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Master Thesis M347

Modeling of Fast-Switching Transformers for Voltage Stability Studies in Python

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> *For knowledge is limited to all we know and understand, while imagination embraces the entire world, and all there will be ever to know and understand.*
>
> — **Albert Einstein**

Preface

Who to thank, which contributions, whatever... Some text to fill a whole line with is some blibla with some explanation making no sense at all but just writing some characters.

- Maximilian Köhler -

Abstract

English version of this Master Thesis abstract.

Kurzfassung

Deutsche Version der Kurzzusammenfassung.

I confirm that I have written this master thesis unaided and without using sources other than those listed and that this thesis has never been submitted to another examination authority and accepted as part of an examination achievement, neither in this form nor in a similar form. All content that was taken from a third party either verbatim or in substance has been acknowledged as such.

Erlangen, 02nd May 2025

Sign

Contents

1 Revitalization of the OLTC

Some blibla as introduction. [1]

1.1 Research Interests

Here are gaps and possible extension of knowledge.

Here are the research objectives and questions.

- Influence of OLTC control on possible operational uses: Short-term voltage stability, long-term voltage stability;
- Can a increased dynamic regulation help machine recovery?
- Does the increased tap ratio gradient harm transient stability of machines? Does it help or harm CCT of machines or machine groups?
- Transformers act as big low-pass filters: Can this behavior be beneficial as well for the interactions of inverters in the grid on AC side (in the sense of Harmonic Stability)? **[Quelle]**

Research Question of this Thesis

How do different control types and characteristics of Tap Changing transformers influence the voltage stability of the given system?

Therefore following questions/steps can be imagined as supportive:

1. How can Voltage stability of a system be classified and be looked at? Which indices, measurements, etc.
2. Which transformer model has to be considered to show influences?
3. Which systems are useful to consider in showing effects? Which circumstances lead to a stability support, which to a decrease? Where can limits be drawn?

Additionally during the process of the thesis, the following question came up as an extension. Is is the second interest of this thesis, and shall be more focused in the later part. Therefore some assessments in the chapter 5 are conducted.

Additional Question of this Thesis

Can the already existing Tap Changer Control of the Fast Switching Module (FSM) be improved towards a more operation oriented control?

This question has following thoughs, concerning the different characteristics and dynamics of the FSM:

1. How do the different time constants of OLTC and FSM influence different stybility aspects in the system?
2. Can the FSM be used as a „damping element“ in the system?
3. Does this possible different behavior of the FSM lead to different operating strategy?
4. What are thoughts on realizing such a strategy with different approaches on the FSM controller?

1.2 Readers Guide

The afore stated research interests in combination with the yet not sufficient framework to use make demands on the structure of this work. Therefore it seems not sufficient trying to apply a completely standard sequence of chapter like „Introduction - Fundamentals - Methods - Results - Discussion“. Instead, the following structure is chosen to fulfill the research interests and to give a clear and understandable overview of the work. This leads to the following structure for the thesis:

- **Chapter 2: Fundamentals,**
  is illustrating and recalling fundamentals for modeling, stability assessments, and discussions;
- **Chapter 3: Methodical Modeling,**
  describes the process of modeling in the tool *diffpssi*, and the implementation of voltage stability indices;
- **Chapter 4: Verification Setup and Result,**
  is showing the verification of the implemented models and tools with the help of common and simple test systems;
- **Chapter 5: Case Study,**
  is looking at the novel control methods from different perspectives and applications;

- **Chapter 6: Discussion,**
  discusses the FSM control strategies, considering the fundamentals, verification, and case study results;
- **Chapter 7: Summary,**
  is summarizing with regards to the research questions, and looking towards research potential and future developments.

When reading this thesis, one might consider its motivation and its prior knowledge for allocating attention to the different chapters. For someone interested in the strategic development of the FSM or OLTCs in general, the introduction with its research interests (section 1.1) are most important. Combined with the Summary and Outlook (chapter 7), the containts of the thesis, answers to the research questions and some perspectives are included. For this level basic knowledge in power system stability and the electrical energy grid is sufficient. When one is also interested in the explanations and thoughts why the answers to the research questions are as they are, the chapter chapter 6 is additionally recommended. Eventaully, the chapter chapter 2 is giving a few basics for an eased understanding of discussion itself. The third level would be a demonstration of practical applications in the case studies (chapter 5). Lastly, if one wants to further improve or develop the tool *diffpssi* or the control strategies in particular, the chapters chapter 3 and chapter 4 are recommended. Additionally the referenced literature and the section A.1 are giving valuable insights and information.

2 Fundamentals

Following chapter shall introduce the basics for implementing an OLTC equipped transformer into a existing PSS framework. This is considering the already existing surrounding, more detailed the electric behavior of the transformer itself and some control engineering theory for the corrosponding OLTC. Thus its main goal is increasing voltage stability [1], main indices and assessment methods are considered as well.

Voltage stability:

- Definitions, classifications, conditions
- PV-curves / QV-curves
- Derivation of Indices: Practical application

Control engineering:

- State-of-the-art control mechanisms for OLTCs
- Chaos theory controllers

2.1 Voltage Stability Basics

A Practical introduction to voltage stability assessment, methods and indices is given in the standard and extending literature of Rueda-Torres, Annakage, Vournas, *et al.* [2], Danish [3], and Cutsem and Vournas [4].

Interesting to note/implement here: Basic classification, definitions, and the nature or conditions of voltage stability. Such as

- Short term vs. long term
- Static vs. dynamic
- Transmission driven vs. load driven vs. generation driven; stability/instability, and/or contributions
- Influence OLTC: Restoring voltage level, but not adding reactive capacities; hence adding risk of voltage collapses
- Load vs. transmission aspects
- Example mechanism: **Collapse effect of the nordic test system** [4], [5]

Table 2.1: Voltage instability types and different time frames with examples; after [Quelle]

No	Type	Cause of incident	Time frames
1	Long-term	Slowly use up of reactive reserves and no outage	Several minutes to several hours
2	Classical	Key outage leads to reactive power shortage	One to five minutes
3	Short-term	Induction motor stalling leads to reactive power shortage	Five to fifteen seconds

2.1.1 Analytical Stability Calculation of Simple Static Power Systems

2.1.2 Stability Limits of Complexer Systems: Continuation Power Flow

2.1.3 Stability Indices for Time Series Calculation

The idea behind stability indices is monitoring the current voltage stability state of the power system in relation to the critical point. This applies not only to static load flow cases, but for time series calculations of short circuits, load shedding or other disturbances. Therefore the dynamic contribution can be included as well. [Quelle]

Reviewing possible indices, either for online resp. real time monitoring or for subsequent analysis after a simulation, is out of the scope of this thesis. Danish [3] has already included an extensive review in his work, with reference to Doig Cardet [6], which is focussing on indices in interest for this thesis in particular. Jacobian Matrix based interesting, because of the possible observability in *diffpssi*.

One easy idea for obtaining a stable operation is looking at the Jacobian Matrix. Although it is mainly used in load flow calculations, it is possible to derive statements about the voltage stability as well. This can be realized through the relation of real power P, reactive power Q, voltage angle δ, and the voltage resp. voltage gradients ΔV. If the

Jacobian Matrix is getting singular, the System will not remain in a stable operation. Singularity of matrices is checked by following two hypothesis tests:

$$\det(\mathbf{J}) = 0 \tag{2.1}$$

$$J \times J^{-1} \uparrow \tag{2.2}$$

The Jacobian Matrix is defined as:

$$\mathbf{J} = \begin{bmatrix} \Delta\mathbf{P} \\ \Delta\mathbf{Q} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{H} & \mathbf{M'} \\ \hline \mathbf{N} & \mathbf{K'} \end{array}\right] \cdot \begin{bmatrix} \Delta\delta \\ \Delta V/V \end{bmatrix}$$

$$\left[\begin{array}{c} \Delta P_1 \\ \vdots \\ \Delta P_n \\ \hline \Delta Q_1 \\ \vdots \\ \Delta Q_n \end{array}\right] = \left[\begin{array}{ccc|ccc} \frac{\partial P_1}{\partial \delta_1} & \cdots & \frac{\partial P_1}{\partial \delta_n} & V_1 \frac{\partial P_1}{\partial V_1} & \cdots & V_n \frac{\partial P_1}{\partial V_n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial \delta_1} & \cdots & \frac{\partial P_n}{\partial \delta_n} & V_1 \frac{\partial P_n}{\partial V_1} & \cdots & V_n \frac{\partial P_n}{\partial V_n} \\ \hline \frac{\partial Q_1}{\partial \delta_1} & \cdots & \frac{\partial Q_1}{\partial \delta_n} & V_1 \frac{\partial Q_1}{\partial V_1} & \cdots & V_n \frac{\partial Q_1}{\partial V_n} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial \delta_1} & \cdots & \frac{\partial Q_n}{\partial \delta_n} & V_1 \frac{\partial Q_n}{\partial V_1} & \cdots & V_n \frac{\partial Q_n}{\partial V_n} \end{array}\right] \cdot \left[\begin{array}{c} \Delta\delta_1 \\ \vdots \\ \Delta\delta_n \\ \hline \Delta V_1/V_1 \\ \vdots \\ \Delta V_n/V_n \end{array}\right] \tag{2.3}$$

