Scheduling algorithms in cellular MIMO systems

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***Abstract*—In this work, we study the resource allocation problem for downlink in 5G New Radio (NR) mobile networks in the case of orthogonal frequency-division multiple access (OFDMA). One of the key features of 5G mobile networks is the usage of antenna arrays that allows them to form beams in a specific direction and send data to several users at the same transmission time interval (TTI) via the same carrier or resource block (RB). Antenna arrays usually contain up to 64 single antennas. We have been modeled the process of the data transmission from the cell to the user device or user equipment (UE), have been formulated mathematical abstraction of the resource allocation problem, and have been proposed a lightweight algorithm that uses a proportional fair metric.**

***Index Terms*—5G networks, New Radio, Massive MIMO, beamforming, graph coloring, proportional fair, greedy algorithm, round-robin, k-means clustering**

I. Introduction

At present 5G New Radio mobile networks are becoming more and more widespread. The crucial property of such networks is the high speed of data transmission, which is necessary for the successful implementation of many modern technologies such as the Internet of things, unmanned cars, and many others. This property is achieved with the following technologies and methods:

1) **Massive MIMO** [1]. used to multiply capacity of the link between transmitter and receiver via the usage of the multiple antennas on both sides. In this article, we consider the case when the number of antennas on the transmitter (base station) is significantly greater than the amount of the antennas on the receiver that can be interpreted as a smartphone, smartwatches, etc.
2) **Beamforming** [2]. Beamforming is the technology that allows forming beams in specific directions based on the interference of the waves emitted by the antenna arrays.
3) **Non-Orthogonal Multiple Access (NOMA)**. NOMA is the ability to transmit different signals to the different users at the same carrier and time.
4) **Time-division duplexing (TDD)**. TDD is used to arrange uplink and downlink signals on the same carrier using a specific signal pattern. This technology was originally introduced on the late LTE systems instead of frequency-division duplexing (known as FDD) where different carriers are used for uplink and downlink signals. This pattern (or any similar one) is part of the configuration of the base station and does not depend on any scheduling algorithm. It influences to the amount of data as constant parameter of the amount of the data that can be sent during one time frame.

In this article, in order to analyze the radio resource management problem, we consider the first two features: Massive MIMO and Beamforming. Note that TDD technology is used as a configuration of the transmitter device and is constant all over the time and this is not the part of the scheduling algorithm: it takes effect to the total size of the data that can be transmitted to the users at each moment.

II. Problem Overview

Suppose we have one transmitter that is called base station or BS that has to transfer information to the several connected receivers called user equipment or UE.

The signal is transferred during specific transmission time intervals (TTI) or frames which duration we assume is a constant that can be changed from 62.5 µs to 1 ms. At each TTI BS has to select the UEs to which data has to be transmitted.

We will assume the following signal propagation model between two antennas on one carrier:

$$s_r = r_{gain} \cdot t_{gain} \cdot loss \cdot phase \cdot s_t + noise, \tag{1}$$

where $s_r \in C$ is the received signal, $s_t \in C$ is the sent signal, $r_{gain} \in R$ is the receiver antenna gain, $t_{gain} \in R$ is the transmitter antenna gain, $loss \in R$ is the propagation loss between two points in the medium, $phase \in C$ is the change of the phase of the signal, $noise \in C$ is the noise (usually Gaussian) in the medium that may slightly add the random change of the signal.

BS utilizes an antenna array(Fig. 1) to transfer the signal. UE utilizes a similar antenna array with fewer amount of antennas.

