Title: Summing $\mu(n)$: an even faster elementary algorithm

We present a new elementary algorithm for computing $M(x) = \sum_{n \leq x} \mu(n)$, where $\mu(n)$ is the Möbius function. Our algorithm takes

$$\text{time} \;\; O_\epsilon\left(x^{\frac{3}{5}}(\log x)^{\frac{3}{5}+\epsilon}\right) \;\; \text{and} \;\; \text{space} \;\; O\left(x^{\frac{3}{10}}(\log x)^{\frac{13}{10}}\right),$$

which improves on existing combinatorial algorithms. While there is an analytic algorithm due to Lagarias-Odlyzko with computations based on the integrals of $\zeta(s)$ that only takes time $O(x^{1/2+\epsilon})$, our algorithm has the advantage of being easier to implement. The new approach roughly amounts to analyzing the difference between a model that we obtain via Diophantine approximation and reality, and showing that it has a simple description in terms of congruence classes and segments. This simple description allows us to compute the difference quickly by means of a table lookup. This talk is based on joint work with Harald Andrés Helfgott.