$a_1(1260)$ from lattice QCD

Unitarity
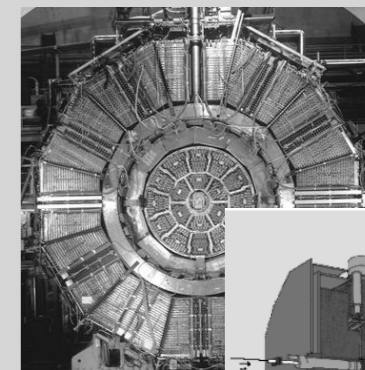
LHC
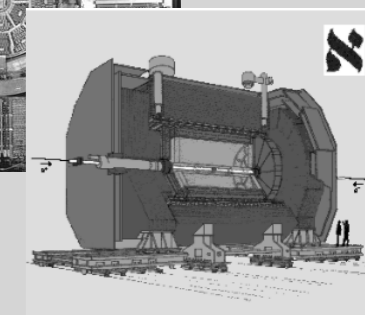
GlueX

GW COLONIAL ONE

QCD

ChPT
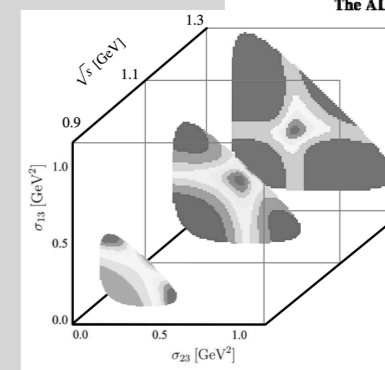
THEORY

PRL(in press)
2107.03973 [hep-lat]

PDG
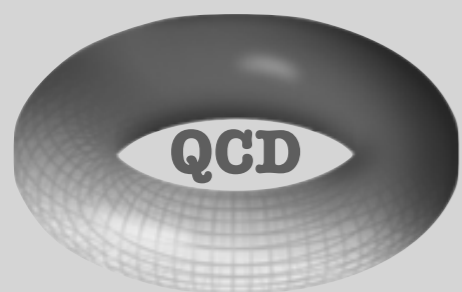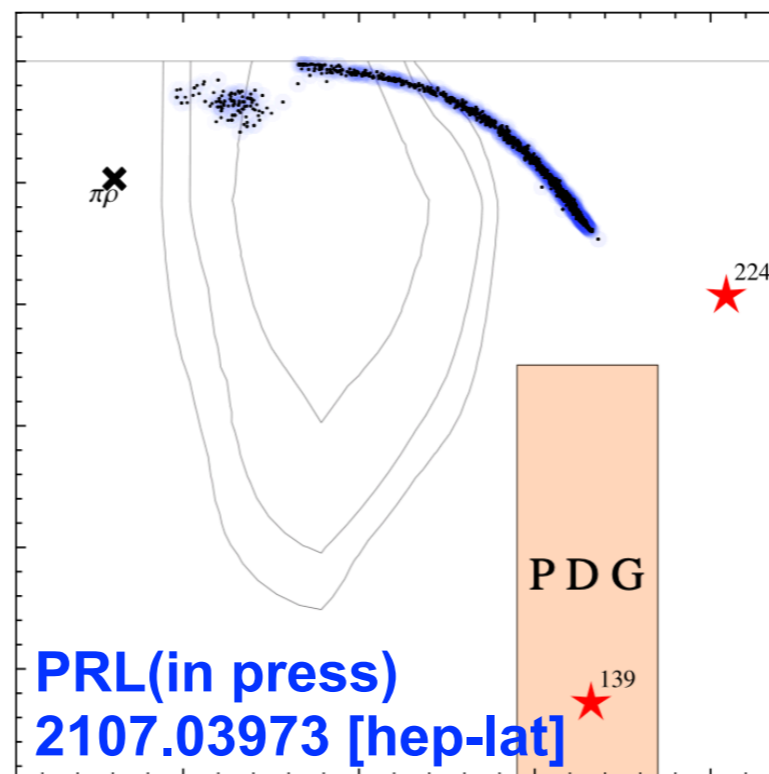
The ALEPH Detector

EXPERIMENT

Maxim Mai

with A. Alexandru*, R. Brett*, C. Culver*, M. Döring,

F. Lee*, D. Sadasivan [*GWQCD]
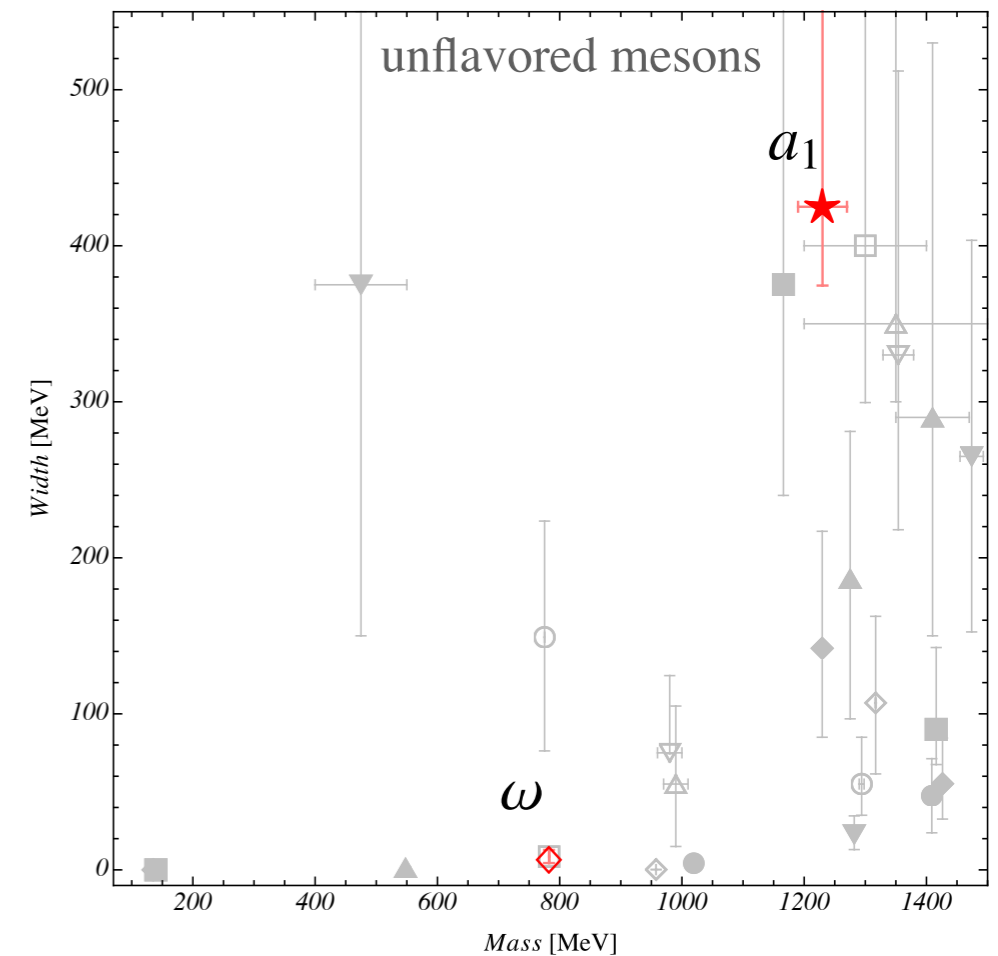
Pattern and **production mechanisms** of many excited hadrons are unresolved issues

Many questions are related to the hadronic 3-body problem

- ω(782), a_1(1260), ...

- Roper resonance *N*(1440)*

- exotic mesons: π_1(1600), ...

(experimental searches @COMPASS, GlueX)

This work: a_1(1260) from lattice QCD

MOTIVATION
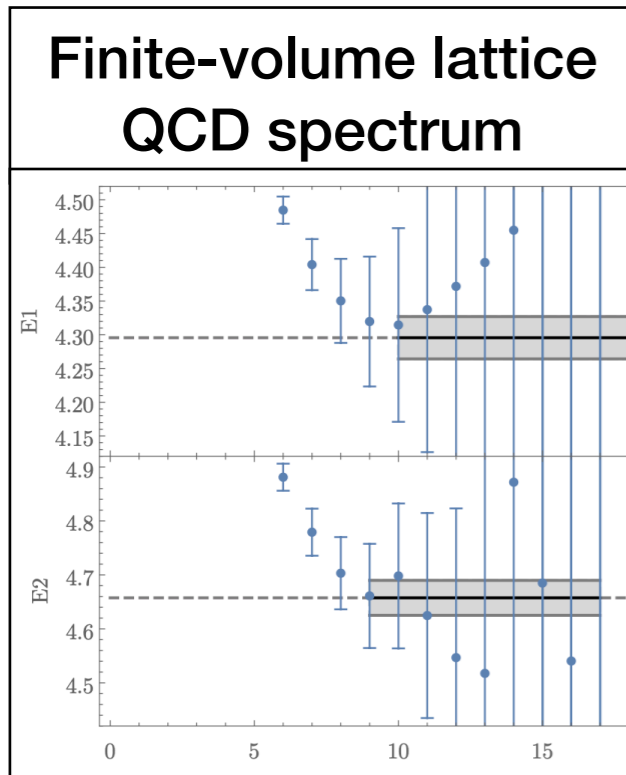
Universal parameters of resonances from poles on Riemann surface

- 3-step procedure:

