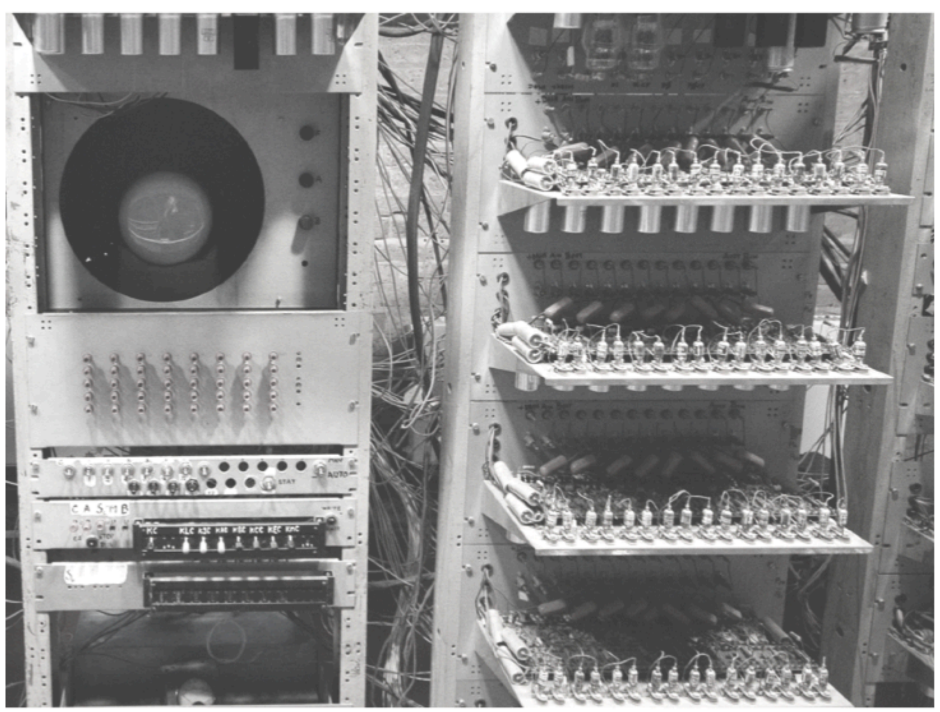
Quantum computing

Maxim Mai

SLIDES/NOTES: maxim-mai.github.io/QC.pdf



Manchester Baby Classical Computer [1948]

IBM Quantum Computer [2022]

Reminder from previous lectures

Hilbert space

... an isolated quantum system is associated with a complex vector space w. inner product

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \quad \{\alpha,\beta \in \mathbb{C} \mid \alpha^2 + \beta^2 = 1\}$$

[QUBIT]

Evolution of a quantum system

... described by a unitary transformation.

e.g. Hademard gate:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}: \quad |0\rangle \mapsto \frac{|0\rangle + |1\rangle}{\sqrt{2}} \quad |1\rangle \mapsto \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

[GATE]

Quantum measurement

... measurement operators act on the space of the system being measured, determines probabilities.

$$P(|\psi\rangle \text{ in state } |0\rangle) = |\langle 0|\psi\rangle|^2$$

Composite quantum system

... tensor product of state spaces of the component systems.

$$|\psi\rangle = \alpha|0\rangle|0\rangle + \beta|1\rangle|0\rangle + \gamma|0\rangle|1\rangle + \delta|1\rangle|1\rangle$$

Summary

Quantum supremacy

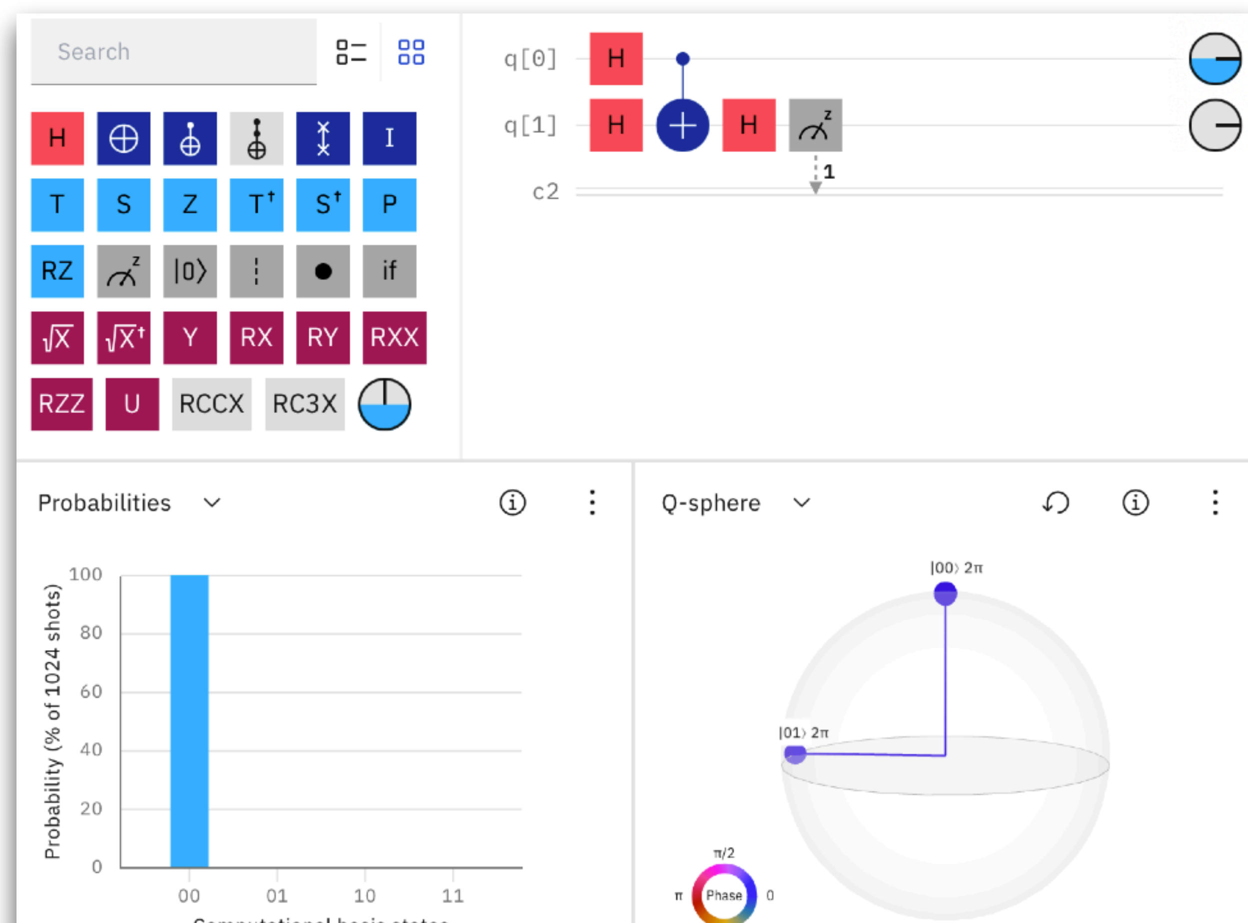
- Quantum Computer acts on quantum systems via unitary transformations
- quantum **parallelism**, quantum **entanglement**
- exponential **speedup** to Classical Computer

