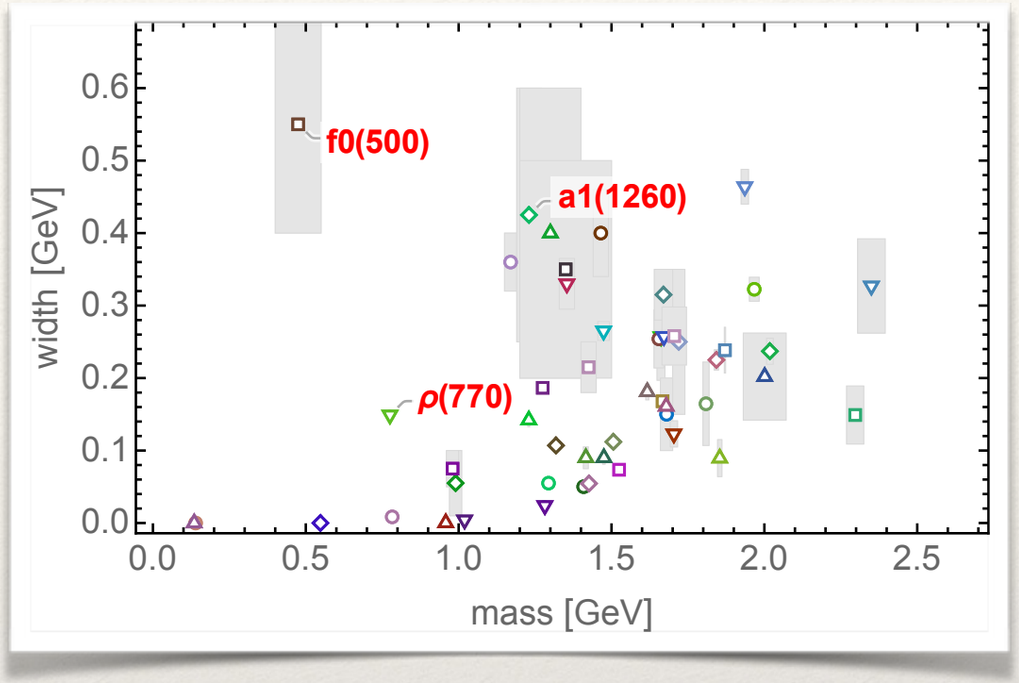
2- and 3-body hadron spectroscopy from lattice QCD

Maxim Mai

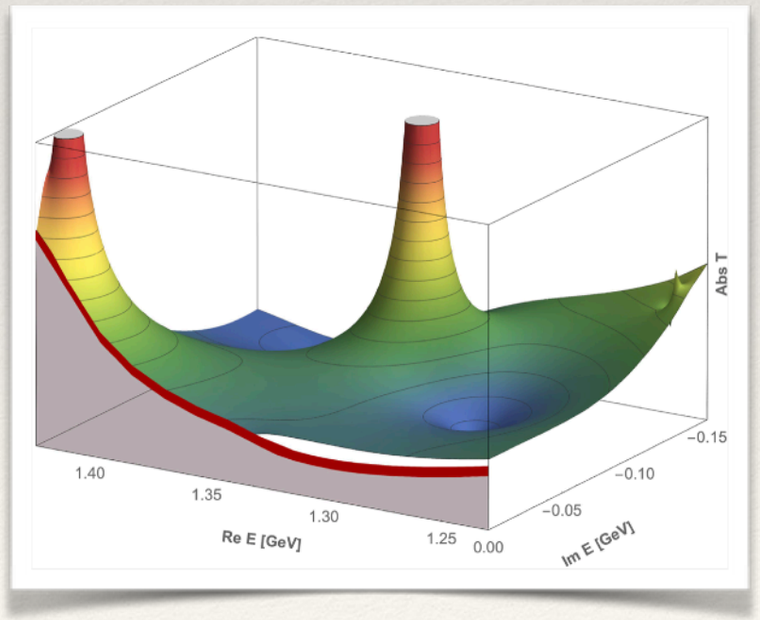
maxim-mai.github.io/exo.pdf

QCD at low energies

- Confinement
- Mass generation
- Intricate spectrum of excited states



- Universal parameters of resonances:
  - analytic properties of *scattering amplitude*
  - pole position = (**mass - 2i width)**
  - residuum ~ (**coupling constant)**



Puzzles

- *a1(1260)*

  - does **not** decay into ππ but only πππ channel

  - test channel for the search for

  spin-exotics → gluonic d.o.f. (GlueX, COMPASS)

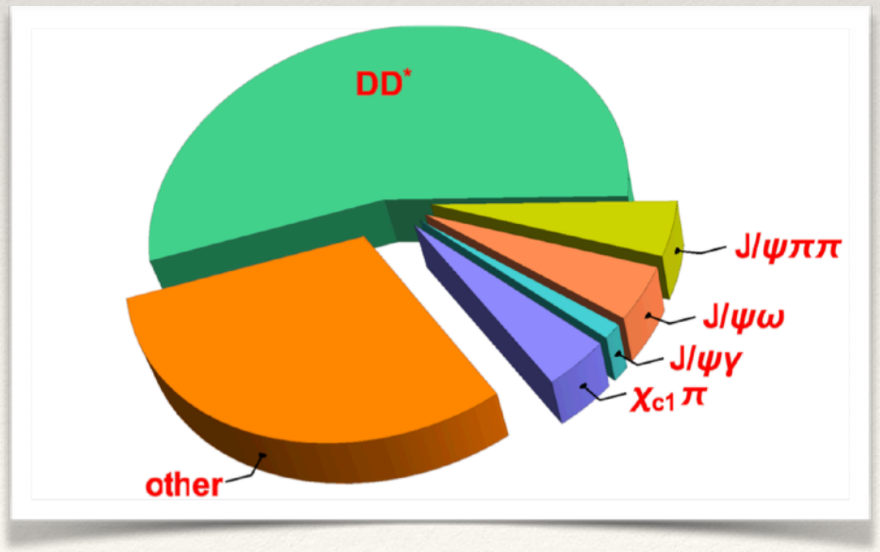
- X(3872)

