

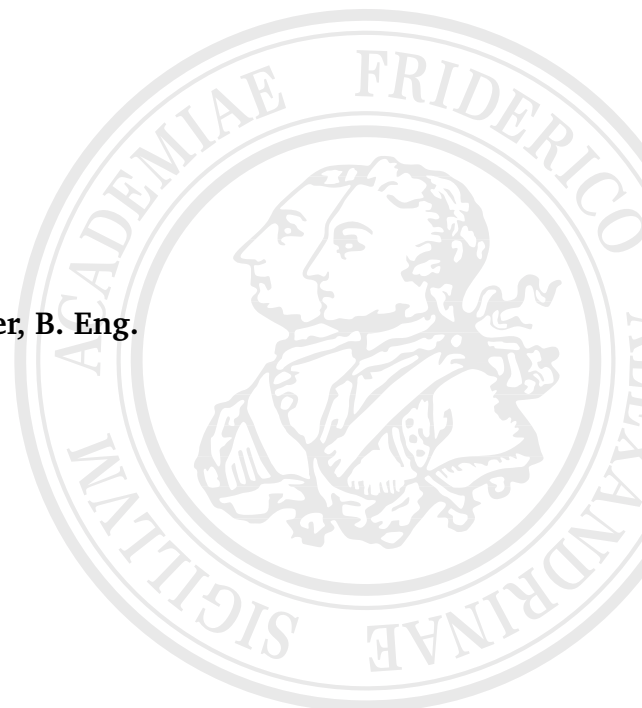
# Modeling of Fast-Switching Transformers for Voltage Stability Studies in Python

MASTER THESIS

by

**Maximilian Markus Veit Köhler, B. Eng.**

May 2nd, 2025



**DISCLAIMER:**

This is not the version for the examination procedure, the chair, and therefore not the graded version. It is a printed version, but its content is completely identical to the submitted version.

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for Voltage Stability Studies in Python,*  
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” Two things are infinite: the universe and human stupidity; and I'm not sure about the universe.

— Albert Einstein



# 1 Introduction

Some blibla as introduction. [1]

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*This is a side note for testing out some options.*

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*This is a second side note for testing out some options.*

[MK1]: Write a nice introduction.

## Research Interests

Here are gaps and possible extension of knowledge.

Here are the research objectives and questions.

- Influence of OLTC control on possible operational uses: Short-term voltage stability, long-term voltage stability;
- Can a increased dynamic regulation help machine recovery?
- Does the increased tap ratio gradient harm transient stability of machines? Does it help or harm CCT of machines or machine groups?
- Transformers act as big low-pass filters: Can this behavior be beneficial as well for the interactions of inverters in the grid on AC side (in the sense of Harmonic Stability)? [Quelle]

### Research Question of this Thesis

How do different control types and characteristics of Tap Changing transformers influence the voltage stability of the given system?

Therefore following questions/steps can be imagined as supportive:

1. How can Voltage stability of a system be classified and be looked at? Which indices, measurements, etc.
2. Which transformer model has to be considered to show influences?
3. Which additional load models, source models, transmission model have to be modeled for an adequate assessment?
4. Which systems are useful to consider in showing effects? Which circumstances lead to a stability support, which to a decrease? Where can limits be drawn?

Additionally during the process of the thesis, the following question came up as an extension. Is is the second interest of this thesis, and shall be more focused in the later part. Therefore some assessments in the [chapter 5](#) are conducted.

### Additional Question of this Thesis

Can the already existing Tap Changer Control of the Fast Switching Module (FSM) be improved towards a more operation oriented control?

## Construction of the Thesis

” So many books, so little time.

— Frank Zappa

## 2 Fundamentals

Following chapter shall introduce the basics for implementing an OLTC equipped transformer into a existing PSS framework. This is considering the already existing surrounding, more detailed the electric behavior of the transformer itself and some control engineering theory for the corresponding OLTC. Thus its main goal is increasing voltage stability [1], main indices and assessment methods are considered as well.

### 2.1 Power System Modeling

Transformer II-Model:

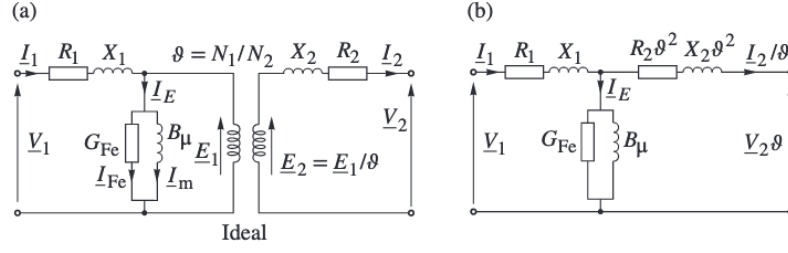
- Deviation from 'normal' circuit
- Placing the impedance elements on the LV side vs. HV side
- Getting into a single line II-model
- Introducing the tapping factor  $\vartheta$ ; if phase shifting it is getting complex -> how to consider this
- Different Y-Bus matrices if LV or HV side is considered
- Per-unit problems with the transformer characteristics
- Short overview of Open-Source power system simulation packages

Voltage stability:

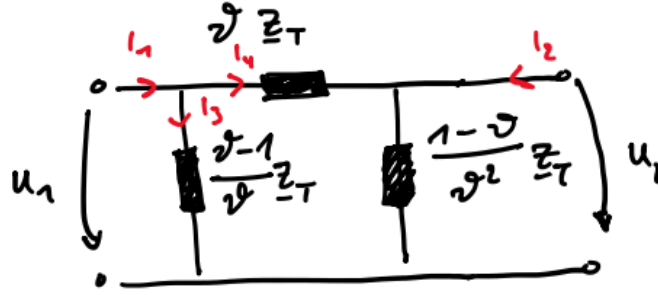
- Definitions, classifications, conditions
- PV-curves / QV-curves
- Derivation of Indices: Practical application

Control engineering:

- State-of-the-art control mechanisms for OLTCs
- Chaos theory controllers



**Figure 2.1:** Two-Winding Transformer Circuit in the Positive Sequence; a) ideal representation with impedances on each HV and LV side and b) related impedances on the XX side; own figure after [1]



**Figure 2.2:** II-representative circuit of a transformer with a longitudinal tap changer; own figure after [1], [2]

### 2.1.1 General and Existing Model

### 2.1.2 Transformer Electric Model and Behavior

$$\underline{I} = \underline{Y} \cdot \underline{U}$$

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{Y}_{11} & \underline{Y}_{12} \\ \underline{Y}_{21} & \underline{Y}_{22} \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} \quad (2.1)$$

The admittance matrix of a two port network can be expressed after Machowski, Lubosny, Bialek, *et al.* [1] as Equation 2.1. For the II-model of an OLTC transformer it is leading to Equation 2.2.

$$\underline{Y}_{\Pi,T} = \begin{bmatrix} \underline{Y}_T & -\vartheta \underline{Y}_T \\ \vartheta^* \underline{Y}_T & -\vartheta^* \underline{Y}_T \end{bmatrix} \quad (2.2)$$

