Fourier Basic Functions

Here we suppose the solution to steady state equation is a linear combination of Fourier basic functions

$$\psi_{1c} = u_0 + u_1 cos(y_1 - s_1) + u_2 cos2(y_1 - s_1) + u_3 sin2(y_1 - s_1)$$

And ψ square:

$$\psi_{1c}^2 = (u_0^2 + \frac{u_1^2}{2} + \frac{u_2^2}{2} + \frac{u_3^2}{2}) + (2u_0u_1 + u_1u_2)cos(y_1 - s_1) + u_1u_3 sin(y_1 - s_1) + (2u_0u_2 + \frac{u_1^2}{2})cos2(y_1 - s_1) + \\ 2u_0u_3 sin2(y_1 - s_1) + u_1u_2 cos3(y_1 - s_1) + u_1u_3 sin3(y_1 - s_1) + (\frac{u_2^2}{2} - \frac{u_3^2}{2})cos4(y_1 - s_1) + u_2u_3 sin4(y_1 - s_1)$$

Congruent Group's Equation

$$\psi(y_1) = \frac{\rho J_{rc}}{D_1}\int dy_2 V(y_1 - y_2, a_0)\psi^2(y_2) + \frac{\rho J_{rp}}{D_2}\int dy_2 V(y_1 - y_2, a_0)\bar{\psi}^2(y_2) + I_1 V(y_1 - x_1, a_0/2) + I_b \quad (1)$$

Global Inhibition

$$D_n = 1 + \omega\int \rho\left[u_n^2(x,k) + J_{int}u_{\bar{n}}^2(x,k)\right]dx = 1 + \pi\omega\rho\left[2u_0^2 + u_1^2 + u_2^2 + u_3^2 + J_{int}(2\bar{u}_0^2 + \bar{u}_1^2 + \bar{u}_2^2 + \bar{u}_3^2)\right]$$

Let's define:

$$B(n,k) = \frac{I_n(k)}{I_0(k)}$$

The square term:

$$\frac{\rho J_{rc}}{D_1}\int dy_2 V(y_1 - y_2, a_0)\psi^2(y_2) = \frac{\rho J_{rc}}{D_1}\Bigg[(u_0^2 + \frac{u_1^2}{2} + \frac{u_2^2}{2} + \frac{u_3^2}{2}) + (2u_0u_1 + u_1u_2)B(1,a_0)cos(y_1 - s_1) \\ + u_1u_3B(1,a_0)sin(y_1 - s_1) + (2u_0u_2 + \frac{u_1^2}{2})B(2,a_0)cos2(y_1 - s_1) + 2u_0u_3B(2,a_0)sin2(y_1 - s_1) + u_1u_2B(3,a_0)cos3(y_1 - s_1) \\ + u_1u_3B(3,a_0)sin3(y_1 - s_1) + (\frac{u_2^2}{2} - \frac{u_3^2}{2})B(4,a_0)cos4(y_1 - s_1) + u_2u_3B(4,a_0)sin4(y_1 - s_1)\Bigg]$$

Projection

Multiply both sides by 1 $cos(y_1 - s_2)$ $sin(y_1 - s_2)$ $cos2(y_1 - s_2)$ $sin2(y_1 - s_2)$ and integrate over y_1

$$u_{10} = \frac{\rho J_{rc}}{D_1}\left[u_{10}^2 + \frac{u_{11}^2}{2} + \frac{u_{12}^2}{2} + \frac{u_{13}^2}{2}\right] + \frac{\rho J_{rp}}{D_2}\left[u_{20}^2 + \frac{u_{21}^2}{2} + \frac{u_{22}^2}{2} + \frac{u_{23}^2}{2}\right] + \frac{I_1}{2\pi} + I_b$$

$$u_{11} = \frac{\rho J_{rc}}{D_1}(2u_{10}u_{11} + u_{11}u_{12})B(1,a_0) + \frac{I_1B(1,a_0/2)}{\pi}cos(x_1 - s_1) + \\ \frac{\rho J_{rp}}{D_2}[(2u_{20}u_{21} + u_{21}u_{22})B(1,a_0)cos(s_2 - s_1) - u_{21}u_{23}B(1,a_0)sin(s_2 - s_1)]$$

$$0 = \frac{\rho J_{rc}}{D_1}u_{11}u_{13}B(1,a_0) + \frac{I_1B(1,a_0/2)}{\pi}sin(x_1 - s_1) + \\ \frac{\rho J_{rp}}{D_2}[(2u_{20}u_{21} + u_{21}u_{22})B(1,a_0)sin(s_2 - s_1) + u_{21}u_{23}B(1,a_0)cos(s_2 - s_1)]$$

