

1. Determine if the following statement is True or False. If the statement is false, provide a counterexample or provide a justification.

(a) If  $F$  and  $G$  are antiderivatives of  $f$ , then  $F = G$ .

**Solution:** False. It can be  $F = G + c$ .

(b) The antiderivative of  $\sec^2(3x)$  is  $\frac{1}{3} \tan(3x)$ .

**Solution:** True. Use change of variable or find derivative of  $\frac{1}{3} \tan(3x)$ .

(c) The indefinite integral of a function  $f$  is the collection of all antiderivatives of  $f$ .

**Solution:** True.

(d) We know how to find the antiderivative of  $e^{x^2}$ , and it is  $e^{x^2}$ .

**Solution:** False.  $\frac{d}{dx}e^{x^2} = 2xe^{x^2}$ .

(e)  $F$  and  $G$  are antiderivatives of  $f$  and  $g$ , then antiderivative of  $FG$  is  $fg$ .

**Solution:** False. Because of the product rule.

2. Evaluate the following indefinite integrals.

(a)  $\int (\sqrt[3]{x} - \frac{1}{x})^3 dx$ .

**Solution.**  $\int (\sqrt[3]{x} - \frac{1}{x})^3 dx = \int (x - \frac{3}{\sqrt[3]{x^2}} + \frac{3}{\sqrt[3]{x^5}} - \frac{1}{x^3}) dx = \frac{x^2}{2} - 9x^{1/3} - \frac{9x^{-2/3}}{2} + \frac{x^{-2}}{2}$ .

(b)  $\int (3^{-x} + e^{-5x}) dx$ .

**Solution.**  $\int (3^{-x} + e^{-5x}) dx = -\frac{3^{-x}}{\log(3)} - \frac{e^{-5x}}{5}$ .

(c)  $\int \frac{e^{\sqrt{2}+x\sqrt{2}}}{\sqrt{x}} dx$ .

**Solution.**  $\int \frac{e^{\sqrt{2}+x\sqrt{2}}}{\sqrt{x}} dx = \frac{-e^{\sqrt{2}}\sqrt{x}}{2} + \frac{x^{\sqrt{2}+\frac{1}{2}}}{\sqrt{2}+\frac{1}{2}}$ .

3. (Optional): Evaluate  $\int \sqrt{\tan(x)} dx$ .

**Solution:** Let  $\tan^2(u) = \tan(x)$ . Then  $2(\tan^2(u) + 1) \tan(u) du = (\tan^2(x) + 1) dx$ . So

$$\int \tan(u) \frac{2(\tan^2(u) + 1) \tan(u) du}{\tan^4(u) + 1}$$

Then use partial fraction.