

1. (Applying the Definite Integral) A marketing company is trying a new campaign. The campaign lasts for three weeks, and during this time, the company finds that it gains customers as a function of time according to the formula:

$$C(t) = 7t - 4t^2$$

where t is time in weeks and the number of customers is given in thousands.

Using the following form of the definite integral,

$$\int_a^b f(t)dt = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f\left(a + \frac{(b-a)i}{n}\right)$$

calculate the average number of customers gained during the three-week campaign.

Also compute the average number of customers using the form

$$\int_a^b f(t)dt = \lim_{n \rightarrow \infty} \frac{b-a}{n} \sum_{i=1}^n f\left(a + \frac{(b-a)(i-1)}{n}\right)$$

Compare the results. What can you confirm from this comparison?

2. Explain why the following property is true:

$$\left| \int_a^b f(t)dt \right| \leq \int_a^b |f(t)|dt$$

Can you find an example where the inequality is strict?

3. Determine if each statement below is true or false.

(a) We always set x_i to be the right-hand endpoint of the i th interval.

(b)

$$\sum_{i=1}^n i^3 = \left(\frac{n(n+1)}{2} \right)^2$$

(c) If $f(x) \geq 0$ on $[a, b]$, then $\int_a^b f(t)dt$ represents the total area bounded by f , $x = a$, $x = b$ and the x -axis.