Although this method seems easy to implement, there are some numerical problems realted to that. Checking if a Matrix is singular with numerical methods, can only be realised as a probablilty expression. A result could be, that the determinant of the matrix is below a certain threshold. The algorithm would propose, that the matrix is probabilistic singular. **[Quelle]**

This problem leads to the necessity of applying other methods or indices for stability assessment. Further, the relation of the system state to the critical voltage collapse point is highly nonlinear in the Jacobian Matrix. This problem is adressed by the indices in various ways, leading to a more or less linearized relation. **[Quelle]** Danish [3] is proposing a few other indices, that are based on the Jacobian Matrix, and shows comparitive characteristics between Jacobian Matrix and system variable based voltage stability indices. These Jacobian Matrix based indices are listed and further described in section A.2, while the comparative characteristics are described in section A.3. The aforeside mentioned work of Doig Cardet [6] is focussing on these indices in particular.

For this thesis, the indices in Table 2.2 are chosen on the basis of the work of Danish [3] and Doig Cardet [6]. They will be implemented in the Python framework in section 3.2, and used for with the theoretical calculated limits of subsection 2.1.1 and subsection 2.1.2.

Table 2.2: Voltage stability indices for the assessment of the power system stability in this thesis; after **[Quelle]**

Index	Basis	Description
VCPI	Jacobian Matrix	The index is based on the Jacobian Matrix, and is showing the proximity of the current system state to the critical voltage collapse point.
VSI	System Variables	The index is based on the system variables, and is showing the proximity of the current system state to the critical voltage collapse point.

2.2 Voltage Angle Stability of Machines

Just short and briefly Describe

- What is Voltage Angle Stability?
- Why relevant to this thesis?
- How to evaluate: EAC and CCT of a generator
- Maybe something like a max. utilization of max power angle?

2.3 Power System Modeling

The simulation of power systems is a crucial tool, not only for stability studies, but for evaluating extensions or modifications in the planning process, the development of Assistance systems for operational management, and many other applications. **[Quelle]** Due to the complexity of the systems, simulations are often simplified. Not only with model constraints, bus as well in the way of calculations. Mainly seperating between Electromagnetic Transient (EMT) and Root Mean Square (RMS) simulations, the latter is used in this thesis. The section A.1 is giving more details, literature, and summarizing the processes behind the used tool *diffpssi*, if the reader is further interested in the topic.

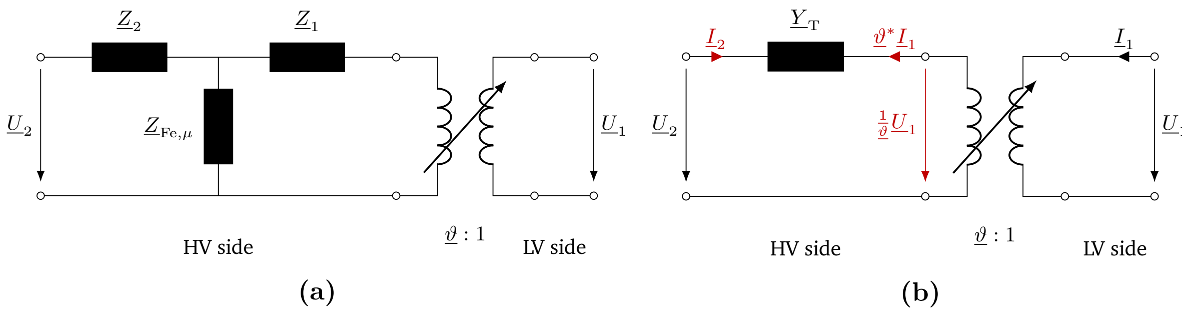

Figure 2.1: Two-Winding Transformer Circuit in the Positive Sequence; a) ideal representation with impedances on the HV side and b) simplifyied circit with only the series impedance related on the HV side; own figure after [1], [7], [8]

2.3.1 Transformer Electric Model and Behavior

Typical for RMS-modeling is the usage of sequence components, especially the positive sequence for symmetrical grid operation and test cases. **[Quelle]** An equivalent circuit for the positive sequence is shown in Figure 2.1 part a), respectively reduced to the transformer ratio, the series impedances of the windings on the LV and HV side, and the shunt branch affected by iron and magnetization losses. [1], [7], [8]

The transformer ratio is typically noted as $\underline{\vartheta}$. Generally speaking it is the ratio between the number of windings of the secondary side to the primary side, as noted in Equation 2.4. With the typically used calculation unit „per unit“[1], the ratio becomes one in the standard case. A transformer ratio, which is only shifting current angles with the shifting angle ϕ, is represented through a complex number using the Euler Identity, as shown in Equation 2.5.

$$\vartheta = \frac{N_2}{N_1} \tag{2.4}$$

$$\underline{\vartheta} = \frac{N_2}{N_1} \cdot \exp\left(j \cdot \phi \cdot \frac{\pi}{180}\right) \tag{2.5}$$

The first simplification is step is considering two assumptions. First, the iron and magnetization losses are neglectable. This can be illustrated with a short-circuit test of the transformer on the secondary side. During this test, one can obtain with the concept of a voltage devider, that

$$\underline{U}_{\mathrm{Fe},\mu} \ll \underline{U}_{\mathrm{T,rated}},$$

[1] means standardization to a reference value; further information on page XIV and Machowski, Lubosny, Bialek, *et al.* [1], Appendix A

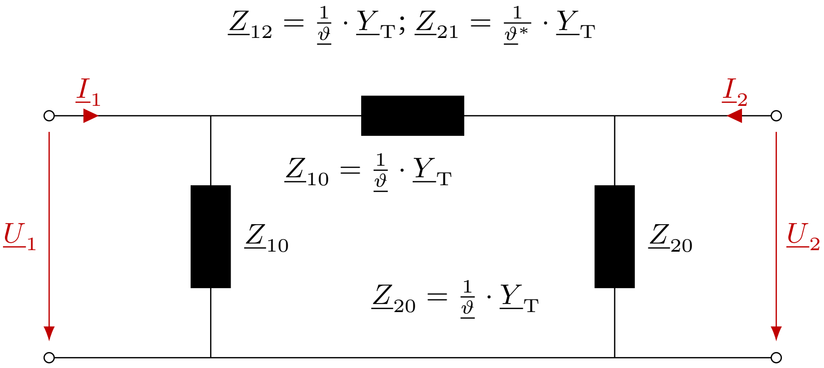

Figure 2.2: Π-representative circuit of an idealized transformer with a tap changer; own figure after [8], [9]

meaning that the shunt branch impedance is much greater that the series impedance of the transformer. Secondly, it is assumed, that the on the primary side related impedance of the secondary side, is equal to the impedance on the primary side. This leads to a symmetrical circuit of the transformer and the positive sequence equivalent circuit simplifies to Figure 2.1 part b). Mathematically this is shortly expressable as Equation 2.6, Equation 2.7, and Equation 2.8. [1], [7], [8]

$$\underline{Z}_1 = R_1 + jX_1; \quad \underline{Z}_2 = R_2\vartheta^2 + jX_2\vartheta^2 \tag{2.6}$$

$$\underline{Z}_1 = \underline{Z}_2 \tag{2.7}$$

$$\underline{Z}_{\mathrm{T}} = \underline{Z}_1 + \underline{Z}_2 \tag{2.8}$$

The afore described simplification leads to only the necessity of considering the series impedance. Considering the afore mentioned normal ratio of $\vartheta = 1$ in the per unit system, the Python framework *diffpssi* has been using this model with only the series impedance and no variable ratio, meaning no shunt branches, before.

