difference lies in the fact that position of the outputs from the second layer will be shifted by π.

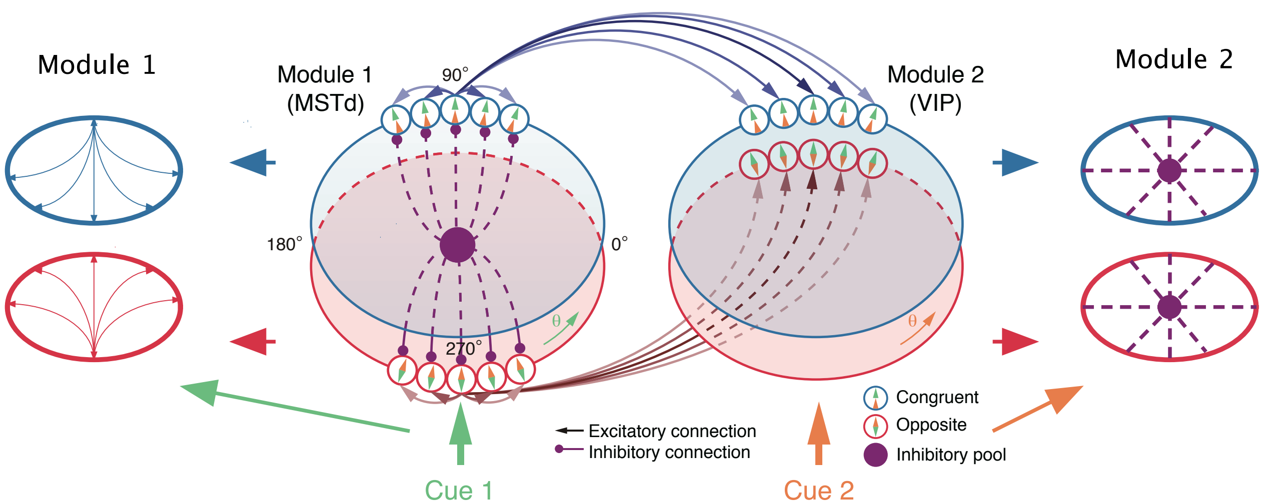

Figure 2: Illustration of decentralized model after modification. New modules in the second layer. No connection between two groups of neurons in each module. The inputs and feedforward inputs from the first layer are rescaled by $1-p_0$ and p_0 respectively. Each group of neurons has their own inhibition pool and there's no reciprocal coupling between two modules.

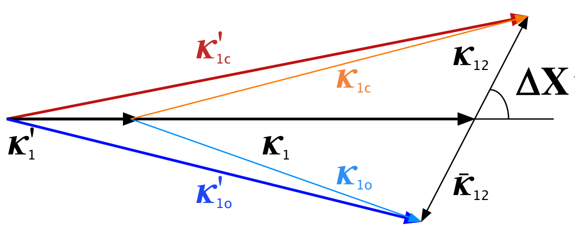

Figure 3: Geometric interpretation of vector space. Colored vectors represent the sum of other vectors (black). Outputs from first layer: κ_{1c} (congruent) and κ_{1o} (opposite), second layer: κ'_{1c} (congruent) and κ'_{1o} (opposite). Note that the vectors (black) have been rescaled by p_0 and $1-p_0$.

Conclusion and Discussion

In this work, we have developed a novel method for revealing the dynamics inside the neural circuit. We discuss the conditions for decentralized model to achieve Bayesian inference, which might be helpful in biological implementation. However, the functions of opposite groups are not fully understood, and the details of the structure on causal inference are missing in this model. That could be our future work.

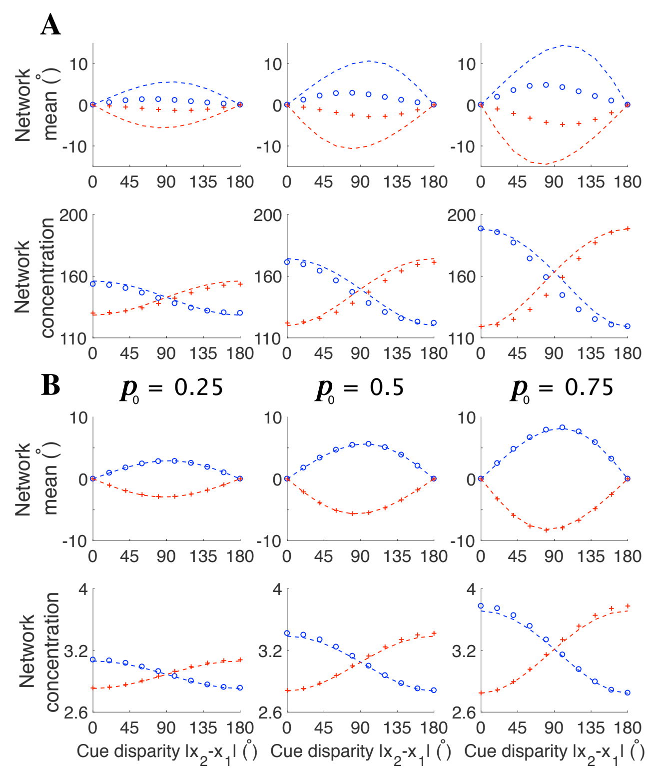

Figure 4: Outputs from second layer with varying probability p_0. Illustration of the population response of congruent and opposite groups in module 1 applying weak inputs (A) and strong inputs (B). Symbols: network results; dash lines: Bayesian prediction. The blue and red colors represent congruent and opposite groups in module 1 respectively.