  - puzzling production mechanism

  - large BR to DbarDπ

  - …similarly for further heavy XYZ exotics

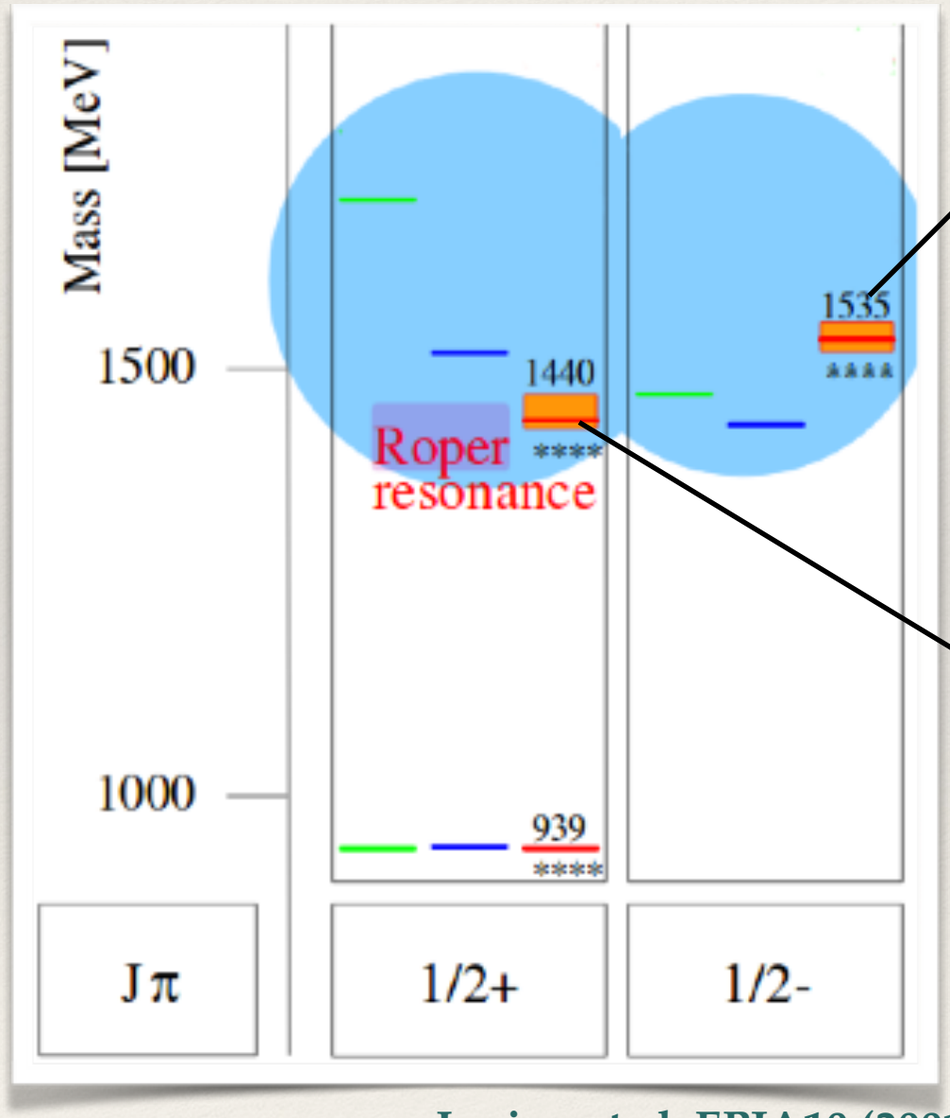
Belle 2003



Puzzles

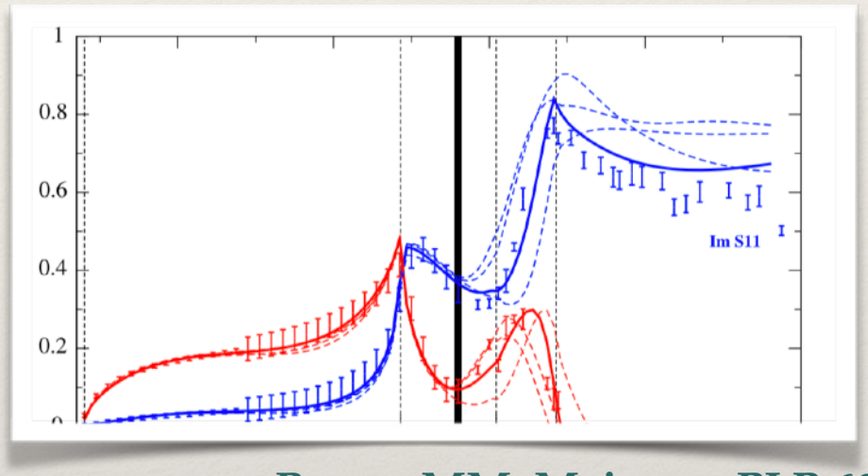
- *Roper-puzzle*

reversed mass pattern cf. constituent Quark Model



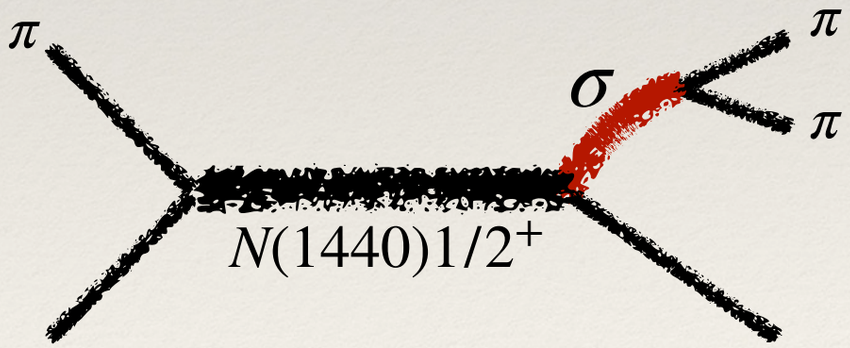
Loring et al. EPJA10 (2001)

N(1535)1/2- easily accessible from dynamical approaches, e.g.,



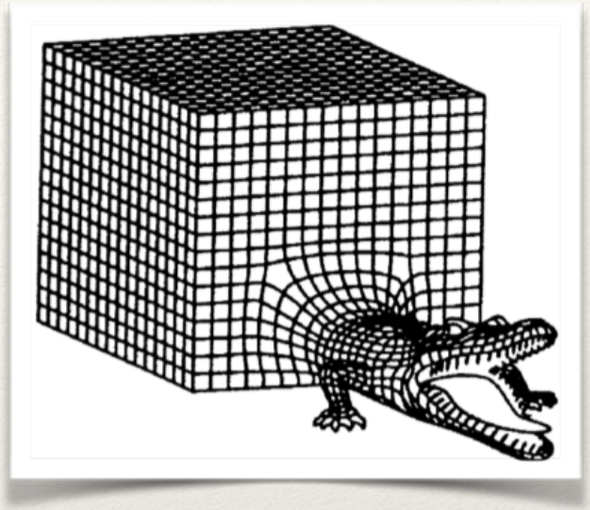
Bruns, MM, Meissner PLB 697(2011)

…but *N(1440)1/2+* has large BR to $\pi\pi N$



Lattice QCD

- the only systematic (non-perturbative) approach to QCD
- numerical ab-initio calculations

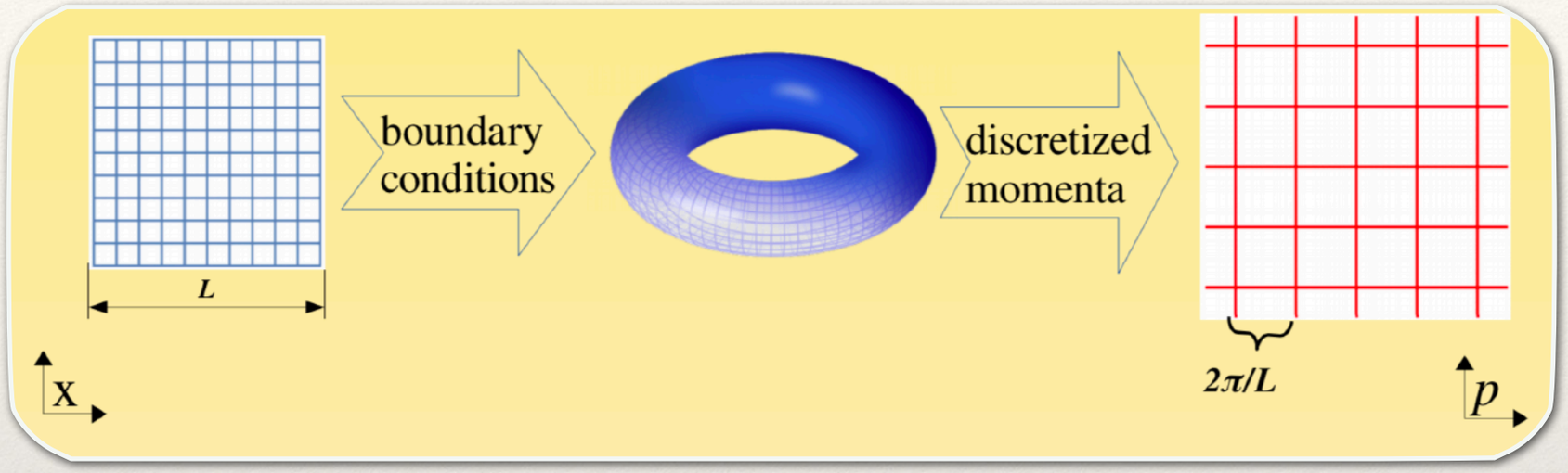


… with some technical hurdles:

1. **discretized Euclidean space-time**
   - continuum limit
2. **unphysical pion mass**
   - well established techniques from ChPT — **Gasser/Leutwyler 1984**
   - extensions to SU(3), baryons, resonances, … — **Becher, Meissner, Pelaez…**
3. **finite volume**
   - well understood for 2 hadrons — **Luescher 1986**
   - new developments for 3-hadron systems — **…later in this talk**

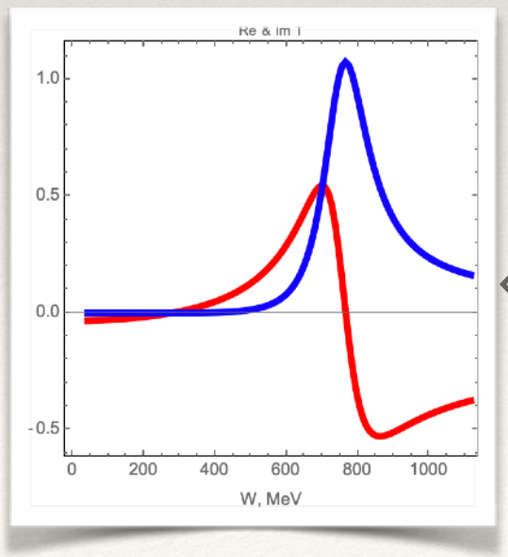
Interaction spectrum in a box

- Lattice calculations are performed in finite volume

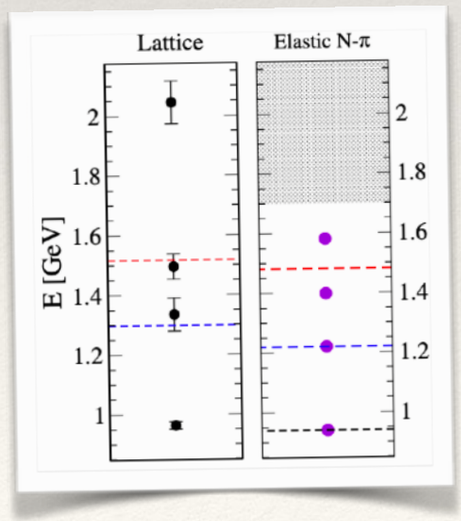


- all momenta are discretized

- interaction spectrum is **real-valued** and **discrete**



QUANTIZATION CONDITION



State of the art

- 2-body case

  - well established (**Luescher's method**) **Lüscher (1986)**

  - **one-to-one mapping** between phase-shifts and energy eigenvalues

  - extensions for coupled-channels, spin, ... **Gottlieb, Rummukainen, Feng, Li, Liu, Doring, Briceno, Rusetsky, Bernard, Meissner...**