When one wants to look at variable transformer ratios, either with representing vector groups, or implementing On-Load Tap Changers (OLTCs), this model of only considering the series impedance has to be extended. Using shunt branches, the variable ratio behavior can be represented in a Π-model, as shown in Figure 2.2. [1], [7], [8]

Looking at the transformer as a black box two-port, with the index one being the LV side, the index two being the HV side, the admittance matrix for the variable ratio behavior can be expressed as in Equation 2.9. The voltages and current are defined as in Figure 2.1 part b). With rearranging the equation, one can obtain the admittance matrix of the Π-model with to the HV side related values as in Equation 2.10. [8], [9]

$$\begin{bmatrix} \underline{\vartheta}^* \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{Y}_\mathrm{T} & -\underline{Y}_\mathrm{T} \\ -\underline{Y}_\mathrm{T} & \underline{Y}_\mathrm{T} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\underline{\vartheta}} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} \tag{2.9}$$

$$\underline{\mathbf{Y}}_{\Pi,\mathrm{T}} = \underline{Y}_\mathrm{T} \cdot \begin{bmatrix} \frac{1}{\underline{\vartheta}\underline{\vartheta}^*} & -\frac{1}{\underline{\vartheta}^*} \\ -\frac{1}{\underline{\vartheta}} & 1 \end{bmatrix} \tag{2.10}$$

For calculation of the individual shunt branches, one can apply the standard representation of two-ports consistent of a linear Π-circuit:

$$\begin{bmatrix} \underline{I}_1 \\ \underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{Y}_{10} + \underline{Y}_{12} & -\underline{Y}_{12} \\ -\underline{Y}_{21} & \underline{Y}_{20} + \underline{Y}_{21} \end{bmatrix} \cdot \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix}$$

When equating this with Equation 2.10, the shunt branches can be calculated respectivly giving the admittances written down as Equation 2.11, Equation 2.12, and Equation 2.13, as they are noted in Figure 2.2 as well. [8], [9]

$$\begin{aligned} \underline{Y}_{12} &= \frac{1}{\underline{\vartheta} \cdot \underline{a}_\mathrm{T}^*} \cdot \underline{Y}_\mathrm{T}, \text{ and} \\ \underline{Y}_{21} &= \frac{1}{\underline{\vartheta} \cdot \underline{a}_\mathrm{T}} \cdot \underline{Y}_\mathrm{T} \end{aligned} \tag{2.11}$$

$$\underline{Y}_{10} = \frac{1}{\underline{\vartheta}} \cdot \left(\frac{1}{\underline{\vartheta}} - \frac{1}{\underline{a}_\mathrm{T}} \right) \cdot \underline{Y}_\mathrm{T} \tag{2.12}$$

$$\underline{Y}_{20} = \left(1 - \frac{1}{\underline{\vartheta} \cdot \underline{a}_\mathrm{T}} \right) \cdot \underline{Y}_\mathrm{T} \tag{2.13}$$

Reactances and resistances are referred to the base voltage and apparent power of the operational unit, such as the transformer. The power system simulation uses its own base voltage and base apparent power, enabling the use of one single calculation domain. This is done to simplify the calculation and to make the results easily comparable to each other. Hence, the reffered values have to be transformed from the equipment based values to the simulation based values. The relation for the transformer admittance is defined as follows. Generally speaking, this thesis is using and reffering to the per unit based values, although it is not denoted in the index of the values.

$$\underline{Y}_{\text{T, based}} = \underline{Y}_{\text{T}} \cdot \frac{S_{\text{n}}}{S_{\text{n, sim}}} \tag{2.14}$$

$$\underline{Z}_{\text{line, based}} = \underline{Z}_{\text{line}} \cdot \frac{S_{\text{n, sim}}}{V^2_{\text{n, sim}}} \tag{2.15}$$

Displayed like in Equation 2.14, the characteristic of the operational unit is referred to the simulation base value. Here, the admittance of the transformer is multiplied with its own rated apparent power, then devided by the apparent power of the simulation system. Similar, the impedance of the lines are calculated via Equation 2.15. This specialities are considered in the tap changer modeling, thus further information is given in [1], Appendix A.

2.3.2 Further Considerations of a Transformer Model

Describe here the asymetric and non-idealized transformer; phase shifting transformers; transformers, which can control logitudinal and transversal ratios; ...

Just briefly desribe the influence of an asymetric vs. an ideal transformer. Why the difference and how the representation could work.

2.3.3 Open-Source Power System Simulation tools

Some information about other open source python power system simulation tools, such as:

- Pandapower,
- TOPS,
- ... .

Maybe relate to this GitHub Repo, comparing different Open-Source Power System Simulation tools. Build up like a scan (see Georg's thesis).

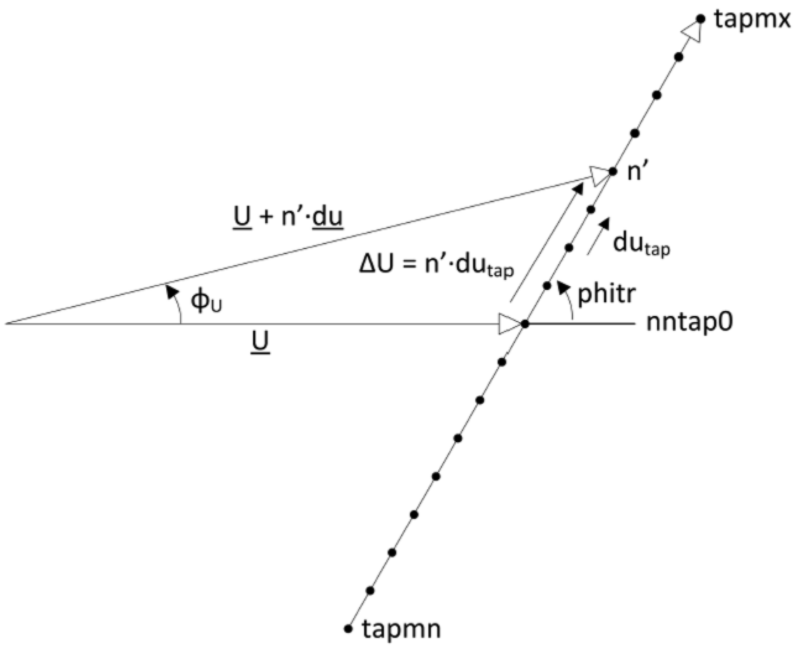

Figure 2.3: Illustration of the tap ratio vector for an ideal and an asymmetric transformer; from the **DIgSILENT Technical Reference Manual** ... [**Quelle**]

2.4 On-Load Tap Changer Controls

2.4.1 Commonly Used On-Load Tap Changer Control

A few basics are in the interest, understanding differences between real world beahavior, or possible ways of building up a OLTC transformer control. This control theory difference can be limiting as well for the results and objectives compared to the actual possible control in the field.

The target voltage is typically set from the control room of the grid operator, coming from pre-calculated load flow analysis. This can be set hours before, or even day-ahead with the estimated loads of the grid. This value is set locally for each operating unit subsequently. The control is then operating locally and without further involvement of the grid operator. [**Quelle**]

Typically the used controller in the field is a discrete controller, which can change tap positions under load within a time frame of around few seconds. Practical tap steps are around 2 % of the overall transforming ratio. The control is set up with a dead band, to avoid unnecessary tap changes. It is necessecary to note here, that this control and its mathematical caracteristics contains logical elements, blocks, and delays, which cannot be translated in a typical control theory transmission function. This leads to the missing possibility to easily obtain mathematical stability for the control of the overall considered power system. **[Quelle]**

2.4.2 Advancement: Fast Switching Module and its Control

Describe the findings of Ilya:

1. Approach and technological (short and not too detailed),
2. Control proposal of Ilya,
3. (Hardware) Development potential.

2.5 Summary in Short and Simple Terms

3 Methodical Modeling

This chapter mehodical modeling focusses on the description of thoughs and structures of the implementation in Python. It is not evolving more than necessary details about the package *diffpssi*, but trying to comprehensible illustrate the structure of the algorithms theirselves and the necessaary bordering interfaces.

3.1 Transformer Equipment Modeling

This section resp. focussed on the dynamics and model behavior of the transformer itself. It is split according to the structure of the implementation itself, into the modeling of the Π-model and the tap changer control. For the last mentioned, there are different control schemes implemented and thus described in the subsequent section.