Universal parameters of resonances from poles on Riemann surface

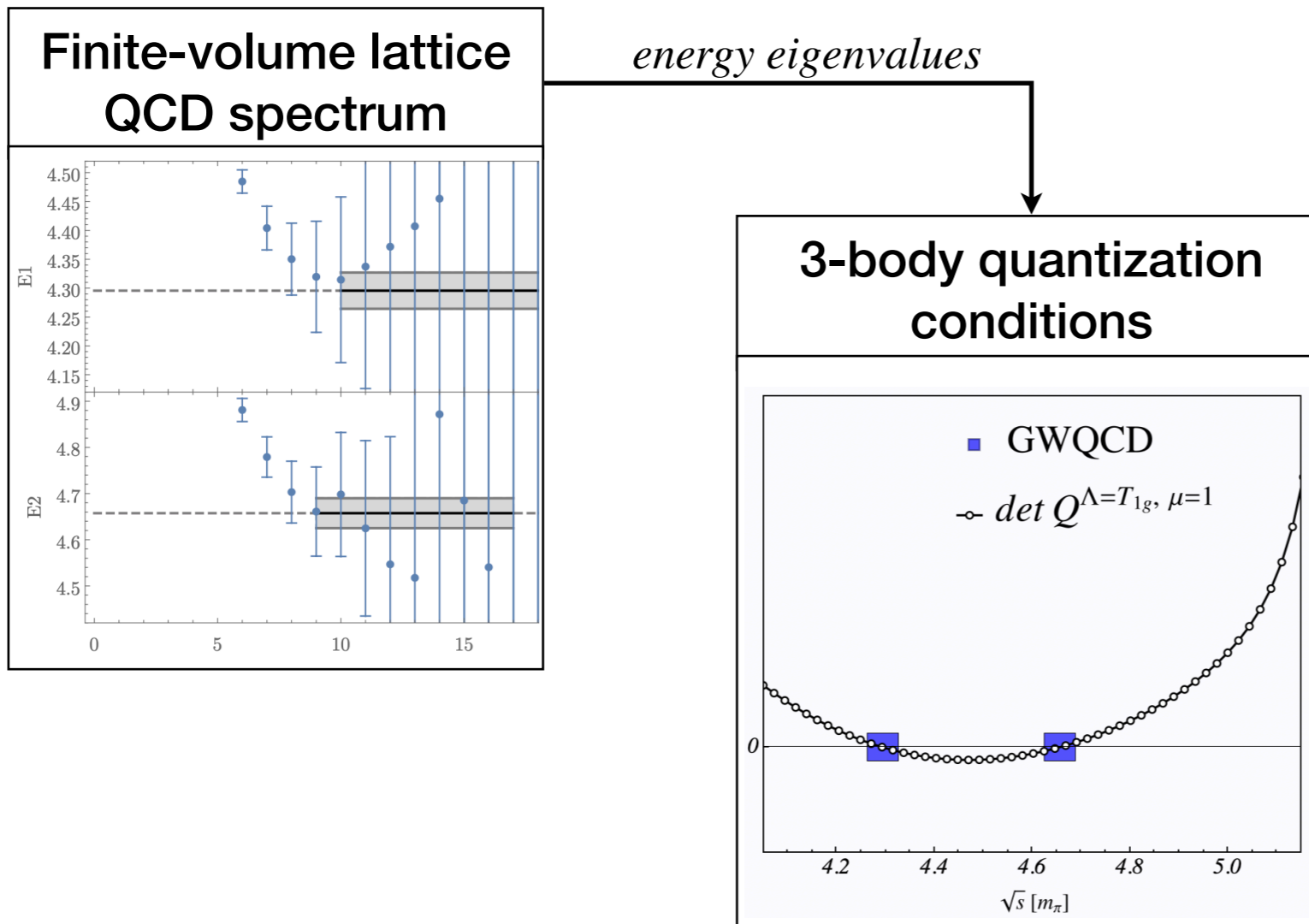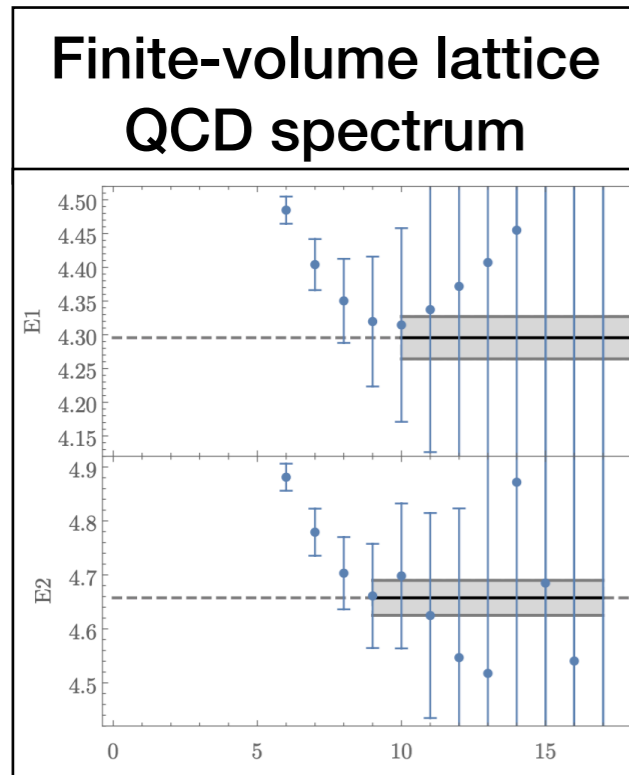- 3-step procedure:

Finite-volume lattice QCD spectrum

MOTIVATION

Universal parameters of resonances from poles on Riemann surface

- 3-step procedure:

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energy eigenvalues

3-body quantization conditions

Universal parameters of resonances from poles on Riemann surface

• 3-step procedure:

Finite-volume lattice QCD spectrum

energy eigenvalues

3-body quantization conditions

volume-independent quantities

Riemann Surface: scattering amplitude

Phenomenological studies

- A clean way: $a_1(1260)$ from τ-decays (ALEPH@CERN)

Mikhashenko et al. [JPAC] (2018) Sadasivan/MM/Döring(2020)

Sadasivan/MM/
Döring(2020)
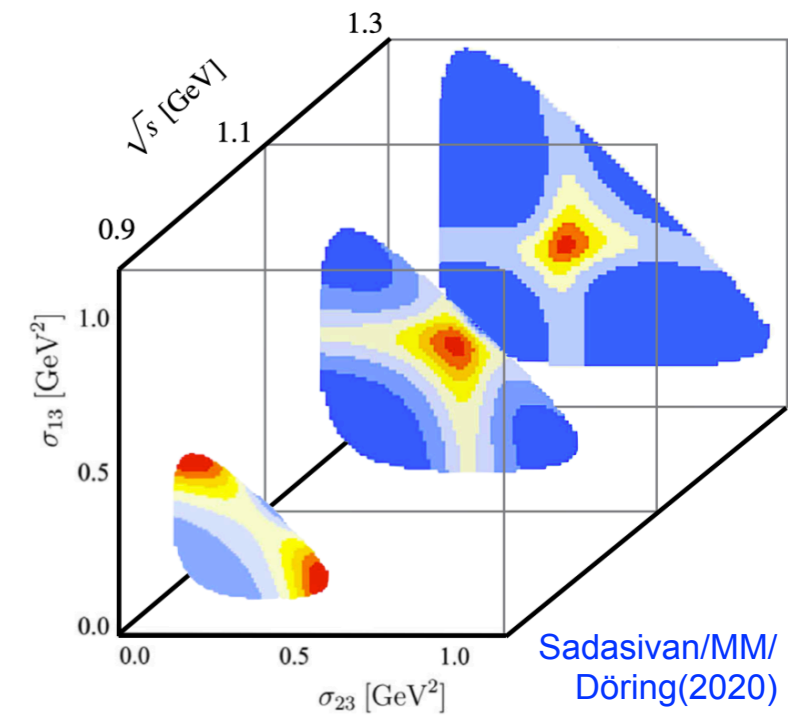
Phenomenological studies

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Sadasivan/MM/
Döring(2020)

Theoretical ab-initio access: lattice QCD

- full quark-gluon dynamics with some technical issues

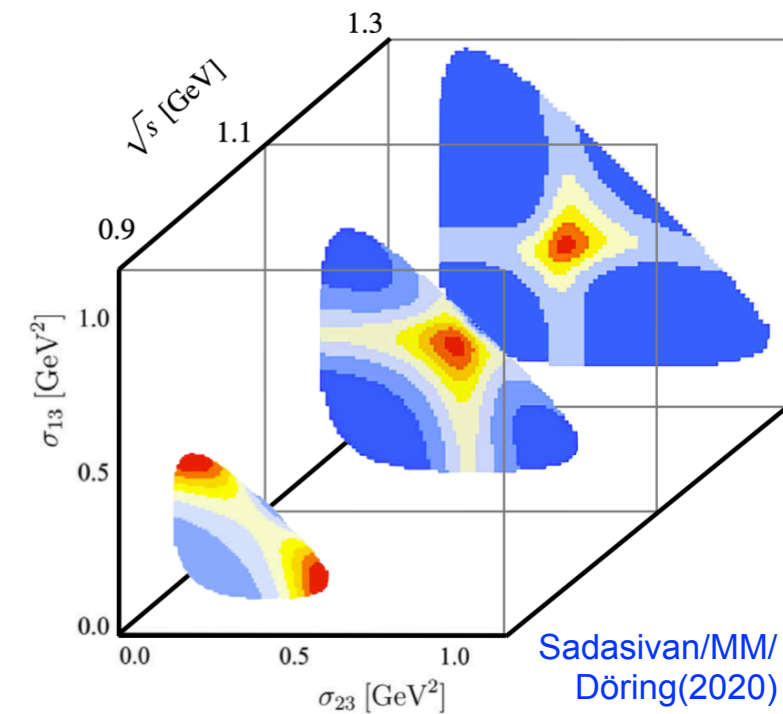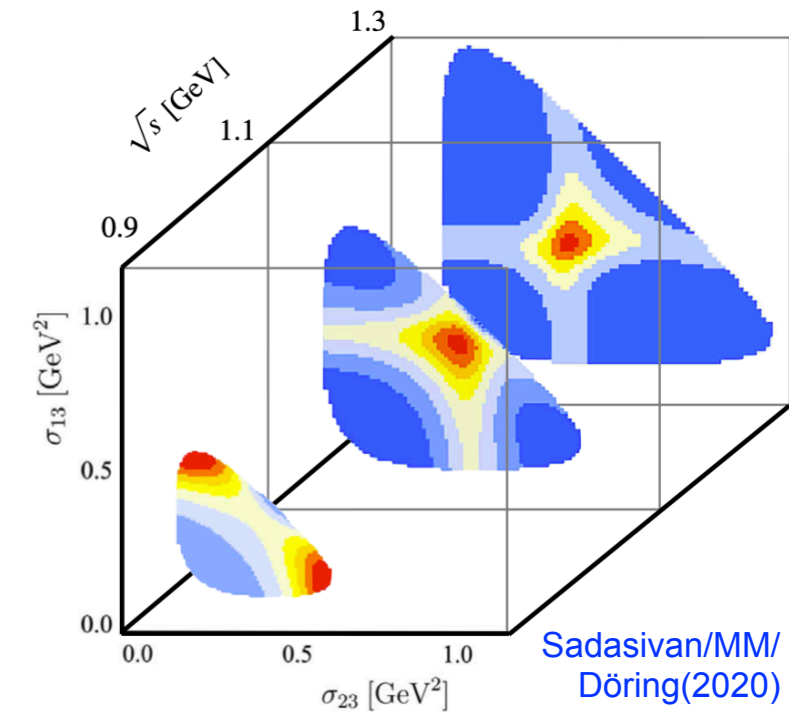 | | | |
  |---|---|---|
  | 1. | Discretized Euclidean space-time | >> continuum limit |
  | 2. | Unphysical pion mass | >> chiral extrapolations |
  | 3. | Finite Volume | >> quantization condition |