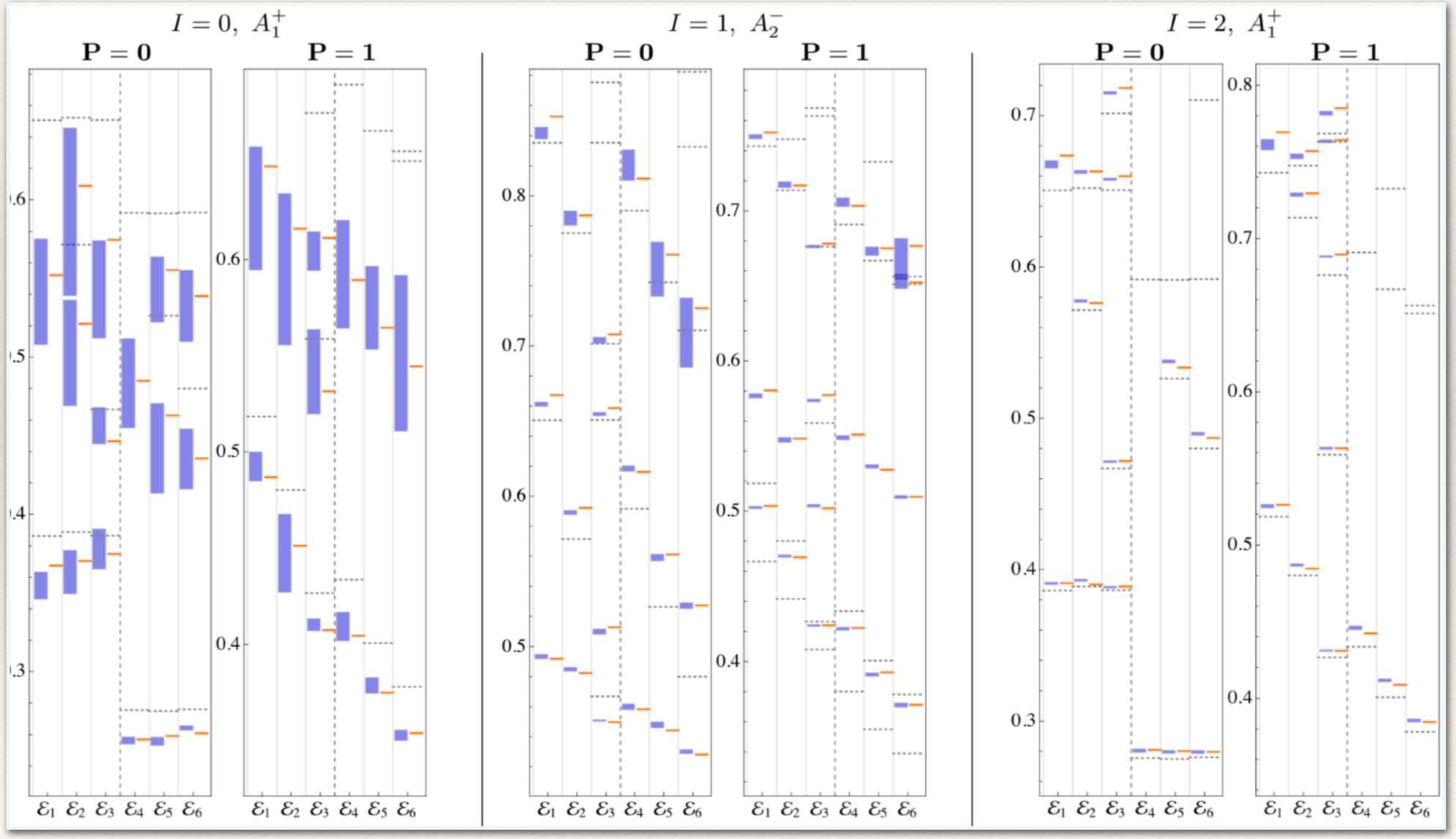
Example: $\pi\pi$ scattering

- numerous calculations of the phase-shift for all three channels

NPLQCD; HadSpec; ETMC; GW-lattice; CP-PACS;….

- structure of $\rho(700)$ and $\sigma(500)$ resonances

GW-Lattice: Guo et al. (2016) Guo et al. (2018) Culver et al. (2019) MM et al.(2019)



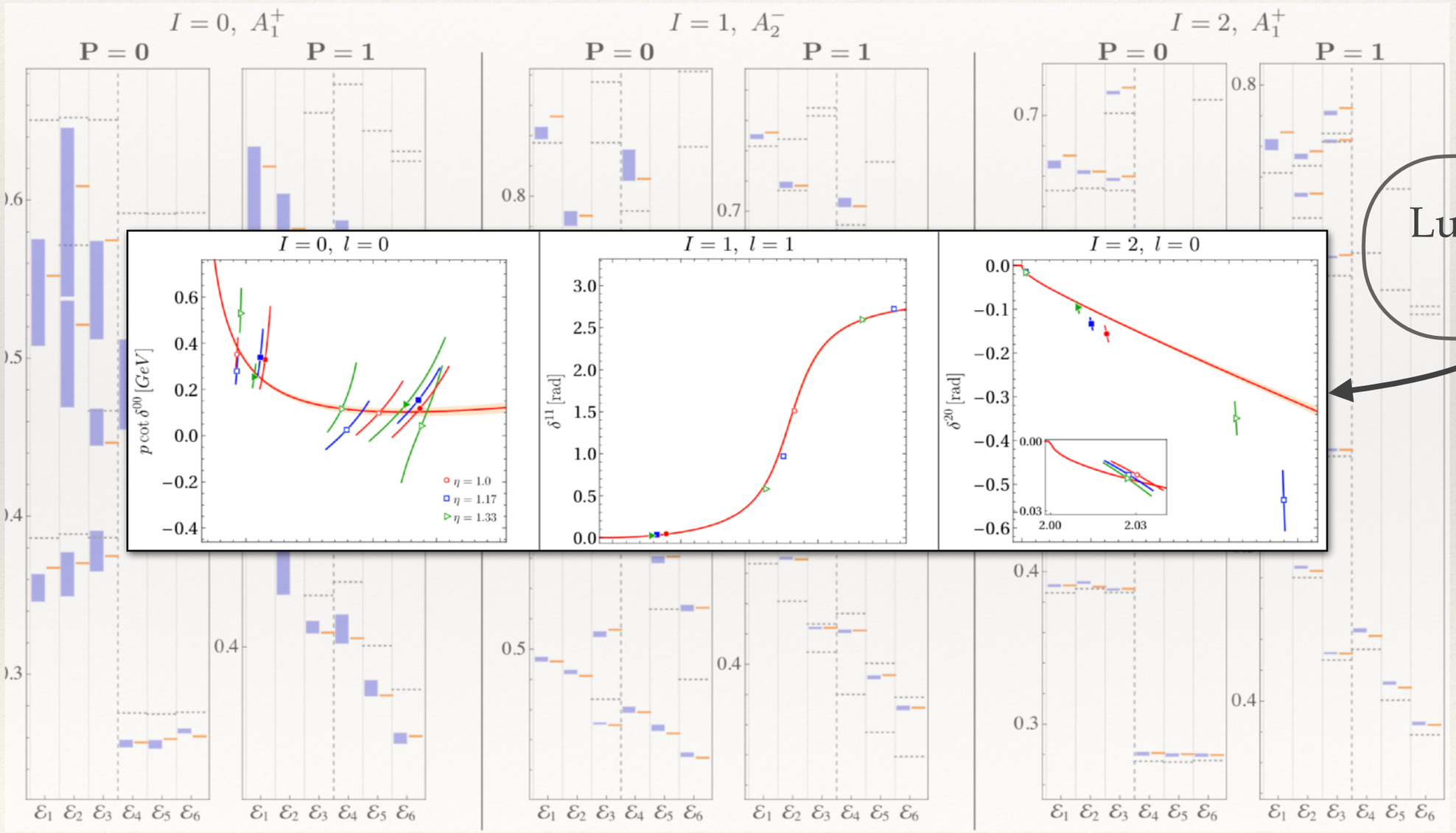
Example: $\pi\pi$ scattering

- numerous calculations of the phase-shift for all three channels

NPLQCD; HadSpec; ETMC; GW-lattice; CP-PACS;….

- structure of $\rho(700)$ and $\sigma(500)$ resonances

GW-Lattice: Guo et al. (2016) Guo et al. (2018) Culver et al. (2019) MM et al.(2019)



Luescher's QC

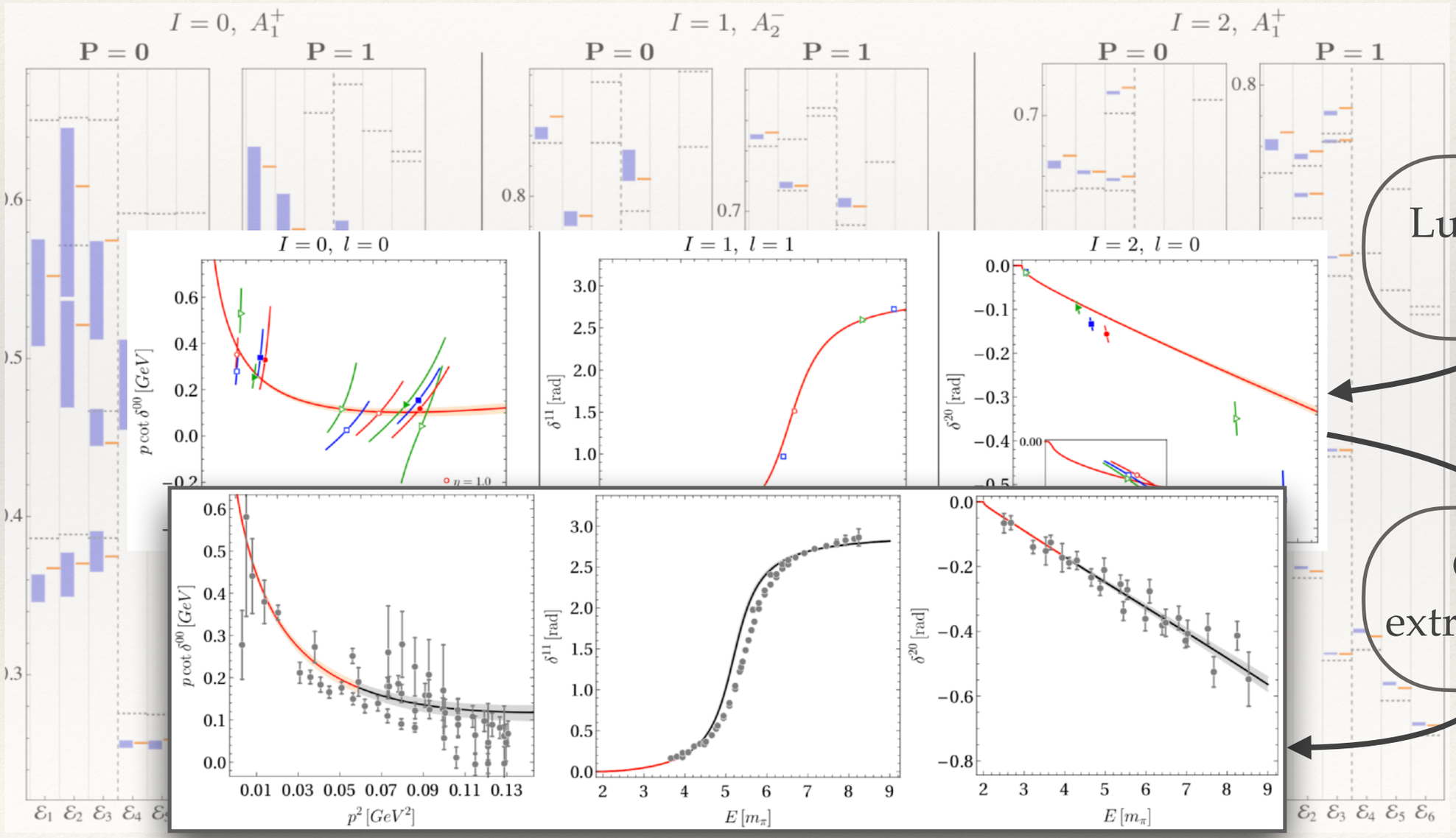
Example: $\pi\pi$ scattering

- numerous calculations of the phase-shift for all three channels

NPLQCD; HadSpec; ETMC; GW-lattice; CP-PACS;….

