Über Problem – "Jerky" Jerry's Jabberwocky Jumper

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Problem

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

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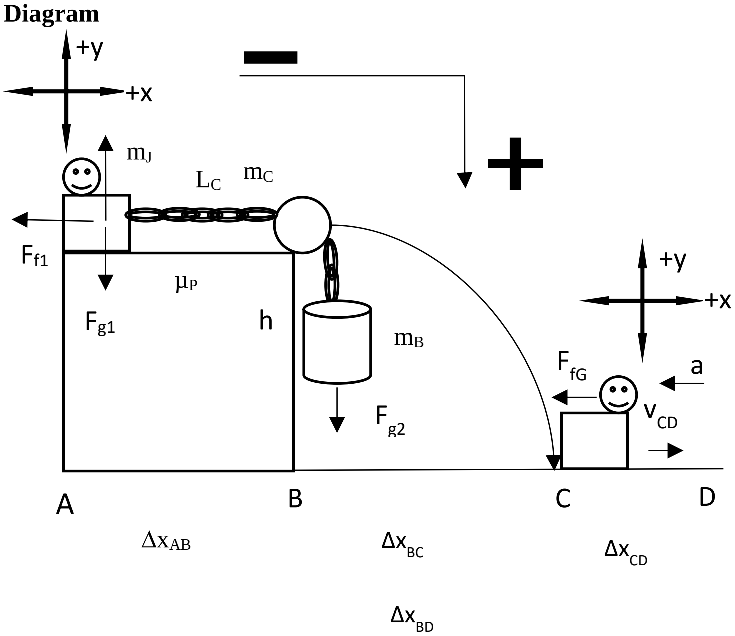
Diagram



Givens and Equations

$m_J = 67$ kg
$m_B = 164$ kg
$m_C = 53$ kg
$L_C = 9$ m
h = 18 m
$\mu_P = 0.24$
$\Delta x_{BD} = 109$ m
$\mu_G = ?$

$$v_f = v_i + at$$
$$x_f = x_i + vt$$
$$y_f = \frac{1}{2}at^2 + v_i t + y_i$$
$$v_f^{\,2} = v_i^{\,2} + 2a\Delta x$$
$$a[x]\,dx = v\,dv$$

Solution and Strategy

I broke up the motion of "Jerky" Jerry's Jabberwocky Jumper into into three stages: stage AB, the jumper moving from point A to point B pulled by a weight on a pulley; stage BC, the jumper moving from point B to point C in projectile motion; and, finally, stage CD, the jumper moving from point C to point D (sliding across the ground).

I broke up each stage into four steps as shown in the right column of this page.

Motion Stage AB

Step 1:

First, use the sum of the forces () acting on the jumper in the y direction to find the normal force () acting on it.

$$\sum F_y = F_{N1} - F_{g1} = m * a_y$$
$$a_y = 0$$
$$F_{N1} = F_{g1}$$
$$F_{N1} = m_J * g$$
$$F_{N1} = 67 * 9.8$$
$$\underline{F_{N1} = 656.6\ N}$$

Step 2:

Then use the normal force () calculated in the previous step and the given coefficient of friction of the platform () to find the forces of friction () acting on the jumper.

$$F_{f1} = \mu_p * F_{N1}$$
$$F_{f1} = 0.24 * 656.6$$
$$\underline{F_{f1} = 157.584\ N}$$

Step 3:

Since the mass off the platform changes over time, make a function for that mass in terms of the horizontal displacement (x) of the jumper. Use that function and the sum of the system forces to find a system acceleration function in terms of x.

$$m[x] = \frac{x}{L_C}(m_C) + m_B$$

$$m[x] = \frac{53x}{9} + 164$$
$$\sum F_S = F_{g2} - F_{f1} = m_{total} * a_s$$
$$(m[x] * g) - F_{f1} = m_{total} * a_s$$
$$\left(\frac{53x}{9} + 164\right)(9.8) - 157.584 = (67 + 164 + 53) * a_s$$
$$a_s[x] = \frac{\left(\frac{53x}{9} + 164\right)(9.8) - 157.584}{67 + 164 + 53}$$

$$a_s[x] = \frac{\left(\frac{519.4x}{9} + 1607.2\right) - 157.584}{284}$$

$$a_s[x] = \frac{\left(\frac{519.4x}{9}\right) + 1449.616}{284}$$

$$a_s[x] = \frac{(519.4x + 13046.544)(1000)}{2556(1000)}$$

$$\underline{a_s[x] = (64925x + 1630818)/319500}$$

Step 4:

Use the acceleration function from the previous step to find the velocity () of the jumper as it comes off the table.

$$\mathrm{a[x]\,dx} = \mathrm{v\,dv}$$
$$\int_{x_0}^{x} \mathrm{a[x]\,dx} = \int_{v_0}^{v} v_B\,\mathrm{dv}$$
$$\int_0^9 \frac{64925x + 1630818}{319500}\,dx = \int_0^v v_B\,dv$$
$$\frac{1}{2}{v_B}^2 = \frac{\left(\frac{1}{2}\right)(64925)(9^2) + 1630818(9)}{319500}$$
$$v_B = \sqrt{\frac{2\left(\left(\frac{1}{2} * 64925 * 9^2\right) + (1630818 * 9)\right)}{319500}}$$
$$v_B \approx 10.4085\ m/s$$

Motion Stage BC

Step 5:

Use a kinematic equation of the form $y_f = \frac{1}{2}at^2 + v_i t + y_i$ to find the time it takes the jumper to reach the ground.

$$y_f = \frac{1}{2}a_y t^2 + v_i t + h$$
$$0 = \frac{1}{2}(-9.8)t^2 + 0 + 18$$
$$0 = -4.9t^2 + 18$$
$$t = \sqrt{\frac{-18}{-4.9}}$$
$$\underline{t \approx 1.191663\ \mathrm{s}}$$

Step 6:

Use the time obtained from the previous step and the velocity of the jumper as it leaves the table to find the horizontal displacement () of the jumper before it hits the ground.

$$\Delta x_{BC} = v_{BX} * t = v_B * t$$
$$\Delta x_{BC} = (10.4085)(1.91663)$$
$$\Delta x_{BC} \approx 19.9492\ m$$

Step 7:

Use a kinematic equation of the form $v_f = v_i + at$ to find the vertical velocity of the jumper when it hits the ground.

$$v_{Cy} = v_{By} + a_y t$$
$$v_{Cy} = 0 + (-9.8)(1.91663)$$
$$v_{Cy} \approx -18.783\ m/s$$

Step 8:

Find the velocity of the jumper at point c using the horizontal component () and the vertical component ().

$$v_{Cx} = v_B$$
$$v_C = \sqrt{{v_{Cx}}^2 + {v_{Cy}}^2} = \sqrt{{v_B}^2 + {v_{Cy}}^2}$$
$$v_C = \sqrt{(10.4085)^2 + (-18.783)^2}$$
$$v_C \approx 21.4741 m/s$$

Motion Stage CD

Step 9:

Use the velocity from the previous step to obtain the initial velocity of the jumper before it starts sliding across the floor.

$$v_{CD} = 0.75 * v_C$$
$$v_{CD} = 0.75(21.4741)$$
$$v_{CD} \approx 16.1056\ m/s$$

Step 10:

Find the acceleration of the jumper as it slides across the floor using the sum of the forces acting on the jumper in the x direction ().

$$\sum F_x = -F_f = m_J * a$$
$$-\mu_G * m_J * g = m_J * a$$
$$a = -9.8 * \mu_G$$

Step 11:

Find the horizontal displacement of the jumper from point C to D ()by using its displacement from point B to C () and from point B to D ().

$$\Delta x_{BD} = \Delta x_{BC} + \Delta x_{CD}$$
$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$
$$\Delta x_{CD} = 109 - 19.9492$$
$$\Delta x_{CD} \approx 89.0508$$

Finally…

Step 12:

Finally, use a kinematic equation of the form ${v_f}^2 = {v_i}^2 + 2a\Delta x$ and solve for the coefficient of friction for the jumper and the ground (μ_G).

$${v_D}^2 = {v_{CD}}^2 + 2a\Delta x_{CD}$$
$$0 = (16.1056)^2 + 2(-9.8 * \mu_G)\Delta x_{CD}$$
$$\mu_G = \frac{-(16.1056)^2}{2 * -9.8 * 89.0508}$$

$$\boxed{\mu_G = 0.1486}$$