Another way of writing down the admittance matrix is shown in Equation 2.3. It is considering, that the matrix can be split up in a symmetric, constant part, and a



variable current injection part. The latter is not symmetrical and depends on the tap position of the transformer. Therefore in some simulation algorithms the static part is used in the admittance matrix, and the variable part is considered in the current injection vector. [1]

$$\begin{aligned} \begin{bmatrix} \underline{I}_1 \\ -\underline{I}_2 \end{bmatrix} &= \begin{bmatrix} \underline{Y}_T & -\underline{Y}_T \\ -\underline{Y}_T & \underline{Y}_T \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} - \begin{bmatrix} \Delta \underline{I}_1 \\ \Delta \underline{I}_2 \end{bmatrix}, \text{ where} \\ \begin{bmatrix} \Delta \underline{I}_1 \\ \Delta \underline{I}_2 \end{bmatrix} &= \begin{bmatrix} 0 & (\vartheta - 1)\underline{Y}_T \\ -(\vartheta^* + 1)\underline{Y}_T & (\vartheta^*\vartheta + 1)\underline{Y}_T \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} \text{ leading to} \\ \underline{\mathbf{Y}}_{\Pi, T} &= \begin{bmatrix} \underline{Y}_T & -\underline{Y}_T \\ -\underline{Y}_T & \underline{Y}_T \end{bmatrix} - \begin{bmatrix} 0 & (\vartheta - 1)\underline{Y}_T \\ -(\vartheta^* + 1)\underline{Y}_T & (\vartheta^*\vartheta + 1)\underline{Y}_T \end{bmatrix} \end{aligned} \quad (2.3)$$

### Per unit system specialities

Reactances and resistances are referred to the base voltage and apparent power of the operational unit, such as the transformer. The power system simulation uses its own base voltage and base apparent power, enabling the use of one single calculation domain. This is done to simplify the calculation and to make the results easily comparable to each other. Hence, the referred values have to be transformed from the equipment based values to the simulation based values. The relations and conversions are defined as follows.

$$\begin{aligned} \underline{Y}_T &= \frac{1}{r_T + x_T \cdot j} \cdot \frac{b_T \cdot j}{2} \\ \underline{Y}_{T, \text{sim}} &= \underline{Y}_T \cdot \frac{S_n}{S_{n, \text{sim}}} \end{aligned} \quad (2.4)$$

$$\underline{U}_{\text{whatever, sim}} = \underline{U}_{\text{whatever}} \cdot \frac{S_n}{S_{n, \text{sim}}} \quad (2.5)$$

Displayed like in [Equation 2.4](#), the characteristic of the operational unit is referred to the simulation base value. Here, the admittance of the transformer is multiplied with its own rated apparent power, then divided by the apparent power of the simulation system. Similar, the voltages are calculated via [Equation 2.5](#). This specialities are considered in the tap changer modeling, thus further information is given in [1], Appendix A.

Additionally to consider:

- D-q transformations (???),
- Frequency domains: reactances and inductances are dependent and can change with the base frequency,
- Torque and power relations.

### 2.1.3 Open-Source Power System Simulation tools

Some information about other open source python power system simulation tools, such as:

- Pandapower,
- TOPS,
- ... .

Build up like a scan (see Georg's thesis).

BUT: As well including the there used implementation of transformers mathematical background and complexity.

## 2.2 Voltage stability basics

### 2.2.1 Voltage stability definitions, classifications, and conditions

A Practical introduction to voltage stability assessment, methods and indices is given in the standard and extending literature of Rueda-Torres, Annakage, Vournas, *et al.* [3], Danish [4], and Cutsem and Vournas [5].

Interesting to note/implement here: Basic classification, definitions, and the nature or conditions of voltage stability. Such as

- Short term vs. long term
- Static vs. dynamic
- Transmission driven vs. load driven vs. generation driven; stability/instability, and/or contributions
- Influence OLTC: Restoring voltage level, but not adding reactive capacities; hence adding risk of voltage collapses
- Load vs. transmission aspects
- Example mechanism: **Collapse effect of the nordic test system** [5], [6]

**Table 2.1:** Voltage instability types and different time frames with examples; after [Quelle]

No	Type	Cause of incident	Time frames
1	Long-term	Slowly use up of reactive reserves and no outage	Several minutes to several hours
2	Classical	Key outage leads to reactive power shortage	One to five minutes
3	Short-term	Induction motor stalling leads to reactive power shortage	Five to fifteen seconds

### 2.2.2 Stability Indices

One easy idea for obtaining a stable operation is looking at the Jacobian Matrix. If this matrix is getting singular, the System will not remain in a stable operation. Singularity of matrices is checked by following two hypothesis tests:

$$\det(\mathbf{J}) = 0 \quad (2.6)$$

$$\mathbf{J} \times \mathbf{J}^{-1} \uparrow \quad (2.7)$$

The Jacobian Matrix is defined as:

$$\mathbf{J} = \begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix} = \begin{bmatrix} \mathbf{H} & \mathbf{M}' \\ \mathbf{N} & \mathbf{K}' \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta \\ \Delta V/V \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_1 \\ \vdots \\ \Delta P_n \\ \Delta Q_1 \\ \vdots \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_n} & V_1 \frac{\partial P_1}{\partial V_1} & \cdots & V_n \frac{\partial P_1}{\partial V_n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_1} & \cdots & \frac{\partial P_n}{\partial \delta_n} & V_1 \frac{\partial P_n}{\partial V_1} & \cdots & V_n \frac{\partial P_n}{\partial V_n} \\ \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_n} & V_1 \frac{\partial Q_1}{\partial V_1} & \cdots & V_n \frac{\partial Q_1}{\partial V_n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_1} & \cdots & \frac{\partial Q_n}{\partial \delta_n} & V_1 \frac{\partial Q_n}{\partial V_1} & \cdots & V_n \frac{\partial Q_n}{\partial V_n} \end{bmatrix} \cdot \begin{bmatrix} \Delta \delta_1 \\ \vdots \\ \Delta \delta_n \\ \Delta V_1/V_1 \\ \vdots \\ \Delta V_n/V_n \end{bmatrix} \quad (2.8)$$

Although this method seems easy to implement, there are some numerical problems related to that. Checking if a Matrix is singular with numerical methods, can only be realised as a probability expression. A result could be, that the determinant of the matrix is below a certain threshold. The algorithm would propose, that the matrix is probabilistic singular. [QUELLE] This problem leads to the necessity of applying other methods or indices for stability assessment. Danish [4] is proposing a few other indices, that are based on the Jacobian Matrix, and shows comparative characteristics between Jacobian Matrix and system variable based voltage stability indices. These

Jacobian Matrix based indices are listed and further described in [section A.2](#), while the comparative characteristics are described in [section A.3](#).

### 2.2.3 Assessment methods

### 2.2.4 Analytical stability calculation of static power systems

## 2.3 Control engineering theory

### 2.3.1 Commonly used on-load tap changer control

A few basics are in the interest, understanding differences between real world behavior, or possible ways of building up a [OLTC](#) transformer control. This control theory difference can be limiting as well for the results and objectives compared to the actual possible control in the field.

The target voltage is typically set from the control room of the grid operator, coming from pre-calculated load flow analysis. This can be set hours before, or even day-ahead with the estimated loads of the grid. This value is set locally for each operating unit subsequently. The control is then operating locally and without further involvement of the grid operator. [\[Quelle\]](#)

Typically the used controller in the field is a discrete controller, which can change tap positions under load within a time frame of around few seconds. Practical tap steps are around 2 % of the overall transforming ratio. The control is set up with a dead band, to avoid unnecessary tap changes. It is necessary to note here, that this control and its mathematical characteristics contains logical elements, blocks, and delays, which cannot be translated in a typical control theory transmission function. This leads to the missing possibility to easily obtain mathematical stability for the control of the overall considered power system. [\[Quelle\]](#)

### 2.3.2 Dynamic voltage stability

Can I really express this as „Controller theory“?

