RESONANT HADRON SYSTEMS FROM:

Symmetries/EFTs

Lattice QCD

Phenomeno logy
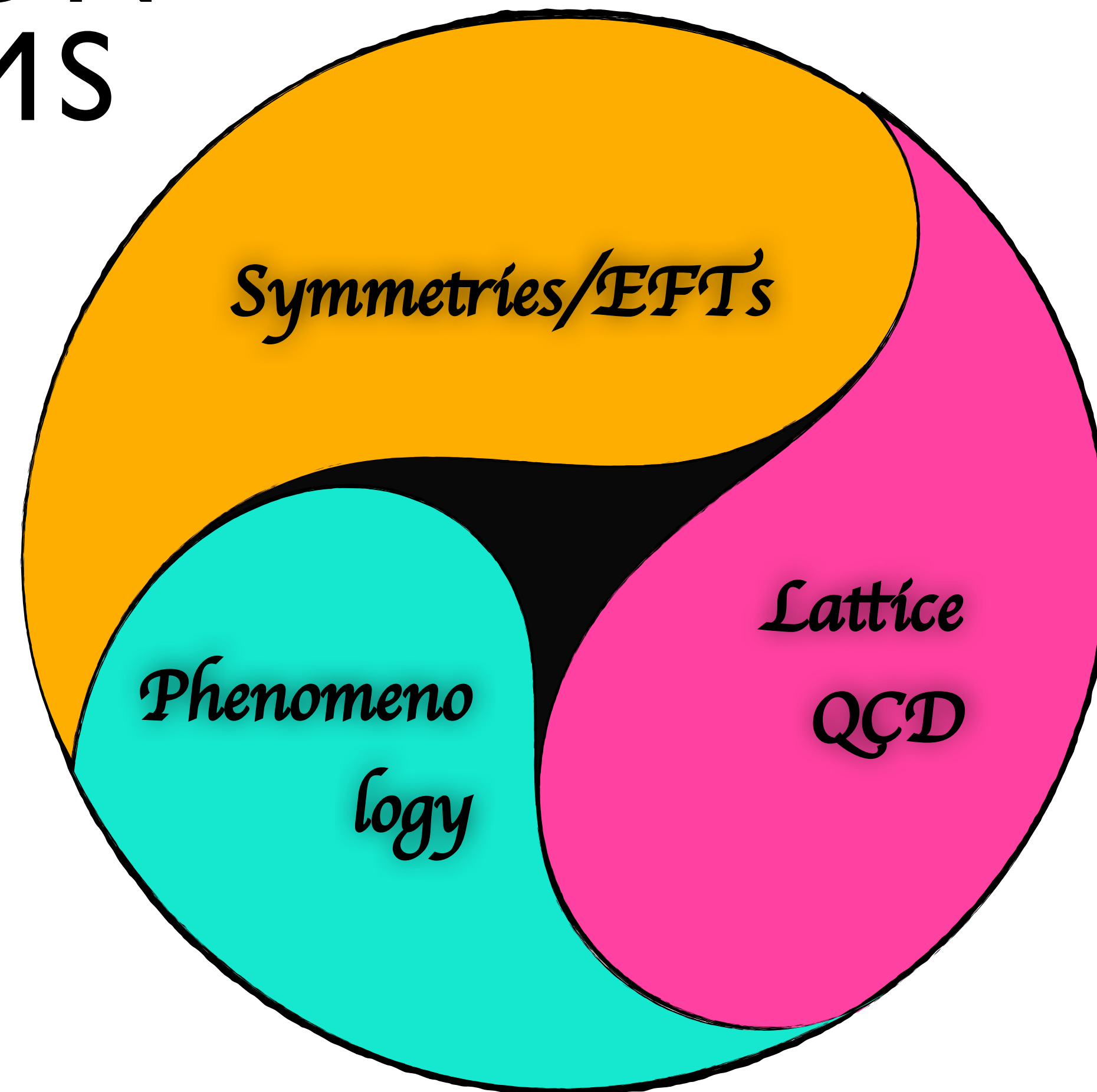
Maxim Mai

University of Bonn

The George Washington University
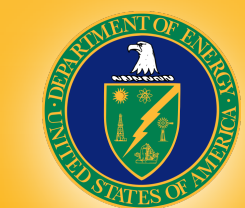
Deutsche Forschungsgemeinschaft

DFG

Effective field theory
Ulf-G Meißner (U. Bonn, FZ Jülich)
Bastian Kubis (U. Bonn)
Evgeny Epelbaum (U. Bochum)
…

Chiral Unitary approaches
Ulf-G Meißner (U. Bonn, FZ Jülich)
Daniel Sadasivan (Ave Maria U.)
LiSheng Geng (Beihang U.)
Aleš Cieply (Rež)
Michael Döring (GWU/JLAB)
…

Data driven tools
Justin Landay (GWU)
Dave Ireland (U. Glasgow)
…

Meson electroproduction
[Jülich-Bonn-Washington
Collaboration]
Ron Workman (JBW, GWU)
Michael Döring (GWU/JLAB)
Deborah Rönchen (FZ Jülich)
Helmut Haberzettl (GWU)

3-body scattering and quantisation condition [FVU]
Michael Döring (GWU/JLAB)
Adam P. Szczepaniak (U. Indiana/JLAB)
Alessandro Pilloni (U. Messina)

Finite-volume effects
Akaki Rusetsky (U. Bonn)
Ulf-G. Meißner (U. Bonn)
Michael Döring (GWU/JLAB)
Daniel Severt (U. Bonn)
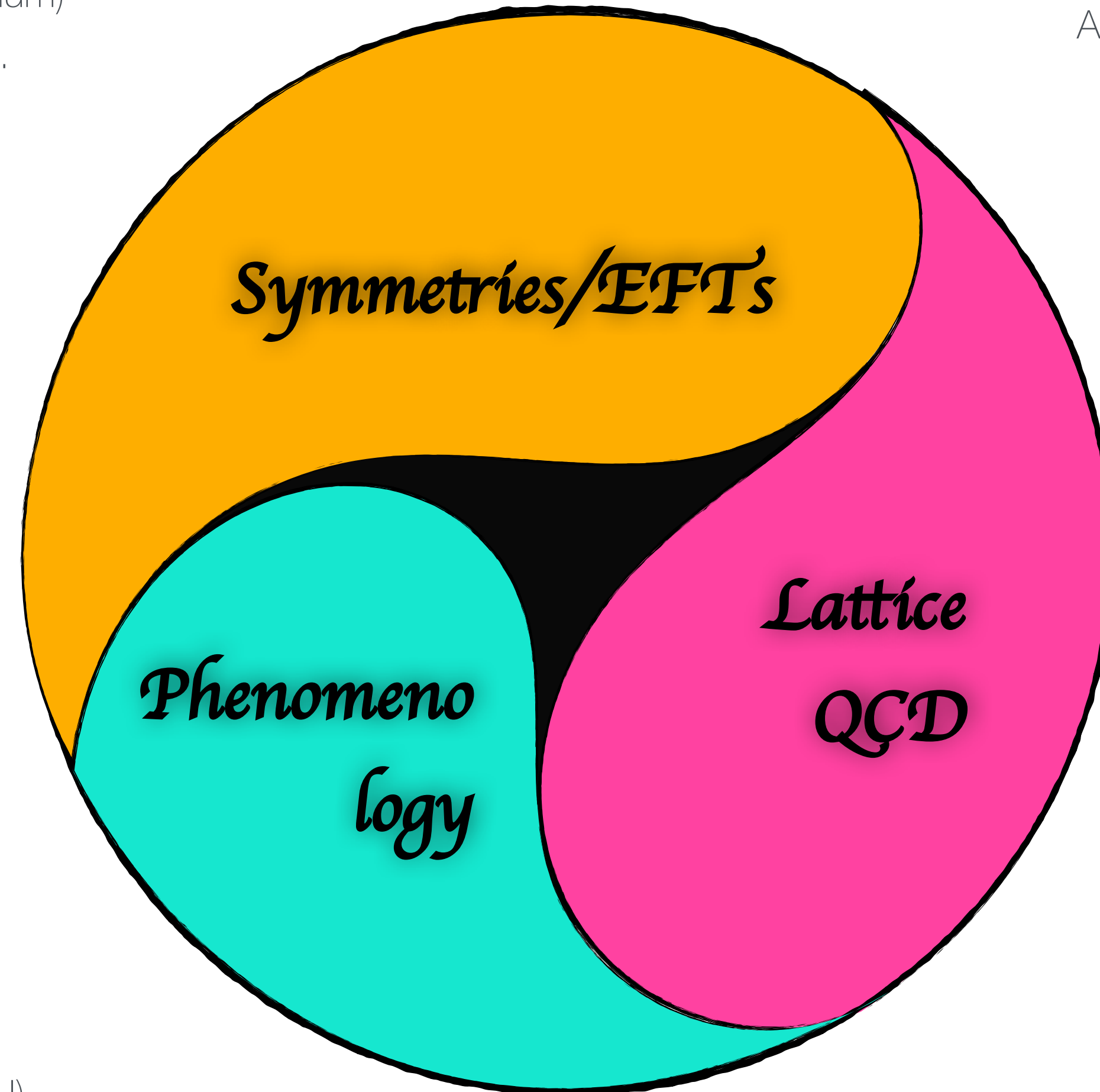
2/3-body lattice spectroscopy
[GWQCD collaboration]
Michael Döring (GWU/JLAB)
Chris Culver (U. Liverpool)
Ruairi Brett (GWU)
Andrei Alexandru (GWU)
Frank X. Lee (GWU)

[Extended Twisted Mass
collaboration]
Carsten Urbach (U. Bonn)
Ferenc Pittler (U. Cyprus)
Marco Garofalo (U. Bonn)
…

Symmetries/EFTs

Phenomeno logy

Lattice QCD
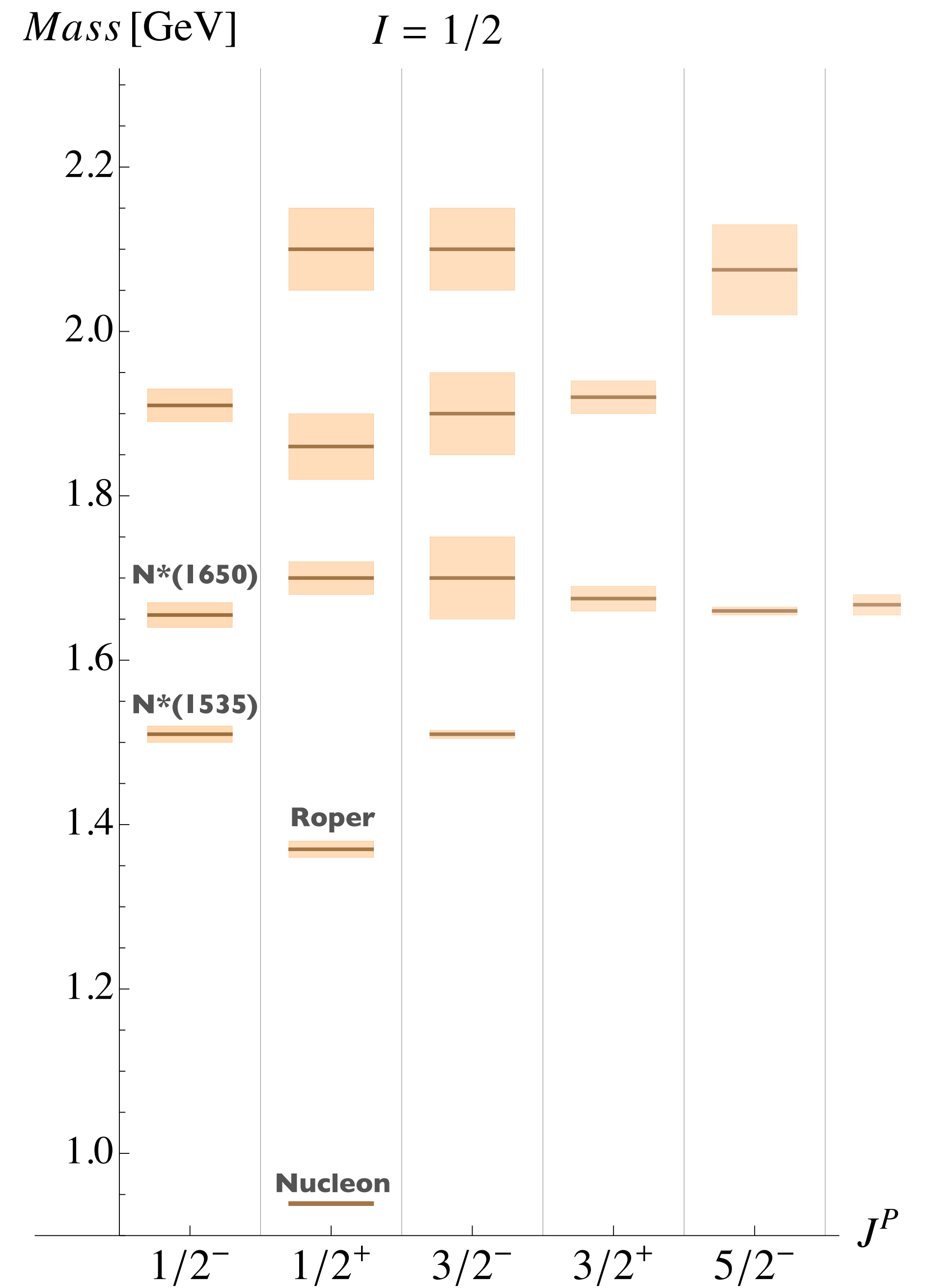
…

HADRON SPECTROSCOPY

Mostly excited states

≈100 mesons & ≈50 baryons (****)

Key questions

"what is the pattern of these states?"
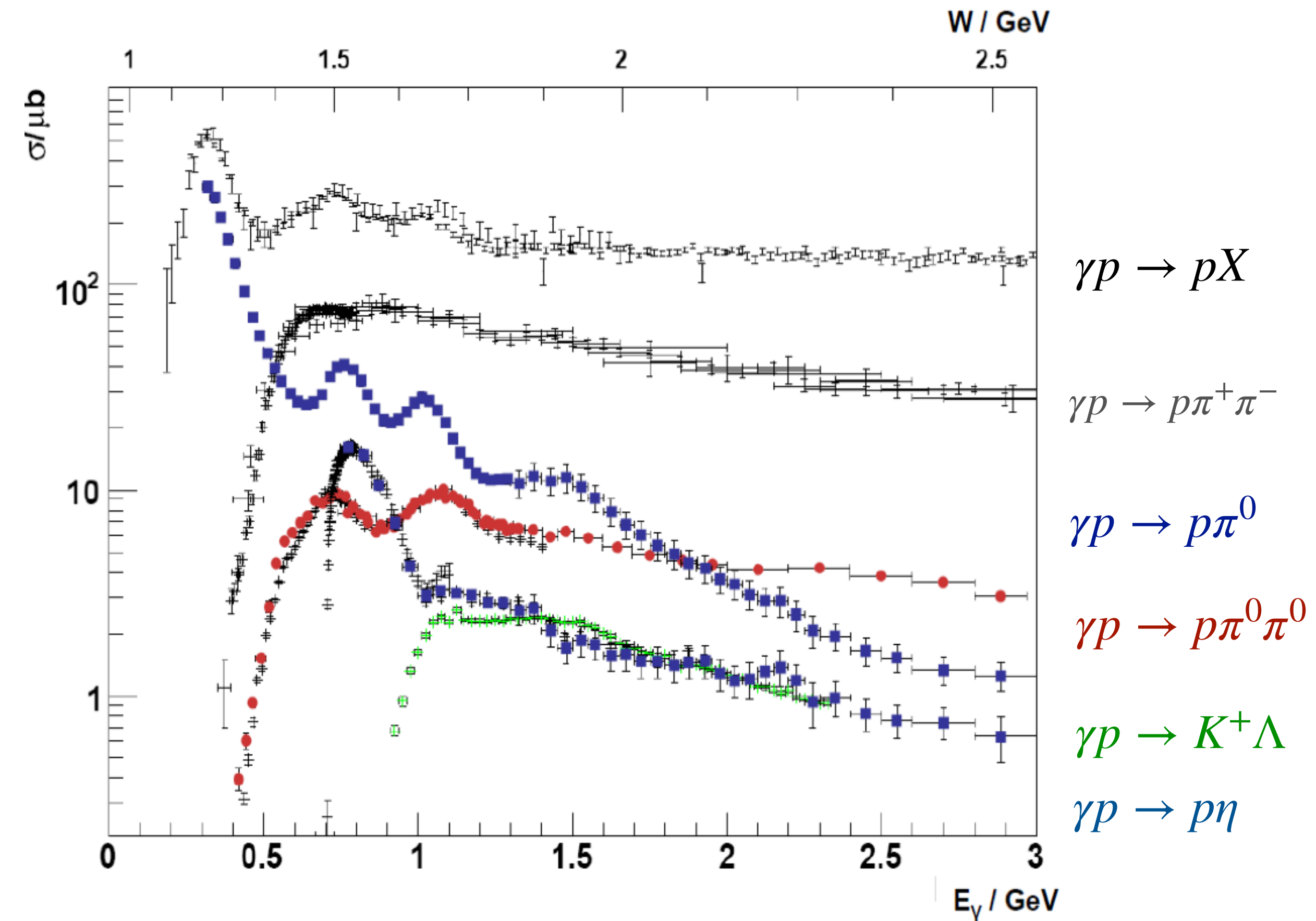
"how they are formed?"

HADRON SPECTROSCOPY

Physical input

- many experimental data

- ongoing experiments[1]

Resonance parameter

*"Not every bump is a resonance, not every resonance is a bump" - R. G. Moorhouse (1960s)*

W / GeV

$\sigma / \mu b$

$\gamma p \to pX$

$\gamma p \to p\pi^+\pi^-$

$\gamma p \to p\pi^0$

$\gamma p \to p\pi^0\pi^0$

$\gamma p \to K^+\Lambda$

$\gamma p \to p\eta$

E_γ / GeV

UNIVERSAL PARAMETERS

Reaction-independent parameters

- pole positions on unphysical Riemann Sheets

- central quantity: **transition amplitudes**

$M^* = (750 - i60)$ MeV

Universal property of the ρ – meson

Review: ``Towards a theory of hadron resonances'' Phys. Rept. 1001 (2023) – MM/Meißner/Urbach

Talks: Szczepaniak, Guo, Rodas, Pelaez, Döring, ...

Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

UNIVERSAL PARAMETERS

Reaction-independent parameters

- pole positions on unphysical Riemann Sheets
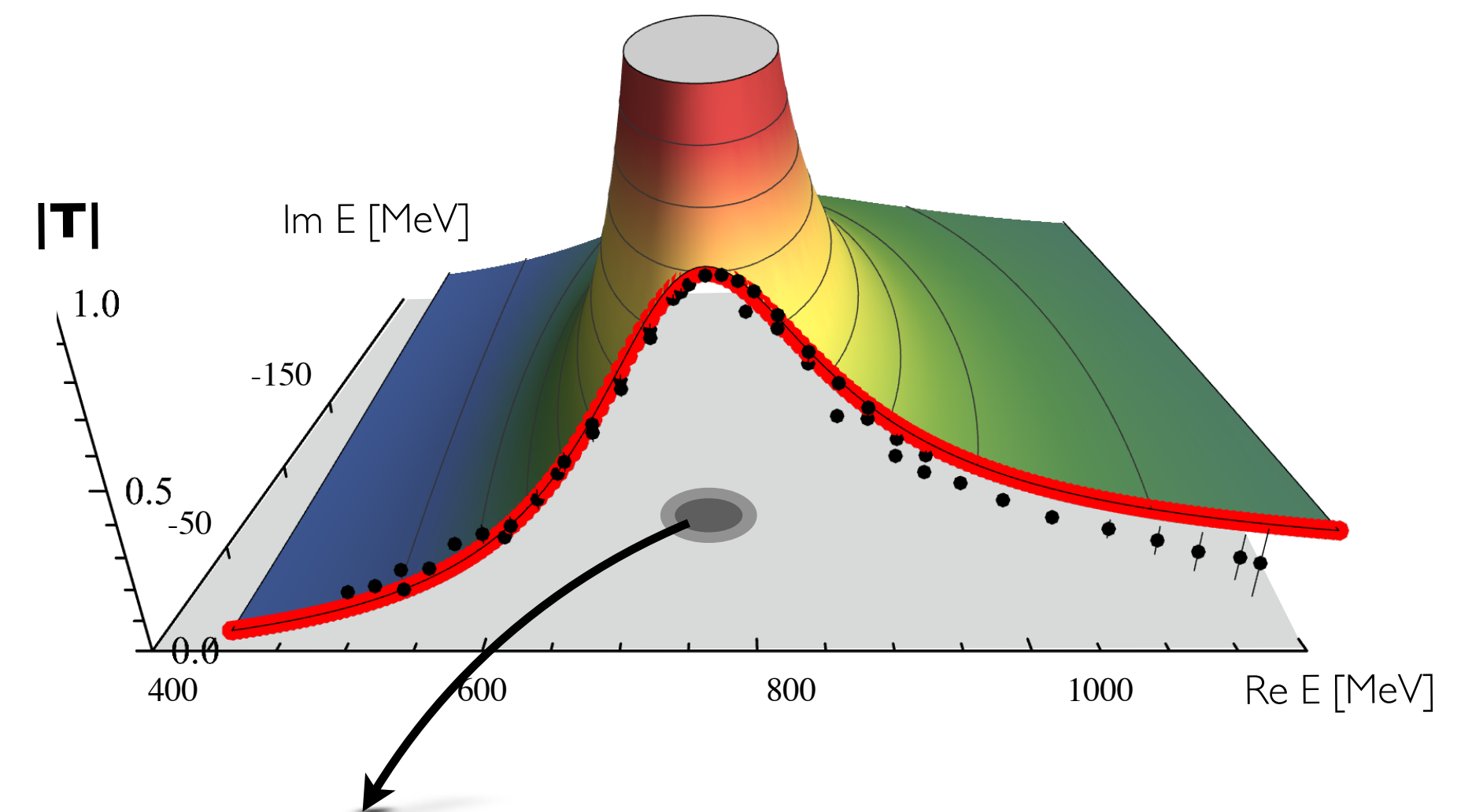
- central quantity: **transition amplitudes**

  → Constraints from S-matrix (Unitarity/Analyticity/Crossing)

  → Constrains from QCD (CHPT/LatticeQCD)

$M^* = (750 - i60)$ MeV
Universal property of the ρ − meson

Review: ``Towards a theory of hadron resonances'' Phys. Rept. 1001 (2023) –
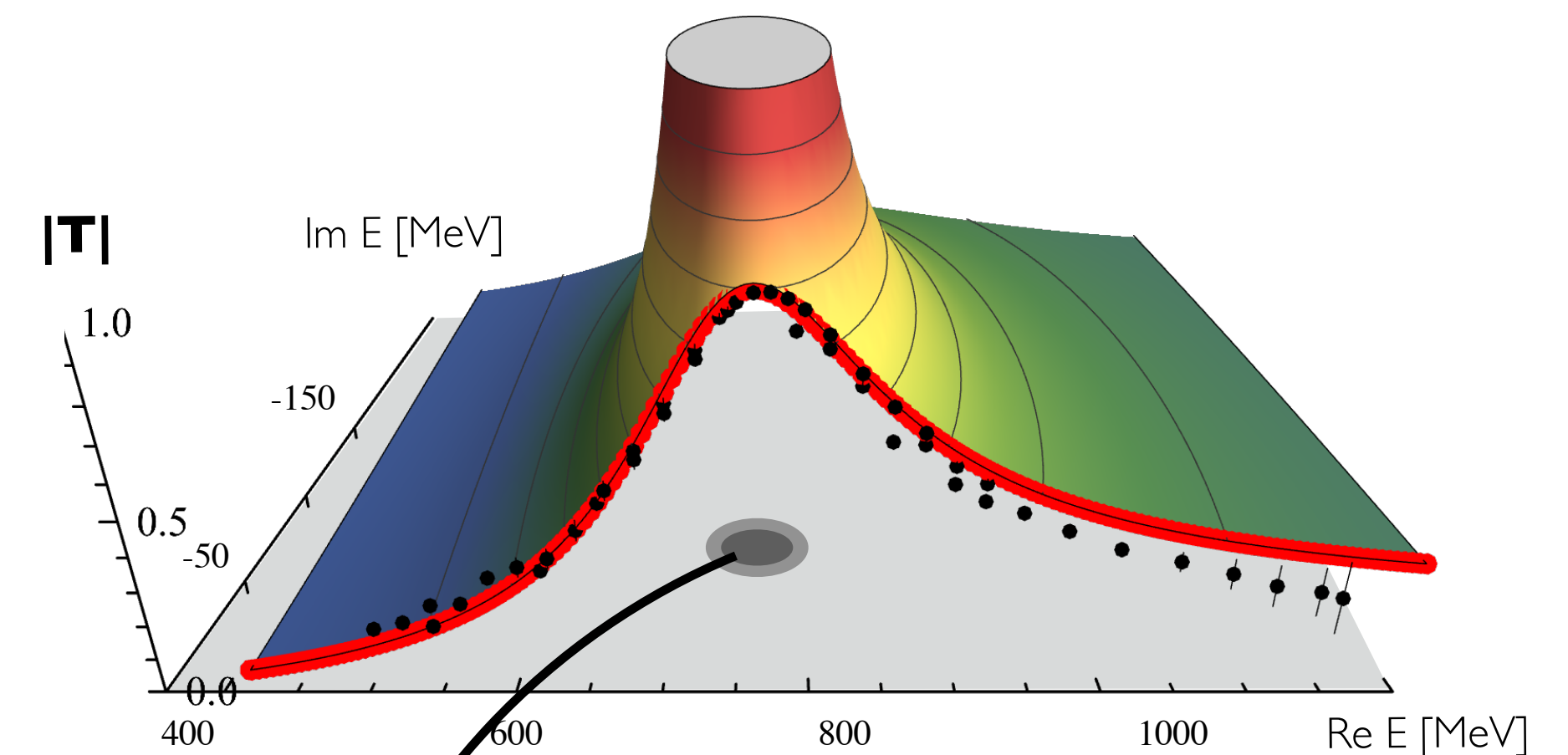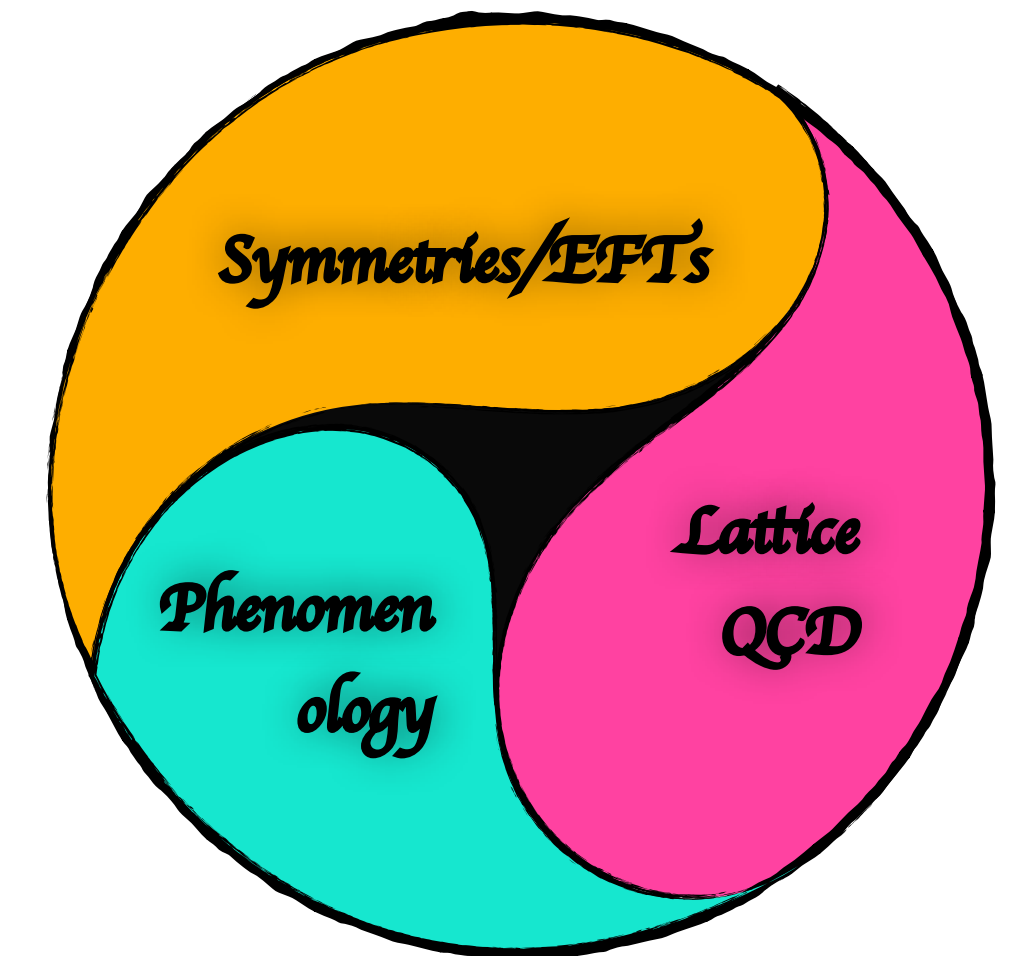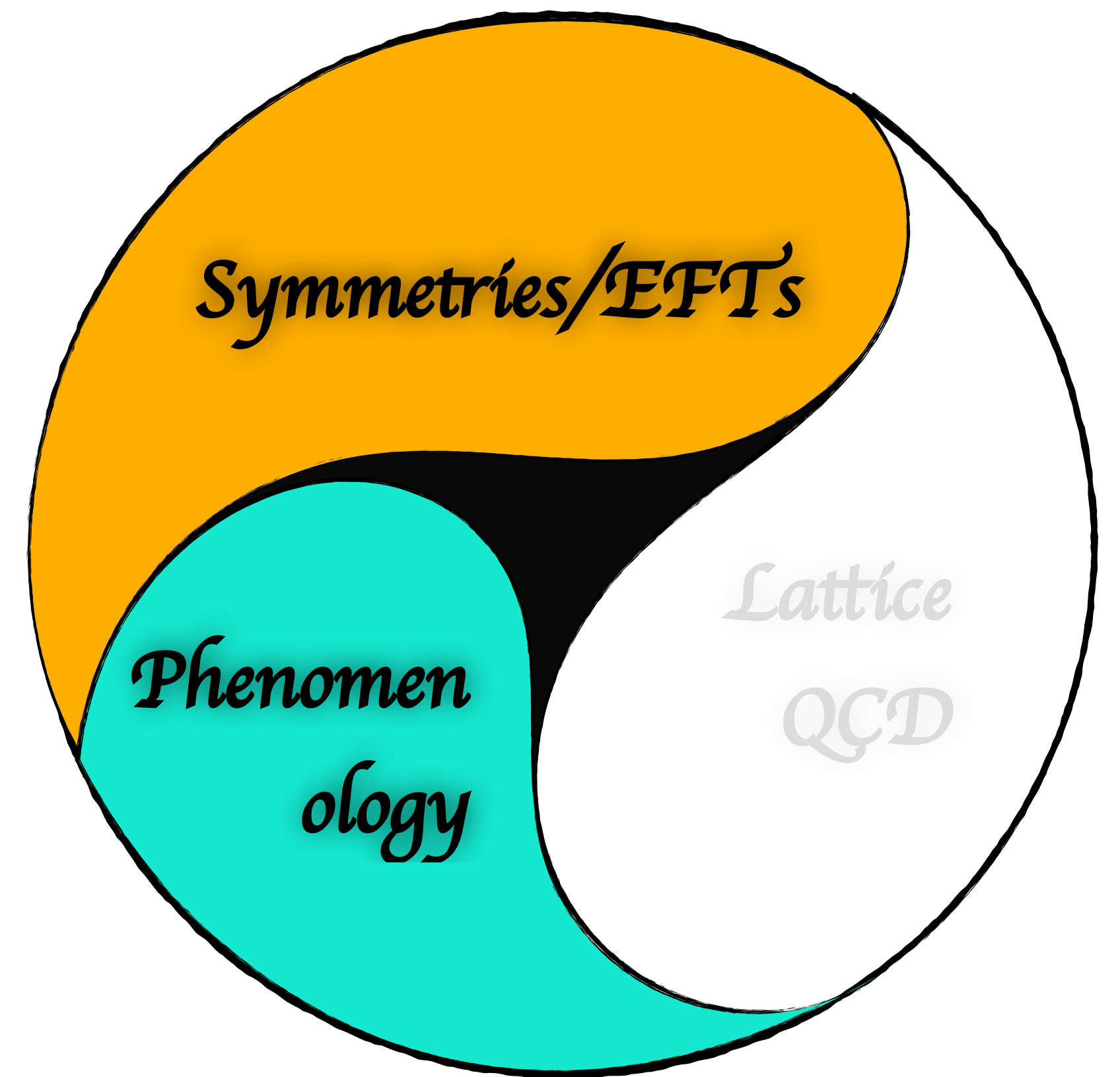MM/Meißner/Urbach

Talks: Szczepaniak, Guo, Rodas, Pelaez, Döring, ...

