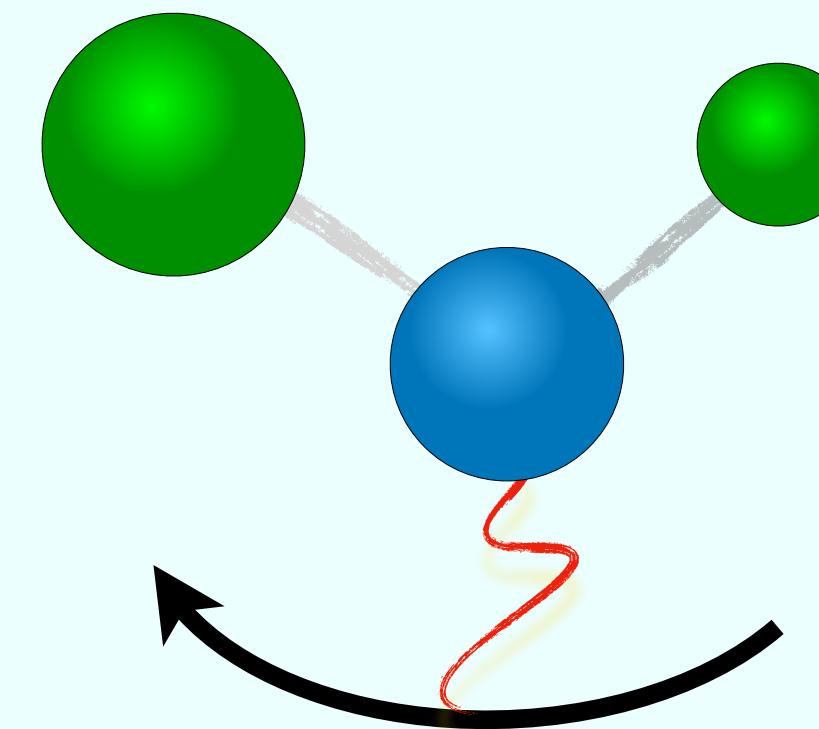




SINGLE MESON ELECTROPRODUCTION



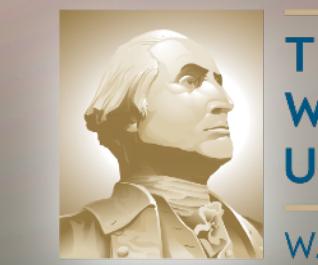
Maxim Mai

[Jülich-Bonn-Washington (JBW) collaboration]

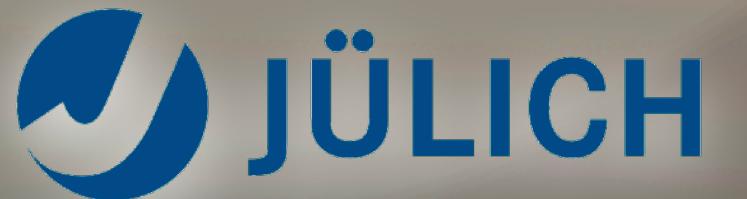
M. Döring, J. Hergenrath, C. Granados, H. Haberzettl, Ulf-G. Meißner, D. Rönchen, I. Strakovsky, R. Workman

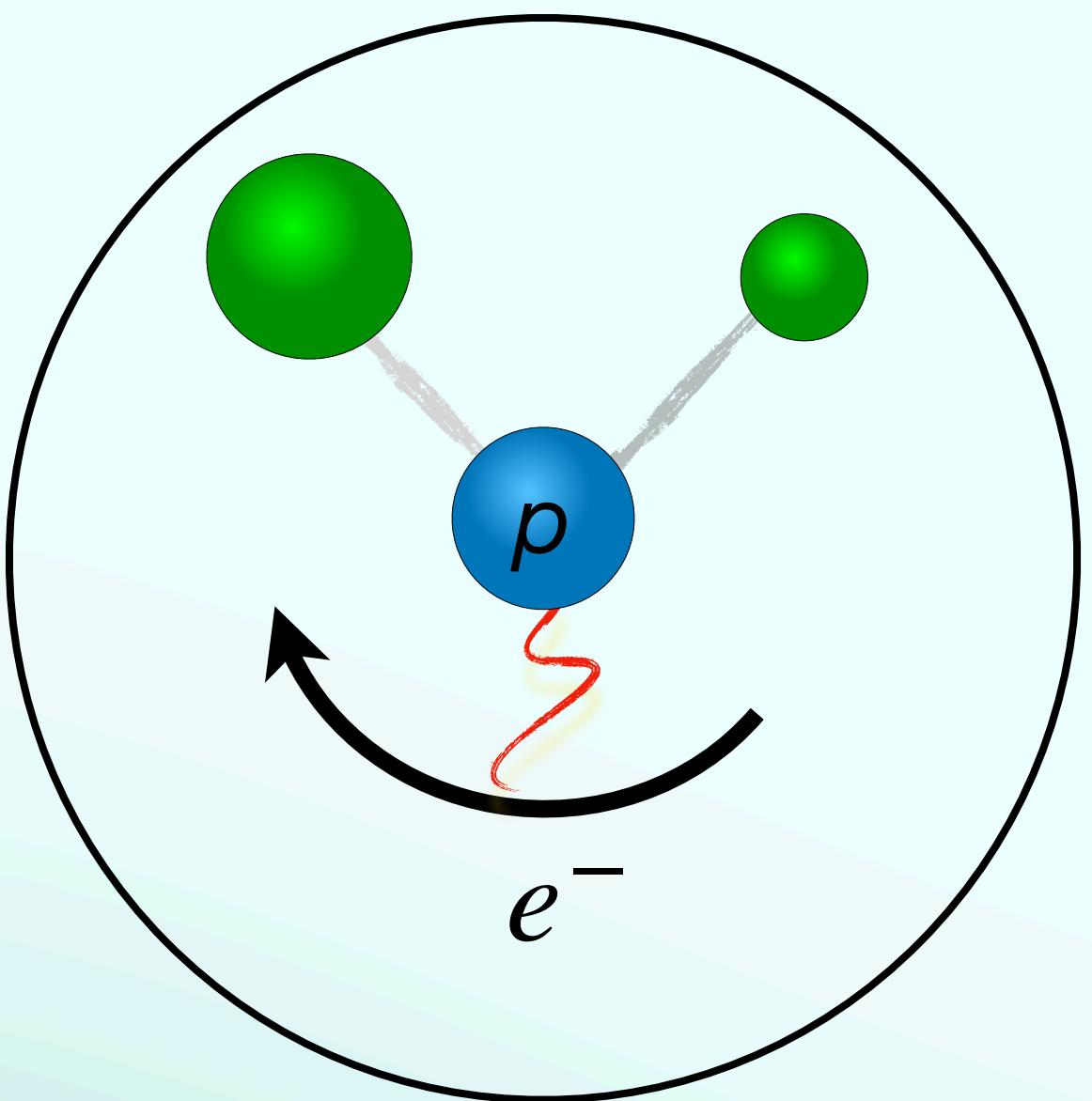


DE-SC0016582
DE-SC0016583

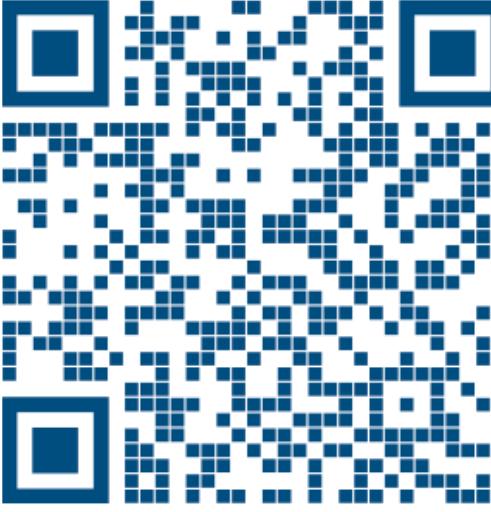


THE GEORGE
WASHINGTON
UNIVERSITY
WASHINGTON, DC





INTRODUCTION



NATURE'S LANGUAGE



Quantum mechanics

- governs subatomic world
- unconventional language
- various interpretations

"If you think you understand quantum mechanics then you don't understand quantum mechanics"

R. P. Feynman

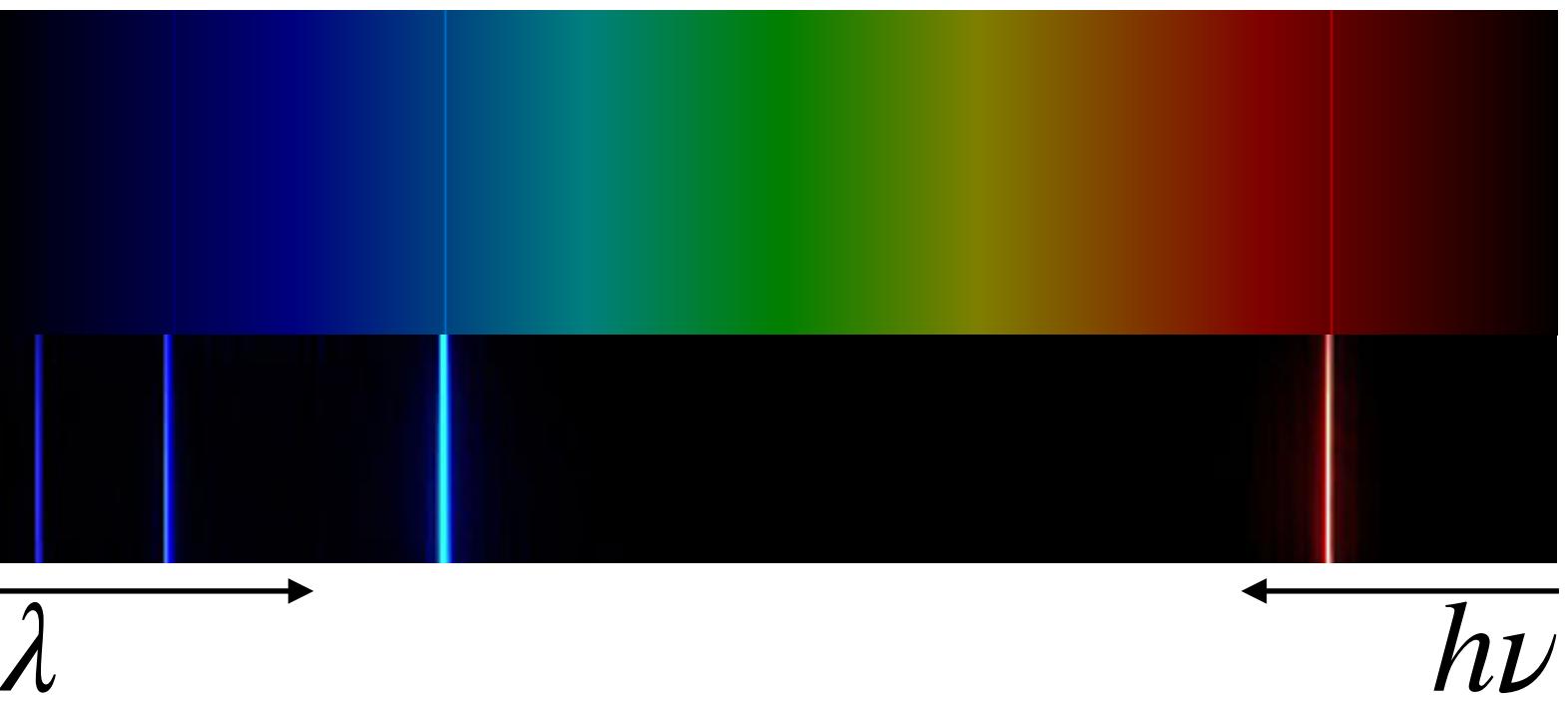
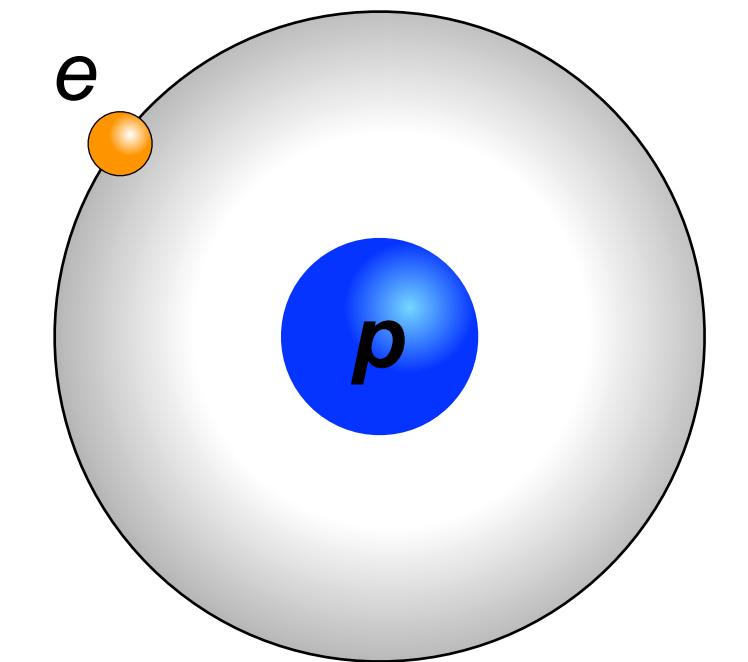


NATURE'S LANGUAGE

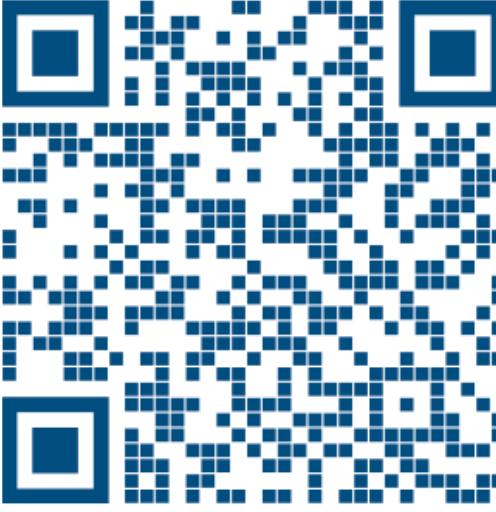


Breakthrough

- explanation of atomic spectra
- discrete excitation energies
- new paradigm of physics



$$\Delta E \sim \frac{1}{n^2} - \frac{1}{m^2}$$

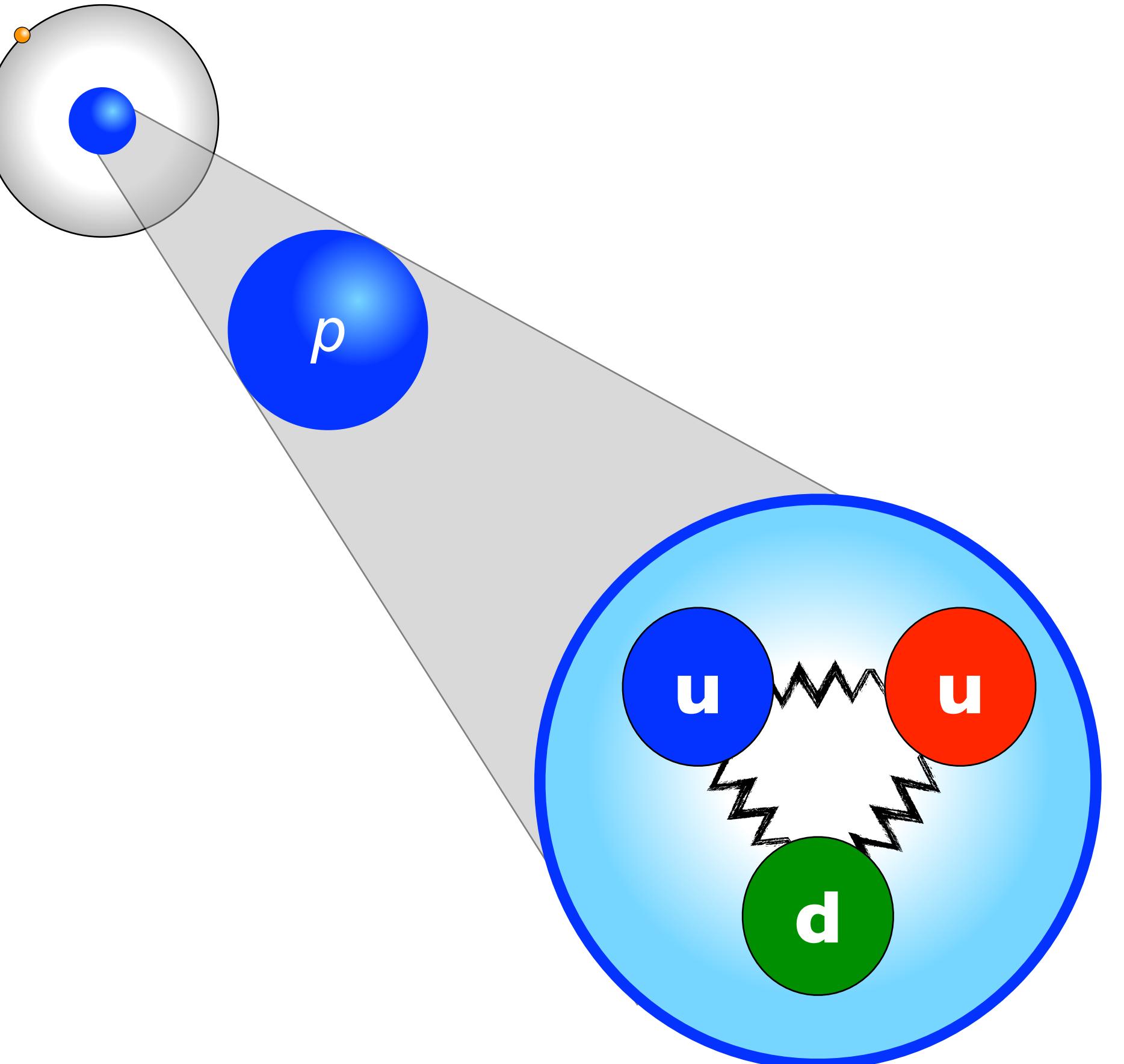


NATURE'S LANGUAGE



Protons/neutrons

- 99% of the mass of visible matter in the universe
- bound and interact via **strong force**
- part of a large class of particles: **hadrons**
- building blocks: **quarks & gluons**



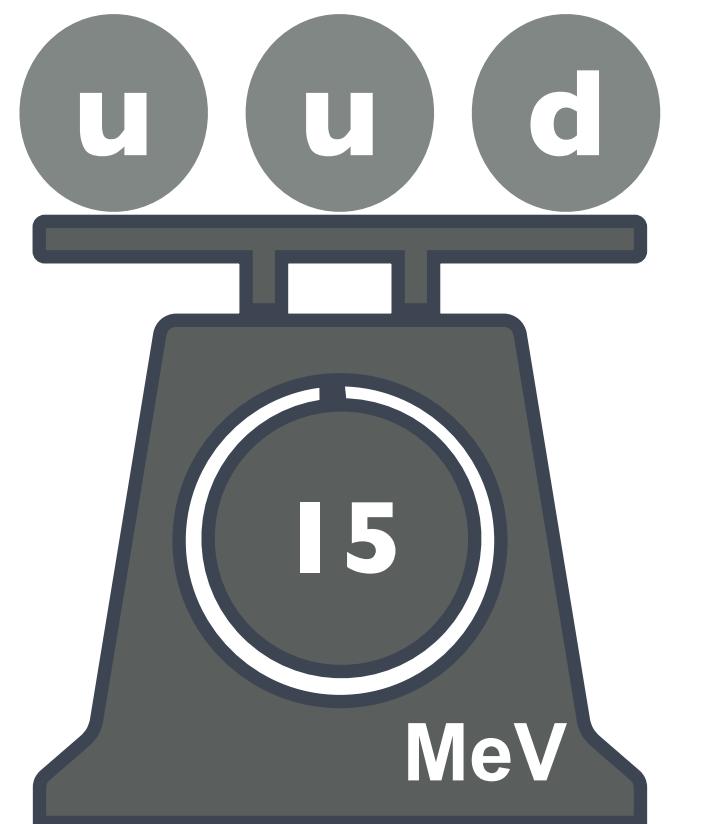


NATURE'S LANGUAGE



Mass puzzle:

