

$e ::=$	expression
c	constant
$op(e_1, \dots, e_n)$	primitive operation
$\text{if } e_1 \text{ then } e_2 \text{ else } e_3$	conditional
$\text{let } x = e_1 \text{ in } e_2$	variable declaration
x	variable reference
$\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2$	declaration of recursive function
$e \ e_1 \ \dots \ e_n$	function call
(e_1, \dots, e_n)	creation of a tuple
$\text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2$	pattern matching against a tuple
$\text{Array.create } e_1 \ e_2$	creation of an array
$e_1.(e_2)$	array element dereference
$e_1.(e_2) \leftarrow e_3$	assignment to an array element

図 1: MinCaml の抽象構文 (型は省略)

Abstract syntax of MinCaml

$\tau ::=$	type
π	primitive type
$\tau_1 \rightarrow \dots \rightarrow \tau_n \rightarrow \tau$	functional type
$\tau_1 \times \dots \times \tau_n$	tuple type
$\tau \text{ array}$	array type
α	type variable (for type inference)

図 2: MinCaml の型

MinCaml types

$\frac{c \text{ is a constant of the } \pi \text{ type}}{\Gamma \vdash c : \pi}$	$\frac{\text{op is a primitive operation that takes values whose types are } \pi_1, \dots, \pi_n \text{ and gives a value of } \pi \text{ type}}{\Gamma \vdash \text{op}(e_1, \dots, e_n) : \pi}$
$\frac{c \text{ は } \pi \text{ 型の定数}}{\Gamma \vdash c : \pi}$	$\frac{op \text{ は } \pi_1, \dots, \pi_n \text{ 型の値を受け取って } \pi \text{ 型の値を返すプリミティブ演算}}{\Gamma \vdash op(e_1, \dots, e_n) : \pi}$
$\frac{\Gamma \vdash e_1 : \text{bool} \quad \Gamma \vdash e_2 : \tau \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash \text{if } e_1 \text{ then } e_2 \text{ else } e_3 : \tau}$	$\frac{\Gamma \vdash e_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash e_2 : \tau_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : \tau_2} \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$
$\frac{\Gamma, x : \tau_1 \rightarrow \dots \rightarrow \tau_n \rightarrow \tau, y_1 : \tau_1, \dots, y_n : \tau_n \vdash e_1 : \tau \quad \Gamma, x : \tau_1 \rightarrow \dots \rightarrow \tau_n \rightarrow \tau \vdash e_2 : \tau'}{\Gamma \vdash \text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2 : \tau'}$	$\frac{\Gamma \vdash e : \tau_1 \rightarrow \dots \rightarrow \tau_n \rightarrow \tau \quad \Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash e \ e_1 \ \dots \ e_n : \tau}$
$\frac{\Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Gamma \vdash e_n : \tau_n}{\Gamma \vdash (e_1, \dots, e_n) : \tau_1 \times \dots \times \tau_n}$	$\frac{\Gamma \vdash e_1 : \tau_1 \times \dots \times \tau_n \quad \Gamma, x_1 : \tau_1, \dots, x_n : \tau_n \vdash e_2 : \tau}{\Gamma \vdash \text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2 : \tau}$
$\frac{\Gamma \vdash e_1 : \text{int} \quad \Gamma \vdash e_2 : \tau}{\Gamma \vdash \text{Array.create } e_1 \ e_2 : \tau \text{ array}}$	
$\frac{\Gamma \vdash e_1 : \tau \text{ array} \quad \Gamma \vdash e_2 : \text{int}}{\Gamma \vdash e_1.(e_2) : \tau}$	$\frac{\Gamma \vdash e_1 : \tau \text{ array} \quad \Gamma \vdash e_2 : \text{int} \quad \Gamma \vdash e_3 : \tau}{\Gamma \vdash e_1.(e_2) \leftarrow e_3 : \text{unit}}$

図 3: MinCaml の型つけ規則

MinCaml's typing rule

$e ::=$ c $op(x_1, \dots, x_n)$ $\text{if } x = y \text{ then } e_1 \text{ else } e_2$ $\text{if } x \leq y \text{ then } e_1 \text{ else } e_2$ $\text{let } x = e_1 \text{ in } e_2$ x $\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2$ $x \ y_1 \ \dots \ y_n$ (x_1, \dots, x_n) $\text{let } (x_1, \dots, x_n) = y \text{ in } e$ $x.(y)$ $x.(y) \leftarrow z$

図 4: MinCaml の K 正規形（外部配列・外部関数適用は省略）

MinCaml's K normal form (external arrays and external function calls are not included)

$\mathcal{K} : \text{Syntax.t} \rightarrow \text{KNormal.t}$

$\mathcal{K}(c)$	$= c$	
$\mathcal{K}(\text{not}(e))$	$= \mathcal{K}(\text{if } e \text{ then false else true})$	
$\mathcal{K}(e_1 = e_2)$	$= \mathcal{K}(\text{if } e_1 = e_2 \text{ then true else false})$	when op is not a logical
$\mathcal{K}(e_1 \leq e_2)$	$= \mathcal{K}(\text{if } e_1 \leq e_2 \text{ then true else false})$	nor comparison operator
$\mathcal{K}(\text{op}(e_1, \dots, e_n))$	$= \text{let } x_1 = \mathcal{K}(e_1) \text{ in } \dots \text{let } x_n = \mathcal{K}(e_n) \text{ in } \text{op}(x_1, \dots, x_n)$	op が論理演算・比較以外の場合
$\mathcal{K}(\text{if not } e_1 \text{ then } e_2 \text{ else } e_3)$	$= \mathcal{K}(\text{if } e_1 \text{ then } e_3 \text{ else } e_2)$	
$\mathcal{K}(\text{if } e_1 = e_2 \text{ then } e_3 \text{ else } e_4)$	$= \text{let } x = \mathcal{K}(e_1) \text{ in let } y = \mathcal{K}(e_2) \text{ in}$ $\text{if } x = y \text{ then } \mathcal{K}(e_3) \text{ else } \mathcal{K}(e_4)$	
$\mathcal{K}(\text{if } e_1 \leq e_2 \text{ then } e_3 \text{ else } e_4)$	$= \text{let } x = \mathcal{K}(e_1) \text{ in let } y = \mathcal{K}(e_2) \text{ in}$ $\text{if } x \leq y \text{ then } \mathcal{K}(e_3) \text{ else } \mathcal{K}(e_4)$	when e1 is not a logical/
$\mathcal{K}(\text{if } e_1 \text{ then } e_2 \text{ else } e_3)$	$= \mathcal{K}(\text{if } e_1 = \text{false} \text{ then } e_3 \text{ else } e_2)$	comparison operator
		e_1 が論理演算・比較以外の場合
$\mathcal{K}(\text{let } x = e_1 \text{ in } e_2)$	$= \text{let } x = \mathcal{K}(e_1) \text{ in } \mathcal{K}(e_2)$	
$\mathcal{K}(x)$	$= x$	
$\mathcal{K}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{let rec } x \ y_1 \ \dots \ y_n = \mathcal{K}(e_1) \text{ in } \mathcal{K}(e_2)$	
$\mathcal{K}(e \ e_1 \ \dots \ e_n)$	$= \text{let } x = \mathcal{K}(e) \text{ in let } y_1 = \mathcal{K}(e_1) \text{ in } \dots \text{let } y_n = \mathcal{K}(e_n) \text{ in}$ $x \ y_1 \ \dots \ y_n$	
$\mathcal{K}(e_1, \dots, e_n)$	$= \text{let } x_1 = \mathcal{K}(e_1) \text{ in } \dots \text{let } x_n = \mathcal{K}(e_n) \text{ in } (x_1, \dots, x_n)$	
$\mathcal{K}(\text{let } (x_1, \dots, x_n) = e_1 \text{ in } e_2)$	$= \text{let } y = \mathcal{K}(e_1) \text{ in let } (x_1, \dots, x_n) = y \text{ in } \mathcal{K}(e_2)$	
$\mathcal{K}(\text{Array.create } e_1 \ e_2)$	$= \text{let } x = \mathcal{K}(e_1) \text{ in let } y = \mathcal{K}(e_2) \text{ in create_array } x \ y$	
$\mathcal{K}(e_1.(e_2))$	$= \text{let } x = \mathcal{K}(e_1) \text{ in let } y = \mathcal{K}(e_2) \text{ in } x.(y)$	
$\mathcal{K}(e_1.(e_2) \leftarrow e_3)$	$= \text{let } x = \mathcal{K}(e_1) \text{ in let } y = \mathcal{K}(e_2) \text{ in let } z = \mathcal{K}(e_3) \text{ in}$ $x.(y) \leftarrow z$	

図 5: K 正規化（論理値の整数化と、`insert_let` による最適化は省略）。右辺に出現していて左辺に出現していない変数は、すべて新しい (fresh) とする。

K normal for (conversion of logical values to numbers and optimization using `insert_let` is abbreviated). A variable that occurs in RHS but not in LHS should be considered a new/fresh.

