$$
\begin{aligned}
F_1 \cdot \arctan(z/n) = \sum_{n=1}^{\infty} \arctan(z/n^2) &= \sum_{n=1}^{\infty} (-1)^{n+1} \zeta(2(2n-1)) \frac{z^{2n-1}}{2n-1} \\
&= \frac{1}{2i} \ln \frac{\sinh(\pi\sqrt{iz})}{i\sin(\pi\sqrt{iz})}
\end{aligned}
$$

$$
\to \left(\frac{\tan(\pi\sqrt{z/2}) + i\tanh(\pi\sqrt{z/2})}{\tan(\pi\sqrt{z/2}) - i\tanh(\pi\sqrt{z/2})} \right) = \frac{\sin(\pi\sqrt{iz})}{\sinh(\pi\sqrt{iz})}
$$

$$
\begin{aligned}
F_1 \cdot \arctan(z/n^3) = \sum_{n=1}^{\infty} \arctan(z/n^4) &= \sum_{n=1}^{\infty} (-1)^{n+1} \zeta(4(2n-1)) \frac{z^{2n-1}}{2n-1} \\
&= \frac{1}{2i} \ln \frac{\sin(\pi i^{3/4} z^{1/4}) \cdot \sinh(\pi i^{3/4} z^{1/4})}{i\sin(\pi(iz)^{1/4}) \cdot \sinh(\pi(iz)^{1/4})}
\end{aligned}
$$