

1. (Applying the Riemann Sum) You are driving when all of a sudden, you see traffic stopped in front of you. You slam the brakes to come to a stop. While your brakes are applied, the velocity of the car is measured, and you obtain the following measurements:

Time since applying breaks (sec) : 0 1 2 3 4 5

Velocity of car (in ft/sec) : 88 60 40 25 10 0

- (a) Plot the points on a curve of velocity vs. time.
 (b) Using the points given, determine upper and lower bounds for the total distance traveled before the car came to a stop.

Solution. $U_f = 88 + 60 + 40 + 25 + 10 = 223$.

$L_f = 60 + 40 + 25 + 10 + 0 = 135$.

2. Estimate the area under the graph of $f(x) = 10 - x^2$ between the lines $x = -3$ and $x = 2$ using $n = 5$ equally spaced subintervals, by finding:

- (a) The upper sum, U_f .

$$U_f = f(-2) + f(-1) + f(0) + f(0) + f(1) = \dots$$

- (b) The lower sum, L_f .

$$L_f = f(-3) + f(-2) + f(-1) + f(1) + f(2) = \dots$$

3. Determine if each statement below is true or false.

- (a) To find the upper sum U_f of a function f on $[a, b]$, after partitioning the interval into n pieces, evaluate f at the right-hand endpoint of each subinterval.

Solution. False. Give an example.

- (b) When the interval $[a, b]$ is partitioned into n pieces, there are exactly n endpoints.

Solution. False. There are $n + 1$ endpoints.

- (c) A partition of the interval $[a, b]$ does not need to be evenly spaced in order to calculate a Riemann Sum.

Solution. True. But we always consider them to be evenly spaced.

- (d) If f is positive and continuous on $[a, b]$, and A is the actual area bounded by f , $x = a$, $x = b$, and the x -axis, then $L_f < A < U_f$.

Solution. True. Check this. Also consider the case when f is not positive.