- structure of $\rho(700)$ and $\sigma(500)$ resonances

GW-Lattice: Guo et al. (2016) Guo et al. (2018) Culver et al. (2019) MM et al.(2019)



Luescher's QC

Chiral extrapolation

State of the art

- 3-body case

  - complex kinematics (*8 variables*)

  - complex interaction structure (*sub-channels*)

  - theoretical developments & numerical investigations

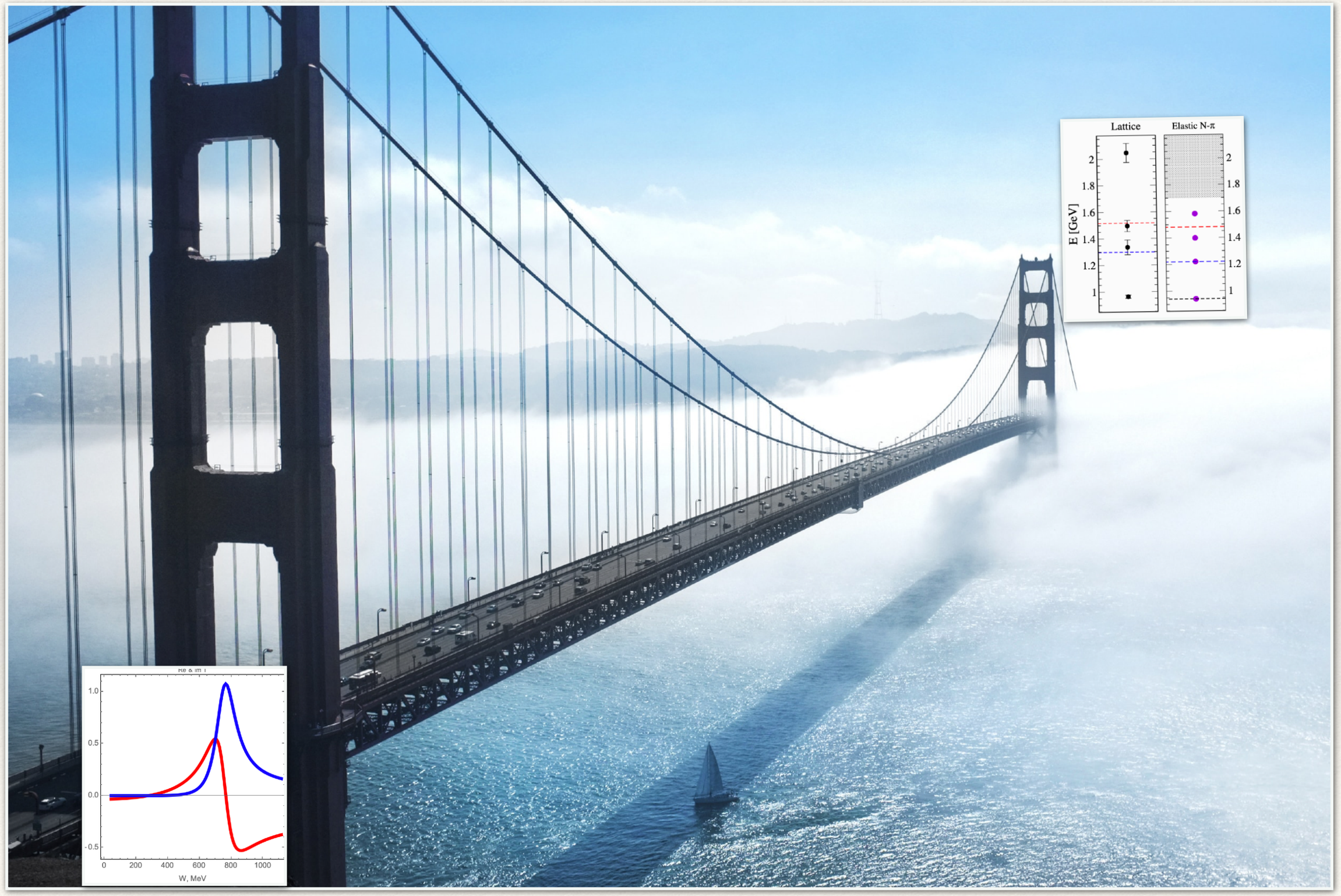
Rusetsky, Polejaeva, Sharpe, Meissner, Davoudi, Hansen, Guo, Briceno, MM, Doring, Hammer, Wu, Blanton, Griesshammer, Romero-López, Pang, …

Review: Hansen,Sharpe (2019)

- first application to Lattice QCD results

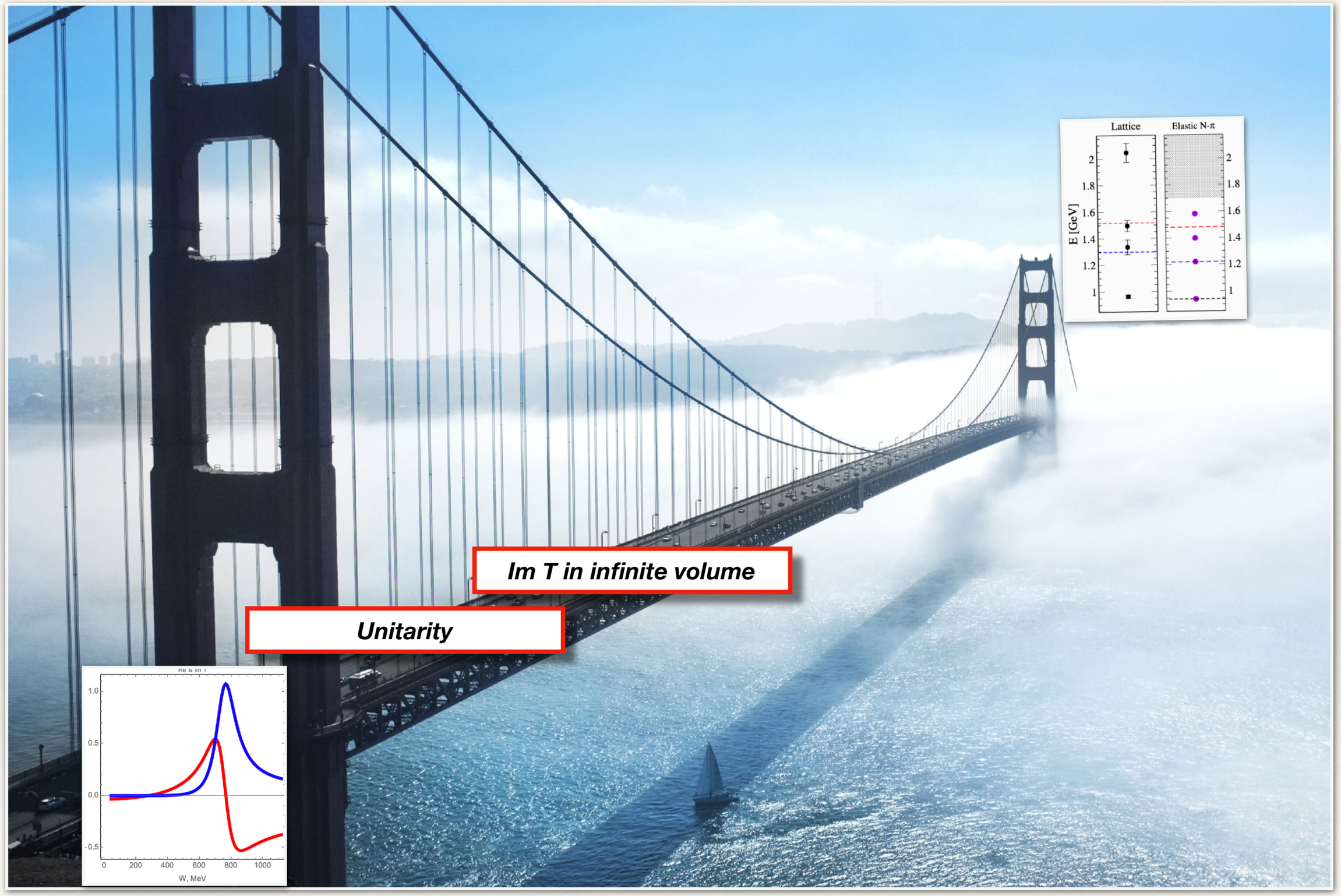
MM, Doring (2018)
Blanton, Romero-López, Sharpe (2019)
MM, Alexandru, Culver, Doering (2019)
Culver, MM, Brett, Doring, Alexandru (2019)