Data: Estabrooks et al. Nucl.Phys.B 79; Protopopescu et al. Phys.Rev.D 7;

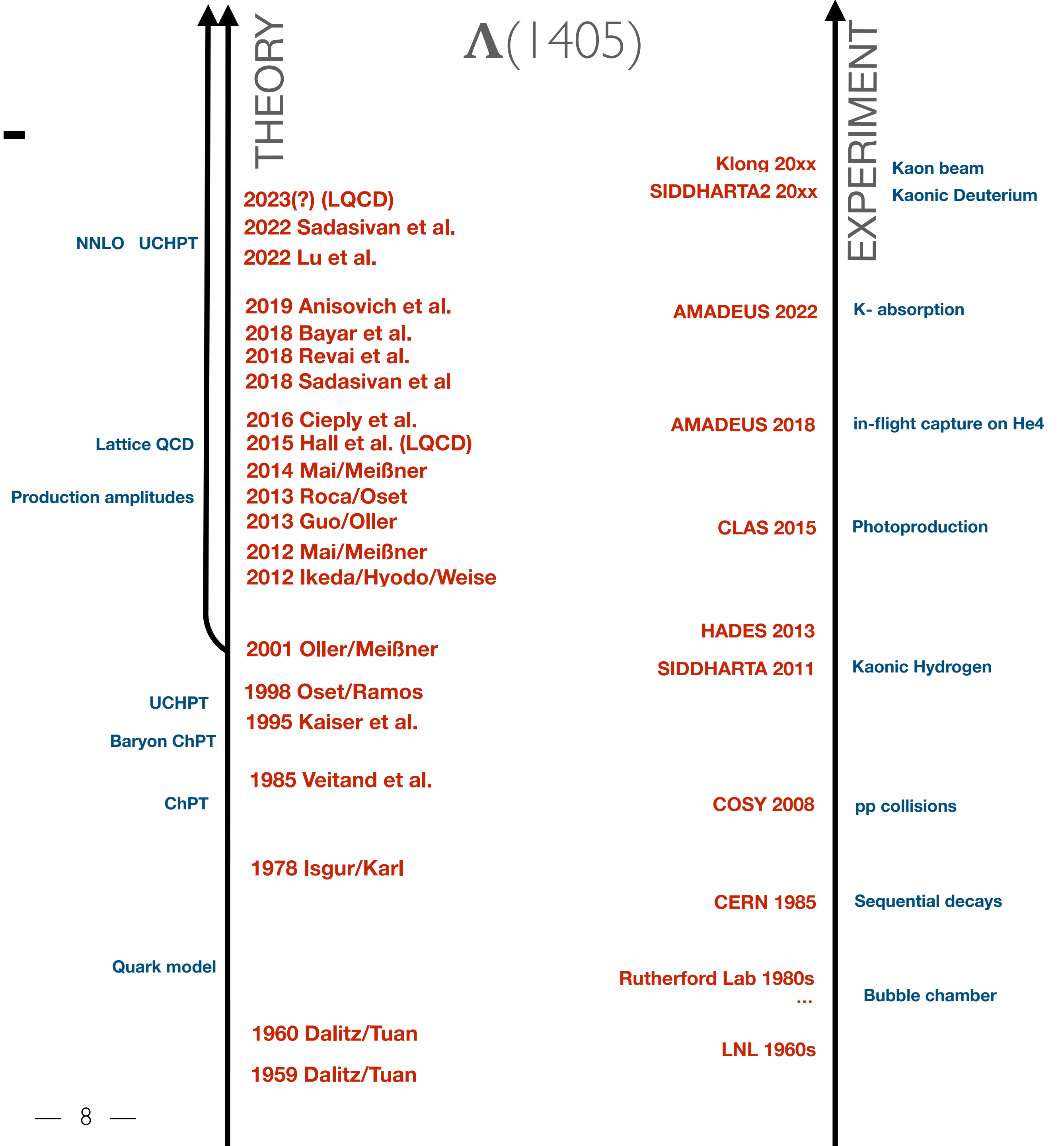INDIVIDUAL STATES

MESON-BARYON RESONANCES
FROM CHPT AND UNITARITY

RESONANT MESON-BARYON SYSTEMS

Many examples:

- N*(1535), N*(1650), …
- Λ(1405), Λ(1380)

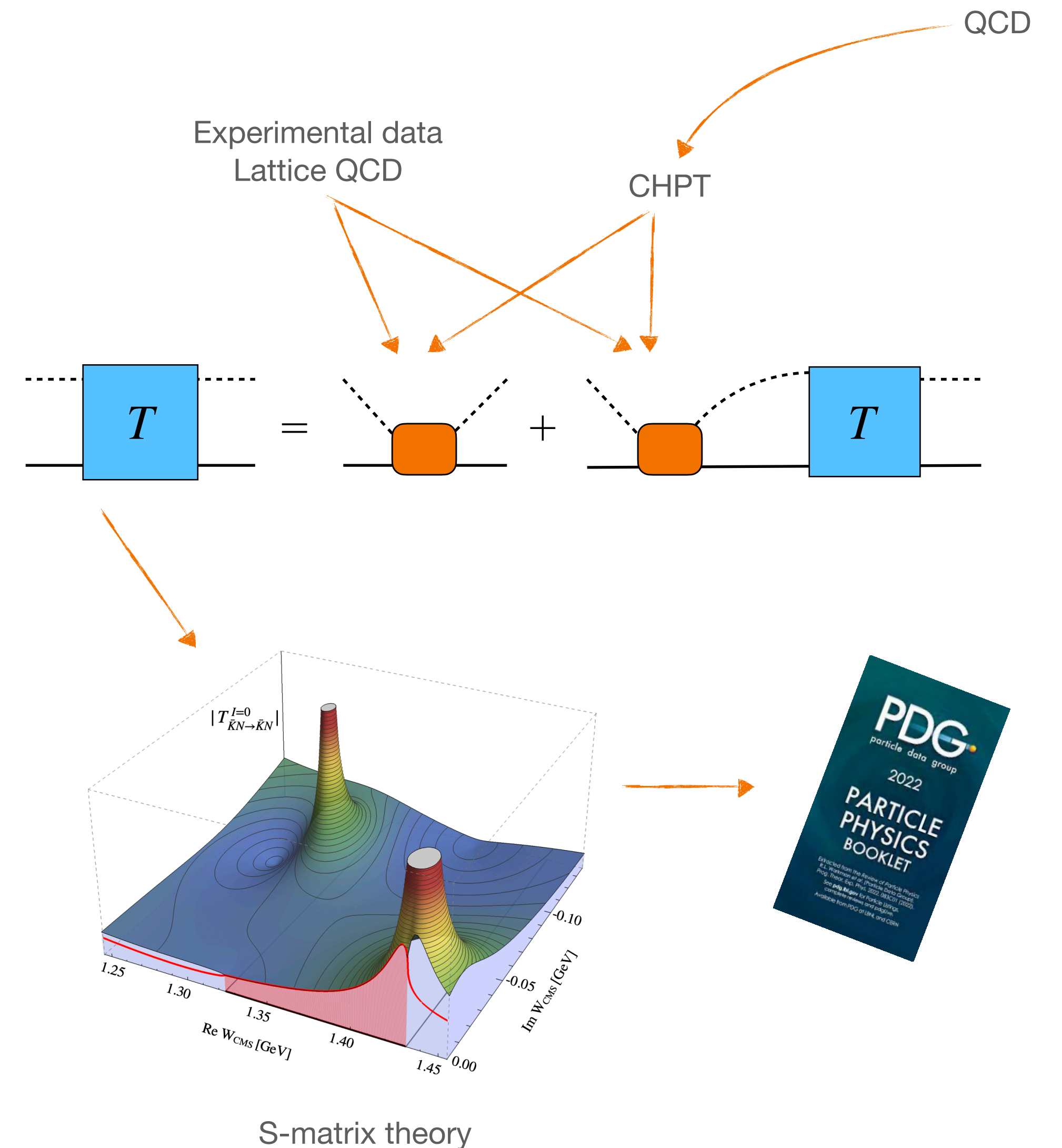 → Long history of experimental and theoretical efforts[1]

[1] **Reviews**: MM Eur.Phys.J.ST 230 (2021); Hyodo/Niiyama Prog.Part.Nucl.Phys. 120 (2021)

Λ(1405)

THEORY

EXPERIMENT

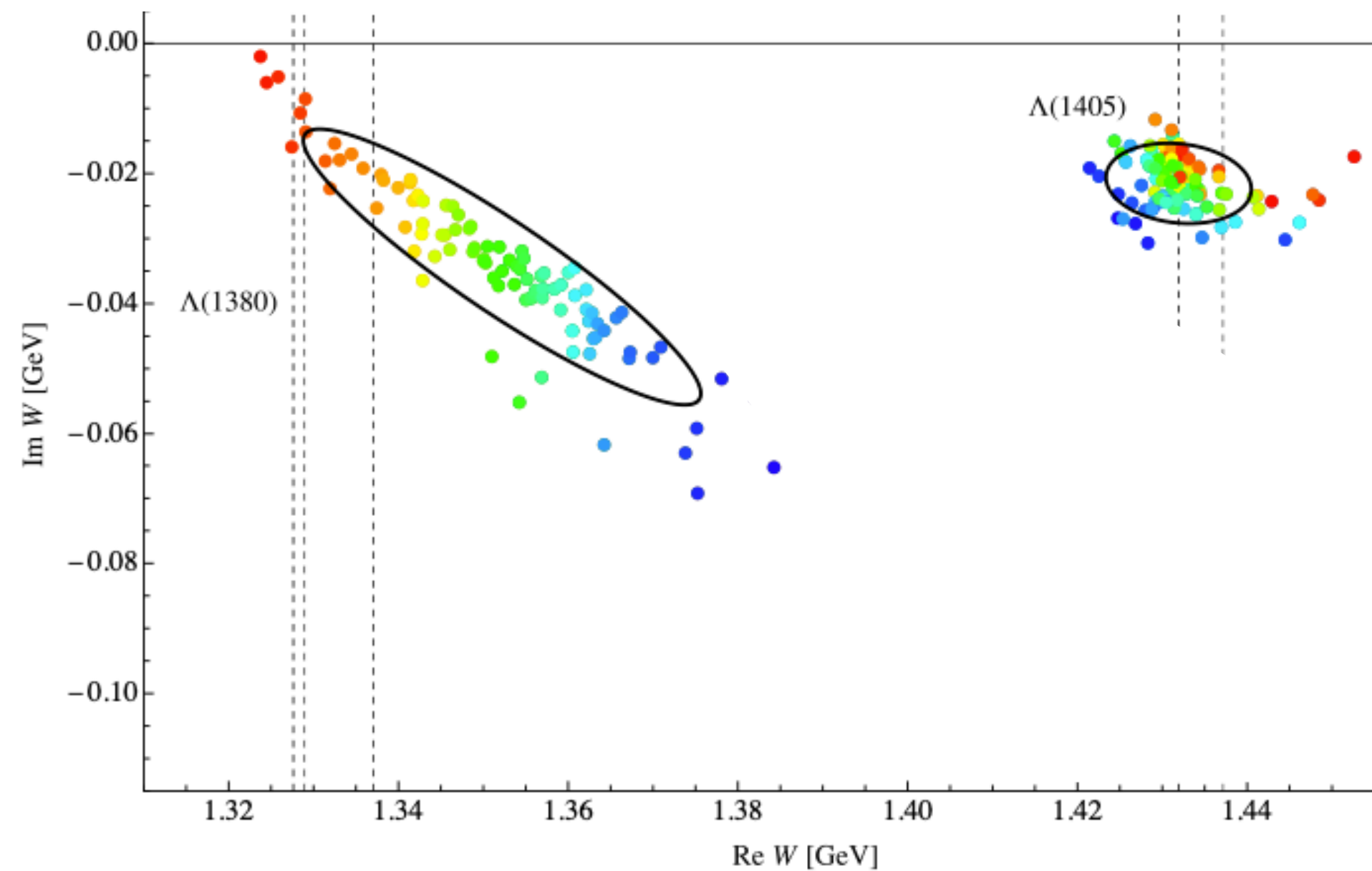| THEORY		EXPERIMENT
		Klong 20xx — Kaon beam
		SIDDHARTA2 20xx — Kaonic Deuterium
NNLO UCHPT	2023(?) (LQCD)	
	2022 Sadasivan et al.	
	2022 Lu et al.	
	2019 Anisovich et al.	AMADEUS 2022 — K- absorption
	2018 Bayar et al.	
	2018 Revai et al.	
	2018 Sadasivan et al	
	2016 Cieply et al.	AMADEUS 2018 — in-flight capture on He4
Lattice QCD	2015 Hall et al. (LQCD)	
	2014 Mai/Meißner	
Production amplitudes	2013 Roca/Oset	
	2013 Guo/Oller	CLAS 2015 — Photoproduction
	2012 Mai/Meißner	
	2012 Ikeda/Hyodo/Weise	
	2001 Oller/Meißner	HADES 2013
		SIDDHARTA 2011 — Kaonic Hydrogen
UCHPT	1998 Oset/Ramos	
	1995 Kaiser et al.	
Baryon ChPT		
	1985 Veitand et al.	
ChPT		COSY 2008 — pp collisions
	1978 Isgur/Karl	
		CERN 1985 — Sequential decays
Quark model		
		Rutherford Lab 1980s ... — Bubble chamber
	1960 Dalitz/Tuan	LNL 1960s
	1959 Dalitz/Tuan	

TRANSITION AMPLITUDE

One way:

- Chiral Perturbation Theory (#QCD#EFT) dictates the form of the interaction at low energies

- Unitary scattering amplitude from the Bethe-Salpeter equation

  → Fit free parameters to experimental data / LQCD

  → Extract complex pole positions for complex energies

QCD

Experimental data
Lattice QCD

CHPT

$$T = \quad + \quad T$$

$$|T^{I=0}_{\bar{K}N \to \bar{K}N}|$$

Re W_{CMS} [GeV]

Im W_{CMS} [GeV]

1.25 1.30 1.35 1.40 1.45

-0.10 -0.05 0.00

PDG
particle data group
2022
PARTICLE
PHYSICS
BOOKLET

S-matrix theory

CURRENT FRONTIER

Statistical tests and new data sources

AMADEUS Phys. Lett. B 782 (2018)
Sadasivan et al Front.Phys. 11 (2023)

- New correlations $\mathbf{\Lambda}$(1405)/$\mathbf{\Lambda}$(1380)

Theory update: NNLO UCHPT

Lu/Geng/Döring/MM Phys.Rev.Lett. 130 (2023)
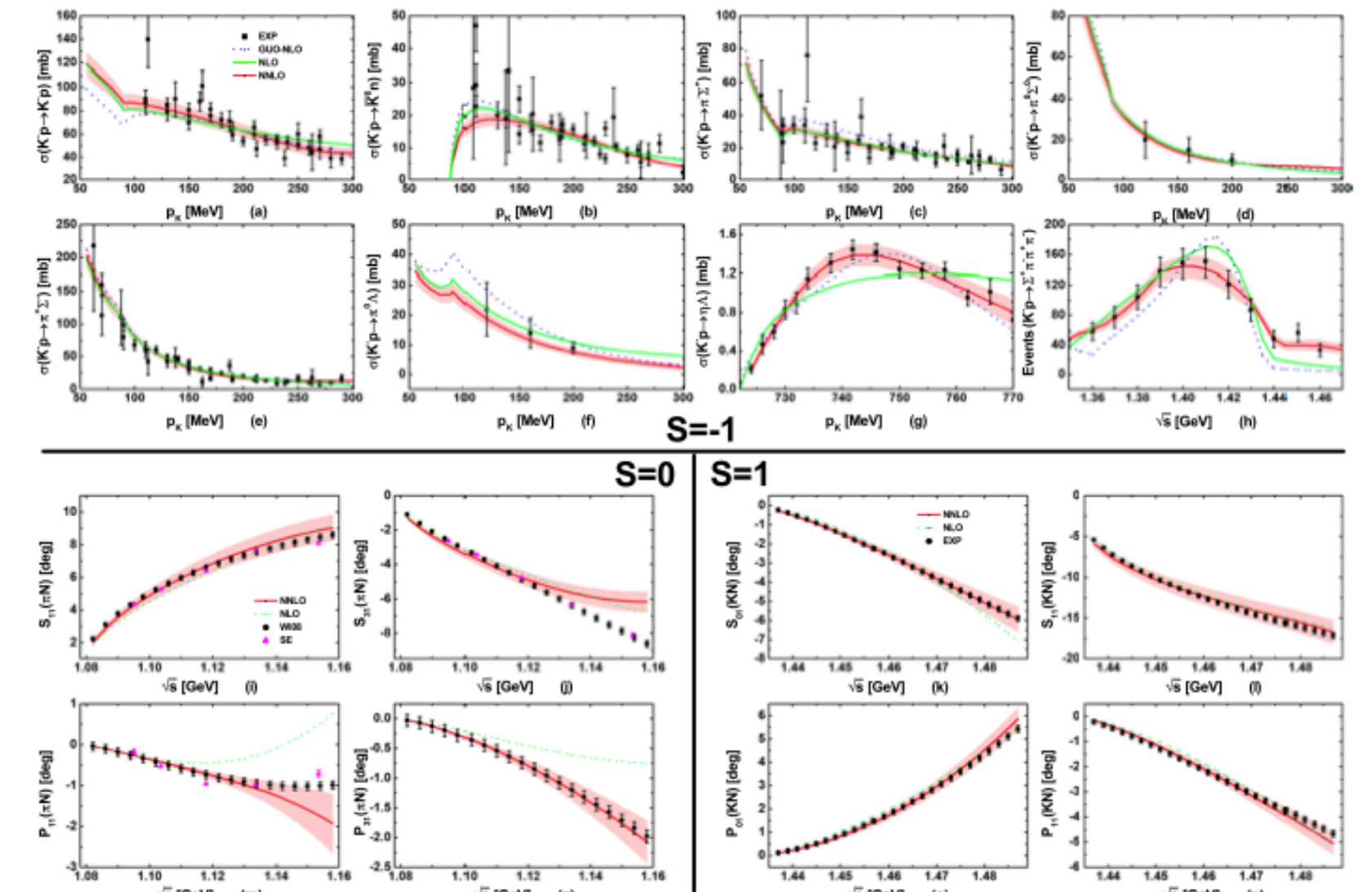
- Simultaneous description of πN/KN/Kbar$N$ scattering
- Two-pole structure $\mathbf{\Lambda}$(1405)/$\mathbf{\Lambda}$(1380)

- **New experimental facilities** (Talks: Francesco Sgaramella/…)
- **Lattice QCD [2 poles]** (Talks: Morningstar/Leinweber)
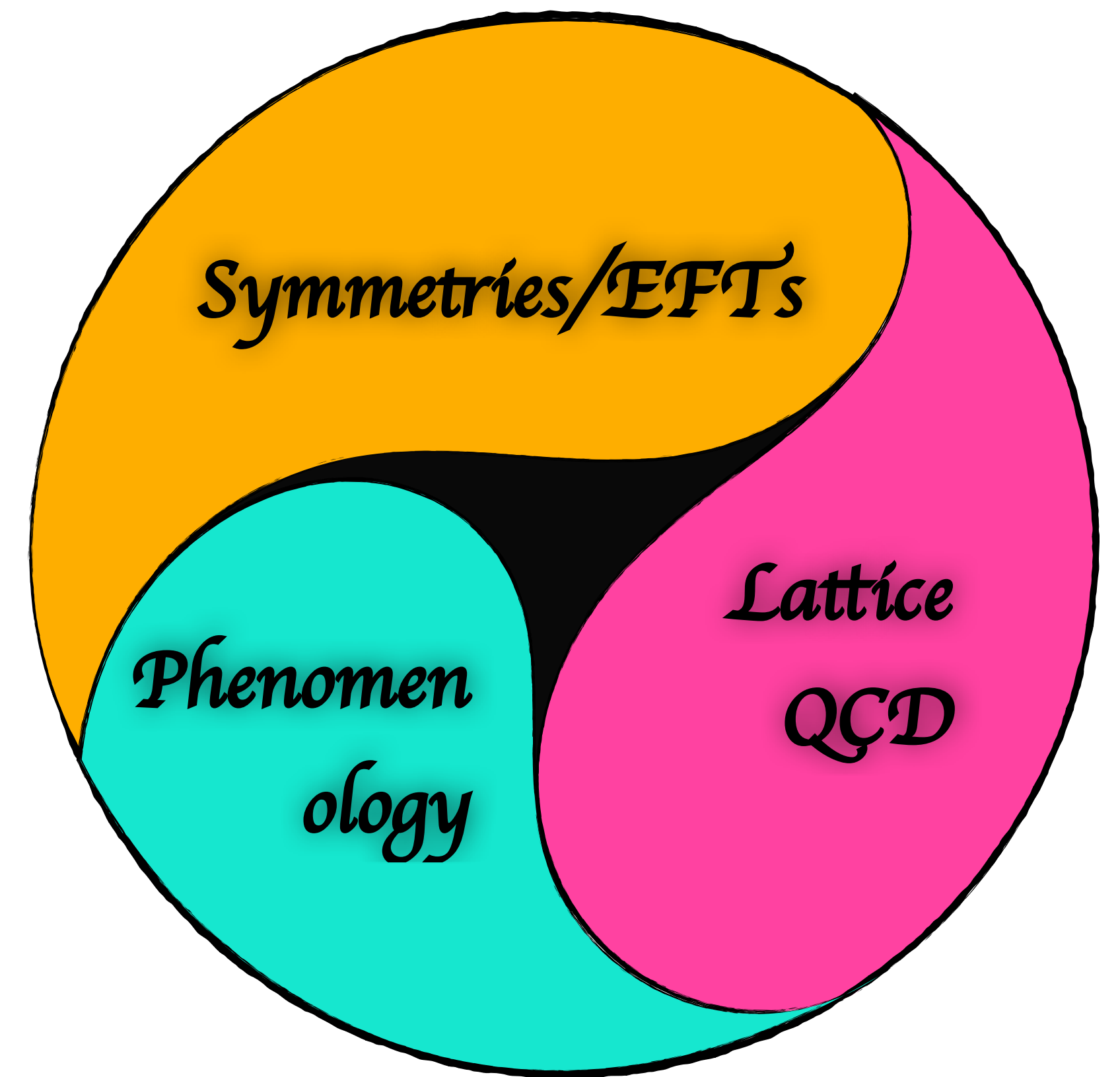
CURRENT FRONTIER

VERY VERY PRELIMINARY

- UCHPT Chiral trajectory

[1] Preliminary results from LQCD: TALK Morningstar
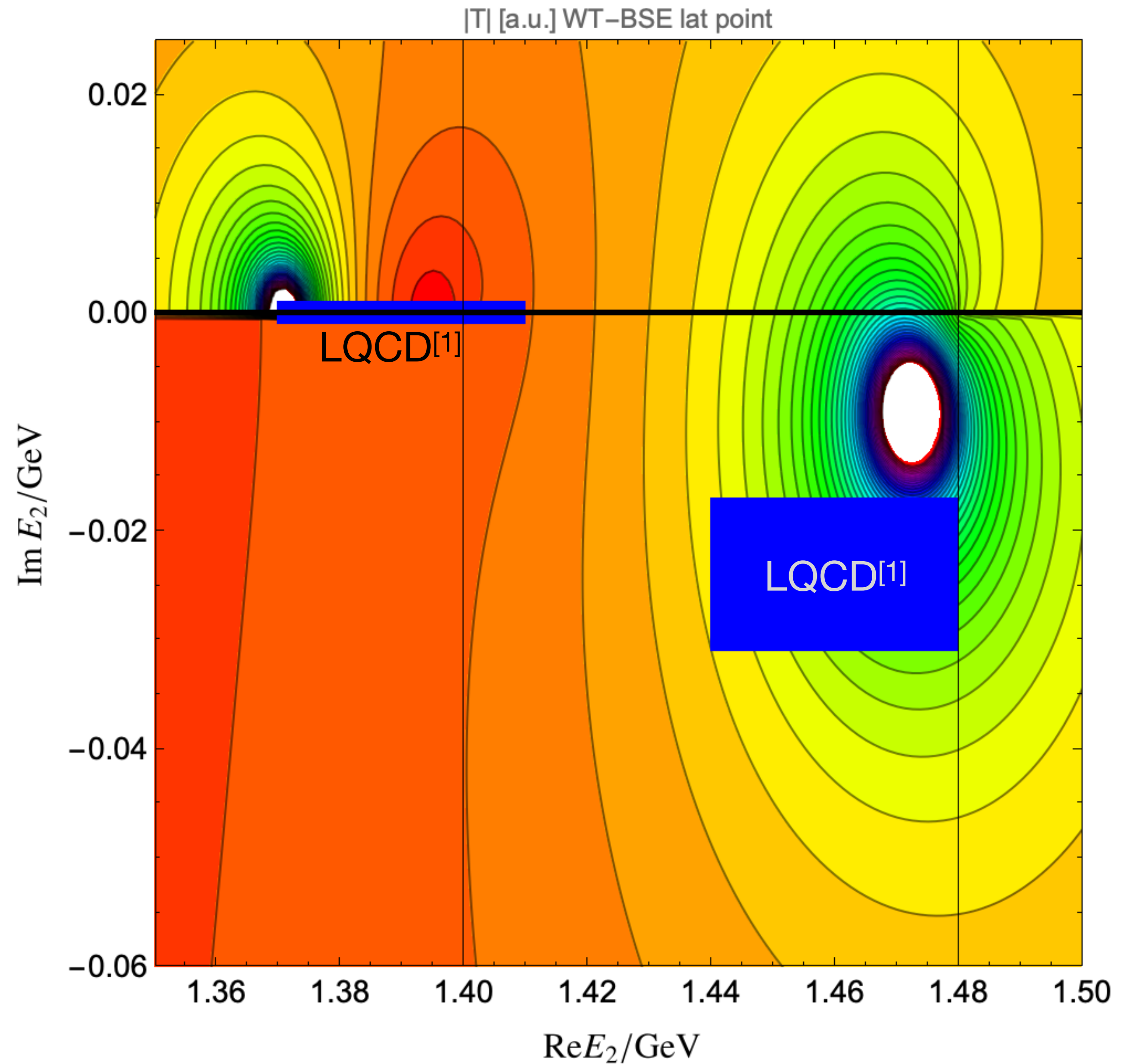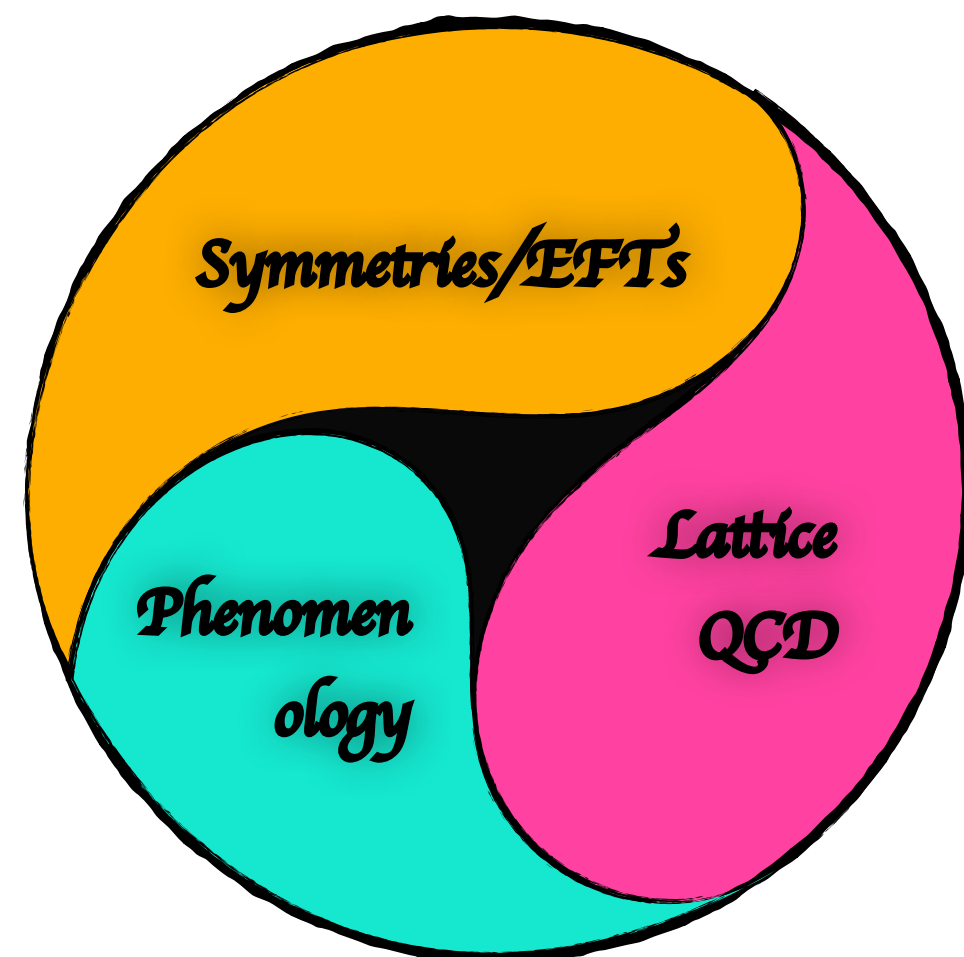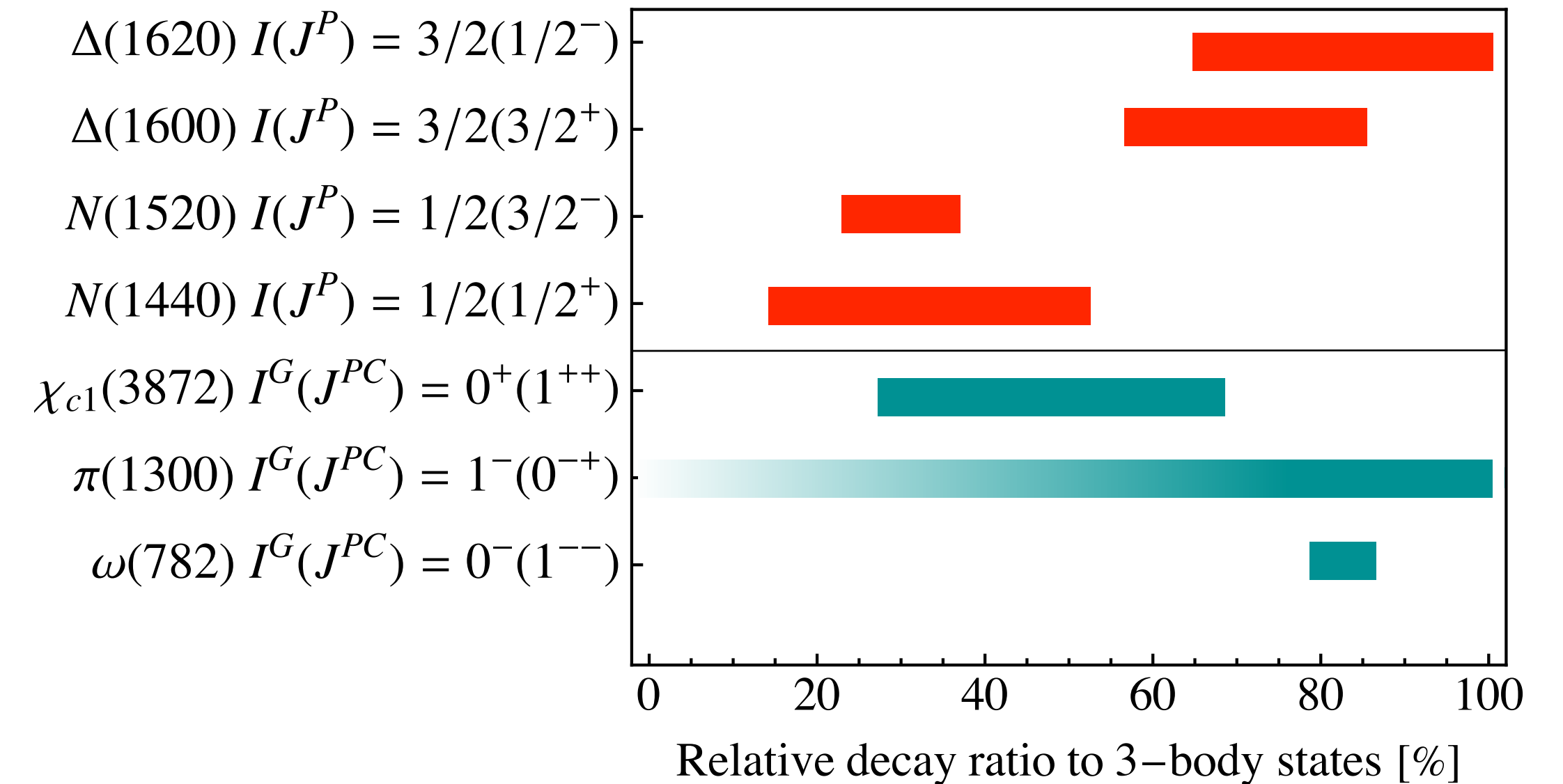
INDIVIDUAL STATES