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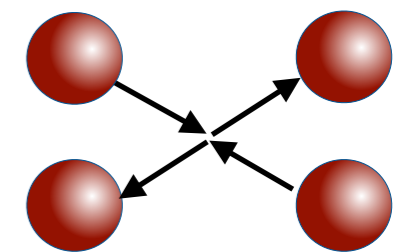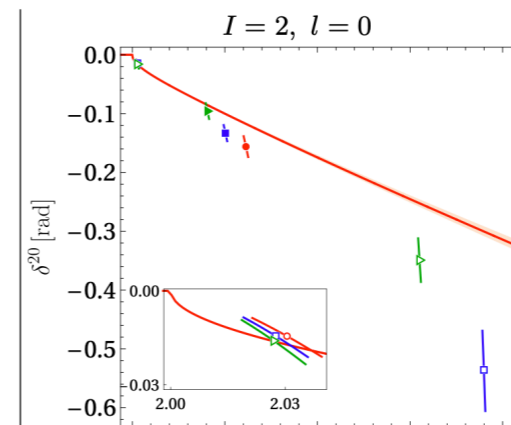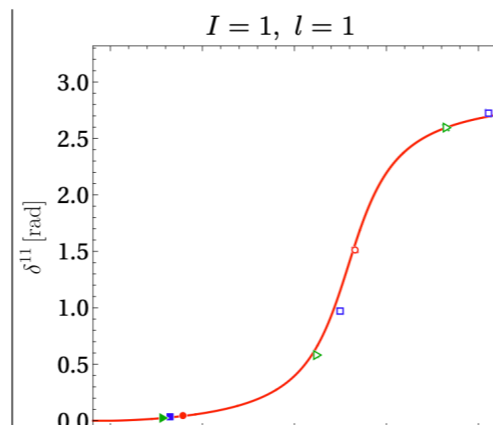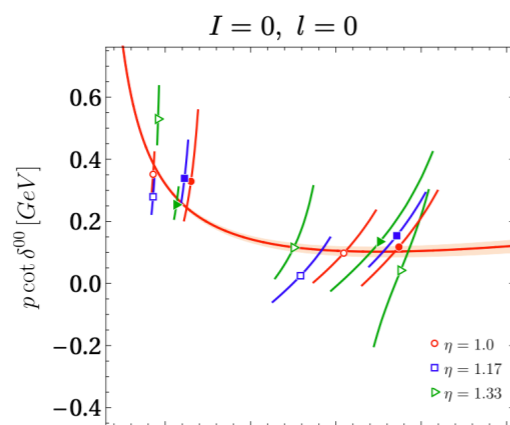

Sadasivan/MM/
Döring(2020)

Theoretical ab-initio access: lattice QCD

- full quark-gluon dynamics with some technical issues

1.	Discretized Euclidean space-time	>> continuum limit
2.	Unphysical pion mass	>> chiral extrapolations
3.	Finite Volume	>> quantization condition

- many studies of 2-body systems   [NPLQCD], [RQCD], [ETMC], [HadSpec], ...   **Review: Rev.Mod.Phys. 90 (2018) 2**

e.g. MM et al. [GWQCD] (2019)

- non-resonant 3-body systems with maximal isospin: $\pi^+\pi^+\pi^+$, $K^-K^-K^-$, ...   [NPLQCD]; Hörz/Hanlon; [GWQCD]; [HadSpec]; Blanton et al.

LATTICE QCD

Key details of GWQCD lattice QCD calculation

- $N_f = 2$ dynamical fermions, LapH smearing

- $\mathbf{P}=(0,0,0)$, $m_\pi=224$ MeV, $m_\pi L=3.3$

- *GEVP with one-/two-/three-meson operators*

- Relevant irrep(O_h) for **a1(1260) I^G ($J^{P\,C}$) = 1^- (1^{++})**:

$\mathbf{P}$	Λ	J^P ($I^G = 1^-$)
$\mathbf{P} = (0,0,0)$	T_{1g}	$1^+, 3^+, \ldots$
	A_{1u}	$0^-, 4^-, \ldots$

LATTICE QCD

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- *Relevant irrep(O_h) for **a1(1260)** I^G (J^{PC}) = 1^- (1^{++}):*

Alexandru, Brett, Culver, Guo, Lee, Pelissier (2013-2021)
PRD87,PRD94,PRD98,PRD96,PRL117,PRD100

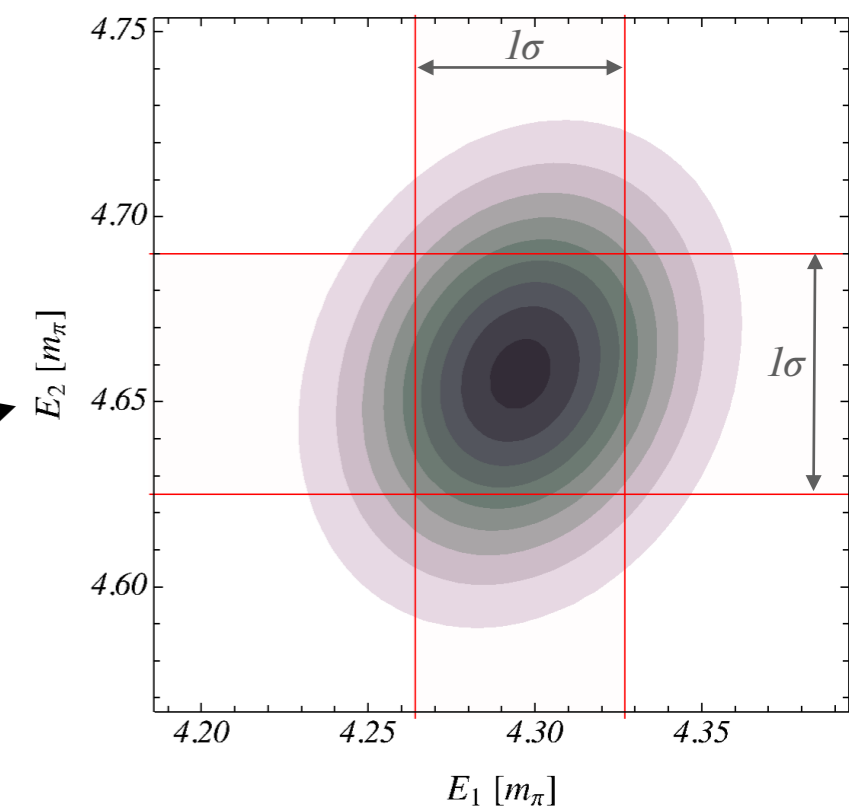| $\mathbf{P}$	Λ	J^P ($I^G = 1^-$)
$\mathbf{P} = (0,0,0)$	T_{1g}	$1^+, 3^+, \ldots$
	A_{1u}	$0^-, 4^-, \ldots$

Key results:

- 3π operators essential for the excited-state extraction
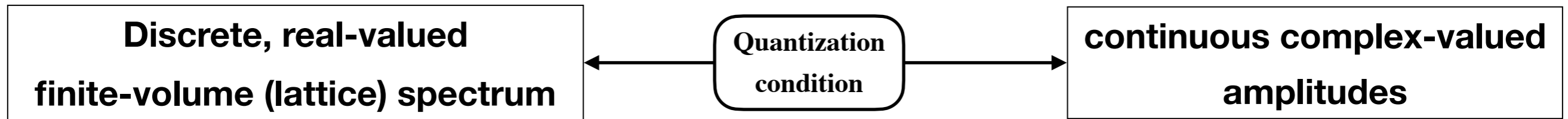 c.f. $\rho\pi$ operators in previous 2-meson a_1 calculation
  Lang et al. JHEP 04, 162 (2014)

- high-momentum states are required:
  $\pi(0,0,0)\pi(+1,+1,0)\pi(-1,-1,0)$ etc..
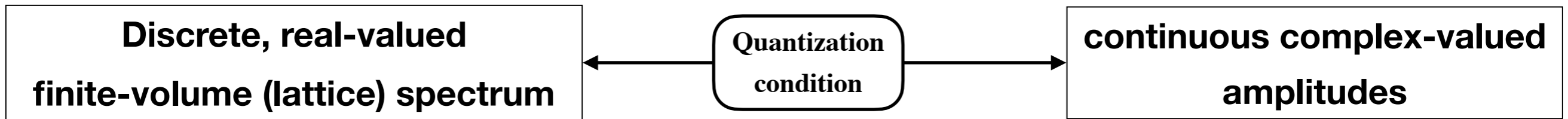
- two levels exist below 5π threshold

QUANTIZATION CONDITION

Discrete, real-valued	Quantization condition	continuous complex-valued
finite-volume (lattice) spectrum	← →	amplitudes

- established in 2-body: Lüscher's method, extensions... Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky...