3.1.1 Implementing a Π-Representative Circuit with Variable Ratio

Before detailing in the software side of the implemetation, some mathematical differences are explained. This results on the one hand from the major differences in the standard literature, especially between Machowski, Lubosny, Bialek, *et al.* [1] and Kundur and Malik [7], resp. Milano [8]. On the other hand, these differences occur as well in the comparative and validation simulation software *DIgSILENT PowerFactory*. The use of these different models is described in its technical reference manual **[Quelle]** .

Mathematical Description and Definitions

Firstly it is important to comment on the use of indices in this thesis, and especially following for this chapter. The index 1 is always referring to the LV side, the index 2 to the HV side. The impedances can be concentrated and related to either the LV, or as usual to the HV side of the transformer. The in subsection 2.3.1 used derivation is using a relation on the HV side. The same accounts for the definition of the OLTC ratio $\underline{\vartheta}$. The OLTC ratio $\underline{\vartheta}$ in this thesis is always placed on the HV side.

This thesis focusses on an ideal tap changer model at first, other possible considerations from subsection 2.3.2 are neglected. As vector groups are as well not considered, the tap ratio stays solely a rational number. Like previously mentioned, and consequently described, the ratio ϑ is then pülaced on the HV side of the transformer. The OLTC ratio ϑ is then defined as:

$$\vartheta_{\mathrm{HV}} = 1 + k \cdot \Delta v \tag{3.1}$$

$$\text{with } k \in [k_{\min}; k_{\max}]; k_{\min} \equiv -k_{\max} \tag{3.2}$$

Within this definition, $k_{\min}$ defines the minimum tap position, $k_{\max}$ the maximum OLTC position. The variable Δv defines the change of the ratio in percent for alterning one position.

Mathematical Different Representations

When one either wants to relate the transformer admittance, or the tap ratio to the LV side, a different admittance matrix definition has to be used. The admittance matrix is then defined as:

$$\underline{\mathbf{Y}}_{\Pi,\mathrm{T}} = \begin{bmatrix} \underline{Y}_{\mathrm{T}} & -\underline{\vartheta}\underline{Y}_{\mathrm{T}} \\ \underline{\vartheta}^{*}\underline{Y}_{\mathrm{T}} & -\underline{\vartheta}^{*}\underline{\vartheta}\underline{Y}_{\mathrm{T}} \end{bmatrix} \tag{3.3}$$

The following mathematical result leads to a necessary change in the software implementation. Either

- the admittance matrix bus indices have to be changed,
- the tap ratio has to be reciprocal according to Equation 3.4, or

- using the HV side admittance matrix, but changing the tap ratio definition and the bus indices.

These different ways of variable and placing definitions also characterize the ways, the admittance matrix of the OLTC transformer is derived from either Machowski, Lubosny, Bialek, *et al.* [1], versus Kundur and Malik [7], Milano [8], or Burlakin, Scheiner, Mehlmann, *et al.* [9]. Another thought or way of representing a transformer with off-nominal ratio is described in the appended section B.4.

$$\vartheta_{\mathrm{LV}} = \frac{1}{1 + k \cdot \Delta v} \tag{3.4}$$

Thoughts, Design, and Implemetation of Algorithmics

Base idea here:

- Show the thought process, design sketches and the implemetation algorithmics in the Python framework.
- Add a class diagram of the transformer model, with all needed interface / logable data, interface methods, and the abstract design.
- Describe addtional methods.
- Show the algorithmic implemetation logic of the Pi model, but not the Tap Changer, as this is seperated.

Additionally interesting extensions:

- How to automatically determine switching direction?
  -> switchin direction dependent on what? (load-flow direction?)
- Controller set points: also dependent on load flow?
- How can I change the transformer control setpoints to be load flow dependent?
- How can I ensure, utilization of the transformer is not $> S_{\mathrm{n}}$?

3.1.2 Tap Changer Control Modeling

This is the description of the ideas, development, and implementation of a OLTC control scheme.

Discrete Control Loop

- Describe implementation
- Describe benefits / drawbacks
- Control scheme
- Switching logic and behavior (voltage tracking)

This control method represents the currently most used and thus representative control scheme for OLTCs. With the mechanic nature of the switching mechanism, the control look can only access discrete ratios within time frames of around a few seconds. Such a discrete control loop is described by Milano [8], [10]. A scheme of this control loop is shown in Figure 3.1.

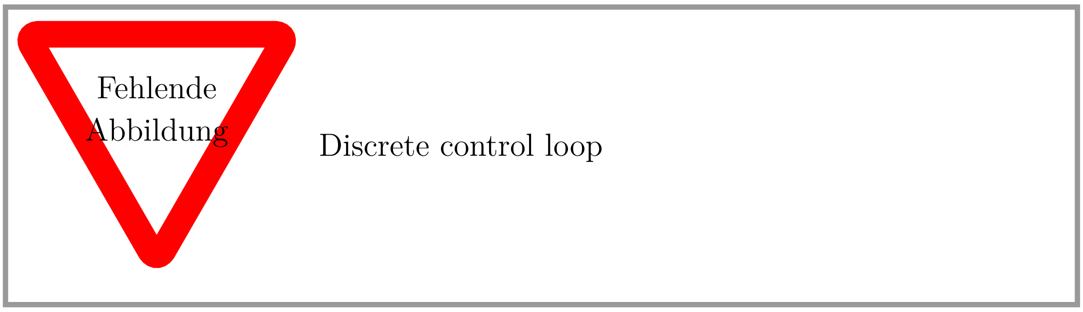

Figure 3.1: Discrete control loop of an OLTC; scheme based on Milano [10]

This control loop type is beneficial due to its accurate representability of current OLTC abilities. It gains access to assess stability within simulation environments, as analytical methods are not suited.

A negative aspect of a discrete control loop is the missing opportunity of generating a transfer function. This blocks the stability assessment with standard control engineering methods. Further, popular analysis methods like eigenvalue analysis is not possible, due to the lack of possibility to form derivatives.

The structure of the implementation is illustrated in the block diagram of Figure 3.2. The controller is actively chainging the algebraic funtions of the simulation environment, therefore it is quasi dynamic. The controller output logic is called, when updating the admittance matrix of the transformer. Additionally, the differential functions of the connected simple controllers, like integrators, PT1-blocks, etc., are called by the solver and are thus part of the differential equations. The logic determines the physical interpretation of the OLTC, and therefore

1. If the OLTC has to switch,

2. When the switching operation is finished, and
3. What the current, or in case after a switching the new, tap ratio is.

It is important to note, that this structure relies on the calculation of the dynamic admittance matrix on each time step.

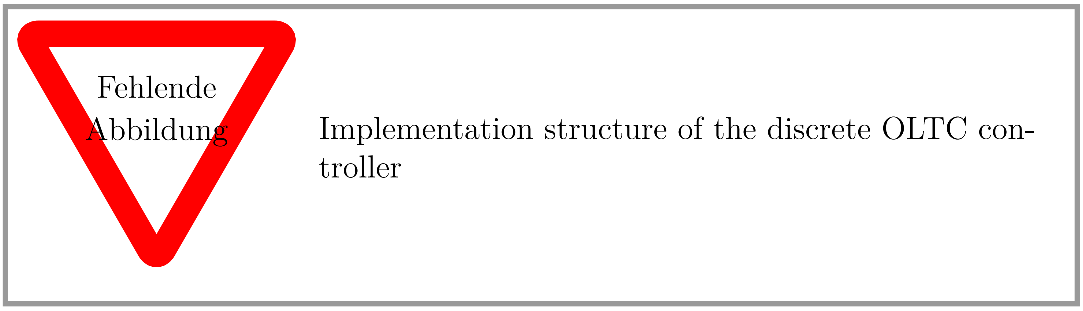

Figure 3.2: Implementation structure of the discrete OLTC controller

The output of the controller is based on the following logic.