$\alpha : \text{Id.t M.t} \rightarrow \text{KNormal.t} \rightarrow \text{KNormal.t}$

$\alpha_\varepsilon(c)$	$= c$
$\alpha_\varepsilon(\text{op}(x_1, \dots, x_n))$	$= \text{op}(\varepsilon(x_1), \dots, \varepsilon(x_n))$
$\alpha_\varepsilon(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } \varepsilon(x) = \varepsilon(y) \text{ then } \alpha_\varepsilon(e_1) \text{ else } \alpha_\varepsilon(e_2)$
$\alpha_\varepsilon(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } \varepsilon(x) \leq \varepsilon(y) \text{ then } \alpha_\varepsilon(e_1) \text{ else } \alpha_\varepsilon(e_2)$
$\alpha_\varepsilon(\text{let } x = e_1 \text{ in } e_2)$	$= \text{let } x' = \alpha_\varepsilon(e_1) \text{ in } \alpha_{\varepsilon, x \mapsto x'}(e_2)$
$\alpha_\varepsilon(x)$	$= \varepsilon(x)$
$\alpha_\varepsilon(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{let rec } x' \ y'_1 \ \dots \ y'_n = \alpha_{\varepsilon, x \mapsto x', y_1 \mapsto y'_1, \dots, y_n \mapsto y'_n}(e_1) \text{ in } \alpha_{\varepsilon, x \mapsto x'}(e_2)$
$\alpha_\varepsilon(x \ y_1 \ \dots \ y_n)$	$= \varepsilon(x) \ \varepsilon(y_1) \ \dots \ \varepsilon(y_n)$
$\alpha_\varepsilon((x_1, \dots, x_n))$	$= (\varepsilon(x_1), \dots, \varepsilon(x_n))$
$\alpha_\varepsilon(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } (x'_1, \dots, x'_n) = \varepsilon(y) \text{ in } \alpha_{\varepsilon, x_1 \mapsto x'_1, \dots, x_n \mapsto x'_n}(e)$
$\alpha_\varepsilon(x.(y))$	$= \varepsilon(x).(\varepsilon(y))$
$\alpha_\varepsilon(x.(y) \leftarrow z)$	$= \varepsilon(x).(\varepsilon(y)) \leftarrow \varepsilon(z)$

図 6: α 変換。 ε は α 変換前の変数を受け取って、 α 変換後の変数を返す写像。右辺に出現していて左辺に出現していない変数 (x' など) は、すべて fresh とする。

α conversion: ε is a mapping that takes a variable name and gives its α -converted name. A name that occurs only in RHS should be considers new/fresh name.

$\beta : \text{Id.t M.t} \rightarrow \text{KNormal.t} \rightarrow \text{KNormal.t}$

$\beta_\varepsilon(c)$	$= c$
$\beta_\varepsilon(\text{op}(x_1, \dots, x_n))$	$= \text{op}(\varepsilon(x_1), \dots, \varepsilon(x_n))$
$\beta_\varepsilon(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } \varepsilon(x) = \varepsilon(y) \text{ then } \beta_\varepsilon(e_1) \text{ else } \beta_\varepsilon(e_2)$
$\beta_\varepsilon(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } \varepsilon(x) \leq \varepsilon(y) \text{ then } \beta_\varepsilon(e_1) \text{ else } \beta_\varepsilon(e_2)$
$\beta_\varepsilon(\text{let } x = e_1 \text{ in } e_2)$	$= \beta_{\varepsilon, x \mapsto y}(e_2) \quad \beta_\varepsilon(e_1) \text{ is a variable "y"}$
$\beta_\varepsilon(\text{let } x = e_1 \text{ in } e_2)$	$= \text{let } x = \beta_\varepsilon(e_1) \text{ in } \beta_\varepsilon(e_2) \quad \beta_\varepsilon(e_1) \text{ is not a variable}$
$\beta_\varepsilon(x)$	$= \varepsilon(x)$
$\beta_\varepsilon(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{let rec } x \ y_1 \ \dots \ y_n = \beta_\varepsilon(e_1) \text{ in } \beta_\varepsilon(e_2)$
$\beta_\varepsilon(x \ y_1 \ \dots \ y_n)$	$= \varepsilon(x) \ \varepsilon(y_1) \ \dots \ \varepsilon(y_n)$
$\beta_\varepsilon((x_1, \dots, x_n))$	$= (\varepsilon(x_1), \dots, \varepsilon(x_n))$
$\beta_\varepsilon(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } (x_1, \dots, x_n) = \varepsilon(y) \text{ in } \beta_\varepsilon(e)$
$\beta_\varepsilon(x.(y))$	$= \varepsilon(x).(\varepsilon(y))$
$\beta_\varepsilon(x.(y) \leftarrow z)$	$= \varepsilon(x).(\varepsilon(y)) \leftarrow \varepsilon(z)$

図 7: β 簡約。 ε は β 簡約前の変数を受け取って、 β 簡約後の変数を返す写像。 $\varepsilon(x)$ が定義されていない場合は、 $\varepsilon(x) = x$ とみなす。

β reduction: ε is a mapping that takes a variable name and gives its β -converted name. We consider $\varepsilon(x) = x$ when ε is not defined for x .

$\mathcal{A} : \text{KNormal.t} \rightarrow \text{KNormal.t}$

$\mathcal{A}(c)$	$= c$
$\mathcal{A}(op(x_1, \dots, x_n))$	$= op(x_1, \dots, x_n)$
$\mathcal{A}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x = y \text{ then } \mathcal{A}(e_1) \text{ else } \mathcal{A}(e_2)$
$\mathcal{A}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x \leq y \text{ then } \mathcal{A}(e_1) \text{ else } \mathcal{A}(e_2)$
$\mathcal{A}(\text{let } x = e_1 \text{ in } e_2)$	$= \text{let } \dots \text{ in let } x = e'_1 \text{ in } \mathcal{A}(e_2)$
$\mathcal{A}(e_1)$ has a form $\text{let } \dots \text{ in } e'$ (let ... is a sequence of one or more let's) and e'_1 is not a let form.	
$\mathcal{A}(x)$	$= x$
$\mathcal{A}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{let rec } x \ y_1 \ \dots \ y_n = \mathcal{A}(e_1) \text{ in } \mathcal{A}(e_2)$
$\mathcal{A}(x \ y_1 \ \dots \ y_n)$	$= x \ y_1 \ \dots \ y_n$
$\mathcal{A}((x_1, \dots, x_n))$	$= (x_1, \dots, x_n)$
$\mathcal{A}(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } (x_1, \dots, x_n) = y \text{ in } \mathcal{A}(e)$
$\mathcal{A}(x.(y))$	$= x.(y)$
$\mathcal{A}(x.(y) \leftarrow z)$	$= x.(y) \leftarrow z$

⊠ 8: Reduction of nested let's

$\mathcal{I} : (\text{Id.t list} \times \text{KNormal.t}) \text{M.t} \rightarrow \text{KNormal.t} \rightarrow \text{KNormal.t}$