### 2.3.3 Bifurcations and Chaos theory control

Is this necessary or already out of scope?

1. Fuzzy Control mechanisms,
2. Neural Networks,
3. Bifurcations.

*Typical presets are manually set*

*Discrete controllers are used in the field*

” All models are wrong, but some are useful.

— Albert Einstein

## 3 Methodical Modeling

### 3.1 Transformer Equipment Modeling

Some literature and fundamentals about transformers, control, stability assessment, fast-switching modules, and analysis in Python.

Things to mention:

- First model: Admittance Matrix manipulations
- Second model: Current Injection Model

#### 3.1.1 Implementing a $\Pi$ -Representative Circuit with Variable Ratio

##### Mathematical Description and Definitions

Describe the Transformer circuit, the  $\Pi$ -model, simplification of impedances, relation to the specific side, ...

1. Show the admittance matrix deduction process for placing the impedances on one side
2. How to interpret  $\underline{v}$ ; especially if complex and a mixture of longitudinal and angle ratio  
-> e.g. Asymmetrical shifter, vector groups, or only tapping on the current-voltage angle  $\phi$
3. Illustrate differences if the impedance relation is switching sides (Machowski vs. Kundur / Milano)  
-> refer the admittances to the lv or hv side: different definition of  $\underline{v}$ , different admittance matrix of the transformer
4. Dependency of connections? (which side, step-up, step-down transformer?)

##### Mathematical for MACHOWSKI / SIMULATION

Similar to the mentioned typical assumptions from [1]

- Neglecting (normally big) shunt branches in the circuit,
- Assuming the impedances on the HV and LV side are symmetrical (the same), and

- (...more?),

these are considered in this paper as well.

Procedure for accounting the Admittance Matrix:

1. Set constraints as Ohms law for the admittance matrix and the tap ratio,
2. Re-arrange equation to set currents and voltages independent of the transformer ratio,
3. calculate the series admittance and the shunt admittances after the given standard formula.

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{Y}_{12} + \underline{Y}_{10} & -\underline{Y}_{12} \\ -\underline{Y}_{21} & \underline{Y}_{21} + \underline{Y}_{20} \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} \quad (3.1)$$

OLTC tap ratio definition:

$$\underline{\vartheta} = 1 + k \cdot \Delta v \quad (3.2)$$

$$\text{with } k \in [k_{\min}; k_{\max}]; k_{\min} \equiv -k_{\max} \quad (3.3)$$

Angle definition as multiplication to the tap ratio:

$$\underline{a}_T = \exp\left(j \cdot \phi \cdot \frac{\pi}{180}\right) \quad (3.4)$$

### Mathematical for Paper ILYA and KUNDUR / MILANO

#### Definition OLTC ratio

Relavant points for the definition of the OLTC ratio:

- Complex vs. Rational number: Angle or Asymetric shifter
- How can angles and logitudinal ratios be mathematically expressed?
- How to understand when Relation to hv or lv side?

The OLTC ratio  $\underline{\vartheta}$  when placing the replacement impedance  $\underline{Z}_T$ , respectively the replacement admittance  $\underline{Y}_T$ , on the high voltage side is defined as:

$$\underline{\vartheta} = 1 + k \cdot \Delta v \quad (3.5)$$

$$\text{with } k \in [k_{\min}; k_{\max}]; k_{\min} \equiv -k_{\max} \quad (3.6)$$

Within this definition,  $k_{\min}$  defines the minimum tap position,  $k_{\max}$  the maximum OLTC postition. When relating the replacement impedance to the low voltage side,

the tap ratio is defined differently, while looking at the tap position in Equation 3.6 in the same manner.

$$\underline{v} = \frac{1}{1 + k \cdot \Delta v} \quad (3.7)$$

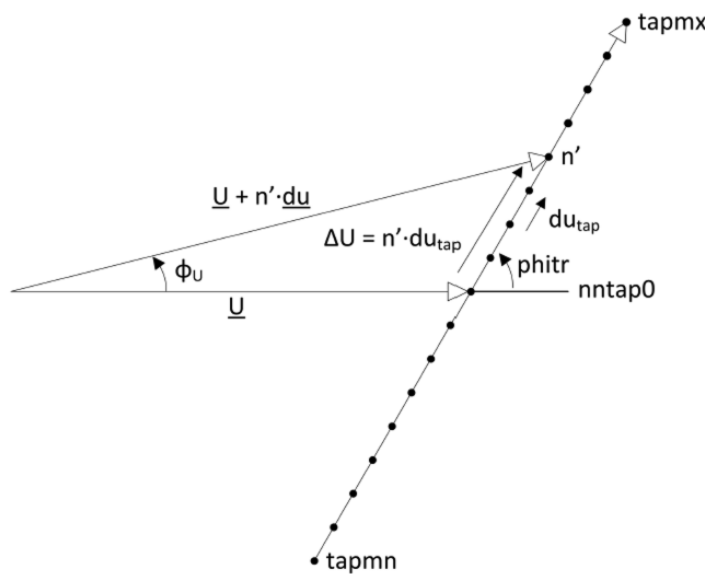
Voltage angle shifting through the influence of vector groups, meaning a different wiring and thus magnetic coupling of the transformer can be expressed within the transformer model. By mathematically applying a turning vector with the length of one to the overall tap ratio, this can be included in the model. Mathematically, this is expressed by the following equation. The characteristic number  $n_T$  is relating to to angle, with one step being equal to  $30^\circ$  angle ratio.

$$\underline{a}_T = \exp\left(j \cdot n_T \cdot \frac{\pi}{6}\right) \quad (3.8)$$

*Representation of  
Vector Groups*

Just briefly describe the influence of an asymmetric vs. an ideal transformer. Why the difference and how the representation could work.

*Ideal vs. asymmetric  
ratio*



**Figure 3.1:** Illustration of the tap ratio vector for an ideal and an asymmetric transformer; from the **DIgSILENT Technical Reference Manual** ... [Quelle]

### Additional Algorithmics

- How to automatically determine switching direction?  
-> switchin direction dependent on what? (load-flow direction?)
- Controller set points: also dependent on load flow?
- How can I change the transformer control setpoints to be load flow dependent?
- How can I ensure, utilization of the transformer is not  $> S_n$ ?

### 3.1.2 Tap Changer Control Modeling

This is the description of the ideas, development, and implementation of a OLTC control scheme.

#### Discrete Control Loop

- Describe implementation
- Describe benefits / drawbacks
- Control scheme
- Switching logic and behavior (voltage tracking)

This control method represents the currently most used and thus representative control scheme for OLTCs. With the mechanic nature of the switching mechanism, the control look can only access discrete ratios within time frames of around a few seconds. Such a discrete control loop is described by Milano [7], [8]. A scheme of this control loop is shown in Figure 3.2.