THREE-BODY SYSTEMS

HADRONIC 3-BODY PROBLEM

- Many known states have large 3-body content:

  → Roper(1440)/X(3872)/a₁(1260)/…

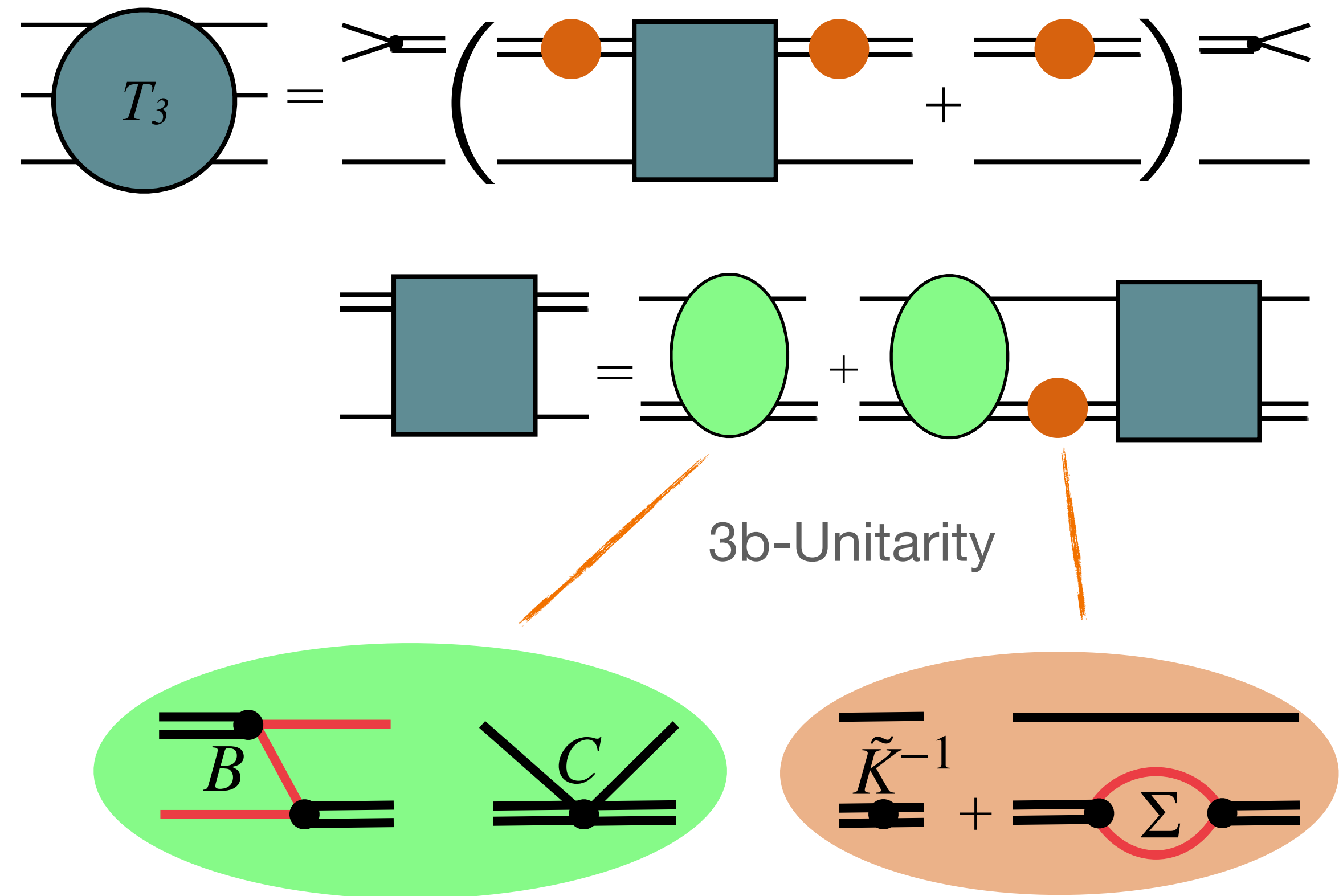- Beyond Standard Model searches (τ-EDM/…)

- Exotic states of matter[1]

Relative decay ratio to 3−body states [%]

R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

TRANSITION AMPLITUDE

Three-body scattering amplitude[1][2]

- constructed from unitarity
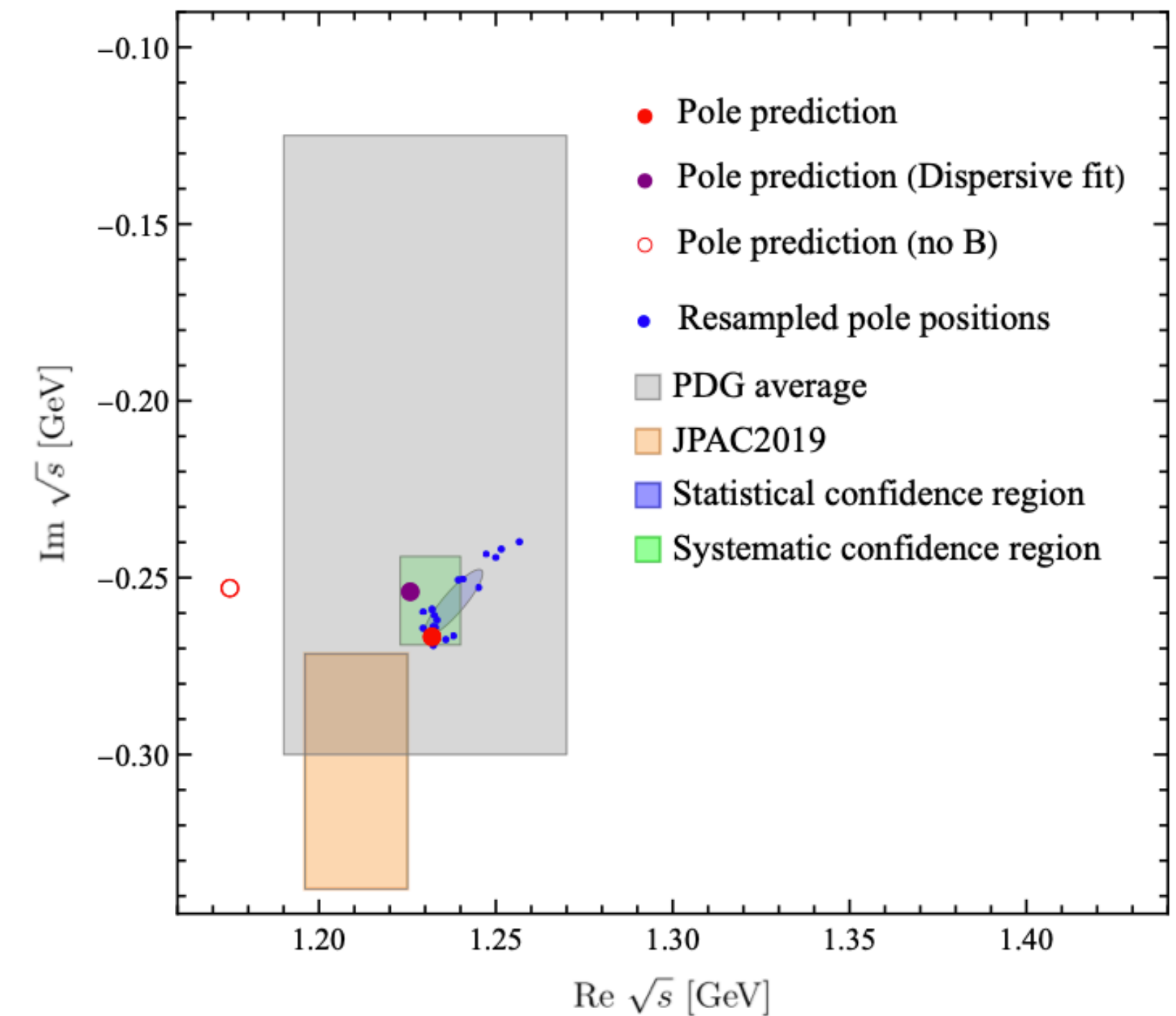
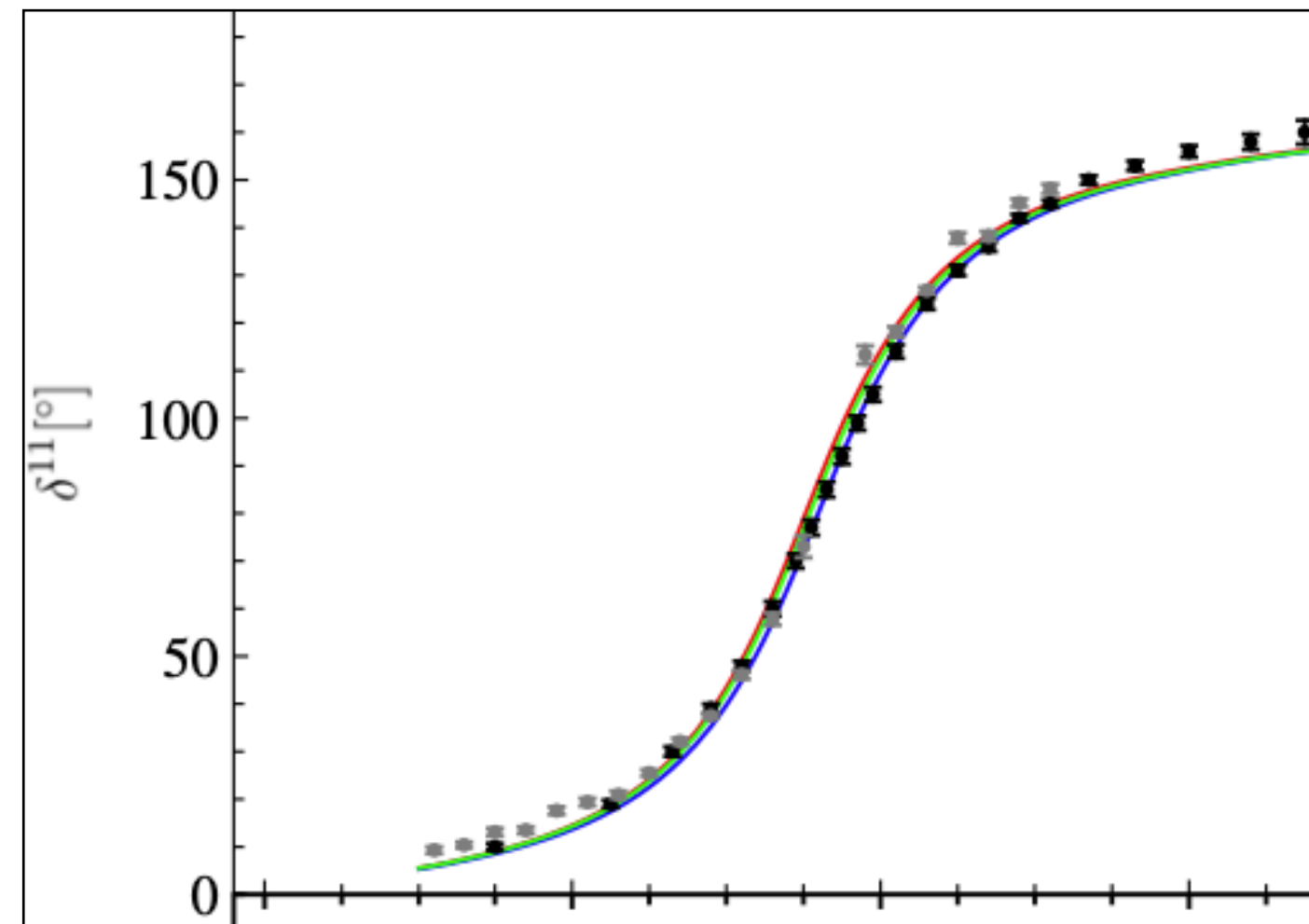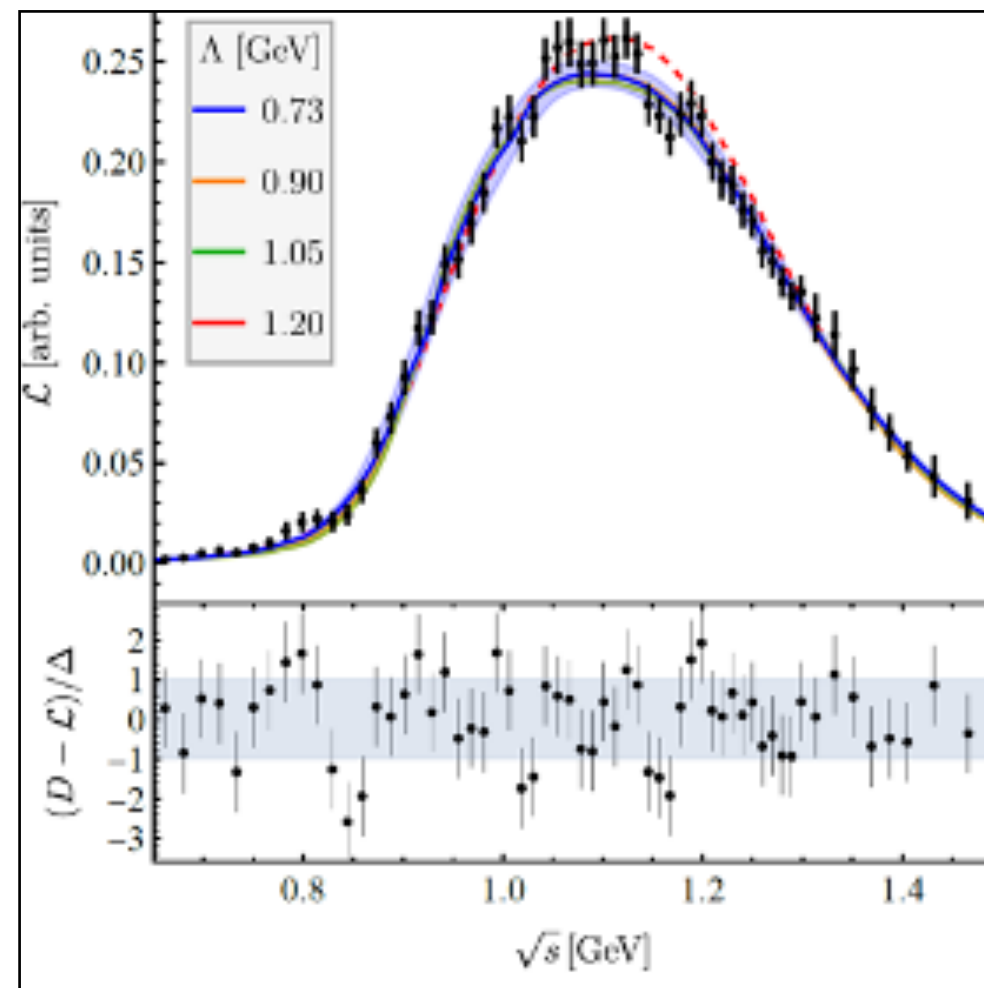- novel result from the S-matrix theory
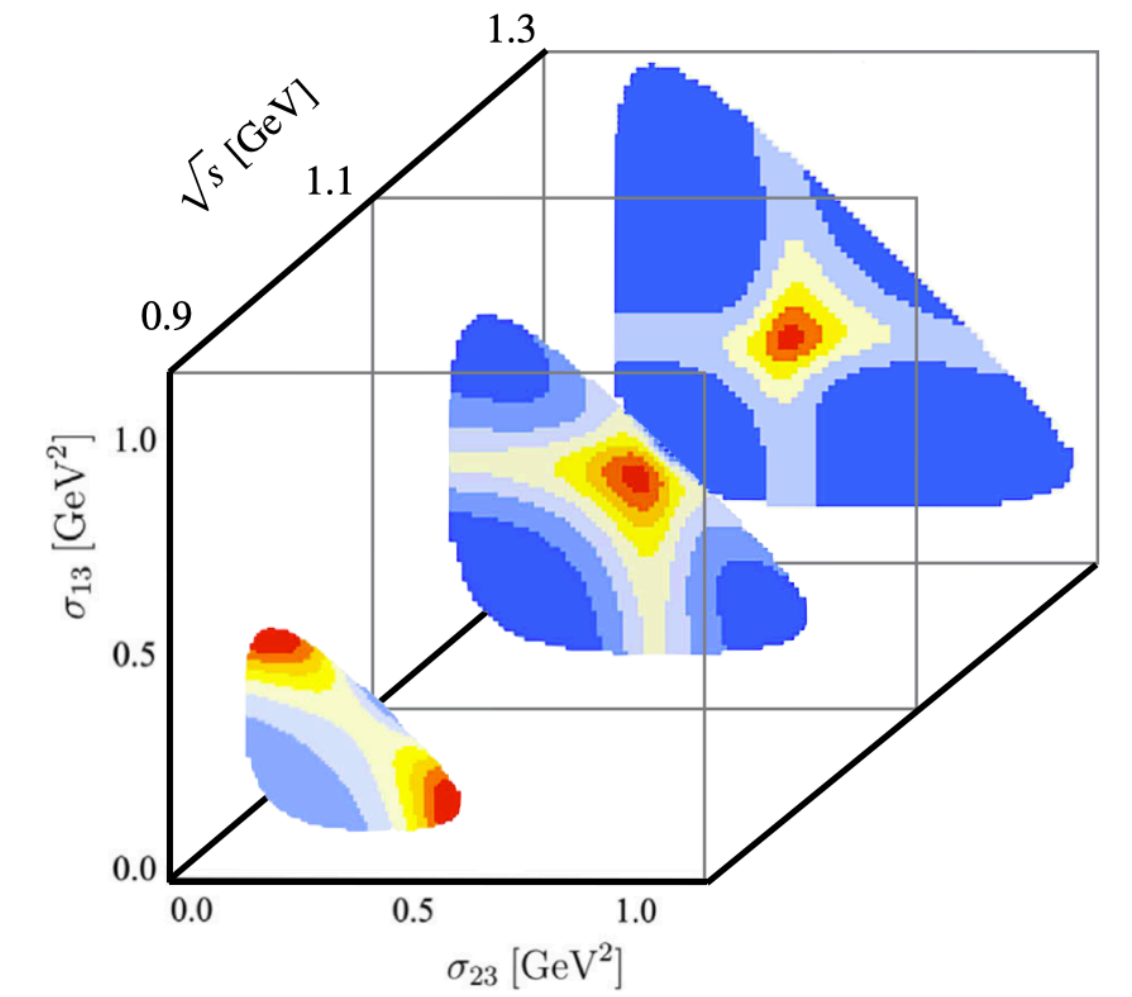
3b-Unitarity

[1] MM/Hu/Döring/Pilloni/Szczepaniak Eur.Phys.J.A 53 (2017)

[2] Related approaches: Wunderlich et al. JHEP 08 (2019); Jackura et al. Eur.Phys.J.C 79 (2019);

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

Data: Schael [ALEPH] Phys.Rept. 421 (2005); Estabrooks et al. Nucl.Phys.B 79;

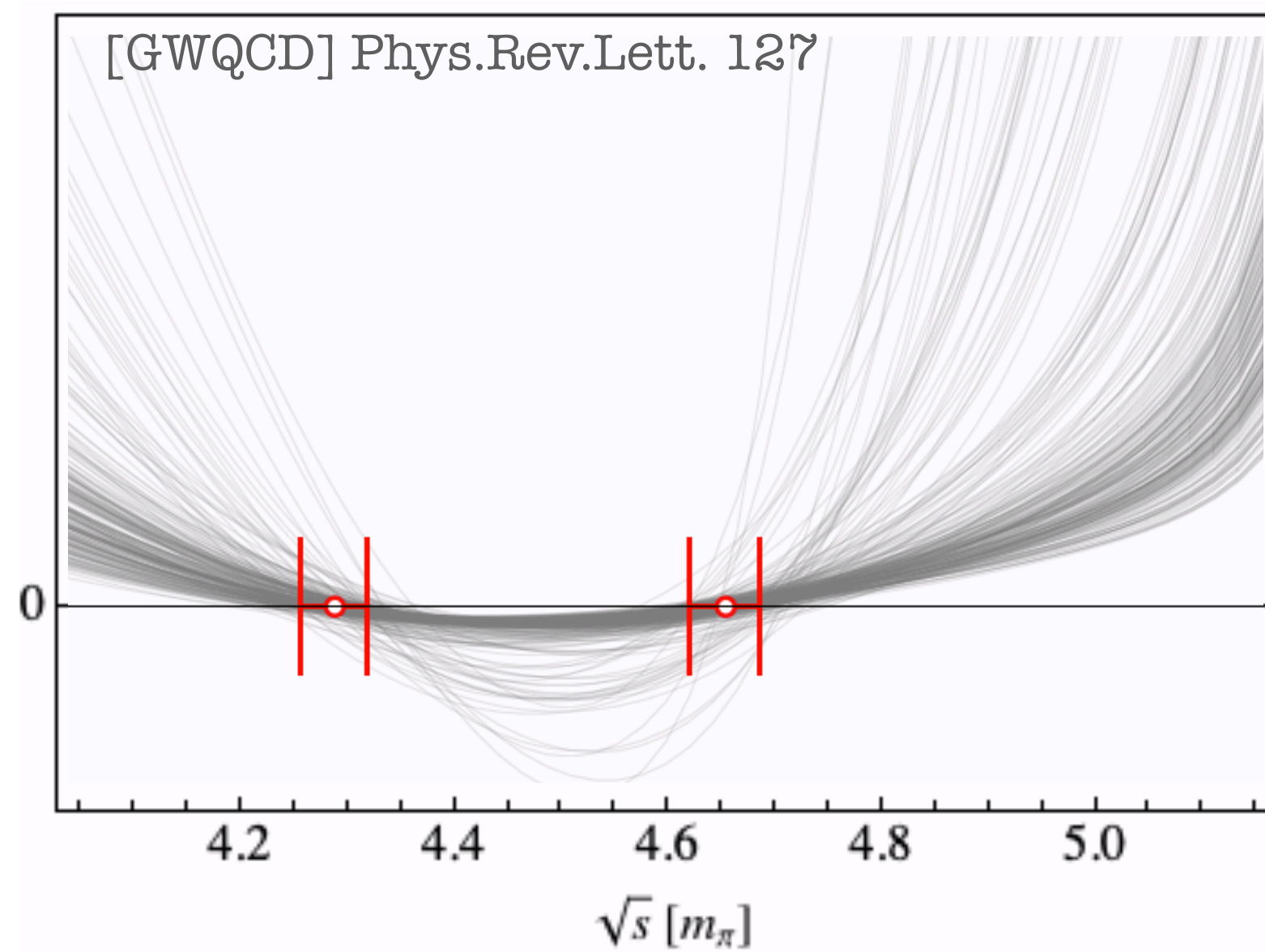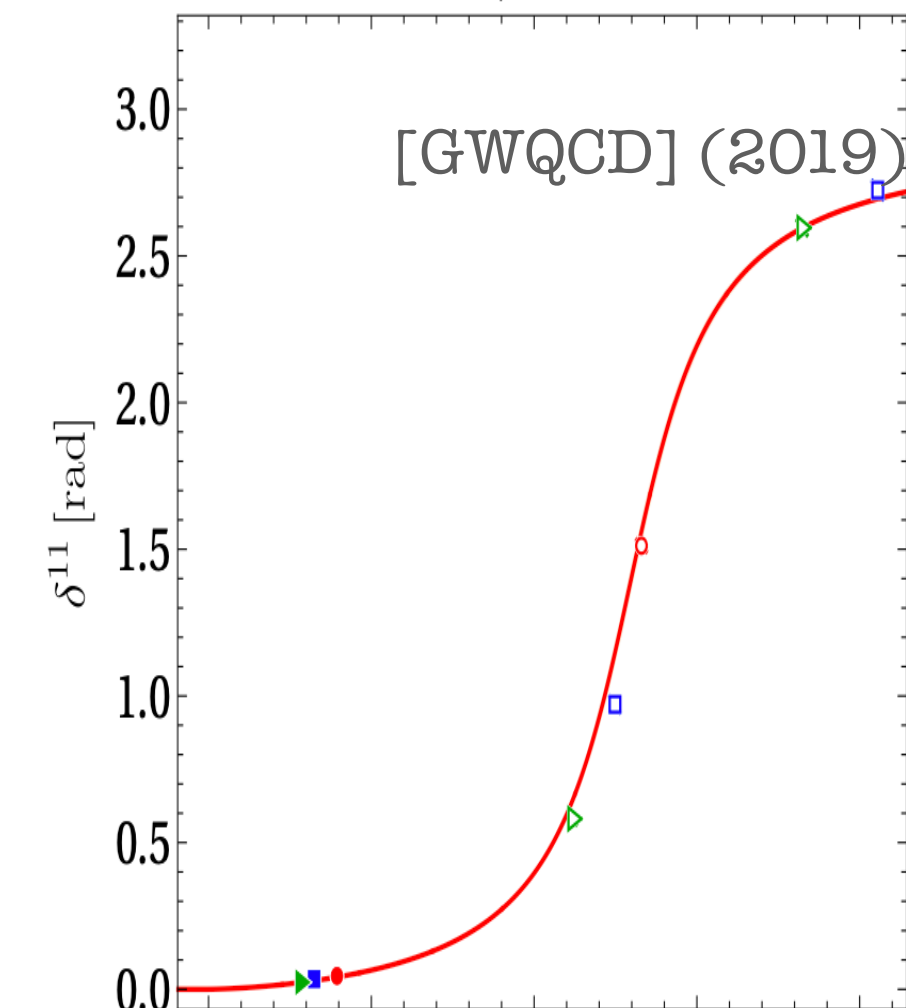Protopopescu et al. Phys.Rev.D 7;

Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020)

3-body quantisation condition — FVU [1][2]

a1(1260)
"Heavy Universe" $(M_\pi : 138 \text{ MeV} \rightarrow 224 \text{ MeV})$

$$\det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p'p}} = 0$$

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3} \frac{(B+C)}{2E_l} \frac{1}{\tilde{K}_n^{-1} - \Sigma_n} T^c$$

$I = 1,\ l = 1$

[GWQCD] (2019)

[GWQCD] Phys.Rev.Lett. 127

$\sqrt{s}\ [m_\pi]$

[1] MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

[2] **Reviews:** Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky
Eur.Phys.J.ST 230 (2021);

**More in
M. Döring's talk #1**

3-body quantisation condition — FVU [1][2]

$$\det\left[2L^3 E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p'p}} = 0$$

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$I = 1, l = 1$

δ^{11} [rad]

[GWQCD] (2019)

[GWQCD] Phys.Rev.Lett. 127

$\sqrt{s}$ [m_π]