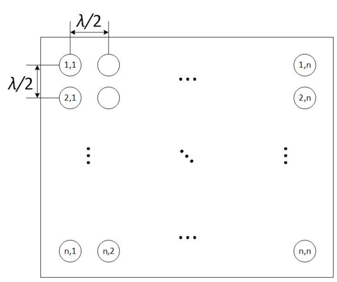

Fig. 1: Antenna array

Suppose BS has n antennas and each UE u has $m_u < n$ antennas thus equation (1) can be extended:

$$s_u^r = H_u \cdot s^t + noise_u, \tag{2}$$

where $s^r \in C^{m_u}$ is the received signal, $s^t \in C^n$ is the sent signal, $H \in C^{m_u \times n}$ is the channel matrix, which describes the model of the signal change while its transfer through the medium and $noise_u$ is the noise (usually Gaussian). According to (1) we can define it as follows:

$$H_u = R_{u,gain} . \times T_{u,gain} . \times Loss_u . \times Phase_u, \tag{3}$$

where $R_{u,gain} \in R^{m_u \times n}$, $T_{gain} \in R^{m_u \times n}$, $Loss \in R^{m_u \times n}$, $Phase \in C^{m_u \times n}$ are the corresponding matrixes and operation $.\times$ is component-wise multiplication of the two matrices. Later we will describe how to calculate each component of the channel matrix. But now we consider the problem statement in detail.

Signals are sent to the several devices simultaneously on the same career. Suppose we have the set U users to schedule and one selected the set $A \subset U$ users to send the signal. Each user has its own channel matrix H_u (see equation 3) and $s_u \in R^{m'_u}$ is the signal sent to the user u, where $m'_u < m_u$ is the amount of the simultaneously sent signals to the specific user (its maximum size is the amount of the antennas m_u on the UE but can be less). Since we expect the corresponding values on receiver u we have to "precode" the signal to form a specific beam to this user - it means that we will distribute the signal on the transmitters antenna array in a special way using the matrix $W_u \in C^{n \times m'_u}$ called precoding matrix. The resulting signal to send will be:

$$s^t = \sum_{u \in A} W_u s_u \tag{4}$$

One can see that UE will receive a modified signal and has to manually extract it to receive the actual value sent from the transmitter. This process is called **post processing** and is modeled by multiplication of the special post processing matrix $F_u \in C^{m'_u \times m^u}$ by the received signal. The result is decoded signal $s^{r,u}$ received by user u:

$$s^{r,u} = F_u \cdot (H_u \cdot \sum_{j \in A} W_j s_j + noise) \tag{5}$$

The ability of the UE to correctly decode the particular signal received is modeled usually via the $SINR$ - the power ratio between useful signal and other signals including noise. This ratio can be written in the following form:

$$\begin{aligned} SINR_{u,j} &= \frac{usefull\_signal}{interference\_signal + noise\_signal}; \\ usefull\_signal &= |f_{u,j} H_u w_{u,j}|^2 (s_{u,j})^2; \\ interference\_signal &= \sum_{u \in A} \sum_{i=1}^{m'_k} |f_{u,j} H_k w_{k,i}|^2 (s_{k,i})^2 - \\ &\quad - |f_{u,j} H_u w_{u,j}|^2 (s_{u,j})^2; \\ noise\_signal &= N \| f_{u,j} \|^2. \end{aligned} \tag{6}$$

In (6) $f_{u,j}$ is the j-th row from the matrix F_u, $w_{u,j}$ is the j-th column from the precoding matrix W_u.

$SINR$ is a very important parameter. It represents the quality of the channel / connection between two devices. Moreover, according to the $SINR$ value BS defines the modulation scheme to send the data to the user and thus the amount of data that will be sent at a particular TTI. This dependency can be modeled in the next form:

$$Bytes \approx const \cdot \ln{(1 + SINR)}, \tag{7}$$

where $const$ is some constant defined by base station configuration (for example TDD pattern used).

Our purpose is to maximize data transfer rate and thus on the basis of the (7) we can formulate the following optimization to be solved to select the users for data transmission at particular TTI:

$$Utility(A) = \sum_{u \in A} \sum_{j=1}^{m_u} \ln\left(1 + SINR_{u,j}\right) \to \max_{A \subset U}, \quad (8)$$

where U is the set of the connected users for which BS has data to send, $SINR_{u,j}$ is the signal interference noise ratio for specific user u at his channel j.

In order to finalize the problem we have to define the precoding matrix W and postprocessing matrix F. There are several ways to calculate precoding matrix, each one has its own pros and cons:

1) Maximum ratio transmission (MRT) [6];
2) Zero forcing (ZF) [9];
3) Wiener transmit filter or regularized ZF.