$$u_{12} = \frac{\rho J_{rc}}{D_1}(2u_{10}u_{12} + \frac{u_{11}^2}{2})B(2,a_0) + \frac{I_1 B(2,a_0/2)}{\pi}cos2(x_1 - s_1) +$$
$$\frac{\rho J_{rp}}{D_2}\left[(2u_{20}u_{22} + \frac{u_{21}^2}{2})B(2,a_0)cos2(s_2 - s_1) - 2u_{20}u_{23}B(2,a_0)sin2(s_2 - s_1)\right]$$

$$u_{13} = \frac{\rho J_{rc}}{D_1}2u_{10}u_{13}B(2,a_0) + \frac{I_1 B(2,a_0/2)}{\pi}sin2(x_1 - s_1) +$$
$$\frac{\rho J_{rp}}{D_2}\left[(2u_{20}u_{22} + \frac{u_{21}^2}{2})B(2,a_0)sin2(s_2 - s_1) + 2u_{20}u_{23}B(2,a_0)cos2(s_2 - s_1)\right]$$

Mean

Firing rate $R_i(s_1, u_0, u_1, u_2, u_3) = \frac{\psi_i^2}{D_i}$, then $\hat{s}$ will be

$$\hat{s}_i = arg(\sum_{-\pi}^{\pi} R_i e^{j\theta})$$

$Real\ Part = \pi(2u_0u_1 + u_1u_2)cos(s) - \pi u_1u_3 sin(s)$
$Imaginary\ Part = \pi(2u_0u_1 + u_1u_2)sin(s) + \pi u_1u_3 cos(s)$

So

$$\hat{s} = atan2[(2u_0u_1 + u_1u_2)sin(s) + u_1u_3cos(s), (2u_0u_1 + u_1u_2)cos(s) - u_1u_3sin(s)]$$

Concentration

Next we need to take noise into consideration.

$$\tau\frac{\partial}{\partial t}\psi(y_1) = -\psi(y_1) + \frac{\rho J_{rc}}{D_1}\int dy_2 V(y_1 - y_2, a_0)\psi^2(y_2) + \frac{\rho J_{rp}}{D_2}\int dy_2 V(y_1 - y_2, a_0)\bar{\psi}^2(y_2) + I_1 V(y_1 - x_1, a_0/2) + I_b$$
$$+ \sqrt{FI_1 V(y_1 - x_1, a_0/2)}\xi_1 + \sqrt{FI_b}\epsilon_1$$

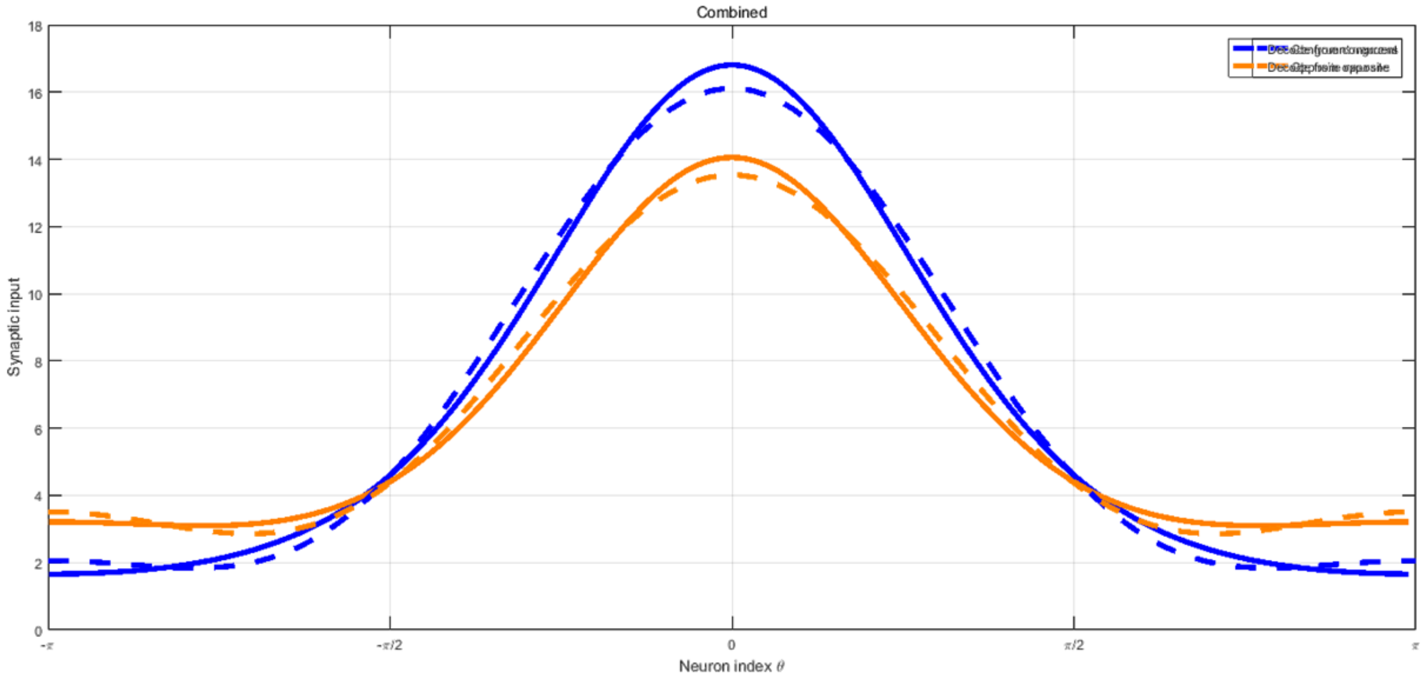
Consider the dynamics of the displacement mode and multiply both sides by $sin(y_1 - s_1)$, integrate over y_1