1σ-region

Im $\sqrt{s}$ [MeV]
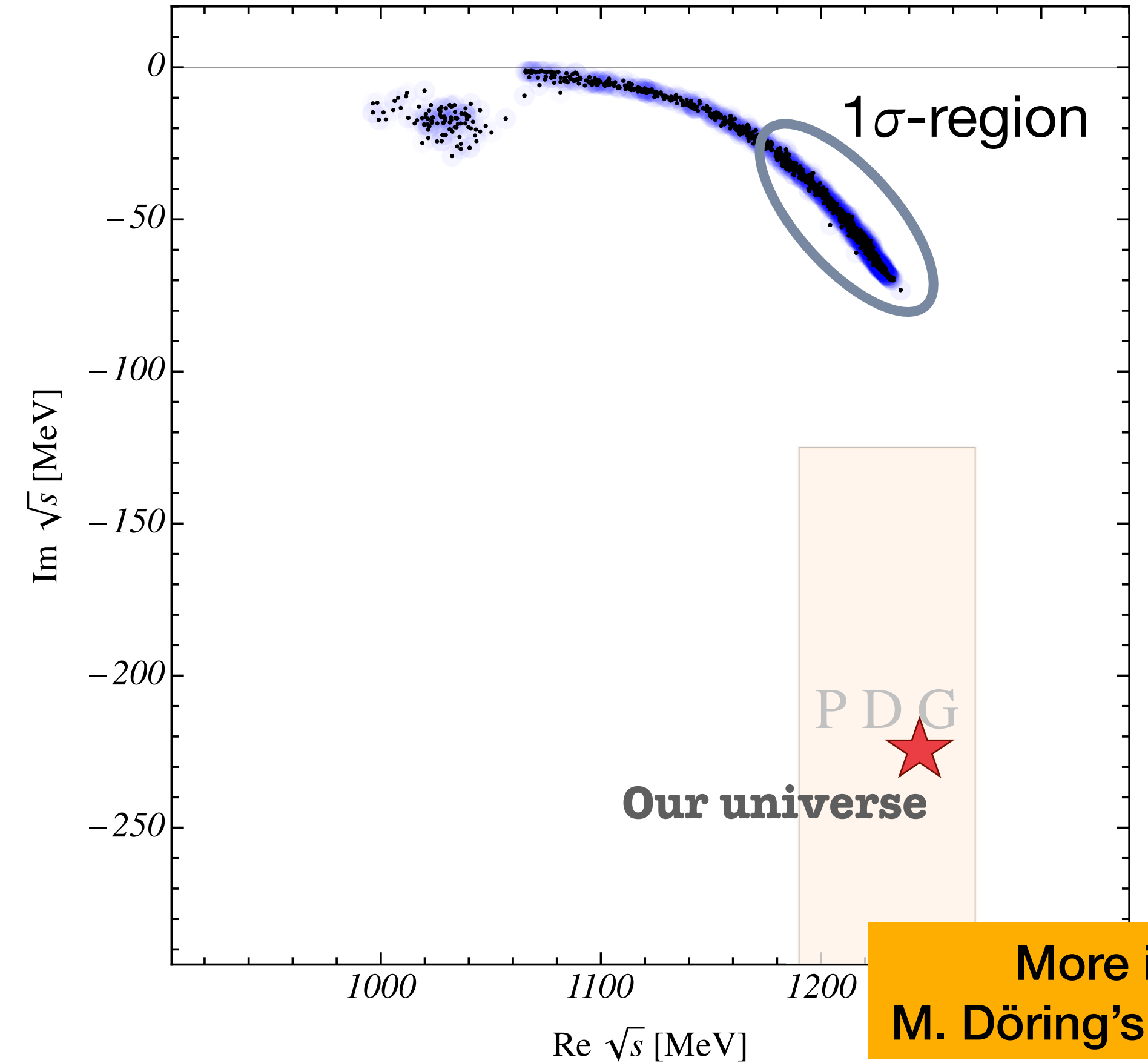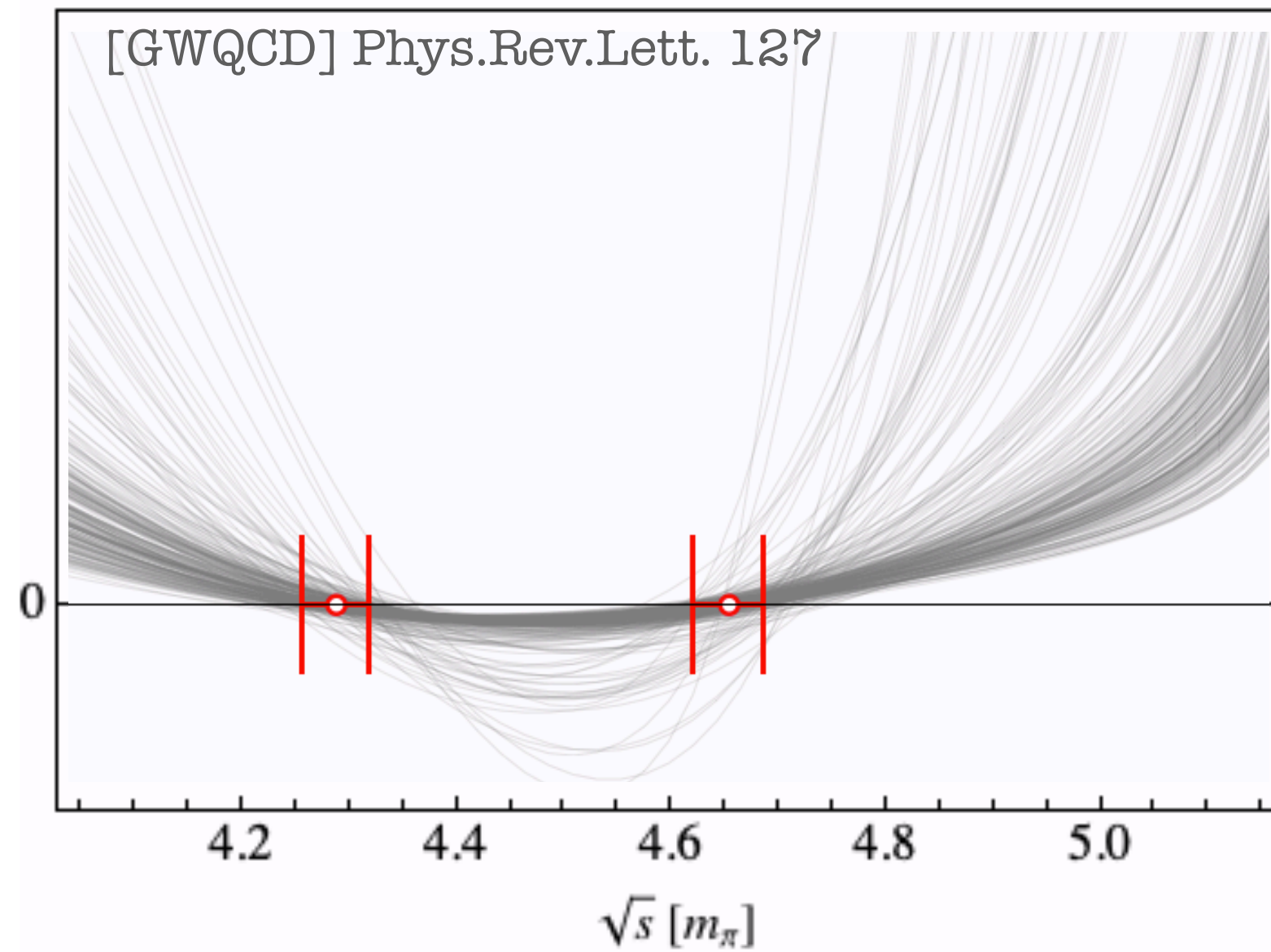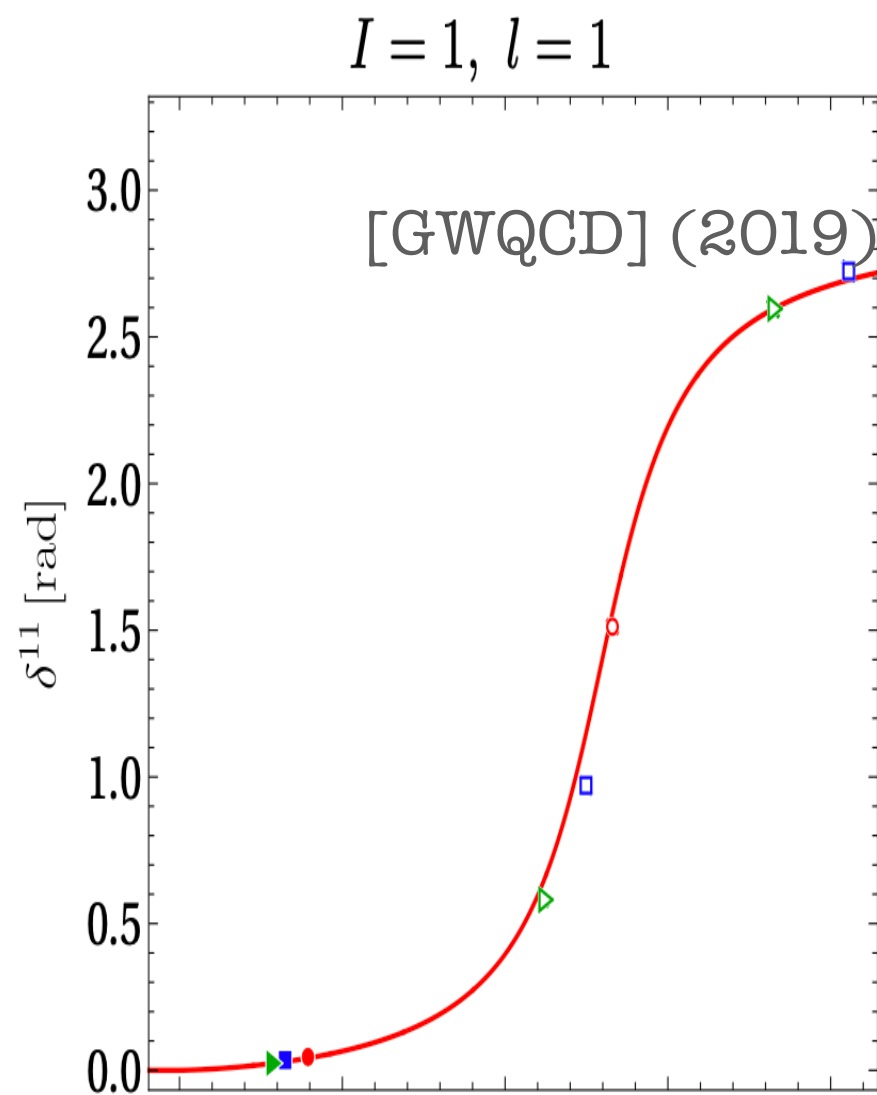
PDG

Our universe

Re $\sqrt{s}$ [MeV]

[1] MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

[2] **Reviews:** Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky
Eur.Phys.J.ST 230 (2021);

More in
M. Döring's talk #1

GLOBAL PROPERTIES

MESON-ELECTROPRODUCTION
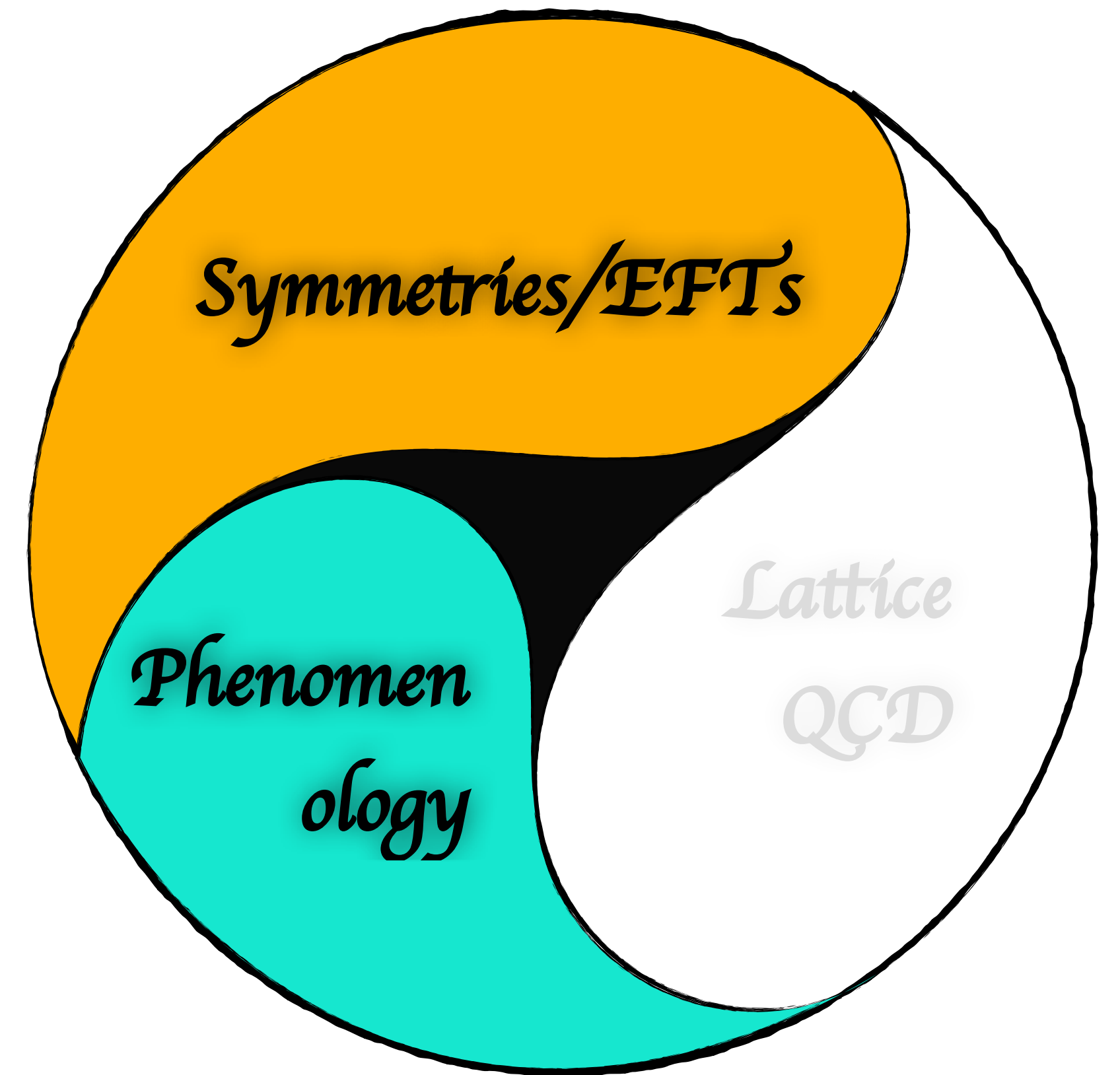
PHOTON-INDUCED EXCITATION

New probe

e^-
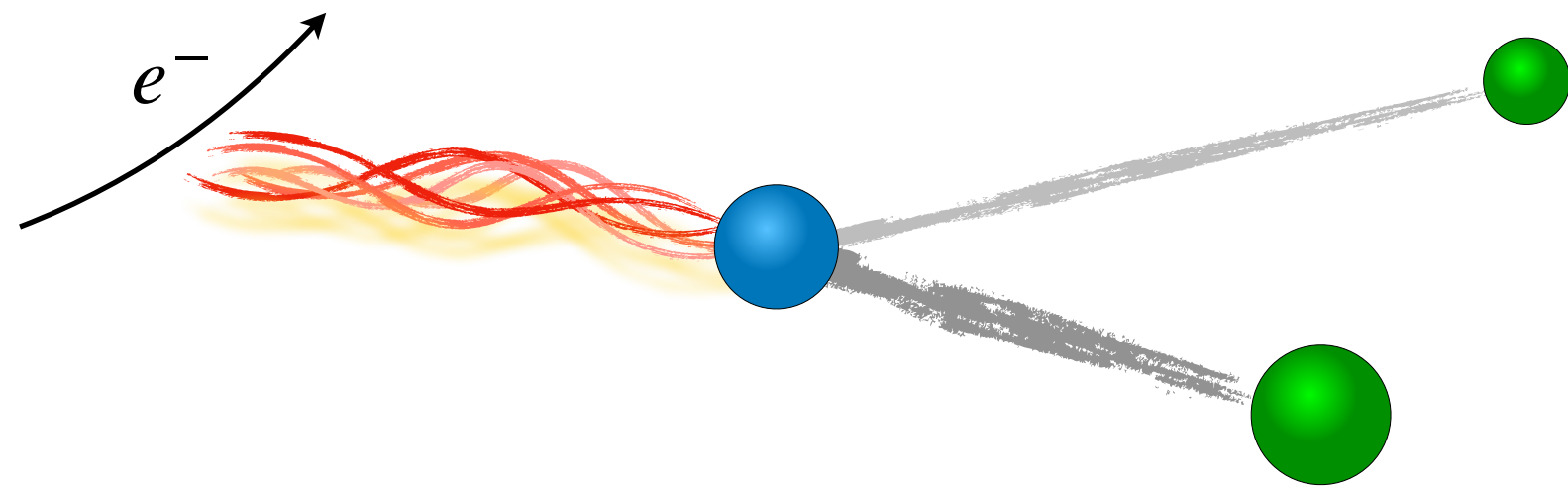
- 5 independent variables
- Momentum transfer dependence
- State compositeness[1] (?)

Experimental accessibility

W / GeV

$\sigma/\mu b$

pX

$p\pi^+\pi^-$

$p\pi^0\pi^0$

$p\pi^0$

$K^+\Lambda$

$p\eta$

E_γ / GeV
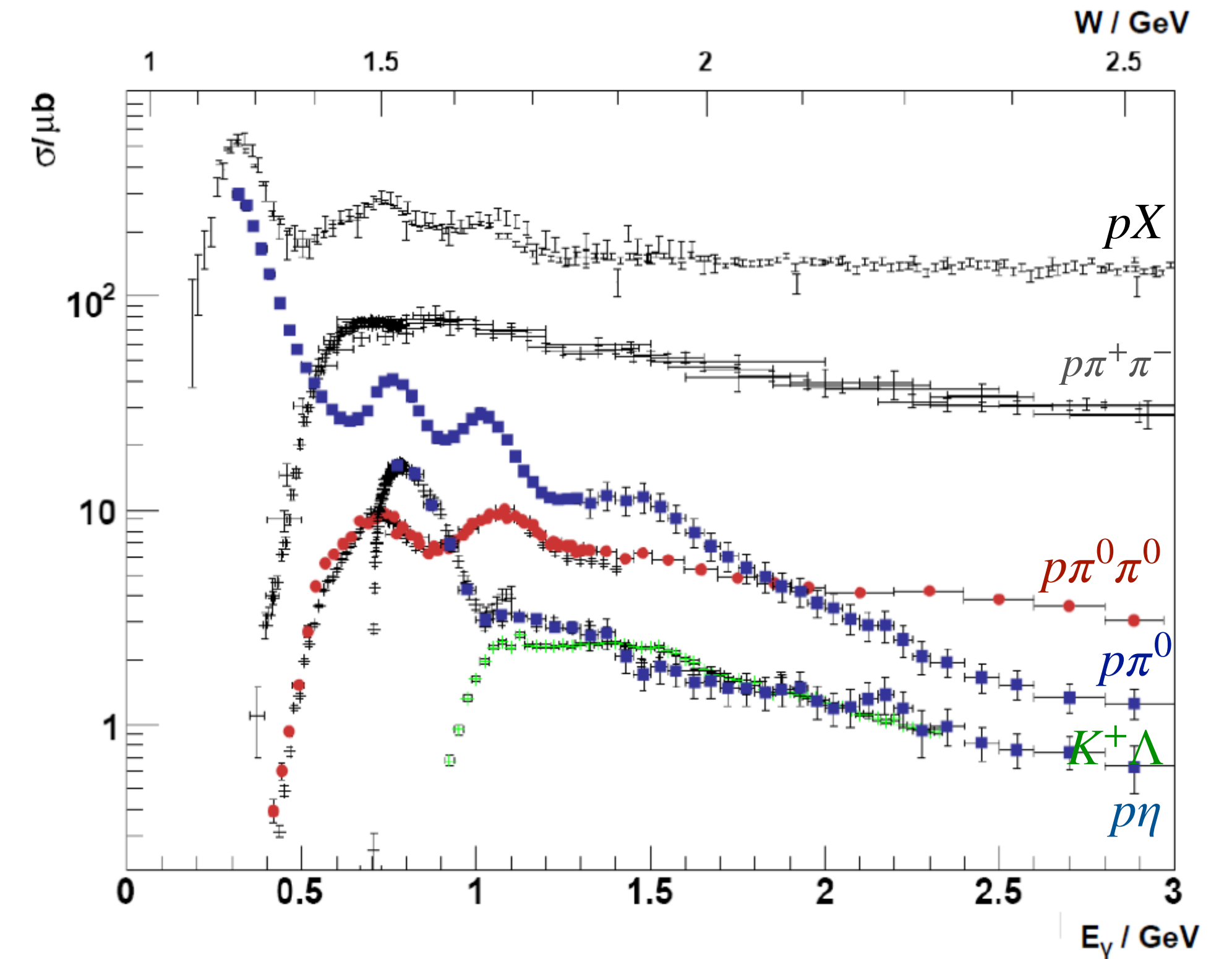
- Large amount of data (10^5)
- more data coming up[1] (Q^2=5-12 GeV²)

[1] e.g., **Review:** Burkert, Roberts, Rev.Mod.Phys. 91 (2019)

[2] Carman, Joo, Mokeev, Few Body Syst. 61, 29 (2020) ... ; [CLAS] Phys.Rev.C 105 (2022) 065201; ...

TRANSITION AMPLITUDES: PREVIOUS APPROACHES

ANL Osaka[1]

(eta)(Kaon) MAID[2]

Ghent group[4]

...

SAID[3]
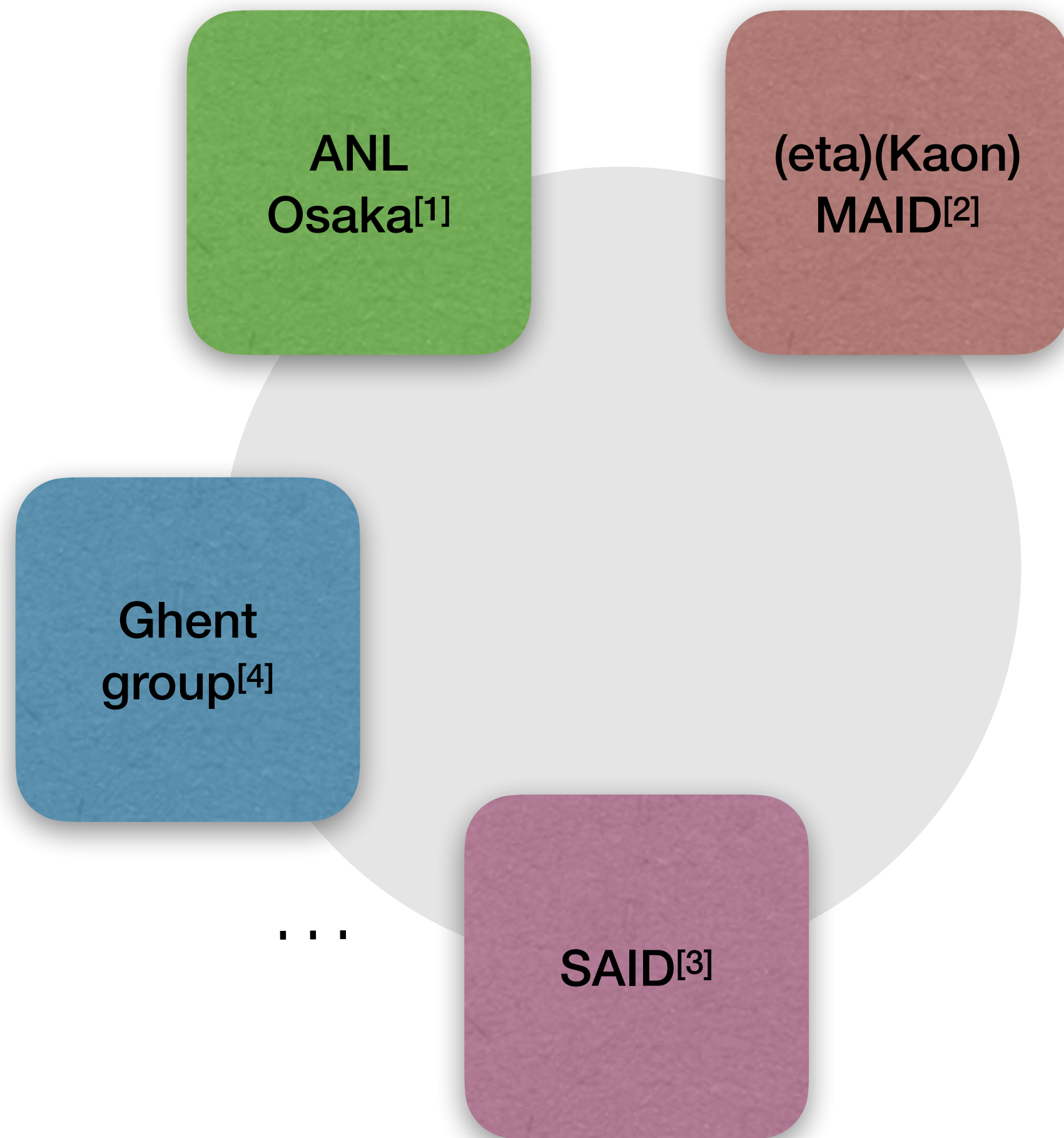
Some highlights

→ simultaneous description of pion photo-/ electroproduction (MAID)

→ low-energy constraints from CHPT (chiral MAID)

→ …

[1] ANL-Osaka PRC 80(2009), Few-Body Syst. 59(2018),... [2] MAID2007, EPJA 34(2007) EtaMAID2018, EPJA 54(2018) [3] SAID, PiN Newsletter 16(2002) [4] Gent group PRC 89(2014),... Aznauryan et al., PRC 80(2009), IJMP(2013),...

TRANSITION AMPLITUDES: NEW APPROACH

ANL Osaka[1]

(eta)(Kaon) MAID[2]

Ghent group[4]

JBW[5]

SAID[3]

...
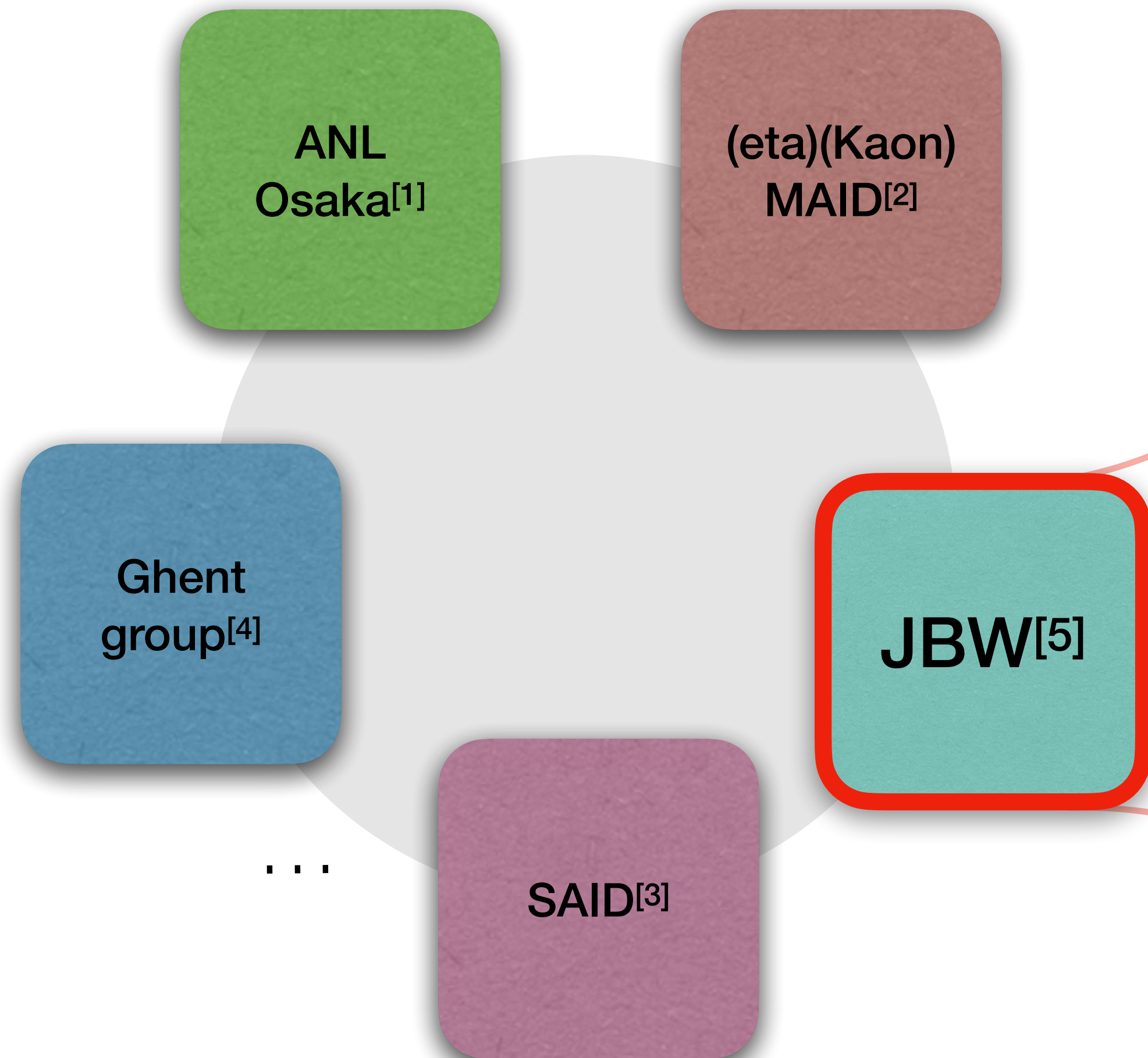
Jülich-Bonn-Washington

- coupled-channel approach (πN, ηN, $K\Lambda$, $K\Sigma$, ...)

