Gmail **MDS <maxieds@gmail.com>**

Offering a preemptive new revision of my JNT article?

25 messages

Maxie Schmidt <maxieds@gmail.com> Fri, Dec 17, 2021 at 11:52 PM
To: Steven J Miller <sjm1@williams.edu>

Steven,

I talked postdocs and the Mertens article over with Jeff Lagarias at Michigan for a couple of hours online today. He had a number of substantive comments about how I need to modify the probabilistic arguments given in Section 4.3. I am also planning to push a new version of the article to the arXiv before my talk at JMM. Is there anything you can do to open up my submission to JNT for another revision even though the current one still does not have referee feedback yet?

Maxie

Steven J Miller <sjm1@williams.edu> Fri, Dec 17, 2021 at 11:57 PM
To: Maxie Schmidt <maxieds@gmail.com>

reading now
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Sat, Dec 18, 2021 at 12:00 AM
To: Maxie Schmidt <maxieds@gmail.com>

I've just told the system to resend to you

I've asked the referee if they are willing to look at a new version

I am glad the paper is constantly improving, but it is a long, technical paper and it is hard to keep iterating

On Fri, 17 Dec 2021, Maxie Schmidt wrote:

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Wed, Dec 22, 2021 at 2:04 PM
To: Steven J Miller <sjm1@williams.edu>

Steven,

I am planning to submit the new revision (and push it to the arXiv) when I get back to Atlanta from my holiday visit with family in Florida. It will be the first of the new year.

The paper is very technical in places, believe me understand this well! I have also worked to add some additional exposition in places and shorten technical proofs that are unnecessary for understanding (e.g., the formerly long proof in Section 5.3 to demonstrate the cancellation expected in the formula for G^{-1}(x) is now much better explained). I have also worked to improve some of the cumbersome notation used throughout the article.

Jeff took issue last week with the probabilistic method arguments and exposition in Section 4.3 of the last revision. He warned me not to let it appear like that in a top journal like JNT. It turns out that these Erdos-Kac theorem variants are very difficult to prove (certainly with analytic arguments alone, in analog to the proof for the distribution of \Omega(n) from MV), and the justification using an underlying probabilistic method proof is really more suitable for a separate follow-up article. I restated these theorems in two distinct forms as a conjecture, which should suffice since they are

not used as lemmas in the rest of the article.

I am attaching a tentative version in case you feel like taking a look and to reassure you that there will not be an infinite time delay in processing on my part. I think the article is in much better shape this time. I am still waiting on some feedback for the draft. That, and the holiday season with my lovely family, are going to force me to wait until new years to resubmit.

See you at JMM. Best,

Maxie
[Quoted text hidden]

mertens-lower-bounds-2021.12.21-v1.pdf
634K

Steven J Miller <sjm1@williams.edu> Wed, Dec 22, 2021 at 4:10 PM
To: Maxie Schmidt <maxieds@gmail.com>

will reply in a day
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Thu, Dec 23, 2021 at 9:14 AM
To: Maxie Schmidt <maxieds@gmail.com>

now you know why I wanted to wait, or maybe you don't

JMM has been postponed :[

more details in the next few days

take your time on the paper

no rush

enjoy the time with the family
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Sat, Jan 1, 2022 at 9:30 PM
To: Steven J Miller <sjm1@williams.edu>

Steven,

Happy 2022! I hope the new year is treating you well so far.

I was informed locally from the GT folks that JMM is now tentatively virtual. I have no idea whether I will still get to present the invited talk when the online conference happens.

When I returned to ATL yesterday I decided to make a tentative final pass on the article revision after letting it sit for nearly a week over the holidays. A copy of the revised version that will appear publicly on the arXiv this coming Tuesday is attached. I think it is in very good shape with technical proofs checked over and shortened exposition in several arguments. You might be interested in pages 10-11 that offer interpretations of the near regularity of |g^{-1}(n)| at its average order for a.e. sufficiently large (integers) x. I also decided to reproduce a couple of plots of the scaled sums of the new functions in the manuscript to enhance the tables of the "nicer" values of these sequences compared to the mystery of the classical partial sums. These appear in Appendix B (page 37).

Can you open up the new revision application for JNT? When I logged into the author site I was unable to upload a new version (with the attached screenshot for reference).

I have talked with several experts that seem to pay attention to the arguments in this paper while corresponding about upcoming postdocs this year. It's a stressfully uncertain process and an awful year to graduate in general.