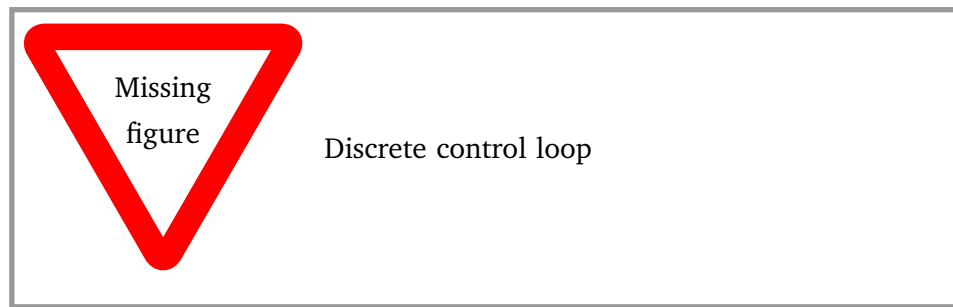


Figure 3.2: Discrete control loop of an OLTC; scheme based on Milano [7]

This control loop type is beneficial due to its accurate representability of current OLTC abilities. It gains access to assess stability within simulation environments, as analytical methods are not suited.

A negative aspect of a discrete control loop is the missing opportunity of generating a transfer function. This blocks the stability assessment with standard control engineering methods. Further, popular analysis methods like eigenvalue analysis is not possible, due to the lack of possibility to form derivatives.

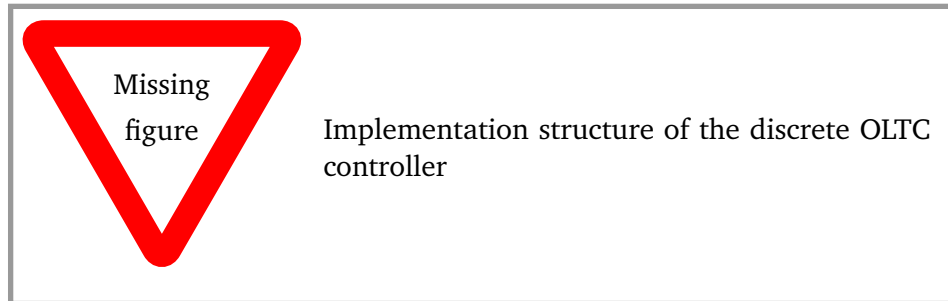
The structure of the implementation is illustrated in the block diagram of Figure 3.3. The controller is actively changing the algebraic functions of the simulation environment, therefore it is quasi dynamic. The controller output logic is called, when updating the admittance matrix of the transformer. Additionally, the differential functions of the connected simple controllers, like integrators, PT1-blocks, etc., are called by the solver and are thus part of the differential equations. The logic determines the physical interpretation of the OLTC, and therefore

1. If the OLTC has to switch,



2. When the switching operation is finished, and
3. What the current, or in case after a switching the new, tap ratio is.

It is important to note, that this structure relies on the calculation of the dynamic admittance matrix on each time step.



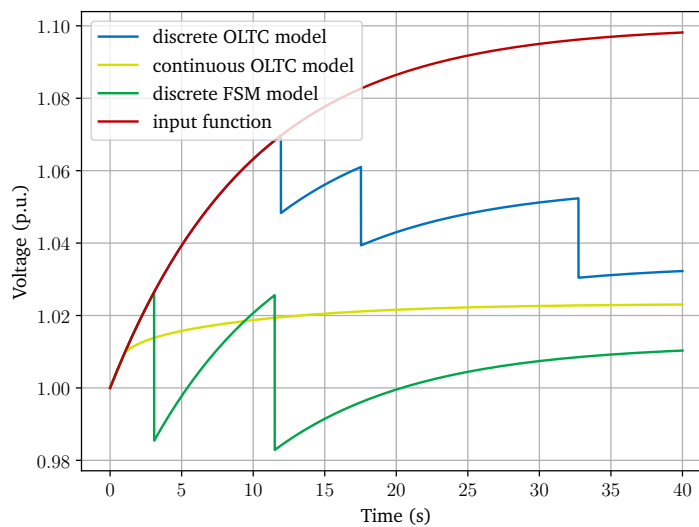
**Figure 3.3:** Implementation structure of the discrete OLTC controller

The output of the controller is based on the following logic.

*Output logic*

*Characterization plot and validation*

### Continuous Control Loop



**Figure 3.4:** Characterization of the OLTC control loop; the input function simulates the to be regulated voltage, the output functions are characterized by  $o(t) = i(t) \cdot \underline{\vartheta}_{\text{trafo}}$

### Control Schemes for the Fast Switching module

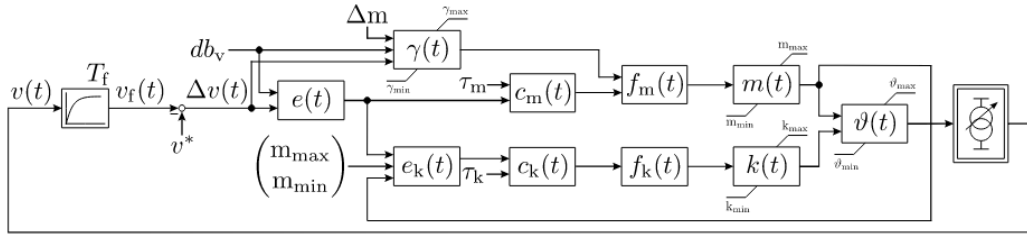
- Describe implementation
- Describe benefits / drawbacks
- Control scheme
- Switching logic and behavior (voltage tracking)

Functional basics of a  
FSM

Describe the operational logic and structure of the Fast Switching Module (FSM) first.

Control logic

A control logic for a so called FSM has been presented from Burlakin, Scheiner, Mehlmann, *et al.* [2], and illustrated in Figure 3.5.



**Figure 3.5:** Control loop of a FSM; scheme based on Burlakin, Scheiner, Mehlmann, *et al.* [2]

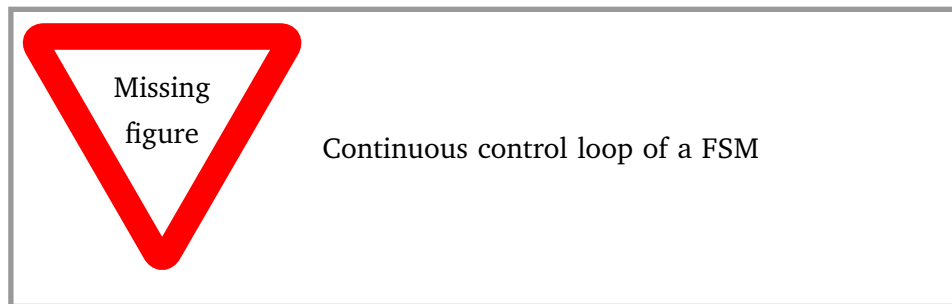
Implementation  
differences

However, the implementation logic in Python is slightly differing from the presented scheme in [2], simply for not overcomplication of the code and therefore debugging. The implementation is similar to the afore discussed one of a standard OLTC controller.

Characterization and  
validation

### Discrete Control Loop as most Representative

A continous control loop for a FSM is presented within Burlakin, Scheiner, Mehlmann, *et al.* [2], [9]. Similar to the solely OLTC loop, it represents the real behavior best, but is obstructive for stability assessments. The scheme of the logic is shown in Figure 3.6.



**Figure 3.6:** Continuous control loop of a FSM; scheme based on Burlakin, Scheiner, Mehlmann, *et al.* [2]

## Continuous Control Loop for best Stability Assessment

### 3.1.3 Experimental: Extended Ideas and Improvements

#### Operational Oriented FSM Control

#### Alternative Tap Skipping Logic

#### Varying the Voltage Setpoint and Target Calculation

Here, another idea of control target creation shall be mentioned. Instead of a fixed bus voltage reference, the difference of both bus voltages is considered. Further, the sign of that difference is used to determine the direction of the tap change.