Continous Control Loop

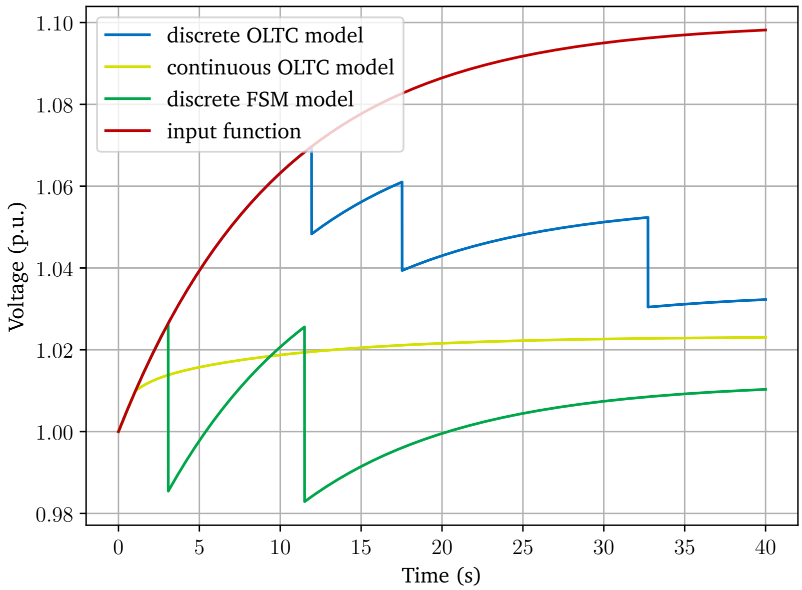

Figure 3.3: Characterization of the OLTC control loop; the input function simulates the to be regulated voltage, the output functions are characterized by $o(t) = i(t) \cdot \underline{\vartheta}_{\text{trafo}}$

Control Schemes for the Fast Switching module

- Describe implementation

- Describe benefits / drawbacks
- Control scheme
- Switching logic and behavior (voltage tracking)

Describe the operational logic and structure of the Fast Switching Module (FSM) first.

A control logic for a so called FSM has been presented from Burlakin, Scheiner, Mehlmann, *et al.* [9], and illustrated in Figure 3.4.

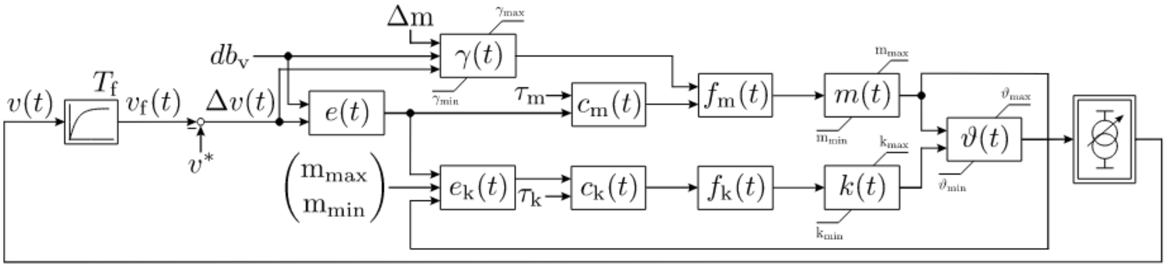

Figure 3.4: Control loop of a FSM; scheme based on Burlakin, Scheiner, Mehlmann, *et al.* [9]

However, the implementation logic in Python is slightly differing from the presented scheme in [9], simply for not overcomplication of the code and therefeore debugging. The implementation is similar to the afore discussed one of a standard OLTC controller.

3.1.3 Experimental: Extended Ideas and Improvements

Operational Oriented FSM Control

Alternative Tap Skipping Logic

Varying the Voltage Setpoint and Target Calculation

Here, another idea of control target creation shall be mentioned. Instead of a fixed bus voltage reference, the difference of both bus voltages is considered. Further, the sign of that difference is used to determine the direction of the tap change.

3.2 Application of Voltage Stability

3.2.1 Generation of Nose Curves

This section describes the implementation of a prevoiusly discussed static voltage analysis tool. The generation of nose curves helps in finding the critical loading of the system at the bus of interest, although it is static nature.

Basic Simplification Idea

Ajjarapu and Christy [11] and Ajjarapu [12] are presenting a method for numerical calculation of nose curves in their work. It is called *Continuation Power Flow* and is based on a modified Newton-Raphson method. The differences rely in a slightly different definition of the power flow equations, considering a load factor λ. Combined with a predictor-corrector iterative solver method, this algorithm is capable of nose curve calulation, and finding the critical loading of the system. While in the first work [11], only the upper part of the curve including the critical point is calculated, the second work [12] is capable of calculating the complete curve with both solutions. As the trade off between implementation effort and the benefits, this method is not exchanging the reduced and simplyfied one.

While this method would be appealing to implement, an additional load flow algorithm, solver, and wrapper seem not profitable for this thesis. An idea was occuring, just iteratively using the available implemented standard Newton-Raphson algorithm, and implementing a wrapper around it. The proposed result should be the upper and stable nose curve branch, with the critical point of active power loading. This shall seem sufficient, as the lower branch solutions are not stable load flow solutions.

The often used parameterization of a function of voltage dependent on the active power and the power angle ϕ should be implemented. In mathematical term, this is expressed as Equation 3.5.

$$|\underline{V}| : P \mapsto f(P, \phi) \tag{3.5}$$

$$Q : \underline{V} \mapsto f(\underline{V}, \phi) \tag{3.6}$$

Under consideration of a complex representation of voltage and powers, this algorithm can calculate $V - Q$ curves as well. Mathematically this is expressable as Equation 3.6.

Implementation Details

The implementation of the nose curve generation is realized as a class in the package *diffpssi.stability_lib.voltage*. Its class diagram with all attributes and methods is shown in Figure 3.5, an extended version is included in section B.2. For an easy and generic use of the *diffpssi* package, *PowerSystemSimulation* objects are used, as well as the function *do_load_flow()* from the package.

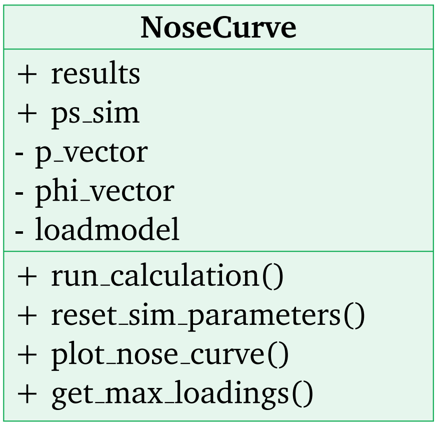

Figure 3.5: Class diagram of the NoseCurve class in the package diffpssi

As the afore mentioned idea, the method for running the calculation is a iterative wrapper of the load flow calculation. This can be as well applied for mutiple busses as a list input. At first, the grid and therefore models of the *PowerSystemSimulation* object has to be cleared with the method *reset_sim_parameters()*. Then the active power vector is iterated as load input, together with the power angle ϕ for the reactive power in the model. Important to note here, is the usage of an ***kwargs* argument. The callable for the model is called with load parameters for each load bus as the Bus name, and a list with active and reactive power. The initials of this grid callable are used as the standard values, so only one bus can be varied at a time. The result is saved as a *pandas DataFrame* in a dict, with the keys being the bus names.

The method *plot_nose_curve()* is used to plot the results, and is using the *matplotlib* package. Further, the method *get_max_loadings()* can provide details about the critical point. Giving back a dict with keys as bus names, the values itself are dicts with key of the power angle parameter $\tan \phi$ and the values as *pandas DataFrame*. The contained details are maximum active power $P_{\max}$, the reactive power Q at this point, and the voltage magnitude $|\underline{V}|$ at the bus.

Results of the Nose Curve Generation

The following figure Figure 3.6 shows the generated nose curve for a simple grid as illustrated in Figure 4.3. The grid is characterized at Bus 1, with a varying power angle as parameter $\tan\phi$. The power angle $\tan\phi$ is used to vary the power factor of the load, thus representing different load characteristics, as

$$\tan\phi = \frac{Q}{P}.$$

Displayed are a few combinations with different load characteristics, leading to a different possible maximum acitve power transfer. Figure 3.7 shows the comparison between the analytical calculation and the implemented solution. The analytical calculation is carryied out with the method described in subsection 2.1.1. For this specific example, the complete calculation, including the set of used parameters, is shown in section B.3. What seems conspicious is the missing lower part of the curve, meaning the second possible solution when solving the power flow equations. Although this seems like a major drawback, the resulting curve contains all the necessary parts, where a stable solution can occur. [Quelle] The solution is reaching exactly until the critical point of power transfer.

