1. Determine if each integral below can be evaluated using a method we have learned so far (formula, u-substitution, integration by parts, or trig identities). If so, evaluate the integral. If not, explain why it cannot be evaluated.

   (a) $\int \tan^4(x)dx$.
   Solution. $\frac{1}{3}\tan^3(x) - \tan(x) + x + C$. Help them to undertand why $\tan^2 + 1$ is important here.

   (b) $\int \sin(x^2)dx$.
   Solution. Cannot be evaluated. Please explain that it cannot be evaluated with any elementary method (not just the ones that we discussed so far).

   (c) $\int e^{2x}\sin(3x)dx$.
   Solution. $\frac{2}{13}e^{2x}\sin(3x) - \frac{3}{13}e^{2x}\cos(3x) + C$. I solved a similar one in lecture. But it is good to see another one. Two IBP.

   (d) $\int \frac{x^2}{(x^2+4)^{3/2}}dx$.
   Solution. $\ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| - \frac{x}{\sqrt{x^2+4}} + C$. Very important to see the effect of $x = 2\tan(\theta)$.

   (e) $\int (x^2+1)e^{2x}dx$.
   Solution. $\frac{1}{2}(x^2+1)e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$. Needs two IBP.

   (f) $\int \frac{\sqrt{1-x^2}}{x^4}dx$.
   Solution. $-\frac{1}{3}\frac{(1-x^2)^{3/2}}{x^3} + C$. Trig $u-$sub with $x = \cos(\theta)$.

   (g) $\frac{dx}{e^x\sqrt{e^{2x}-9}}dx$.
   Solution. $\frac{\sqrt{e^{2x}-9}}{9e^x} + C$. Trig $u-$sub with $e^x = 3\sec(\theta)$.

   (h) $\int \sin^2(x)\cos^2(x)dx$.
   Solution. $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$.

   (i) $\int \frac{x+3}{(x-1)(x^2-4x+4)}dx$.
   Solution. $4\ln\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + C$. They need to use partial fraction.

2. Determine if the following statements below are always true or sometimes false.

   (a) If an integral contains the term $a^2 + x^2$, The best choice is to use the substitution $x = a\sec(\theta)$.
   Comment. False. The correct sub would be $a\tan(\theta)$.

(b) If we use the trig substitution $x = \sin(\theta)$, then it is possible that $\sqrt{1-x^2} = -\cos(\theta)$.

Comment. True. Taking square root always will ends up positive. So if we have definite integral, we need to check θ and it is possible that we need a negative sign to make cos to be positive.