Maxie
[Quoted text hidden]

2 attachments

Screen Shot 2022-01-01 at 9.17.33 PM.png
50K

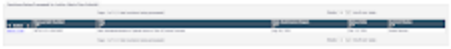
mertens-lower-bounds.pdf
703K

Steven Miller <sjm1@williams.edu> Sat, Jan 1, 2022 at 11:01 PM
To: Maxie Schmidt <maxieds@gmail.com>
Cc: Steve Miller <sjm1@williams.edu>

In queue
Sent from my iPad

> On Jan 1, 2022, at 9:30 PM, Maxie Schmidt <maxieds@gmail.com> wrote:
>
> [Quoted text hidden]

2 attachments



Screen Shot 2022-01-01 at 9.17.33 PM.png
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mertens-lower-bounds.pdf
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Miller, Steven <sjm1@williams.edu> Sun, Jan 2, 2022 at 9:47 AM
To: Maxie Schmidt <maxieds@gmail.com>, Steven Miller <sjm1@williams.edu>

I have returned the paper to you; I put it in this mode to make sure nothing could happen accidentally on any end.

Whom have you been talking to about the paper? Glad to hear it is getting good reads.
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Sun, Jan 2, 2022 at 4:30 PM
To: "Miller, Steven" <sjm1@williams.edu>

Steven,

Thank you. I think the paper is worth taking the time for all of these revisions by the end of the process. I appreciate the support and flexibility to submit yet another revised manuscript to correct, and definitely better phrase, some of the

results in this long and technical article.

So far I have had video chats with a few professors at the universities I applied for postdoc positions this year. The first conversation I had was with Bob Vaughan, who ended up sponsoring my NSF fellowship application with Penn State. He or Montgomery would have been in the short list of professors I want to work with. Peter Winker at Dartmouth was also interested when I talked to him on Zoom this fall. He said he would be delighted to have me, but that he had to ask some others in the NT and combinatorics groups to get onboard as he has recently had a postdoc (there are three spots in total there, I believe, for the upcoming year). I also had the opportunity to talk with Jeff Lagarias for a couple of hours before the break. He had some excellent suggestions about presentation of the probabilistic method arguments which I now state as conjectures in the manuscript (better for follow-up articles, and non-trivial to prove in this shortened context as well). I do not yet have any definite offers, but the season is young. One of my advisors said to wait until February when the NSF announcements are in, and big name universities like Brown start reviewing their application queue. I sincerely hope to God that my tenure at a standard order Google job does not happen to me this year, even though they pay you a freaking fortune starting salary to hack source code and occasionally do math...

Maxie
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Mon, Jan 3, 2022 at 7:46 AM
To: Maxie Schmidt <maxieds@gmail.com>

thnx

did you originally say the probabilistic items were theorems in the earlier draft and proved them, and now are just saving that for a later article, or did you think you had a proof but now think it is not quite there yet?
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Mon, Jan 3, 2022 at 9:44 AM
To: Steven J Miller <sjm1@williams.edu>

Steven,

It's a little of both. In the first revision, and editing it for a while before I made the second one with the initial probabilistic argument, I came very close to an analytic proof of part (A) of the conjecture (for the scaled functions). I realized after talking with Jeff for a while about this that the statement needs to be more precise with an exact probabilistic model underneath to define the rvs used with the Lindeberg CLT (non obvious construction, and I haven't spent enough time on it just yet). The statement in part (B) is motivated along the lines of Erdos-Kac. After talking with JL, I finally sat down and worked out the variance type sums (second moments) of the deterministic function inputs starting from the average order formulas I worked so hard to prove in the previous couple of subsections. This is the important, and new, conjectured component that will need to most likely be proved with probabilistic methods moving forward. Because of the way \Omega(n) behaves uniformly on 1 \leq k \leq R\log\log x (R < 2) and much less so above this point, it is more complicated to state the probabilistic model underneath as the rvs I was considering are parameterized in multiple variables and cases (the n and some k=\Omega(n)). I need to spend more time with the problem and motivating the proof method that turns out to work the best some months later. It is definitely the subject of a follow-up article. It was also suggested to me that I tried to do too much rigorously in a single paper.

I will work on trying to resubmit the arXiv preprint version that appears on the servers tomorrow sometime soon.

Thanks again for feedback and correspondence.