- quarks are too light \Rightarrow constituent quark model?
- Lattice QCD¹: ~90% of mass is generated by dynamical effects

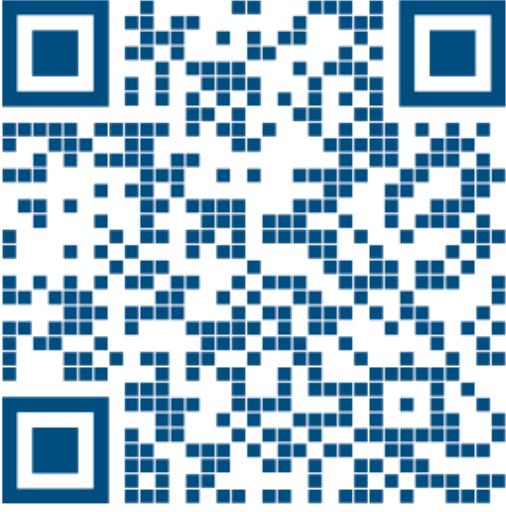


Higgs mechanism



Dynamics of gluons
and quarks

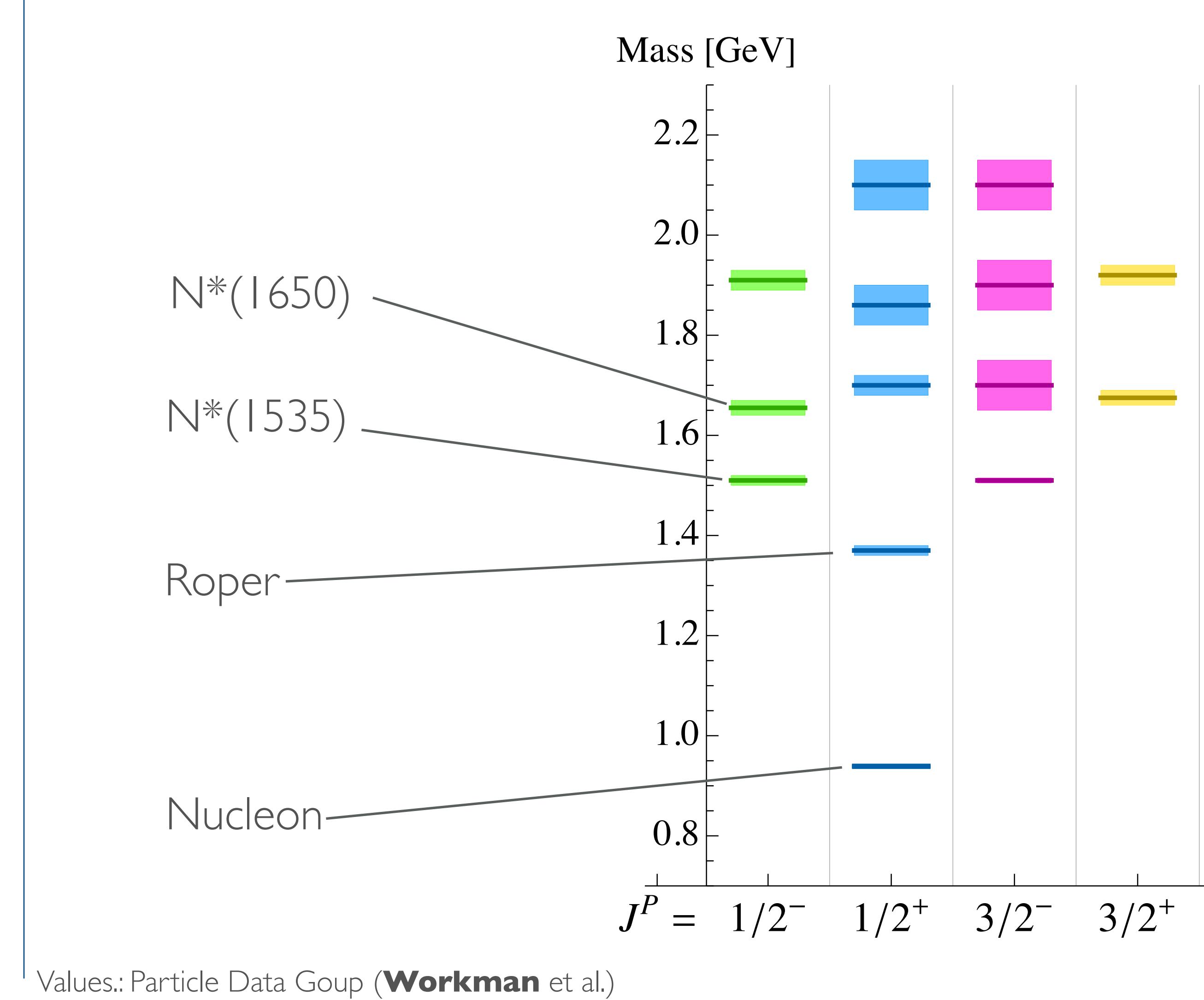
¹) Durr et al. Science 322 (2008)



HADRON SPECTRUM



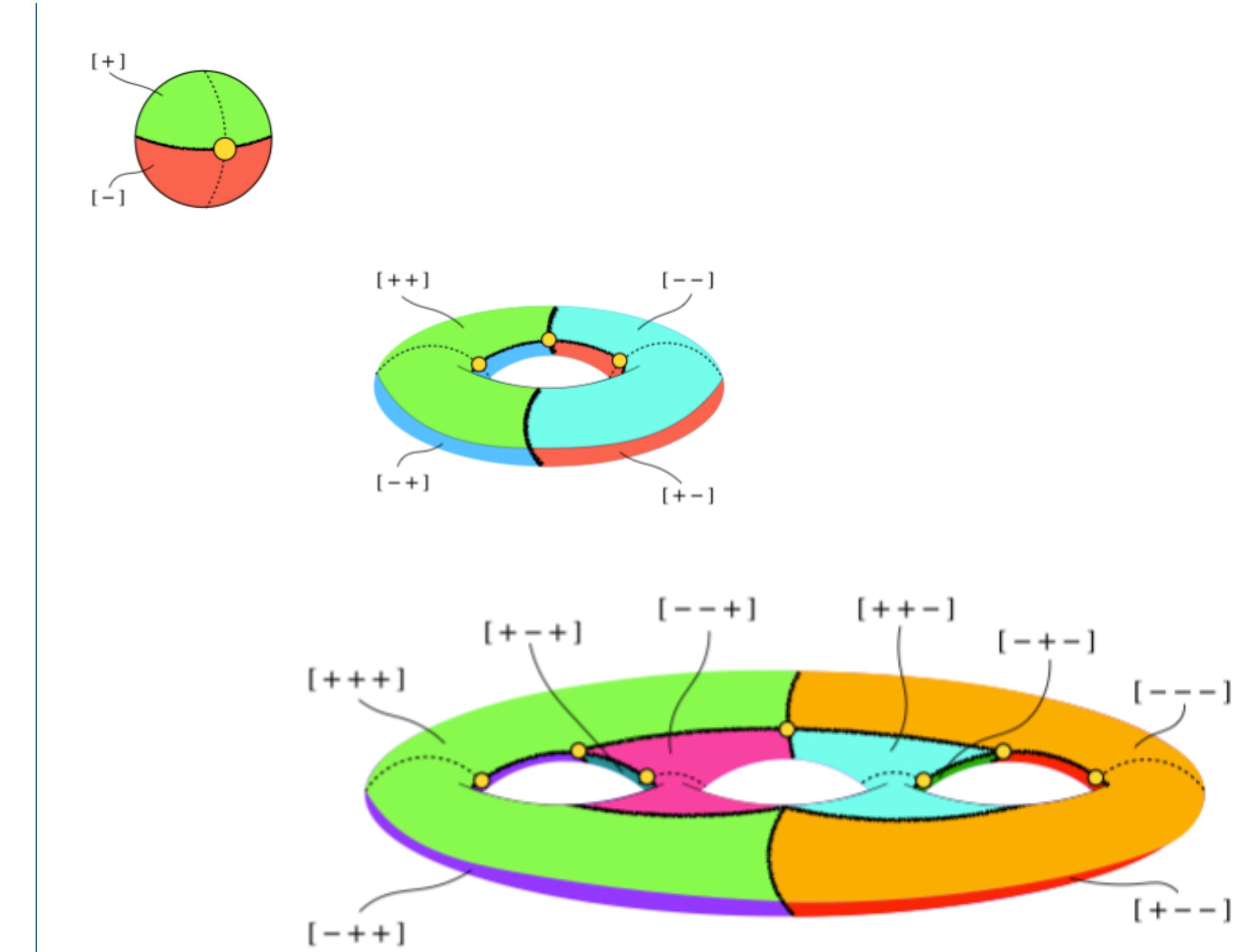
- PDG: ~100(50) excited meson(baryon) states (***)





HADRON SPECTRUM

- reaction-independent (*universal*) parameters:
> poles on the Riemann Surface

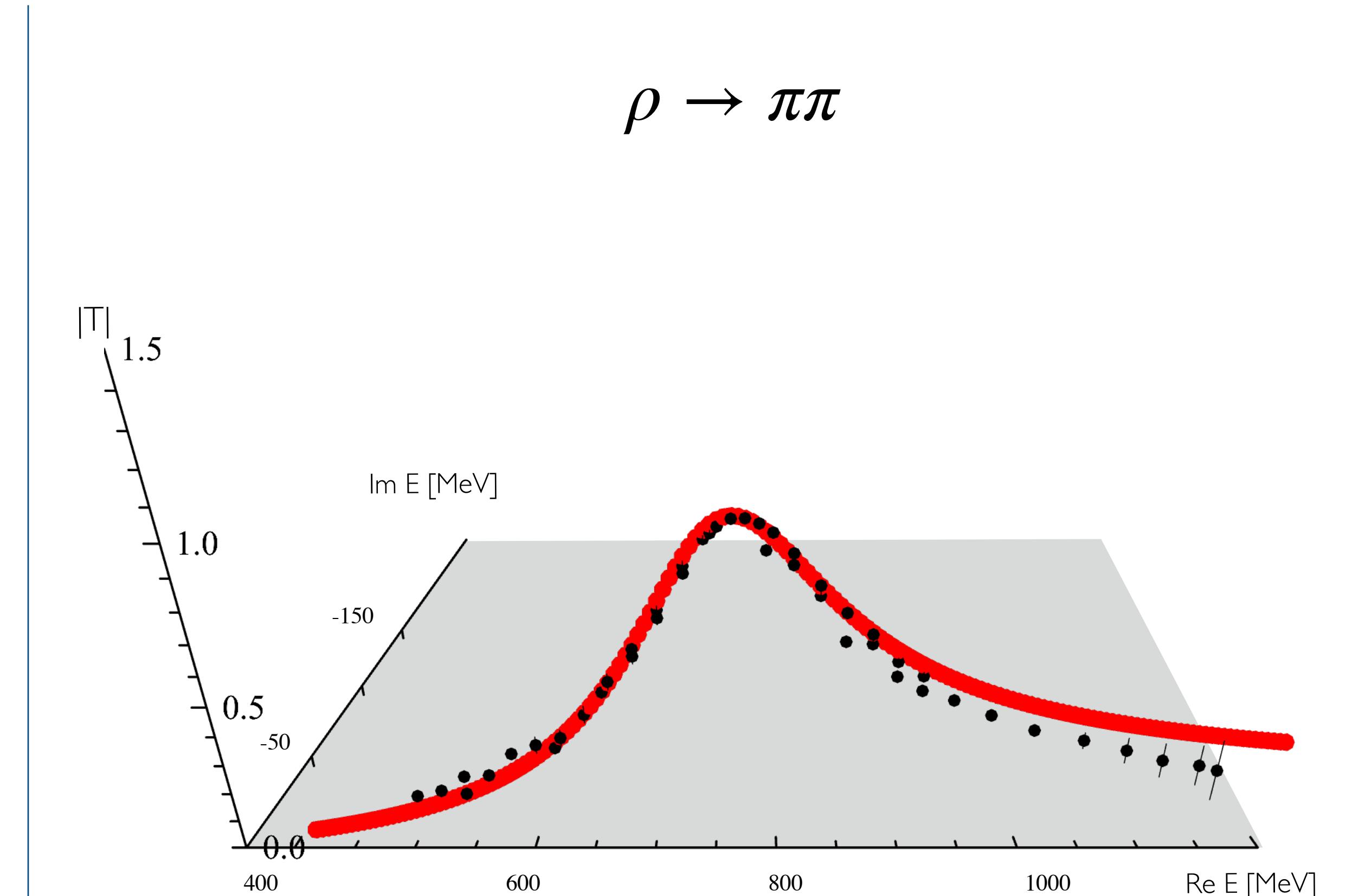


MM/Meißner/Urbach 2206.01477 under review in Phys. Rept.



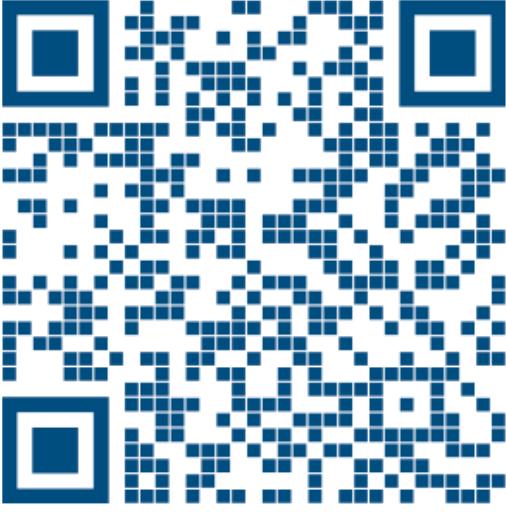
HADRON SPECTRUM

- physical information ($E \in \mathbb{R}$)
 - > theory -- Lattice QCD (review¹)
 - > experiment



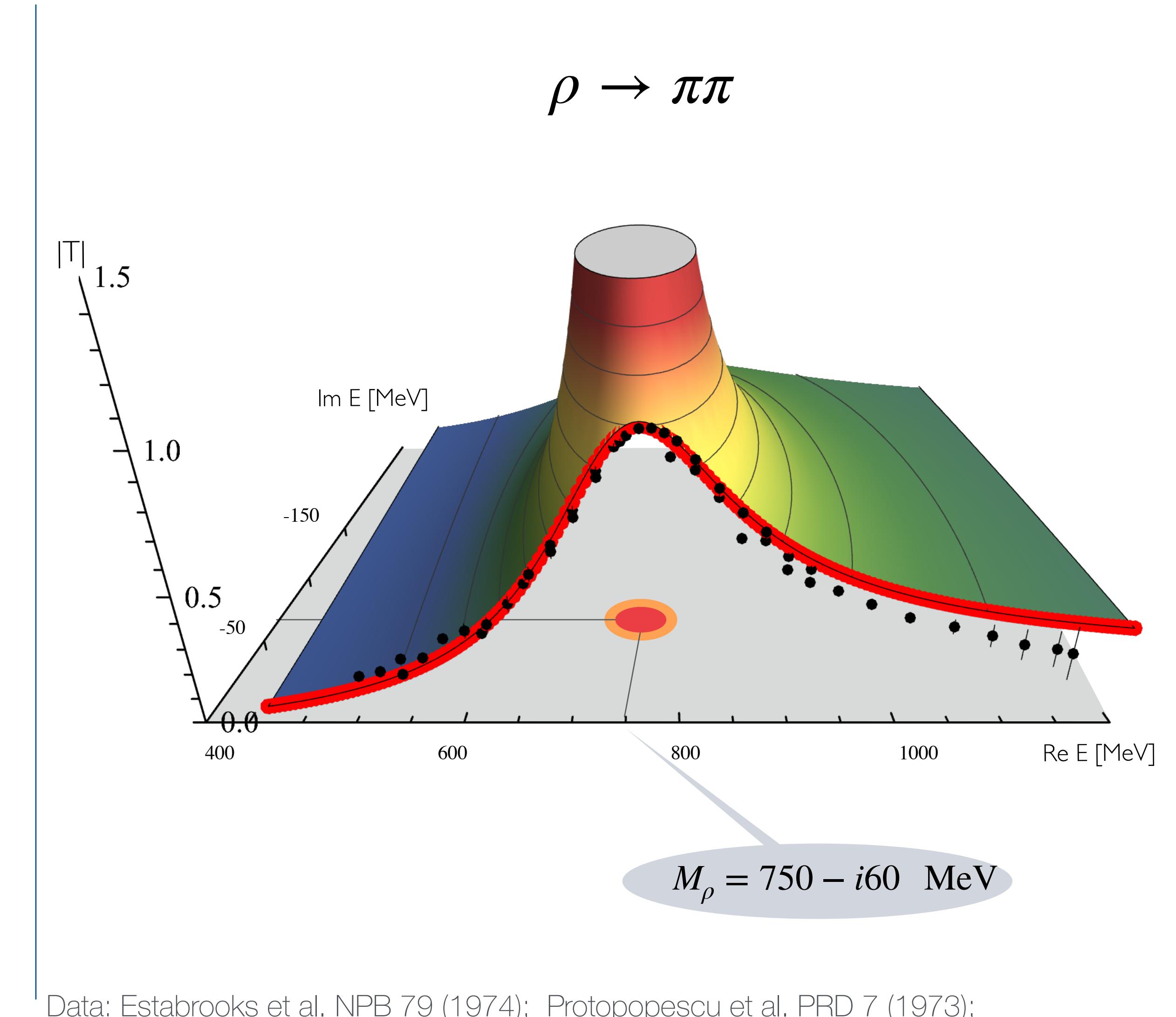
1) MM/Meißner/Urbach 2206.01477 under review in Phys. Rept.