$\mathcal{I}_\varepsilon(c)$	$= c$
$\mathcal{I}_\varepsilon(\text{op}(x_1, \dots, x_n))$	$= \text{op}(x_1, \dots, x_n)$
$\mathcal{I}_\varepsilon(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x = y \text{ then } \mathcal{I}_\varepsilon(e_1) \text{ else } \mathcal{I}_\varepsilon(e_2)$
$\mathcal{I}_\varepsilon(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x \leq y \text{ then } \mathcal{I}_\varepsilon(e_1) \text{ else } \mathcal{I}_\varepsilon(e_2)$
$\mathcal{I}_\varepsilon(\text{let } x = e_1 \text{ in } e_2)$	$= \text{let } x = \mathcal{I}_\varepsilon(e_1) \text{ in } \mathcal{I}_\varepsilon(e_2)$
$\mathcal{I}_\varepsilon(x)$	$= x$
$\mathcal{I}_\varepsilon(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \varepsilon' = \varepsilon, x \mapsto ((y_1, \dots, y_n), e_1) \text{ として}$ $\text{let rec } x \ y_1 \ \dots \ y_n = \mathcal{I}_{\varepsilon'}(e_1) \text{ in } \mathcal{I}_{\varepsilon'}(e_2)$ $\text{size}(e_1) \leq \text{th}$
$\mathcal{I}_\varepsilon(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{let rec } x \ y_1 \ \dots \ y_n = \mathcal{I}_\varepsilon(e_1) \text{ in } \mathcal{I}_\varepsilon(e_2)$ $\text{size}(e_1) > \text{th}$
$\mathcal{I}_\varepsilon(x \ y_1 \ \dots \ y_n)$	$= \alpha_{y_1 \mapsto z_1, \dots, y_n \mapsto z_n}(e) \quad \varepsilon(x) = ((z_1, \dots, z_n), e)$
$\mathcal{I}_\varepsilon(x \ y_1 \ \dots \ y_n)$	$= x \ y_1 \ \dots \ y_n \quad \varepsilon(x) \text{ is undefined}$
$\mathcal{I}_\varepsilon((x_1, \dots, x_n))$	$= (x_1, \dots, x_n)$
$\mathcal{I}_\varepsilon(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } (x_1, \dots, x_n) = y \text{ in } \mathcal{I}_\varepsilon(e)$
$\mathcal{I}_\varepsilon(x.(y))$	$= x.(y)$
$\mathcal{I}_\varepsilon(x.(y) \leftarrow z)$	$= x.(y) \leftarrow z$
$\text{size}(c)$	$= 1$
$\text{size}(\text{op}(x_1, \dots, x_n))$	$= 1$
$\text{size}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= 1 + \text{size}(e_1) + \text{size}(e_2)$
$\text{size}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= 1 + \text{size}(e_1) + \text{size}(e_2)$
$\text{size}(\text{let } x = e_1 \text{ in } e_2)$	$= 1 + \text{size}(e_1) + \text{size}(e_2)$
$\text{size}(x)$	$= 1$
$\text{size}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= 1 + \text{size}(e_1) + \text{size}(e_2)$
$\text{size}(x \ y_1 \ \dots \ y_n)$	$= 1$
$\text{size}((x_1, \dots, x_n))$	$= 1$
$\text{size}(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= 1 + \text{size}(e)$
$\text{size}(x.(y))$	$= 1$
$\text{size}(x.(y) \leftarrow z)$	$= 1$

Inline expansion: ε is a mapping that takes the name of a smaller-sized function and gives its formal arguments and body. "th" is a user-specified threshold, that specifies the maximum function size that can be expanded.

$\mathcal{F} : \text{KNormal.t M.t} \rightarrow \text{KNormal.t} \rightarrow \text{KNormal.t}$

$\mathcal{F}_\varepsilon(c)$	$= c$	X の場合: when X それ以外の場合: otherwise
$\mathcal{F}_\varepsilon(\text{op}(x_1, \dots, x_n))$	$= c$	$\text{op}(\varepsilon(x_1), \dots, \varepsilon(x_n)) = c$ の場合
$\mathcal{F}_\varepsilon(\text{op}(x_1, \dots, x_n))$	$= \text{op}(x_1, \dots, x_n)$	それ以外の場合
$\mathcal{F}_\varepsilon(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \mathcal{F}_\varepsilon(e_1)$	$\varepsilon(x)$ と $\varepsilon(y)$ are the same constants
$\mathcal{F}_\varepsilon(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \mathcal{F}_\varepsilon(e_2)$	$\varepsilon(x)$ と $\varepsilon(y)$ are different constants
$\mathcal{F}_\varepsilon(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x = y \text{ then } \mathcal{F}_\varepsilon(e_1) \text{ else } \mathcal{F}_\varepsilon(e_2)$	それ以外の場合
$\mathcal{F}_\varepsilon(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \mathcal{F}_\varepsilon(e_1)$	$\varepsilon(x)$ と $\varepsilon(y)$: constants & $\varepsilon(x) \leq \varepsilon(y)$ の場合
$\mathcal{F}_\varepsilon(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \mathcal{F}_\varepsilon(e_2)$	$\varepsilon(x)$ と $\varepsilon(y)$: constants & $\varepsilon(x) > \varepsilon(y)$ の場合
$\mathcal{F}_\varepsilon(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x \leq y \text{ then } \mathcal{F}_\varepsilon(e_1) \text{ else } \mathcal{F}_\varepsilon(e_2)$	それ以外の場合
$\mathcal{F}_\varepsilon(\text{let } x = e_1 \text{ in } e_2)$	$= e'_1 = \mathcal{F}_\varepsilon(e_1)$ として $\text{let } x = e'_1 \text{ in } \mathcal{F}_{\varepsilon, x \mapsto e'_1}(e_2)$	
$\mathcal{F}_\varepsilon(x)$	$= x$	
$\mathcal{F}_\varepsilon(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{let rec } x \ y_1 \ \dots \ y_n = \mathcal{F}_\varepsilon(e_1) \text{ in } \mathcal{F}_\varepsilon(e_2)$	
$\mathcal{F}_\varepsilon(x \ y_1 \ \dots \ y_n)$	$= x \ y_1 \ \dots \ y_n$	
$\mathcal{F}_\varepsilon((x_1, \dots, x_n))$	$= (x_1, \dots, x_n)$	
$\mathcal{F}_\varepsilon(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } x_1 = y_1 \text{ in } \dots \text{let } x_n = y_n \text{ in } \mathcal{F}_\varepsilon(e)$	$\varepsilon(y) = (y_1, \dots, y_n)$ の場合
$\mathcal{F}_\varepsilon(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } (x_1, \dots, x_n) = y \text{ in } \mathcal{F}_\varepsilon(e)$	
$\mathcal{F}_\varepsilon(x.(y))$	$= x.(y)$	
$\mathcal{F}_\varepsilon(x.(y) \leftarrow z)$	$= x.(y) \leftarrow z$	

Constant folding: ε is a mapping that takes a variable and gives a constant.
 its associated expression ~~and gives a constant.~~

$\mathcal{E} : \text{KNormal.t} \rightarrow \text{KNormal.t}$

$\mathcal{E}(c)$	$= c$	"X の場合" (X no baai): "when X"
$\mathcal{E}(op(x_1, \dots, x_n))$	$= op(x_1, \dots, x_n)$	"それ以外の場合" (sore igai no baai): "otherwise"
$\mathcal{E}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x = y \text{ then } \mathcal{E}(e_1) \text{ else } \mathcal{E}(e_2)$	
$\mathcal{E}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x \leq y \text{ then } \mathcal{E}(e_1) \text{ else } \mathcal{E}(e_2)$	
$\mathcal{E}(\text{let } x = e_1 \text{ in } e_2)$	$= \mathcal{E}(e_2)$	$\text{effect}(\mathcal{E}(e_1)) = \text{false}$ かつ $x \notin FV(\mathcal{E}(e_2))$ の場合
$\mathcal{E}(\text{let } x = e_1 \text{ in } e_2)$	$= \text{let } x = \mathcal{E}(e_1) \text{ in } \mathcal{E}(e_2)$	それ以外の場合
$\mathcal{E}(x)$	$= x$	
$\mathcal{E}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \mathcal{E}(e_2)$	$x \notin FV(\mathcal{E}(e_2))$ の場合
$\mathcal{E}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{let rec } x \ y_1 \ \dots \ y_n = \mathcal{E}(e_1) \text{ in } \mathcal{E}(e_2)$	それ以外の場合
$\mathcal{E}(x \ y_1 \ \dots \ y_n)$	$= x \ y_1 \ \dots \ y_n$	
$\mathcal{E}((x_1, \dots, x_n))$	$= (x_1, \dots, x_n)$	
$\mathcal{E}(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \mathcal{E}(e)$	$\{x_1, \dots, x_n\} \cap FV(\mathcal{E}(e)) = \emptyset$ の場合
$\mathcal{E}(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } (x_1, \dots, x_n) = y \text{ in } \mathcal{E}(e)$	それ以外の場合
$\mathcal{E}(x.(y))$	$= x.(y)$	
$\mathcal{E}(x.(y) \leftarrow z)$	$= x.(y) \leftarrow z$	