Let us briefly describe the first two of the precoding strategies, while the third one is actually a combination of them and will be skipped since the precoding strategy is out of the scope of our work.

MRT precoding strategy corresponds to the idea to maximize the power of the signal sent to the user. Signal $s \in C^n$ transferred to a specific antenna by antenna array will have the form $Hs \in C$ where H is reffered to as vector-row. Its power will be $\|Hs\|^2$ and its maximum will be achieved if distribute original signal as follows:

$$s = (H^*)^T s_{orig} / \|H\|.$$

The value $(H^*)^T$ corresponds to the precoding matrix.

Another type of precoding is ZF-precoding. Its idea is the following: maximize the power of the signal with zero impact to the other users scheduled for the data transfer. Thus it can be written in the following form

$$w_j^i = h_j^i - H_{(i)}^T (H_{(i)} H_{(i)}^T)^{-1} H_{(i)} h_j^i,$$

where $H_{(i)}$ is the concatenated channel matrix of the allocated users without particular channel j for user i.

In this article, we use the MRT precoding strategy to focus on the optimization problem.

The other important item is post processing of the signal or its decoding to the original one. It actually has two parts: precoding modification on BS and post processing multiplication on UE. Let us briefly describe it below.

Suppose for the particular user we have the following channel matrix $H_u \in C^{m \times n}$. The channel vectors from channel matrix H_u are close enough due to their nature and in order to use them, we prepare another m orthogonal vectors on their basis. This is done via singular value decomposition (SVD) of the channel matrix: $H_u = U \Sigma V^*$. Our new channel vectors are the first m vectors from the matrix V. The singular values on the diagonal of the matrix Σ defines the efficiency of the channel. In our experiments, we configure the number of channels to be used for all users. The transferring power of the signal is distributed uniformly over the selected channels. Thus the result precoding matrix W_u for the user u will be the first m_u vectors of the matrix V from SVD decomposition.

As described before on the receiver, we should do an inverse operation to SVD one. This is done via the MMSE scheme and has the following form:

$$F_u = (H_u W_u)^* ((H_u W_u)(H_u W_u)^* + NI)^{-1}, \quad (9)$$

where I is identity matrix, N is the noise estimate.

Note that $F \in C^{m_u \times m_u}$. If $m'_u < m_u$ then we will use only the first m'_u vectors from the matrix F to post process the signal and calculate the SINR.

Note that utility function (8) uses only immediate rates. In this case, BS will select only the users with the best SINR values to send the data and fairness requirement will not meet (i.e. there can be a case when some even good users will not receive data at all and loss the connection if there will be a set of very good users). In order to take it into account special "fairness" coefficient is usually introduced. In our artilce this coefficient is the inverse historical rate of the data transferred during a specific amount of the previous TTIs. Thus we will consider utility function in the following form:

$$Utility(A) = \sum_{u \in A} pf_u \sum_{j=1}^{m'_u} \ln\left(1 + SINR_{u,j}\right) \to \max_{A \subset U}, \quad (10)$$

where $pf_u = 1/(1 + total\_size)$, $total\_size$ is the amount of bytes sent to the users in the last k TTIs. Thus the max value of pf_u is 1 while the min value is defined by the maximum immediate data transfer rate for this user.

Maximization of the function (10) forms the criteria to select the users at the specific TTI.

III. Simulation

In the previous section problem statement for radio resource management is described along with the approach to solve it. In order to check the suggested approach, we build a simulator of the data transfer process. The simulator's diagram is presented on the Fig. 2

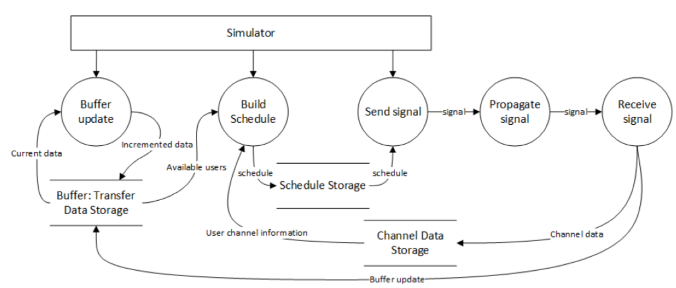

Fig. 2: Simulator scheme

We model only the part processes that happen on the MAC layer and the processes on the physical layer relevant to our research while the other processes are out of the scope of the presented model. We also assume that all users considered are connected to the base station.