Data: Estabrooks et al. NPB 79 (1974); Protopopescu et al. PRD 7 (1973);



HADRON SPECTRUM

- physical information ($E \in \mathbb{R}$)
 - > theory -- Lattice QCD (review¹)
 - > experiment

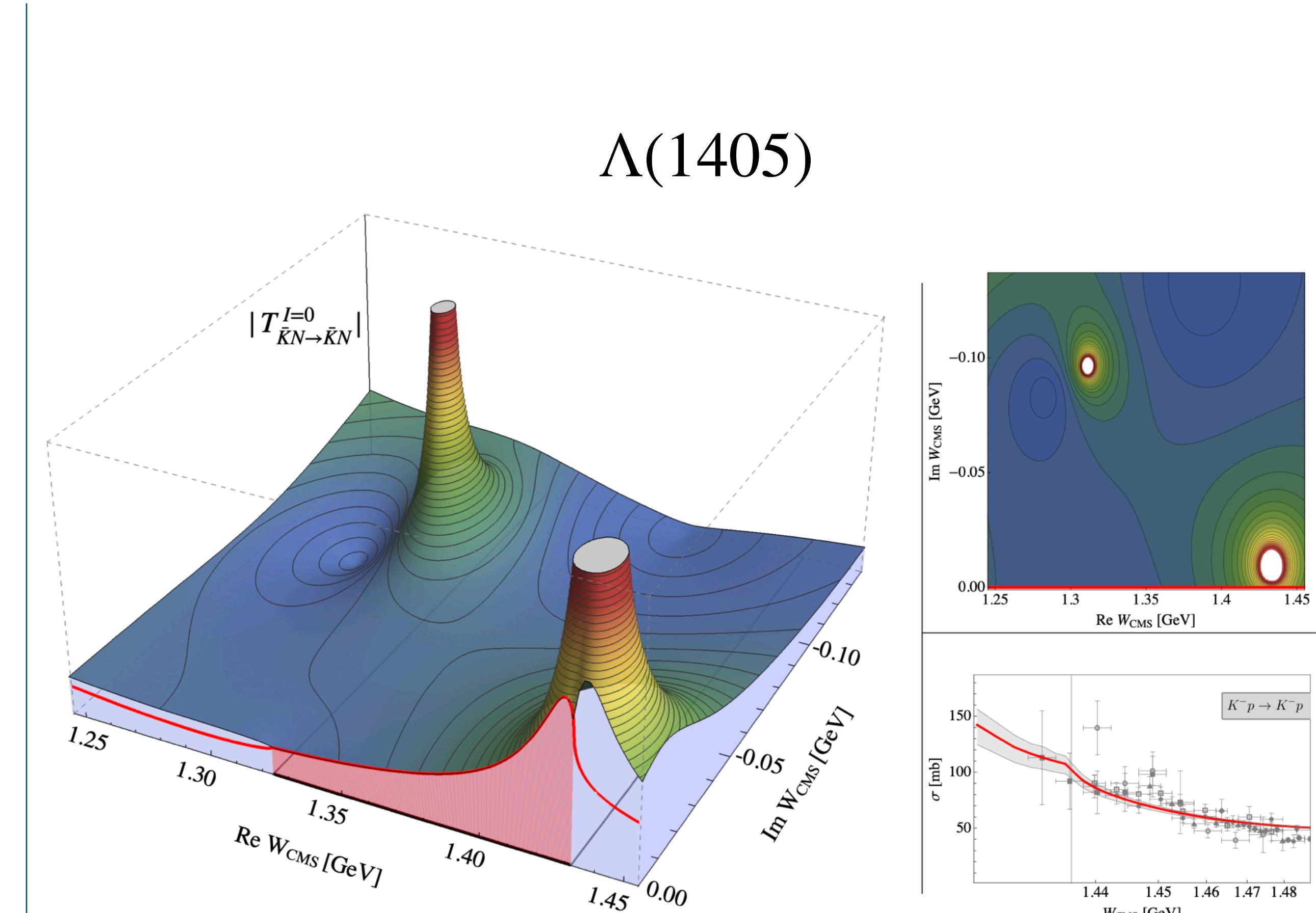


1) MM/Meißner/Urbach 2206.01477 under review in Phys. Rept.

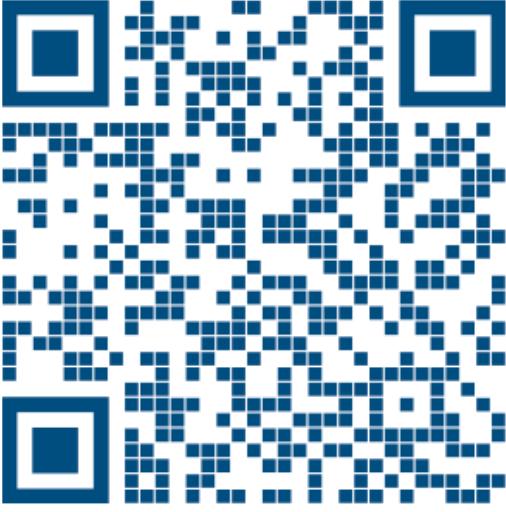


HADRON SPECTRUM

- physical information ($E \in \mathbb{R}$)
 - > theory -- *Lattice QCD (review¹)*
 - > experiment



1) MM/Meißner/Urbach 2206.01477 under review in Phys. Rept.

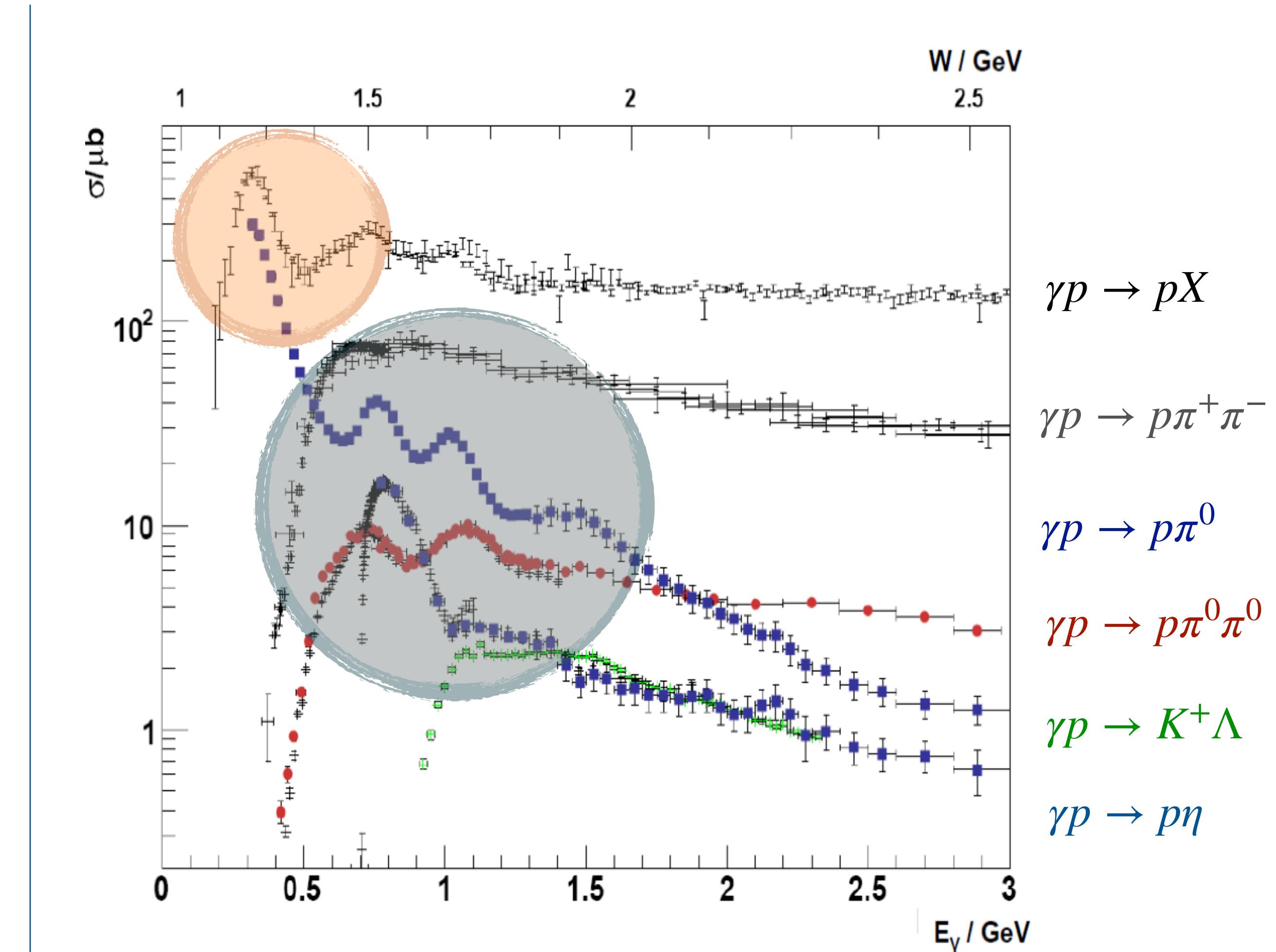


HADRON SPECTRUM

Photon-induced excitation

via meson photo-/electroproduction

- > large amount of data (10^5 for $\gamma p \rightarrow \pi N$)
- > bumps are **not** necessarily resonances
- > many more data to emerge at JLab¹
($Q^2=5-12$ GeV 2)



1) Carman, Joo, Mokeev, Few Body Syst. 61, 29 (2020) ... ;
[CLAS] Phys.Rev.C 105 (2022) 065201; ...

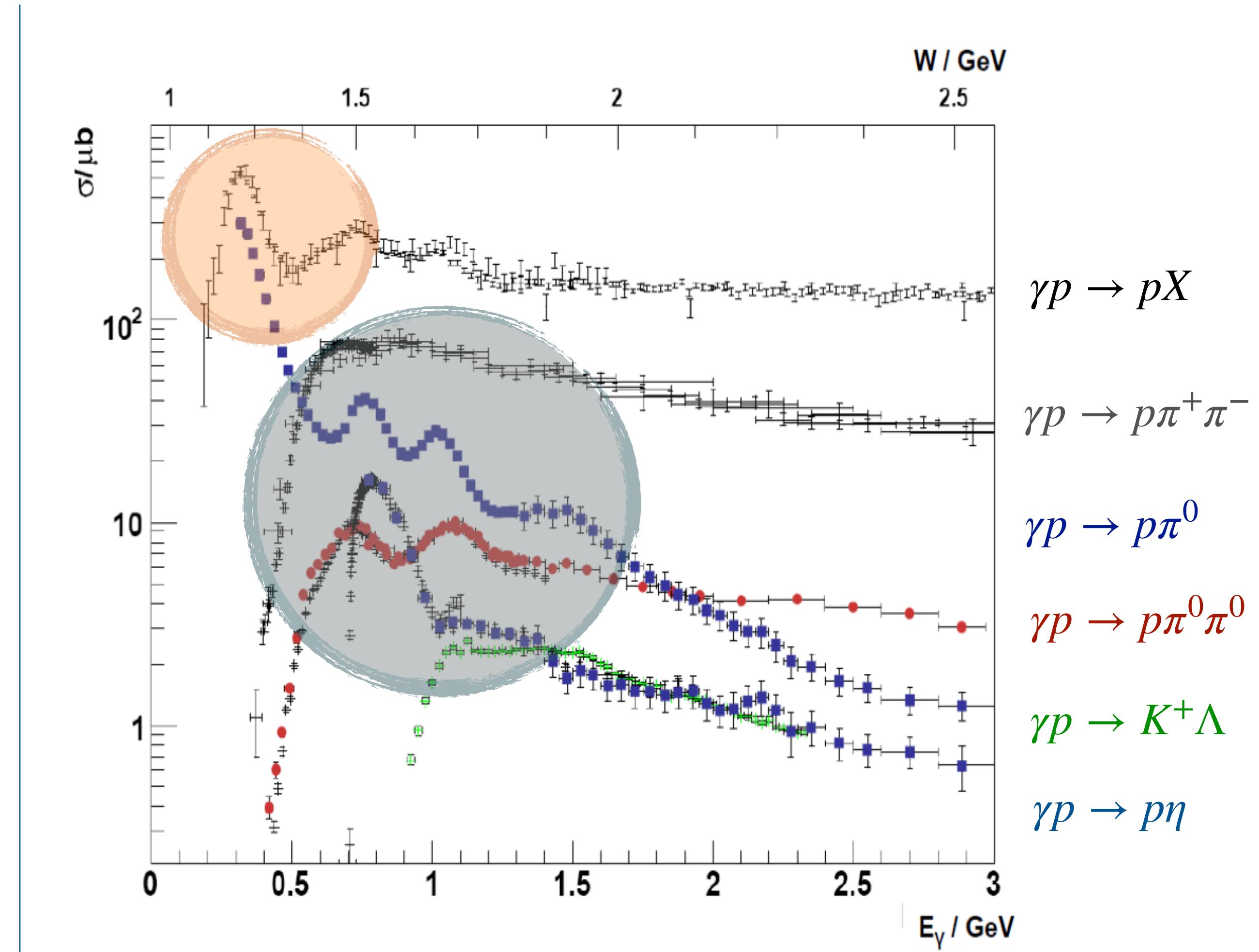
Data: Jefferson Laboratory, ELSA, MAMI



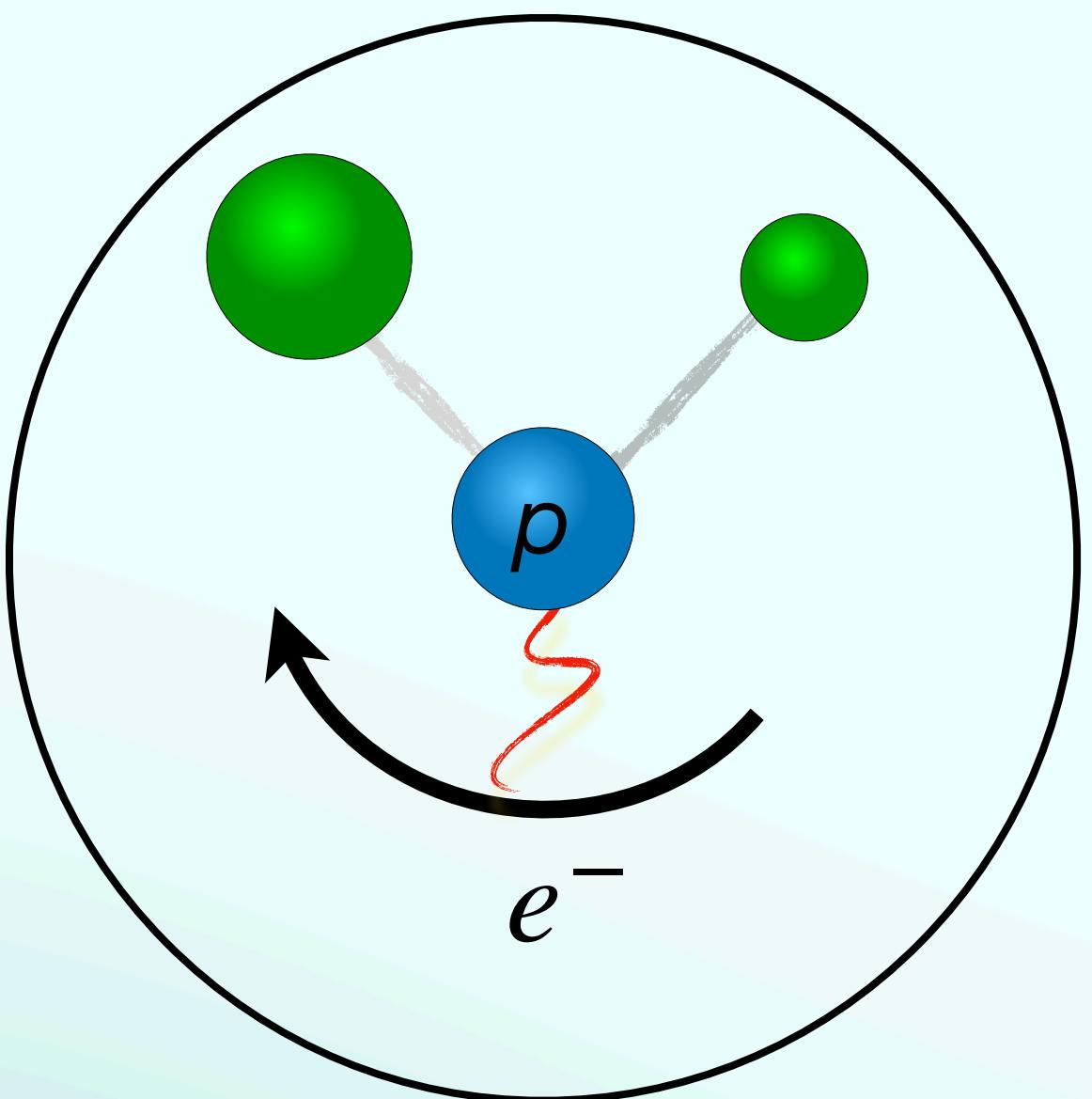
HADRON SPECTRUM

Key questions:

- > can we describe the data consistently with the scattering data?
- > can we extract universal information about the hadron spectrum?



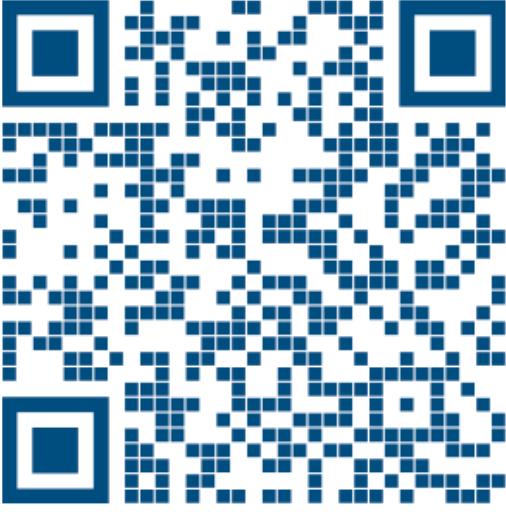
Data: Jefferson Laboratory, ELSA, MAMI



THEORY

[JBW] MM, M.Döring, C.Granados, H.Haberzettl, J.Hergenrather,
U.Meißner, D.Rönchen, I.Strakovsky, R.Workman

Phys.Rev.C 103 (2021) 6, 065204



EXISTING APPROACHES

- ANL-Osaka¹
- (eta)(kaon)MAID²
- SAID³ [Ron's talk]
- ...⁴

Some highlights

- > Simultaneous description of pion photo- and electroproduction (MAID)
- > Low-energy constraints from CHPT (chiral MAID)
- > Roper form factor from single and double pion electroproduction⁵

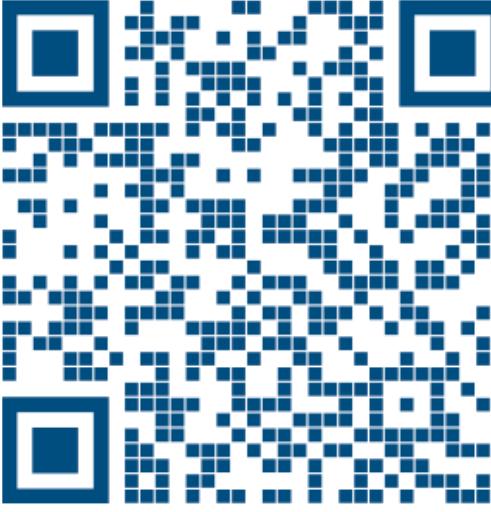
1) ANL-Osaka PRC 80(2009), Few-Body Syst. 59(2018),...

2) MAID2007, EPJA 34(2007) EtaMAID2018, EPJA 54(2018)

3) SAID, PiN Newsletter 16(2002)

4) Gent group PRC 89(2014),... Aznauryan et al., PRC 80(2009), IJMP(2013),...