$\text{effect} : \text{KNormal.t} \rightarrow \text{bool}$

$\text{effect}(c)$	$= \text{false}$
$\text{effect}(op(x_1, \dots, x_n))$	$= \text{false}$
$\text{effect}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{effect}(e_1) \vee \text{effect}(e_2)$
$\text{effect}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{effect}(e_1) \vee \text{effect}(e_2)$
$\text{effect}(\text{let } x = e_1 \text{ in } e_2)$	$= \text{effect}(e_1) \vee \text{effect}(e_2)$
$\text{effect}(x)$	$= \text{false}$
$\text{effect}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \text{effect}(e_2)$
$\text{effect}(x \ y_1 \ \dots \ y_n)$	$= \text{true}$
$\text{effect}((x_1, \dots, x_n))$	$= \text{false}$
$\text{effect}(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{effect}(e)$
$\text{effect}(x.(y))$	$= \text{false}$
$\text{effect}(x.(y) \leftarrow z)$	$= \text{true}$

図 11: 不要定義削除 (1/2)

Elimination of redundant definition

$FV : \text{KNormal.t} \rightarrow \text{S.t}$

$FV(c)$	$= \emptyset$
$FV(op(x_1, \dots, x_n))$	$= \{x_1, \dots, x_n\}$
$FV(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \{x, y\} \cup FV(e_1) \cup FV(e_2)$
$FV(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \{x, y\} \cup FV(e_1) \cup FV(e_2)$
$FV(\text{let } x = e_1 \text{ in } e_2)$	$= FV(e_1) \cup (FV(e_2) \setminus \{x\})$
$FV(x)$	$= \{x\}$
$FV(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= ((FV(e_1) \setminus \{y_1, \dots, y_n\}) \cup FV(e_2)) \setminus \{x\}$
$FV(x \ y_1 \ \dots \ y_n)$	$= \{x, y_1, \dots, y_n\}$
$FV((x_1, \dots, x_n))$	$= \{x_1, \dots, x_n\}$
$FV(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \{y\} \cup (FV(e) \setminus \{x_1, \dots, x_n\})$
$FV(x.(y))$	$= \{x, y\}$
$FV(x.(y) \leftarrow z)$	$= \{x, y, z\}$

图 12: 不要定义消除 (2/2)

Elimination of redundant definition

$P ::=$	The whole program
$(\{D_1, \dots, D_n\}, e)$	Definitions of the top-level functions and the main routine
$D ::=$	Definitions of the top-level functions
$L_x(y_1, \dots, y_m)(z_1, \dots, z_n) = e$	Function: label/formal arguments/free variables/body
$e ::=$	
c	
$op(x_1, \dots, x_n)$	
$\text{if } x = y \text{ then } e_1 \text{ else } e_2$	
$\text{if } x \leq y \text{ then } e_1 \text{ else } e_2$	
$\text{let } x = e_1 \text{ in } e_2$	
x	
$\text{make_closure } x = (L_x, (y_1, \dots, y_n)) \text{ in } e$	Closure creation
$\text{apply_closure}(x, y_1, \dots, y_n)$	Function call using a closure
$\text{apply_direct}(L_x, y_1, \dots, y_n)$	Function call without using a closure
(x_1, \dots, x_n)	(for known functions)
$\text{let } (x_1, \dots, x_n) = y \text{ in } e$	
$x.(y)$	
$x.(y) \leftarrow z$	

图 13: The closure language

$\mathcal{C} : \text{KNormal.t} \rightarrow \text{Closure.t}$

$\mathcal{C}(c)$	$= c$
$\mathcal{C}(op(x_1, \dots, x_n))$	$= op(x_1, \dots, x_n)$
$\mathcal{C}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x = y \text{ then } \mathcal{C}(e_1) \text{ else } \mathcal{C}(e_2)$
$\mathcal{C}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x \leq y \text{ then } \mathcal{C}(e_1) \text{ else } \mathcal{C}(e_2)$
$\mathcal{C}(\text{let } x = e_1 \text{ in } e_2)$	$= \text{let } x = \mathcal{C}(e_1) \text{ in } \mathcal{C}(e_2)$
$\mathcal{C}(x)$	$= x$
$\mathcal{C}(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2)$	$= \mathcal{D} \text{ に } \mathbf{L}_x(y_1, \dots, y_n)(z_1, \dots, z_m) = e'_1 \text{ を加え、}$ $\text{make_closure } x = (\mathbf{L}_x, (z_1, \dots, z_m)) \text{ in } e'_2 \text{ を返す}$ $\text{ただし } e'_1 = \mathcal{C}(e_1), e'_2 = \mathcal{C}(e_2),$ $FV(e'_1) \setminus \{x, y_1, \dots, y_n\} = \{z_1, \dots, z_m\}$
$\mathcal{C}(x \ y_1 \ \dots \ y_n)$	$= \text{apply_closure}(x, y_1, \dots, y_n)$
$\mathcal{C}((x_1, \dots, x_n))$	$= (x_1, \dots, x_n)$
$\mathcal{C}(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \text{let } (x_1, \dots, x_n) = y \text{ in } \mathcal{C}(e)$
$\mathcal{C}(x.(y))$	$= x.(y)$
$\mathcal{C}(x.(y) \leftarrow z)$	$= x.(y) \leftarrow z$

$FV : \text{Closure.t} \rightarrow \text{S.t}$

$FV(c)$	$= \emptyset$
$FV(op(x_1, \dots, x_n))$	$= \{x_1, \dots, x_n\}$
$FV(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \{x, y\} \cup FV(e_1) \cup FV(e_2)$
$FV(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \{x, y\} \cup FV(e_1) \cup FV(e_2)$
$FV(\text{let } x = e_1 \text{ in } e_2)$	$= FV(e_1) \cup (FV(e_2) \setminus \{x\})$
$FV(x)$	$= \{x\}$
$FV(\text{make_closure } x = (\mathbf{L}_x, (y_1, \dots, y_n)) \text{ in } e)$	$= \{y_1, \dots, y_n\} \cup (FV(e) \setminus \{x\})$
$FV(\text{apply_closure}(x, y_1, \dots, y_n))$	$= \{x, y_1, \dots, y_n\}$
$FV(\text{apply_direct}(\mathbf{L}_x, y_1, \dots, y_n))$	$= \{y_1, \dots, y_n\}$
$FV((x_1, \dots, x_n))$	$= \{x_1, \dots, x_n\}$
$FV(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \{y\} \cup (FV(e) \setminus \{x_1, \dots, x_n\})$
$FV(x.(y))$	$= \{x, y\}$
$FV(x.(y) \leftarrow z)$	$= \{x, y, z\}$

図 14: 賢くない Closure 変換 $\mathcal{C}(e)$ 。 \mathcal{D} はトップレベル関数定義の集合を記憶しておくためのグローバル変数。