- constraints from scattering data and theory:

  → gauge invariance

  → (pseudo)threshold behaviour

  → Final-state unitarity

- web-interface: **https://jbw.phys.gwu.edu/**

More in
M. Döring's talk #2 [6 Jun 2023, 17:20 DAD/5L]

[1] ANL-Osaka PRC 80(2009), Few-Body Syst. 59(2018),... [2] MAID2007, EPJA 34(2007) EtaMAID2018, EPJA 54(2018) [3] SAID, PiN Newsletter 16(2002) [4] Gent group PRC 89(2014),... Aznauryan et al., PRC 80(2009), IJMP(2013),...   [5][JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

INTERPOLATIONS AND PREDICTIONS

Fits accomplished: π^0p/π^+n/ηp/(+$K\Lambda$ upcoming)[1]

Example: Joo data[2]

- not measured quantities can be estimated

- **interpolator** over observable types and kinematics

[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

[2] Joo et al. [CLAS] PRC (2003), PRL (2002)

INTERPOLATIONS AND PREDICTIONS

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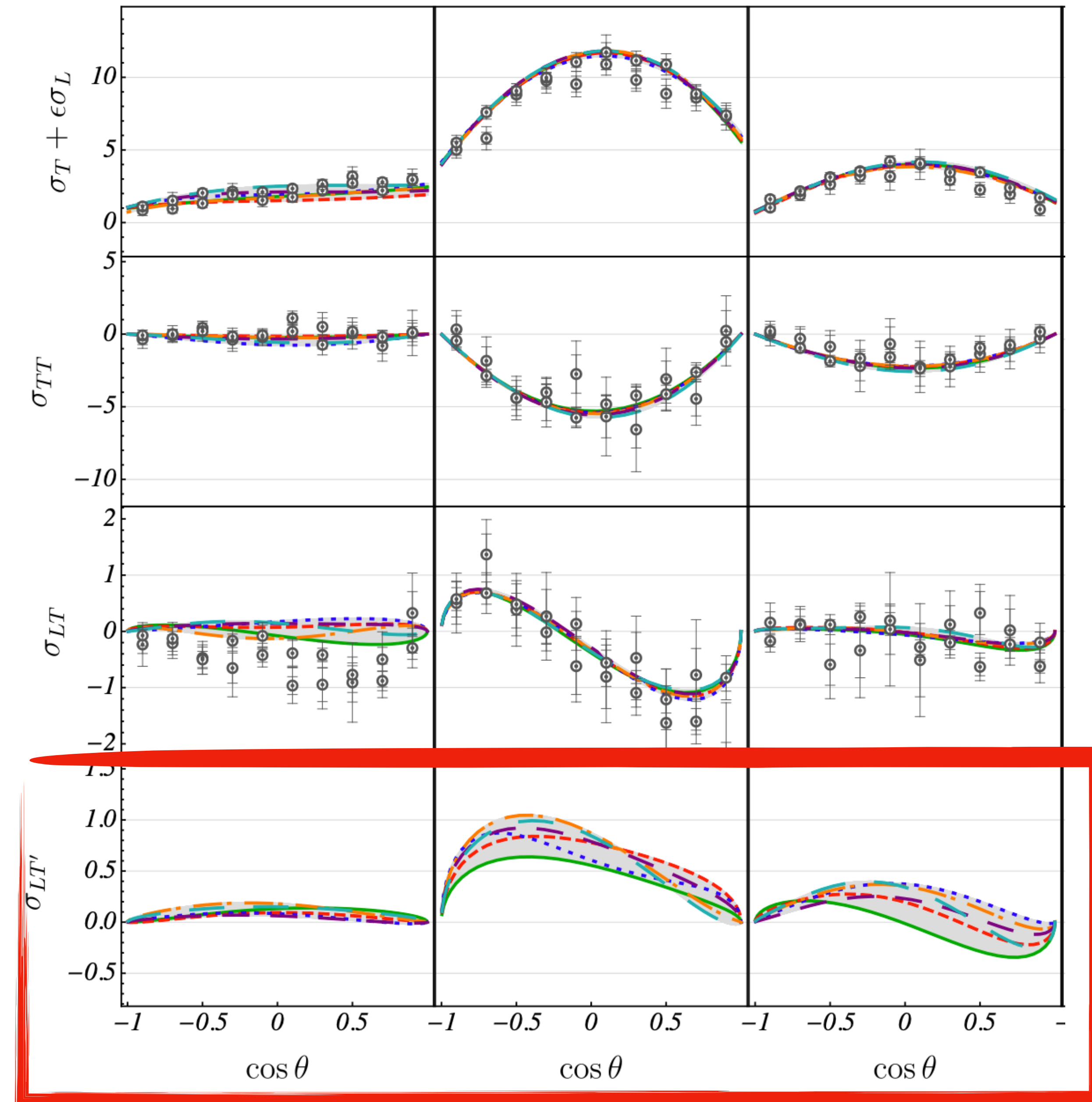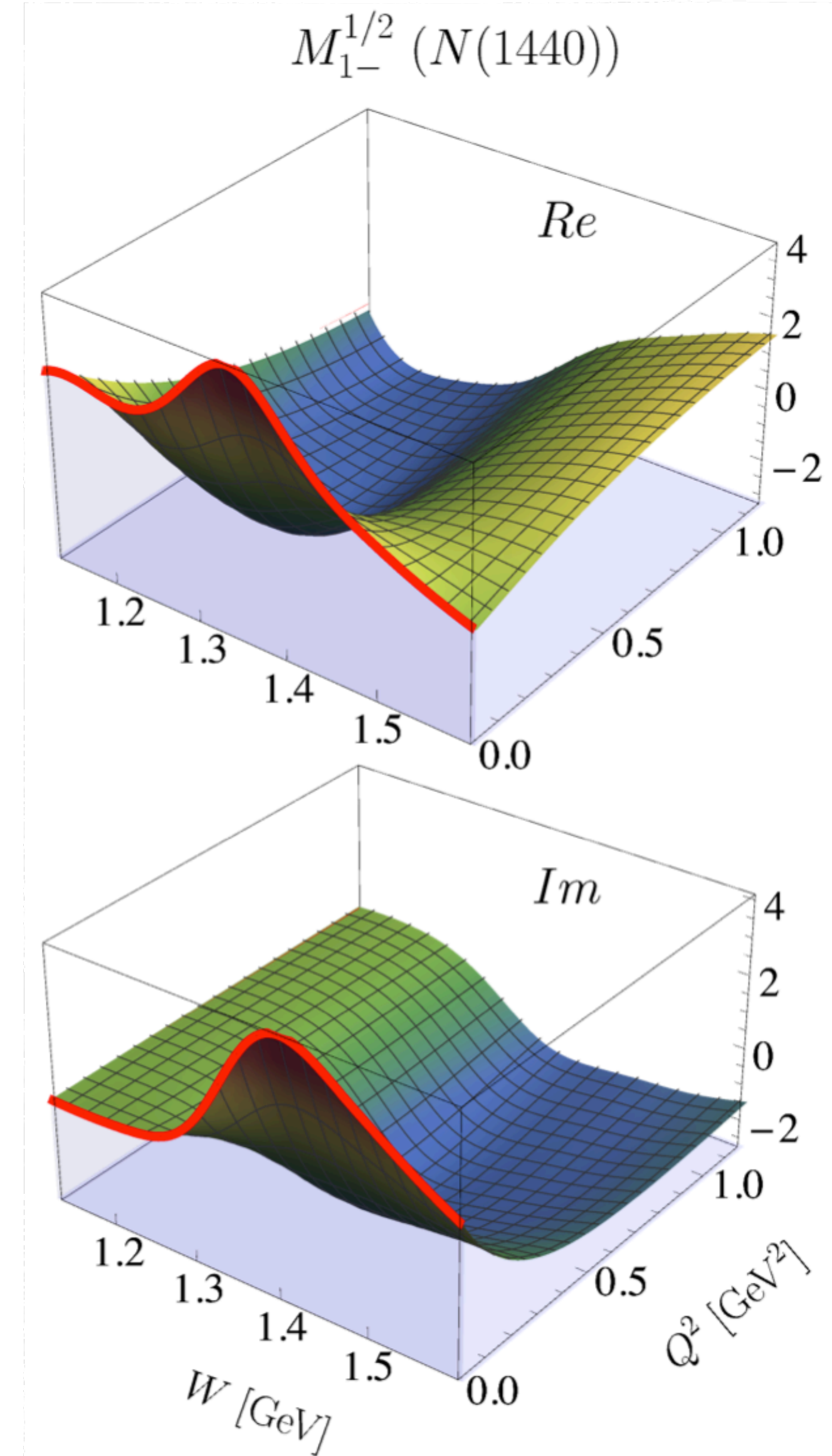[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

[2] Joo et al. [CLAS] PRC (2003), PRL (2002)

INTERPOLATIONS AND PREDICTIONS

Example: Roper(1440)

- Non-trivial Q^2 behaviour

  → complex structure (**3-body effects, …**)

  → Helicity couplings (upcoming)

[1] [JBW] MM et al. Phys.Rev.C 103 (2021) 6 / Phys.Rev.C 106 (2022) 015201

New synergetic approaches to universal parameters of resonance become available
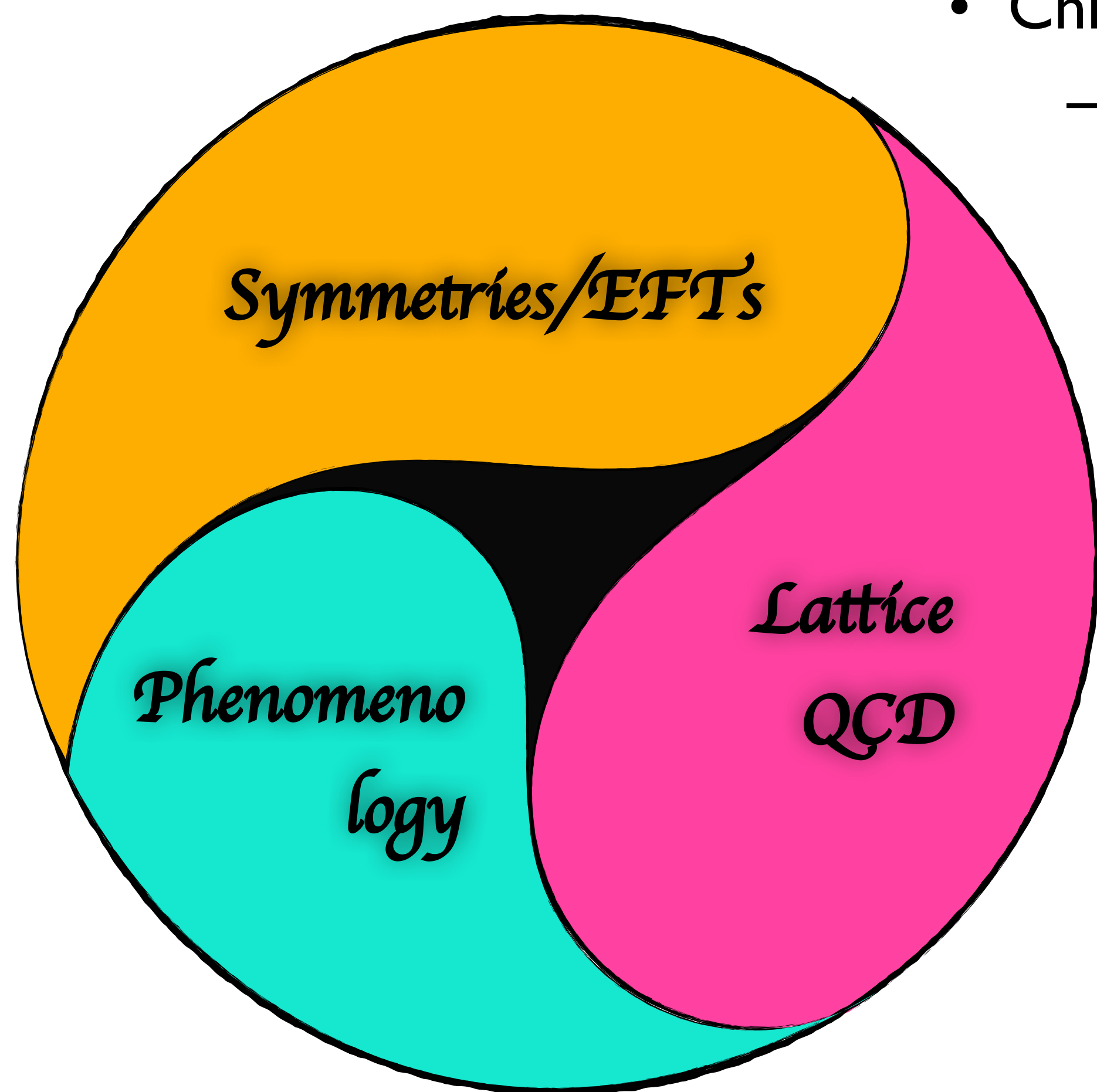
- **Chiral unitary models**
  - →QCD symmetries constraints to hadron-hadron dynamics
  - →Strong predictive power

- **Lattice hadron spectroscopy**
  - →Novel 3-body methodology has matured
  - →EFTs and S-matrix theory: bridge to real world physics

- **Phenomenological models**
  - →Link between observables and transition amplitudes

THANK YOU

TRANSITION AMPLITUDE

Final-state unitarity

- Jülich-Bonn dynamical coupled-channel model[1]
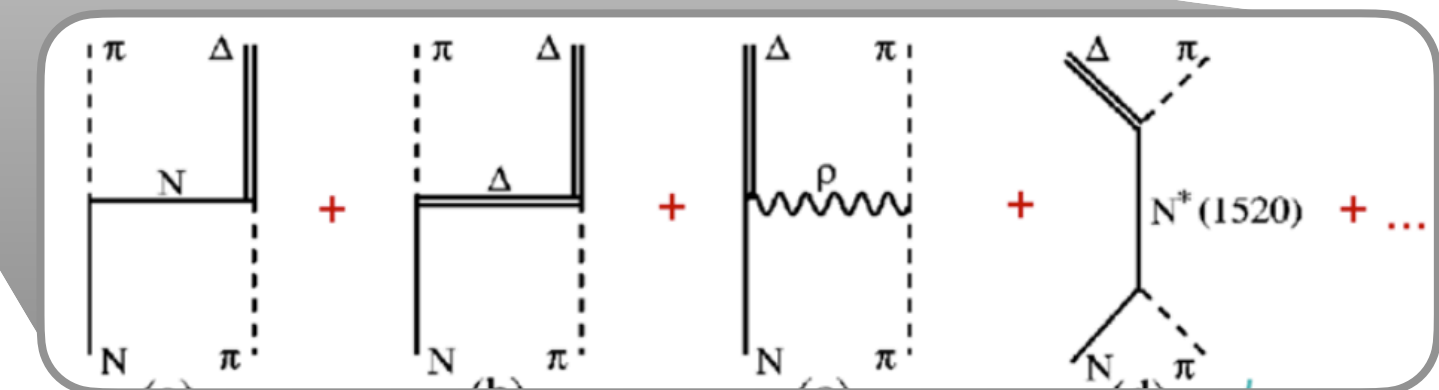- Amplitudes fixed from scattering and photoproduction
  data

  $\pi N \rightarrow xX$ and $\gamma N \rightarrow xX$ (~60k data)
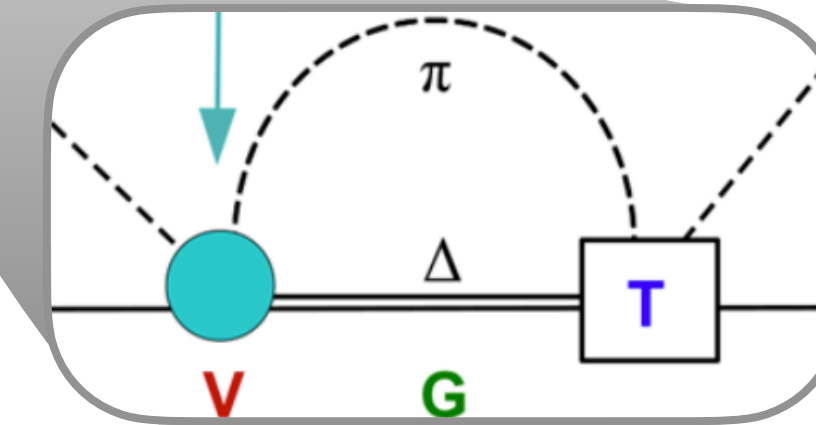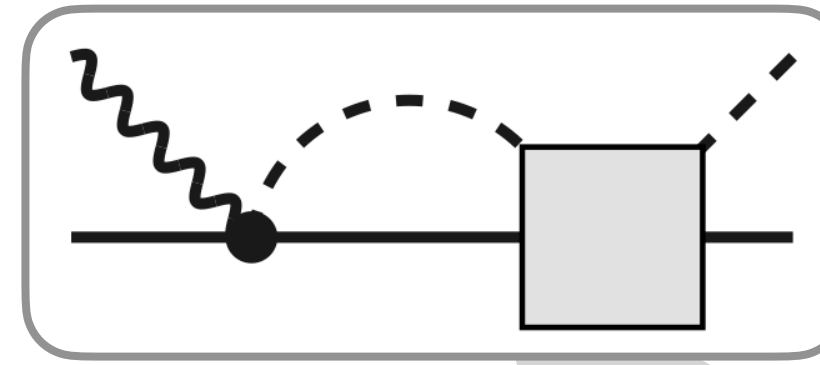
SYMMETRIES

Five kinematical variables (3*(2+3)-10=5)

- total energy: **W**

- photon virtuality: **Q²**

- angles: $\theta_e, \theta, \varphi$

Underlying objects[1,2]:

- Helicity amplitudes:

- Multipoles:

$$\{H_i(W, Q^2, \theta) \mid i = 1..8\}$$

$$\mathcal{M}_{\ell\pm}(W, Q^2) \in \{E_{\ell\pm}(W, Q^2), L_{\ell\pm}(W, Q^2), M_{\ell\pm}(W, Q^2)\}$$
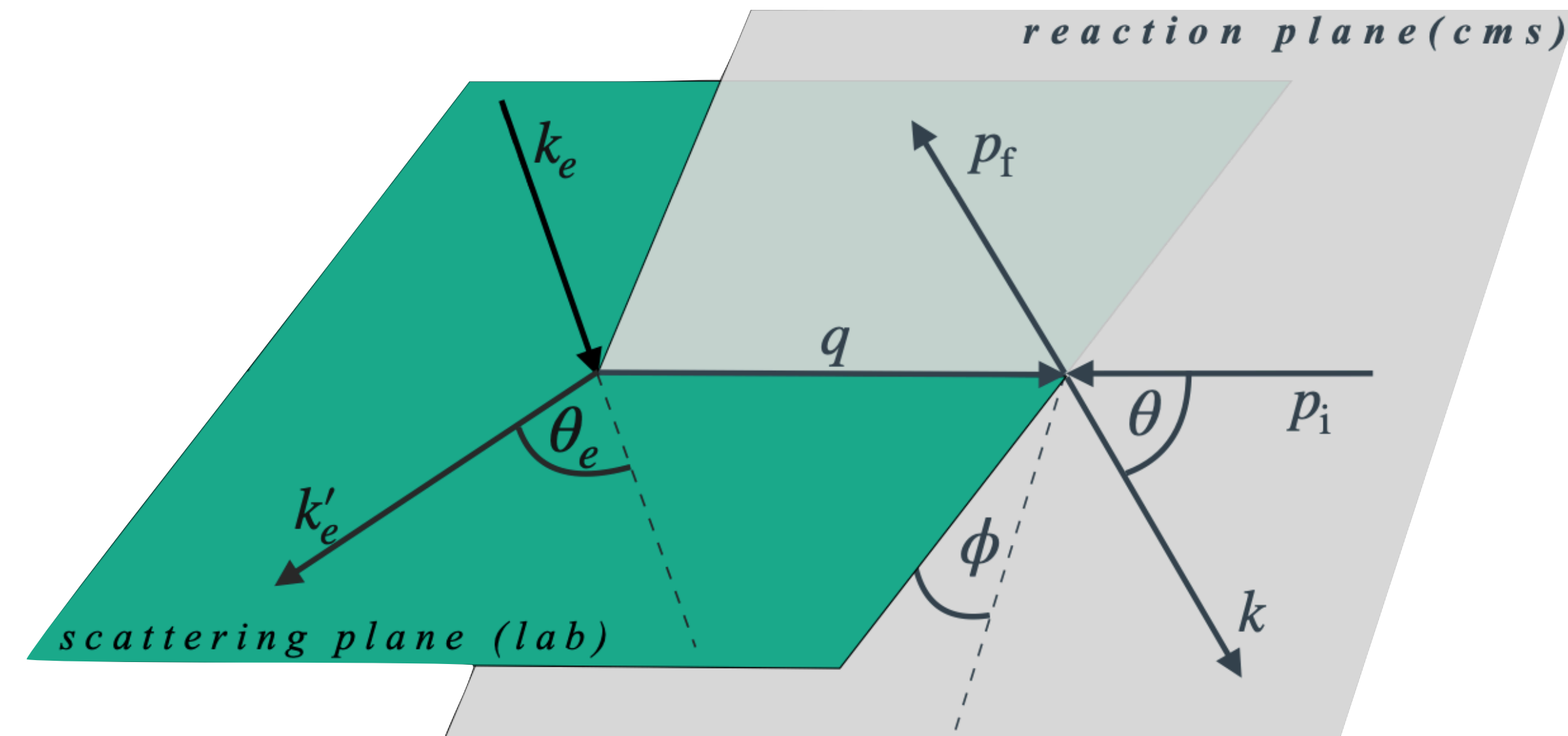
1) Chew et al. Phys.Rev. 106 (1957); Dennery Phys.Rev. 124 (1961); Berends et al. Nucl.Phys.B 4 (1967);

2) (for explicit formulas) MM et al. Phys.Rev.C 103 (2021)

THEORETICAL CONSTRAINTS

Gauge invariance

- manifest implementation[1] too costly
- instead Ward-Takahashi identity by construction

$$H_7 = \sum_{i=1}^{6} a_i H_i \quad H_8 = \sum_{i=1}^{6} b_i H_i$$

Pseudo/threshold constraints:

Siegert's theorem[2]

- Long-wavelength limit: electric and magnetic multipoles

  are related

  ... fewer parameters needed 👍

$$lim_{k \to 0} E_{\ell+} = k^{\ell} \dots$$

$$L_{\ell\pm} \sim E_{\ell\pm} \quad \textbf{for} \quad q = 0$$

1) Afnan et al.(1995); Kvinikhidze et al.(1999); Haberzettl(19xx-2021); Borasoy et al.
   (2007); Ruic et al.(2011); MM et al. (2012); Bruns, Cieplý, MM 2206.08767 [nucl-th]
2) Siegert(1973) Amaldi et al.(1979) Tiator(2016)

MULTIPOLES

$$\mathscr{M}_{\mu\gamma*}(W, Q^2) = R_{\ell'}(\lambda, q/q_\gamma)\left[V_{\mu\gamma*}(W, Q^2) + \sum_\kappa \int_0^\infty dp\, p^2\, T_{\mu\kappa}^{\text{JUBO}}(p, W)G_\kappa(p, W)V_{\kappa\gamma*}(p, W, Q^2)\right]$$

$$V_{\mu\gamma}^{\text{JUBO}}(W)e^{-\beta_\mu^0 Q^2/m_p^2}\left(1 + Q^2/m_p^2\beta_\mu^1 + (Q^2/m_p^2)^2\beta_\mu^2\right)$$

👍 Fulfils:

- Final state unitarity / Gauge invariance / Siegert's theorem / Threshold behaviour

👍 Describes:

- Scattering and photoproduction data -- parameters (λ, β) from fits to electroproduction data

🙈 Parametrization dependence due to incomplete data
    ... even for a truncated complete electroproduction experiment
    ... in future: Bias-variance tradeoff with statistical criteria

Tiator et al.(2017)

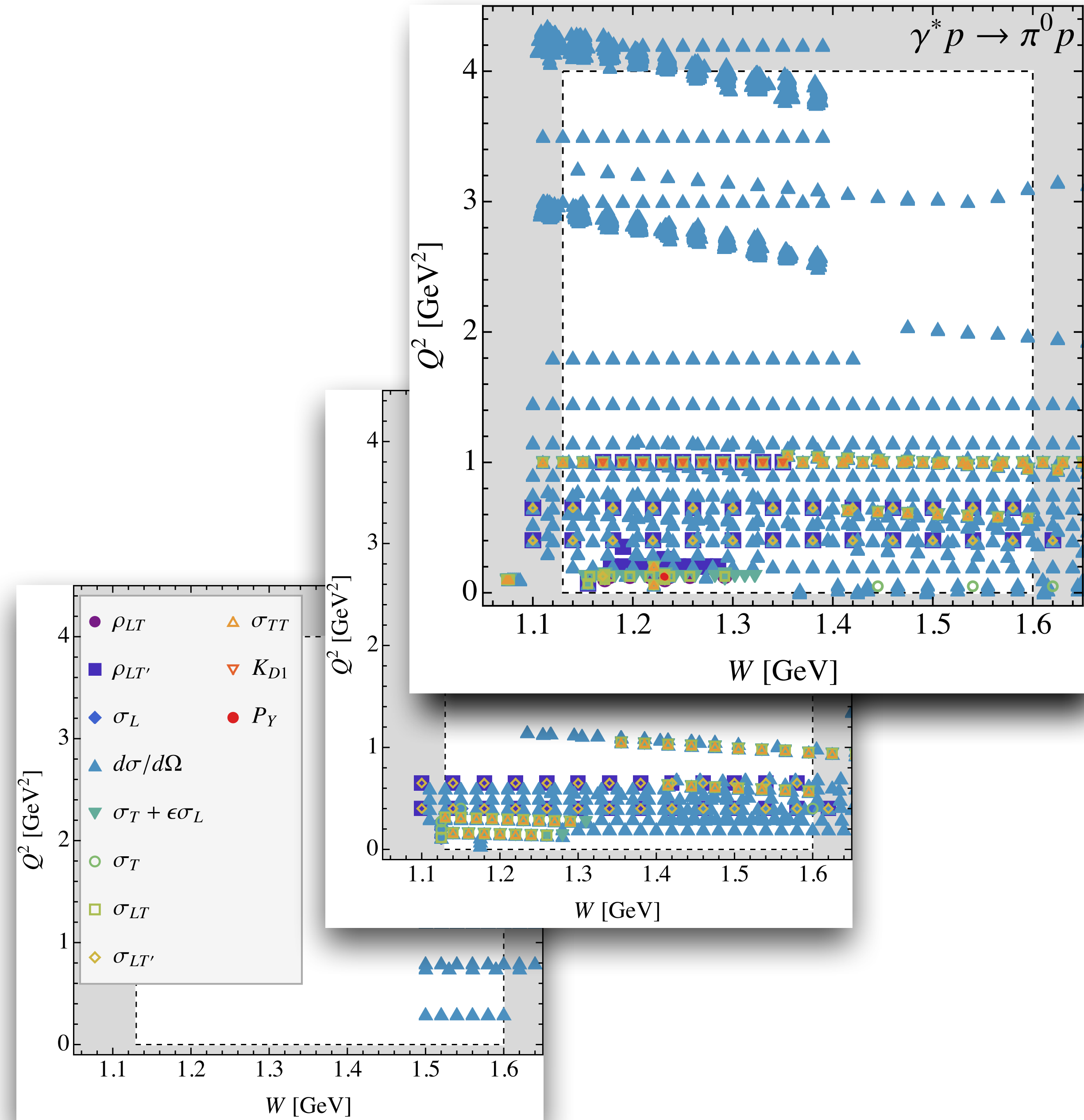Landay et al., Phys.Rev.D (2019), 1810.00075 [nucl-th]

(2019)

DATA AND PARAMETERS

Experimental data

- 45k(π^0p) + 37k(π^+n) + 2k(ηp) = 84k data

Parameters $\{\lambda, \beta\}$

- S/P/D waves

- 26 multipoles × (10..13 pars) = 257 pars

1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

DATA AND PARAMETERS

Fits[1]:

- multiple solutions obtained

- systematic uncertainties studied

	χ^2/dof	$\chi^2_{\pi^0 p/\text{data}}$	$\chi^2_{\pi^+ n/\text{data}}$	$\chi^2_{\eta p/\text{data}}$
$\mathfrak{F}_1^{\text{reg}}$	1.66	1.68	1.61	1.77
$\mathfrak{F}_2^{\text{reg}}$	1.73	1.71	1.71	2.29
$\mathfrak{F}_3^{\text{reg}}$	1.69	1.69	1.66	1.89
$\mathfrak{F}_4^{\text{reg}}$	1.69	1.7	1.64	2.05
$\mathfrak{F}_1^{\text{wt}}$	1.54	1.74	1.63	1.25
$\mathfrak{F}_2^{\text{wt}}$	1.63	1.82	1.79	1.27

1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

GLOBAL FEATURES

Predictive power

- example: Joo data[2]
- not measured quantities can be estimated:
  - **interpolator** over observable types and kinematics
  - already usable through web-interface

https://jbw.phys.gwu.edu/
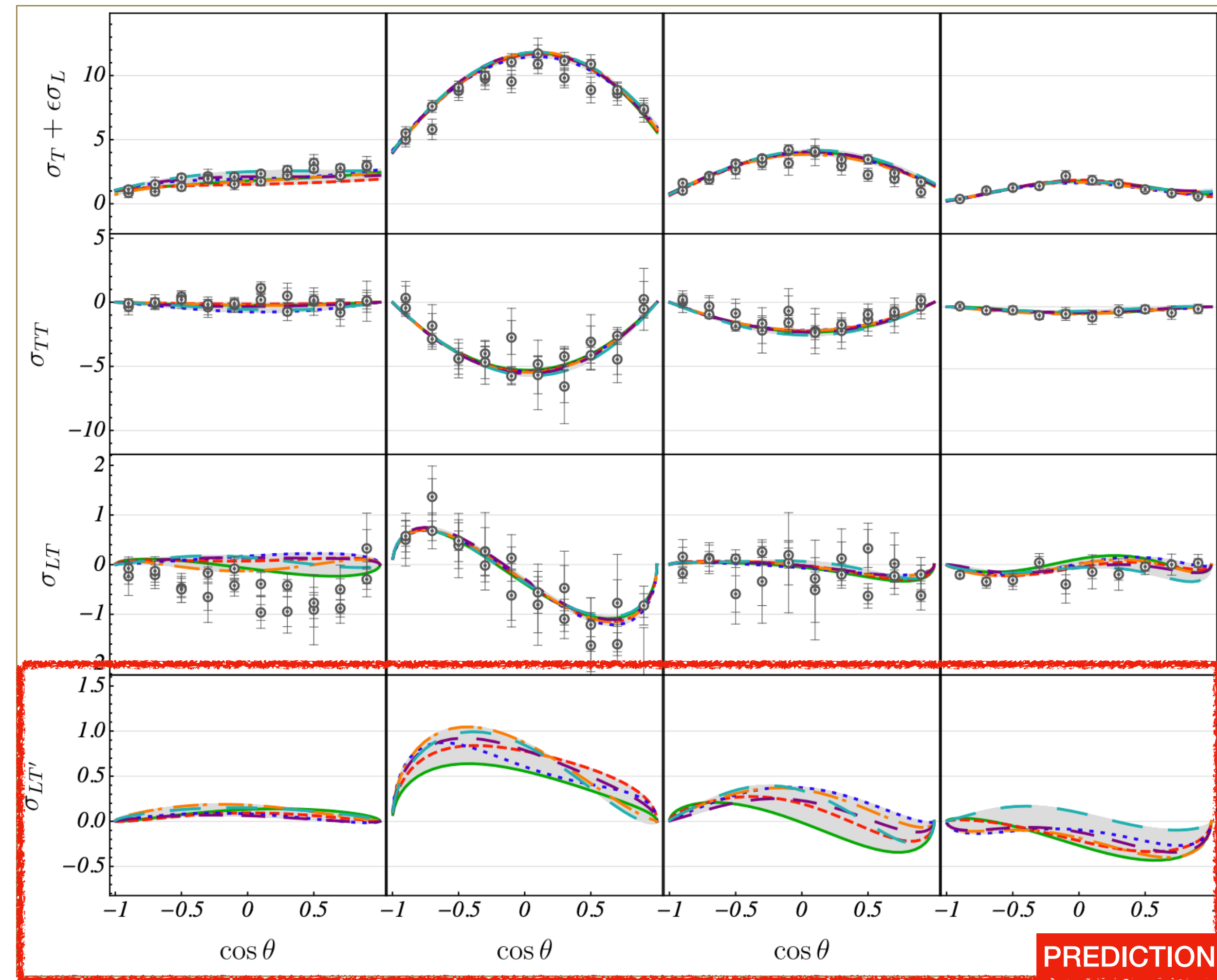
1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

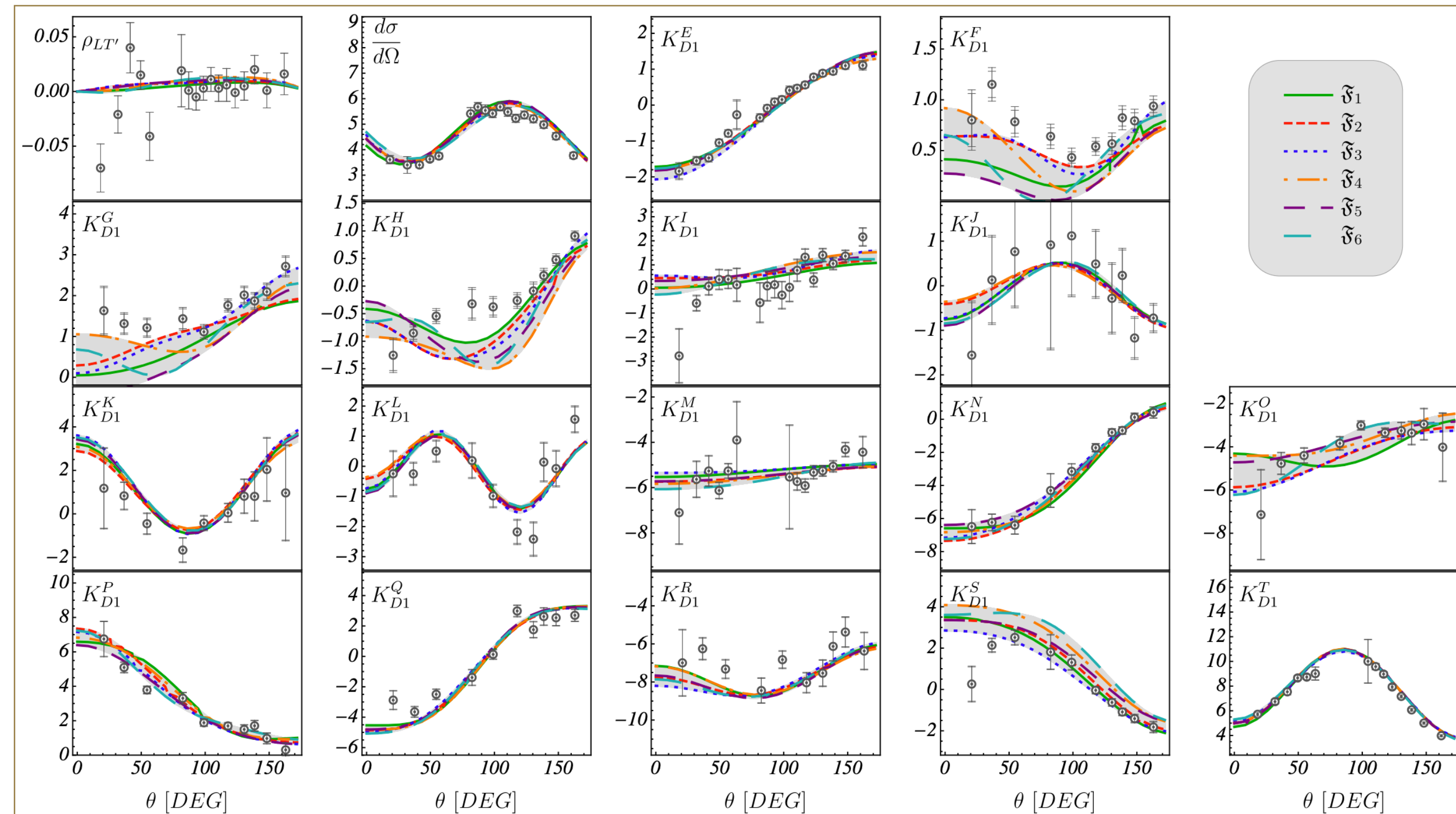2) Joo et al. [CLAS] PRC (2003), PRL (2002)