5) Burkert, Roberts, Rev.Mod.Phys. 91 (2019)

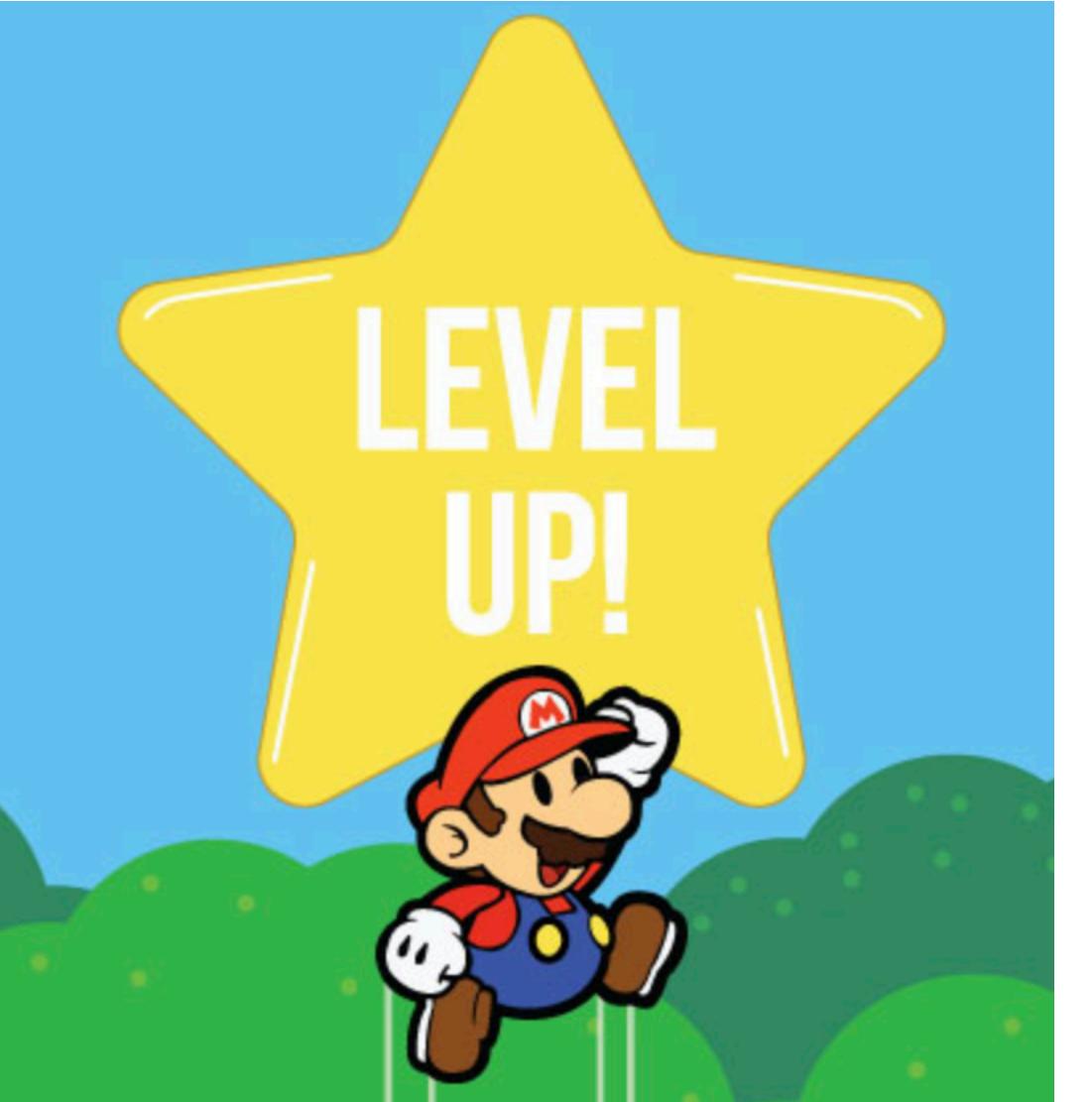


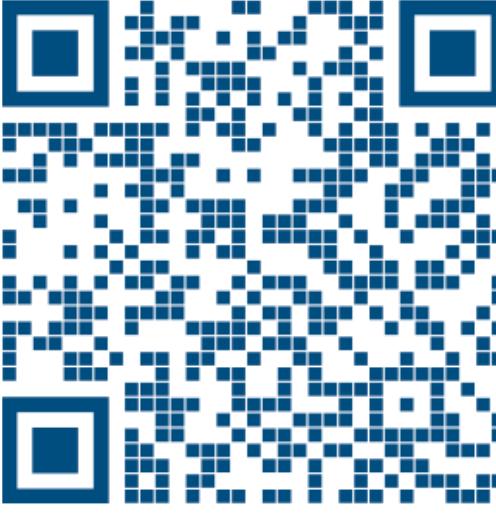
EXISTING APPROACHES



Level up the game

- coupled-channel approach. Universality \Leftrightarrow simultaneous description of πN , ηN , $K\Lambda$ channels
- threshold constraints, gauge invariance, ...
- constraints from scattering data





KINEMATICS

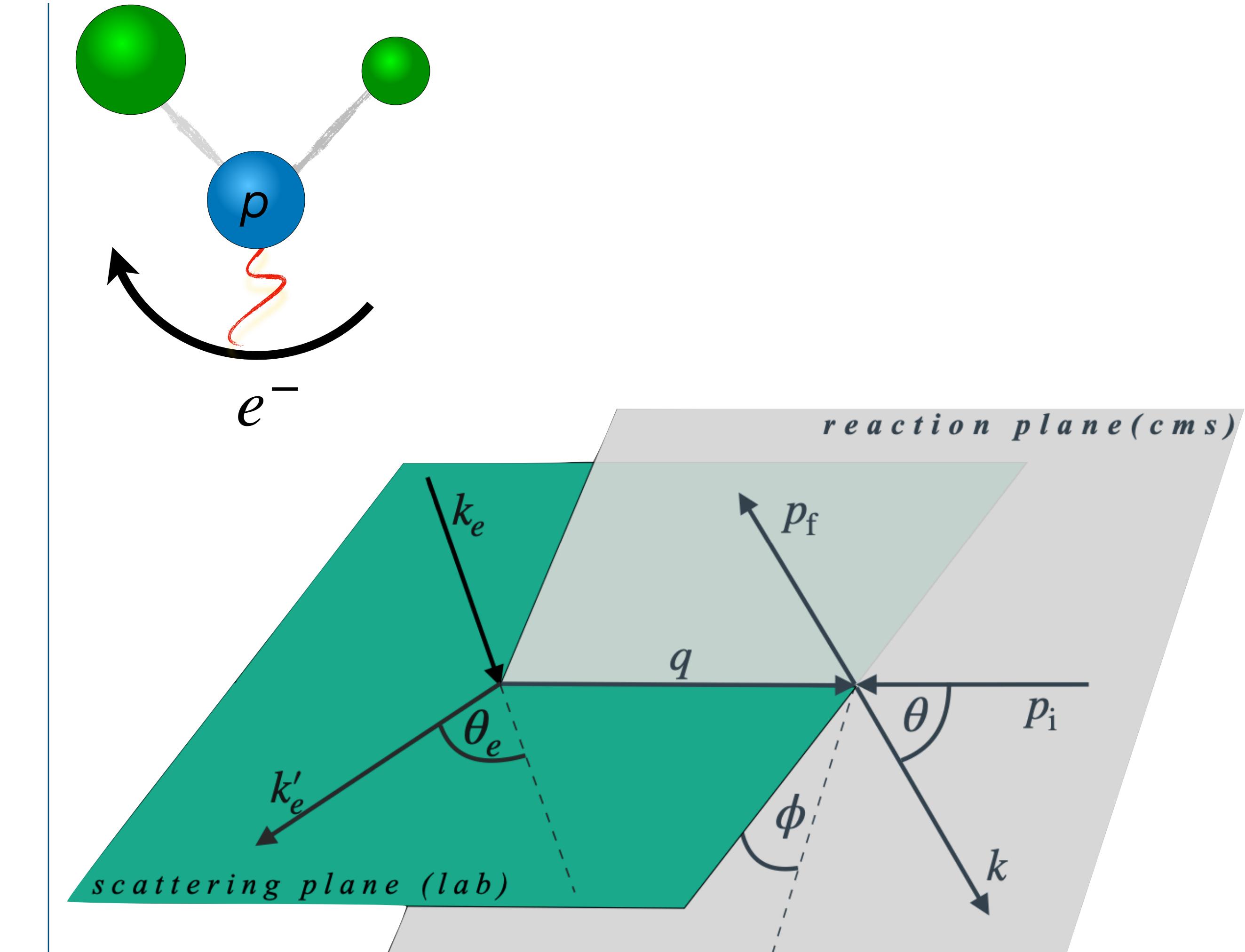


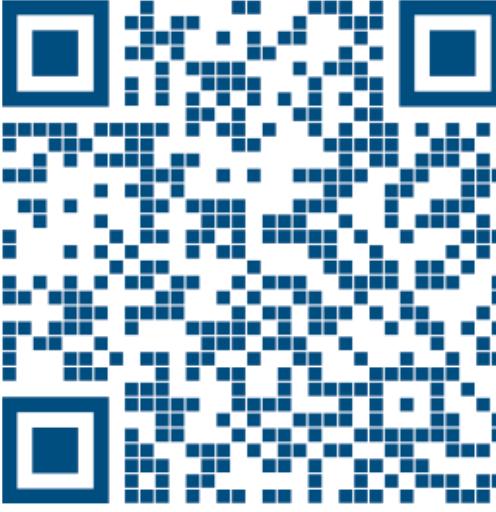
$3^*(2+3)-10 = \text{five independent variables}$

- total energy: W
- photon virtuality: Q^2
- transverse photon polarization:

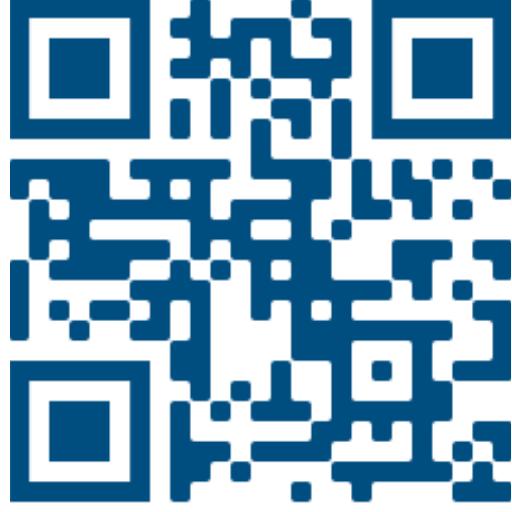
$$\epsilon = 1 + 2q_L^2/Q^2 \tan^2 \theta_e/2$$

- production angles: θ, φ



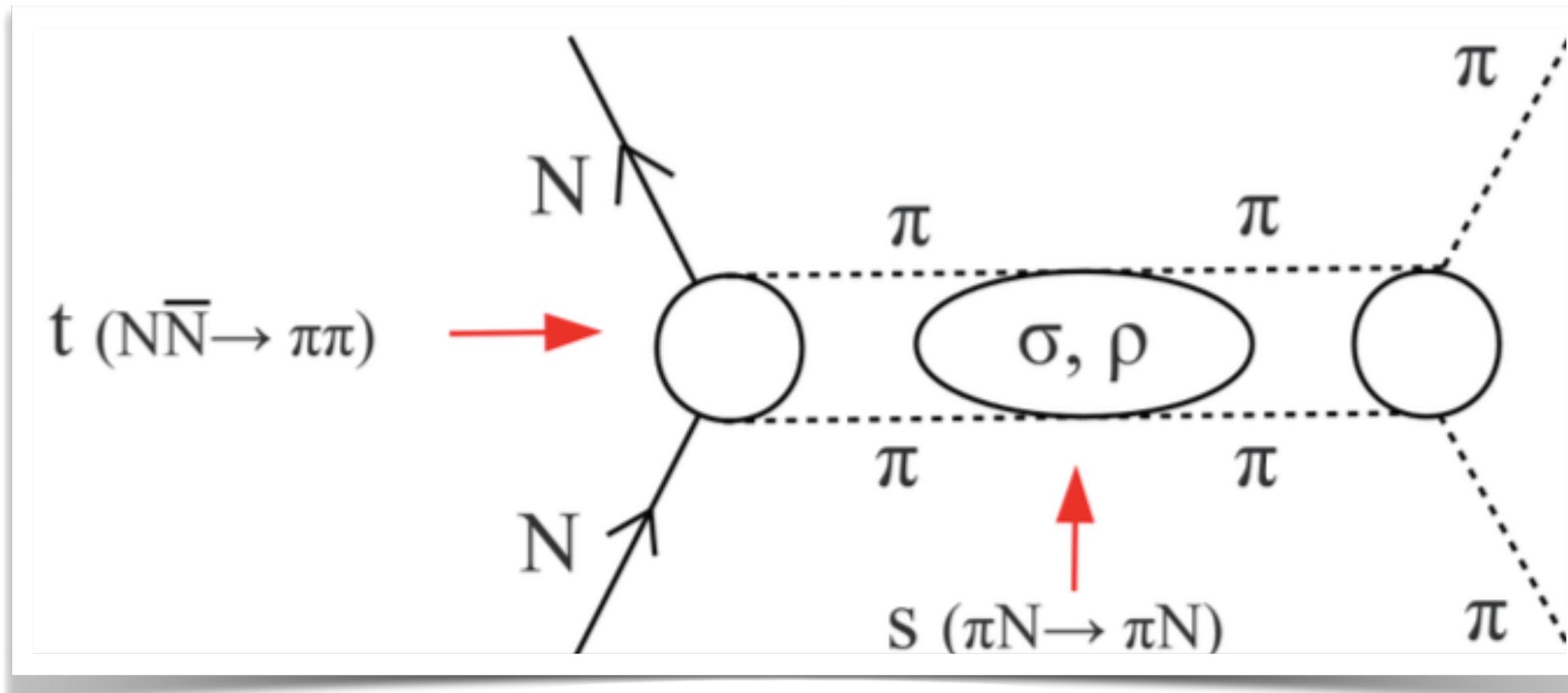


GENERAL PRINCIPLES



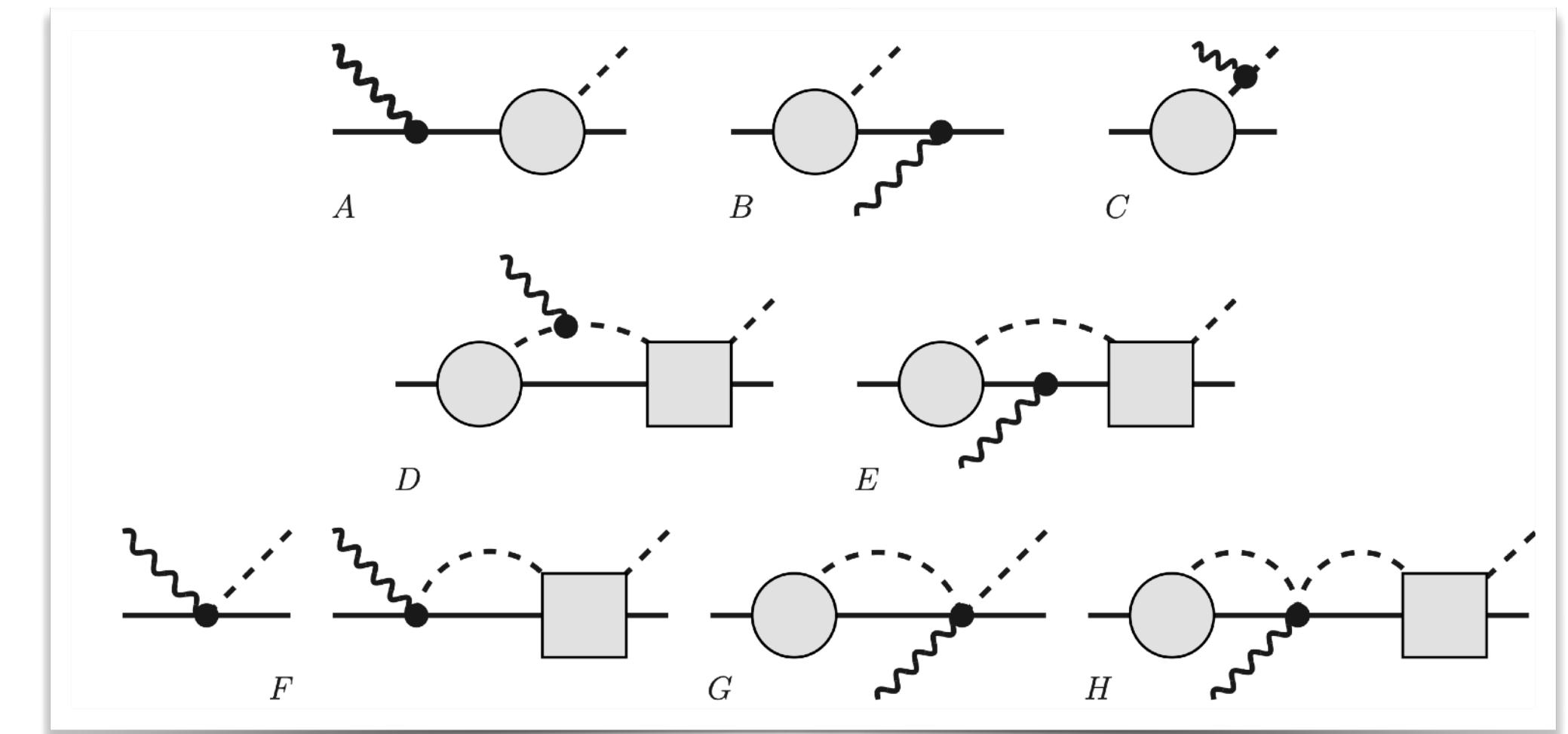
Pion-induced reactions

- Unitarity/Analyticity/Crossing symmetry
- Underlying objects: scattering amplitudes

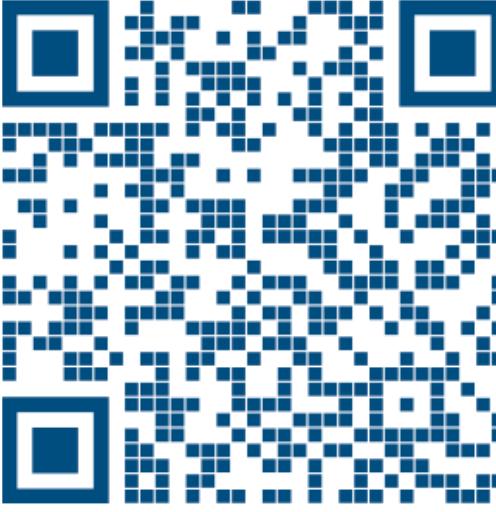


Photon-induced reactions

- Final state unitarity/Gauge invariance¹
- Underlying objects: transition matrix element



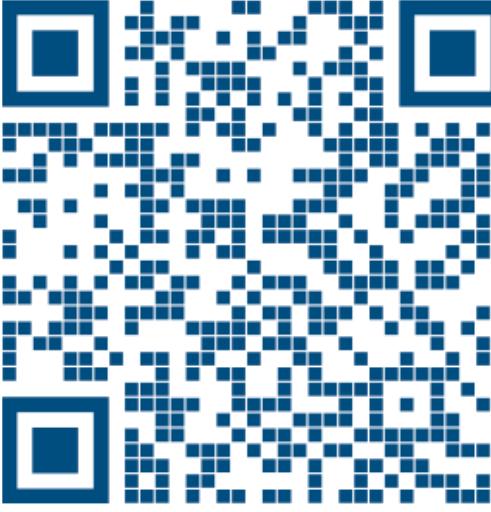
Afnan et al.(1995) Kvinikhidze et al.(1999) Haberzettl(19xx-2021)
Borasoy et al.(2007) Ruic et al.(2011) MM et al.(2012)



MULTIPOLES

Observable (e.g. cross section)

$$\frac{d\sigma^\nu}{d\Omega}(W, Q^2, \epsilon, \theta, \phi) = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \dots$$