$\mathcal{C} : \text{S.t} \rightarrow \text{KNormal.t} \rightarrow \text{Closure.t}$

$$\begin{aligned}
\mathcal{C}_s(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2) &= \mathcal{D} \text{ に } \mathbf{L}_x(y_1, \dots, y_n)() = e'_1 \text{ を加え、} \\
&\quad \text{make_closure } x = (\mathbf{L}_x, ()) \text{ in } e'_2 \text{ を返す} \\
&\quad \text{ただし } e'_1 = \mathcal{C}_{s'}(e_1), e'_2 = \mathcal{C}_{s'}(e_2), s' = s \cup \{x\}, \\
&\quad FV(e'_1) \setminus \{y_1, \dots, y_n\} = \emptyset \text{ の場合} \\
\mathcal{C}_s(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2) &= \mathcal{D} \text{ に } \mathbf{L}_x(y_1, \dots, y_n)(z_1, \dots, z_m) = e'_1 \text{ を加え、} \\
&\quad \text{make_closure } x = (\mathbf{L}_x, (z_1, \dots, z_m)) \text{ in } e'_2 \text{ を返す} \\
&\quad \text{ただし } e'_1 = \mathcal{C}_s(e_1), e'_2 = \mathcal{C}_s(e_2), \\
&\quad FV(e'_1) \setminus \{y_1, \dots, y_n\} \neq \emptyset, \\
&\quad FV(e'_1) \setminus \{x, y_1, \dots, y_n\} = \{z_1, \dots, z_m\} \text{ の場合} \\
\mathcal{C}_s(x \ y_1 \ \dots \ y_n) &= \text{apply_closure}(x, y_1, \dots, y_n) && x \notin s \text{ の場合} \\
\mathcal{C}_s(x \ y_1 \ \dots \ y_n) &= \text{apply_direct}(\mathbf{L}_x, y_1, \dots, y_n) && x \in s \text{ の場合}
\end{aligned}$$

図 15: やや賢い Closure 変換 $\mathcal{C}_s(e)$ 。 s は自由変数がないとわかっている関数の名前の集合。

$\mathcal{C} : \text{S.t} \rightarrow \text{KNormal.t} \rightarrow \text{Closure.t}$

$$\begin{aligned}
\mathcal{C}_s(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2) &= \mathcal{D} \text{ に } \mathbf{L}_x(y_1, \dots, y_n)() = e'_1 \text{ を加え、} \\
&\quad \text{make_closure } x = (\mathbf{L}_x, ()) \text{ in } e'_2 \text{ を返す} \\
&\quad \text{ただし } e'_1 = \mathcal{C}_{s'}(e_1), e'_2 = \mathcal{C}_{s'}(e_2), s' = s \cup \{x\}, \\
&\quad FV(e'_1) \setminus \{y_1, \dots, y_n\} = \emptyset \text{ かつ } x \in FV(e'_2) \text{ の場合} \\
\mathcal{C}_s(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2) &= \mathcal{D} \text{ に } \mathbf{L}_x(y_1, \dots, y_n)() = e'_1 \text{ を加え、} e'_2 \text{ を返す} \\
&\quad \text{ただし } e'_1 = \mathcal{C}_{s'}(e_1), e'_2 = \mathcal{C}_{s'}(e_2), s' = s \cup \{x\}, \\
&\quad FV(e'_1) \setminus \{y_1, \dots, y_n\} = \emptyset \text{ かつ } x \notin FV(e'_2) \text{ の場合} \\
\mathcal{C}_s(\text{let rec } x \ y_1 \ \dots \ y_n = e_1 \text{ in } e_2) &= \mathcal{D} \text{ に } \mathbf{L}_x(y_1, \dots, y_n)(z_1, \dots, z_m) = e'_1 \text{ を加え、} \\
&\quad \text{make_closure } x = (\mathbf{L}_x, (z_1, \dots, z_m)) \text{ in } e'_2 \text{ を返す} \\
&\quad \text{ただし } e'_1 = \mathcal{C}_s(e_1), e'_2 = \mathcal{C}_s(e_2), \\
&\quad FV(e'_1) \setminus \{y_1, \dots, y_n\} \neq \emptyset, \\
&\quad FV(e'_1) \setminus \{x, y_1, \dots, y_n\} = \{z_1, \dots, z_m\} \text{ の場合} \\
\mathcal{C}_s(x \ y_1 \ \dots \ y_n) &= \text{apply_closure}(x, y_1, \dots, y_n) && x \notin s \text{ の場合} \\
\mathcal{C}_s(x \ y_1 \ \dots \ y_n) &= \text{apply_direct}(\mathbf{L}_x, y_1, \dots, y_n) && x \in s \text{ の場合}
\end{aligned}$$

図 16: もっと賢い Closure 変換 $\mathcal{C}_s(e)$

$P ::=$	
$(\{D_1, \dots, D_n\}, E)$	
$D ::=$	
$L_x(y_1, \dots, y_n) = E$	
$E ::=$	命令の列
$x \leftarrow e; E$	代入
e	返値
$e ::=$	式
c	即値
L_x	ラベル
$op(x_1, \dots, x_n)$	算術演算
$\text{if } x = y \text{ then } E_1 \text{ else } E_2$	比較&分岐
$\text{if } x \leq y \text{ then } E_1 \text{ else } E_2$	比較&分岐
x	mov 命令
$\text{apply_closure}(x, y_1, \dots, y_n)$	クロージャを用いた関数呼び出し
$\text{apply_direct}(L_x, y_1, \dots, y_n)$	クロージャを用いない関数呼び出し
$x.(y)$	ロード
$x.(y) \leftarrow z$	ストア
save (x, y)	変数 x の値をスタック位置 y に退避する
restore (y)	スタック位置 y から値を復元する

図 17: 仮想マシンコードの構文

$\mathcal{V} : \text{Closure.prog} \rightarrow \text{SparcAsm.prog}$	
$\mathcal{V}(\{\{D_1, \dots, D_n\}, e\})$	$= (\{\mathcal{V}(D_1), \dots, \mathcal{V}(D_n)\}, \mathcal{V}(e))$
$\mathcal{V} : \text{Closure.fundef} \rightarrow \text{SparcAsm.fundef}$	
$\mathcal{V}(\text{L}_x(y_1, \dots, y_n)(z_1, \dots, z_n) = e)$	$= \text{L}_x(y_1, \dots, y_n) = z_1 \leftarrow \text{R}_0.(4); \dots; z_n \leftarrow \text{R}_0.(4n); \mathcal{V}(e)$
$\mathcal{V} : \text{Closure.t} \rightarrow \text{SparcAsm.t}$	
$\mathcal{V}(c)$	$= c$
$\mathcal{V}(op(x_1, \dots, x_n))$	$= op(x_1, \dots, x_n)$
$\mathcal{V}(\text{if } x = y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x = y \text{ then } \mathcal{V}(e_1) \text{ else } \mathcal{V}(e_2)$
$\mathcal{V}(\text{if } x \leq y \text{ then } e_1 \text{ else } e_2)$	$= \text{if } x \leq y \text{ then } \mathcal{V}(e_1) \text{ else } \mathcal{V}(e_2)$
$\mathcal{V}(\text{let } x = e_1 \text{ in } e_2)$	$= x \leftarrow \mathcal{V}(e_1); \mathcal{V}(e_2)$
$\mathcal{V}(x)$	$= x$
$\mathcal{V}(\text{make_closure } x = (\text{L}_x, (y_1, \dots, y_n)) \text{ in } e)$	$= x \leftarrow \text{R}_{\text{hp}}; \text{R}_{\text{hp}} \leftarrow \text{R}_{\text{hp}} + 4(n+1); z \leftarrow \text{L}_x; x.(0) \leftarrow z;$ $x.(4) \leftarrow y_1; \dots; x.(4n) \leftarrow y_n; \mathcal{V}(e)$
$\mathcal{V}(\text{apply_closure}(x, y_1, \dots, y_n))$	$= \text{apply_closure}(x, y_1, \dots, y_n)$
$\mathcal{V}(\text{apply_direct}(\text{L}_x, y_1, \dots, y_n))$	$= \text{apply_direct}(\text{L}_x, y_1, \dots, y_n)$
$\mathcal{V}((x_1, \dots, x_n))$	$= y \leftarrow \text{R}_{\text{hp}}; \text{R}_{\text{hp}} \leftarrow \text{R}_{\text{hp}} + 4n;$ $y.(0) \leftarrow x_1; \dots; y.(4(n-1)) \leftarrow x_n; y$
$\mathcal{V}(\text{let } (x_1, \dots, x_n) = y \text{ in } e)$	$= \{x_1, \dots, x_n\} \cap FV(e) = \{x_{i_1}, \dots, x_{i_m}\} \text{ として}$ $x_{i_1} \leftarrow y.(4(i_1-1)); \dots; x_{i_m} \leftarrow y.(4(i_m-1)); \mathcal{V}(e)$
$\mathcal{V}(x.(y))$	$= y' \leftarrow 4 \times y; x.(y')$
$\mathcal{V}(x.(y) \leftarrow z)$	$= y' \leftarrow 4 \times y; x.(y') \leftarrow z$