Maxie
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Mon, Jan 3, 2022 at 10:15 AM
To: Maxie Schmidt <maxieds@gmail.com>

ok

what worries me is did you roiginally think you had a rigorous proof of these results, and then later realize that you did not.
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Mon, Jan 3, 2022 at 11:26 AM
To: Steven J Miller <sjm1@williams.edu>

Let me explain:

The analytic proof I had of the conjecture part (A) should suffice to prove that result. It is long and technical to the point that I am comfortable leaving a hint at the proof without elaborating in complete detail in Section 4.3. The version that I worked on for a while (months overall the article material) up until submitting the second revision in December did not appear publicly nor end up submitted back to JNT. It corrects an issue that would have been in the corresponding revision 1 version of that proof. I can go back and dig into my typeset, date-stamped versions of the article I have locally if you think it would add something for the reviewer to consider. The statement of part (B) is newer, and is something I was only able to write down after summing the second moment of C_{\Omega}(n). This happened after the good chat with JL, who is much more of an expert in the probabilistic method arguments than I have sat down with before, a few weeks ago.

The proof of part (B) definitely is more fit to approach in a separate follow up article. You know the history behind Erdos-Kac and how its was formulated and established with all due rigor... Kac had a probabilistic proof (semi-ansatz that was very new at that time) of the result. Then Erdos, and all the genius weight he brings on Ritalin, was able to give a non conditional proof with sieves and analysis. I expect that there are ways to get at part (B) by both, and if I sit down when I have free time in coming months, maybe by the end of the summer I will have a solid typeset manuscript that elaborates sufficiently on both proof methods. I should keep this in confidence, but JL is actively working with this type of argument on functions that are similar, though I do not know precise details about his work (maybe just yet). His suggestions are good and I took them to heart while making this last round of revisions.

One thing that I do want to comment on is why the new statement (now that I finally realized the denominator / variance asymptotics) of part (B) is important and a good thing to have readily in the article moving forward for whenever it gets published. I am not adding in new things ad hoc without having thought through the exposition and the best ways to leave things in this already long manuscript. I mentioned this to Lacey over email in slightly different wording to point out to him that I was right about the gut intuition on how regularly centered near its average order the unsigned function |g^{-1}(n)| is for nearly every large n (remarked on in context on pp. 10-11 of the introduction section that the referee asked me to modify to include more exposition and motivation last time I heard back). In effect, even though this function is nowhere near strongly additive itself, it inherits the regularity and "nicer" properties much like the distributions of the classic \omega(n) and \Omega(n) functions. Then through the new exact formula in (14), asymptotics of M(x) depend on sums of the auxiliary partial sums of G^{-1}(x). Since G^{-1}(x) = \sum_{n \leq x} \lambda(n) |g^{-1}(n)|, we basically get a sign-weighted combination of summands that are asymptotically about \pm 1 each half of the time, and summands that are not only characterized by a probability distribution underneath, but also have that strikingly normal tendency towards the same (logarithmically weighted) values in successive values along smaller intervals. This suggests, as expected by random-walk type models for M(x), that there is usually a huge amount of cancellation that happens for the function, but with some exceptional ability to have a larger variance at large x. I want to eventually make precise what happens along subsequences where the |g^{-1}(n)| hit extremal values on n<=x, and without getting any corresponding cancellation from nearby terms. But this is WAY outside of the scope of the work in this manuscript!

A long justification of the key changes, hoping that it saves time or makes sense down the line.

Maxie
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Mon, Jan 3, 2022 at 11:30 AM
To: Maxie Schmidt <maxieds@gmail.com>

thanks

again, as an analytic number theorist, I know how painful it is to work in the subject

you can easily have true statements that are given incomplete proofs

with all the iterations and comments, I just want to get a clear sense of di dyou previously think you had complete proofs, and if so, you need to be extra careful in the submission here
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Mon, Jan 3, 2022 at 12:41 PM
To: Steven J Miller <sjm1@williams.edu>

I am confident that all of the proofs in the new version are accurate and complete and error free. Is it going to hurt my chances in the review statement that I changed what was formerly labeled a theorem to a conjecture on solid advice from an external expert? There is no formal proof given of the initial conjectured results. I do however prove a second corollary that is correct provided that the conjecture holds.