RESULTS

πN data fits[1]:

→all strategies converge

→different minima (systematic uncertainties)

→Kelly data[2]

1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

2) Jefferson Lab Hall A Collaboration Phys.Rev.Lett. 95 (2005) 102001

RESULTS

Fits[1]:

→ all strategies converge

→ different minima (systematic uncertainties)

Fit	σ_L		$d\sigma/d\Omega$		$\sigma_T + \epsilon\sigma_L$		σ_T		σ_{LT}		$\sigma_{LT'}$		σ_{TT}		K_{D1}		P_Y		ρ_{LT}		$\rho_{LT'}$		χ^2_{dof}
	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	$\pi^0 p$	$\pi^+ n$	
$\mathfrak{F}_1$	–	9	65355	53229	870	418	87	88	1212	133	862	762	4400	251	4493	–	234	–	525	–	3300	10294	**1.77**
$\mathfrak{F}_2$	–	4	69472	55889	1081	619	65	78	1780	150	1225	822	4274	237	4518	–	325	–	590	–	3545	10629	**1.69**
$\mathfrak{F}_3$	–	8	66981	54979	568	388	84	95	1863	181	1201	437	3934	339	4296	–	686	–	687	–	3556	9377	**1.81**
$\mathfrak{F}_4$	–	22	63113	52616	562	378	153	107	1270	146	1198	1015	4385	218	5929	–	699	–	604	–	3548	11028	**1.78**
$\mathfrak{F}_5$	–	20	65724	53340	536	528	125	81	1507	219	1075	756	4134	230	5236	–	692	–	554	–	3580	11254	**1.81**
$\mathfrak{F}_6$	–	18	71982	58434	1075	501	29	68	1353	135	1600	1810	3935	291	5364	–	421	–	587	–	3932	11475	**1.78**

1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201
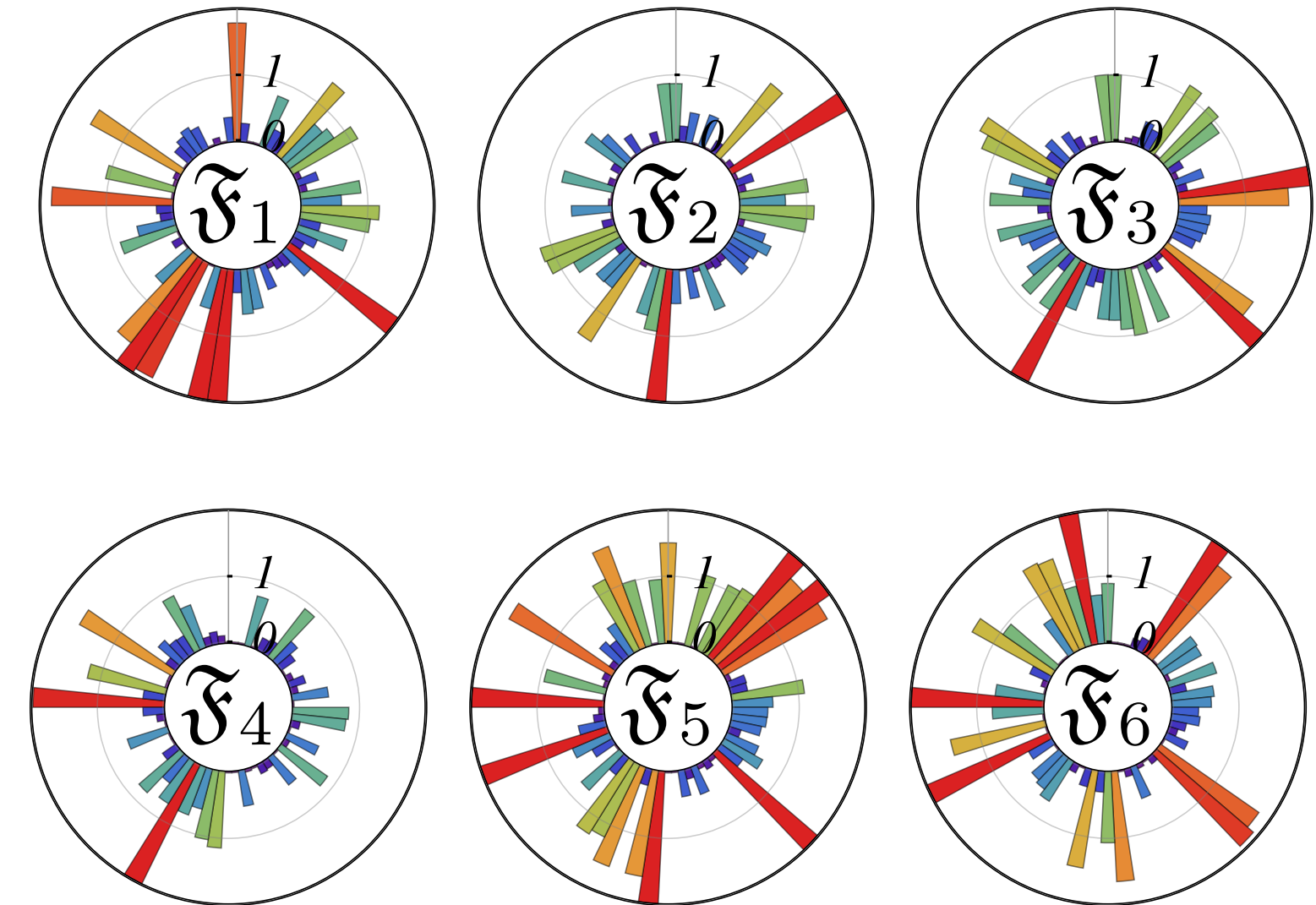
MULTIPOLES

Observable (e.g. cross section)
$$\frac{d\sigma^v}{d\Omega}\left(W, Q^2, \epsilon, \theta, \phi\right) = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \ldots$$

Structure functions
$$\sigma_T\left(W, Q^2, \theta\right) = k/q_\gamma\left(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2\right)/2, \ldots$$

Helicity amplitudes
$$H_1\left(W, Q^2, \theta\right) = \sin\theta\cos\theta/2(-\mathscr{F}_3 - \mathscr{F}_4)/\sqrt{2}, \ldots$$

CGLN amplitudes
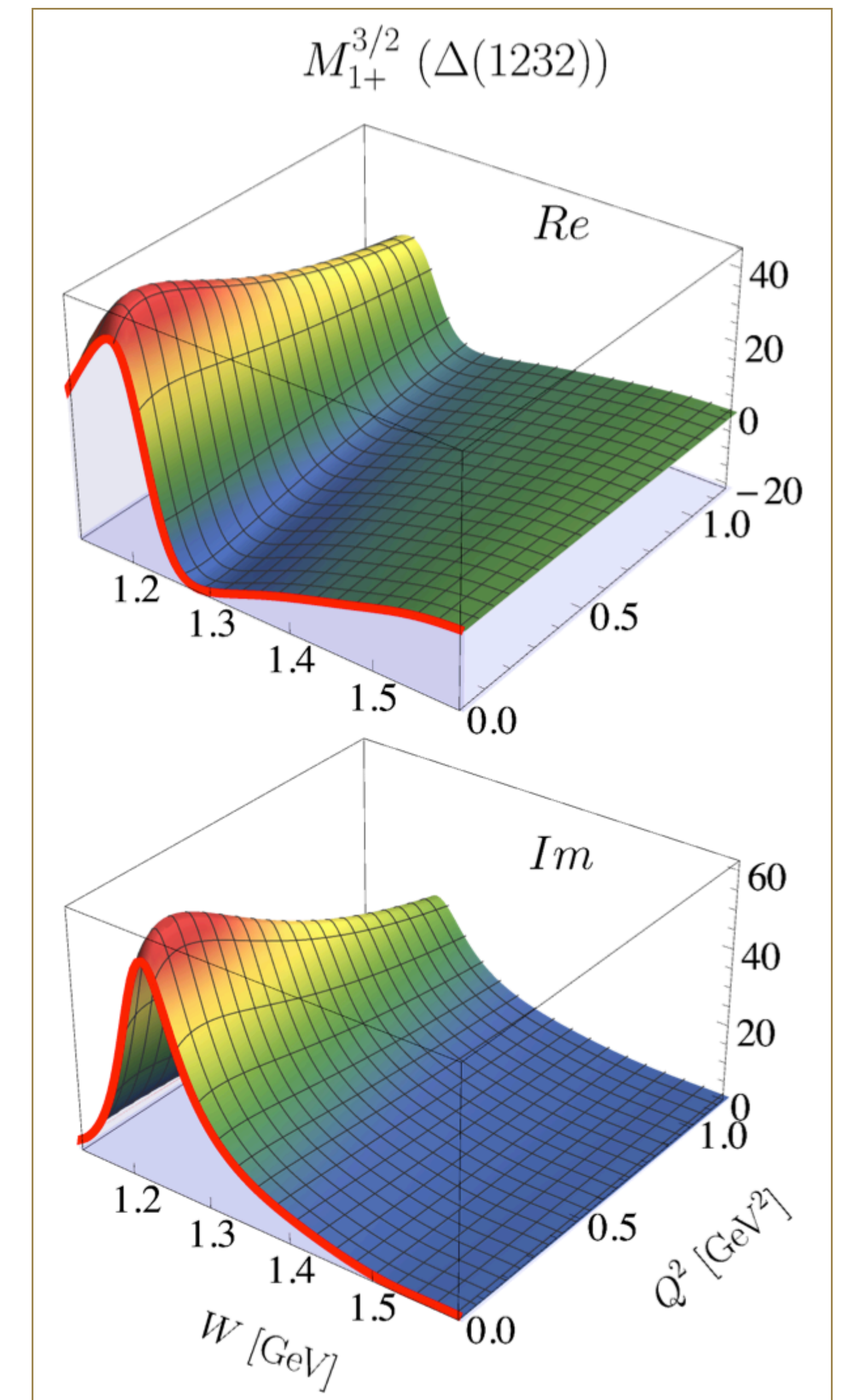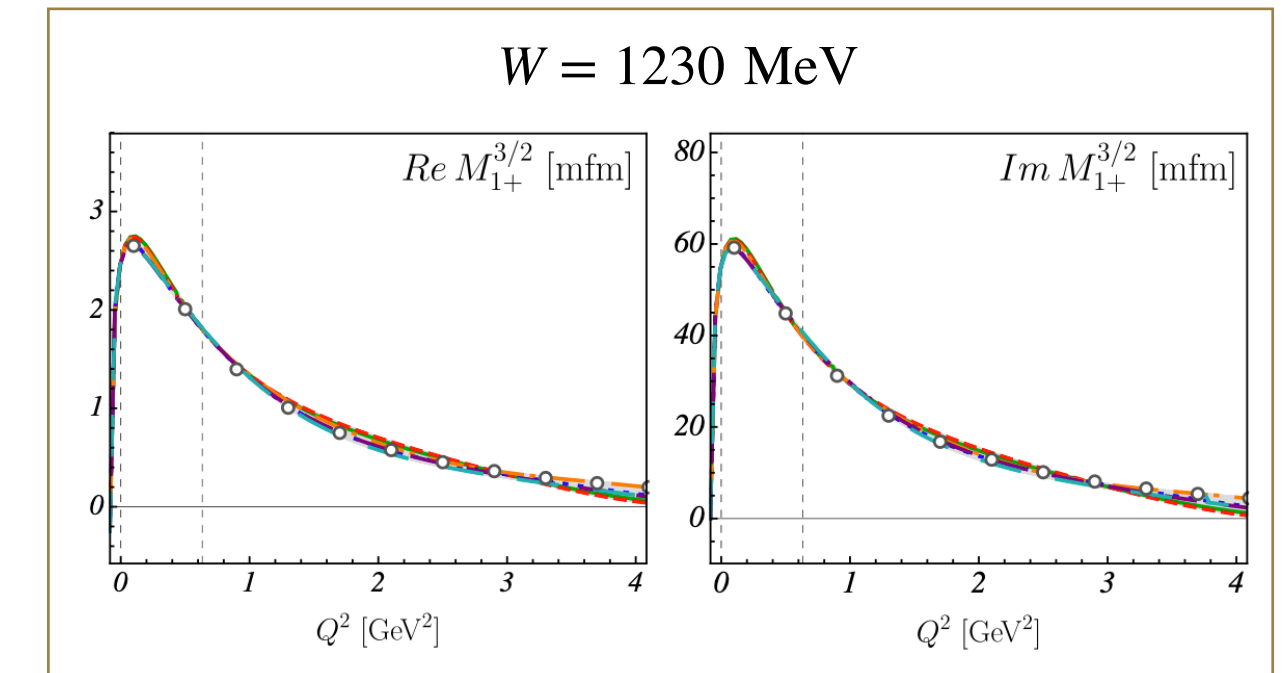$$\mathscr{F}_1\left(W, Q^2, \theta\right) = \sum_{\ell>0}\ell M_{\ell+}(W, Q^2)P'_{\ell+1}(\cos\theta) + \ldots$$

Multipoles
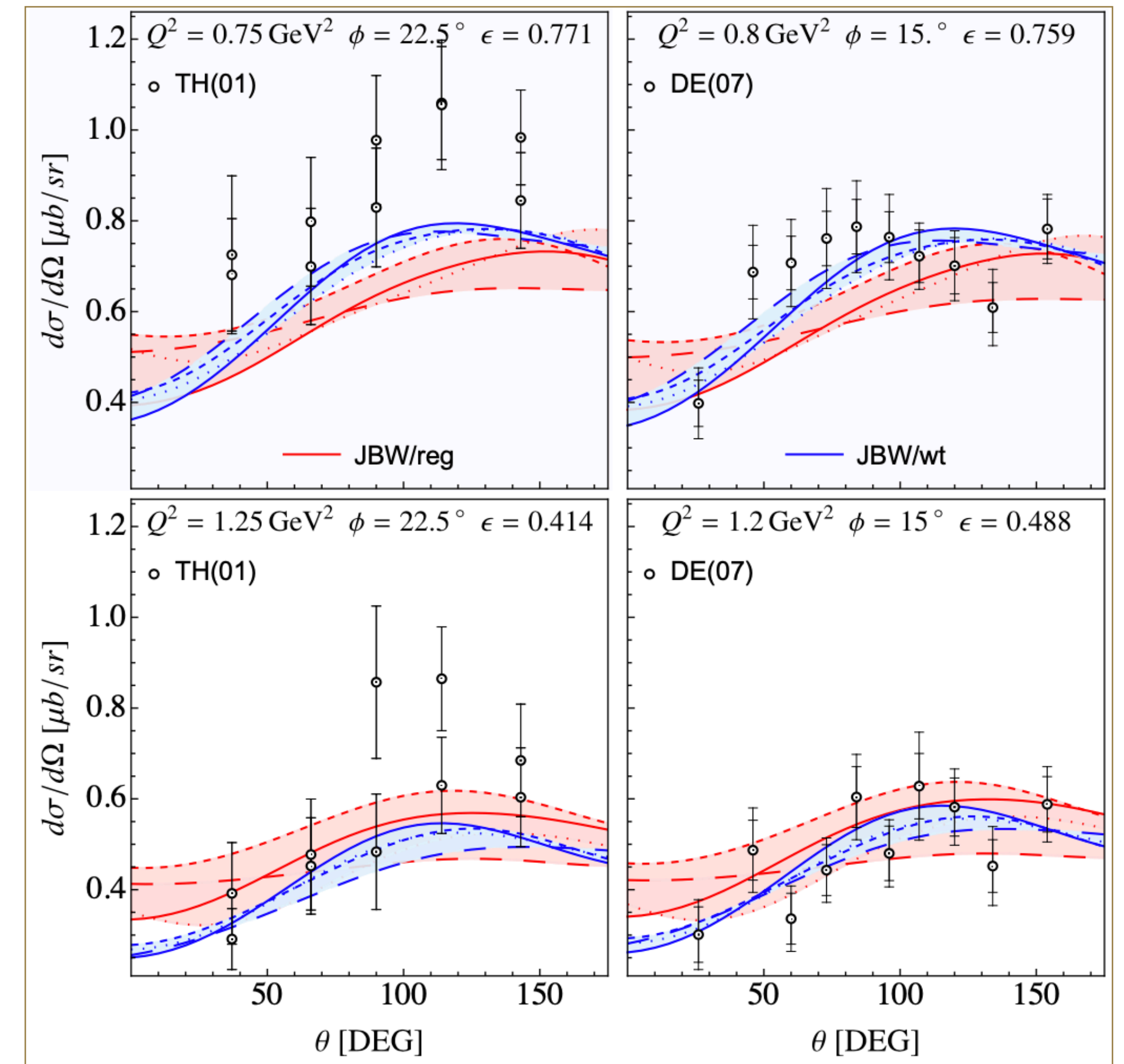$$\{E_{\ell\pm}(W, Q^2), L_{\ell\pm}(W, Q^2), M_{\ell\pm}(W, Q^2)\}$$

Chew et al. Phys.Rev. 106 (1957); Dennery Phys.Rev. 124 (1961); Berends et al. Nucl.Phys.B 4 (1967); ... (for explicit formulas) MM et al. Phys.Rev.C 103 (2021)

RESULTS

$W = 1230$ MeV

$Re\ M_{1+}^{3/2}$ [mfm] $Im\ M_{1+}^{3/2}$ [mfm]

Q^2 [GeV^2] Q^2 [GeV^2]

Delta(1232):

- Large multipoles well determined
- simple Q^2 dependence

$M_{1+}^{3/2}$ ($\Delta(1232)$)

Re

Im

W [GeV] Q^2 [GeV^2]

[JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

RESULTS

Ambiguities in the data:

- example ηN data[2]

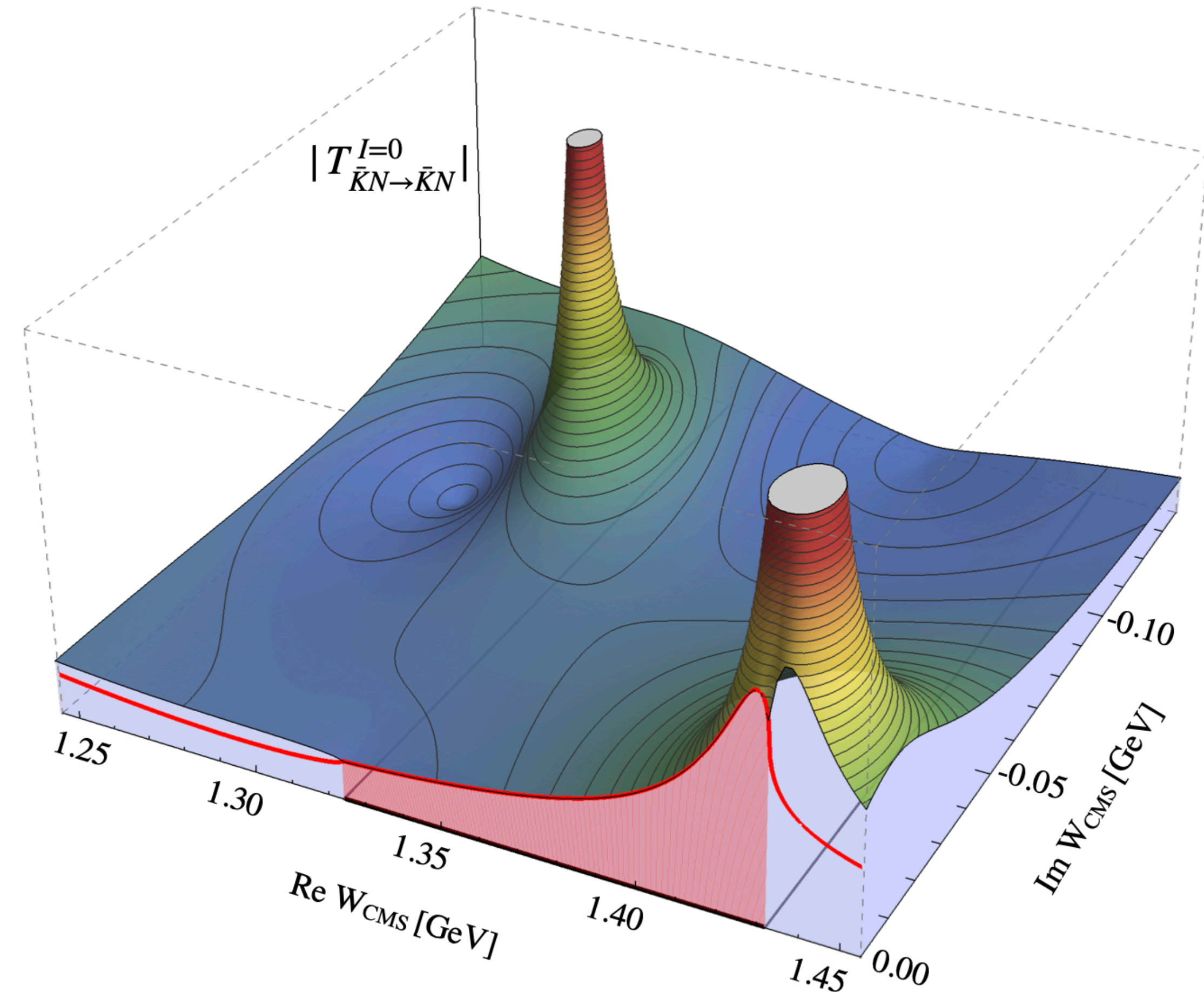1) [JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

2) H. Denizli et al. (CLAS) PRC 76, 015204 (2007); Thompson et al. (CLAS), PRL86, 1702-1706 (2001); ...

RESONANCE POLE(S)

- Inclusion of chiral symmetry constants demands a second state[1]:

$$W* = (1325\ldots1381)-i(56\ldots114)\ MeV$$

→ Common phenomenon in hadron physics[2]

$|T_{\bar{K}N\to\bar{K}N}^{I=0}|$

Im W_{CMS} [GeV]

Re W_{CMS} [GeV]

-0.10

-0.05

0.00

1.25 1.30 1.35 1.40 1.45

1)Oller/Meißner (2001); Ikeda/Hyodo/Weise(2011); MM/Meißner(2012); Guo/Oller(2012),...
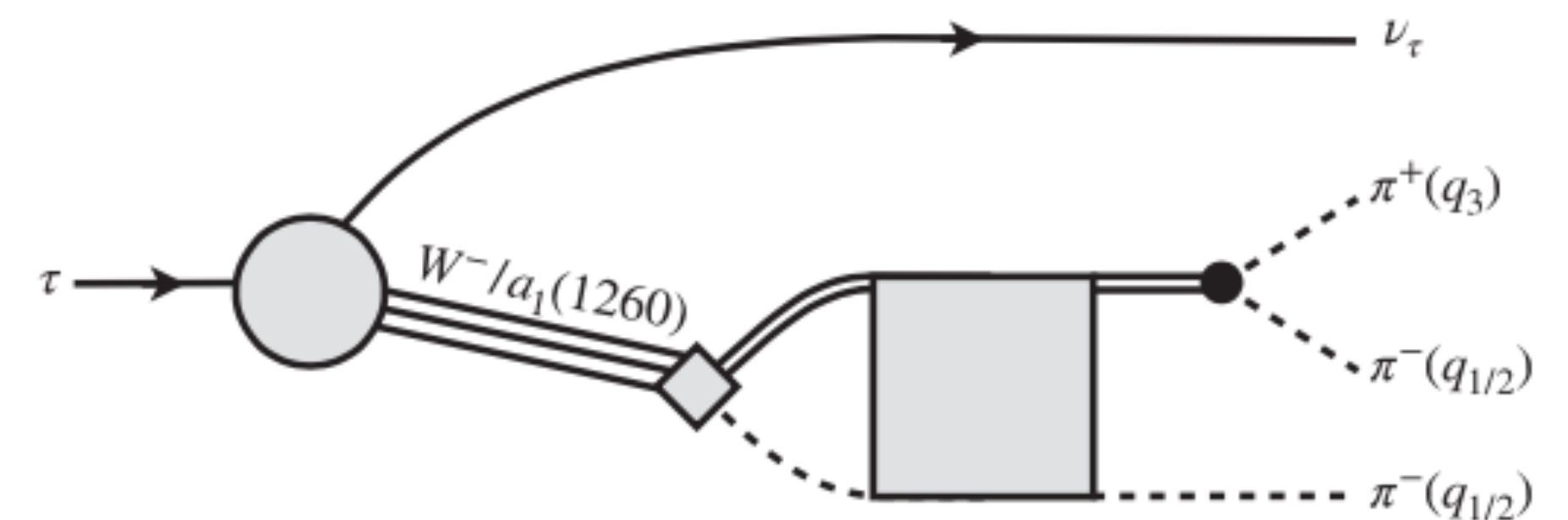2)Meißner, Symmetry 12 (2020) 6, 981

HADRONIC 3-BODY PROBLEM: IMPACT

Intricate kinematics/dynamics

- 8 variables
- 2-body sub-channel dynamics

Hadron spectroscopy riddles

- Roper(1440) → $\pi\pi$N [first FV evaluations[1]]

- X(3872) → DbarDπ

- a_1(1260) → $\pi\pi\pi$

- ...

- Beyond Standard Model: τ-EDM

- Precision physics: rare hadronic W-decays[2]
- Exotic states of matter[3]

1) Severt/MM/Meißner JHEP04(2023)   >>>   PHD talk on Friday

2) Sirunyan et al. [CMS@CERN] PRL122

3) Experimental programs: GlueX@JLAB; COMPASS@CERN;

BROADER IMPACT

Twice non-perturbative regime of QCD
- too low for perturbative QCD
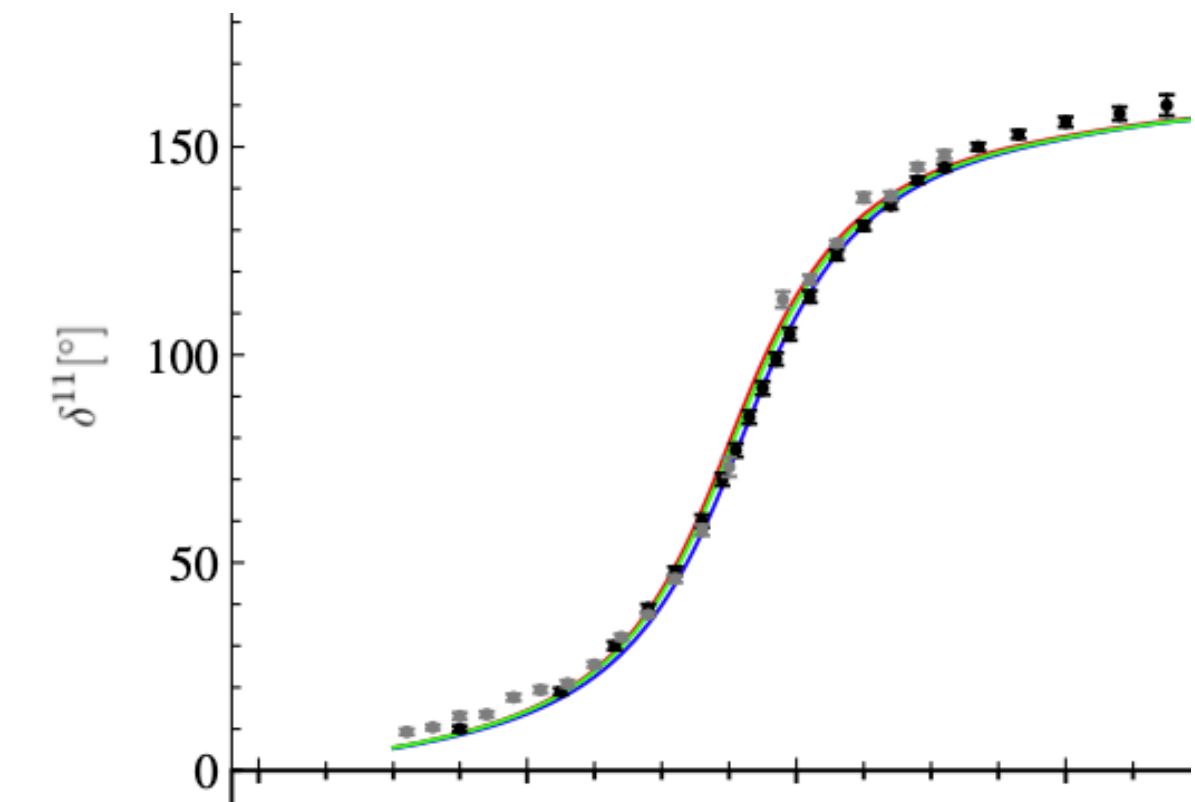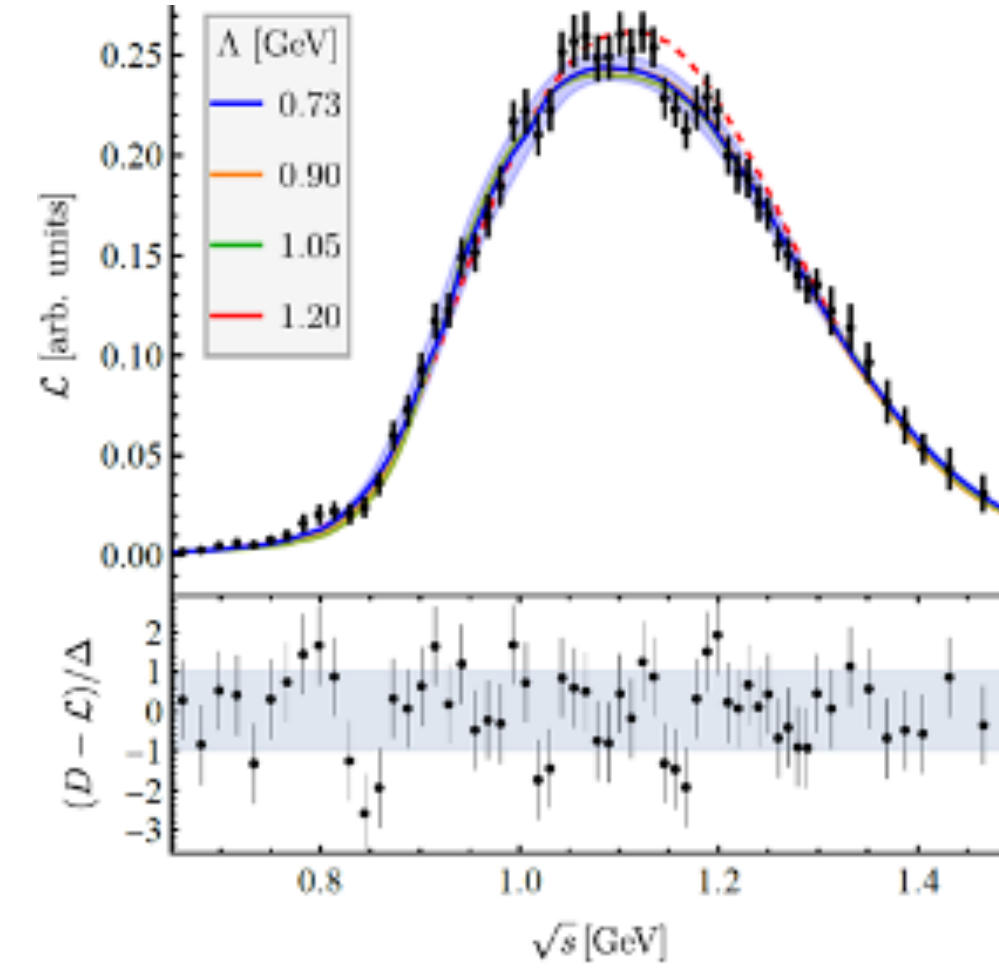- too high for low-energy EFT