## 3.2 Supplementary Modeling and Advancements

As the python framework is currently missing some representations of components, this chapter aims to describe the implementation of those. Mainly focussing on source and load models, as the later considered test, benchmark, and use case networks require alternative behaviors.

### 3.2.1 ZIP Load Model

#### Why important?

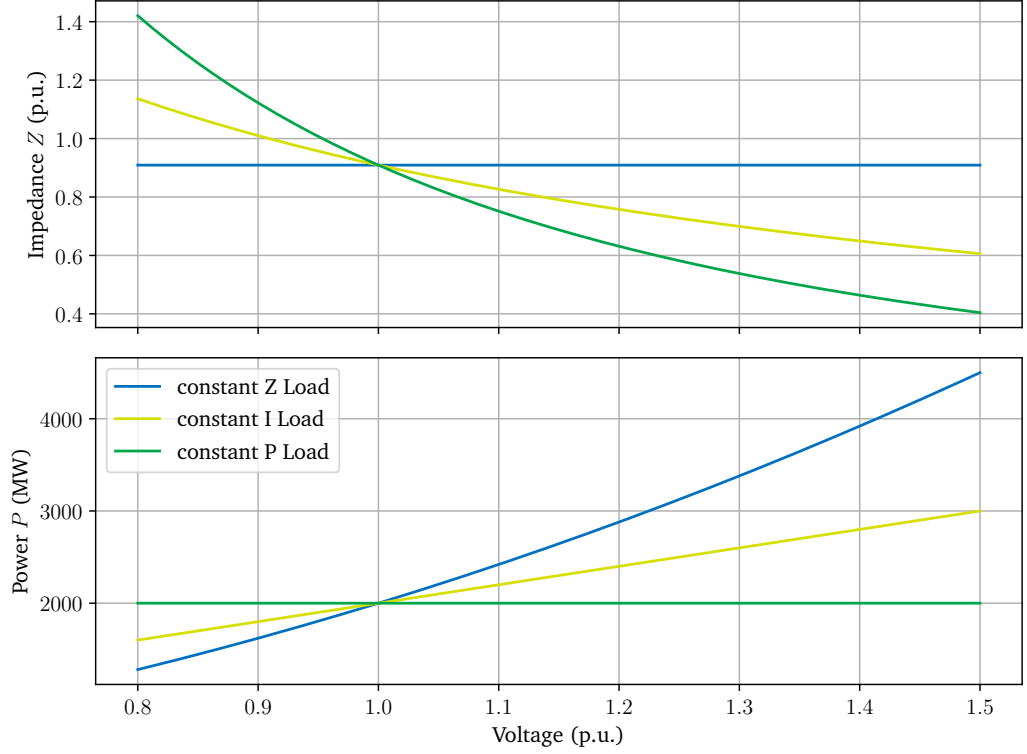
Mostly, a polynomial load model is used. It is called ZIP-model, as there are individual contributions to constant impedance  $\underline{Z}$ , constant current  $\underline{I}$ , and constant power  $P$ , or respectively  $Q$ , are considered. The model is described by *IEEE Guide for Load Modeling and Simulations for Power Systems* [10]. Either two ways of mathematical description are considered valid, dependent on the allowed influence of the frequency deviation. The use of periodized phasor representation, typical for a RMS simulation, is missing or often neglecting this frequency information. Therefore the set of Equation 3.9 and Equation 3.10 is considered sufficient and implemented in the Python framework.

$$P = P_n \cdot \left[ p_1 \left( \frac{U}{U_n} \right)^2 + p_2 \left( \frac{U}{U_n} \right) + p_3 \right] \quad (3.9)$$

$$Q = Q_n \cdot \left[ q_1 \left( \frac{U}{U_n} \right)^2 + q_2 \left( \frac{U}{U_n} \right) + q_3 \right] \quad (3.10)$$

with  $p_i \in [0, 1]$  and  $q_i \in [0, 1]$

#### Characteristics?



**Figure 3.7:** Characterization of the ZIP load model; with (upper) the result of the impedances dependent on the voltage at the connected bus, and (lower) the resulting power consumption of the different models, representative only the real power  $P$

How does it look like in the simulation environment?

$$P = P_n \cdot \left[ p_1 \left( \frac{U}{U_n} \right)^2 + p_2 \left( \frac{U}{U_n} \right) + p_3 \right] \cdot (1 + k_{pf} \Delta f) \quad (3.11)$$

$$Q = Q_n \cdot \left[ q_1 \left( \frac{U}{U_n} \right)^2 + q_2 \left( \frac{U}{U_n} \right) + q_3 \right] \cdot (1 + k_{qf} \Delta f) \quad (3.12)$$

$$\text{with } \sum_{i=1}^3 p_i = 1 \quad \text{and} \quad \sum_{i=1}^3 q_i = 1$$

Although the subset of Equation 3.11 and Equation 3.12 would consider a relation to the frequency at the given time, it is not used. The use of periodized phasor representation, typical for a RMS simulation, is missing this information in the framework diffpssi. Therefore the set of Equation 3.9 and Equation 3.10 is implemented in the Python framework. It has to be mentioned, because the comparison tool PowerFactory is using this load model, though in some parts of the simulation results are showing a slightly bigger error.

### 3.2.2 Induction Machine Models

As one of the most important loads to consider, especially for many load driven instability mechanisms, the Induction Machine (IM) is a crucial component, [Quelle]

Just briefly:

- Why is it crucial?
- How do the instability mechanisms work and look like?
- What are the different types of IMs modeling (complete and dynamic, static, ...)

Three main ways of IM modeling are relevant to mention in this section:

1. Static model as introduced in *IEEE Guide for Load Modeling and Simulations for Power Systems* [10],
2. a dynamic 'fixed-speed' IM model, and
3. a doubly fed IM model.

The last ones are mentioned and further described in Machowski, Lubosny, Bialek, *et al.* [1]. Least model requires very detailed information, and shall be suitable for SMIB models for machine behavior studies or similar. The second model is suitable for network analysis and machine behaviors. The first model applies for high perception of IMs in total loading of the network. As referencing to *IEEE Guide for Load Modeling and Simulations for Power Systems* [10], is is similar implemented as the before mentioned ZIP load model, considering characteristic equations for its real power  $P$  and reactive power  $Q$ . Both models shall be described in the following section.

#### Static Model of Induction Machines

For this operational unit type is a detailed dynamic modeling possible. With some considerations, it can be sufficient, modeling this equipment just with the The model is described by *IEEE Guide for Load Modeling and Simulations for Power Systems* [10] as formulated in following set of equations.

$$P = \left( R_s + \frac{R_r}{s} \right) \cdot \frac{U^2}{\left( R_s + \frac{R_r}{s} \right)^2 + (X_{\gamma s} + X_{\gamma r})^2} \quad (3.13)$$

$$Q = (X_{\gamma s} + X_{\gamma r}) \cdot \frac{U^2}{\left( R_s + \frac{R_r}{s} \right)^2 + (X_{\gamma s} + X_{\gamma r})^2} + \frac{U^2}{X_s} \quad (3.14)$$

Briefly describe the implementation.

[MK2]: Is here really a difference between the two s in the equations?