There are the following elements in the simulator:

- Simulator. It orchestrates the simulation process via starting of the processes: Buffer update, Build Schedule and Send Signal.
- Buffer: Transfer Data Storage. This storage is used to store information about the size of the data to be transferred to a specific user.
- Schedule Storage. This storage is used to collect the schedules created during the simulation at each TTI.
- Channel Data Storage. This storage is used to collect information about the channel at each TTI, i.e. channel matrix for each connected user, and channel quality indicator (CQI) value (and thus measured SINR value).

The elements above forms the information stored on the base station. The other elements in diagram 2 are the processes that simulate: receive reception of the new data (Buffer update process), the schedule build process (Build schedule process. According to the information it selects the users at a particular TTI.), signal preparation and send process (Send Signal process), propagation of the signal from the base station to the particular user device (Propagate signal process) and analysis of the signal received (Receive signal process).

Let us describe them in a few words:

- Buffer update process. While this process simulator specifies the size of the data that has to be sent to the specific UE.
- Build schedule process. According to the information it selects the users at a particular TTI.
- Send Signal process. According to the schedule formed before it: 1) prepares precoding matrix, 2) prepares the signal to be sent at specific TTI, 3) distributes the signal over antenna array, 4) invokes the propagate signal process.
- Propagate Signal Process. At this process the signal is changed according to the selected propagation model. This propagation model takes into account the following elements: base station antenna model (see section antenna model), propagation model (see section propagation model), receiver antenna model (see section antenna model).
- Receive Signal Process. At this moment simulation of the signal processing on UE is simulated. It includes the following steps for each user: calculation of the post processing matrix F, calculation of the $SINR$, identification if the signal is received successfully, calculation of the channel quality indicator (CQI) and updating channel information in the related storage[1].

IV. Propagation Model

In order to simulate signal propagation between two devices we use the 3GPP study [4] which contains detailed discussion of this problem.

Below we describe the propagation process in the assumption of the single antenna on the receiver. This result in the simulator is extended for the case of the antenna array.

[1]Note, that channel information is updated with specific frequency

Let us consider the components of the signal propagation process:

1) Transmitter antenna array;
2) Signal propagation through the media;
3) Receiver antenna.

We assume that the antenna array (see Fig. 1) contains n' rows and n' columns (i.e. $n = n' \times n'$) and the distance between adjacent antennas is $\lambda/2$, where λ is a wavelength and any particular antenna on the transmitter is a parabolic one, while UE antenna is isotropic one.

Parabolic antenna pattern is modeled as follows:

$$\begin{aligned} A(\theta,\varphi) &= -\min\left\{-A_{hor}(\theta) - A_{ver}(\varphi), A_{max}\right\}; \\ A_{hor}(\theta) &= -\min\left\{12\left(\frac{\theta - 90^\circ}{\theta_{base}}\right), SLA_V\right\}; \\ A_{ver}(\varphi) &= -\min\left\{12\left(\frac{\varphi}{\varphi_{base}}\right), A_{max}\right\}. \end{aligned} \tag{11}$$

The following parameters are used: $\theta_{base} = 65^\circ$, $SLA_V = 30dB$, $\phi_{base} = 65^\circ$, $A_{max} = 30dB$, $\theta \in [0^\circ, 180^\circ]$, $\varphi \in [-180^\circ, 180^\circ]$. The related angles (θ, φ) used in antenna pattern are presented in the Fig. 3.