Maxie
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Mon, Jan 3, 2022 at 12:44 PM
To: Maxie Schmidt <maxieds@gmail.com>

no, it won't hurt your chances

let me be very explicit to make sure I understand the history:

Did you believe everything was rigorously proved in the previous version, and after talking with experts do you now realize some of your arguments were not complete, and thus moved a theorem to a conjecture?

I believe you would answer 'yes'.
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Wed, Jan 5, 2022 at 5:56 PM
To: Steven J Miller <sjm1@williams.edu>

Steven,

Sorry for the delay in response. I have had a lot going on with family this holiday season. I totally crashed yesterday.

Yes, is my answer. I believed my arguments were correct, then realized a conjecture was the better way to state things in this manuscript.

Maxie
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Thu, Jan 6, 2022 at 6:43 AM
To: Maxie Schmidt <maxieds@gmail.com>

this is still not a well-written response

when you say you realized a conjecture was a better way to state things, I believe what you mean is that what you wrote was NOT correct, it was not a rigorous and complete proof.

I am not trying to be difficult; this is just a very technical work that has gone thru many iterations, and before a referee spends, again, a lot of time looking at it I want to be as comfortable as possible that it is correct. That is VERY hard to do for long, technical papers in analytic number theory...
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Thu, Jan 6, 2022 at 6:01 PM
To: Steven J Miller <sjm1@williams.edu>

Steven,

Let me respond to what I believe you are asking me for as clarifications:

After talking with an expert outside of the review process, the conjecture replaces what was previously stated as a theorem. The result (the first one) given in the conjecture is still correct. The justification I sketched via a probabilistic method was not precise in terms of the underlying probability model and so had some flaws underneath that I did not realize at the time I resubmitted the revision. I realize now that to give a complete and rigorous proof, it will take many more pages than the sketched method via the Lindeberg CLT to make it completely well justified.

Does that answer the questions you have? I can elaborate more in a follow-up email.

MDS
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Fri, Jan 7, 2022 at 7:50 AM
To: Maxie Schmidt <maxieds@gmail.com>

ok, so what you are saying is the original submission was wrong, wrong because of incomplete arguments, but wrong

this is as I've said not unusual, but it is a warning sign and you need to be very careful in how you write everything up, making sure you include all the steps

I would thus tone down the exposition a bit and increase the detail in the proofs. you can move some of the arguments to the appendices, but you shouldn't skip steps especially as you've had an incomplete argument before

you should also have lots of computations backing your results. this provides more confidence, and makes your paper a fit for the computational section, which has an easier path to acceptance
[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Fri, Jan 7, 2022 at 11:47 AM
To: Steven J Miller <sjm1@williams.edu>

I will keep that in mind when I revise.

Thanks.

Maxie
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Fri, Jan 7, 2022 at 12:38 PM
To: Maxie Schmidt <maxieds@gmail.com>

you're welcome

I think you're investigating some very interesting objects

take the time to do it carefully, add as much data as you can to support your results

[Quoted text hidden]

Maxie Schmidt <maxieds@gmail.com> Thu, Jan 13, 2022 at 7:00 PM
To: Steven J Miller <sjm1@williams.edu>

Steven,

I mentioned before that I have been corresponding with Jeff Lagarias who has proven to be very opinionated about clear writing style. I have been working to revise things according to his suggestions. The result is substantially easier to read already. I am still working on the revised draft.

One issue he suggested is to change the title. My new tentative title is "*Characterizations of partial sums of the M\"obius function by signed sums of additively structured auxiliary unsigned sequences*". What do you think? Is it too wordy for the title?

Maxie
[Quoted text hidden]

Steven J Miller <sjm1@williams.edu> Thu, Jan 13, 2022 at 7:01 PM
To: Maxie Schmidt <maxieds@gmail.com>

I'm fine with wordy titles

he is a giant, whatever time / advice he gives you, view yourself as moses with a few tablets that can be filled :]

On Thu, 13 Jan 2022, Maxie Schmidt wrote:

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>
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> On Fri, Jan 7, 2022 at 12:39 PM Steven J Miller <sjm1@williams.edu> wrote:
>> you're welcome
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>> take the time to do it carefully, add as much data as you can to support
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> > > > of M(x) depend
> > > > > on sums of the auxiliary partial sums of G^{-1}(x). Since G^{-1}(x)
= \sum_{n \leq x}
> \lambda(n)
> > |g^{-1}(n)|, we
> > > basically
> > > > get a
> > > > > sign-weighted combination of summands that are asymptotically about
\pm 1 each half of the
> time, and
> > summands that
> > > are not
> > > > only
> > > > > characterized by a probability distribution underneath, but also
have that strikingly normal
> tendency
> > towards the
> > > same
> > > > (logarithmically
> > > > > weighted) values in successive values along smaller intervals. This
suggests, as expected by
> > random-walk type models
> > > for
> > > > M(x), that there
> > > > > is usually a huge amount of cancellation that happens for the
function, but with some
> exceptional
> > ability to have a
> > > larger
> > > > variance at
> > > > > large x. I want to eventually make precise what happens along
subsequences where the
> |g^{-1}(n)| hit
> > extremal values
> > > on
> > > > n<=x, and without
> > > > > getting any corresponding cancellation from nearby terms. But this
is WAY outside of the scope
> of the
> > work in this
> > > > manuscript!
> > > > >
> > > > > A long justification of the key changes, hoping that it saves time
or makes sense down the
> line.
> > > > > >
> > > > > > Maxie
> > > > > >
> > > > > > On Mon, Jan 3, 2022 at 10:15 AM Steven J Miller <sjm1@williams.edu>
wrote:
> > > > > > >
> > > > > > > ok

> > > > >
> > > > > what worries me is did you roiginally think you had a rigorous proof of
> > > > > these results, and then later realize that you did not.
> > > > > >
> > > > > On Mon, 3 Jan 2022, Maxie Schmidt wrote:
> > > > > >
> > > > > > Steven,
> > > > > > >
> > > > > > It's a little of both. In the first revision, and editing it for a while before I made
> the
> > > second one with
> > > > > the
> > > > > > > initial probabilistic argument, I came very close to an analytic proof of part (A) of
> the
> > > conjecture (for
> > > > > the
> > > > > > > scaled functions). I realized after talking with Jeff for a while about this that the
> > > statement needs to be
> > > > > more
> > > > > > > precise with an exact probabilistic model underneath to define the rvs used with the
> Lindeberg
> > > CLT (non
> > > > > obvious
> > > > > > > construction, and I haven't spent enough time on it just yet). The statement in part
> (B) is
> > > motivated along
> > > > > the
> > > > > > > lines of Erdos-Kac. After talking with JL, I finally sat down and worked out the
> variance type
> > > sums (second
> > > > > > > moments) of the deterministic function inputs starting from the average order formulas
> I
> > > worked so hard to
> > > > > prove
> > > > > > > in the previous couple of subsections. This is the important, and new, conjectured
> component
> > > that will need
> > > > > to
> > > > > > > most likely be proved with probabilistic methods moving forward. Because of the way
> \Omega(n)
> > > behaves
> > > > > uniformly
> > > > > > > on 1 \leq k \leq R\log\log x (R < 2) and much less so above this point, it is more
> complicated
> > > to state the
> > > > > > > probabilistic model underneath as the rvs I was considering are parameterized in
> multiple
> > > variables and

> > > cases
> > > > > > (the n and some k=\Omega(n)). I need to spend more time with the problem and
> > motivating the
> > > proof method
> > > > that
> > > > > > turns out to work the best some months later. It is definitely the subject of a
> > follow-up
> > > article. It was
> > > > also
> > > > > > suggested to me that I tried to do too much rigorously in a single paper.
> > > > > > >
> > > > > > I will work on trying to resubmit the arXiv preprint version that appears on the
> > servers
> > > tomorrow sometime
> > > > soon.
> > > > > > >
> > > > > > Thanks again for feedback and correspondence.
> > > > > > >
> > > > > > Maxie
> > > > > > >
> > > > > > On Mon, Jan 3, 2022 at 7:46 AM Steven J Miller <sjm1@williams.edu> wrote:
> > > > > > >
> > > > > > > thnx
> > > > > > >
> > > > > > > did you originally say the probabilistic items were theorems in the
> > > > > > > earlier draft and proved them, and now are just saving that for a later
> > > > > > > article, or did you think you had a proof but now think it is not quite
> > > > > > > there yet?
> > > > > > >
> > > > > > > On Sun, 2 Jan 2022, Maxie Schmidt wrote:
> > > > > > >
> > > > > > > > Steven,
> > > > > > > >
> > > > > > > > Thank you. I think the paper is worth taking the time for all of these
> > revisions by
> > > the end of the
> > > > > > > > process. I
> > > > > > > > appreciate the support and flexibility to submit yet another revised
> > manuscript to
> > > correct, and
> > > > > > > > definitely
> > > > > > > > better phrase, some of the results in this long and technical article.
> > > > > > > >
> > > > > > > > So far I have had video chats with a few professors at the universities I
> > applied for
> > > postdoc
> > > > > > > > positions this
> > > > > > > > year. The first conversation I had was with Bob