### Dynamic 'fixed-speed' Induction Machine model'

#### From ChatGPT:

The dynamic model of IMs is essential for accurately representing their behavior under various operating conditions. This model includes the differential equations that describe the machine's electrical and mechanical dynamics. The equations are typically derived from the machine's equivalent circuit and can be expressed in the d-q reference frame.

The dynamic model can be represented by the following set of equations:

$$\frac{d\psi_d}{dt} = v_d - R_s i_d + \omega \psi_q \quad (3.15)$$

$$\frac{d\psi_q}{dt} = v_q - R_s i_q - \omega \psi_d \quad (3.16)$$

$$\frac{d\omega}{dt} = \frac{1}{J}(T_m - T_e - B\omega) \quad (3.17)$$

where:

- $\psi_d, \psi_q$  are the d-q axis flux linkages
- $v_d, v_q$  are the d-q axis voltages
- $i_d, i_q$  are the d-q axis currents
- $R_s$  is the stator resistance
- $\omega$  is the rotor angular velocity
- $T_m$  is the mechanical torque
- $T_e$  is the electromagnetic torque
- $J$  is the moment of inertia
- $B$  is the damping coefficient

The electromagnetic torque  $T_e$  can be calculated as:

$$T_e = \frac{3}{2}p(\psi_d i_q - \psi_q i_d) \quad (3.18)$$

where  $p$  is the number of pole pairs.

This dynamic model allows for the simulation of the IMs transient response to changes in voltage, frequency, and load conditions. It is particularly useful for studying stability and control strategies in power systems.

Briefly describe the implementation.

## 3.3 Application of Voltage Stability

### 3.3.1 Influences of other device characteristics

Just look on other mutual influences in the power system (simulation), such as:

- Load characteristics and types of modeling
- Maximum thermal currents of cables and operating components
- Asynchronous machines (or called „induction motors“?)

### 3.3.2 Observing the current state of the system

#### Static and Dynamic Indices

- Which indices can be implemented?
- Which make sense?
- Implementation and calculation of them?

#### Stability Monitoring

- Index combination and „traffic light“ monitoring
- Restoration options and opportunities
- Local mapping
- Weak point identification





” If you tell the truth, you don’t have to remember anything.

— Mark Twain

## 4 Verification setup and results

### 4.1 Representative Electrical Networks

The following section shall introduce the used power systems in the simulation with the Python framework, considering verification, and also extension meaning the performed case studies in [chapter 5](#). The models are chosen to represent different network sizes and complexities, thus allowing the objective of graded interaction levels of the developed (transformer) model. The models are based on the work of Machowski, Lubosny, Bialek, *et al.* [1], Kundur and Malik [11], *IEEE Guide for Load Modeling and Simulations for Power Systems* [10], and Van Cutsem, Glavic, Rosehart, *et al.* [6].

#### Single Machine Infinite Bus (SMIB) Model

One very popular and thus powerful electrical network for the verification of power system stability is the [SMIB](#) model. It is a compact and simplified model of a power system, allowing easy analytical calculation, verification and development. Mutual influences are comparably simple to understand and calculate, as the infinite bus bus is acting as a fixed grid connection point with a large adjoining grid. The generator is connected to the bus bar via a transmission line and a transformer. The model was largely discussed by Kundur and Malik [11], and is shown in [Figure 4.1](#). The generator and the [IBB](#) are represented by synchronous machines, developed and discussed by Kordowich and Jaeger [12]. The specific model details are included in ??, additionally the simulation setup for verification is described in [Table 4.1](#).

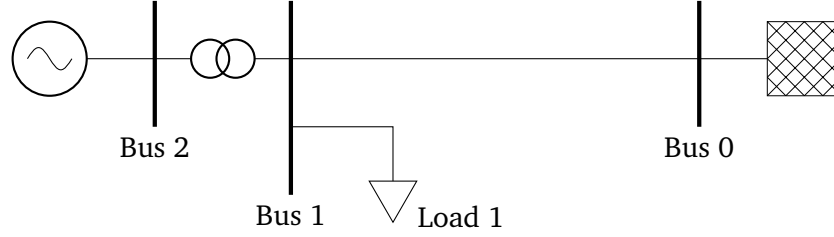


**Figure 4.1:** Single Machine Infinite Bus ([SMIB](#)) model for verification and validation of the Python framework; own figure after [1], [11]

**Table 4.1:** Simulation Setup for validation of the  $\Pi$ -modeled transformer; considering a transforming ratio  $\underline{v} \neq 1$  and  $\underline{v} \in \mathbb{C}$

Parameter	Value
Generator inertia $H$	3.5 s
Generator damping $D$	0.1 p.u.
Generator resistance $R$	0.01 p.u.
Generator reactance $X$	0.1 p.u.
Transformer resistance $R$	0.01 p.u.
Transformer reactance $X$	0.1 p.u.
Transmission line resistance $R$	0.01 p.u.
Transmission line reactance $X$	0.1 p.u.

Further, this model shall be slightly modified according to Figure 4.2. A load is added at the secondary bus of the transformer, the rest of the system is kept. Table 4.1 already contains this modification.

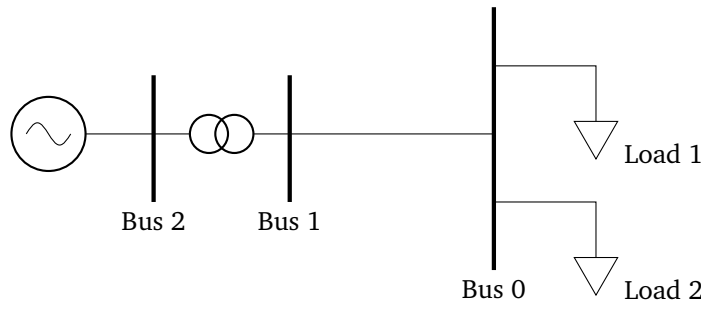


**Figure 4.2:** Modified Single Machine Infinite Bus (SMIB) model with additional load

### Simple Single Machine Load Model

Following model is often recommended [Quelle] for easy voltage control studies, in explicit for OLTCs. Similar to the SMIB model, it consists from one synchronous generator, busses, and lines in a single branch. The IBB is thus removed and changed to a load. This two element type o configuration allows for an easy analytical calculation of voltage stability and control. Although this thesis is focussing on OLTC transformers, the model is extended with one in between. A single line representation is depicted in Figure 4.3.

Further details about its configuration and simulation setup are included in ?? . It should be noted, that simple load models are not useful for simulation of this example network. Usually constant Z models are used as loads, therefore simulation results can be misleading and not showing desired effects or voltage instability mechanisms [Quelle] . The simulation framework is extended with XX types of load models, to



**Figure 4.3:** Single line representation of a simple single machine load model; own illustration with characteristics from **[Quelle]**