Quantization condition in a nutshell



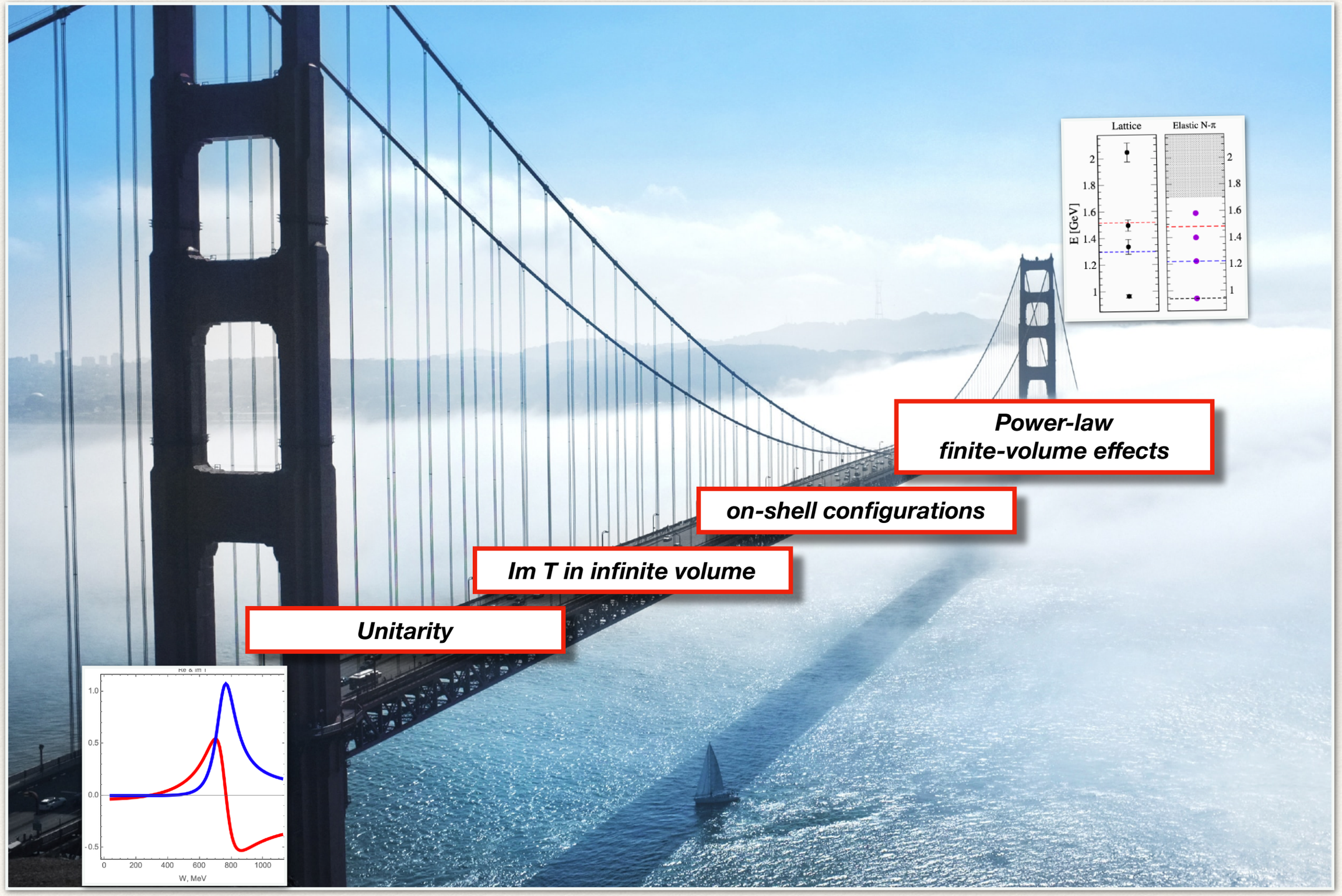
Quantization condition in a nutshell

Quantization condition in a nutshell



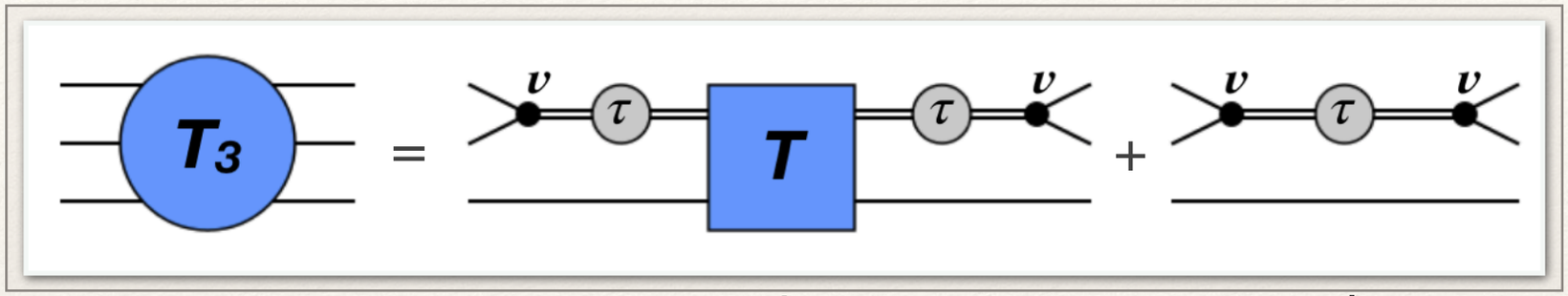
Quantization condition in a nutshell

Quantization condition in a nutshell



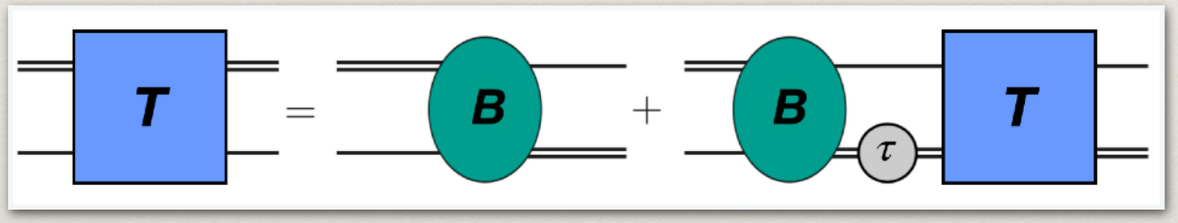
Relativistic unitary scattering amplitude

MM, Hu, Doring, Pilloni, Szczepaniak Eur.Phys.J. A53 (2017)



connected part

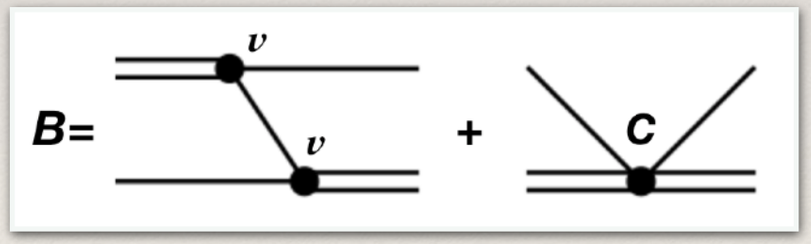
- general 4d BSE-like equation w.r.t kernel B

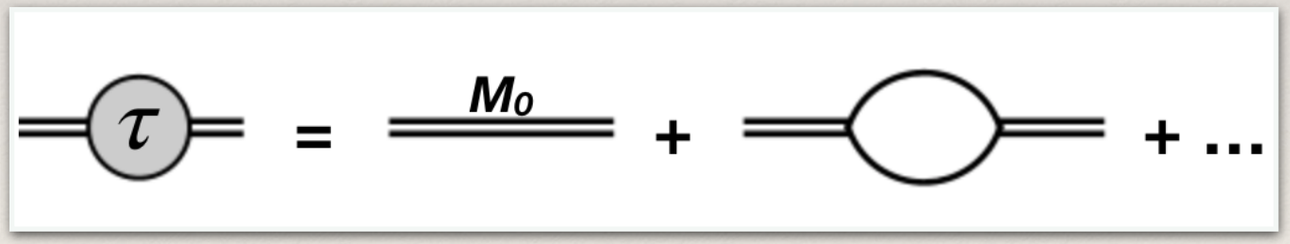


disconnected part

- spectator + tower of functions $\tau(Minv)$ with correct right-hand-singularities
- coupling $v(q,p)$ is cut-free