- 3-body methods matured

Bedaque, Blanton, Briceño, Davoudi, Döring, Grießhammer, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe, Wu

Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

QUANTIZATION CONDITION

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Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

Finite Volume Unitarity (FVU)   MM, Döring EPJA (2017) PRL (2019)

- *extended to higher spin and coupled-channels: new degree of freedom (helicity, λ)*

$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]^{\Lambda}_{(\lambda'\lambda)(\mathbf{p'p})}$$
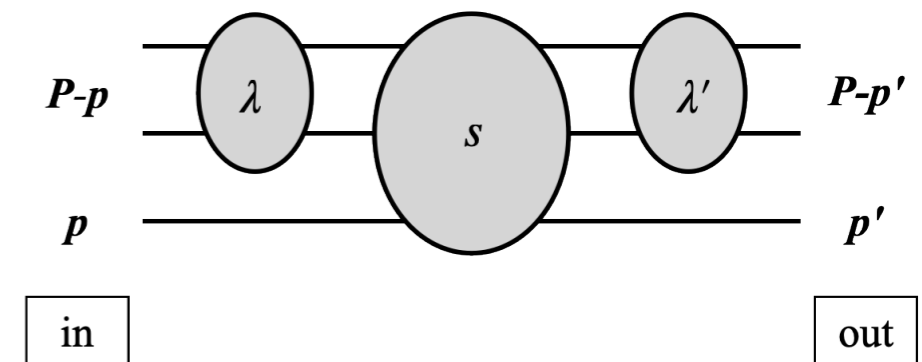
- *∞-dim. determinant equation in $\mathbf{p} \in \dfrac{2\pi}{L}\mathbf{Z}^3$*   ⟶   *practical applications require truncation*
  ⟶   *common to all quantization conditions*

QUANTIZATION CONDITION

$$0 = \det \left[\; {\color{blue} B(s)} + {\color{red} C(s)} - 2L^3 E_{\mathbf{p}} \left({\color{green} \tilde{K}_2^{-1}(s)} - {\color{purple} \Sigma_2^L(s)} \right) \; \right]_{(\lambda' \lambda)(\mathbf{p}' \mathbf{p})}^{\Lambda}$$
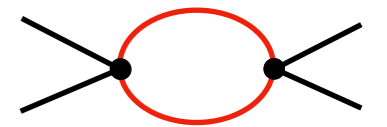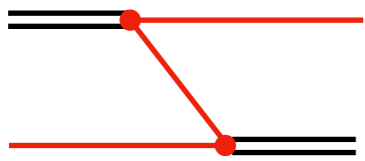
MM, Döring EPJA (2017)
PRL (2019)

QUANTIZATION CONDITION
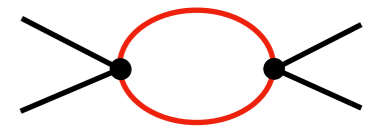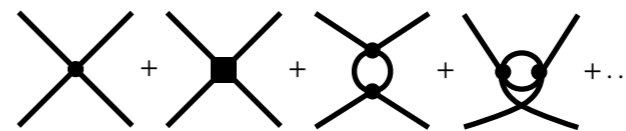
$$0 = \det \left[\, B(s) + C(s) - 2L^3 E_{\mathbf{p}} \left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s) \right) \, \right]^{\Lambda}_{(\lambda' \lambda)(\mathbf{p'p})}$$

MM, Döring EPJA (2017)
PRL (2019)

two-body self-energy

- fixed by 2b-unitarity

- no free parameters

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two-body kernel

- dynamics of $l{=}1$ $\pi\pi$ system

$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

- regular function $\Rightarrow$ polynomial

MM et al. [GWQCD] (2019)

- parameters (a_0, a_1) from cross-channel fit to $\pi\pi$ GWQCD levels

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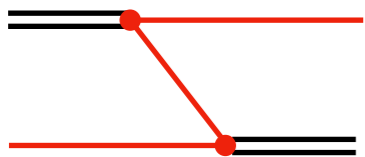
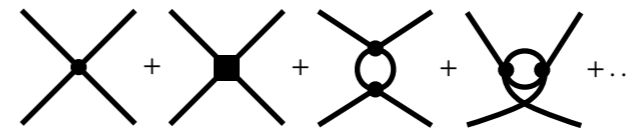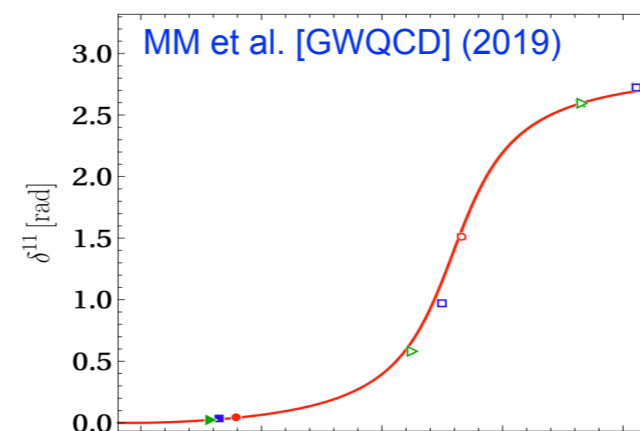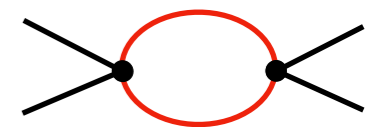- no free parameters

three-body force

- dynamics of $\rho\pi$ system
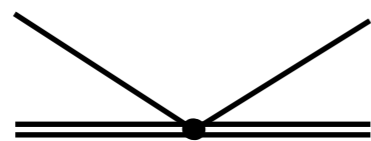
- regular function ⇒ Laurent series

$$C_{\ell'\ell}(s, \mathbf{p'}, \mathbf{p}) = \sum_{i=-1}^{\infty} c_{\ell'\ell}^{(i)}(\mathbf{p'}, \mathbf{p})(\mathbf{s} - \mathbf{m_{a_1}^2})^{\mathbf{i}}$$

- fit to 3-body levels → *next slide*

two-body kernel

- dynamics of $l=1$ $\pi\pi$ system

$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

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- no free parameters

FROM LATTICE EIGENVALUES TO POLES

Last unknown (volume-independent) quantity: 3-body term C

- a_1 pole is generated *with/without explicit pole-term*

- best description (with large correlations)

$$C_{\ell'\ell} = g_{\ell'} |\mathbf{p}'|^{\ell'} \frac{1}{s - m_{a_1}^2} g_\ell |\mathbf{p}|^\ell + c\, \delta_{\ell'0}\delta_{\ell 0}$$

- full resampling... accessing statistical errors
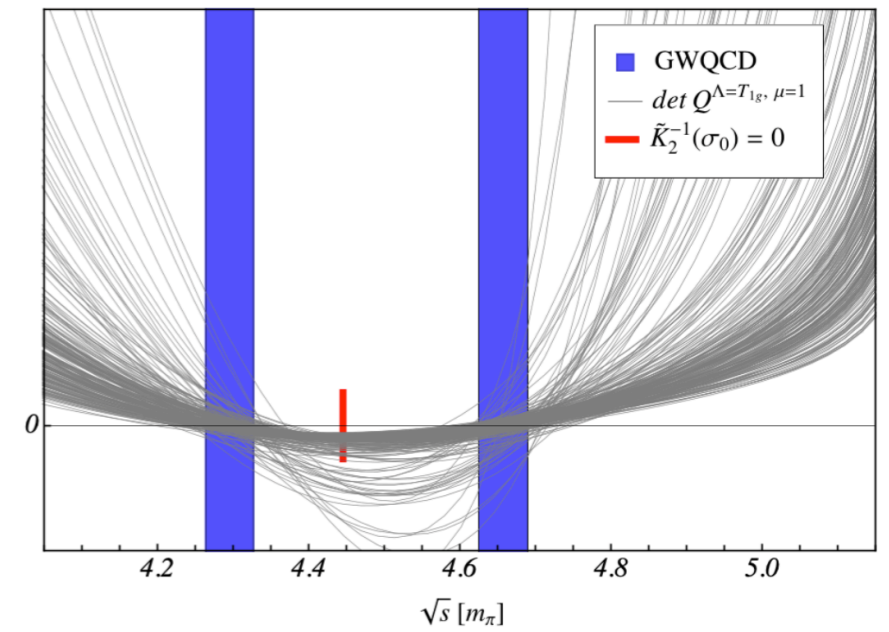
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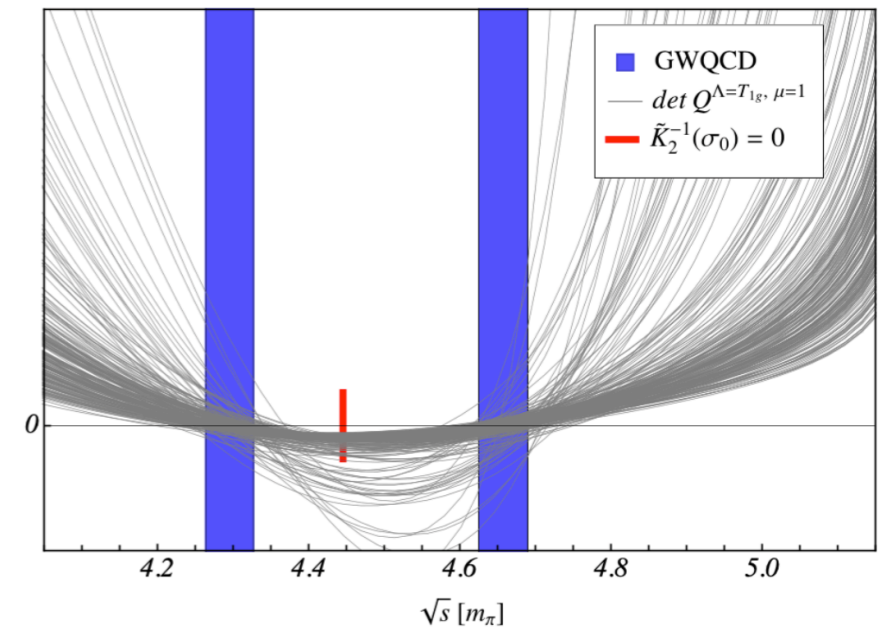
Infinite-volume scattering amplitude