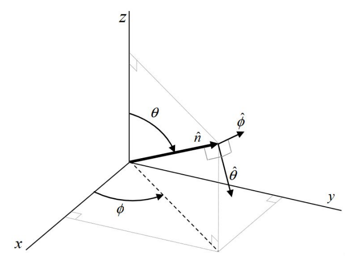

Fig. 3: Antenna angles

To calculate the signal propagation loss we use the ABG model [7]:

$$\begin{aligned} PL^{ABG}(d,f) = &\ 10\alpha\log_{10}(d/1m)) + \beta \\ + &\ 10\gamma\log_{10}(f/1GHz)), \end{aligned} \tag{12}$$

where f is the frequency and d is the distance between the transmitter and receiver and consider related UMa LOS scenario: $\alpha = 2.8$, $\beta = 11,4dB$ and $\gamma = 2.3$.

Both antenna pattern and propagation loss defines the change of the power of the signal. The phase shift ps for each signal we will define as follows:

$$ps(d) = \exp\left\{i\frac{2\pi}{\lambda}\cdot d\right\}. \tag{13}$$

Thus the signal change $h_{k,l}$ for the k-th antenna on transmitter and l-th antenna on receiver is defined by the following equation:

$$h_{k,l} = exp((A(\theta_{k,l},\varphi_{k,l}) + PL^{ABG}(d_{k,l}))/10) * ps(d_{k,l}).$$

The elements $h_{k,l}$ form the channel matrix H for the specific UE and finalizes the propagation model overview as follows:

$$y = Hx,$$

where x is the signal sent by transmitter and y is the signal received by receiver.

V. Scheduling Algorithms

Let us consider resource allocation problem in the form (10).

If $|U| \sim 30$ (it is the usual amount of the users on the base station), then amount of the possible variants to solve the problem is 2^{30}. Thus this problem requires special algorithms. To solve this problem we use the following algorithms:

1) Round Robin algorithm modification;
2) Local search on the binary cube;
3) Local search on the binary cube with; randomized direction selection criteria;
4) Graph algorithm;
5) Clustering based algorithm.

In the subsections below this algorithms are described.

A. Modified Round Robin algorithm

Modified Round Robin algorithm [13], [14] represents user scheduling greedy algorithm (we will also call it a baseline algorithm). Its input is the ordered by PF metric set of the users. The algorithms steps are following:

1) Set $A_0 = \emptyset$, $f(A_0) = 0$, $i = 1$;
2) Set $A_i = A_{i-1} \cap i$;
3) If $f(A_i) < f(A_{i-1})$ then $A_i = A_{i-1}$ (remove user i);
4) Set $i = i + 1$;
5) If $i < |U|$ then go to step 2 otherwise set $A = A_i$ and stop.

It is obvious that this algorithm is not the optimal one but illustrates the easiest scheme for the greedy algorithms that can be used for these scheduling problems. Its plus is its simplicity - it has linear dependency on the number of users. However, this algorithm has a straight relationship on the criteria users ordered and can give completely different results for the different criteria(PF metric, User ID and etc).

B. Local search algorithm

Local search algorithm represents the other possible set of algorithms that can be used to solve the problem 10. Any allocation A can be represented by the binary vector x where $x_i = 1$ if $i \in A$ and $x_i = 0$ otherwise. Thus $A = A(x)$ The input for this algorithm will be ordered by pf metric set of the users. To fulfill the fairness requirement (all users should receive the data) we also require that by any means the value $x_1 = 1$ at any step of the algorithm. Define also the probability function $rand(x)$ which value is 1 with probability $x \in [0, 1]$. Thus the algorithm can be described by the following steps:

1) Set $next = true$, Set $x_i = rand(pf_i/pf_1)$, $i = 1, ...n$; $f* = f(A(x))$, $A = A(x)$;
2) if $!next$ then stop and return x, $A(x)$, f^*;
3) set next = false, $y = x$;
4) for $j \in [2 : |U|]$ do:
   a) Set $x^j = x$; $x^j_j = !x^j_j$;
   b) if $f^* < f(A(x^j))$ then: $f^* = f(A(x^j))$, $y = x^j$, next = true;
5) set $x = y$ go to step 2

It is obvious that this algorithm is the local one and converges to some local optimal solution. In the worst case the algorithm should "go" through all vertexes of the binary cube, however one can expect due to the nature of the optimization problem that its execution time has linear dependency on the size of the users set.