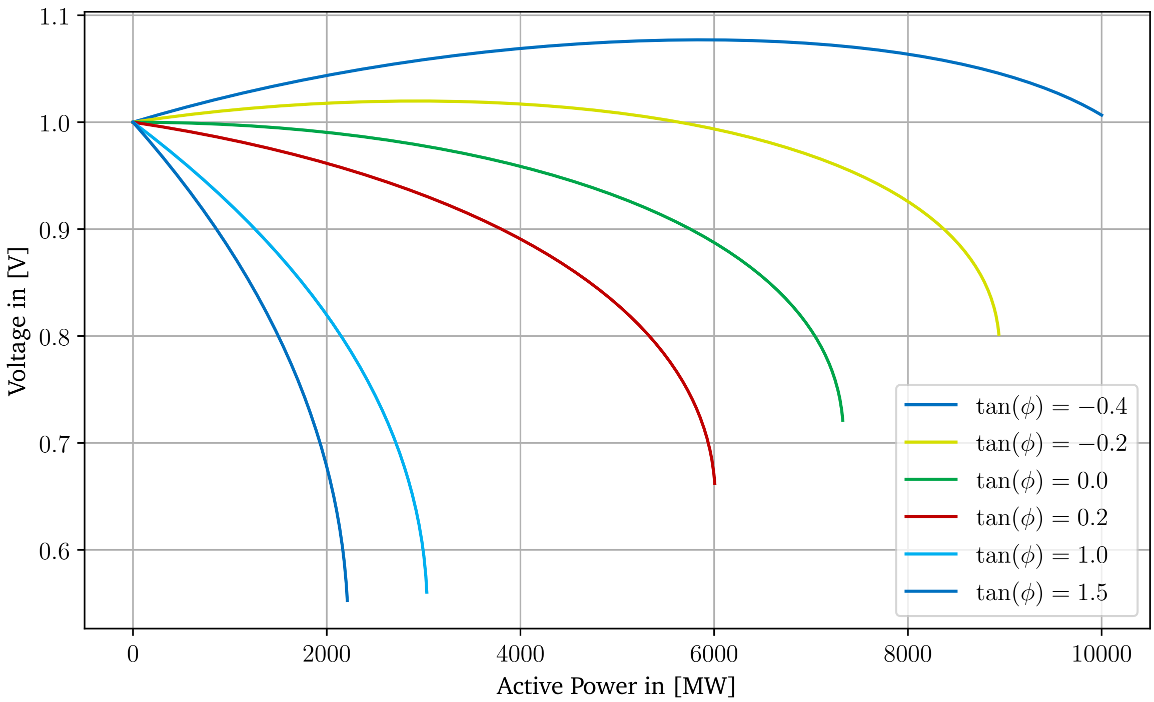

Figure 3.6: Examplary generated nose curve for a simple generator - load grid for various power angle level parameters $\tan\phi$; Applied on the grid of Figure 4.3 with a characterization at Bus 1

3.3 Summary in Short and Simple Terms

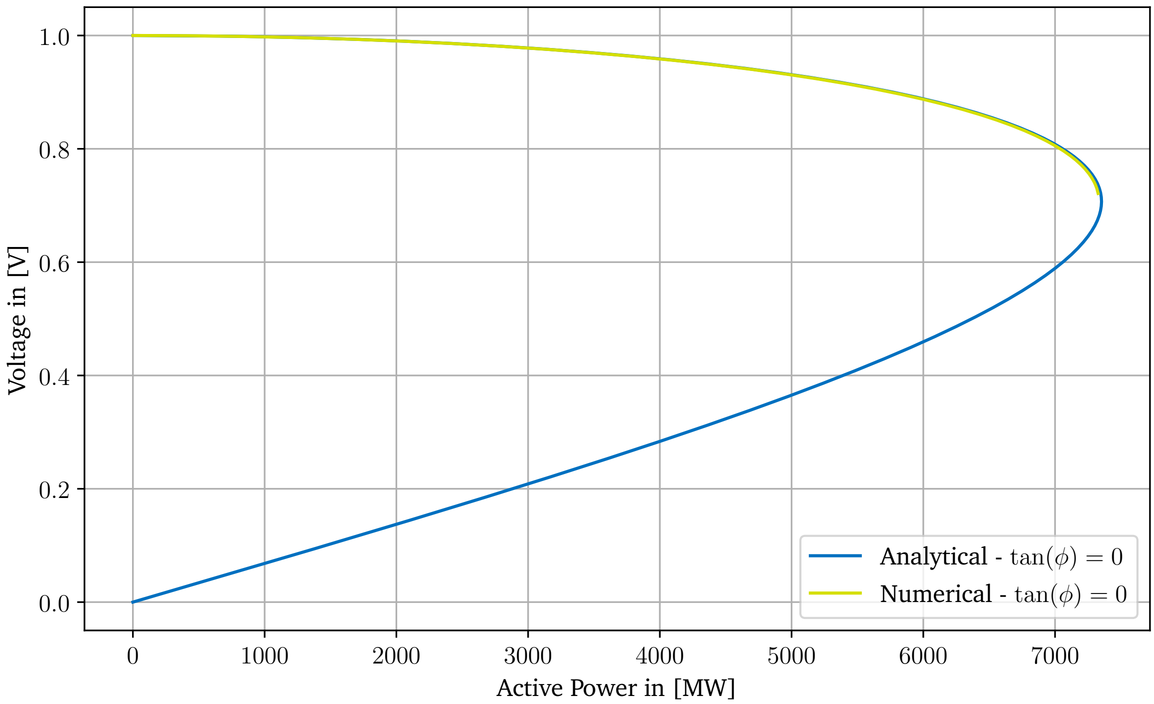

Figure 3.7: Comparison between the analytical calculation and the implemented solution

4 Verification Setup and Results

4.1 Representative Electrical Networks

The following section shall introduce the used power systems in the simulation with the Python framework, considering verification, and also extension meaning the performed case studies in chapter 5. The models are chosen to represent different network sizes and complexities, thus allowing the objective of graded interaction levels of the developed (transformer) model. The models are based on the work of Machowski, Lubosny, Bialek, *et al.* [1], Kundur and Malik [7], *IEEE Guide for Load Modeling and Simulations for Power Systems* [13], and Van Cutsem, Glavic, Rosehart, *et al.* [5].

Single Machine Infinite Bus (SMIB) Model

One very popular and thus powerful electrical network for the verification of power system stability is the SMIB model. It is a compact and simplified model of a power system, allowing easy analytical calculation, verification and development. Mutual influences are comparably simple to understand and calculate, as the infinite bus bus is acting as a fixed grid connection point with a large adjoining grid. The generator is connected to the bus bar via a transmission line and a transformer. The model was largely discussed by Kundur and Malik [7], and is shown in Figure 4.1. The generator and the IBB are represented by synchronous machines, developed and discussed by Kordowich and Jaeger [14]. The specific model details are included in **??**, additionally the simulation setup for verification is described in Table 4.1.

Figure 4.1: Single Machine Infinite Bus (SMIB) model for verification and validation of the Python framework; own figure after [1], [7]

Table 4.1: Simulation Setup for validation of the Π-modeled transformer; considering a transforming ratio $\underline{\vartheta} \neq 1$ and $\underline{\vartheta} \in \mathbb{C}$

Parameter	Value
Generator inertia H	3.5 s
Generator damping D	0.1 p.u.
Generator resistance R	0.01 p.u.
Generator reactance X	0.1 p.u.
Transformer resistance R	0.01 p.u.
Transformer reactance X	0.1 p.u.
Transmission line resistance R	0.01 p.u.
Transmission line reactance X	0.1 p.u.

Further, this model shall be slightly modified according to Figure 4.2. A load is added at the secondary bus of the transformer, the rest of the system is kept. Table 4.1 already contains this modification.

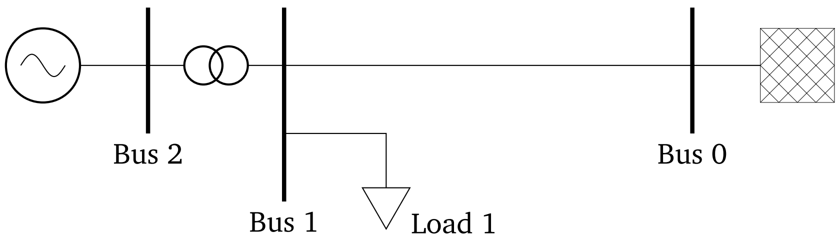

Figure 4.2: Modified Single Machine Infinite Bus (SMIB) model with additional load

Simple Single Machine Load Model

Following model is often recommended [Quelle] for easy voltage control studies, in explicit for OLTCs. Similar to the SMIB model, it consists from one synchronous generator, busses, and lines in a single branch. The IBB is thus removed and changed to a load. This two element type o configuration allows for an easy analytical calculation of voltage stability and control. Although this thesis is focussing on OLTC transformers, the model is extended with one in between. A single line representation is depicted in Figure 4.3.