Vaughan, who ended up
> sponsoring my
> > NSF fellowship
> > > > > > application
> > > > > > > with Penn State. He or Montgomery would have been in
the short list of
> professors I
> > want to work
> > > > > > with. Peter
> > > > > > > Winker at Dartmouth was also interested when I
talked to him on Zoom this
> fall. He
> > said he would
> > > > > > be delighted to
> > > > > > > have me, but that he had to ask some others in the
NT and combinatorics groups
> to get
> > onboard as
> > > > > > he has recently
> > > > > > > had a postdoc (there are three spots in total there,
I believe, for the
> upcoming
> > year). I also had
> > > > > > the
> > > > > > > opportunity to talk with Jeff Lagarias for a couple
of hours before the break.
> He had
> > some
> > > > > > excellent suggestions
> > > > > > > about presentation of the probabilistic method
arguments which I now state as
> > conjectures in the
> > > > > > manuscript
> > > > > > > (better for follow-up articles, and non-trivial to
prove in this shortened
> context as
> > well). I do
> > > > > > not yet have
> > > > > > > any definite offers, but the season is young. One of
my advisors said to wait
> until
> > February when
> > > > > > the NSF
> > > > > > > announcements are in, and big name universities like
Brown start reviewing
> their
> > application
> > > > > > queue. I sincerely
> > > > > > > hope to God that my tenure at a standard order
Google job does not happen to
> me this
> > year, even
> > > > > > though they pay
> > > > > > > you a freaking fortune starting salary to hack
source code and occasionally do
> math...
> > > > > > >
> > > > > > > Maxie
> > > > > > >
> > > > > > > On Sun, Jan 2, 2022 at 9:47 AM Miller, Steven

<sjm1@williams.edu> wrote:
> > > > > > > I have returned the paper to you; I put it in this mode to make sure
> nothing
> > could happen
> > > > > > > accidentally on any end.
> > > > > > > Whom have you been talking to about the paper? Glad to hear it is getting good
> reads.
> > > > > > >
> > > > > > > On Sat, Jan 1, 2022 at 9:30 PM Maxie Schmidt <maxieds@gmail.com> wrote:
> > > > > > > Steven,
> > > > > > >
> > > > > > > Happy 2022! I hope the new year is treating you well so far.
> > > > > > >
> > > > > > > I was informed locally from the GT folks that JMM is now tentatively virtual.
> I have
> > no idea
> > > > > > whether
> > > > > > > I will still get to present the invited talk when the online conference
> happens.
> > > > > > >
> > > > > > > When I returned to ATL yesterday I decided to make a tentative final pass on
> the
> > article revision
> > > > > > > after letting it sit for nearly a week over the holidays. A copy of the
> revised
> > version that will
> > > > > > > appear publicly on the arXiv this coming Tuesday is attached. I think it is in
> very
> > good shape
> > > > > > with
> > > > > > > technical proofs checked over and shortened exposition in several arguments.
> You might
> > be
> > > > > > interested
> > > > > > > in pages 10-11 that offer interpretations of the near regularity of
> |g^{-1}(n)| at its
> > average
> > > > > > order
> > > > > > > for a.e. sufficiently large (integers) x. I also decided to reproduce a couple
> of
> > plots of the
> > > > > > > scaled sums of the new functions in the manuscript to enhance the tables of
> the
> > "nicer" values of
> > > > > > > these sequences compared to the mystery of the classical partial sums. These
> appear in