- integral equation with all inputs fixed from lattice QCD results

$$T^c = B + C + \int\frac{d^3\ell}{(2\pi)^3}\frac{(B+C)}{2E_l}\frac{1}{\tilde{K}_n^{-1}-\Sigma_n}T^c$$

- contour deformation of spectator momenta

- analytic continuation to 2.Riemann sheet
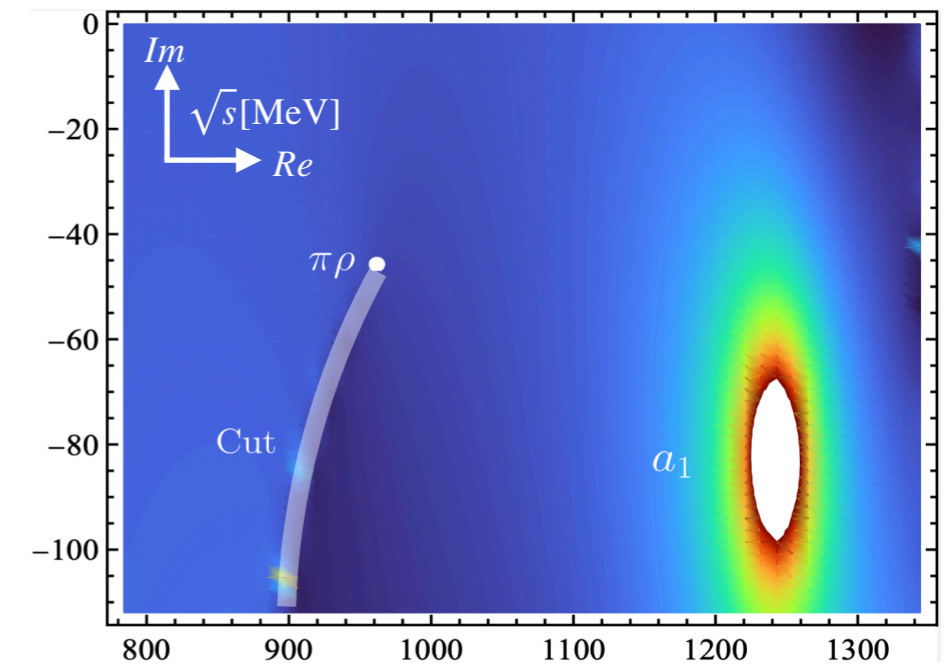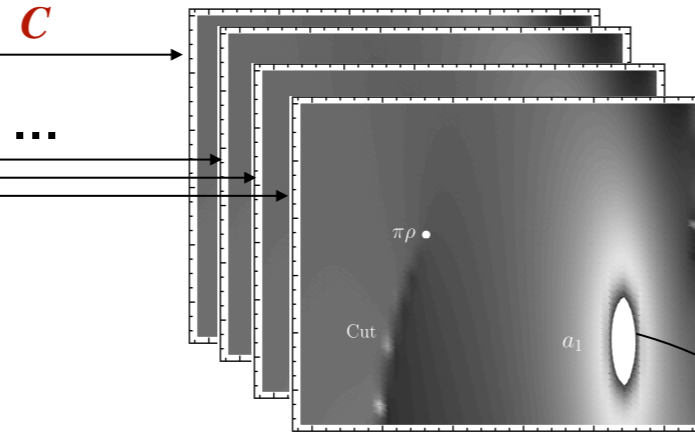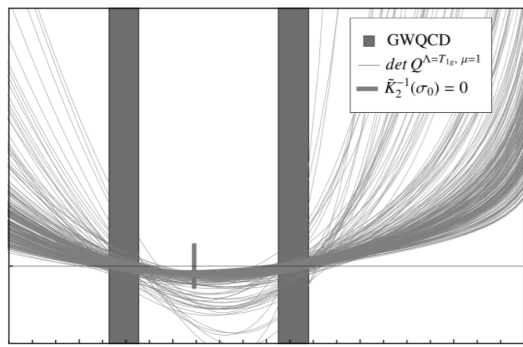
- m_π=224 MeV

- 2000 resampling sets

- distribution of poles is finite
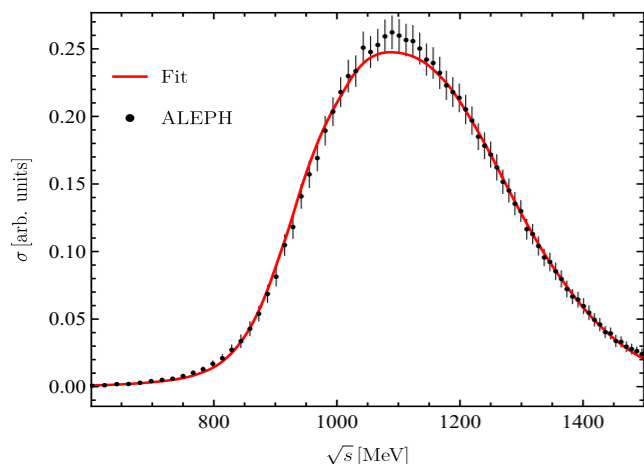
- 90% @ ~((1110..1210) - (20..70)i) MeV
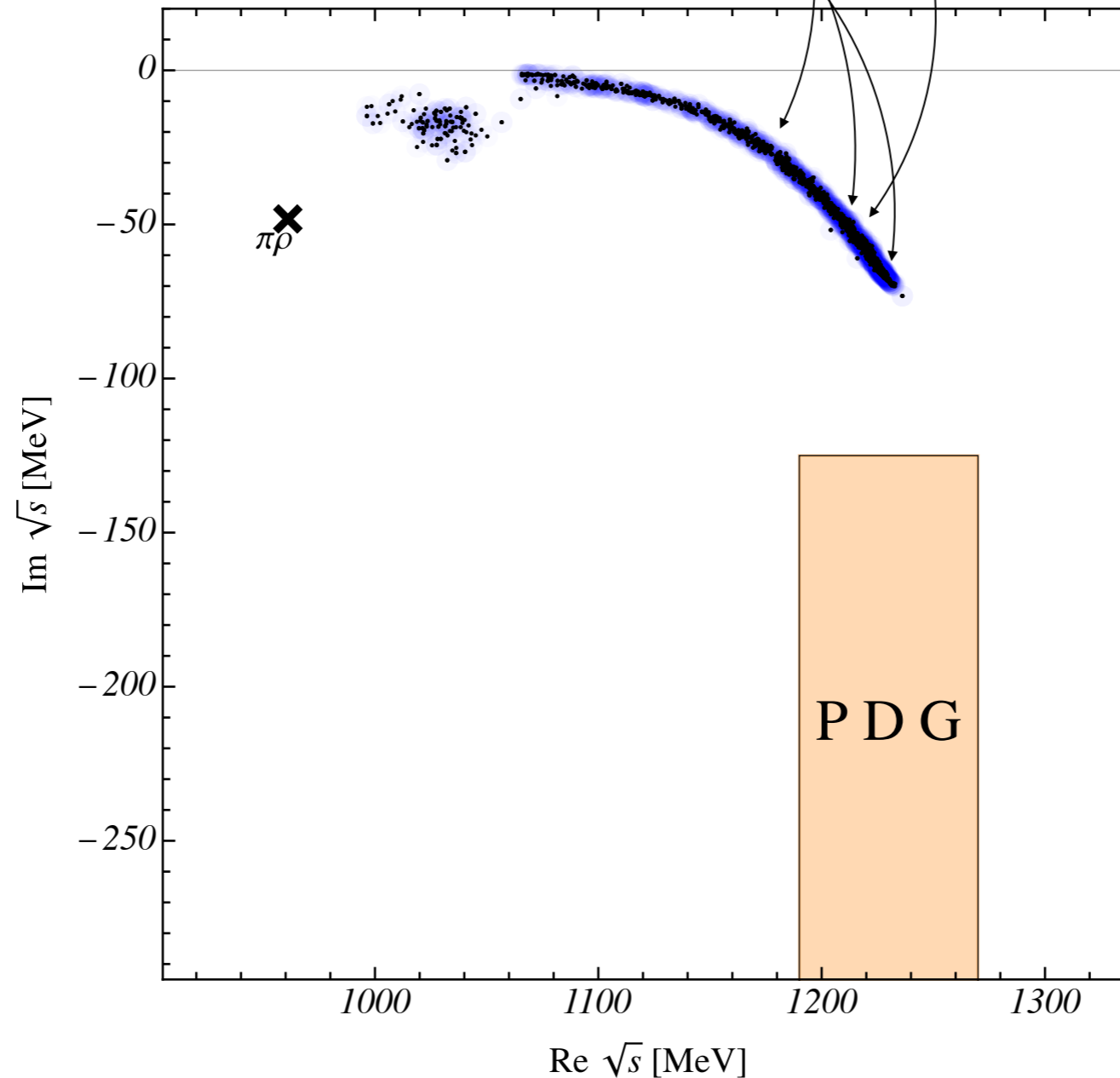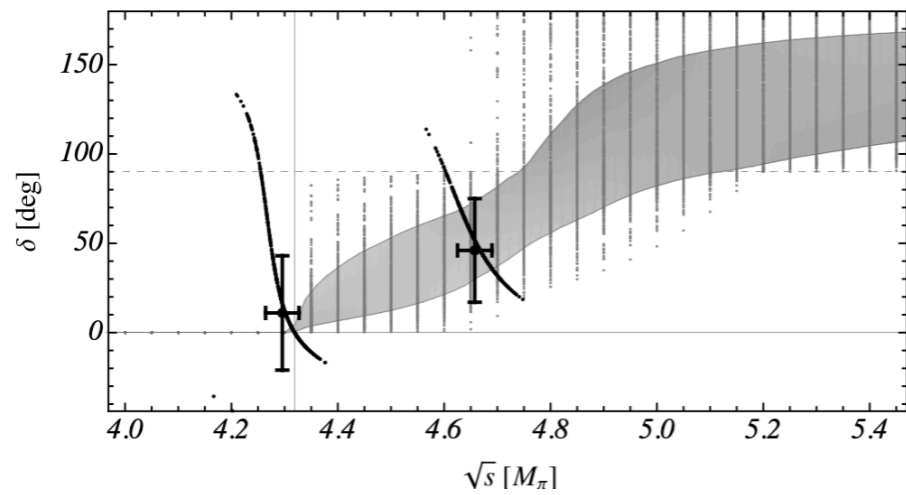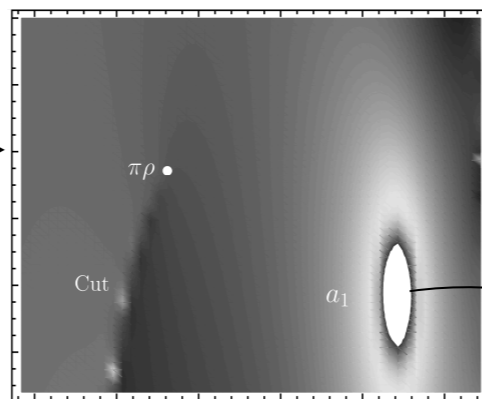
Can one use two-body approximation?