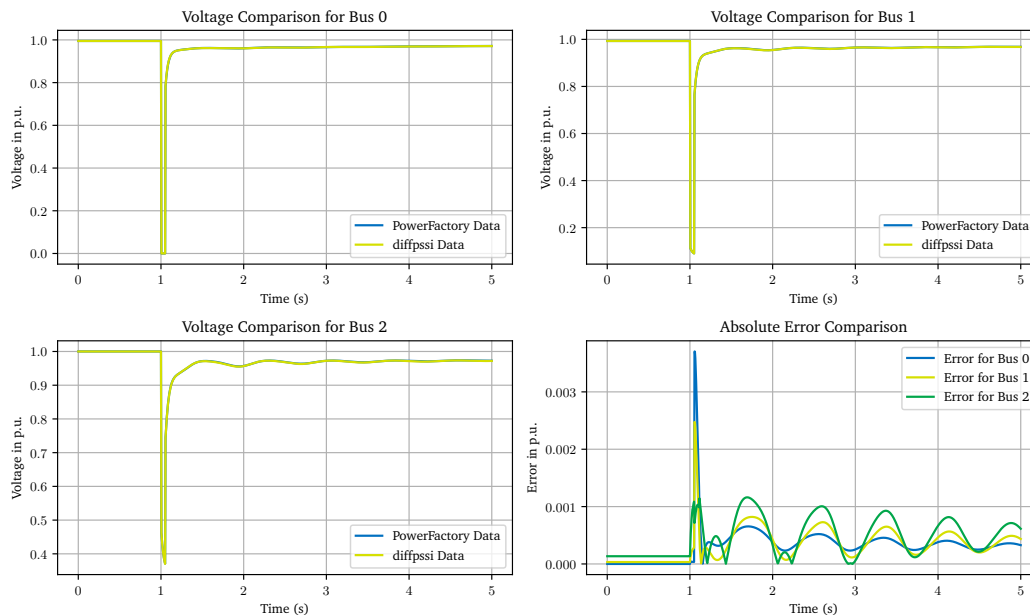
satisfy the requirements of the single machine load model, and a connected stability assessment.

IEEE nine-bus system

Nordic test system

## 4.2 Validation Steps

### 4.2.1 Validation of the $\Pi$ -modeled Transformer with Variable Tap Position



**Figure 4.4:** Comparison of the  $\Pi$ -modeled transformer in the **SMIB** model between PowerFactory and the Python framework

#### 4.2.2 Validation of the ZIP-Load Model

#### 4.2.3 Validation of the OLTC control schemes

Standard Discrete OLTC Control

Fast Switching OLTC Control

#### 4.2.4 Validation of the Induction Machine Model

Place results here, looking at: off nominal tap ratio, and with off nominal phase shifting (e.g.  $110^\circ$ )

### 4.3 Discussion of Model Limitations and Improvements

” *Insanity is doing the same thing, over and over again, but expecting different results.*

— Narcotics Anonymous

## 5 Case study

In the interest of investigation / the Case Study are:

- Influence of switching times on stability margin/begin of destabilization,
- Influence of max. ratio change per switching event, and
- Influence on different test systems (destabilization mechanisms).

### 5.1 Scenario setting

Does it make sense to structure like that? (Scenarios - Simulation - Results)

Or is it a better idea thinking in terms of specific „use cases“ as sections:

- What happens under strong grid conditions? -> Section: Strong grid condition behavior
- What happens under weak grid conditions? -> Section: Weak grid condition behavior
- Strongly interconnected grids
- Widely extended linear string grids
- Section: Use case of Wind farm integration
- Influence on transient stability: SMIB model with and without OLTC

#### **Influence of FSM on Machines and their stability criterions**

Thinking of Rotor Angle Stability, maybe considered by an EAC implementation?  
What does the fast Switching, esp. at up to 8% of the nominal voltage, do to the machines?

#### **Novel Control Strategy FSM**

Thinking of fast and slow voltage gradients: fast gradients are compensated by the FSM, slow gradients are compensated by the OLTC. Therefore optimal utilisation of injected damping moment of the FSM.

Also thinking of different presets of the OLTC and FSM, which are tried to keep constant. Different grid operators can utilize for typical grid conditions of over- or undervoltage at [PCC](#).

Following contains:

- Implementation of different logic
- Testing of presets and switchin logic
- Damping moment beneficial?

#### **Possible Extension for Power Flow congruent Control**

Extension in the Control Algorithm to decide which Bus has to be regulated, to avoid contrary actions of the OLTC and FSM against the power flow. Therefore not decrease of stability, but increase. Possible Application: Grid coupling Transformers, Battery Storage assisted Virtual powerplants, etc.

For this thinking maybe another control strategy is relevant:

- No setpoint from a load flow day-ahead or similar time frame; but rather current load and bus voltages are considered
- Not the deviation of one transformer bus voltage from a setpoint, but the deviation between the two bus voltages is relevant
- Maybe the absolut deviation to the optimal bus voltages at the current load situation is relevant

Big general problem: In which direction does the OLTC / the FSM have to swith? In some cases, the direction is not correct, in some it is correct.

## **5.2 Simulation**

## **5.3 Results**

## 6 Discussion of the results





## 7 Summary and outlook

Some conclusion.

Some outlook and nice blibla.



# Acronyms

<b>FSM</b>	Fast Switching Module
<b>HV</b>	High Voltage
<b>IBB</b>	Infinite Bus Bar
<b>IM</b>	Induction Machine
<b>LV</b>	Low Voltage
<b>OLTC</b>	On-Load Tap Changer
<b>PCC</b>	Point of Common Coupling
<b>PSS</b>	Power System Simulation
<b>RMS</b>	Root Mean Square
<b>SG</b>	Synchronous Generator
<b>SMIB</b>	Single Machine Infinite Bus



# Symbols

$\delta$	$^{\circ} / \text{deg}$	power angle (or power angle difference)
$\Delta\omega$	$\frac{1}{s}$	change of rotor angular speed
$\underline{\theta}$	-	transformer ratio; complex if phase shifting
$A$	-	acceleration or deceleration area
$\underline{E}$	V	voltage of <a href="#">SG</a> or <a href="#">IBB</a>
$H_{\text{gen}}$	s	inertia constant of a Synchronous Generator ( <a href="#">SG</a> )
$\underline{I}$	A	current
$P$	W	effective power; electrical or mechanical
$Q$	var	reactive power
$R$	$\Omega$	ohmic resistance
$\underline{S}$	VA	apparent power
$\underline{V}$	V	voltage
$\underline{X}$	$\Omega$	reactance
$\underline{Y}$	$\frac{1}{\Omega} / S$	admittance
$\underline{Z}$	$\Omega$	impedance

The different symbols are used with different indices, these are semantic and explained in the surrounding context. Following notation is commonly used for mathematical and physical symbols:

- Phasors or complex quantities are underlined (e.g.  $\underline{I}$ )
- Arrows on top mark a spatial vector (e.g.  $\vec{F}$ )
- Boldface denotes matrices or vectors (e.g.  $\mathbf{F}$ )
- Roman typed symbols are units (e.g. s)
- Lower case symbols denote instantaneous values (e.g.  $i$ )
- Upper case symbols denote [RMS](#) or peak values (e.g.  $\underline{I}$ )
- Subscripts relating to physical quantities or numerical variables are written italic (e.g.  $\underline{I}_1$ )

In the simulations and calculations the per unit system (p.u.) is preferred, thus normalizing all values with a base value. Where necessary, absolute units are added to indicate the explicit use of the normal unit system. For more information about this per-unit system please refer to Machowski, Lubosny, Bialek, *et al.* [1], specifically Appendix A.1 provides a detailed description and explanation.