図 18: 仮想マシンコード生成 $\mathcal{V}(P)$, $\mathcal{V}(D)$ および $\mathcal{V}(e)$ 。右辺に出現して左辺に出現しない変数は fresh とする。 R_{hp} はヒープポインタ (専用レジスタ)。 $e_1; e_2$ はダミーの変数 x について $x \leftarrow e_1; e_2$ の略記。 $x \leftarrow E_1; E_2$ は、 $E_1 = (x_1 \leftarrow e_1; \dots; x_n \leftarrow e_n; e)$ として、 $x_1 \leftarrow e_1; \dots; x_n \leftarrow e_n; x \leftarrow e; E_2$ の略記。

$$\begin{aligned}
& FV : \mathbf{S.t} \rightarrow \mathbf{SparcAsm.t} \rightarrow \mathbf{S.t} \\
& FV_s(x \leftarrow e; E) = s' = FV_s(E) \setminus \{x\} \text{ として } FV_{s'}(e) \\
& FV_s(e) = FV_s(e) \\
\\
& FV : \mathbf{S.t} \rightarrow \mathbf{SparcAsm.exp} \rightarrow \mathbf{S.t} \\
& FV_s(c) = s \\
& FV_s(\mathbf{L}_x) = s \\
& FV_s(op(x_1, \dots, x_n)) = \{x_1, \dots, x_n\} \cup s \\
& FV_s(\mathbf{if } x = y \mathbf{ then } E_1 \mathbf{ else } E_2) = \{x, y\} \cup FV_s(E_1) \cup FV_s(E_2) \\
& FV_s(\mathbf{if } x \leq y \mathbf{ then } E_1 \mathbf{ else } E_2) = \{x, y\} \cup FV_s(E_1) \cup FV_s(E_2) \\
& FV_s(x) = \{x\} \cup s \\
& FV_s(\mathbf{apply_closure}(x, y_1, \dots, y_n)) = \{x, y_1, \dots, y_n\} \cup s \\
& FV_s(\mathbf{apply_direct}(\mathbf{L}_x, y_1, \dots, y_n)) = \{y_1, \dots, y_n\} \cup s \\
& FV_s(x.(y)) = \{x, y\} \cup s \\
& FV_s(x.(y) \leftarrow z) = \{x, y, z\} \cup s \\
& FV_s(\mathbf{save}(x, y)) = \{x\} \cup s \\
& FV_s(\mathbf{restore}(y)) = s
\end{aligned}$$

図 19: 命令の列 E および式 e において生きている変数の集合 $FV_s(E)$ および $FV_s(e)$ 。 s は E や e の後で使われる変数の集合。以後の $FV(E)$ は $FV_\emptyset(E)$ の略記。

$$\begin{aligned}
& \mathcal{R} : \text{SparcAsm.prog} \rightarrow \text{SparcAsm.prog} \\
& \mathcal{R}(\{\{D_1, \dots, D_n\}, E\}) = (\{\mathcal{R}(D_1), \dots, \mathcal{R}(D_n)\}, \mathcal{R}_\emptyset(E, x, ())) \quad x \text{ はダミーの fresh な変数} \\
\\
& \mathcal{R} : \text{SparcAsm.fundef} \rightarrow \text{SparcAsm.fundef} \\
& \mathcal{R}(\text{L}_x(y_1, \dots, y_n) = E) = \text{L}_x(\text{R}_1, \dots, \text{R}_n) = \mathcal{R}_{x \mapsto \text{R}_0, y_1 \mapsto \text{R}_1, \dots, y_n \mapsto \text{R}_n}(E, \text{R}_0, \text{R}_0) \\
\\
& \mathcal{R} : \text{Id.t M.t} \rightarrow \text{SparcAsm.t} \times \text{Id.t} \times \text{SparcAsm.t} \rightarrow \text{SparcAsm.t} \times \text{Id.t M.t} \\
& \mathcal{R}_\varepsilon((x \leftarrow e; E), z_{\text{dest}}, E_{\text{cont}}) = E'_{\text{cont}} = (z_{\text{dest}} \leftarrow E; E_{\text{cont}}), \\
& \quad \mathcal{R}_\varepsilon(e, x, E'_{\text{cont}}) = (E', \varepsilon'), \\
& \quad r \notin \{\varepsilon'(y) \mid y \in \text{FV}(E'_{\text{cont}})\}, \\
& \quad \mathcal{R}_{\varepsilon', x \mapsto r}(E, z_{\text{dest}}, E_{\text{cont}}) = (E'', \varepsilon'') \text{ として} \\
& \quad ((r \leftarrow E'; E''), \varepsilon'') \quad x \text{ がレジスタでない場合} \\
& \mathcal{R}_\varepsilon((r \leftarrow e; E), z_{\text{dest}}, E_{\text{cont}}) = E'_{\text{cont}} = (z_{\text{dest}} \leftarrow E; E_{\text{cont}}), \\
& \quad \mathcal{R}_\varepsilon(e, r, E'_{\text{cont}}) = (E', \varepsilon'), \\
& \quad \mathcal{R}_{\varepsilon'}(E, z_{\text{dest}}, E_{\text{cont}}) = (E'', \varepsilon'') \text{ として} \\
& \quad ((r \leftarrow E'; E''), \varepsilon'') \\
& \mathcal{R}_\varepsilon(e, x, E_{\text{cont}}) = \mathcal{R}_\varepsilon(e, x, E_{\text{cont}}) \quad (\text{次図参照})
\end{aligned}$$

図 20: 単純なレジスタ割り当て $\mathcal{R}(P)$, $\mathcal{R}(D)$ および $\mathcal{R}_\varepsilon(E, z_{\text{dest}}, E_{\text{cont}})$ 。 ε は変数からレジスタへの写像、 z_{dest} は E の結果をセットする変数、 E_{cont} は E の後に実行される命令の列。 $\mathcal{R}_\varepsilon(E, x, E_{\text{cont}})$ の返り値はレジスタ割り当てされた命令の列 E' と、 E の後のレジスタ割り当てを表す写像 ε' の組。 [ファイル `regAlloc.notarget-nospill.ml` 参照]

$\mathcal{R} : \text{Id.t M.t} \rightarrow \text{SparcAsm.exp} \times \text{Id.t} \times \text{SparcAsm.t} \rightarrow \text{SparcAsm.t} \times \text{Id.t M.t}$	
$\mathcal{R}_\varepsilon(c, z_{\text{dest}}, E_{\text{cont}})$	$= (c, \varepsilon)$
$\mathcal{R}_\varepsilon(\text{L}_x, z_{\text{dest}}, E_{\text{cont}})$	$= (\text{L}_x, \varepsilon)$
$\mathcal{R}_\varepsilon(\text{op}(x_1, \dots, x_n), z_{\text{dest}}, E_{\text{cont}})$	$= (\text{op}(\varepsilon(x_1), \dots, \varepsilon(x_n)), \varepsilon)$
$\mathcal{R}_\varepsilon(\text{if } x = y \text{ then } E_1 \text{ else } E_2, z_{\text{dest}}, E_{\text{cont}})$	$= \mathcal{R}_\varepsilon(E_1, z_{\text{dest}}, E_{\text{cont}}) = (E'_1, \varepsilon_1),$ $\mathcal{R}_\varepsilon(E_2, z_{\text{dest}}, E_{\text{cont}}) = (E'_2, \varepsilon_2),$ $\varepsilon' = \{z \mapsto r \mid \varepsilon_1(z) = \varepsilon_2(z) = r\},$ $\{z_1, \dots, z_n\} =$ $(FV(E_{\text{cont}}) \setminus \{z_{\text{dest}}\} \setminus \text{dom}(\varepsilon')) \cap \text{dom}(\varepsilon) \text{ として}$ $((\text{save}(\varepsilon(z_1), z_1); \dots; \text{save}(\varepsilon(z_n), z_n);$ $\text{if } \varepsilon(x) \leq \varepsilon(y) \text{ then } E'_1 \text{ else } E'_2), \varepsilon')$
$\mathcal{R}_\varepsilon(\text{if } x \leq y \text{ then } E_1 \text{ else } E_2, z_{\text{dest}}, E_{\text{cont}})$	$= \text{同様}$
$\mathcal{R}_\varepsilon(x, z_{\text{dest}}, E_{\text{cont}})$	$= (\varepsilon(x), \varepsilon)$
$\mathcal{R}_\varepsilon(\text{apply_closure}(x, y_1, \dots, y_n), z_{\text{dest}}, E_{\text{cont}})$	$= \{z_1, \dots, z_n\} = (FV(E_{\text{cont}}) \setminus \{z_{\text{dest}}\}) \cap \text{dom}(\varepsilon) \text{ として}$ $((\text{save}(\varepsilon(z_1), z_1); \dots; \text{save}(\varepsilon(z_n), z_n);$ $\text{apply_closure}(\varepsilon(x), \varepsilon(y_1), \dots, \varepsilon(y_n))), \emptyset)$
$\mathcal{R}_\varepsilon(\text{apply_direct}(\text{L}_x, y_1, \dots, y_n), z_{\text{dest}}, E_{\text{cont}})$	$= \text{同様}$
$\mathcal{R}_\varepsilon(x.(y), z_{\text{dest}}, E_{\text{cont}})$	$= (\varepsilon(x).(\varepsilon(y)), \varepsilon)$
$\mathcal{R}_\varepsilon(x.(y) \leftarrow z, z_{\text{dest}}, E_{\text{cont}})$	$= (\varepsilon(x).(\varepsilon(y)) \leftarrow \varepsilon(z), \varepsilon)$
$\mathcal{R}_\varepsilon(\text{save}(x, y), z_{\text{dest}}, E_{\text{cont}})$	$= (\text{save}(\varepsilon(x), y), \varepsilon)$
$\mathcal{R}_\varepsilon(\text{restore}(y), z_{\text{dest}}, E_{\text{cont}})$	$= (\text{restore}(y), \varepsilon)$