- Breit-Wigner parametrization

- 2-body Lüscher quantization condition
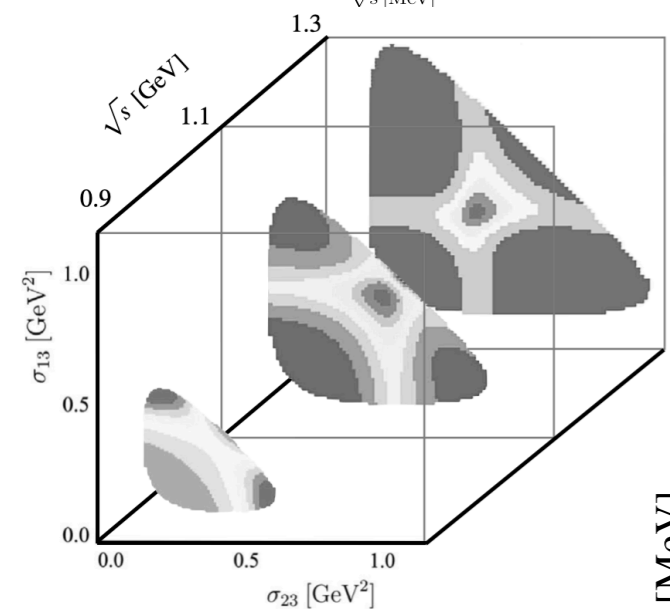
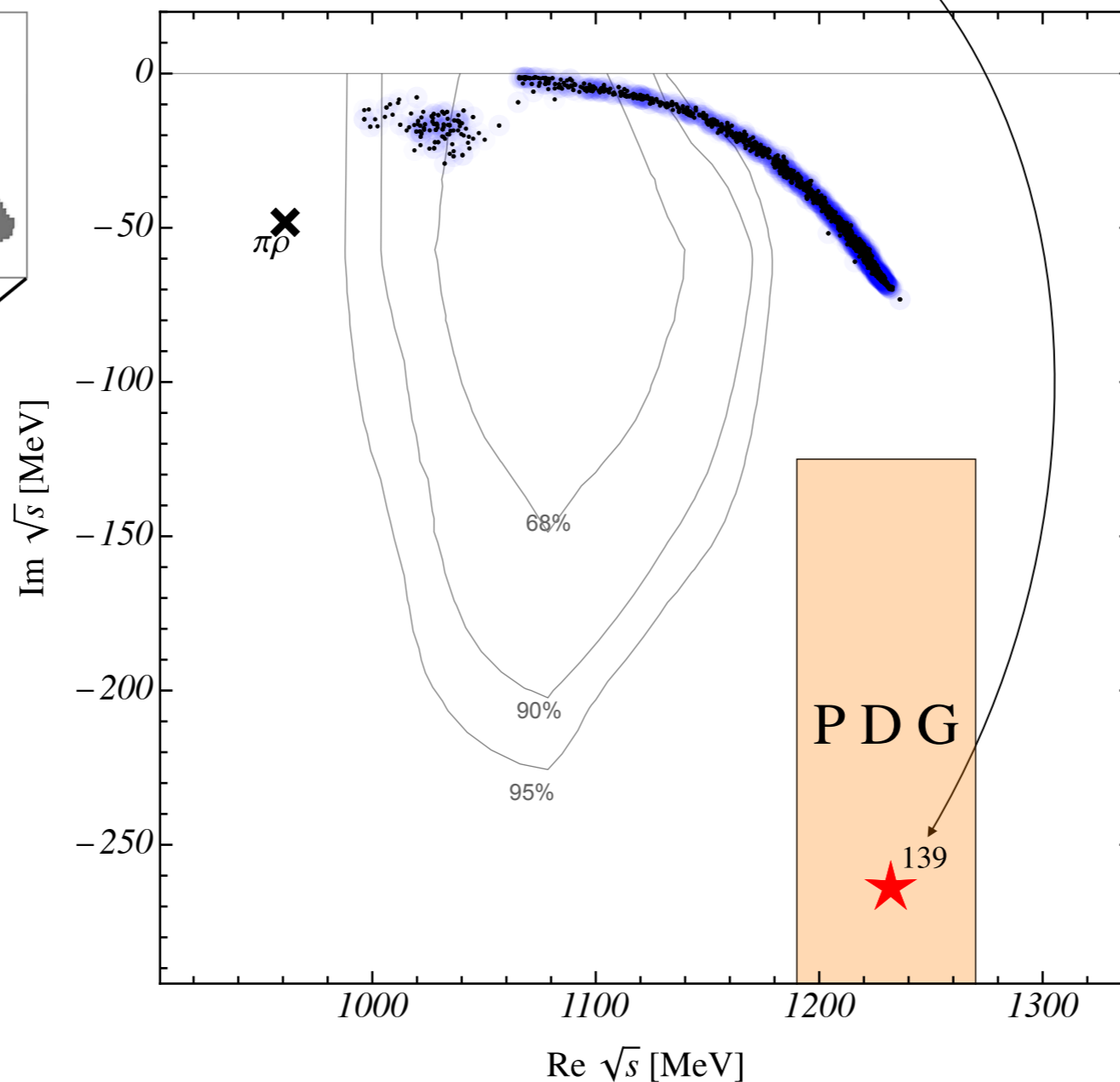$\Rightarrow$ very crude approximation!
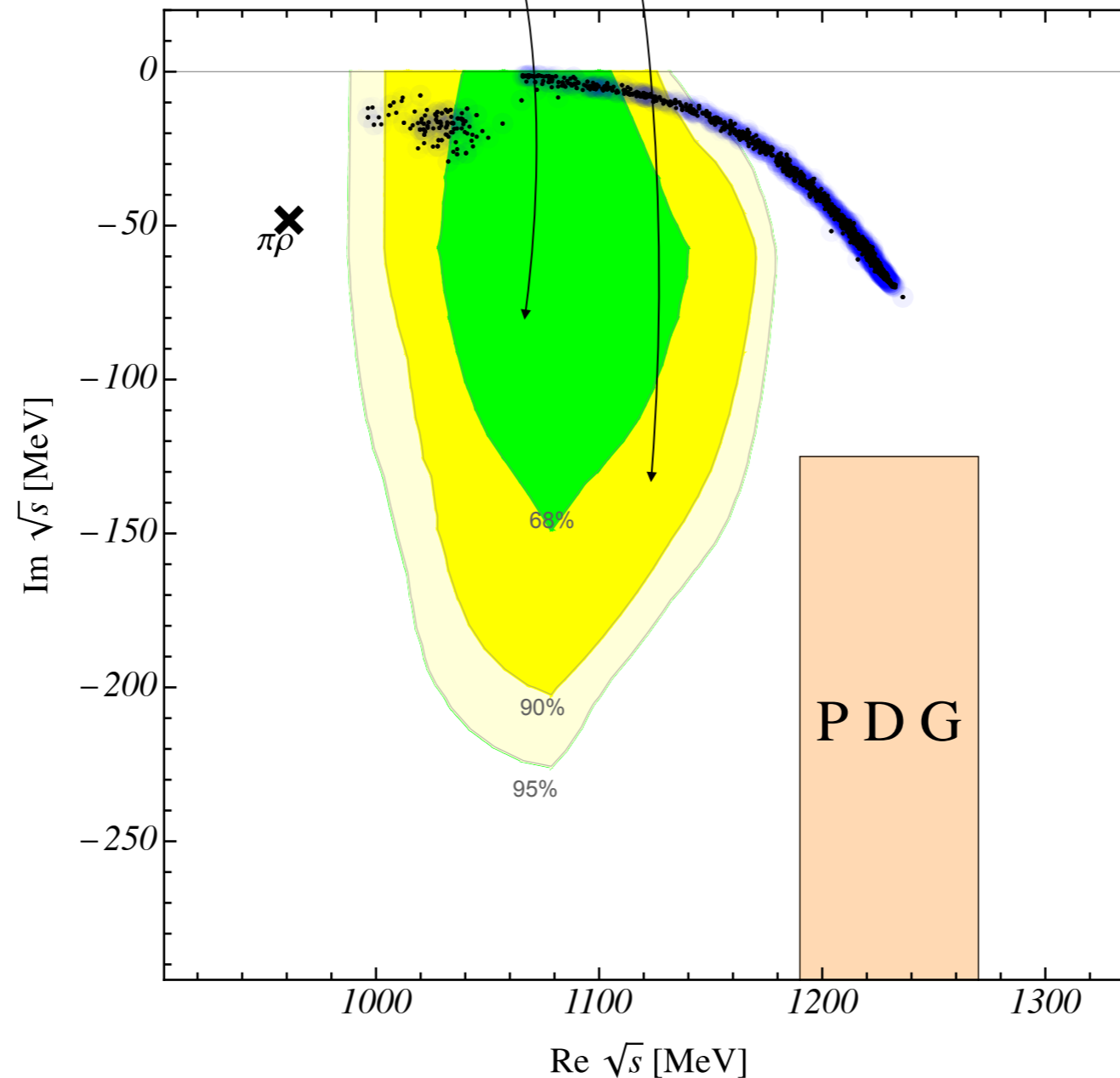
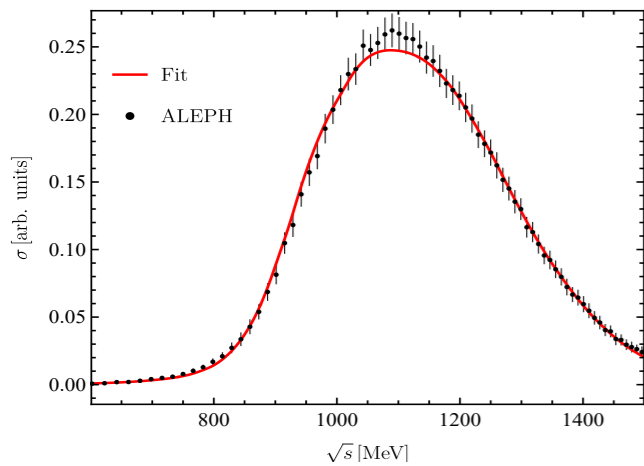# RESULTS & CHECKS