KbarNN & KbarNNN bound states
- dominated by KbarN interaction
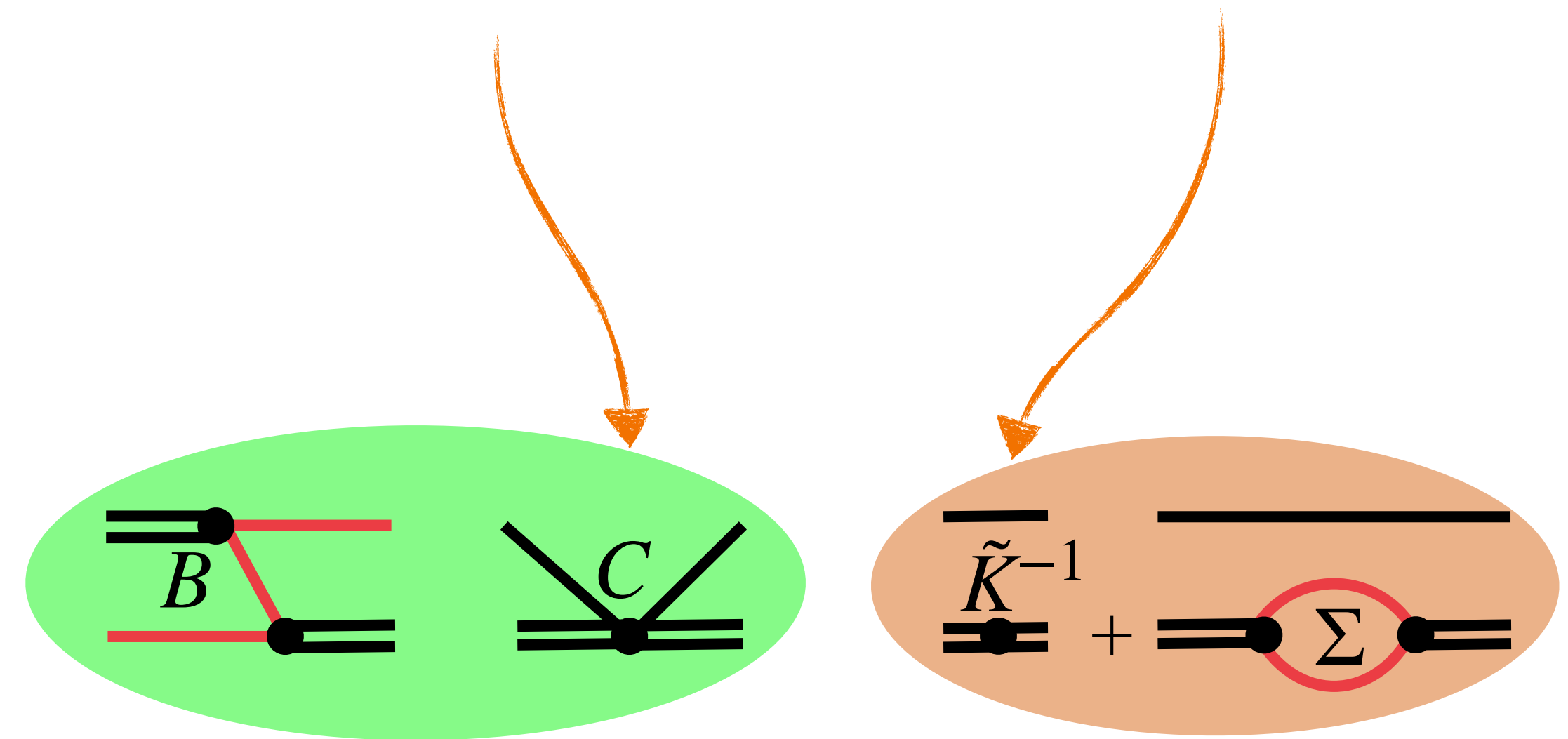- KbarN input is critical for interpretation

Antikaons in nuclear medium
- Strangeness in the EoS of neutron stars
- *K*-condensate can change EoS-stiffness

a₁(1260) PHENOMENOLOGY

- Fix quantum numbers to **a₁(1260) → πππ**

- solution via complex spectator momentum

- unknown parameter from fits[1] to data

1) Sadasivan/MM/Döring/Alexandru/Culver/Lee Phys.Rev.D 101 (2020)

Data: Schael [ALEPH] Phys.Rept. 421 (2005); Estabrooks et al. Nucl.Phys.B 79;
Protopopescu et al. Phys.Rev.D 7;

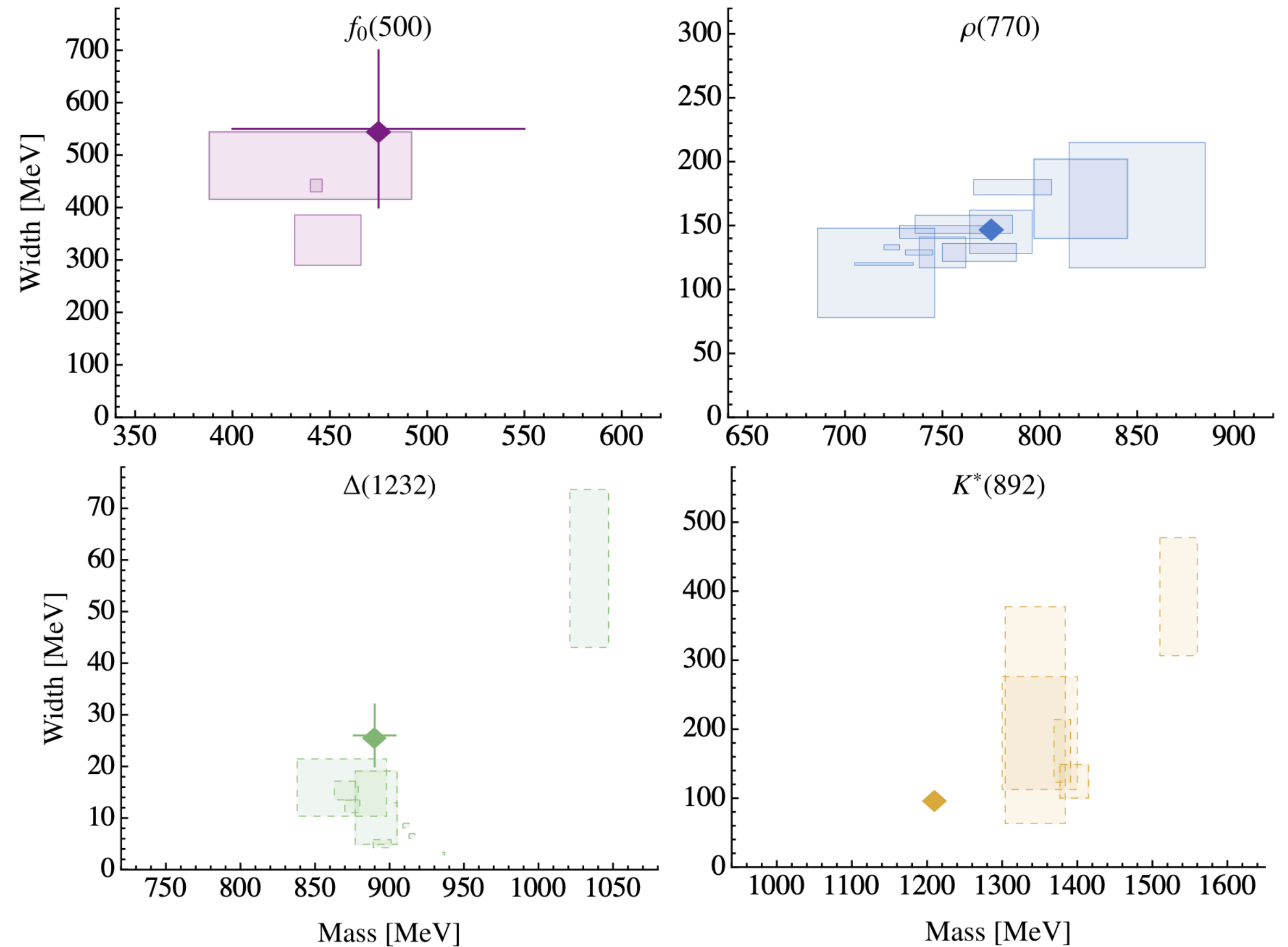# LATTICE HADRON SPECTROSCOPY

$$\mathcal{L}_{\mathrm{QCD}} = \sum \bar{q}_f^a \left(i\slashed{D}_{ab} - m_f \delta_{ab} \right) q_f^b - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- Numerical evaluation of QCD Green's functions

LATTICE HADRON SPECTROSCOPY

$f_0(500)$

$\rho(770)$

$\Delta(1232)$

$K^*(892)$

Width [MeV]
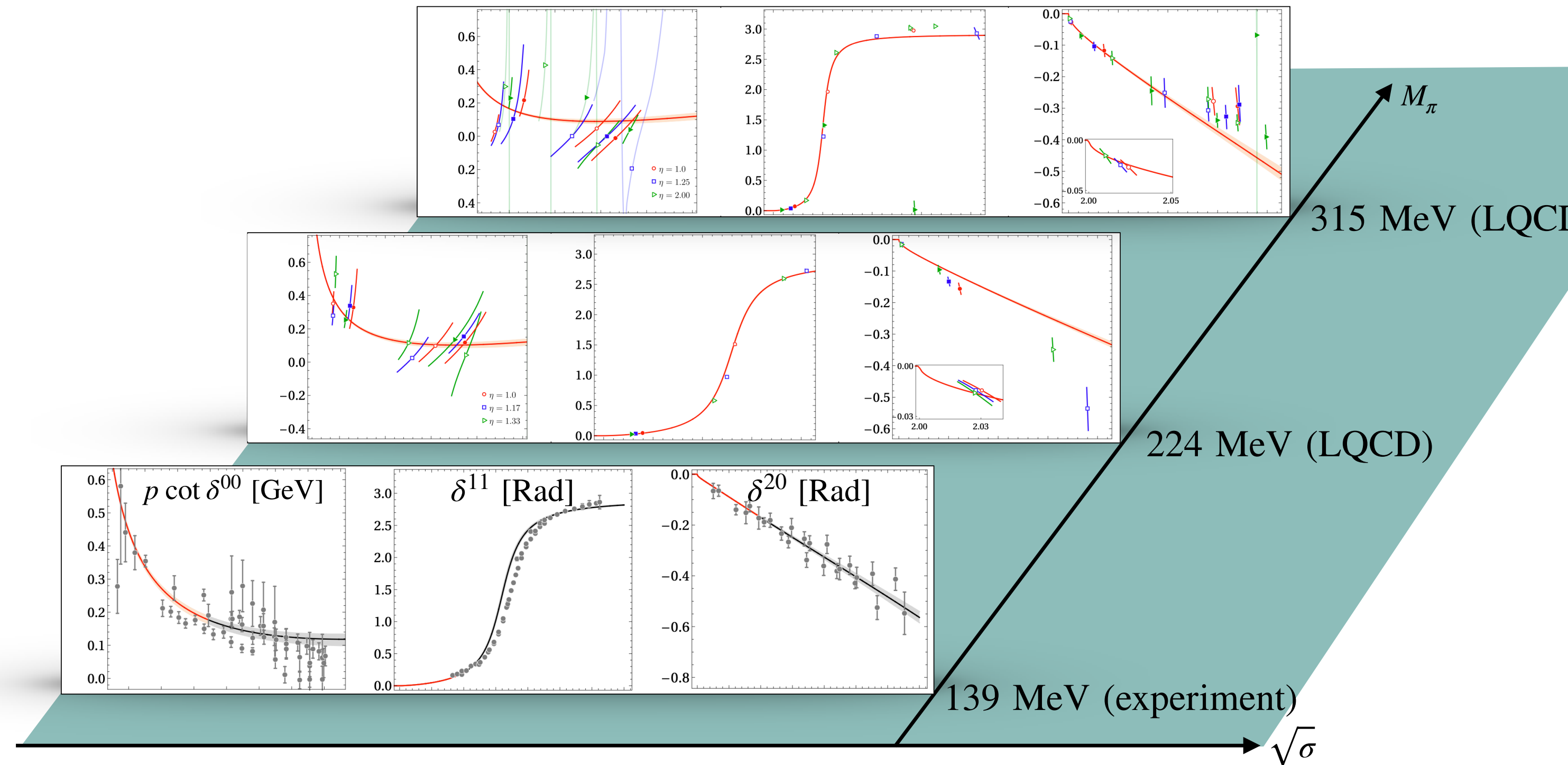
Mass [MeV]
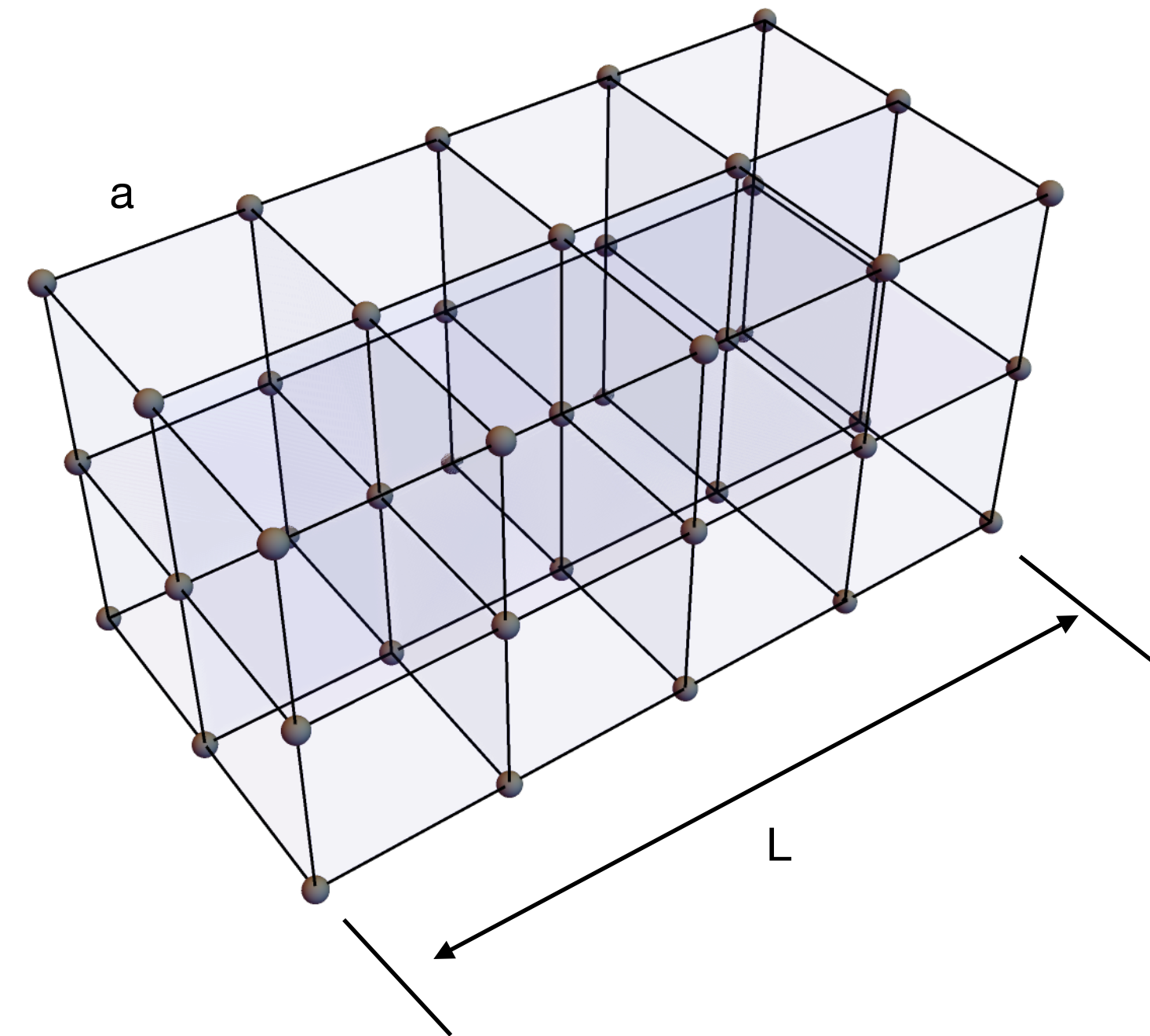
- Many studies of 2-body systems[1]

MM/Meißner/Urbach 2206.01477 Phys.Rept. 1001 (2023)

1)[NPLQCD], [RQCD], [ETMC], [HadSpec], ...

2)Reviews: Briceño/Dudek/Young Rev.Mod.Phys. 90 (2018); MM/Meißner/Urbach Phys.Rept. 1001 (2023)

LATTICE HADRON SPECTROSCOPY

- Experimentally inaccessible scenarios:
  - → Unconventional quantum numbers
  - → Three-body scattering
  - → Unphysical pion mass (chiral trajectories)
  - …

MM/Culver/Brett/Alexandru/Döring/Lee Phys.Rev.D

Review: MM/Döring/Rusetsky EPJ ST (2021)
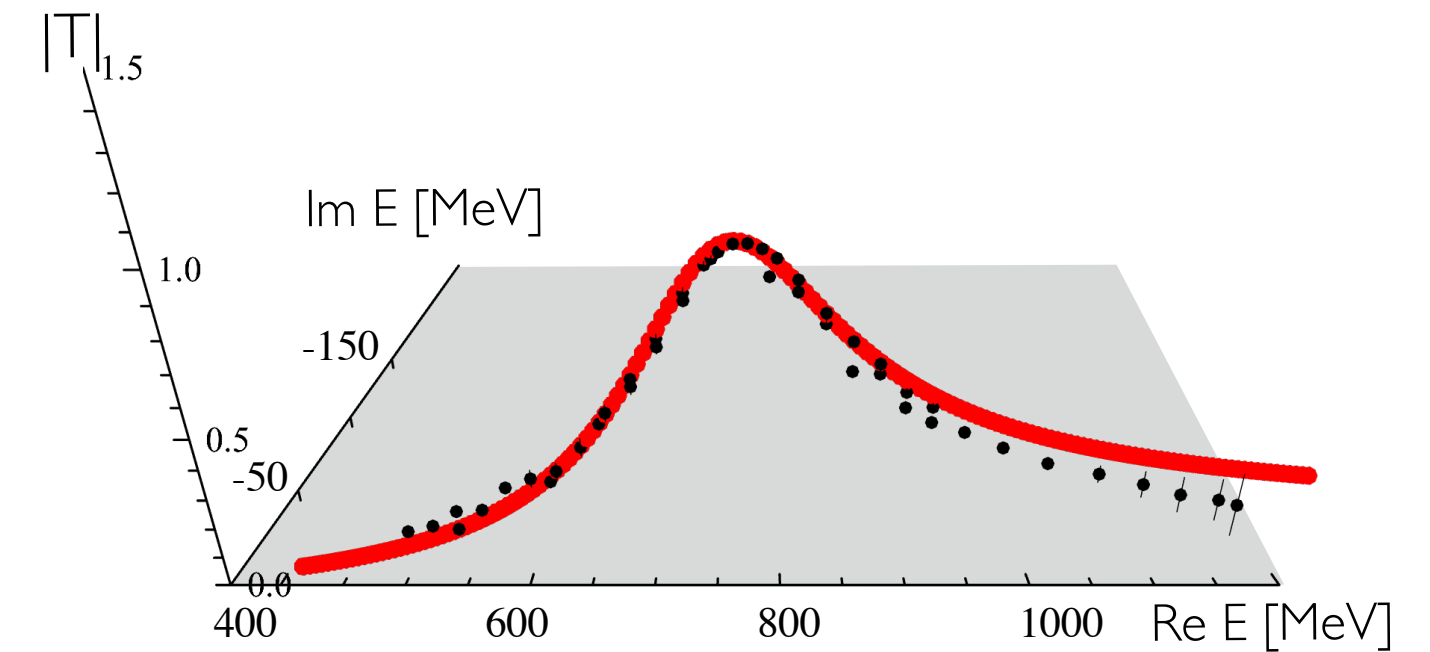
1)[NPLQCD], [RQCD], [ETMC], [HadSpec], …

2)Reviews: Briceño/Dudek/Young Rev.Mod.Phys. 90 (2018); MM/Meißner/Urbach Phys.Rept. 1001 (2023)
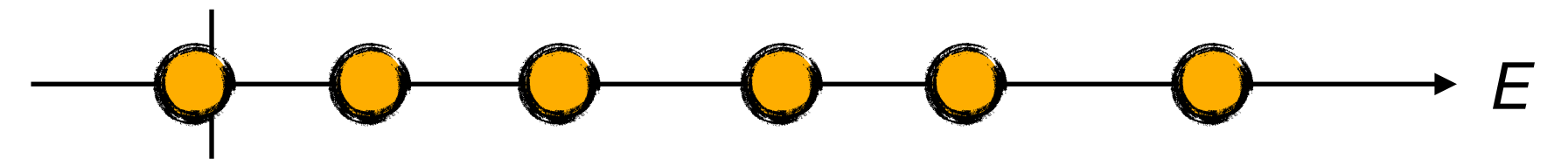
HADRONS IN A BOX

|T|[1.5]

Im E [MeV]

1.0

-150

0.5

-50

0.0

400   600   800   1000   Re E [MeV]

🤗 A unitary "T-matrix" accounts for all O(mL) effects!

🤗 Heavily simplified:

on-shell particle-configurations: $\Delta E \sim mL$

off-shell particle-configurations: $\Delta E \sim e^{-mL}$

E

1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen,

MM, Blanton, …

2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

3-BODY QUANTISATION CONDITION

Reminder

Finite-volume unitarity (FVU)[1,2]

- separates volume dependent terms
- volume independent terms connect infinite/finite-volume spectra

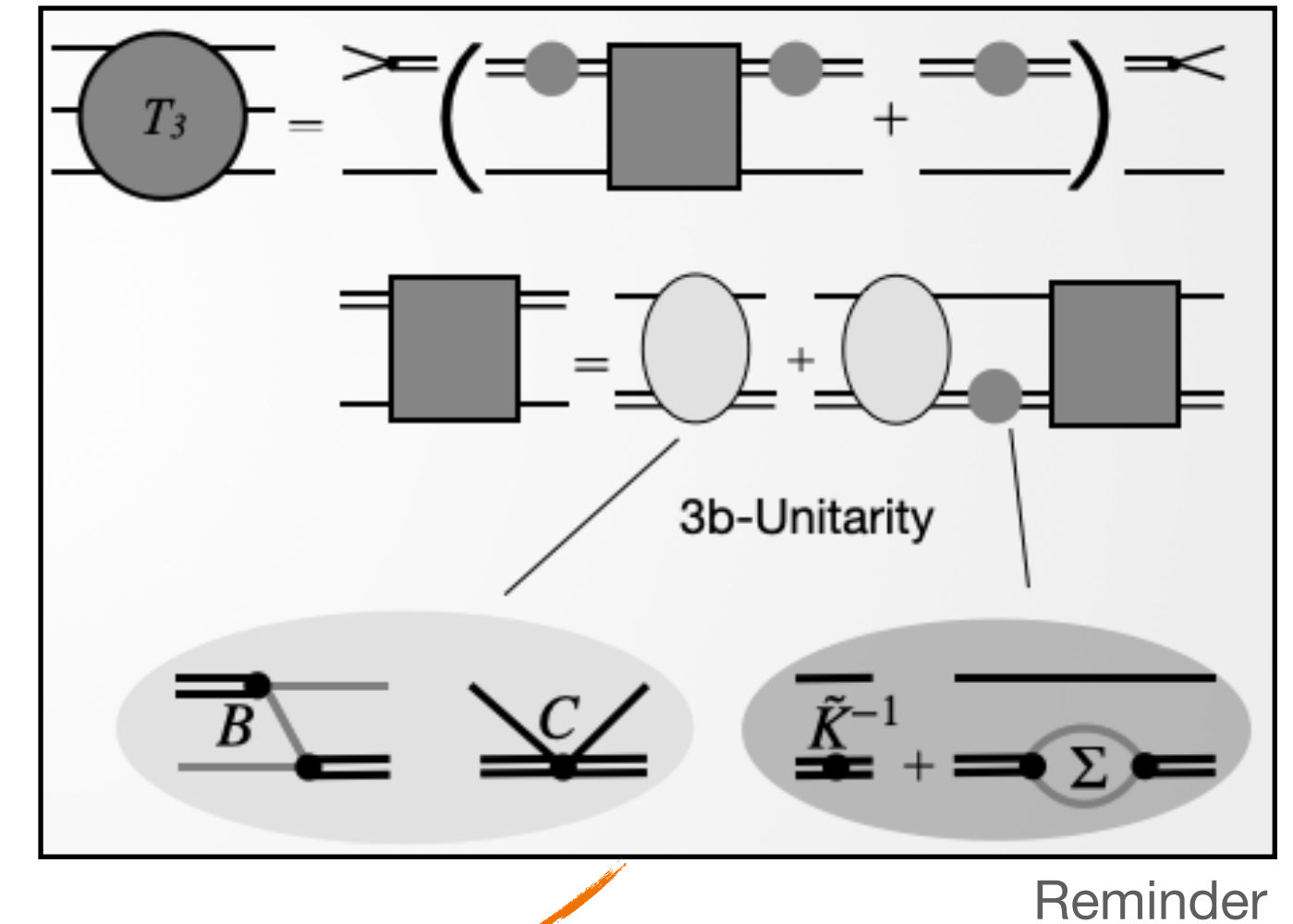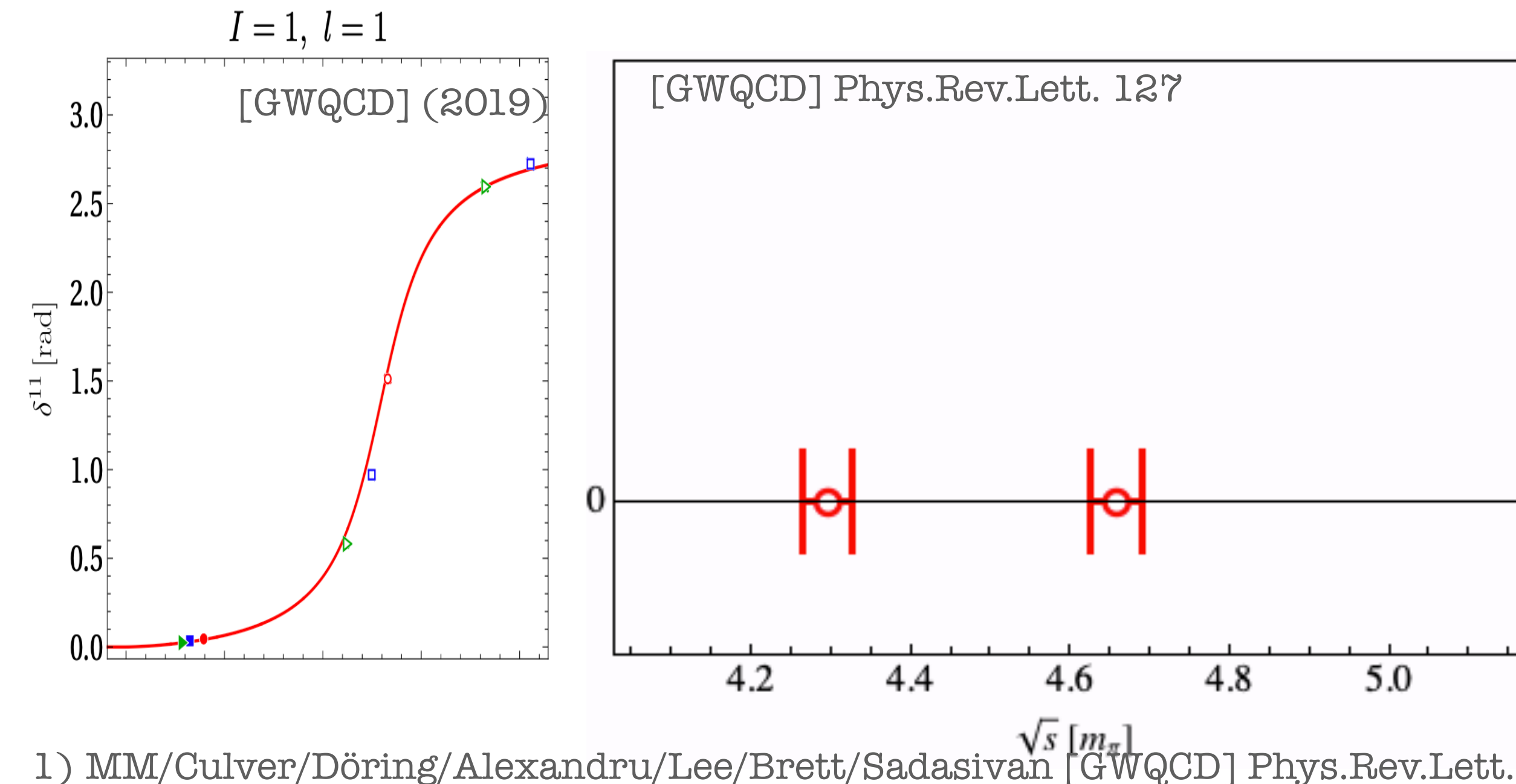$$0 = \det \left[2L^3 E \left(\tilde{K}_n^{-1} - \Sigma \right) - B - C \right]_{\mathbf{p'p}}$$

1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner, Rusetsky, Hansen, MM, Blanton, ...