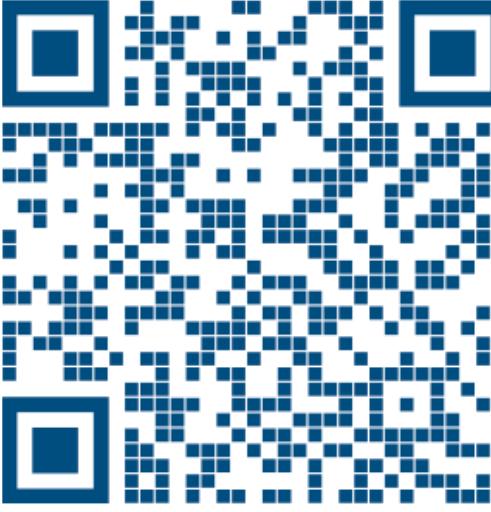
MULTIPOLES

Observable (e.g. cross section)

$$\frac{d\sigma^\nu}{d\Omega}(W, Q^2, \epsilon, \theta, \phi) = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \dots$$

Structure functions

$$\sigma_T(W, Q^2, \theta) = k/q_\gamma \left(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2 \right)/2, \dots$$



MULTIPOLES

Observable (e.g. cross section)

$$\frac{d\sigma^v}{d\Omega}(W, Q^2, \epsilon, \theta, \phi) = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \dots$$

Structure functions

$$\sigma_T(W, Q^2, \theta) = k/q_\gamma \left(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2 \right)/2, \dots$$

Helicity amplitudes

$$H_1(W, Q^2, \theta) = \sin\theta\cos\theta/2(-\mathcal{F}_3 - \mathcal{F}_4)/\sqrt{2}, \dots$$



MULTIPOLES

Observable (e.g. cross section)

$$\frac{d\sigma^\nu}{d\Omega}(W, Q^2, \epsilon, \theta, \phi) = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \dots$$

Structure functions

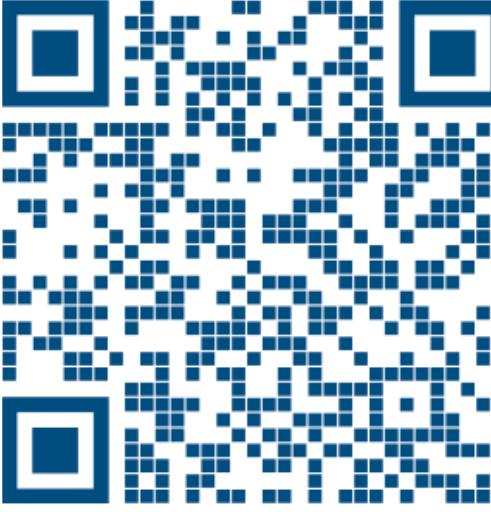
$$\sigma_T(W, Q^2, \theta) = k/q_\gamma \left(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2 \right)/2, \dots$$

Helicity amplitudes

$$H_1(W, Q^2, \theta) = \sin\theta\cos\theta/2(-\mathcal{F}_3 - \mathcal{F}_4)/\sqrt{2}, \dots$$

CGLN amplitudes

$$\mathcal{F}_1(W, Q^2, \theta) = \sum_{\ell>0} \ell M_{\ell+}(W, Q^2) P'_{\ell+1}(\cos\theta) + \dots$$



MULTIPOLES

Observable (e.g. cross section)

$$\frac{d\sigma^\nu}{d\Omega}(W, Q^2, \epsilon, \theta, \phi) = \sigma_T + \epsilon\sigma_L + \sqrt{2\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi + \dots$$

Structure functions

$$\sigma_T(W, Q^2, \theta) = k/q_\gamma \left(|H_1|^2 + |H_2|^2 + |H_3|^2 + |H_4|^2 \right)/2, \dots$$

Helicity amplitudes

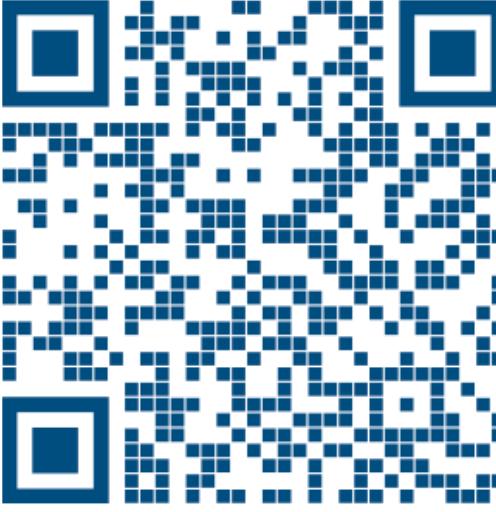
$$H_1(W, Q^2, \theta) = \sin\theta\cos\theta/2(-\mathcal{F}_3 - \mathcal{F}_4)/\sqrt{2}, \dots$$

CGLN amplitudes

$$\mathcal{F}_1(W, Q^2, \theta) = \sum_{\ell>0} \ell M_{\ell+}(W, Q^2) P'_{\ell+1}(\cos\theta) + \dots$$

Multipoles

$$\{E_{\ell\pm}(W, Q^2), L_{\ell\pm}(W, Q^2), M_{\ell\pm}(W, Q^2)\}$$



MULTIPOLES



Gauge invariance

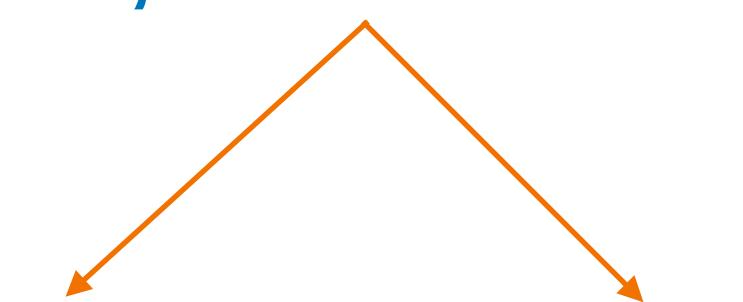
- manifest implementation¹
- include by design (this work)

Ward-Takahashi identity:

$$k_\mu T^\mu = 0$$

$$H_7 = \sum_{i=1}^6 a_i H_i$$

$$H_8 = \sum_{i=1}^6 b_i H_i$$



1) Afnan et al.(1995) Kvinikhidze et al.(1999) Haberzettl(19xx-2021)
Borasoy et al.(2007) Ruic et al.(2011) MM et al.(2012)



MULTIPOLES



Final-state unitarity and photoproduction limit

- start from Jülich-Bonn coupled-channel model¹
 - > Potential V from an effective Lagrangian
 - > T^P genuine resonance states in the s-channel diagrams
 - > T^{NP} dynamically generated poles: t/u-channel

$$\langle L'S'p' | \mathcal{T}_{\mu\nu}^{IJ} | LSp \rangle = \langle L'S'p' | \mathcal{V}_{\mu\nu}^{IJ} | LSp \rangle + \sum_{\gamma, L''S''} \int_0^\infty dq \frac{q^2/E}{\langle L'S'p' | \mathcal{V}_{\mu\gamma}^{IJ} | L''S''q \rangle} \frac{1}{E - E_\gamma(q) + i\epsilon} \langle L''S''q | \mathcal{T}_{\gamma\nu}^{IJ} | LSp \rangle$$

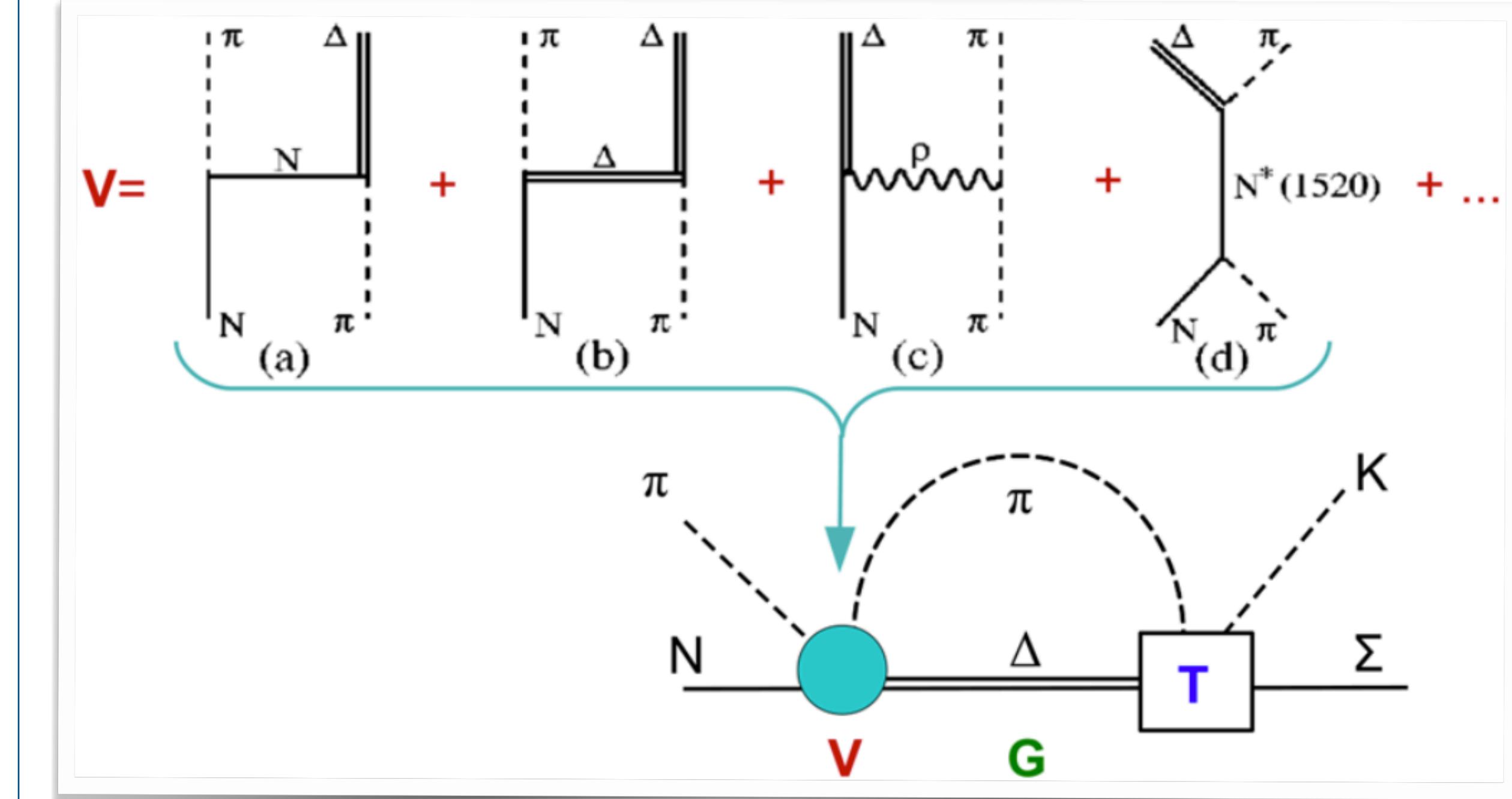
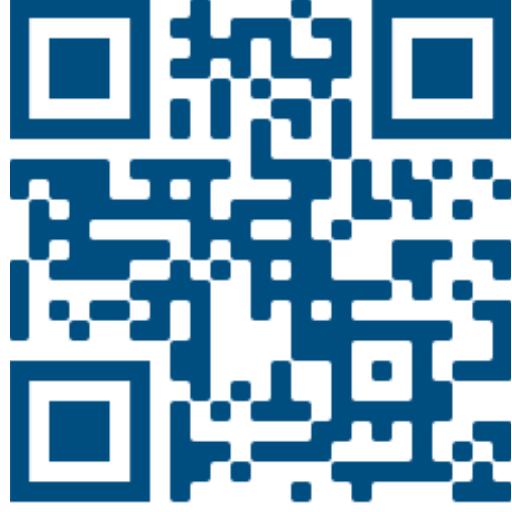


Fig: D. Rönchen

1) Rönchen et al., EPJA 49, 44 (2013)



MULTIPOLES

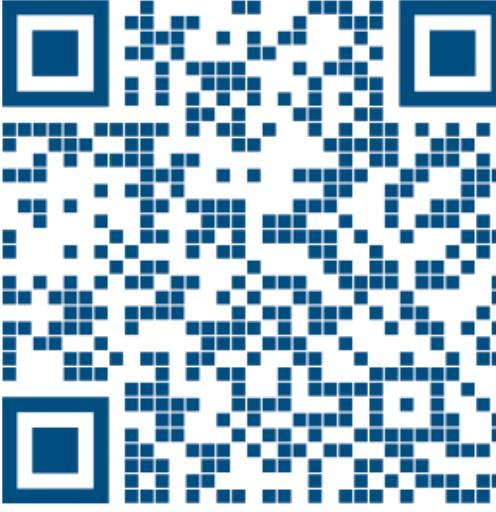


$$\mathcal{M}_{\mu\gamma^*}(k, W, Q^2) = R_\ell(\lambda, q/q_\gamma) \left(V_{\mu\gamma^*}(k, W, Q^2) + \sum_{\kappa} \int_0^\infty dp p^2 T_{\mu\kappa}(k, p, W) G_\kappa(p, W) V_{\kappa\gamma^*}(p, W, Q^2) \right)$$

For $Q^2=0$ (real photons) identical to
Jülich-Bonn photoproduction amplitude

$$V_{\mu\gamma^*}(k, W, Q^2) = V_{\mu\gamma}^{\text{JUBO}}(k, W) \cdot \tilde{F}_D(Q^2) \cdot e^{-\beta_\mu^0 Q^2/m_p^2} \left(1 + Q^2/m_p^2 \beta_\mu^1 + (Q^2/m_p^2)^2 \beta_\mu^2 \right)$$

- 1) Siegert(1973) Amaldi et al.(1979) Tiator(2016)
- 2) Tiator et al.(2017)
- 3) Landay et al.(2017) (2019)



MULTIPOLES

$$\mathcal{M}_{\mu\gamma^*}(k, W, Q^2) = R_\ell(\lambda, q/q_\gamma) \left(V_{\mu\gamma^*}(k, W, Q^2) + \sum_{\kappa} \int_0^\infty dp p^2 T_{\mu\kappa}(k, p, W) G_\kappa(p, W) V_{\kappa\gamma^*}(p, W, Q^2) \right)$$

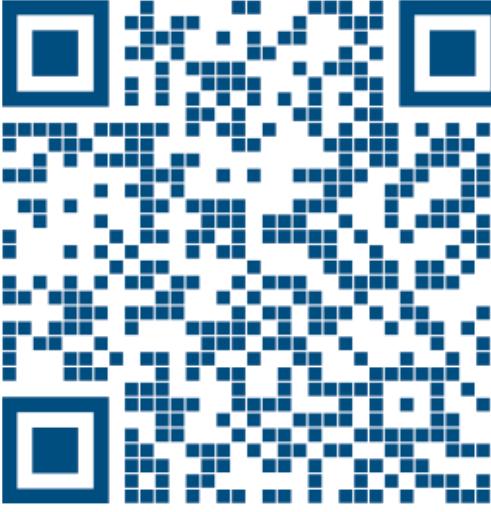
For $Q^2=0$ (real photons) identical to
Jülich-Bonn photoproduction amplitude

$$V_{\mu\gamma^*}(k, W, Q^2) = V_{\mu\gamma}^{\text{JUBO}}(k, W) \cdot \tilde{F}_D(Q^2) \cdot e^{-\beta_\mu^0 Q^2/m_p^2} \left(1 + Q^2/m_p^2 \beta_\mu^1 + (Q^2/m_p^2)^2 \beta_\mu^2 \right)$$

Siegert's theorem¹
...at pseudo-threshold:

$$V^{L_{\ell^\pm}} = (\text{const.}) \cdot V^{E_{\ell^\pm}}$$

- 1) Siegert(1973) Amaldi et al.(1979) Tiator(2016)
- 2) Tiator et al.(2017)
- 3) Landay et al.(2017) (2019)



MULTIPOLES

$$\mathcal{M}_{\mu\gamma^*}(k, W, Q^2) = R_\ell(\lambda, q/q_\gamma) \left(V_{\mu\gamma^*}(k, W, Q^2) + \sum_{\kappa} \int_0^\infty dp p^2 T_{\mu\kappa}(k, p, W) G_\kappa(p, W) V_{\kappa\gamma^*}(p, W, Q^2) \right)$$

(Pseudo)-threshold behavior with
meson/photon momenta

$$\begin{aligned} \lim_{k \rightarrow 0} E_{\ell+} &= k^\ell \\ \lim_{q \rightarrow 0} L_{\ell+} &= q^\ell \\ \dots \end{aligned}$$

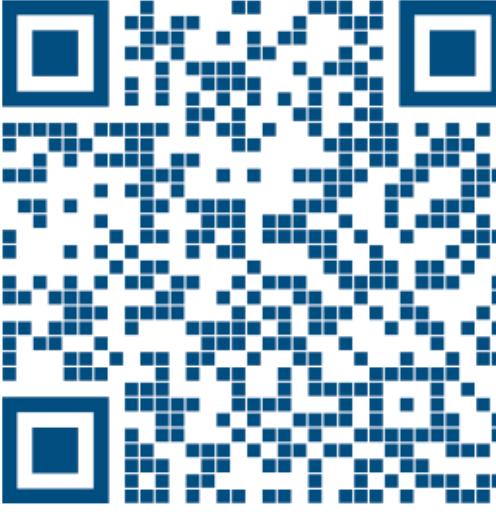
For $Q^2=0$ (real photons) identical to
Jülich-Bonn photoproduction amplitude

$$V_{\mu\gamma^*}(k, W, Q^2) = V_{\mu\gamma}^{\text{JUBO}}(k, W) \cdot \tilde{F}_D(Q^2) \cdot e^{-\beta_\mu^0 Q^2/m_p^2} \left(1 + Q^2/m_p^2 \beta_\mu^1 + (Q^2/m_p^2)^2 \beta_\mu^2 \right)$$

Siegert's theorem¹
...at pseudo-threshold:

$$V^{L_{\ell^\pm}} = (\text{const.}) \cdot V^{E_{\ell^\pm}}$$

- 1) Siegert(1973) Amaldi et al.(1979) Tiator(2016)
- 2) Tiator et al.(2017)
- 3) Landay et al.(2017) (2019)



MULTIPOLES

$$\mathcal{M}_{\mu\gamma^*}(k, W, Q^2) = R_\ell(\lambda, q/q_\gamma) \left(V_{\mu\gamma^*}(k, W, Q^2) + \sum_{\kappa} \int_0^\infty dp p^2 T_{\mu\kappa}(k, p, W) G_\kappa(p, W) V_{\kappa\gamma^*}(p, W, Q^2) \right)$$

(Pseudo)-threshold behavior with meson/photon momenta

$$\begin{aligned} \lim_{k \rightarrow 0} E_{\ell+} &= k^\ell \\ \lim_{q \rightarrow 0} L_{\ell+} &= q^\ell \\ \dots \end{aligned}$$

For $Q^2=0$ (real photons) identical to Jülich-Bonn photoproduction amplitude

$$V_{\mu\gamma^*}(k, W, Q^2) = V_{\mu\gamma}^{\text{JUBO}}(k, W) \cdot \tilde{F}_D(Q^2) \cdot e^{-\beta_\mu^0 Q^2/m_p^2} \left(1 + Q^2/m_p^2 \beta_\mu^1 + (Q^2/m_p^2)^2 \beta_\mu^2 \right)$$