$$\tau\frac{\partial}{\partial t}\delta s_1 = -\delta s_1 + \frac{\rho J_{rc}}{D_1 u_{11}}(2u_{10}u_{11} + u_{11}u_{12})B(1,a_0)\delta s_1$$
$$+ \frac{\rho J_{rp}}{D_2 u_{11}}[(2u_{20}u_{21} + u_{21}u_{22})B(1,a_0)cos(s_2 - s_1) - u_{21}u_{23}B(1,a_0)sin(s_2 - s_1)]\,\delta s_2$$
$$+ \frac{\sqrt{FI_1}}{\pi u_{11}}\int\sqrt{V(y_1 - x_1, a_0/2)}sin(y_1 - s_1)\xi_1 dy_1 + \frac{\sqrt{FI_b}}{\pi u_{11}}\int sin(y_1 - s_1)\epsilon_1 dy_1$$

Noise Temperature

$$T_1 = \frac{F}{2\pi^2\rho u_{11}^2}\left[I_1\int V(y_1 - x_1, a_0/2)sin^2(y_1 - s_1)dy_1 + I_b\int sin^2(y_1 - s_1)dy_1\right]$$
$$= \frac{F}{2\pi^2\rho u_{11}^2}\left[(\frac{I_1}{2} + \pi I_b) - \frac{I_1}{2}B(2,a_0/2)cos2(x_1 - s_1)\right]$$