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# Bibliography

- [1] J. Machowski, Z. Lubosny, J. W. Bialek, and J. R. Bumby, *Power System Dynamics: Stability and Control*, Third edition. Hoboken, NJ, USA: John Wiley, 2020, 1 p., ISBN: 978-1-119-52636-0 978-1-119-52638-4.
- [2] I. Burlakin, E. Scheiner, G. Mehlmann, *et al.*, “Enhanced Voltage Control in Offshore Wind Farms with Fast-Tapping on-Load Tap-Changers,” presented at the 23rd Wind & Solar Integration Workshop, Helsinki, Finland, Oct. 11, 2024.
- [3] J. L. Rueda-Torres, U. Annakage, C. Vournas, *et al.*, “Evaluation of Voltage Stability Assessment Methodologies in Modern Power Systems with Increased Penetration of Inverter-Based Resources (TR 126),” 2024. DOI: [10.17023/A3K-AZ76](https://doi.org/10.17023/A3K-AZ76). [Online]. Available: [https://resourcecenter.ieee-pes.org/publications/technical-reports/pes\\_tr\\_126\\_psdp\\_110724](https://resourcecenter.ieee-pes.org/publications/technical-reports/pes_tr_126_psdp_110724) (visited on 12/02/2024).
- [4] M. S. S. Danish, *Voltage Stability in Electric Power System: A Practical Introduction*. Berlin: Logos-Verl, 2015, 219 pp., ISBN: 978-3-8325-3878-1.
- [5] T. Cutsem and C. Vournas, *Voltage Stability of Electric Power Systems*. Boston, MA: Springer US, 1998, ISBN: 978-0-387-75535-9 978-0-387-75536-6. DOI: [10.1007/978-0-387-75536-6](https://doi.org/10.1007/978-0-387-75536-6). [Online]. Available: <http://link.springer.com/10.1007/978-0-387-75536-6> (visited on 10/24/2024).
- [6] T. Van Cutsem, M. Glavic, W. Rosehart, *et al.*, “Test Systems for Voltage Stability Studies,” *IEEE Transactions on Power Systems*, vol. 35, no. 5, pp. 4078–4087, Sep. 2020, ISSN: 0885-8950, 1558-0679. DOI: [10.1109/TPWRS.2020.2976834](https://doi.org/10.1109/TPWRS.2020.2976834). [Online]. Available: <https://ieeexplore.ieee.org/document/9018172/> (visited on 11/08/2024).
- [7] F. Milano, “Hybrid Control Model of Under Load Tap Changers,” *IEEE Transactions on Power Delivery*, vol. 26, no. 4, pp. 2837–2844, Oct. 2011, ISSN: 0885-8977, 1937-4208. DOI: [10.1109/TPWRD.2011.2167521](https://doi.org/10.1109/TPWRD.2011.2167521). [Online]. Available: <http://ieeexplore.ieee.org/document/6029301/> (visited on 09/02/2024).
- [8] F. Milano, *Power System Modelling and Scripting* (Power Systems), 1. ed. Heidelberg: Springer, 2010, 556 pp., ISBN: 978-3-642-13668-9.
- [9] I. Burlakin, E. Scheiner, G. Mehlmann, *et al.*, “Enhancing Variable Shunt Reactors with a Power Electronic Fast-Switching Module,” in *Power Transformers and Reactors (A2)*, Paris: CIGRE, Aug. 2024.
- [10] *IEEE Guide for Load Modeling and Simulations for Power Systems*, 2022. DOI: [10.1109/IEEESTD.2022.9905546](https://doi.org/10.1109/IEEESTD.2022.9905546). [Online]. Available: <https://ieeexplore.ieee.org/document/9905546/> (visited on 11/08/2024).

- 
- [11] P. S. Kundur and O. P. Malik, *Power System Stability and Control*, Second edition. New York Chicago San Francisco Athens London Madrid Mexico City Milan New Delhi Singapore Sydney Toronto: McGraw Hill, 2022, 948 pp., ISBN: 978-1-260-47354-4.
- [12] G. Kordowich and J. Jaeger. “A Physics Informed Machine Learning Method for Power System Model Parameter Optimization.” arXiv: [2309.16579](https://arxiv.org/abs/2309.16579) [cs, eess]. (Sep. 28, 2023), [Online]. Available: <http://arxiv.org/abs/2309.16579> (visited on 09/02/2024), pre-published.

# Appendix

## A Fundamentals [c](#)

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A.2 Jacobian based voltage stability criteria [c](#)

A.3 Comparison of System based and Jacobian based indices [d](#)

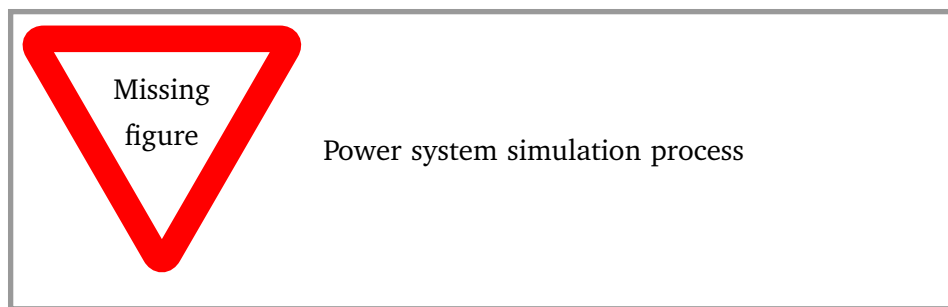




# A Fundamentals

## A.1 Description of the Power System Simulation process

In this appendix section, the general process of power system simulation is described. As this thesis is aiming to understand voltage stability and processes in longer periods of time, these explanations apply to pointer-based simulations, called RMS simulations. Meaning that the considered effects are slower electromechanical nature instead of faster electromagnetic ones. The in this thesis used Python framework „diffpssi“ is based on this type of simulation, and due to its open-source based nature traceable.



**Figure A.1:** Power system simulation process; own illustration

Really basic: (?)

- Phasor description
- Symmetricak Components
- RMS vs EMT simulation (-> meaning one cannot simulate other faults than 3ph w/o ground)

Less basic and more advanced:

- routines in the framework
- two types: Algebraic and Differential equations have to be solved at each time step -> What is which? Which operational equipment is typically described with which type of equation?

## A.2 Jacobian based voltage stability criterions

Danish [4] is showing, describing, and referencing some voltage stability indices based on the Jacobian matrix. The following table is a collection of these indices.

### **A.3 Comparison of System based and Jacobian based indices**

Table A.1: Jacobian based voltage stability criterions; after Danish [4]

Index	Abbreviation	Calculation	Stability Threshold	Reference
Tangent Vector Index	TVI	$\text{TVI}_i = \left  \frac{dV_i}{d\lambda} \right ^{-1}$	depending on load increase	
Test Function		$t_{cc} =  e_c^T \cdot \mathbf{J} \times \mathbf{J}_{cc}^{-1} \cdot e_c $	details are given in reference	
	$i$	$i = \frac{1}{i_0} \cdot \sigma_{\max} \cdot \left( \frac{d\sigma_{\max}}{d\lambda_{\text{total}}} \right)^{-1}$	$i > 0$	
Minimum Eigenvalue		$\Delta V = \sum_i \frac{\xi_i \eta_i}{\lambda_i} \Delta Q$	all eigenvalues should be positive	
Minimum Singular Value		$\begin{bmatrix} \Delta \vartheta \\ \Delta V \end{bmatrix} = \mathbf{V} \sum^{-1} \mathbf{U}^T \begin{bmatrix} \Delta F \\ \Delta G \end{bmatrix}$	details are given in reference	
Predicting Voltage Collapse		$\frac{V}{V_0}$	the smallest index value	
Impedance Ratio		$\frac{Z_i i}{Z_i}$	$\frac{Z_i i}{Z_i} \leq 1$	