B & τ from 3-body unitarity

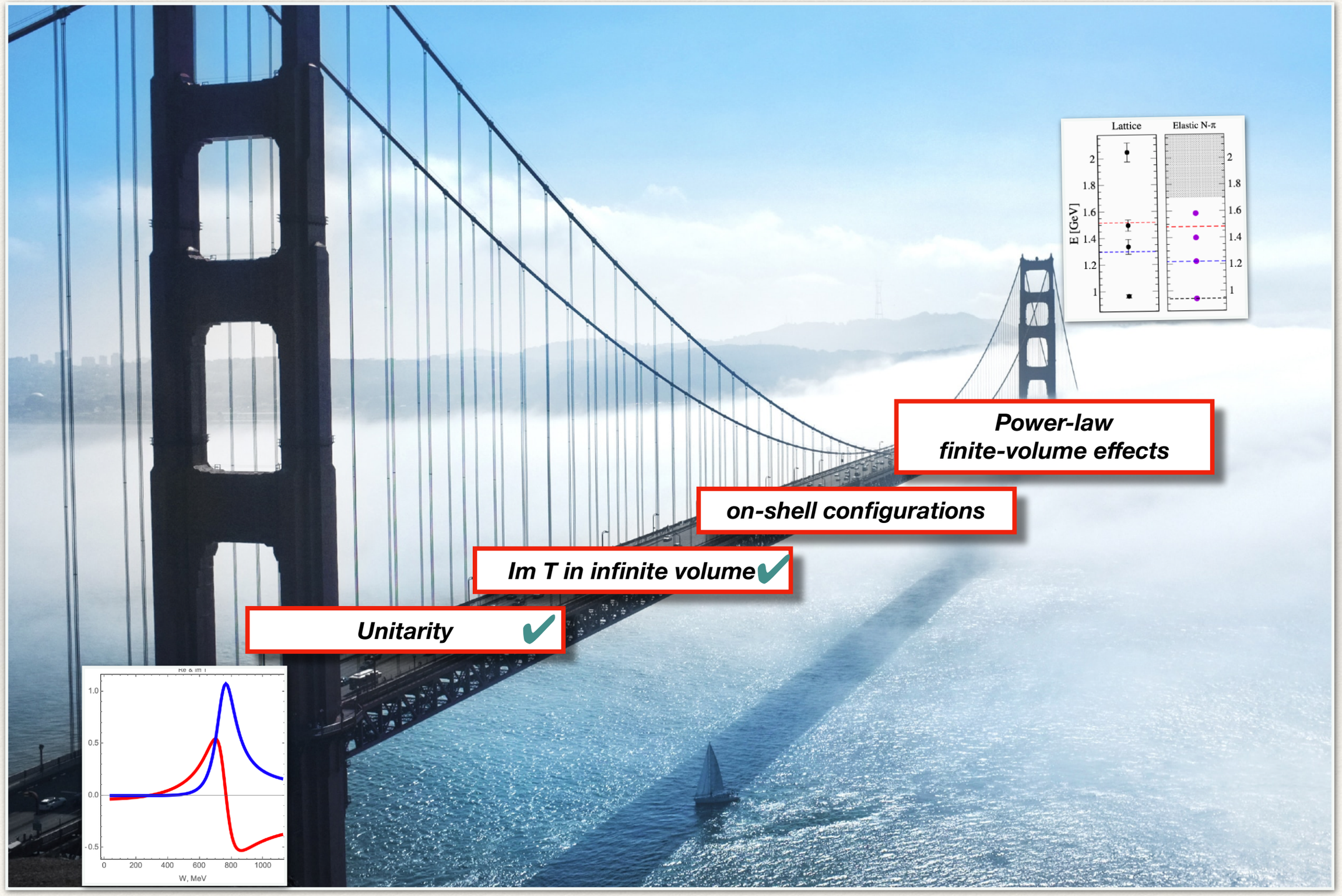




- analytic properties and generalization

Jackura et al. [JPAC] EPJ C79 (2019)

Quantization condition in a nutshell



Relativistic 3b QC

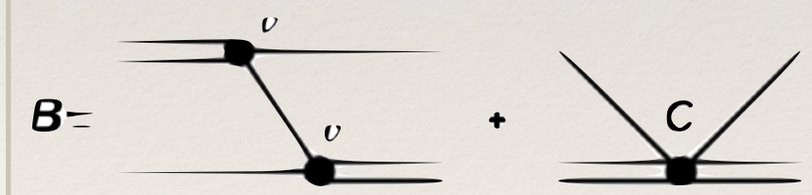
- In *"finite volume world"* momenta are discrete → replace integrals by sums

  ⇒ scattering amplitude is **real-valued** and **singular matrix equation**

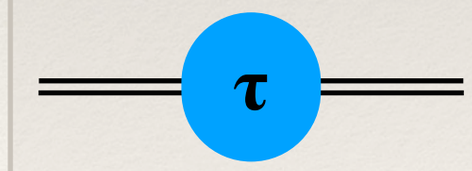
  ⇒ singularities, iff total energy (E^*) is such that 3 pions are **on-shell**

  ⇒ **relativistic 3-body quantization condition**

$$\left\langle \mathbf{v}(\mathbf{p}_1,\mathbf{p}_2)\ \left[\mathbf{B}(\mathbf{E}^*)+\mathbf{E}_\mathbf{L}\ \tau_{\mathbf{LP}}^{-1}(\mathbf{E}^*)\right]^{-1}_{\mathbf{p}_3,\mathbf{q}_3}\ \mathbf{v}(\mathbf{q}_1,\mathbf{q}_2)\right\rangle_{\mathbf{p}_i,\mathbf{q}_j} = \infty$$



$$[E_L]_{\mathbf{pq}} = \delta_{\mathbf{pq}} 2L^3\sqrt{m_\pi^2+\mathbf{p}^2}$$



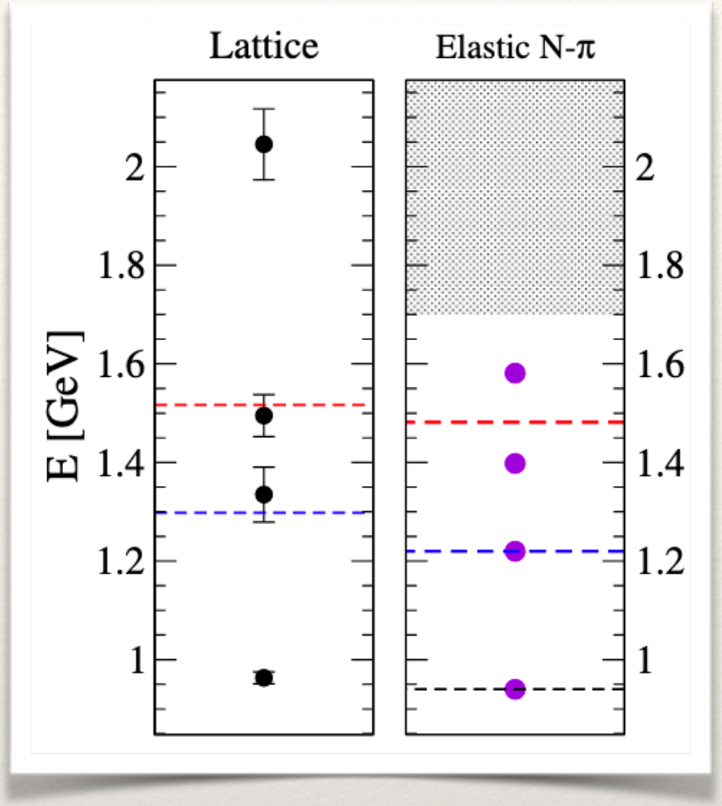
~ OPE and 3-body force

~ 2-b partial wave

Lattice studies of 3b systems

- $I=3$ $\pi\pi\pi$ — Beane et al. [NPLQCD] PRL100 (2008)
- X(2872) — Prelovsek, Leskovec PRL111 (2013)
- $a_1(1260)$ — Lang et al. JHEP 1404
- $N(1440)1/2^+$ — Lang et al. PRD 95(2017)
- $I=2$ $\pi\varrho$ — Woss et al. JHEP 1807 [Hadspec]
- $\pi\omega/\pi\varphi$ — Woss et al (2019)
- $I=3$ $\pi\pi\pi$ — Hörz/Hanlon PRL123 (2019); Culver/MM/Brett/Doering/Alexandru (2019)

… more to expect



Lang et al.(2017)

Lattice studies of 3b systems

- $I=3$ **ππ**π
- X(2872)
- $a_1(1260)$
- $N(1440)1/2^+$
- $I=2$ **πQ**
- **πω / πφ**
- $I=3$ **ππ**π

… more to expect

Beane et al. [NPLQCD] PRL100 (2008)

Prelovsek, Leskovec PRL111 (2013)

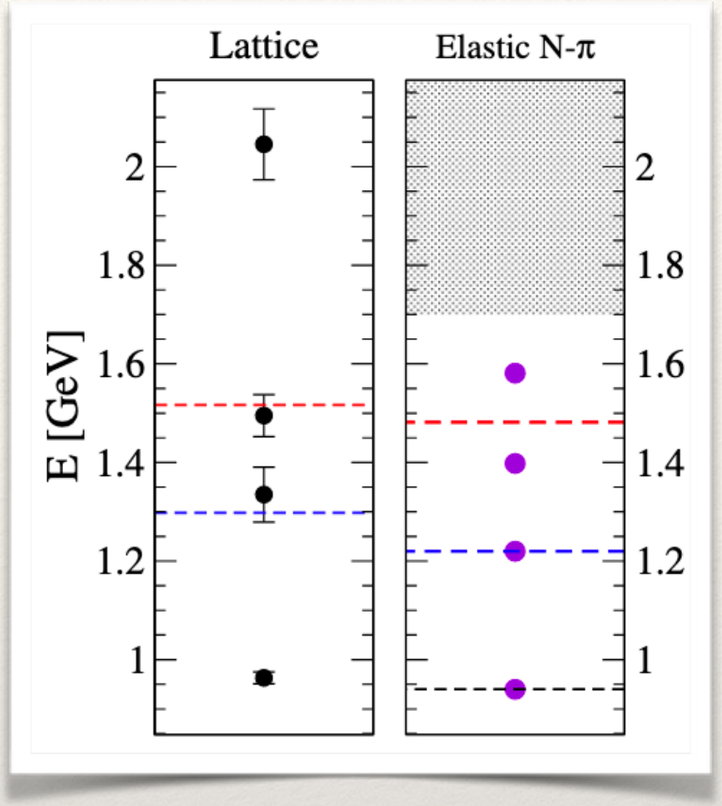
Lang et al. JHEP 1404

Lang et al. PRD 95(2017)

Woss et al. JHEP 1807 [Hadspec]

Woss et al (2019)

Hörz/Hanlon PRL123 (2019)
Culver/MM/Brett/Doering/Alexandru (2019)



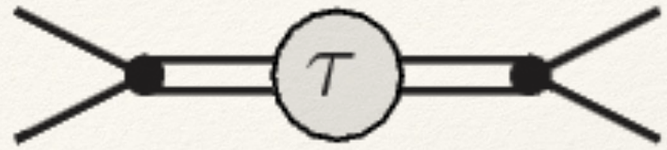
Lang et al.(2017)