Results


Combined
Synaptic input
18
16
14
12
10
8
6
4
2
0
-π
-π/2
0
π/2
π
Neuron index θ

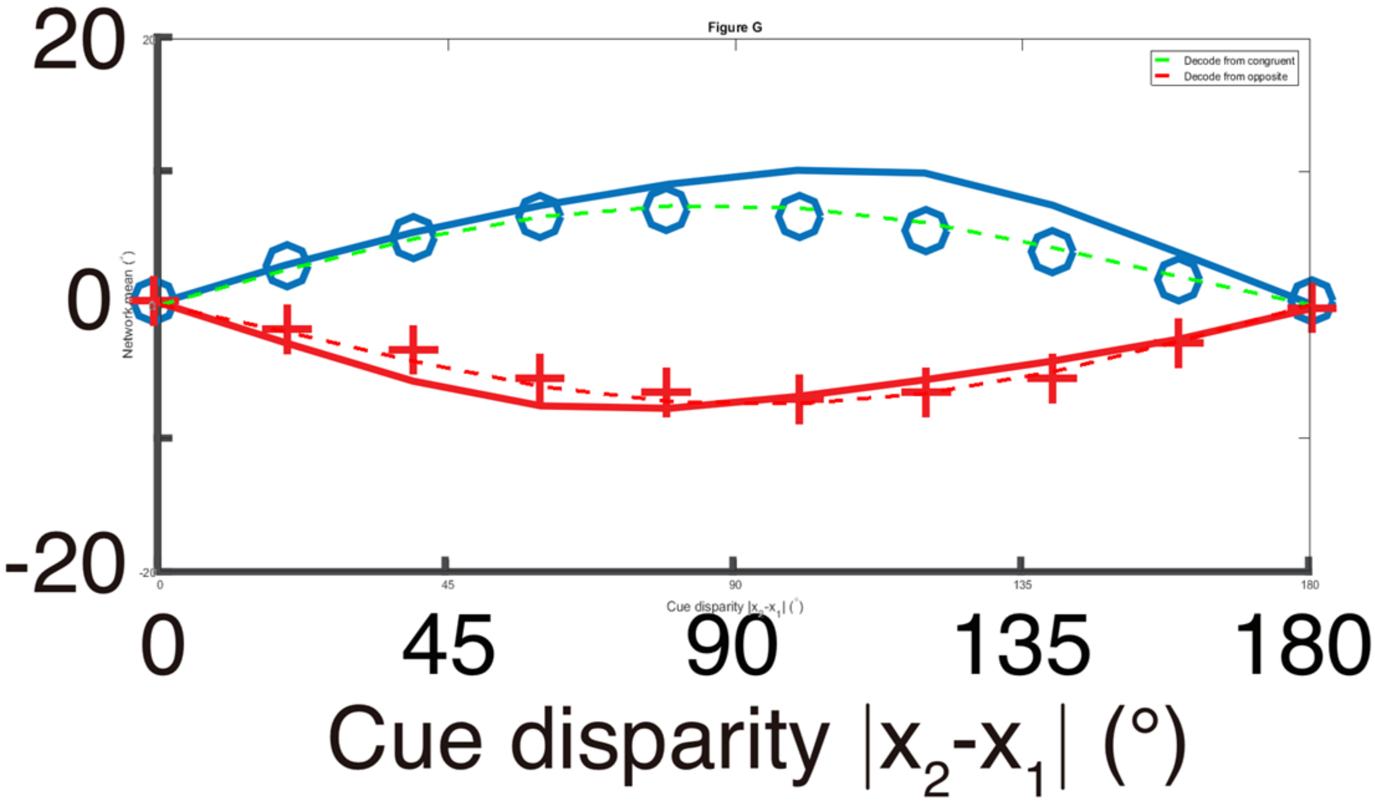
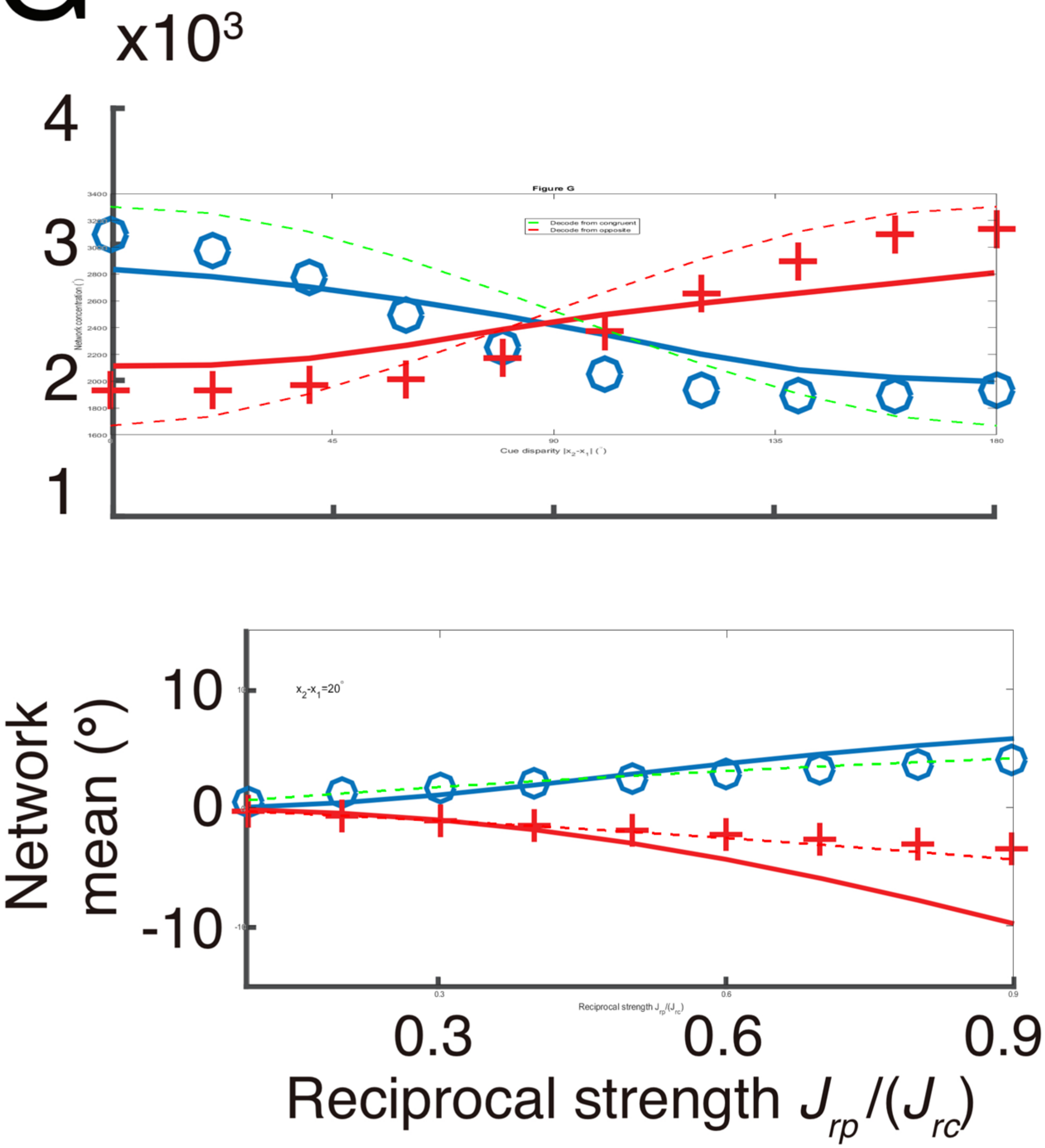