C. Randomized Local search on the binary cube

This algorithm is just a modification of the previous one where index j is selected only $k < |U|$ times as a random element from the set $[2 : |U|]$.

D. Graph algorithm

This algorithm is based on the graphs and represents the further development of the coloring algorithm suggested in [8].

This algorithm builds a graph where:

1) vertexes V are the users from the set $|U|$;
2) edges E identify the possibility to merge users on the same carrier: $e(i, j)$ exists if $|(h^1_i, h^1_j)| > a$. Here $h^1_i = h_i/\|h_i\|$.

The algorithm selects from the graph the set of the vertexes without edges between them.

E. Clustering algorithm

The clustering algorithm represents the further extension of the ideas presented in the graph algorithm: let us group users in the clusters according to their mutual value $|(h^1_i, h^1_j)|$ and then select from each cluster the users with maximum pf metric. After this execute either local search on the binary cube or the round robin modification algorithm. For clustering we use k-means algorithm [15] according to the distance matrix $D = |(h^1_i, h^1_j)|^2$.

Formally we can describe this algorithm as follows:

1) Calculate distance matrix $D = |(h^1_i, h^1_j)|^2$;
2) Cluster (or group) users with kmeans with parameter $k = 6$;
3) Select the users from the clusters received;
4) Identify the allocation with the greedy algorithm described before

VI. Experiments

We tested the suggested algorithms and compared their performance with the modified round-robin algorithm **A** in the following scenario:

- antenna amount on the base station is 8×8;
- amount of steps (duration) is 10000;
- amount of the resource block is 10;
- wave length $\lambda = 0.1m$;
- amount of users is 15;
- noise constant is $3,16228 \cdot 10^{-20}$.

Users are uniformly distributed on the distance of about 200 meters behind the base station. Each user is a "full buffer", i.e. for each user, the traffic is generated starting from some

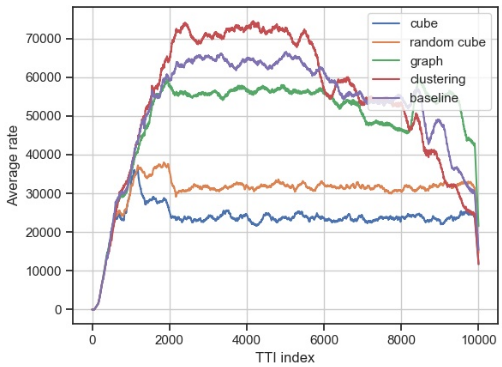

Fig. 4: Data transfer process

step greater than 0, and traffic generation is stopped at another TTI those index is greater then 5000.

Experiment results are presented in Fig. 4, where *baseline* corresponds to modified round-robin algorithm **A**, *cube* means local search algorithm **B**, *random cube* is the randomized local search algorithm **C** *graph* and *cluster* are graph algorithm **D** and clustering algorithm **E** respectively.

In the table I presented the summary of the experiments performed where total means the total amount of bytes transferred and reduced is the amount of bytes transferred between 2000 TTI and 8000 TTI.

	baseline	clustering	graph	random cube
total	522542481	539187344	496635199	305253812
reduced	365858255	393827816	326163629	189661805

TABLE I: Algorithms statistics

The experiment results presented allow to make the following conclusions:

1) the modified round-robin algorithm, graph algorithm and clustering algorithm performances are close to each other where clustering is slightly better, while local search algorithm and its randomized versions looks less efficient. However it can be result of the required additional configuration of the algorithms that may significantly improve their performance
2) *clustering* algorithm shows the better performance during the main data transfer process from 2000 TTI till 6000 TTI. However, it cannot be said that they are really better in any case - more experiments have to be performed.

VII. Conclusion

In this article, we considered four algorithms: local search algorithm and its randomized version, graph algorithm and clustering algorithm that can be used to solve radio resource management allocation problem and compare it with baseline modified round-robin algorithm. All algorithms took into account fairness requirement: the worst user will receive data due to his pf metric. Graph and clustering algorithms shows slightly better results in comparison with modified round robin algorithm and local search algorithm.

VIII. Regards

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