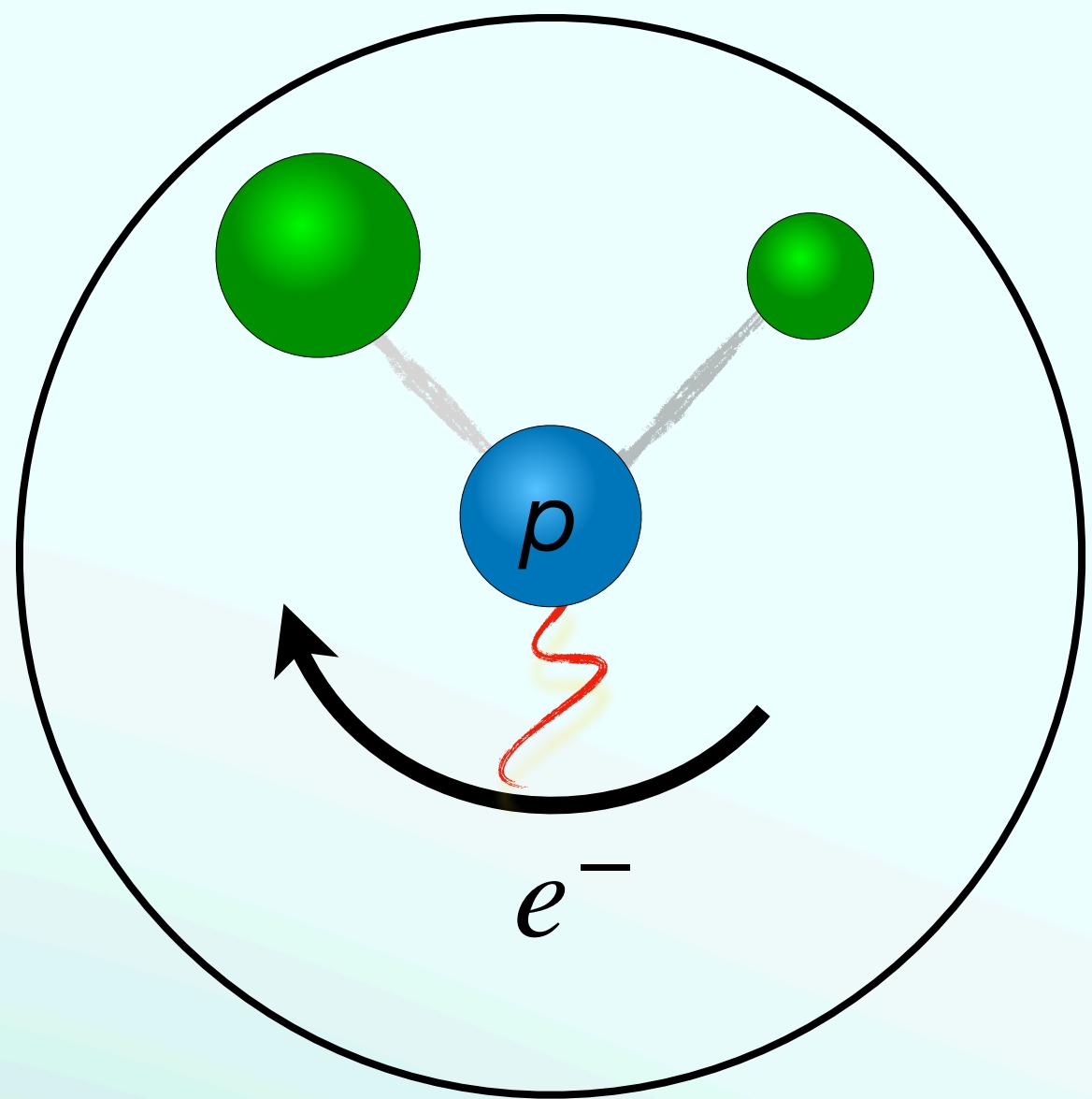
Siegert's theorem¹
...at pseudo-threshold:

$$V^{L_{\ell^\pm}} = (\text{const.}) \cdot V^{E_{\ell^\pm}}$$

Parametrization dependence due to incomplete data

- > even for a truncated complete electroproduction experiment²
- > in future: Bias-variance tradeoff with statistical criteria³

1) Siegert(1973) Amaldi et al.(1979) Tiator(2016)
 2) Tiator et al.(2017)
 3) Landay et al.(2017) (2019)



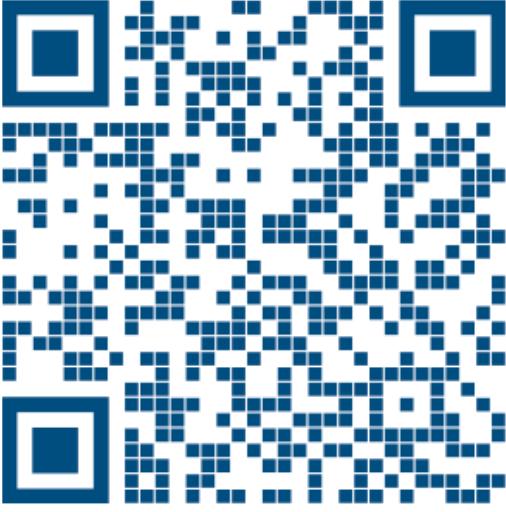
RESULTS

[JBW] MM, M.Döring, C.Granados, H.Haberzettl, J.Hergenrather, Ulf-G. Meißner, D.Rönchen, I.Strakovsky, R.Workman

Phys.Rev.C 103 (2021) 6, 065204
Phys.Rev.C 106 (2022) 015201

INTERACTIVE WEB INTERFACE: <https://jbw.phys.gwu.edu>

DEGREES OF FREEDOM



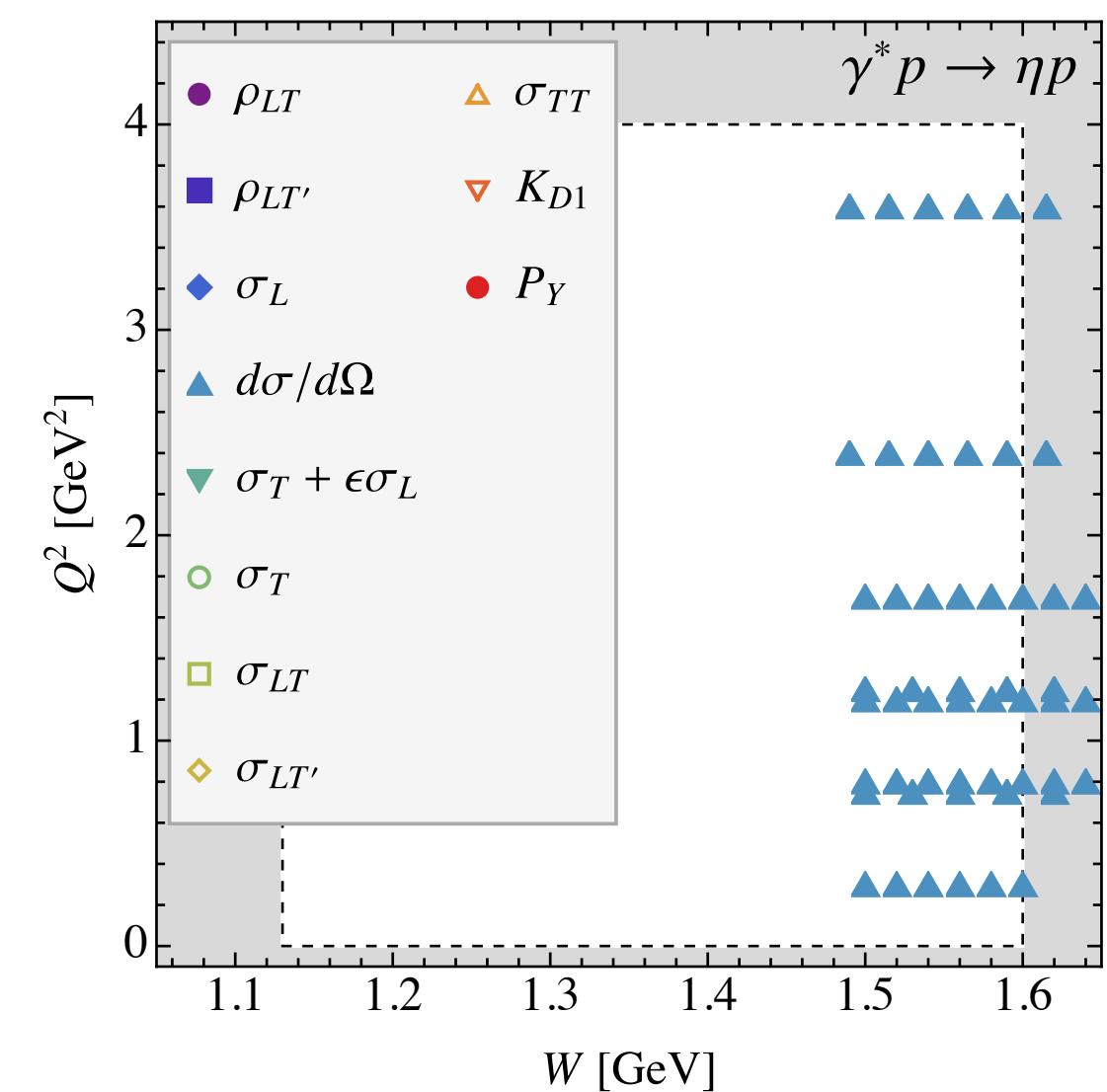
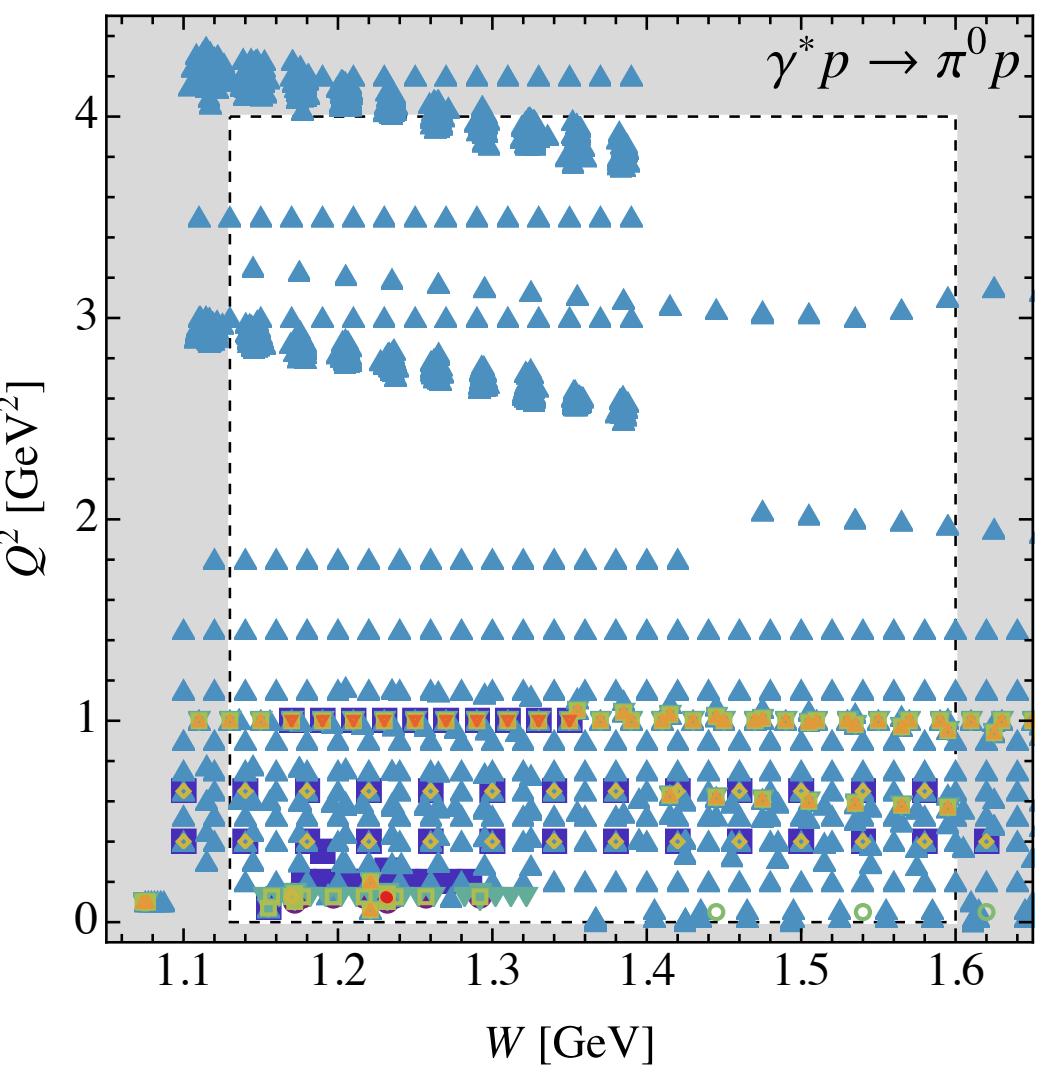
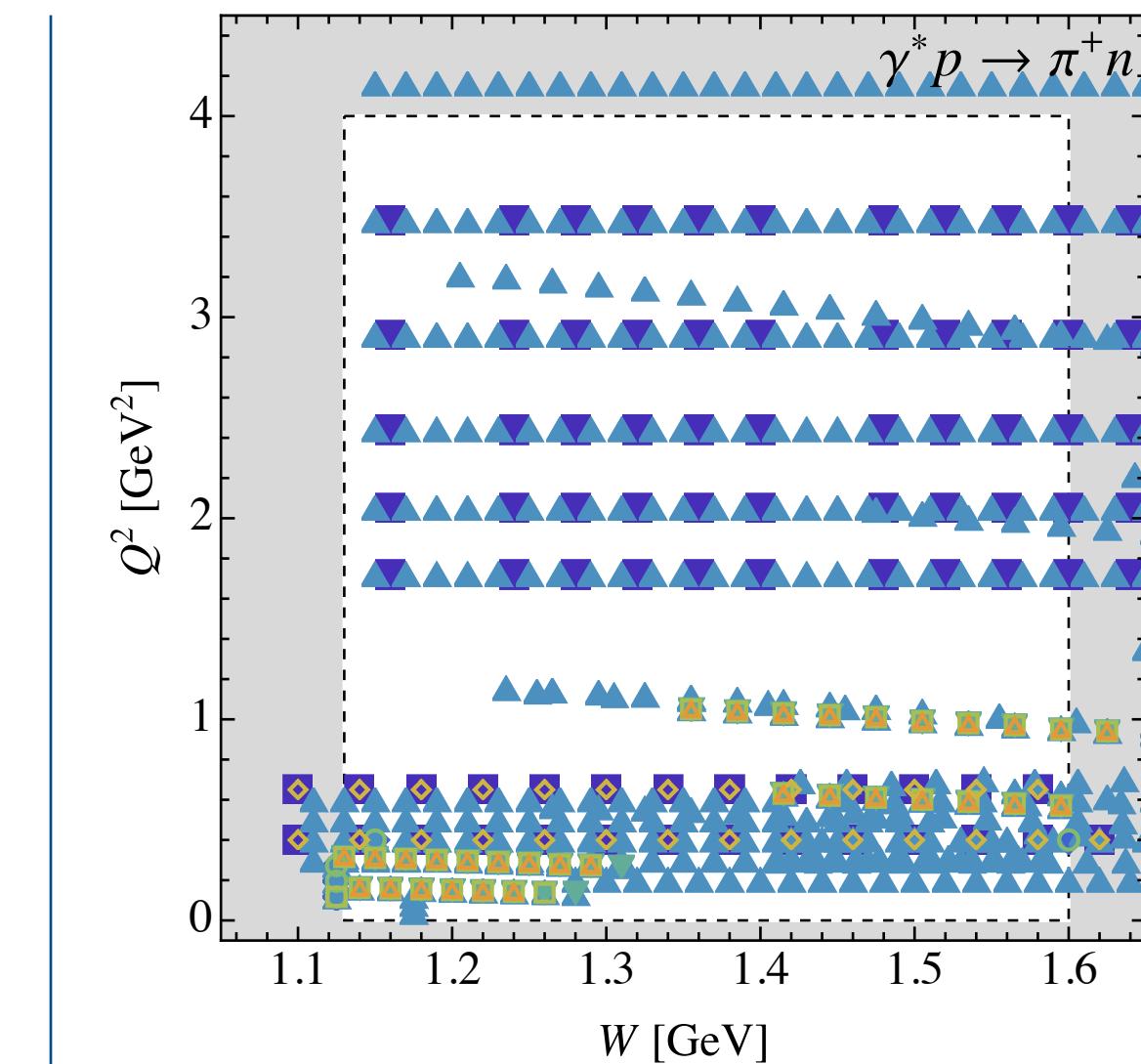
Data ($1.13 < W/\text{GeV} < 1.6$, $Q^2 < 4 \text{ GeV}^2$)

$$45k(\pi^0 p) + 37k(\pi^+ n) + 2k(\eta p) = 84k \text{ data}$$

11 observable types

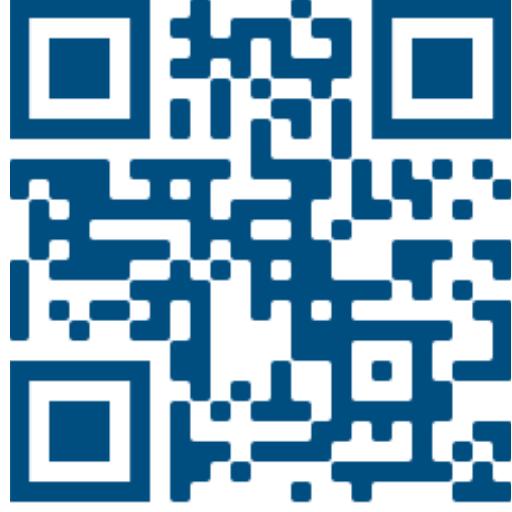
Parameters (S/P/D waves)