2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

- First LQCD calculation[1] of a resonant 3b system
  - $N_f = 2$ dynamical fermions
  - LapH smearing
  - **P**=(0,0,0)
  - m_π=**224** MeV, $m_\pi L$=3.3
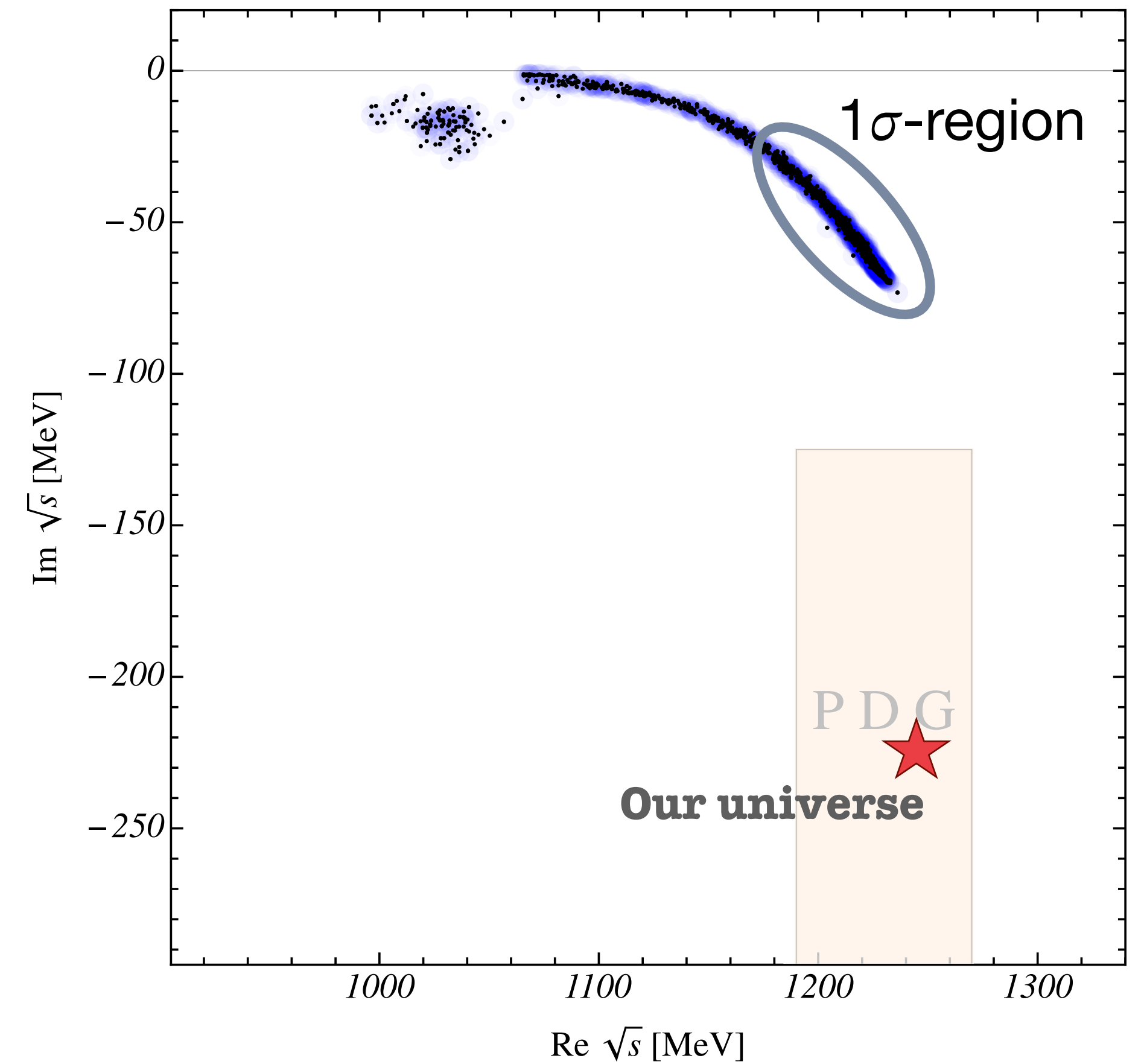 - GEVP with one-/two-/three-meson operators

$I = 1, \, l = 1$

[GWQCD] (2019)

[GWQCD] Phys.Rev.Lett. 127

1) MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127

$$0 = \det\left[2L^3E\left(\tilde{K}_n^{-1} - \Sigma\right) - B - C\right]_{\mathbf{p'p}}$$

$$T^c = B + C + \int \frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1} - \Sigma_n}T^c$$

$I = 1,\ l = 1$

[GWQCD] (2019)

[GWQCD] Phys.Rev.Lett. 127

δ^{11} [rad]

1σ-region

Im $\sqrt{s}$ [MeV]

PDG

Our universe

$\sqrt{s}\ [m_\pi]$

Re $\sqrt{s}$ [MeV]

1) MM/Culver/Döring/Alexandru/Lee/Brett/Sadasivan [GWQCD] Phys.Rev.Lett. 127
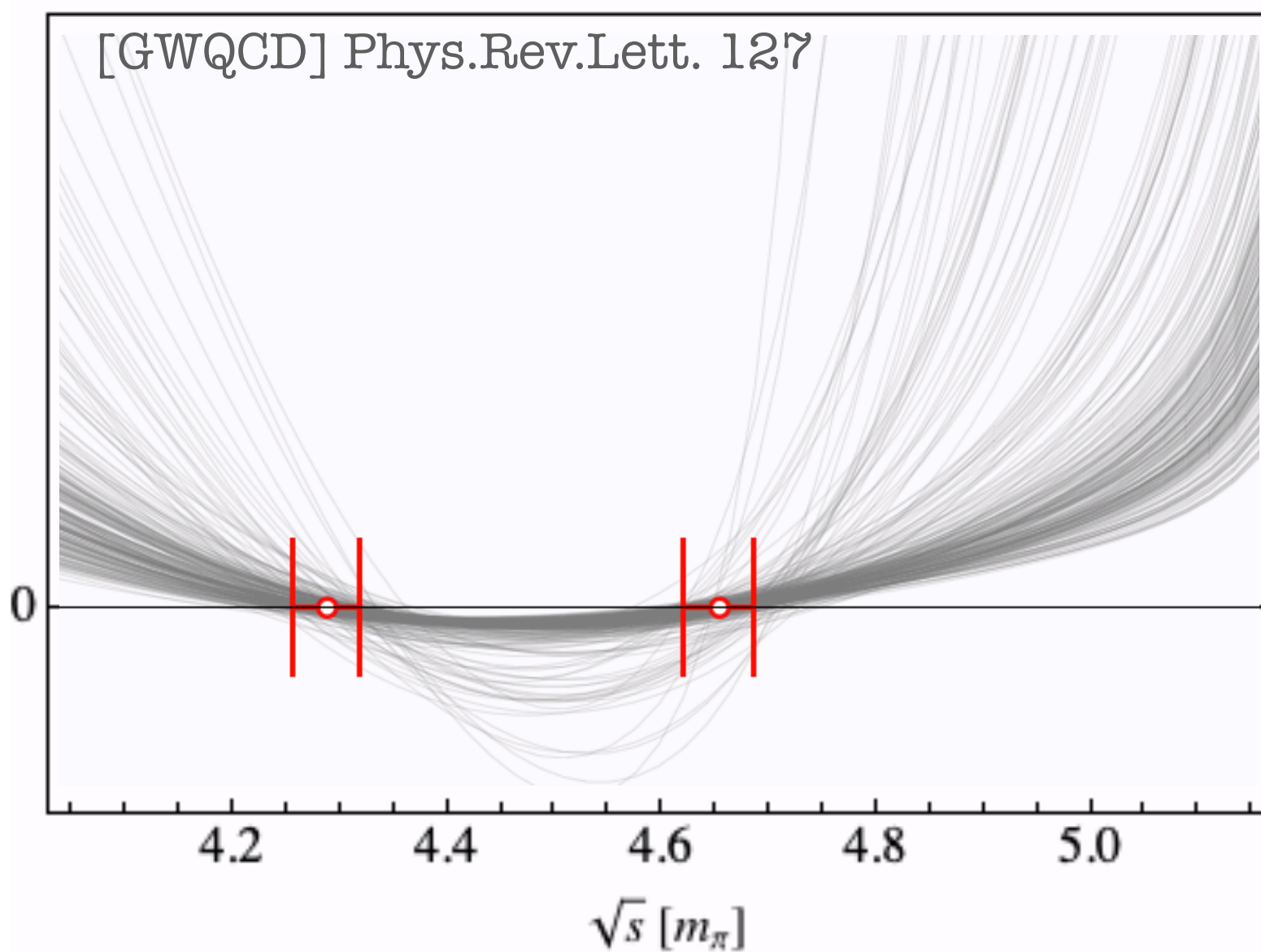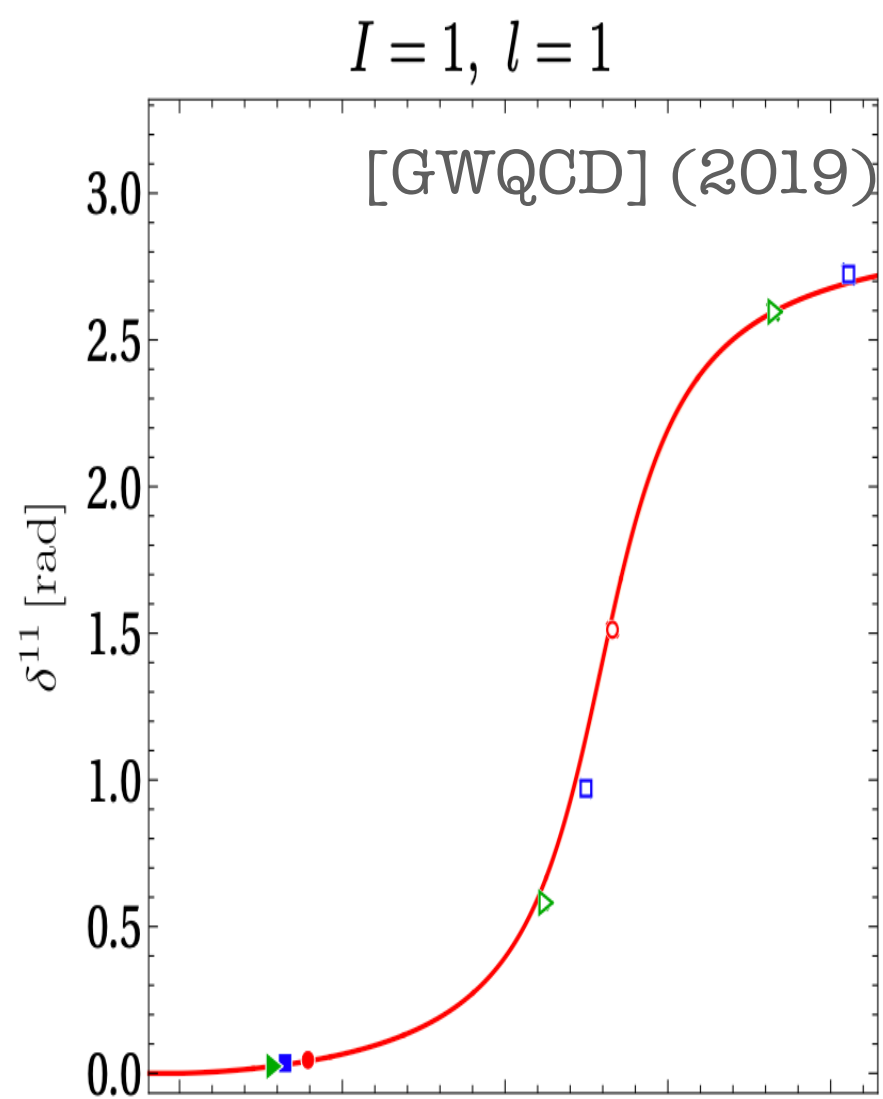
HADRONS IN A BOX

Finite-volume spectrum is real and discrete!

... requires mapping: Quantization condition[1,2]

🤗 Heavily simplified:

on-shell particle-configurations: $\Delta E \sim mL$

off-shell particle-configurations: $\Delta E \sim e^{-mL}$

🤩 *A unitary "T-matrix" accounts for all O(mL) effects!*
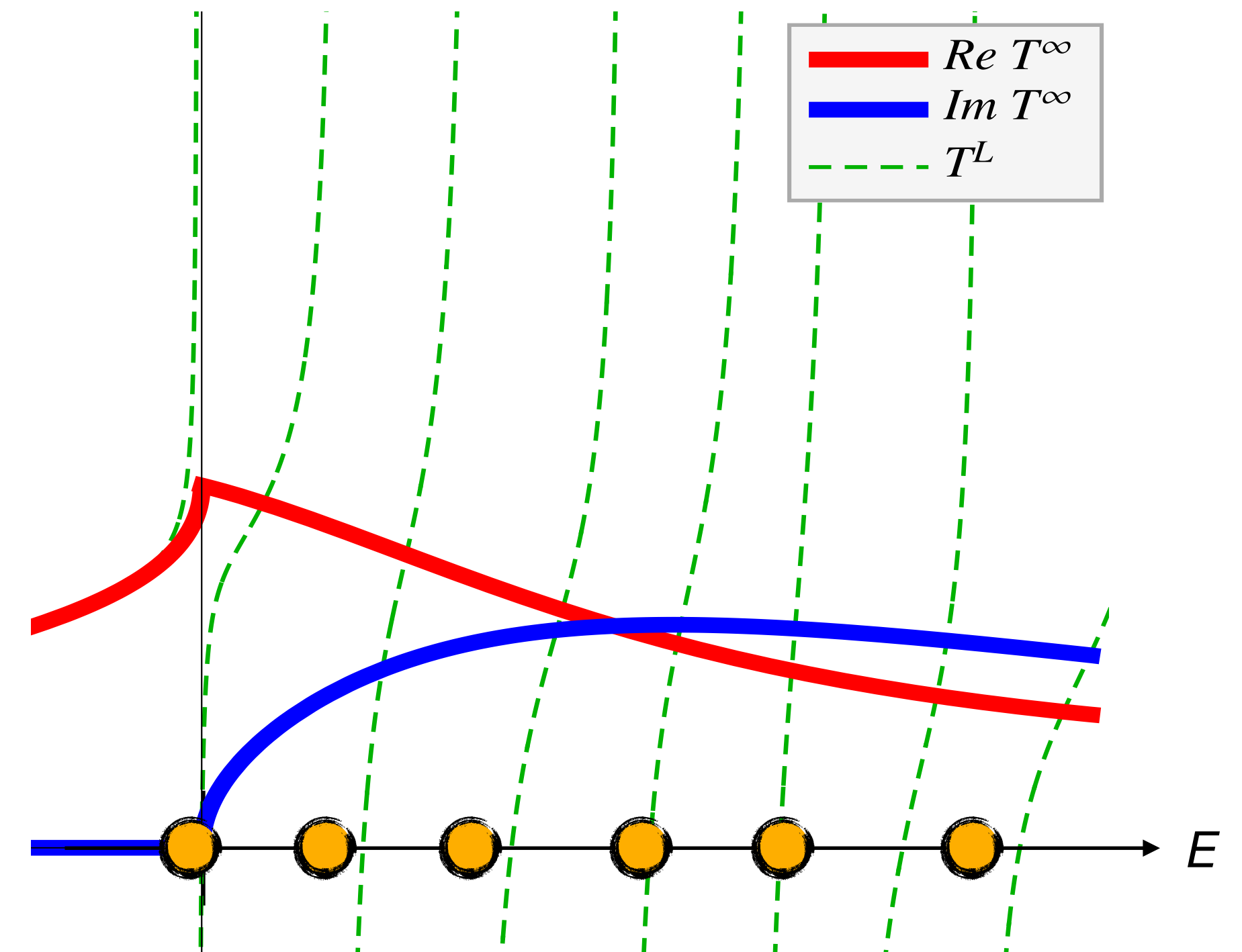
1) Lüscher, Gottlieb, Rummukainen, Feng, Li, Döring, Briceño, Meißner,
   Rusetsky, Hansen, MM, Blanton, ...
2) Reviews: Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019); MM/Döring/

GRAVITATIONAL 3-BODY PROBLEM

… not that we talk about

- birth of mathematical chaos[1]
- no closed solution
- in general non-repeating trajectories
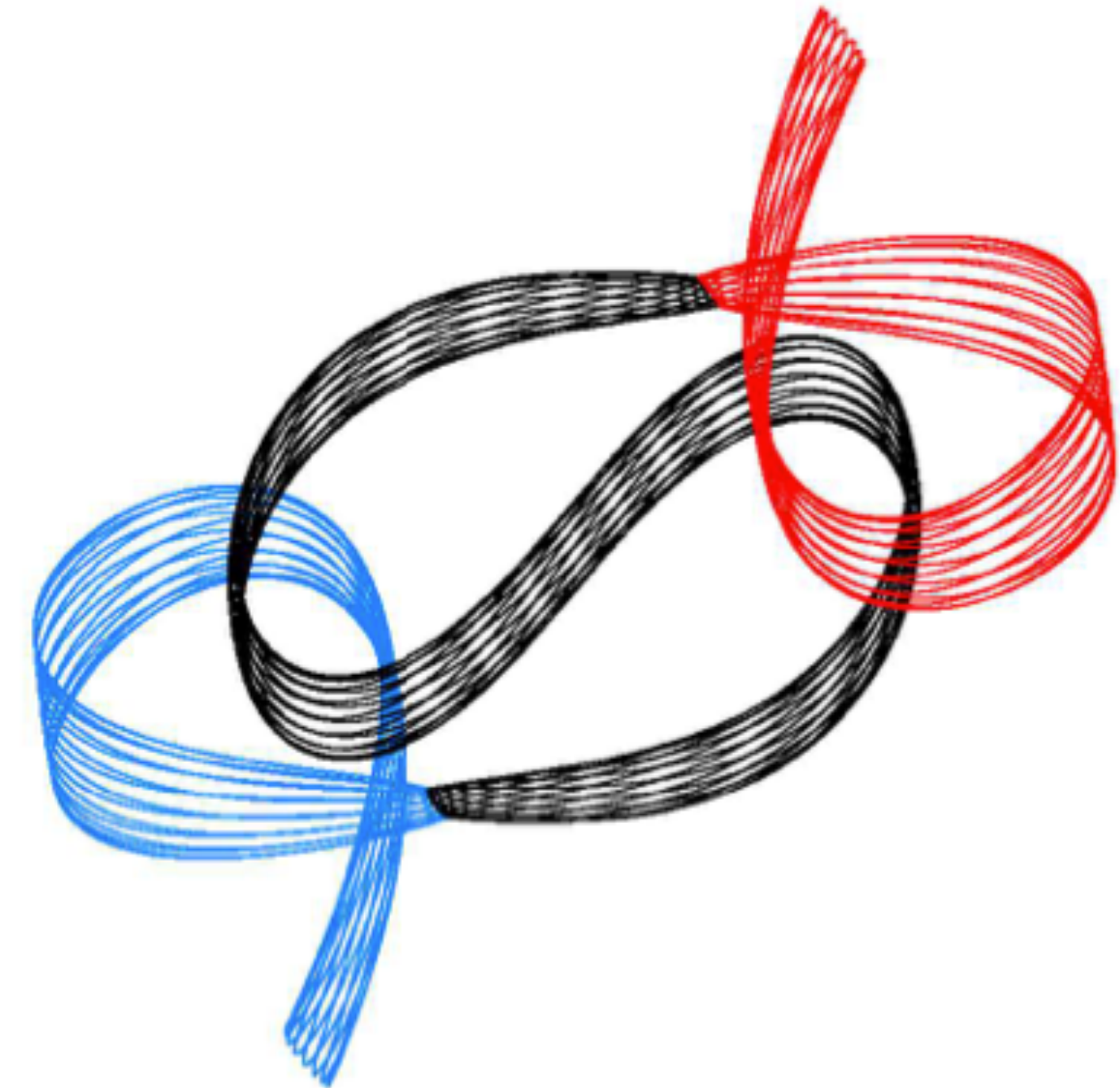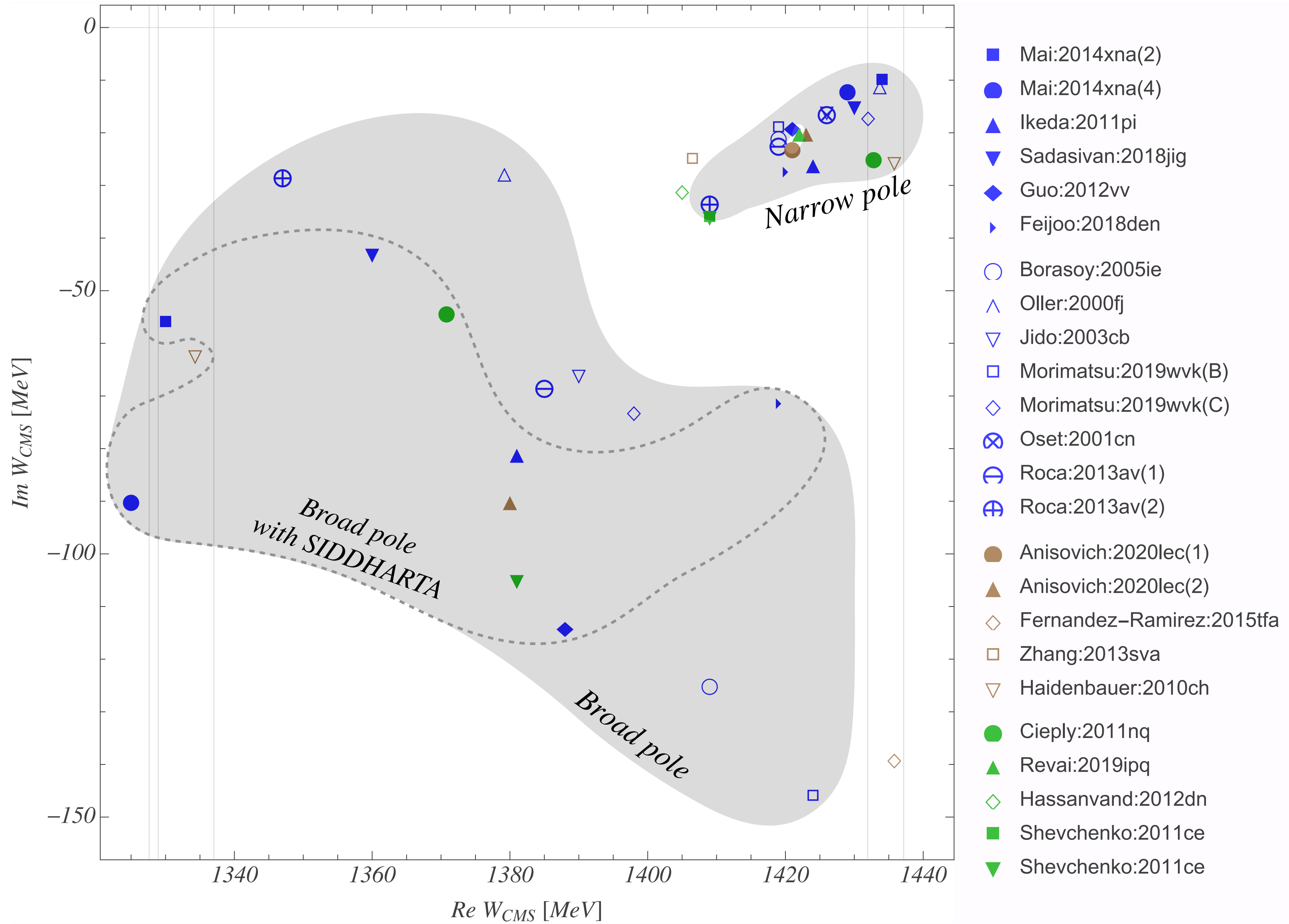

1) H. Poincaré

Fig.: Šuvakov/Dmitrašinović PhysRevLett.110.114301 (2013)

■ Mai:2014xna(2)	
● Mai:2014xna(4)	
▲ Ikeda:2011pi	
▼ Sadasivan:2018jig	
◆ Guo:2012vv	
▶ Feijoo:2018den	
◠ Borasoy:2005ie	
⋀ Oller:2000fj	
▽ Jido:2003cb	
□ Morimatsu:2019wvk(B)	
◇ Morimatsu:2019wvk(C)	
⊗ Oset:2001cn	
⊖ Roca:2013av(1)	
⊕ Roca:2013av(2)	
● Anisovich:2020lec(1)	
▲ Anisovich:2020lec(2)	
◇ Fernandez–Ramirez:2015tfa	
□ Zhang:2013sva	
▽ Haidenbauer:2010ch	
● Cieply:2011nq	
▲ Revai:2019ipq	
◇ Hassanvand:2012dn	
■ Shevchenko:2011ce	
▼ Shevchenko:2011ce	

Current frontier: 3-body dynamics from LQCD

➡ 3-body Quantization Conditions[1]

  ➡ RFT / FVU / NREFT

    ➡ many perturbatively interacting systems are studied[2]

$$0 = \det \left(L^3 \left(\tilde{F}/3 - \tilde{F}(\tilde{K}_2^{-1} + \tilde{F} + \tilde{G})^{-1}\tilde{F} \right)^{-1} + K_{\mathrm{df},3} \right)$$ RFT

$$0 = \det \left(B_0 + C_0 - E_L \left(K^{-1}/(32\pi) + \Sigma_L \right) \right)$$ FVU

1) Rusetsky, Bedaque, Grießhammer, Sharpe, Meißner, Döring, Hansen, Davoudi, Guo....

Reviews:

Hansen/Sharpe Ann.Rev.Nucl.Part.Sci. 69 (2019);

MM/Döring/Rusetsky Eur.Phys.J.ST 230 (2021);

2) MM/Döring PRL122(2019); Blanton et al. PRL 124 (2020); Hansen et al. PRL

━━━ 3-body force   ━━━ 2-body interaction

━━━ one-particle exchange   ━━━ 2-body self-energy

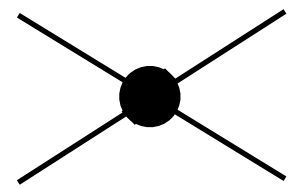| g		a	m_1	c_0	c_1	m_1'	c_0'	c_1'	χ^2_{dof}
5	FVU	$-0.1512(9)$	$3.0229(1)$	$-0.0188(35)$	$-$	$-$	$-$	$-$	2.9
	RFT	$-0.1522(12)$	$-$	$-$	$-$	$3.0232(2)$	$31.6(8.4)$	$-$	2.5
	FVU	$-0.1569(12)$	$3.0233(2)$	$-0.0297(57)$	$2.29(38)$	$-$	$-$	$-$	1.5
	RFT	$-0.1571(10)$	$-$	$-$	$-$	$3.0237(2)$	$37.6(9.0)$	$2789(540)$	1.5
10	FVU	$-0.1521(11)$	$3.0205(2)$	$-0.0475(66)$	$-$	$-$	$-$	$-$	1.7
	RFT	$-0.1531(13)$	$-$	$-$	$-$	$3.0212(3)$	$80(14)$	$-$	1.6
	FVU	$-0.1549(16)$	$3.0205(2)$	$-0.0595(99)$	$0.93(41)$	$-$	$-$	$-$	1.5
	RFT	$-0.1563(27)$	$-$	$-$	$-$	$3.0213(3)$	$97(16)$	$1773(980)$	1.4
20	FVU	$-0.1444(11)$	$3.0184(2)$	$-0.1136(77)$	$-$	$-$	$-$	$-$	1.6
	RFT	$-0.1450(17)$	$-$	$-$	$-$	$3.0199(2)$	$178(17)$	$-$	1.6
	FVU	$-0.1464(14)$	$3.0183(2)$	$-0.1363(148)$	$0.84(39)$	$-$	$-$	$-$	1.3
	RFT	$-0.1484(16)$	$-$	$-$	$-$	$3.0200(2)$	$210(23)$	$2227(600)$	1.2

Variate g($\varphi_1 \rightarrow \varphi_0\varphi_0\varphi_0$) coupling:

- avoided level crossing becomes wider
- RFT and FVU

$$q^* \cot \delta = \frac{1}{aM_0}$$
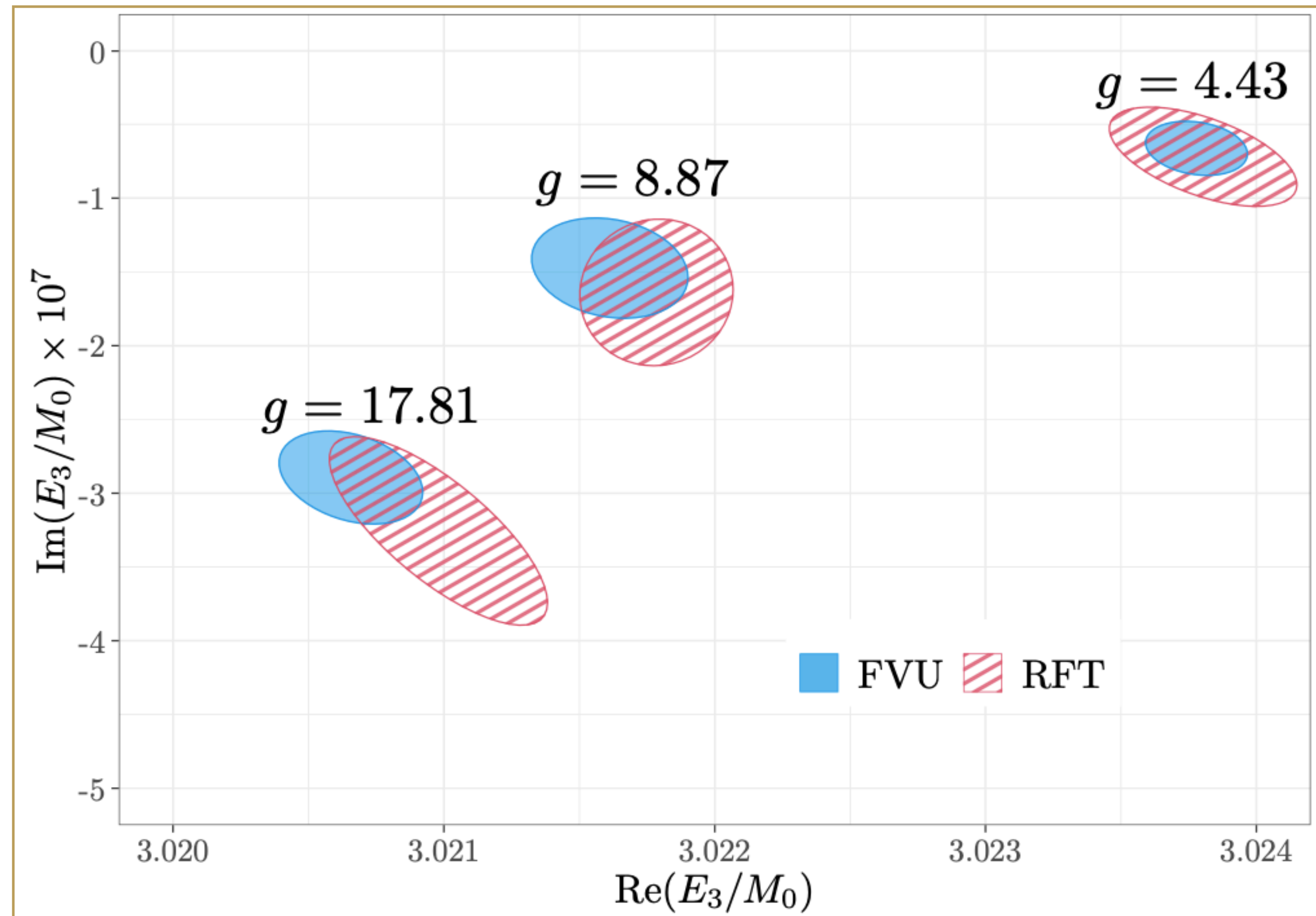
$$C = \frac{c_0}{E_3^3 - m_1^2} + c_1$$
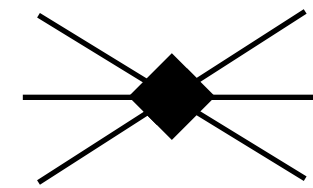
... same fit quality

... observables determined consistently

Pole positions

- FVU: complex energy-plane analysis[1]

  -- resonance width grows ~ **g^2**

  -- avoided level crossing gap >> width

- Similarly from RFT with Breit-Wigner like approximation

1) Sadasivan/MM/.. Phys.Rev.D 101 (2020)

LATTICE QCD

$$\mathcal{L}_{QCD} = \sum \bar{q}_f^a \left(i\not{D}_{ab} - m_f \delta_{ab} \right) q_f^b - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

Lattice QCD: numerical evaluation of QCD Green's functions. But…

- discretized Euclidean space time (*a>0*)
- in finite volume (*L<oo*)