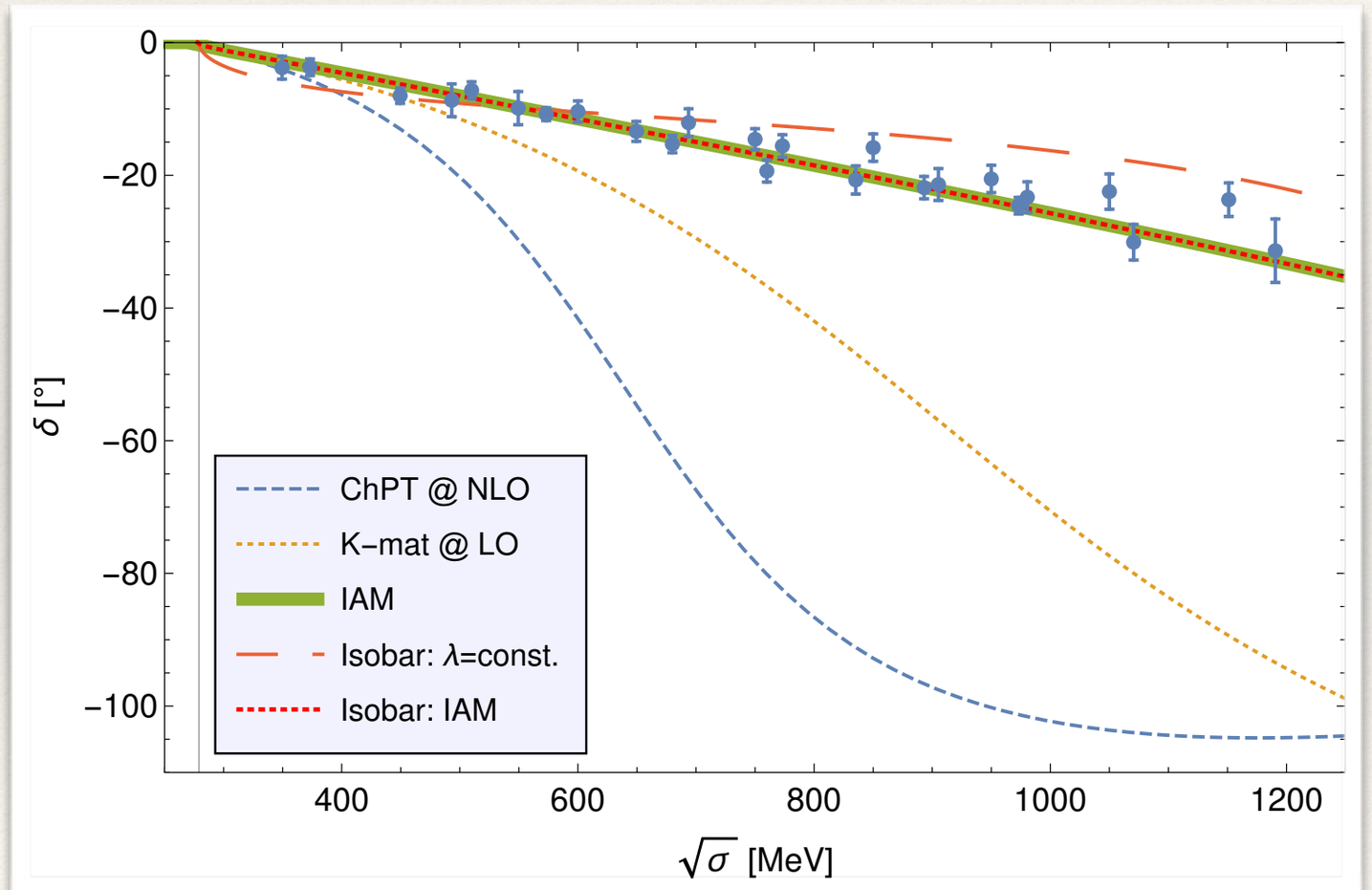
○ = fin vol. analysis accomplished

Three positive pions

MM, Doring PRL 122 (2018)

- 2-body sub-channel
  - One-channel problem: $\pi+\pi+$ system in S-wave
  - How to parametrize the scattering amplitude?





1) ✗ $$T_2 = \frac{-\lambda^2/(32\pi)}{\sigma - M_0^2 - \sum_{\pm}\int \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{\lambda^2}{4E_{\boldsymbol{k}}\sqrt{\sigma}(\sqrt{\sigma}\pm 2E_{\boldsymbol{k}})}}$$

- incorrect $m\pi$ behavior!

2) ✗ $$T_2 = \frac{1}{K^{-1} - ip_{cms}}$$

- works badly for high energies

3) ✓ $$T_2 = \frac{T_{\rm LO}^2}{T_{\rm LO} - T_{\rm NLO}}$$ Truong(1988)

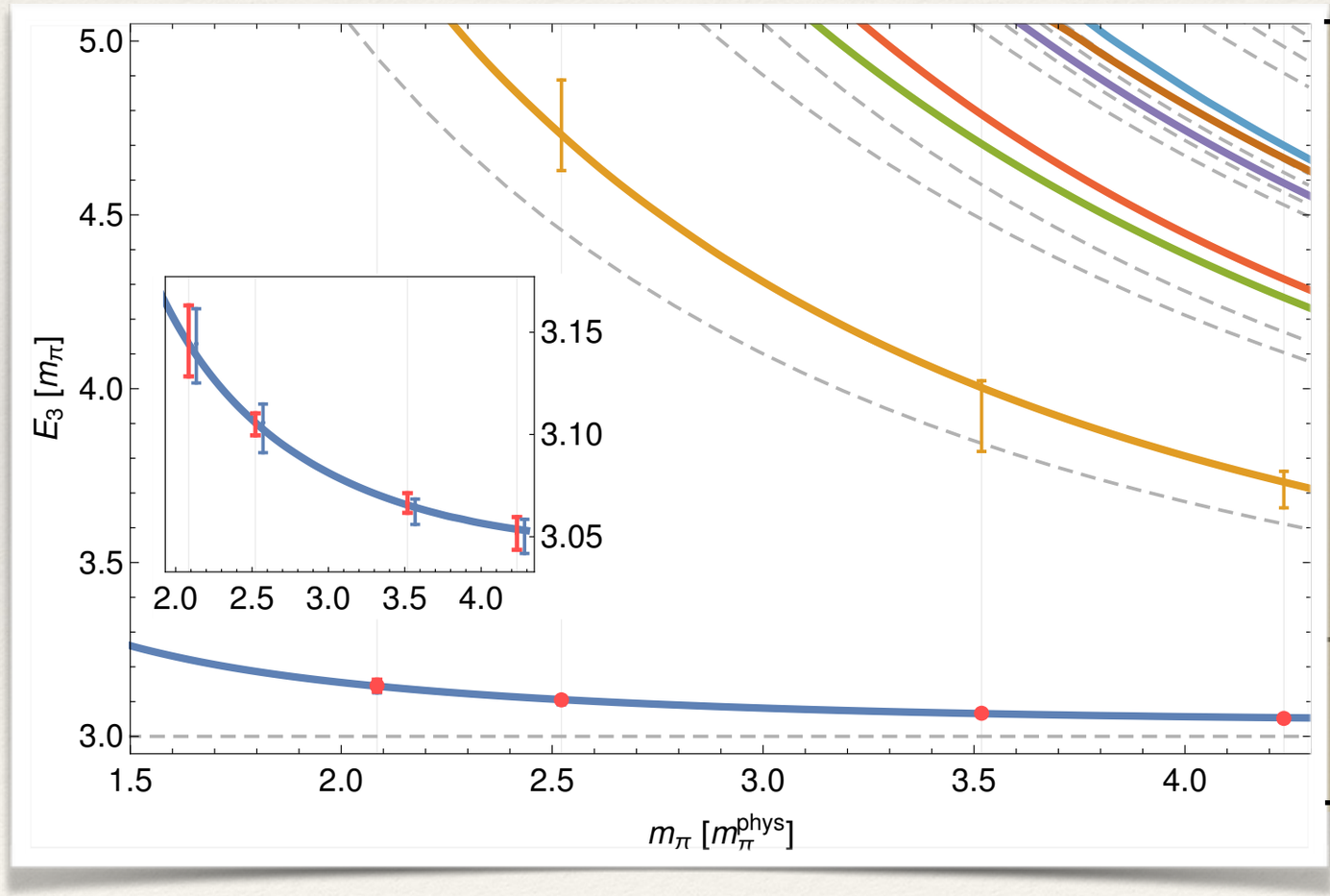
- correct σ and m_π behavior

Three positive pions

MM, Doring PRL 122 (2018)

- 3-body spectrum
  - genuine 3-body force C is not known
  - momenta dependent function

$$\left\langle \mathbf{v}\ \frac{\mathbf{1}}{\mathbf{B}(\mathbf{E}^*) + \mathbf{E_L}\ \tau_{\mathbf{LP}}^{-\mathbf{1}}(\mathbf{E}^*)}\ \mathbf{v} \right\rangle = \infty$$



