1. Evaluate the following integrals using the method of substitution.

   (a) $\int \frac{1}{\ln(x^x)} dx$.
   Solution. Let $u = \ln(x)$. Please remind them the logarithm properties.

   (b) $\int \frac{e^{2x}}{\sqrt{3-4e^{2x}}} dx$.
   Solution. Consider $u = e^{2x}$.

   (c) $\int \frac{1}{\sqrt{4-(x+3)^2}} dx$.
   Solution. Please do it in 2 steps. First let $u = x + 3$. Then pick $2\sin(v) = u$.

2. Suppose that $y = f(x)$ and $y = g(x)$ are both continuous functions on the interval $[a, b]$. Determine if each statement below is always true or sometimes false.

   (a) Suppose that $f(c) > g(c)$ for some number $c \in (a, b)$. Then the area bounded by f, g, $x = a$, and $x = b$ can be found by evaluating the integral $\int_a^b (f(x) - g(x))dx$.
   Comment. False. Please help them understand by showing counterexample. Also, explain the case when $f(x) > g(x)$ for all x.

   (b) If $\int_a^b (f(x) - g(x))dx$ evaluates to -5, then the area bounded by $f, g, x = a$, and $x = b$ is 5.

   Solution. False. There might be intersection points.

   (c) If $f(x) > g(x)$ for every $x \in [a, b]$, then $\int_a^b |f(x) - g(x)|dx = \int_a^b (f(x) - g(x))dx$
   Solution. True.

3. Find the area bounded by the region between the curves $f(x) = x^3 + 2x^2$ and $g(x) = x^2 + 2x$.

   Comment. Please use the 3 steps: finding intersection points, finding the larger function in each subinterval, and computing the subintervals. The final answer is 37/12.

4. Find the area bounded by the region enclosed by the three curves $y = x^3$, $y = -x$, and $y = -1$.

   Comment. Final answer: 5/4

5. Find the area bounded by the curves $y = \cos(x)$ and $y = \sin(2x)$ on the interval $[0, \frac{\pi}{2}]$.

   Comment. Final answer: 1/2.

6. Find the area of the triangle with vertices at the points $(0, 1)$, $(3, 4)$, and $(4, 2)$. USE CALCULUS.

   Comment. Final answer: 4.5, please explain that although it seems not the bext way to find the area, even in this case it might have computational advantage.

7. For each function below: (i) determine which method to use to evaluate the function (formula, u-substitution, or integration by parts, and (ii) evaluate the integral.

   (a) $\int \frac{\sqrt{\ln(x)}}{x} dx$.
   Solution. Using $u-$sub $u = \ln(x)$.

   (b) $\int (\ln(x))^2 \, dx$.
   Solution. We need to take integration by parts. $u = \ln^2(x)$ and $dv = dx$.