図 21: 単純なレジスタ割り当て $\mathcal{R}_\varepsilon(e, z_{\text{dest}}, E_{\text{cont}})$ 。 $\mathcal{R}_\varepsilon(e)$ の右辺で変数 x のレジスタ $\varepsilon(x)$ が定義されていない場合は、 $\mathcal{R}_\varepsilon(e) = \mathcal{R}_\varepsilon(x \leftarrow \text{restore}(x); e)$ とする。ただしレジスタ r については $\varepsilon(r) = r$ とする。
 [ファイル `regAlloc.notarget-nospill.ml` 参照]

$T : \text{Id.t} \rightarrow \text{SparcAsm.t} \times \text{Id.t} \rightarrow \text{bool} \times \text{S.t}$	
$T_x((y \leftarrow e; E), z_{\text{dest}})$	$= T_x(e, y) = (c_1, s_1)$ として、もし c_1 ならば (true, s_1) そうでなければ $T_x(E, z_{\text{dest}}) = (c_2, s_2)$ として $(c_2, s_1 \cup s_2)$
$T_x(e, z_{\text{dest}})$	$= T_x(e, z_{\text{dest}})$
$T : \text{Id.t} \rightarrow \text{SparcAsm.exp} \times \text{Id.t} \rightarrow \text{bool} \times \text{S.t}$	
$T_x(x, z_{\text{dest}})$	$= (\text{false}, \{z_{\text{dest}}\})$
$T_x(\text{if } y = z \text{ then } E_1 \text{ else } E_2, z_{\text{dest}})$	$= T_x(E_1, z_{\text{dest}}) = (c_1, s_1),$ $T_x(E_2, z_{\text{dest}}) = (c_2, s_2)$ として $(c_1 \wedge c_2, s_1 \cup s_2)$
$T_x(\text{if } y \leq z \text{ then } E_1 \text{ else } E_2, z_{\text{dest}})$	$=$ 同上
$T_x(\text{apply_closure}(y_0, y_1, \dots, y_n), z_{\text{dest}})$	$= (\text{true}, \{\mathbf{R}_i \mid x = y_i\})$
$T_x(\text{apply_direct}(\mathbf{L}_y, y_1, \dots, y_n), z_{\text{dest}})$	$=$ 同上
$T_x(e, z_{\text{dest}})$	$= (\text{false}, \emptyset)$ それ以外の場合

図 22: 変数 x に割り当てるレジスタ r を選ぶときに使う targetting $T_x(E, z_{\text{dest}})$ および $T_x(e, z_{\text{dest}})$ 。 E や e で関数呼び出しがあったかどうかを表す論理値 c と、 x を割り当てると良いレジスタの集合 s の組を返す。前々図の「 x がレジスタでない場合」において、 $T_x(E'_{\text{cont}}, z_{\text{dest}}) = (c, s)$ として、できれば $r \in s$ とする。[ファイル `regAlloc.target-nospill.ml` 参照]

$$\begin{aligned}
\mathcal{R} : \text{Id.t} \text{ M.t} &\rightarrow \text{SparcAsm.t} \times \text{Id.t} \times \text{SparcAsm.t} \rightarrow \text{SparcAsm.t} \times \text{Id.t} \text{ M.t} \\
\mathcal{R}_\varepsilon((x \leftarrow e; E), z_{\text{dest}}, E_{\text{cont}}) &= E'_{\text{cont}} = (z_{\text{dest}} \leftarrow E; E_{\text{cont}}), \\
\mathcal{R}_\varepsilon(e, x, E'_{\text{cont}}) &= (E', \varepsilon'), \\
y &\in FV(E'_{\text{cont}}), \\
\mathcal{R}_{\varepsilon' \setminus \{y \mapsto \varepsilon'(y)\}, x \mapsto \varepsilon'(y)}(E, z_{\text{dest}}, E_{\text{cont}}) &= (E'', \varepsilon'') \text{ として} \\
\begin{cases} ((\text{save}(\varepsilon(y), y); \varepsilon'(y) \leftarrow E'; E''), \varepsilon'') & y \in \text{dom}(\varepsilon) \text{ のとき} \\ ((\varepsilon'(y) \leftarrow E'; E''), \varepsilon'') & y \notin \text{dom}(\varepsilon) \text{ のとき} \end{cases} \\
&x \text{ がレジスタでなく、} \\
&r \notin \{\varepsilon'(y) \mid y \in FV(E'_{\text{cont}})\} \text{ なる } r \text{ がない場合}
\end{aligned}$$

図 23: spilling をするレジスタ割り当て $\mathcal{R}_\varepsilon(E, z_{\text{dest}}, E_{\text{cont}})$ [ファイル `regAlloc.target-latespill.ml` 参照]

```

 $S : \text{SparcAsm.prog} \rightarrow \text{string}$ 
 $S(\{D_1, \dots, D_n\}, E) = \begin{array}{l} \text{.section ".text"} \\ S(D_1) \\ \dots \\ S(D_n) \\ \text{.global min\_caml\_start} \\ \text{min\_caml\_start:} \\ \text{save \%sp, -112, \%sp} \\ S(E, \%g0) \\ \text{ret} \\ \text{restore} \end{array}$ 

 $S : \text{SparcAsm.fundef} \rightarrow \text{string}$ 
 $S(L_x(y_1, \dots, y_n) = E) = \begin{array}{l} x: \\ S(E, R_0) \\ \text{retl} \\ \text{nop} \end{array}$ 

 $S : \text{SparcAsm.t} \times \text{Id.t} \rightarrow \text{string}$ 
 $S((x \leftarrow e; E), z_{\text{dest}}) = \begin{array}{l} S(e, x) \\ S(E, z_{\text{dest}}) \end{array}$ 
 $S(e, z_{\text{dest}}) = S(e, z_{\text{dest}})$ 