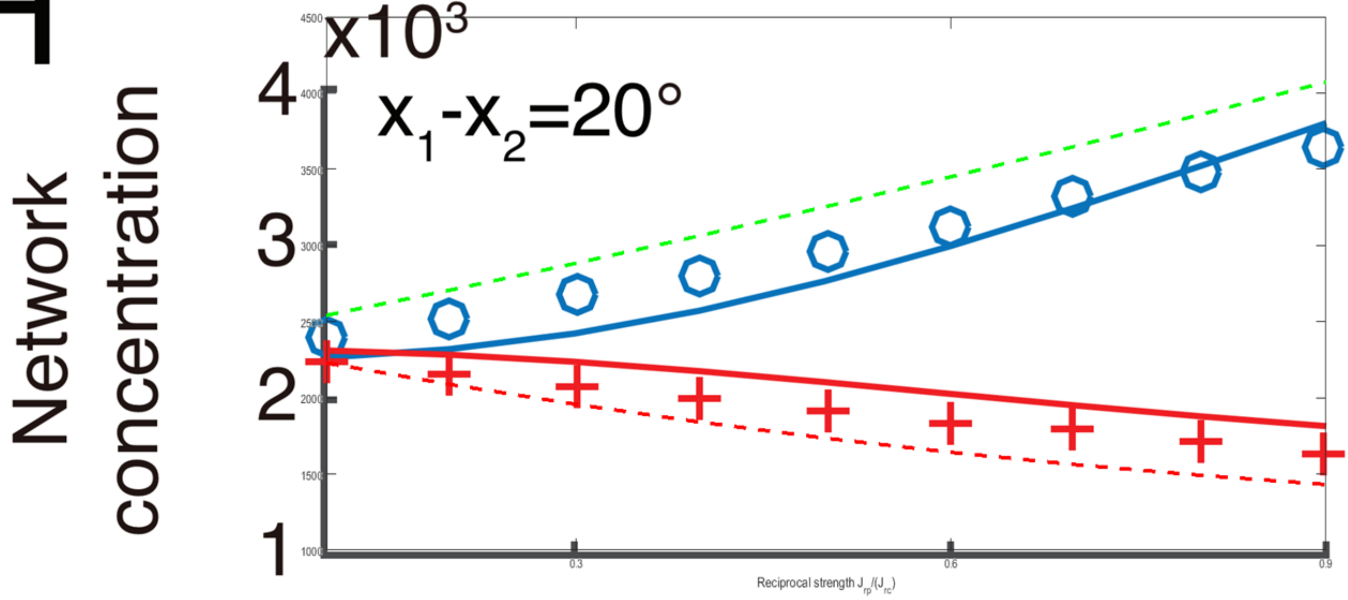
Figure G
Decode from congruent
Decode from opposite
20
0
-20
Network mean (°)
0
45
90
135
180
Cue disparity |x2-x1| (°)

G



x10^3
4
3
2
1
Figure G
Decode from congruent
Decode from opposite
Network concentration (°)
Cue disparity |x2-x1| (°)
0
45
90
135
180
x2-x1=20°
Network mean (°)
10
0
-10
0.3
0.6
0.9
Reciprocal strength Jrp/(Jrc)

H



Network concentration
x10^3
4
3
2
1
4500
4000
3500
3000
2500
2000
1500
1000
x1-x2=20°
0.3
0.6
0.9
Reciprocal strength Jrp/(Jrc)