

1. Evaluate the integrals:

(a) $\int_2^5 \frac{3x-5}{x^3} dx.$

Solution. $\int_2^5 (3x^{-2} - 5x^{-3}) dx = \dots = \frac{-3}{8}.$

(b) $\int_3^5 \frac{dx}{x^2(x-3)}.$

Solution. Partial fraction: $\int_3^5 \frac{ax+b}{x^2} + \frac{c}{x-3} dx = \dots$

(c) $\int_{\pi}^{7\pi/2} \frac{\cot(x) + \sin^2(x)}{4}.$

Solution. For $\cot(x)$ use change of variable or the table. Also $\sin^2(x) = \frac{1 - \cos(2x)}{2}.$

2. Find $F'(3)$ where

$$F(x) = \int_{\cos(4\pi x)}^{e^{1/x}} \frac{3x^2}{x+2} dx$$

Comment. FTC. Please help them understand that because $\sin(12\pi) = 0$, they do not need to do the computation for that term.

3. (Optional) Let $f(1/x) = f(x)$ and f be an odd function. If $\int_{1/2}^{1/4} f(x) \frac{dx}{x^2} = 3$. Then compute

$$\int_{-4}^2 (f(x) + 3x^2 - 5) dx$$

Solution. $\int_{-4}^4 f(x) dx = 0$. So we need to find $\int_2^4 f(x) dx$. If $u = \frac{1}{x}$

$$-3 = \int_{\frac{1}{4}}^{\frac{1}{2}} f(x) \frac{dx}{x^2} = \int_2^4 f(x) dx.$$

The rest is straightforward.

4. a) Given the function below, evaluate $\int_1^9 f(x) dx$.

$$f(x) = \begin{cases} x^2 + 4 & x < 4 \\ \sqrt{x} - x & x \geq 4. \end{cases}$$

b) Would you get the same answer to part (a) if you evaluated $F(9) - F(1)$? What does this tell you about the FTC and continuity?

Comment. Please help them to understand it both by computation and seeing the diagram.

5. (a) Evaluate the expressions:

$$\int x(x+1)dx \quad \text{and} \quad \int xdx \int (x+1)dx$$

- b) Looking at your answer in part (a), what, if anything, can you say in general about $\int f(x)g(x)dx$?

Comment. No. This question is important. Please give them official comment about it.

6. For each integral below, determine if we can evaluate the integral using the method of u-substitution. If the answer is "yes", detect u .

a) $\int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) dx$.

Solution. Yes. We can. $u = \frac{1}{x}$.

b) $\int x \csc^2(x) dx$.

Solution. No. Please help them to check all the options for u .

c) $\int e^{x^2} dx$.

Solution. No. You can also mention that this integral is not solvable using elementary methods.

7. Determine if each statement below is true or false.

- (a) If f is a continuous function, then the function $F(x) = \int_a^x f(t)dt$ is an anti-derivative of f .

Solution. True. This is in fact FTC.

- (b) If F is an anti-derivative of f , then $\int_a^b f(t)dt$ represents the slope of the secant line of $F(x)$ on the interval $[a, b]$.

Solution. False. Please explain the correct notions.

- (c) $\frac{d}{dx} \left(\int_a^b f(t)dt \right) = f(b)$.

Solution. False. Please help them to understand that the LHS is zero.

- (d) Given that f is continuous on $[a, b]$ and $F'(x) = f(x)$, then $F(b) - F(a)$ represents the net area bounded by the graph of $y = f(x)$, the lines $x = a$, $x = b$, and the x -axis.

Solution True. This is in fact FTC.