1. Using the general form of the definite integral, $\int_a^b f(t)dt = \lim_{n\to\infty} \sum_{i=1}^n f(x_i^*)\Delta x$, evaluate:

$$\int_2^4 (x-1)^2 dx$$

Solution.

$$\begin{aligned}\lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^n (2+\frac{2i}{n}-1)^2 &= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^n 1+\frac{4i}{n}+\frac{4i^2}{n^2} \\ &= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^n 1+\frac{8}{n^2}\sum_{i=1}^n i+\frac{8}{n^3}\sum_{i=1}^n i^2 = 2+4+\frac{8}{3}=\frac{26}{3}.\end{aligned}$$

2. Evaluate $\int_0^2 |x-1|dx$ using integral properties from class. (HINT: draw a picture, and use geometry!)

Solution. Use the fact that $|x-1|$ is symmetric with respect to line $x=1$ to conclude that

$$\int_0^2 |x-1|dx = 2\int_1^2 (x-1)dx.$$

Now use Riemann sums to solve the integral. Please draw a picture.

3. Suppose that $f(x)$ is an even function such that $\int_0^2 f(x)dx = 5$ and $\int_0^3 f(x)dx = 8$. Find the value of $\int_{-2}^3 f(x)dx$.

Solution.

$$\int_{-2}^3 f(x)dx = \int_{-2}^0 f(x)dx + \int_0^3 f(x)dx = \int_{-2}^0 f(x)dx + 8$$

Since f is even function,

$$\int_{-2}^0 f(x)dx = \int_0^2 f(x)dx = 5$$

So

$$\int_{-2}^3 f(x)dx = 13$$