What does phenomenology says?

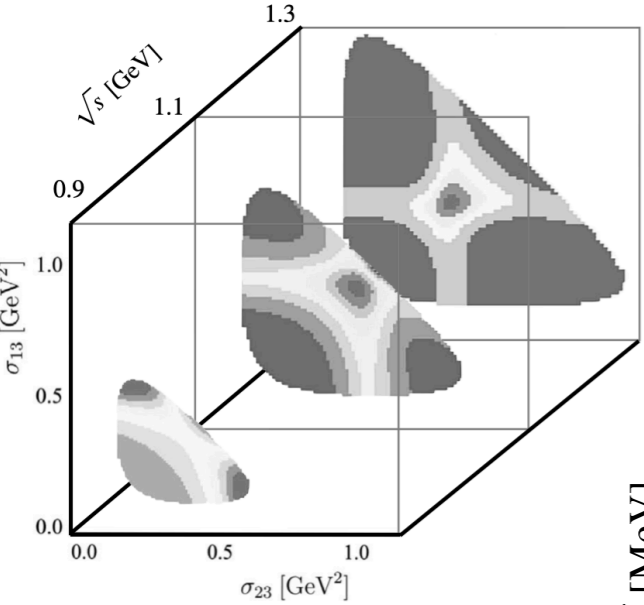- $\tau \to (\pi\pi\pi)\nu_\tau$ from ALEPH@CERN

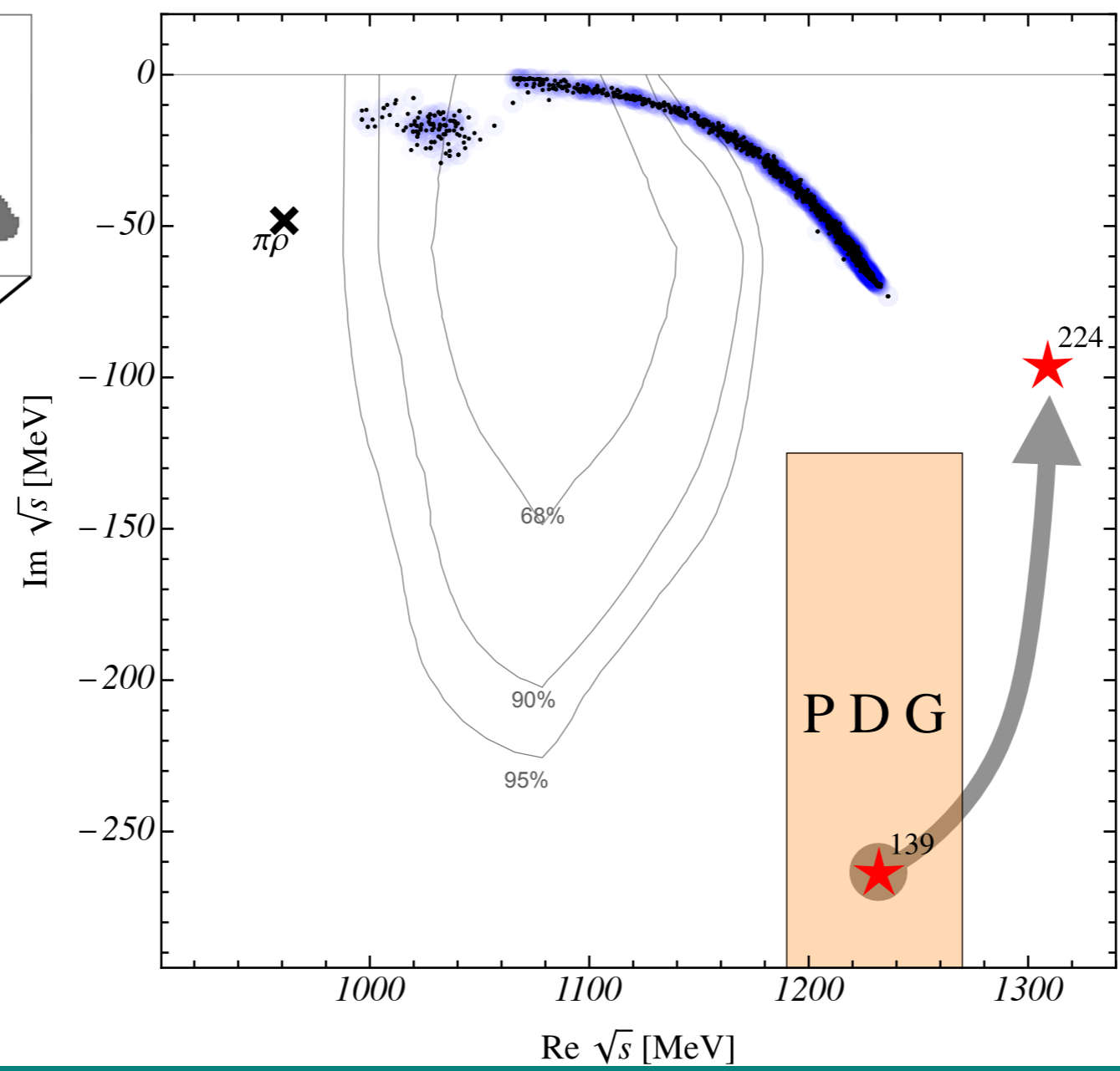- fit to line shape to fix C

cf. Sadasivan/
MM/Döring(2020)

What does phenomenology says?

- $\tau \to (\pi\pi\pi)\nu_\tau$ from ALEPH@CERN

- fit to line shape to fix C

C

cf. Sadasivan/
MM/Döring(2020)

(naive) **chiral extrapolation**
confirms expectations:

- *a_1 becomes heavier*

- *... and narrower*

Summary

- First-ever determination of a1(1260) parameters from lattice QCD

- Lattice QCD levels with 1/2/3-meson operators calculated

- 3-body quantization condition extended to spin-1 systems

- infinite-volume amplitude solved with complex integration contour and for complex energies