Further details about its configuration and simulation setup are included in **??**. It should be noted, that simple load models are not useful for simulation of this example network.

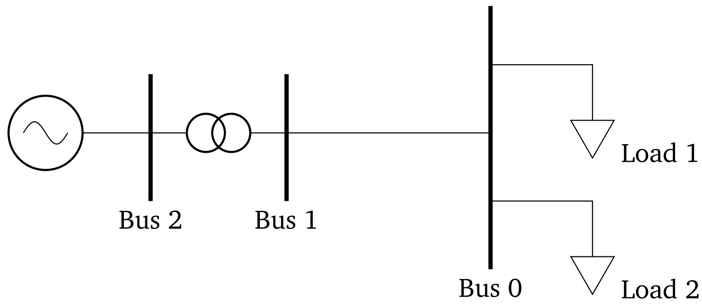

Figure 4.3: Single line representation of a simple single machine load model; own illustration with characterstics from [Quelle]

Usually constant Z models are used as loads, therefore simulation results can be misleading and not showing desired effects or voltage instability mechanisms [Quelle] . The simulation framework is extended with XX types of load models, to satisfy the requirements of the single machine load model, and a connected stability assessment.

IEEE Nine-Bus System

Nordic Test System

4.2 Validation Steps

4.2.1 Validation of the Π-Modeled Transformer with Variable Tap Position

4.2.2 Validation of the OLTC Control Schemes

Standard Discrete OLTC Control

Fast Switching OLTC Control

4.2.3 Voltage Stability Rating Plausibility

Place results here, looking at: off nominal tap ratio, and with off nominal phase shifting (e.g. 110°)

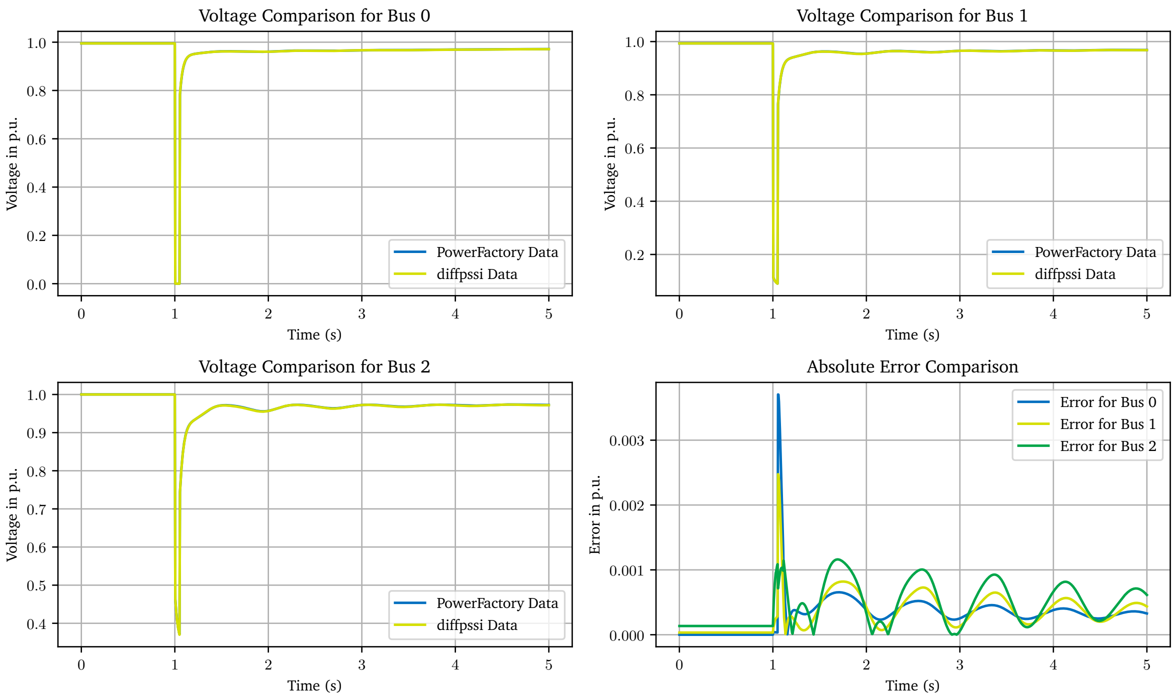

Figure 4.4: Comparison of the Π-modeled transformer in the SMIB model between PowerFactory and the Python framework

4.3 Discussion of Model Limitations and Improvements

4.4 Summary in Short and Simple Terms

5 Case Study

In the interest of investigation / the Case Study are:

- Influence of switching times on stability margin/begin of destabilization,
- Influence of max. ratio change per switching event, and
- Influence on different test systems (destabilization mechanisms).

Does it make sense to structure like that? (Scenarios - Simulation - Results)

Or is it a better idea thinking in terms of specific „use cases“ as sections:

- What happens under strong grid conditions? -> Section: Strong grid condition behavior
- What happens under weak grid conditions? -> Section: Weak grid condition behavior
- Strongly interconnected grids
- Widely extended linear string grids

5.1 Influence of FSM on Voltage Angle Stability of Machines

Thinking of Rotor Angle Stability, maybe considered by an EAC implementation?

What does the fast Switching, esp. at up to 8% of the nominal voltage, do to the machines?

Comparison between standard Transformer / OLTC and FSM control.

5.2 Utilization of FSM with a Power Swing Damping Effect

Thinking of the damping effect of the FSM, especially in the case of power swings, caused by shortages and following swinging of Generators.

5.3 Novel Control Strategy FSM

Thinking of fast and slow voltage gradients: fast gradients are compensated by the FSM, slow gradients are compensated by the OLTC. Therefore optimal utilisation of injected damping moment of the FSM.

Also thinking of different presets of the OLTC and FSM, which are tried to keep constant. Different grid operators can utilize for typical grid conditions of over- or undervoltage at PCC.

Following contains:

- Implemetation of different logic
- Testing of presets and switchin logic
- Damping moment beneficial?

5.3.1 Scenario Setting

5.3.2 Simulation

5.3.3 Results

5.4 Possible Extension for Power Flow Congruent Control

Extension in the Control Algorithm to decide which Bus has to be regulated, to avoid contrairy actions of the OLTC and FSM against the power flow. Therefore not decrease of stability, but increase. Possible Application: Grid coupling Transformers, Battery Storage assisted Virtual powerplants, etc.

For this thinking maybe another control strategy is relevant:

- No setpoint from a load flow day-ahead or similar time frame; but rather current load and bus voltages are considered
- Not the deviation of one transformer bus voltage from a setpoint, but the deviation between the two bus voltages is relevant

- Maybe the absolut deviation to the optimal bus voltages at the current load situation is relevant

Big general problem: In which direction does the OLTC / the FSM have to swith? In some cases, the direction is not correct, in some it is correct.

5.5 Summary in Short and Simple Terms

6 Discussion of the Results

6.1 Transformer Model and Validation

6.2 Voltage Stability Rating and Assessment

6.3 Case Studies and Practical Applications

6.4 Summary in Short and Simple Terms

7 Summary and Outlook

Some conclusion.

1. How do different control types and characteristics of Tap Changing transformers influence the voltage stability of the given system?

Some summary about Research Question 1. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

2. Can the already existing Tap Changer Control of the Fast Switching Module (FSM) be improved towards a more operation oriented control?