> > Appendix B
> > > > > > > (page 37).
> > > > > > > >
> > > > > > > Can you open up the new revision application for JNT? When I logged into the
> author
> > site I was
> > > > > > > unable to upload a new version (with the attached screenshot for reference).
> > > > > > > >
> > > > > > > I have talked with several experts that seem to pay attention to the arguments
> in this
> > paper while
> > > > > > > corresponding about upcoming postdocs this year. It's a stressfully uncertain
> process
> > and an awful
> > > > > > > year to graduate in general.
> > > > > > > >
> > > > > > > Maxie
> > > > > > > >
> > > > > > > On Thu, Dec 23, 2021 at 9:14 AM Steven J Miller <sjm1@williams.edu> wrote:
> > > > > > > >
> > > > > > > now you know why I wanted to wait, or maybe you don't
> > > > > > > >
> > > > > > > JMM has been postponed :[
> > > > > > > >
> > > > > > > more details in the next few days
> > > > > > > >
> > > > > > > take your time on the paper
> > > > > > > >
> > > > > > > no rush
> > > > > > > >
> > > > > > > enjoy the time with the family
> > > > > > > >
> > > > > > > On Wed, 22 Dec 2021, Maxie Schmidt wrote:
> > > > > > > >
> > > > > > > > Steven,
> > > > > > > >
> > > > > > > > I am planning to submit the new revision (and push it to the arXiv)
> when I get
> > back to
> > > > > > > Atlanta from my holiday visit with family in Florida. It
> > > > > > > > will be the first of the new year.
> > > > > > > >
> > > > > > > > The paper is very technical in places, believe me understand this
> well! I have
> > also
> > > > > > > worked to add some additional exposition in places and shorten
> > > > > > > > technical proofs that are unnecessary for understanding (e.g., the
> formerly
> > long proof

> > > > > > > in Section 5.3 to demonstrate the cancellation expected in
> > > > > > > > the formula for G^{-1}(x) is now much better explained). I have also
> > worked to
> > > improve
> > > > > > > > some of the cumbersome notation used throughout the article.
> > > > > > > > >
> > > > > > > > Jeff took issue last week with the probabilistic method arguments and
> > > exposition in
> > > > > > > > Section 4.3 of the last revision. He warned me not to let it
> > > > > > > > appear like that in a top journal like JNT. It turns out that these
> > Erdos-Kac
> > > theorem
> > > > > > > > variants are very difficult to prove (certainly with analytic
> > > > > > > > arguments alone, in analog to the proof for the distribution of
> > \Omega(n) from
> > > MV),
> > > > > > > > and the justification using an underlying probabilistic method
> > > > > > > > proof is really more suitable for a separate follow-up article. I
> > restated
> > > these
> > > > > > > > theorems in two distinct forms as a conjecture, which should
> > > > > > > > suffice since they are not used as lemmas in the rest of the article.
> > > > > > > > >
> > > > > > > > I am attaching a tentative version in case you feel like taking a look
> > and to
> > > reassure
> > > > > > > > you that there will not be an infinite time delay in
> > > > > > > > processing on my part. I think the article is in much better shape
> > this time.
> > > I am
> > > > > > > > still waiting on some feedback for the draft. That, and the
> > > > > > > > holiday season with my lovely family, are going to force me to wait
> > until new
> > > years to
> > > > > > > > resubmit.
> > > > > > > > >
> > > > > > > > See you at JMM. Best,
> > > > > > > > >
> > > > > > > > Maxie
> > > > > > > > >
> > > > > > > > On Sat, Dec 18, 2021 at 12:00 AM Steven J Miller <sjm1@williams.edu>
> > wrote:

> > > > > > > > I've just told the system to resend to
you
> > > > > > > >
> > > > > > > > I've asked the referee if they are
willing to look at a new
> version
> > > > > > > >
> > > > > > > > I am glad the paper is constantly
improving, but it is a long,
> technical
> > > > > > > > paper and it is hard to keep iterating
> > > > > > > >
> > > > > > > > On Fri, 17 Dec 2021, Maxie
> > > > > > > > Schmidt wrote:
> > > > > > > >
> > > > > > > > > Steven,
> > > > > > > > >
> > > > > > > > > I talked postdocs and the Mertens
article over with Jeff
> Lagarias at
> > Michigan
> > > > > > > for a couple of hours online today. He had a
number of
> > > > > > > > substantive
> > > > > > > > > comments about how I need to modify
the probabilistic
> arguments given
> > in
> > > > > > > Section 4.3. I am also planning to push a new
version of the
> > > > > > > > article to the
> > > > > > > > > arXiv before my talk at JMM. Is
there anything you can do to
> open up
> > my
> > > > > > > submission to JNT for another revision even
though the
> > > > > > > > current one still does
> > > > > > > > > not have referee feedback yet?
> > > > > > > > >
> > > > > > > > > Maxie
> > > > > > > > >
> > > > > > > > >
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