PREDICTION

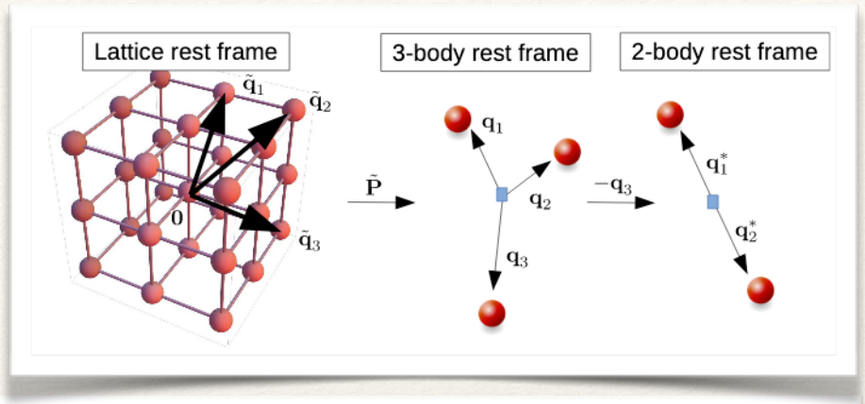
FIT

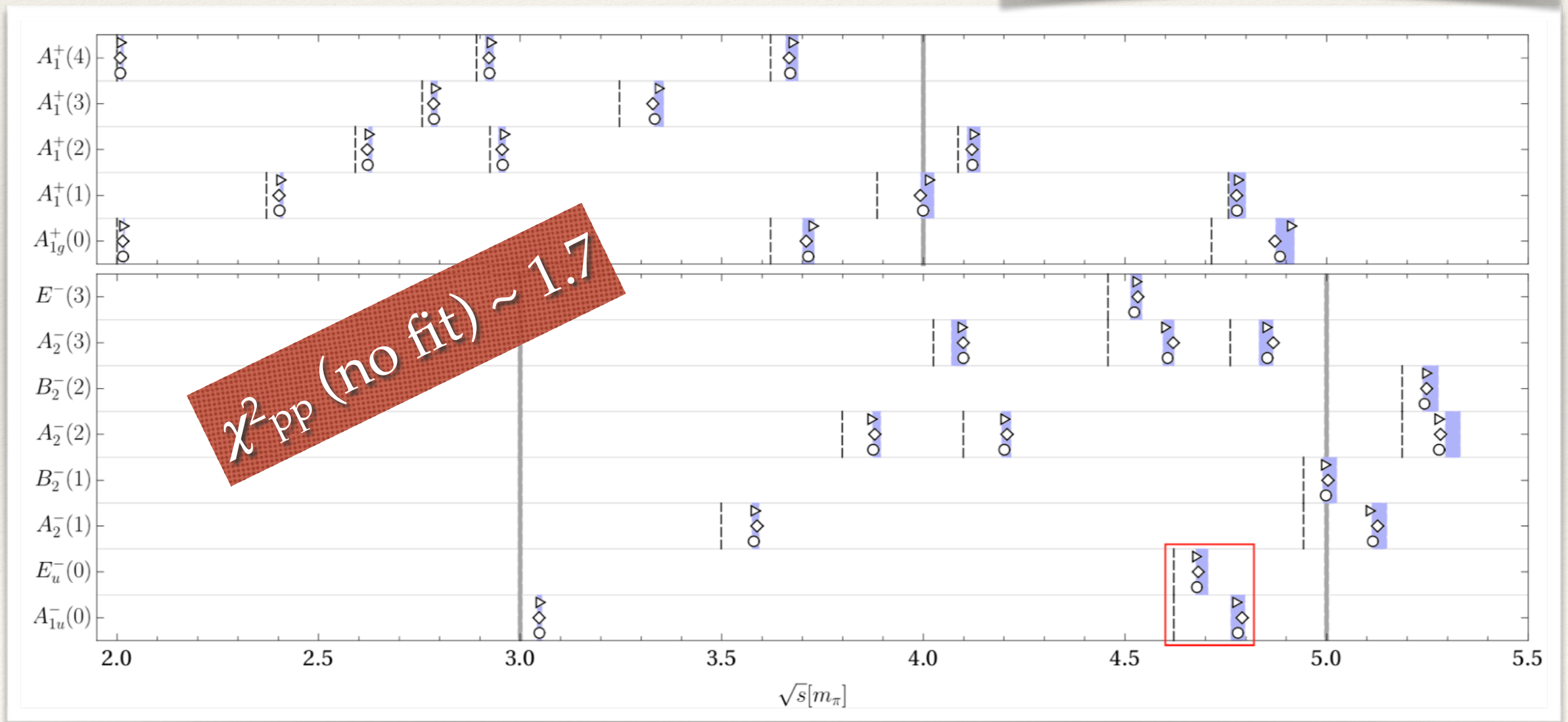
$c=0.2\pm1.5$

Three positive pions

MM et al. (2019)

- New data is available — Hörz/Hanlon PRL123 (2019)
  - Extend to arbitrary irreps & boosts
  - Use the same parametric input



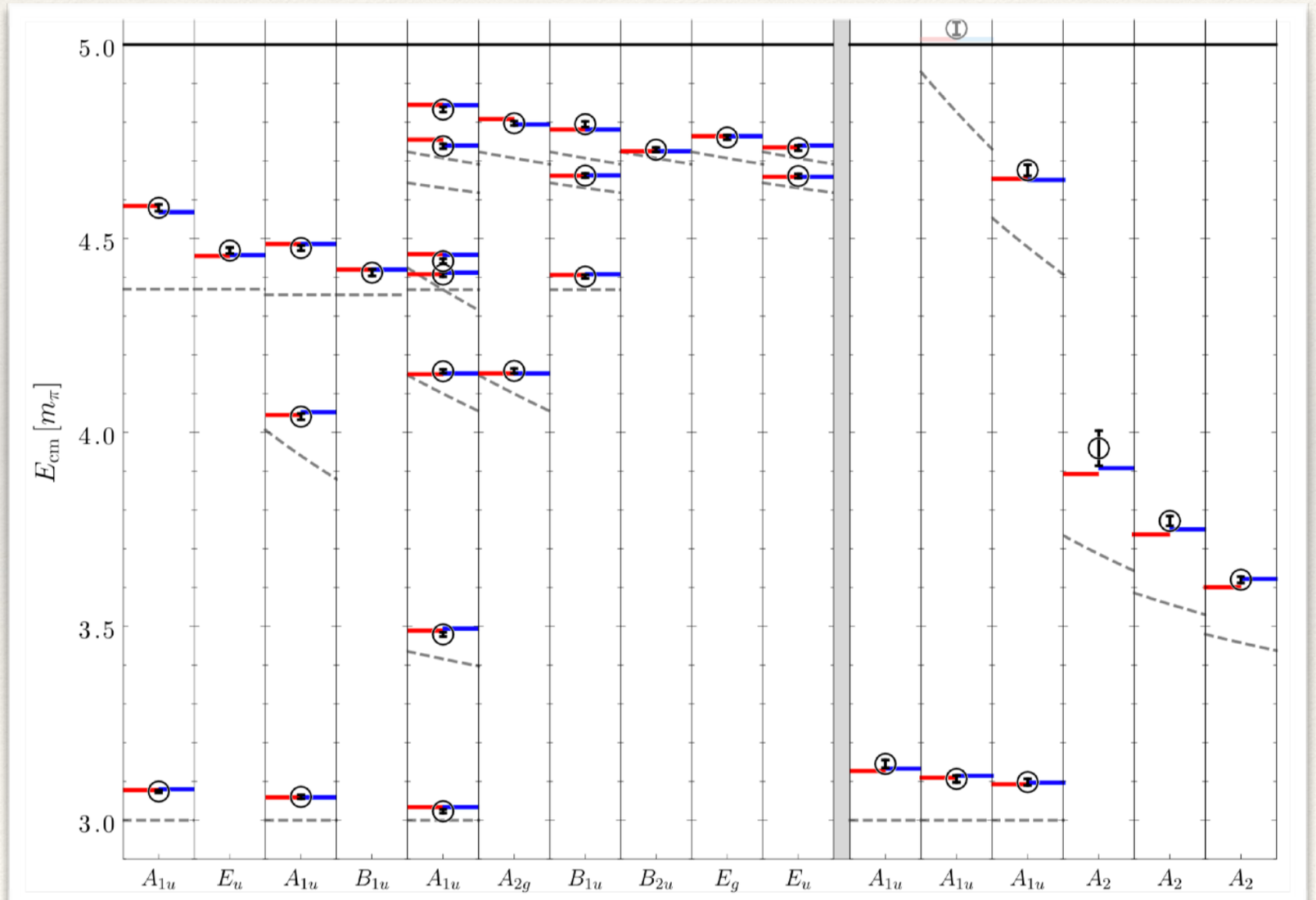


χ^2_{pp} (no fit) ~ 1.7

Three positive pions

- **Newer** data is available
  - very dense spectrum from elongated boxes
  - different pion masses (chiral extrapolations?)

Culver, MM, Brett, Alexandru, Doring (2019)
arXiv:1911.09047



predictions from MM/Doring(2018)

lattice calculation

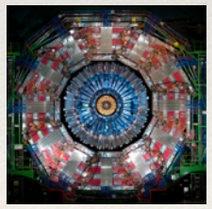
$\chi^2_{\rm pp}$ (no fit) ~ 2

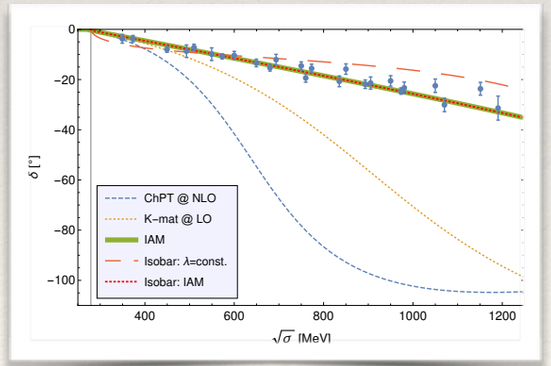
SUMMARY

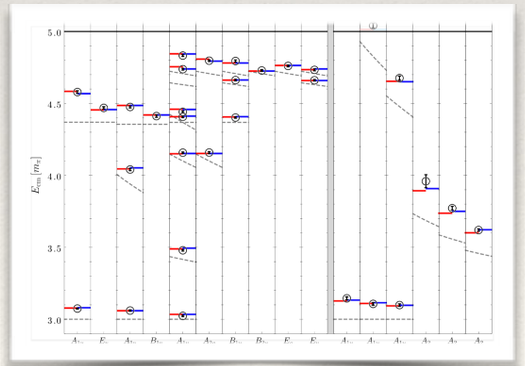
- **2-body spectroscopy from Lattice is highly evolved:**
  - new high-precision information on properties of resonances
  - complementary insights to experimental studies

    e.g. m_π *dependence, extreme quantum numbers,…*

- **3-body spectroscopy from Lattice is maturing quickly**
  - many theoretical tools have been developed
  - first data-driven studies are performed (details/statistics important)







QCD