Some summary about Research Question 2. Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

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Acronyms

EMT	Electromagnetic Transient
FSM	Fast Switching Module
HV	High Voltage
IBB	Infinite Bus Bar
LV	Low Voltage
OLTC	On-Load Tap Changer
PCC	Point of Common Coupling
PSS	Power System Simulation
RMS	Root Mean Square
SG	Synchronous Generator
SMIB	Single Machine Infinite Bus

Symbols

δ	° / deg	power angle (or power angle difference)
$\Delta\omega$	$\frac{1}{\mathrm{s}}$	change of rotor angular speed
$\underline{\vartheta}$	-	transformer ratio; complex if phase shifting
A	-	acceleration or deceleration area
$\underline{E}$	V	voltage of SG or IBB
H_{gen}	s	inertia constant of a Synchronous Generator (SG)
$\underline{I}$	A	current
P	W	effective power; electrical or mechanical
Q	var	reactive power
R	Ω	ohmic resistance
$\underline{S}$	VA	apparent power
$\underline{V}$	V	voltage
$\underline{X}$	Ω	reactance
$\underline{Y}$	$\frac{1}{\Omega}$ / S	admittance
$\underline{Z}$	Ω	impedance

The different symbols are used with different indices, these are semantic and explained in the surrounding context. Following notation is commonly used for mathematical and physical symbols:

- Phasors or complex quantities are underlined (e.g. $\underline{I}$)
- Arrows on top mark a spatial vector (e.g. $\overrightarrow{F}$)
- Boldface denotes matrices or vectors (e.g. $\mathbf{F}$)
- Roman typed symbols are units (e.g. s)
- Lower case symbols denote instantaneous values (e.g. $\underline{i}$)
- Upper case symbols denote RMS or peak values (e.g. $\underline{I}$)
- Subscripts relating to physical quantities or numerical variables are written italic (e.g. $\underline{I}_1$)

In the simulations and calculations the per unit system (p.u.) is preferred, thus normalizing all values with a base value. Where necessary, absolute units are added to indicate the explicit use of the normal unit system. For more information about this per-unit system please refer to Machowski, Lubosny, Bialek, *et al.* [1], specifically Appendix A.1 provides a detailed description and explanation.

List of Figures

List of Tables

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Appendix

A Fundamentals

A.1 Description of the Power System Simulation process

In this appendix section, the general process of power system simulation is described. As this thesis is aiming to understand voltage stability and processes in longer periods of time, these explanations apply to pointer-based simulations, called RMS simulations. Meaning that the considered effects are slower electromechanical nature instead of faster electromagnetic ones. The in this thesis used Python framework „*diffpssi*“ is based on this type of simulation, and due to its open-source based nature traceable.

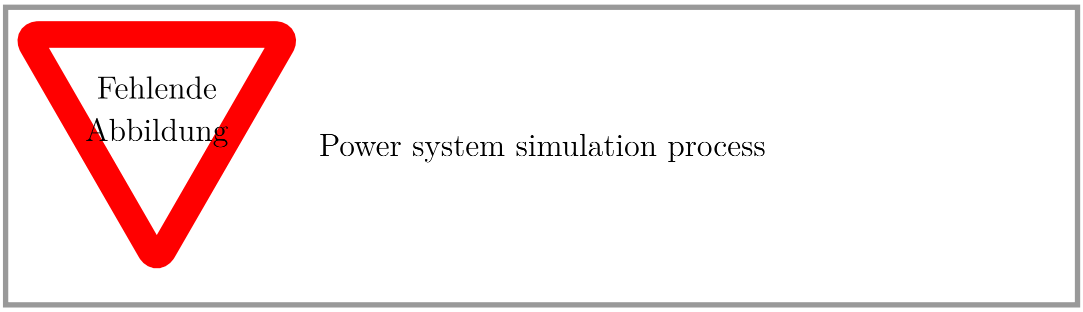

Figure A.1: Power system simulation process; own illustration

Really basic: (?)

- RMS vs EMT simulation (-> meaning one cannot simulate other faults than 3ph w/o ground)
- Phasor description
- Basic formulation: Static (algebraic) and dynamic (differential) equations
- Using of solvers (Integrators) for time domain simulation
- Using of different optimizatinon algorithms for steady state (load flow) simulation -> initial values

Less basic and more advanced:

- rountines in the framework

- two types: Algebraic and Differential equations have to be solved at each time step -> What is which? Which operational equipment is typically described with which type of equation?
- per unit system applying for easier simulation (different voltage levels)
- ...

A.2 Jacobian based voltage stability criterions

Danish [3] is showing, describing, and referencing some voltage stability indices based on the Jacobian matrix. The following table is a collection of these indices.

A.3 Comparison of System based and Jacobian based indices

Table A.1: Jacobian based voltage stability criterions; after Danish [3]

Index	Abbreviation	Calculation	Stability Threshold	Reference
Tangent Vector Index	**TVI!**	$\mathrm{TVI}_i = \left\lvert \frac{\mathrm{d}V_i}{\mathrm{d}\lambda} \right\rvert^{-1}$	depending on load increase	
Test Function		$t_{cc} = \left\lvert e_c^T \cdot \mathbf{J} \times \mathbf{J}_{cc}^{-1} \cdot e_c \right\rvert$	details are given in reference	
Second Order Index	i	$i = \frac{1}{i_0} \cdot \sigma_{\max} \cdot \left(\frac{\mathrm{d}\sigma_{\max}}{\mathrm{d}\lambda_{\mathrm{total}}}\right)^{-1}$	$i > 0$	
Minimum Eigenvalue		$\Delta V = \sum_i \frac{\xi_i \eta_i}{\lambda_i} \Delta Q$	all eigenvalues should be positive	
Minimum Singular Value		$\begin{bmatrix} \Delta\vartheta \\ \Delta V \end{bmatrix} = \mathbf{V} \sum^{-} 1 \mathbf{U}^T \begin{bmatrix} \Delta F \\ \Delta G \end{bmatrix}$	details are given in reference	
Predicting Voltage Collapse		$\frac{V}{V_0}$	the smallest index value	
Impedance Ratio		$\frac{Z_i i}{Z_i}$	$\frac{Z_i i}{Z_i} \leq 1$	

B Modeling

B.1 Admittance Calculation of a Two-Port

Follwing part shall just give a short, but complete and clear overview, how the admittance matrix of a two-port system is calculated. Therefore the main focus of this thesis, a two-port with variable translation ratio, is kept.

B.2 Class Diagram of the Class Nose Curves

NoseCurve	
+ results:	dict[DataFrame]
+ ps_sim:	diffpssi.PowerSystemSimulation
- p_vector:	list
- phi_vector:	list
- loadmodel:	callable
+ run_calculation(bus: list[str]):	dict[pd.DataFrame]
+ reset_sim_parameters():	None
+ plot_nose_curve(busses: list[str], size: tuple = (12, 6), title: bool = True, save_path: str = None):	None
+ get_max_loadings(busses: list[str]):	dict[dict[DataFrame]]

Figure B.1: Complete class diagram of the class Nose Curves; including all attributes and methods with data types, returns, and inputs

B.3 Analytical Calculation of Simple Nose Curves

Some blibla and equations about the analytical calculation of simple nose curves.

B.4 Alternative Current Injection Model

Machowski, Lubosny, Bialek, *et al.* [1] describes another way of modeling a OLTC transformer with variable ratio. This model is looking at the shunt brnaches as current injections, which are added to the individual busses. Beneficial, the system admittance matrix is staying symmetrical, while the different transformer state(s) are represented by the different current injections. This can be mathematically expressed by following set of equations:

$$\begin{bmatrix} \underline{I}_1 \\ -\underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{Y}_\mathrm{T} & -\underline{Y}_\mathrm{T} \\ -\underline{Y}_\mathrm{T} & \underline{Y}_\mathrm{T} \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} - \begin{bmatrix} \Delta\underline{I}_1 \\ \Delta\underline{I}_2 \end{bmatrix}, \text{ where}$$

$$\begin{bmatrix} \Delta\underline{I}_1 \\ \Delta\underline{I}_2 \end{bmatrix} = \begin{bmatrix} \underline{0} & (\underline{\vartheta} - 1)\underline{Y}_\mathrm{T} \\ -(\underline{\vartheta}^* + 1)\underline{Y}_\mathrm{T} & (\underline{\vartheta}^*\underline{\vartheta} + 1)\underline{Y}_\mathrm{T} \end{bmatrix} \begin{bmatrix} \underline{U}_1 \\ \underline{U}_2 \end{bmatrix} \text{ leading to}$$

$$\underline{\mathbf{Y}}_{\Pi,\mathrm{T,Current\ Injection}} = \begin{bmatrix} \underline{Y}_\mathrm{T} & -\underline{Y}_\mathrm{T} \\ -\underline{Y}_\mathrm{T} & \underline{Y}_\mathrm{T} \end{bmatrix} - \begin{bmatrix} \underline{0} & (\underline{\vartheta} - 1)\underline{Y}_\mathrm{T} \\ -(\underline{\vartheta}^* + 1)\underline{Y}_\mathrm{T} & (\underline{\vartheta}^*\underline{\vartheta} + 1)\underline{Y}_\mathrm{T} \end{bmatrix}$$