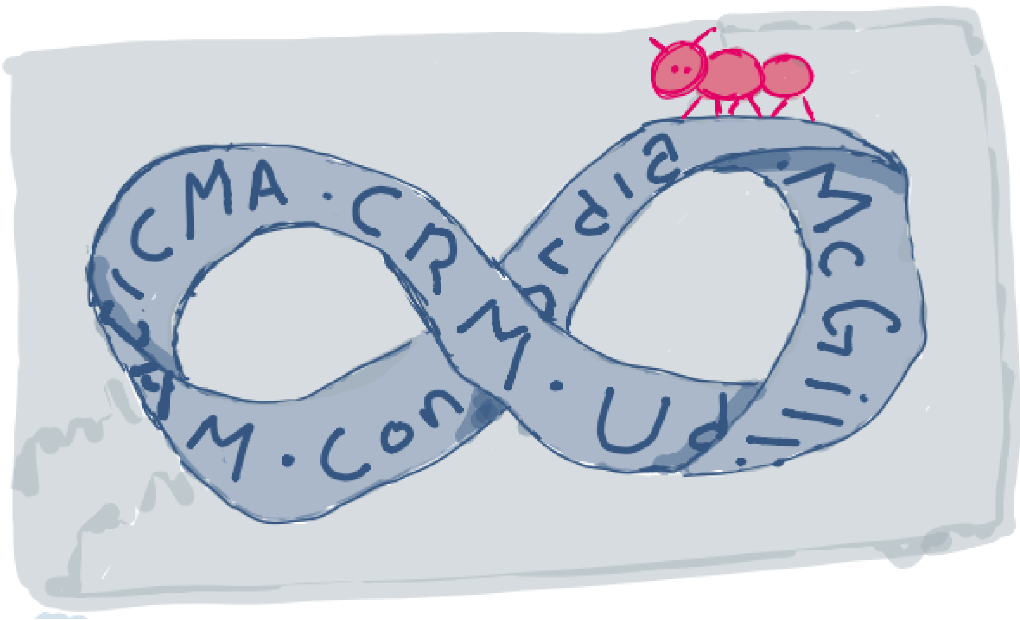

Exact formulas for partial sums of the Möbius function expressed by partial sums of weighted Liouville functions

by: Maxie Schmidt
Georgia Tech

Virtually: See e-mail for link

Abstract

The Mertens function, $M(x) := \sum_{n \leq x} \mu(n)$, is defined as the summatory function of the classical Möbius function for $x \geq 1$. The Dirichlet inverse function $g(n) := (\omega + \mathbb{1})^{-1}(n)$ is defined in terms of the shifted strongly additive function $\omega(n)$ that counts the number of distinct prime factors of n without multiplicity. Discrete convolutions of the partial sums of $g(n)$ with the prime counting function provide new exact formulas for $M(x)$ that are weighted sums of the Liouville function involving $|g(n)|$ for $n \leq x$. We study the distribution of the unsigned function $|g(n)|$ through the auxiliary unsigned sequence $C_\Omega(n)$ whose Dirichlet generating function is given by $(1 - P(s))^{-1}$ for $\Re(s) > 1$ where $P(s) = \sum_p p^{-s}$ is the prime zeta function. An application of the Selberg-Delange method yields asymptotics for the restricted sums of $C_\Omega(n)$ over all $n \leq x$ such that $\Omega(n) = k$ uniformly for $1 \leq k \leq \frac{3}{2} \log\log x$. We use these formulas to prove precise formulas for the average order of both $C_\Omega(n)$ and $|g(n)|$. Higher-order moments of these functions are predicted numerically by the conjecture that there is a limiting probability measure on $\mathbb{R}$ whose cumulative density function gives the distribution of the distinct values of each function over $n \leq x$ as $x \to \infty$.