```

図 24: 単純なアセンブリ生成 $S(P)$, $S(D)$ および $S(E, z_{\text{dest}})$

$\mathcal{S} : \text{SparcAsm.exp} \times \text{Id.t} \rightarrow \text{string}$	
$\mathcal{S}(c, z_{\text{dest}})$	<code>= set c, z_{dest}</code>
$\mathcal{S}(L_x, z_{\text{dest}})$	<code>= set L_x, z_{dest}</code>
$\mathcal{S}(op(x_1, \dots, x_n), z_{\text{dest}})$	<code>= op x₁, ..., x_n, z_{dest}</code>
$\mathcal{S}(\text{if } x = y \text{ then } E_1 \text{ else } E_2, z_{\text{dest}})$	<code>= cmp x, y</code> <code>bne b₁</code> <code>nop</code> $\mathcal{S}(E_1, z_{\text{dest}})$ <code>b b₂</code> <code>nop</code> $b_1 :$ $\mathcal{S}(E_2, z_{\text{dest}})$ $b_2 :$
$\mathcal{S}(\text{if } x \leq y \text{ then } E_1 \text{ else } E_2, z_{\text{dest}})$	<code>= 同様</code>
$\mathcal{S}(x, z_{\text{dest}})$	<code>= mov x, z_{dest}</code>
$\mathcal{S}(\text{apply_closure}(x, y_1, \dots, y_n), z_{\text{dest}})$	<code>= shuffle((x, y₁, ..., y_n), (R₀, R₁, ..., R_n))</code> <code>st R_{ra}, [R_{st} + 4#ε]</code> <code>ld [R₀], R_{n+1}</code> <code>call R_{n+1}</code> <code>add R_{st}, 4(#ε + 1), R_{st} ! delay slot</code> <code>sub R_{st}, 4(#ε + 1), R_{st}</code> <code>ld [R_{st} + 4#ε], R_{ra}</code> <code>mov R₀, z_{dest}</code>
$\mathcal{S}(\text{apply_direct}(L_x, y_1, \dots, y_n), z_{\text{dest}})$	<code>= shuffle((y₁, ..., y_n), (R₁, ..., R_n))</code> <code>st R_{ra}, [R_{st} + 4#ε]</code> <code>call x</code> <code>add R_{st}, 4(#ε + 1), R_{st} ! delay slot</code> <code>sub R_{st}, 4(#ε + 1), R_{st}</code> <code>ld [R_{st} + 4#ε], R_{ra}</code> <code>mov R₀, z_{dest}</code>
$\mathcal{S}(x.(y), z_{\text{dest}})$	<code>= ld [x + y], z_{dest}</code>
$\mathcal{S}(x.(y) \leftarrow z, z_{\text{dest}})$	<code>= st z, [x + y]</code>
$\mathcal{S}(\text{save}(x, y), z_{\text{dest}})$	<code>= もし $y \notin \text{dom}(\varepsilon)$ なら ε に $y \mapsto 4\#\varepsilon$ を加えて</code> <code>st x, [R_{st} + ε(y)]</code>
$\mathcal{S}(\text{restore}(y), z_{\text{dest}})$	<code>= ld [R_{st} + ε(y)], z_{dest}</code>

図 25: 単純なアセンブリ生成 $\mathcal{S}(e, z_{\text{dest}})$ 。ε はスタック位置を記憶するグローバル変数。#ε は ε の要素の個数。 $\text{shuffle}((x_1, \dots, x_n), (r_1, \dots, r_n))$ は x_1, \dots, x_n を r_1, \dots, r_n に適切な順序で移動する命令。

$$\begin{aligned}
& \mathcal{S} : \mathbf{S.t} \rightarrow \mathbf{SparcAsm.t} \times \mathbf{Id.t} \rightarrow \mathbf{S.t} \times \mathbf{string} \\
& \mathcal{S}_s((x \leftarrow e; E), z_{\text{dest}}) = \mathcal{S}_s(e, x) = (s', S), \\
& \quad \mathcal{S}_{s'}(E, z_{\text{dest}}) = (s'', S') \text{ として} \\
& \quad (s'', SS') \\
& \mathcal{S}_s(e, z_{\text{dest}}) = \mathcal{S}_s(e, z_{\text{dest}}) \\
\\
& \mathcal{S} : \mathbf{S.t} \rightarrow \mathbf{SparcAsm.exp} \times \mathbf{Id.t} \rightarrow \mathbf{S.t} \times \mathbf{string} \\
& \mathcal{S}_s(\text{if } x = y \text{ then } E_1 \text{ else } E_2, z_{\text{dest}}) = \mathcal{S}_s(E_1, z_{\text{dest}}) = (s_1, S_1), \\
& \quad \mathcal{S}_s(E_2, z_{\text{dest}}) = (s_2, S_2) \text{ として} \\
& \quad (s_1 \cap s_2, \\
& \quad \text{cmp } x, y \\
& \quad \text{bne } b_1 \\
& \quad \text{nop} \\
& \quad S_1 \\
& \quad \text{b } b_2 \\
& \quad \text{nop} \\
& \quad b_1 : \\
& \quad S_2 \\
& \quad b_2 :) \\
& \mathcal{S}_s(\text{if } x \leq y \text{ then } E_1 \text{ else } E_2, z_{\text{dest}}) = \text{同様} \\
& \mathcal{S}_s(\text{save}(x, y), z_{\text{dest}}) = (s, \text{nop}) \quad y \in s \text{ の場合} \\
& \mathcal{S}_s(\text{save}(x, y), z_{\text{dest}}) = \text{もし } y \notin \text{dom}(\varepsilon) \text{ なら } \varepsilon \text{ に } y \mapsto 4\#\varepsilon \text{ を加えて} \\
& \quad (s \cup \{y\}, \text{st } x, [\text{R}_{\text{st}} + \varepsilon(y)]) \quad y \notin s \text{ の場合} \\
& \mathcal{S}_s(e, z_{\text{dest}}) = (s, \text{以前と同様}) \quad \text{上述以外の場合}
\end{aligned}$$

図 26: 無駄な save を省略するアセンブリ生成 $\mathcal{S}_s(E, z_{\text{dest}})$ および $\mathcal{S}_s(e, z_{\text{dest}})$ 。 s はすでに save された変数の名前の集合。以前の $\mathcal{S}(E, z_{\text{dest}})$ は $\mathcal{S}_\emptyset(E, z_{\text{dest}}) = (s, S)$ として S の略記とする。

$\mathcal{S} : \text{SparcAsm.fundef} \rightarrow \text{string}$
 $\mathcal{S}(\text{L}_x(y_1, \dots, y_n) = E) = \mathcal{S}_\emptyset(E, \text{tail}) = (s, S) \text{ として}$
 $x:$
 S

$\mathcal{S} : \text{S.t} \rightarrow \text{SparcAsm.exp} \times \text{Id.t} \rightarrow \text{S.t} \times \text{string}$
 $\mathcal{S}_s(\text{if } x = y \text{ then } E_1 \text{ else } E_2, \text{tail}) = \mathcal{S}_s(E_1, \text{tail}) = (s_1, S_1),$
 $\mathcal{S}_s(E_2, \text{tail}) = (s_2, S_2) \text{ として}$
 $(\emptyset,$
 $\text{cmp } x, y$
 $\text{bne } b$
 nop
 S_1
 $b:$
 $S_2)$

$\mathcal{S}_s(\text{if } x \leq y \text{ then } E_1 \text{ else } E_2, \text{tail}) = \text{同様}$
 $\mathcal{S}_s(\text{apply_closure}(x, y_1, \dots, y_n), \text{tail}) = (\emptyset,$
 $\text{shuffle}((x, y_1, \dots, y_n), (\text{R}_0, \text{R}_1, \dots, \text{R}_n))$
 $\text{ld } [\text{R}_0], \text{R}_{n+1}$
 $\text{jmp } \text{R}_{n+1}$
 $\text{nop})$

$\mathcal{S}_s(\text{apply_direct}(\text{L}_x, y_1, \dots, y_n), \text{tail}) = (\emptyset,$
 $\text{shuffle}((y_1, \dots, y_n), (\text{R}_1, \dots, \text{R}_n))$
 $\text{b } x$
 $\text{nop})$

$\mathcal{S}_s(e, \text{tail}) = \mathcal{S}_s(e, \text{R}_0) = (s', S) \text{ として}$
 $(\emptyset,$
 S
 retl
 $\text{nop})$

上述以外の場合

図 27: 末尾呼び出し最適化をするアセンブリ生成 $\mathcal{S}_s(D)$ および $\mathcal{S}_s(e, z_{\text{dest}})$ 。 $z_{\text{dest}} = \text{tail}$ の場合が末尾。