Challenges

- preparing operators is not trivial
- decoherence: decay of prepared states over time. Thermalization, cosmic rays...

Hands-on quantum computing links

- https://quantum-computing.ibm.com
- https://qiskit.org



QUANTUM COMPUTING

INTRO

- Hello, I am Maxim Mai, and I welcome you to the second part of my presentation as part of the Application for a Professorship position in Bochum.
- All materials are online.

GOALS

- "Quantum Computing" is the topic of today's lecture.
- It is a hot topic. Some people argue that we are on the verge of a revolution happened before with classical computing in 1950's SLIDE 0
- to get a taste of what makes QC so powerful we will see an example today.
- to begin with like every lecture, I made a small cheat sheet reminding on previous lecture.

SLIDE 1

- postulates
- principles

} talk through

Computation requires:

1) Information unit:

Qubit: quantum state with two pure states $|0\rangle$ & $|1\rangle$

[maybe spin system, or excited state of an atom]

[QC will be probabilistic]

2) Operations

Gates: Unitary operators

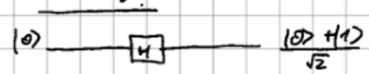
$$\alpha|0\rangle + \beta|1\rangle \xmapsto{NOT} \beta|0\rangle + \alpha|1\rangle$$

or for $\begin{pmatrix}\alpha\\\beta\end{pmatrix}$ representation: $\begin{pmatrix}\alpha\\\beta\end{pmatrix} \xmapsto{NOT} \begin{pmatrix}\beta\\\alpha\end{pmatrix}$

$NOT = \sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}$. Pauli matrix.

$XX^+ = \mathbb{1}$ ✓

Circuits:



[lets make the advantage even more apparent]

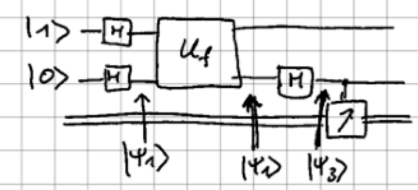
Algorithm:

- we are given a function $f: \{0,1\} \to \{0,1\}$
- the function is either constant:
  ⓐ $f(1) = f(0)$ or ⓑ $f(1) \neq f(0)$.

- HOW MANY EVALUATION OF f DO WE NEED TO DECIDE IF IT IS ⓐ or ⓑ

I CC: two times!

II QC: define a unitary transfo $U_f |x,y\rangle \mapsto |x, y \oplus_2 f(x)\rangle$ and following circuit:



$$|\psi_1\rangle = H_1 H_2 |01\rangle = H|0\rangle H|1\rangle = \frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$$

$$|\psi_2\rangle = U_f|\psi_1\rangle = \frac{1}{2}\underbrace{U_f|0\rangle(|0\rangle - |1\rangle)}_{A} + \frac{1}{2}\underbrace{U_f|1\rangle(|0\rangle - |1\rangle)}_{B}$$

$$A = |0\rangle|(0-1) \oplus_2 f(0)\rangle = (-1)^{f(0)}|0\rangle(|0\rangle - |1\rangle)$$

$$B = |1\rangle|(0-1) \oplus_2 f(1)\rangle = (-1)^{f(1)}|1\rangle(|0\rangle - |1\rangle)$$

$$= \begin{cases} \pm \frac{1}{2}(|0\rangle + |1\rangle)(|0\rangle - |1\rangle) & f(0) = f(1) \\ \pm \frac{1}{2}(|0\rangle - |1\rangle)(|0\rangle - |1\rangle) & f(0) \neq f(1) \end{cases}$$

Overall phase never matters

$$|\psi_3\rangle = H_2|\psi_2\rangle = \begin{cases} \frac{1}{\sqrt{2}} \pm|0\rangle(|0\rangle - |1\rangle) & f(0) = f(1) \\ \frac{1}{\sqrt{2}} \pm|1\rangle(|0\rangle - |1\rangle) & f(0) \neq f(1) \end{cases}$$

one iteration of a U_f is needed to det ⓐ/ⓑ