- pole positions and couplings are determined
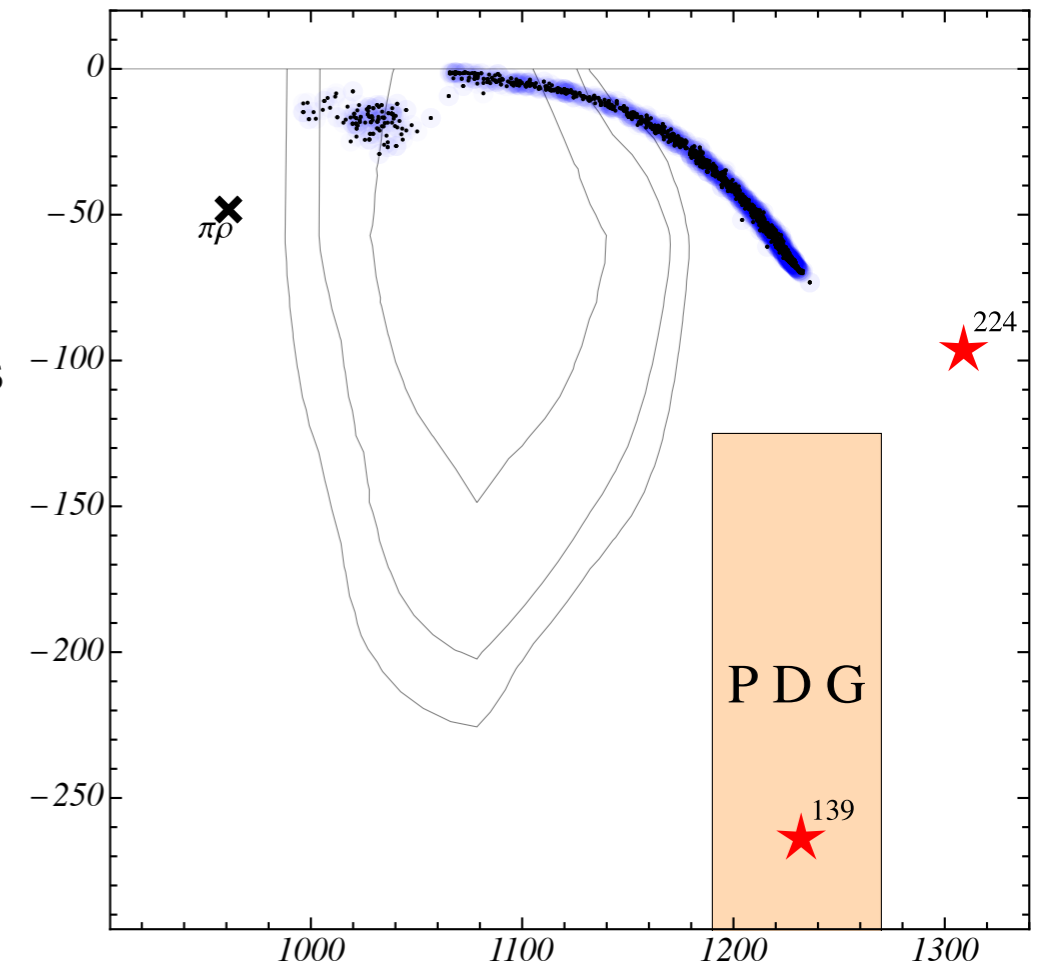
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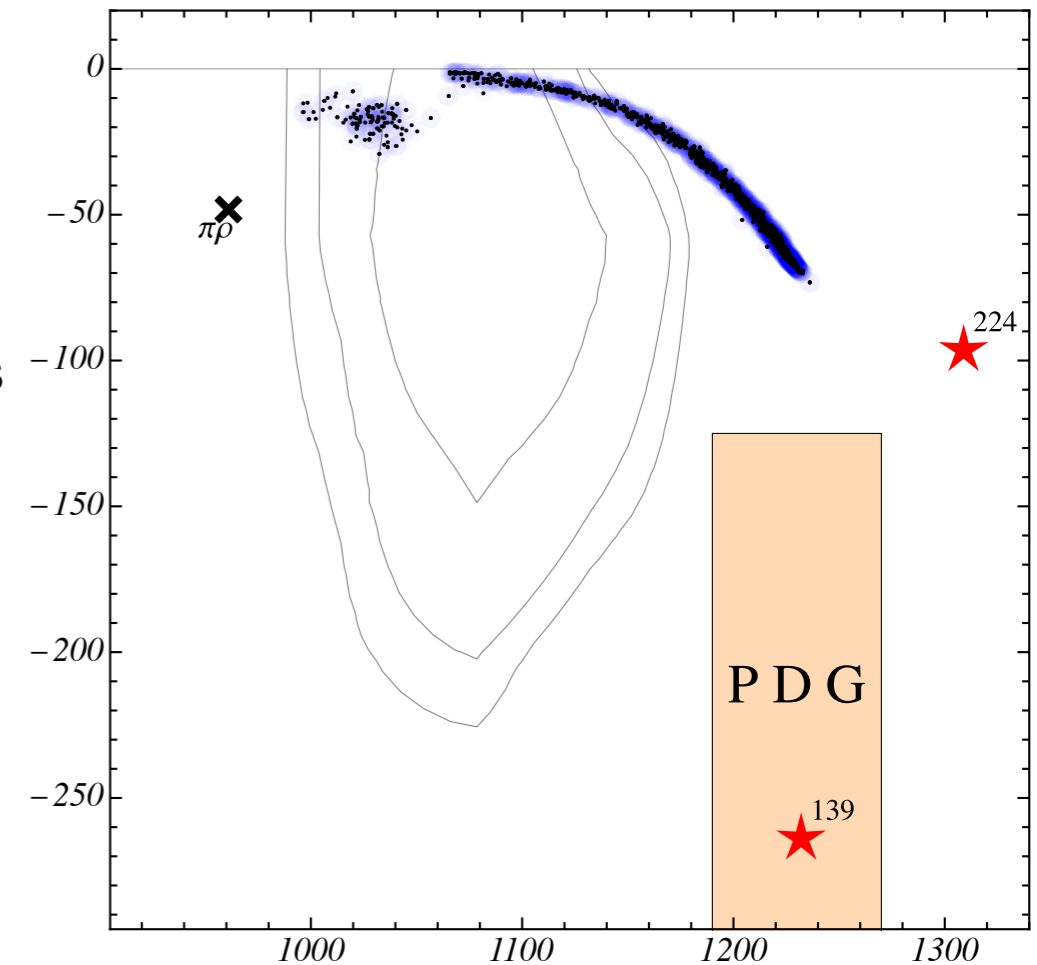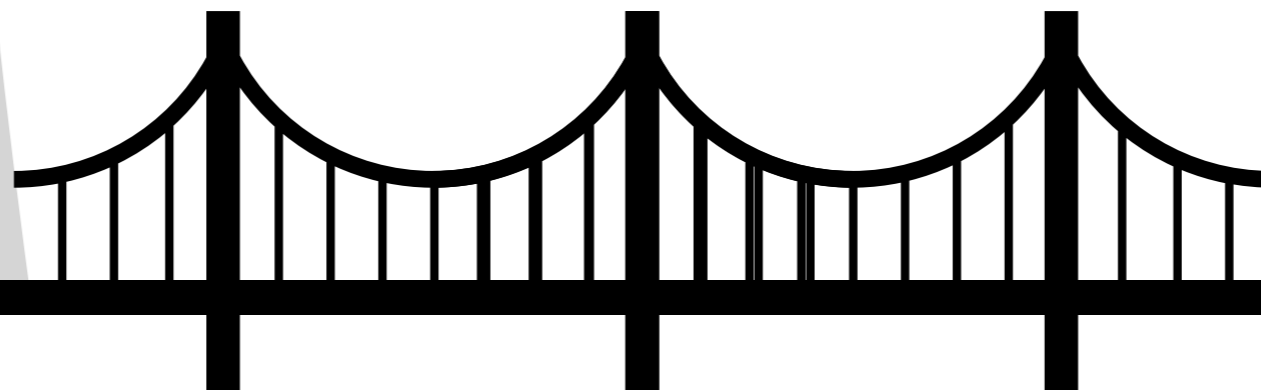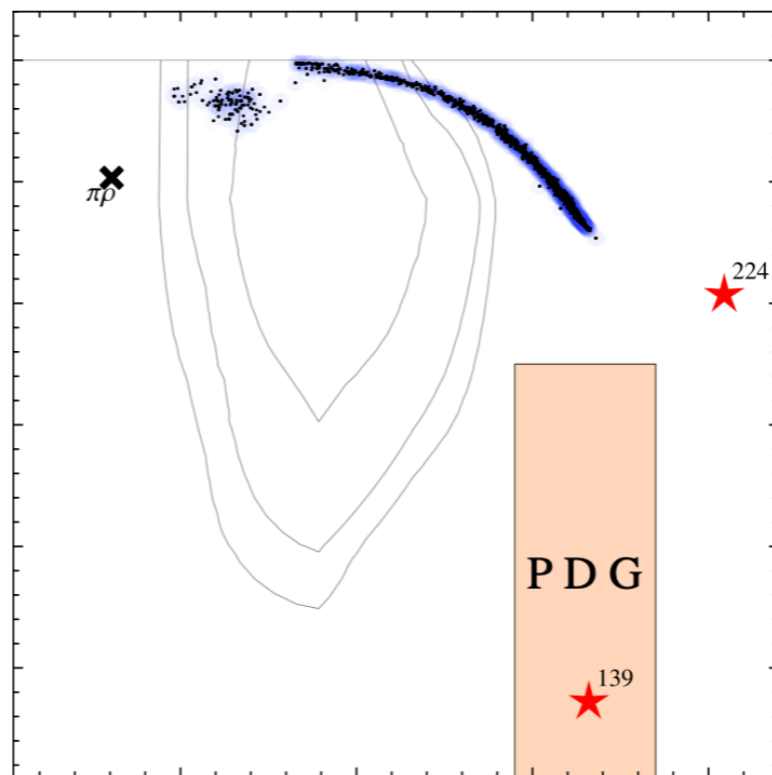
Outlook

- More lattice levels (another setup: m_π, L):

  - precise determination of a_1 parameters

  - importance of the $\sigma\pi$ channels
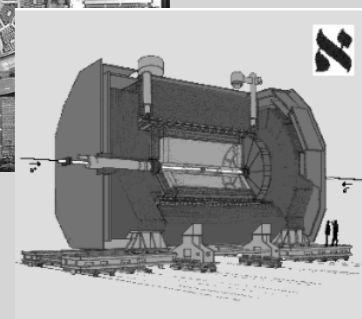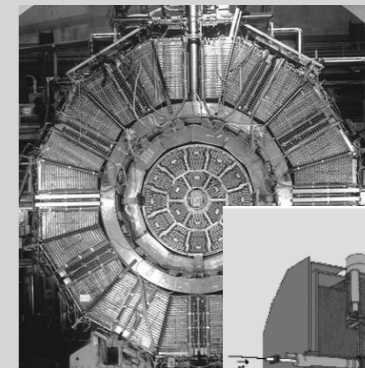
  - chiral trajectory of three-body term *C*
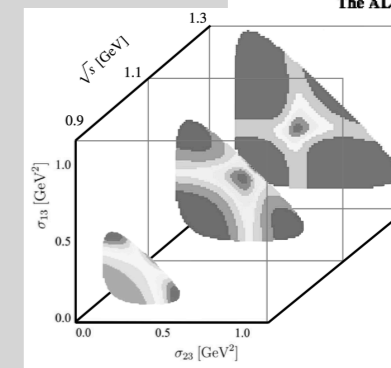
Unitarity

QCD

ChPT

THEORY

PDG
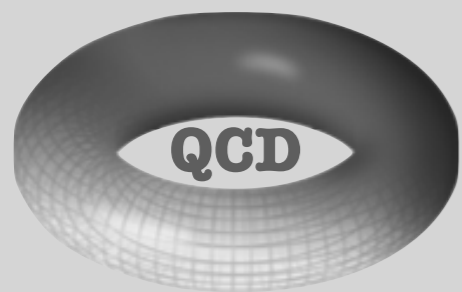
LHC

GlueX

The ALEPH Detector

EXPERIMENT

THANK YOU