$$26 \text{ multipoles} * (10..13 \text{ pars}) = 257 \text{ pars}$$



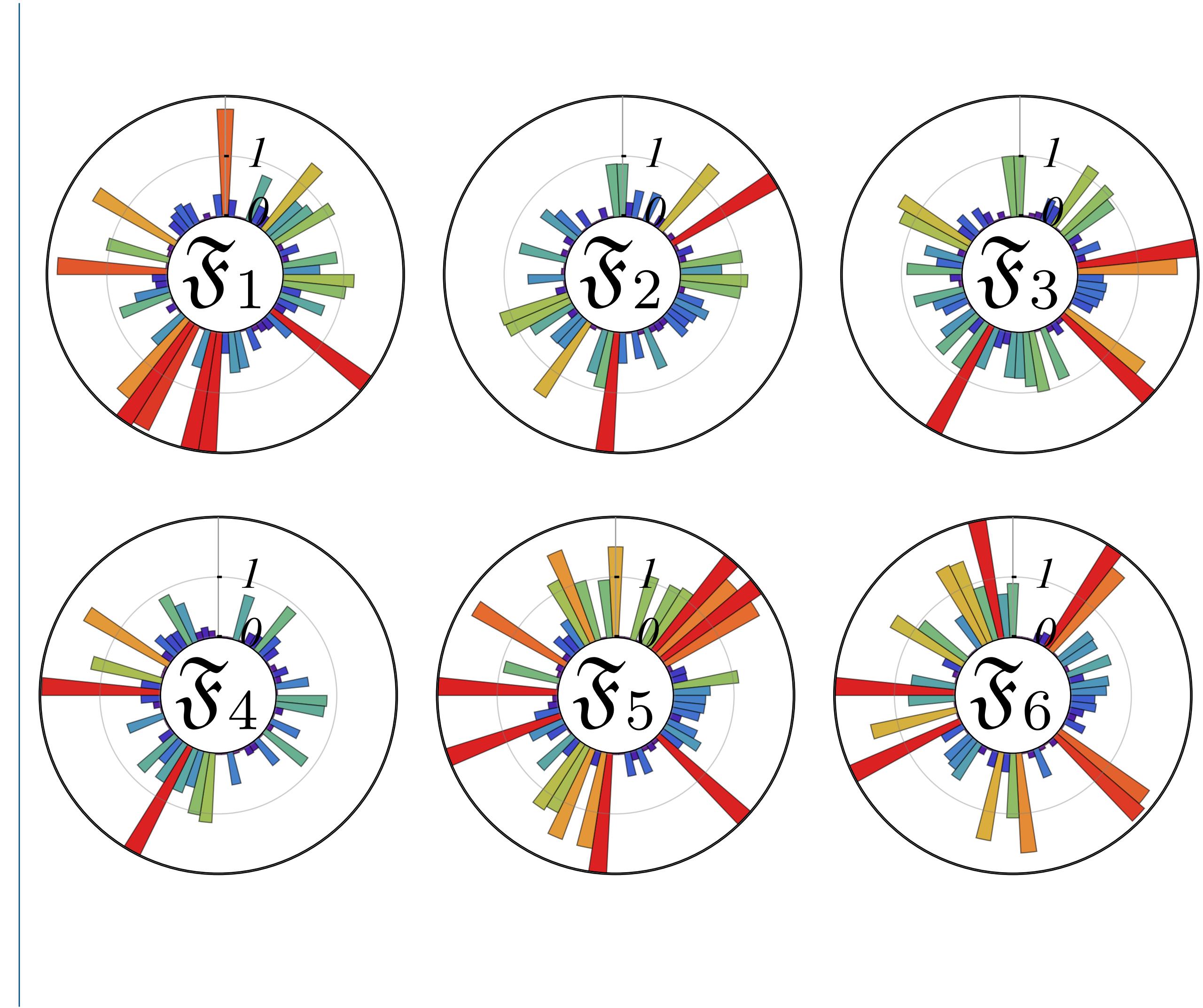


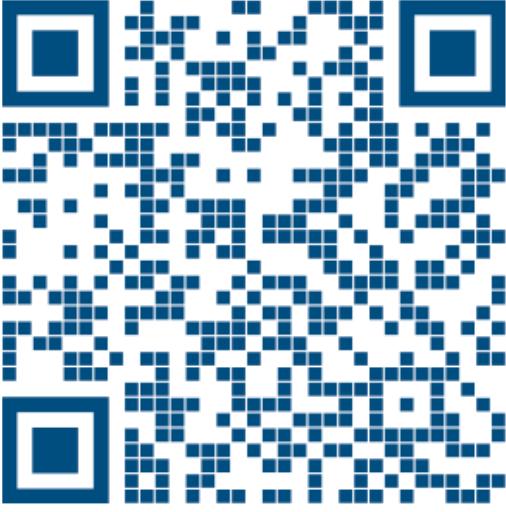
DEGREES OF FREEDOM



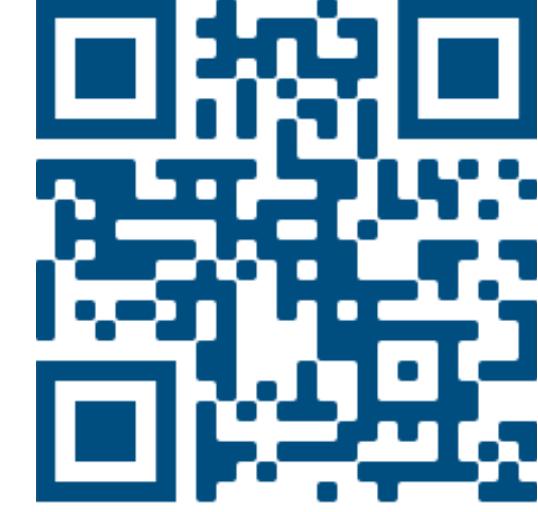
Systematic uncertainties:

- many local minima:
 - > different fit strategies (sequential, all-in fits)





DEGREES OF FREEDOM



Systematic uncertainties:

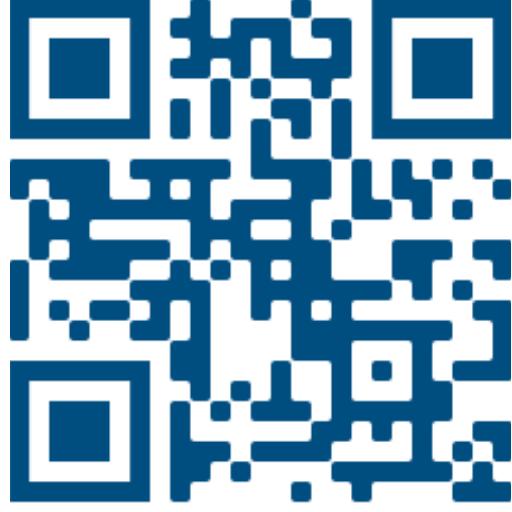
- pIN data is dominant:
 > weighted or unweighted fits

$$\chi^2_{\text{reg}} = \sum_{i=1}^{N_{\text{all}}} \left(\frac{\mathcal{O}_i^{\text{exp}} - \mathcal{O}_i}{\Delta_i^{\text{stat}} + \Delta_i^{\text{syst}}} \right)^2$$

$$\chi^2_{\text{wt}} = \sum_{j \in \{\pi^0 p, \pi^+ n, \eta p\}} \frac{N_{\text{all}}}{3N_j} \sum_{i=1}^{N_j} \left(\frac{\mathcal{O}_{ji}^{\text{exp}} - \mathcal{O}_{ji}}{\Delta_{ji}^{\text{stat}} + \Delta_{ji}^{\text{syst}}} \right)^2$$



RESULTS



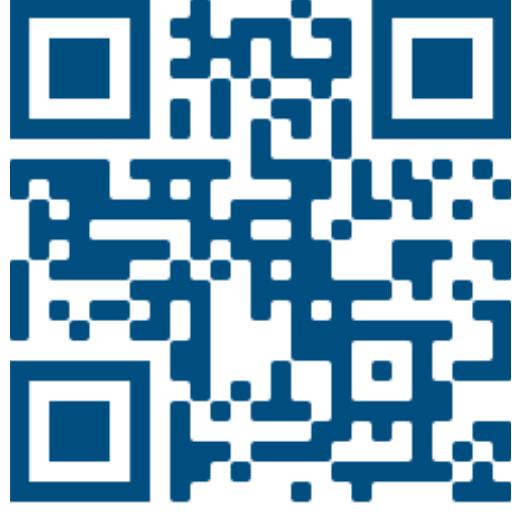
πN data fits:

> all strategies converge

> different minima (systematic uncertainties)

Fit	σ_L $\pi^0 p \pi^+ n$	$d\sigma/d\Omega$ $\pi^0 p \pi^+ n$	$\sigma_T + \epsilon\sigma_L$ $\pi^0 p \pi^+ n$	σ_T $\pi^0 p \pi^+ n$	σ_{LT} $\pi^0 p \pi^+ n$	$\sigma_{LT'}$ $\pi^0 p \pi^+ n$	σ_{TT} $\pi^0 p \pi^+ n$	K_{D1} $\pi^0 p \pi^+ n$	P_Y $\pi^0 p \pi^+ n$	ρ_{LT} $\pi^0 p \pi^+ n$	$\rho_{LT'}$ $\pi^0 p \pi^+ n$	χ^2_{dof}
\mathfrak{F}_1	- 9	65355 53229	870 418	87 88	1212 133	862 762	4400 251	4493 -	234 -	525 -	3300 10294	1.77
\mathfrak{F}_2	- 4	69472 55889	1081 619	65 78	1780 150	1225 822	4274 237	4518 -	325 -	590 -	3545 10629	1.69
\mathfrak{F}_3	- 8	66981 54979	568 388	84 95	1863 181	1201 437	3934 339	4296 -	686 -	687 -	3556 9377	1.81
\mathfrak{F}_4	- 22	63113 52616	562 378	153 107	1270 146	1198 1015	4385 218	5929 -	699 -	604 -	3548 11028	1.78
\mathfrak{F}_5	- 20	65724 53340	536 528	125 81	1507 219	1075 756	4134 230	5236 -	692 -	554 -	3580 11254	1.81
\mathfrak{F}_6	- 18	71982 58434	1075 501	29 68	1353 135	1600 1810	3935 291	5364 -	421 -	587 -	3932 11475	1.78

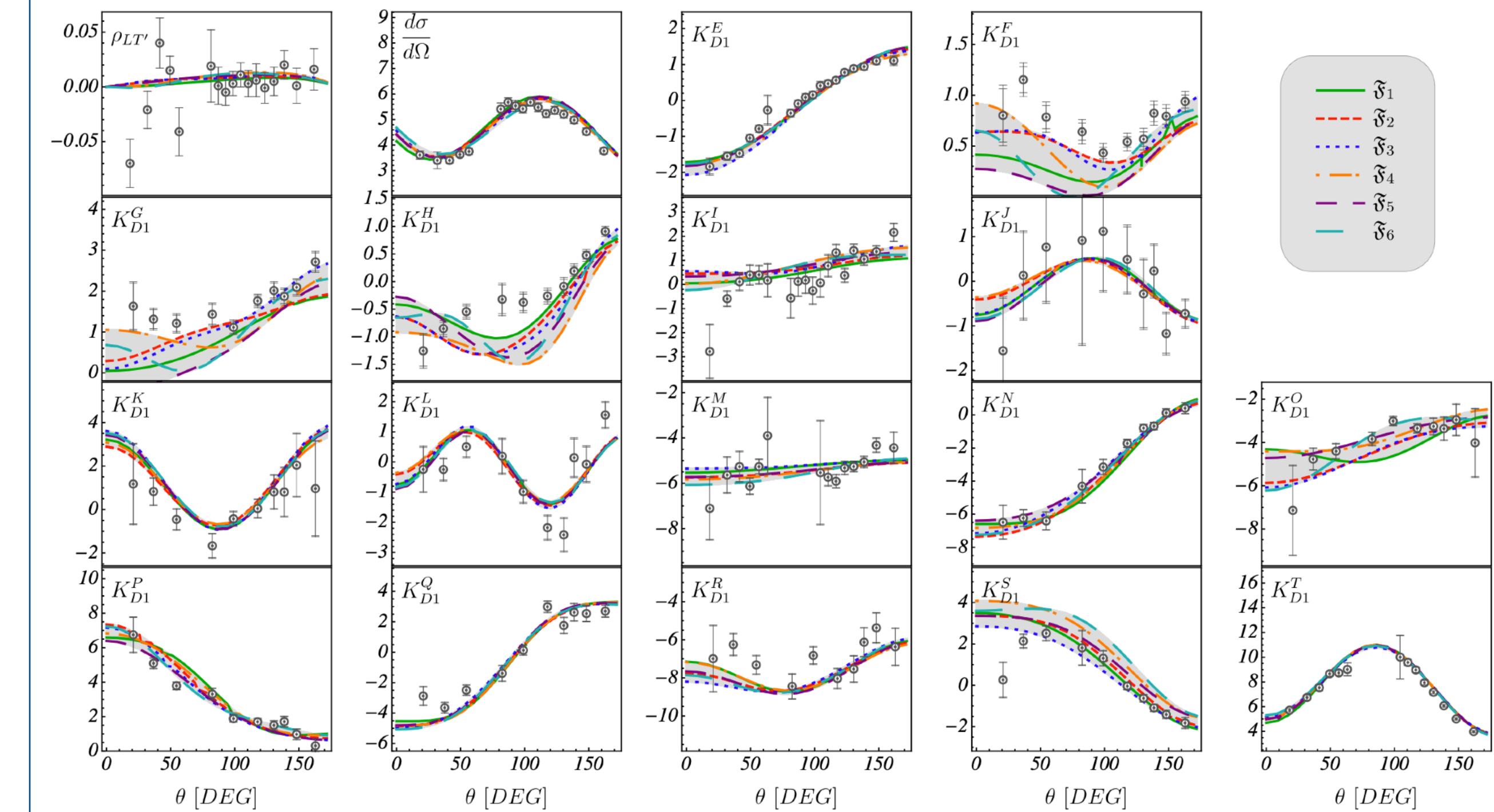
RESULTS



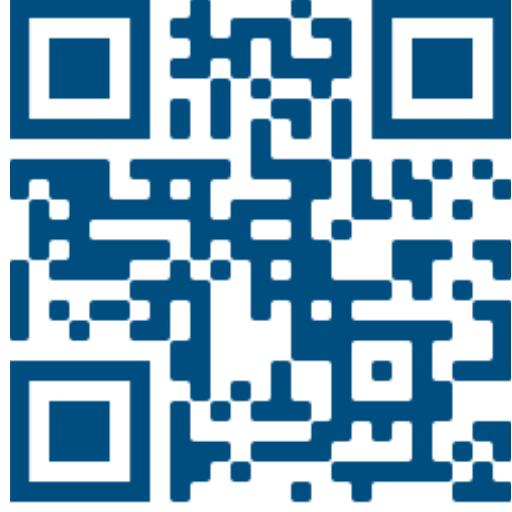
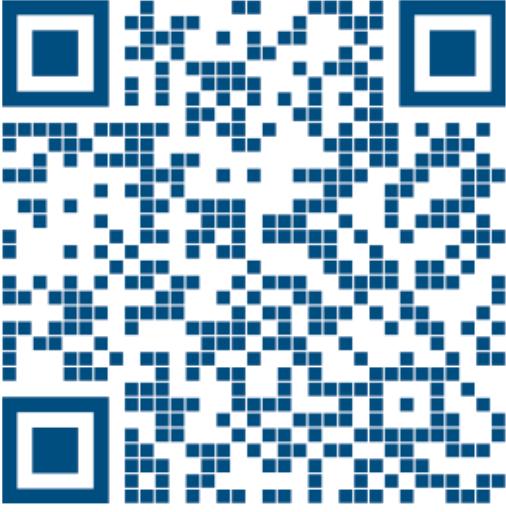
πN data fits:

> all strategies converge

> different minima (systematic uncertainties)

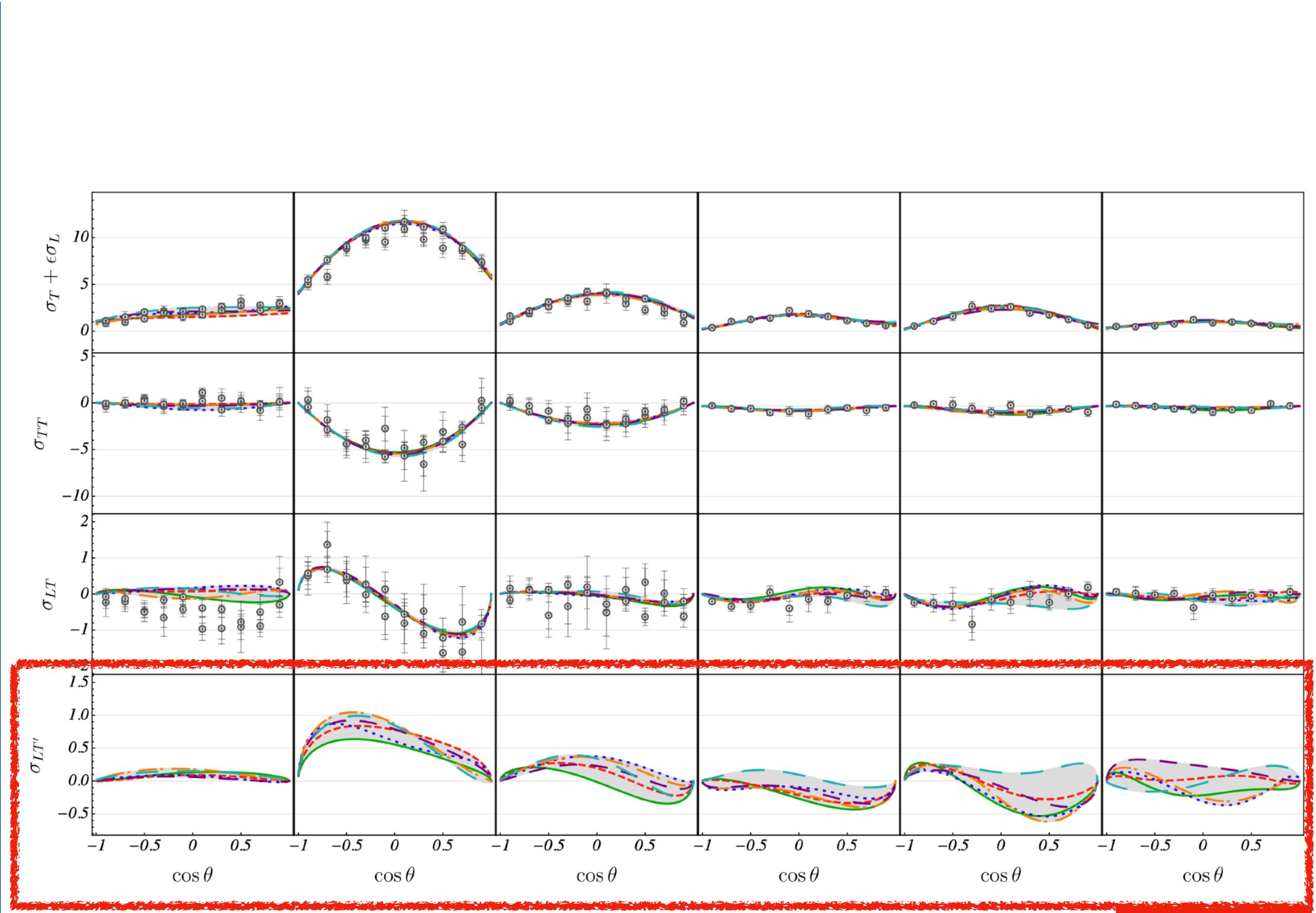


RESULTS



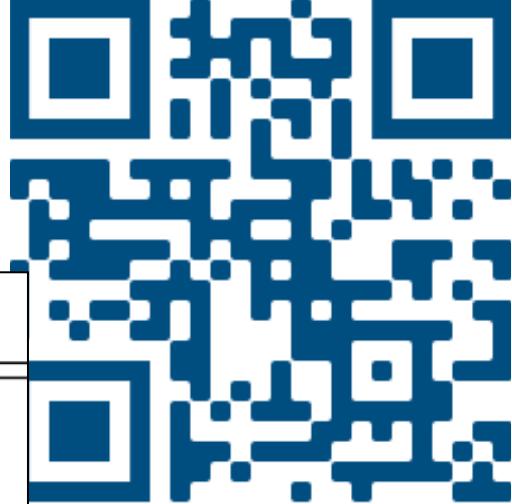
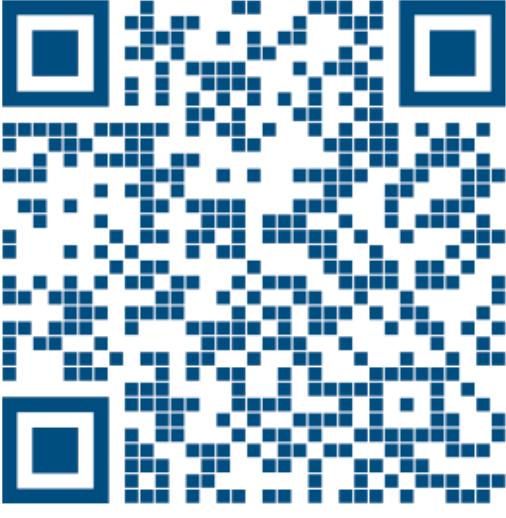
πN data fits:

- > all strategies converge
- > different minima (systematic uncertainties)



PREDICTION

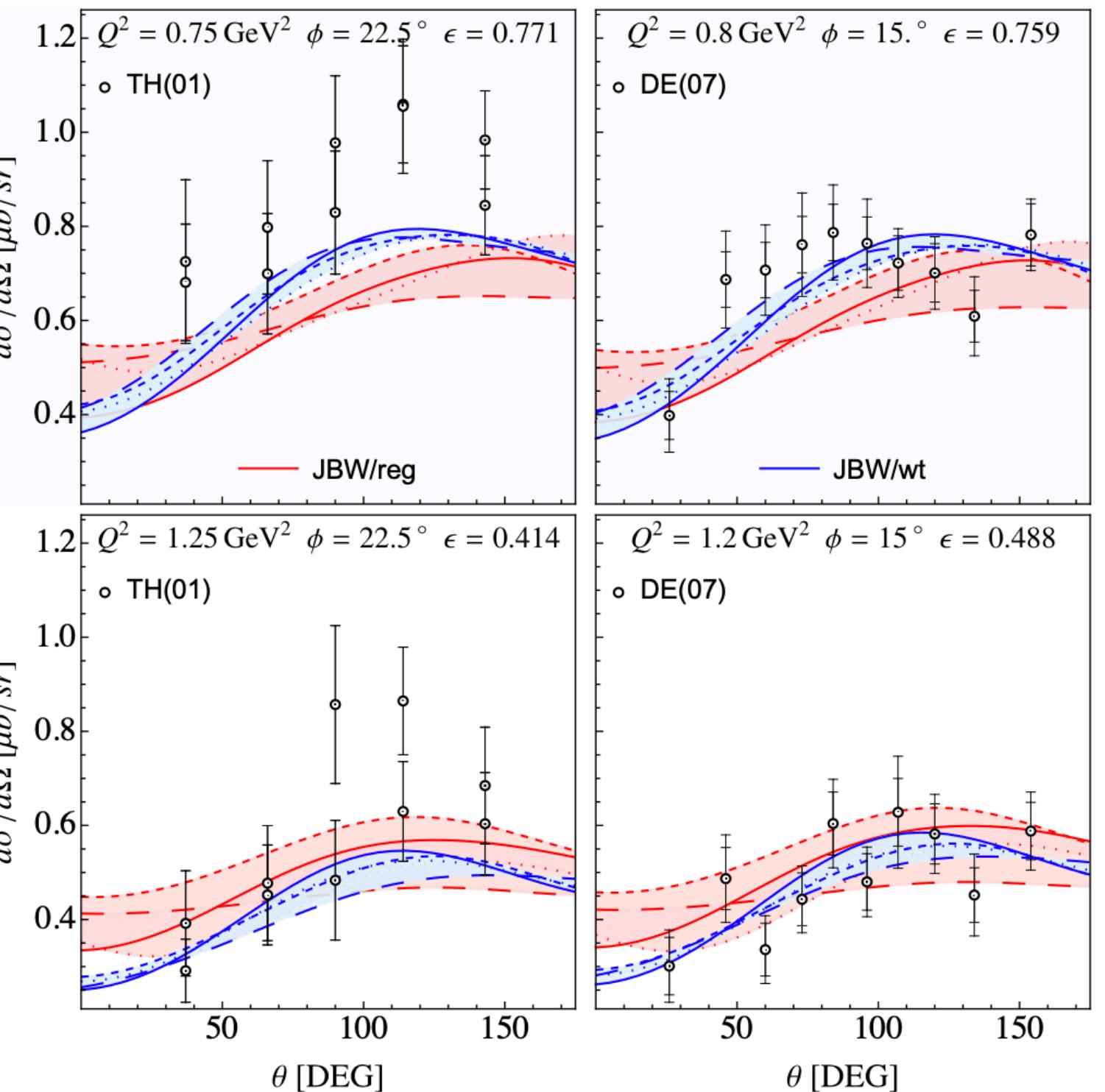
RESULTS



- ηN data fits:**
- > all strategies converge
 - > different minima (systematic uncertainties)

[JBW] MM et al. *Phys.Rev.C* 103 (2021) 6; *Phys.Rev.C* 106 (2022) 015201

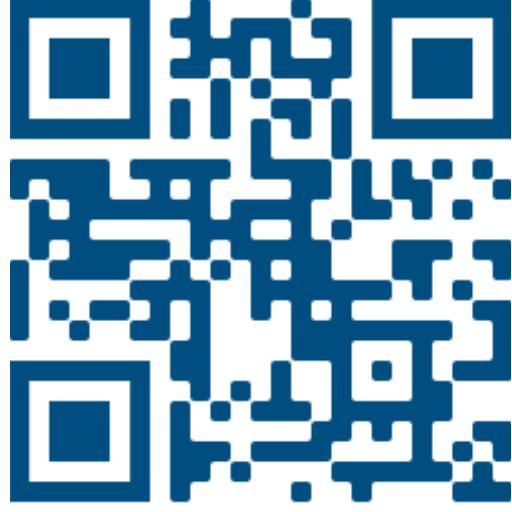
	χ^2/dof	$\chi^2_{\pi^0 p}/\text{data}$	$\chi^2_{\pi^+ n}/\text{data}$	$\chi^2_{\eta p}/\text{data}$
$\mathfrak{F}_1^{\text{reg}}$	1.66	1.68	1.61	1.77
$\mathfrak{F}_2^{\text{reg}}$	1.73	1.71	1.71	2.29
$\mathfrak{F}_3^{\text{reg}}$	1.69	1.69	1.66	1.89
$\mathfrak{F}_4^{\text{reg}}$	1.69	1.7	1.64	2.05
$\mathfrak{F}_1^{\text{wt}}$	1.54	1.74	1.63	1.25
$\mathfrak{F}_2^{\text{wt}}$	1.63	1.82	1.79	1.27



Data: H. Denizli et al. (CLAS) PRC 76, 015204 (2007);
Thompson et al. (CLAS), PRL86, 1702–1706 (2001)



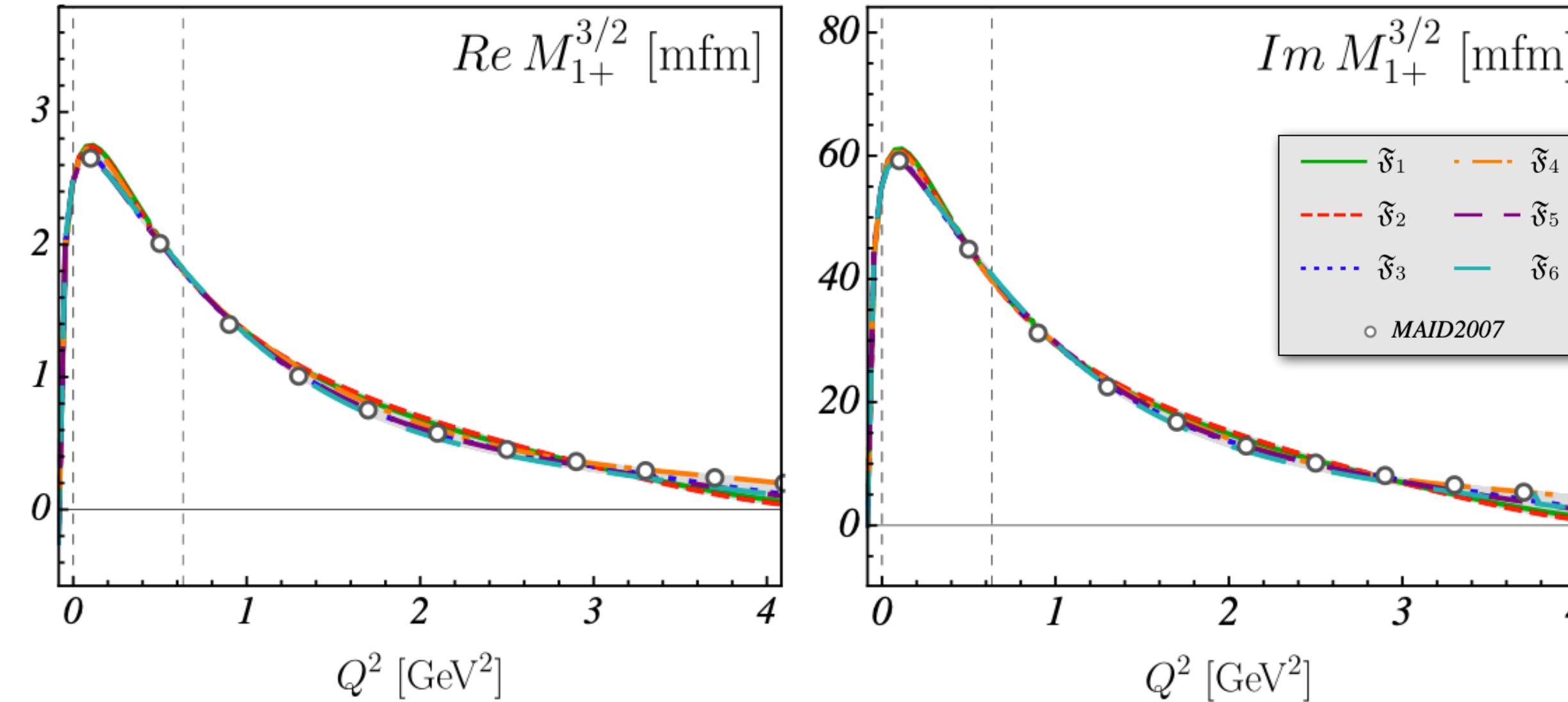
MULTIPOLES



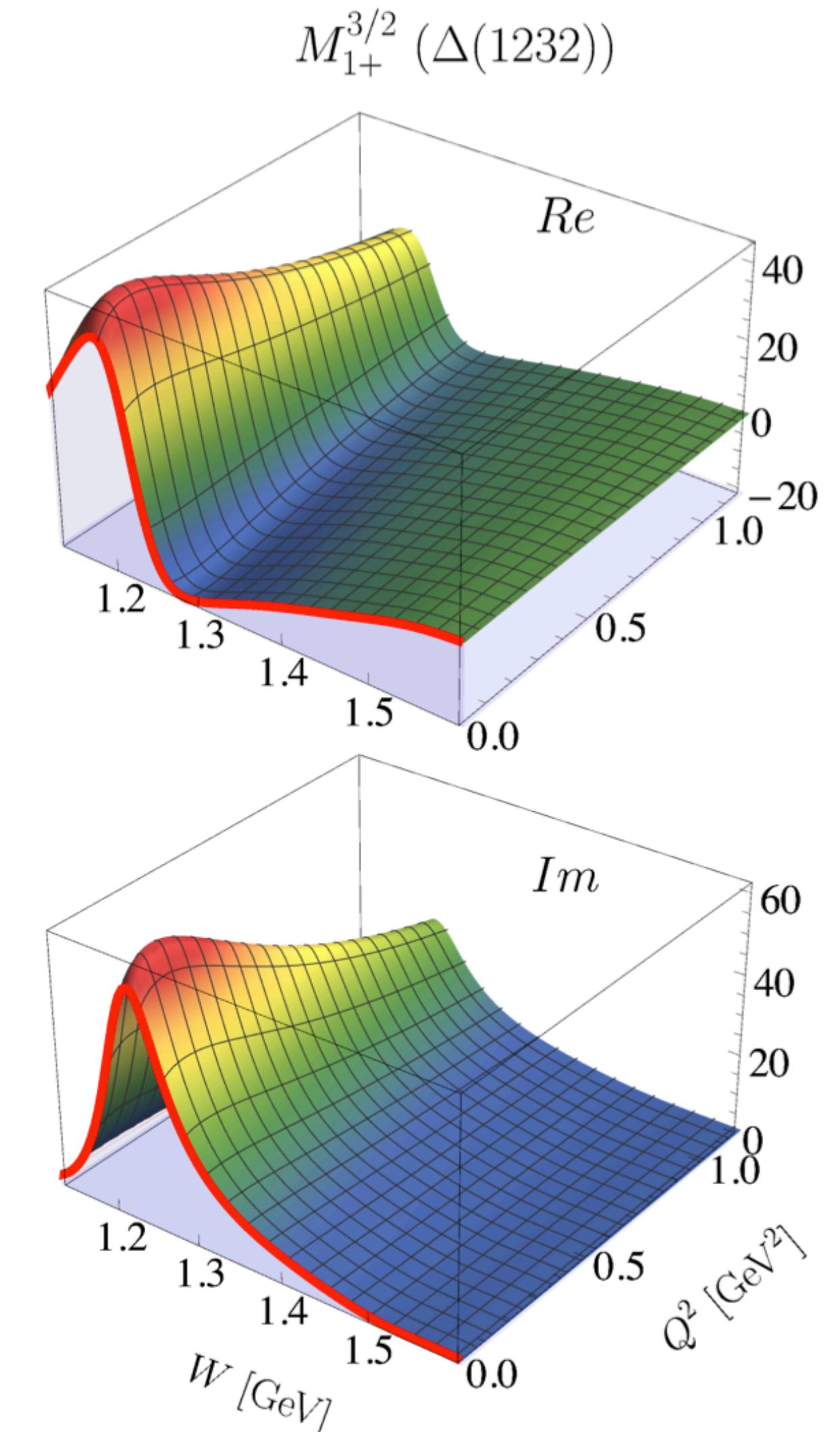
Delta:

- Large multipoles well determined small systematic uncertainties

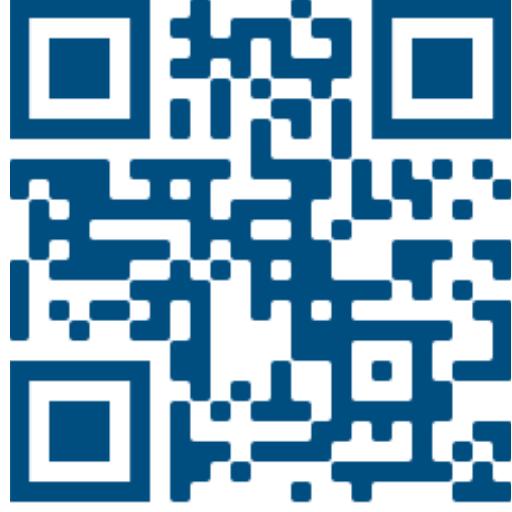
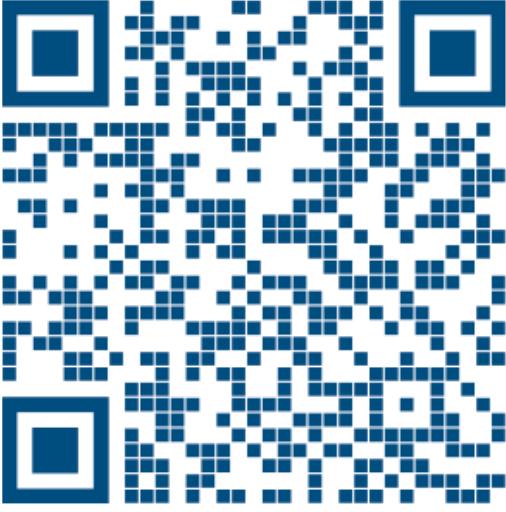
$W = 1230 \text{ MeV}$



[JBW] MM et al. Phys.Rev.C 103 (2021) 6; Phys.Rev.C 106 (2022) 015201

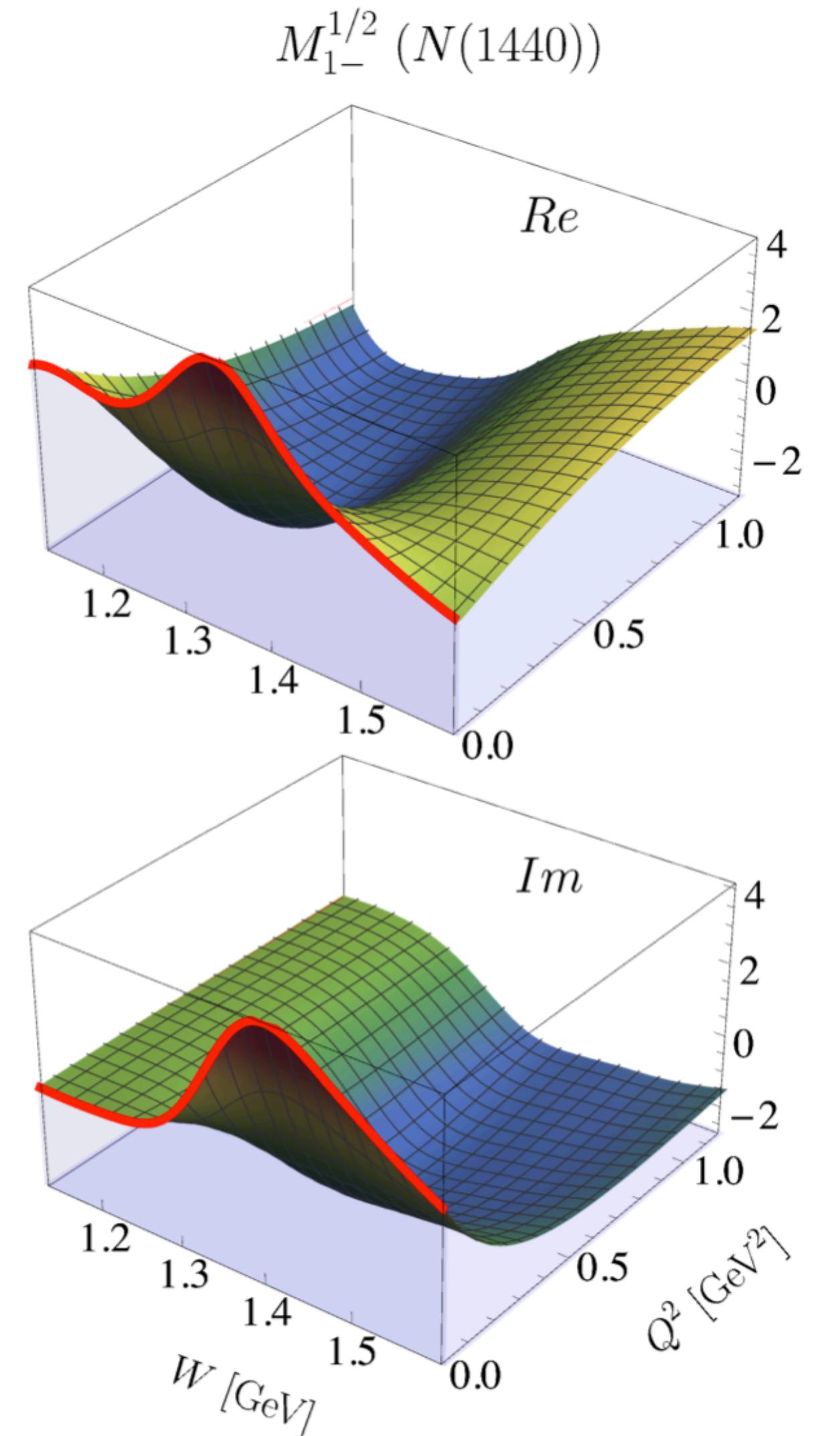
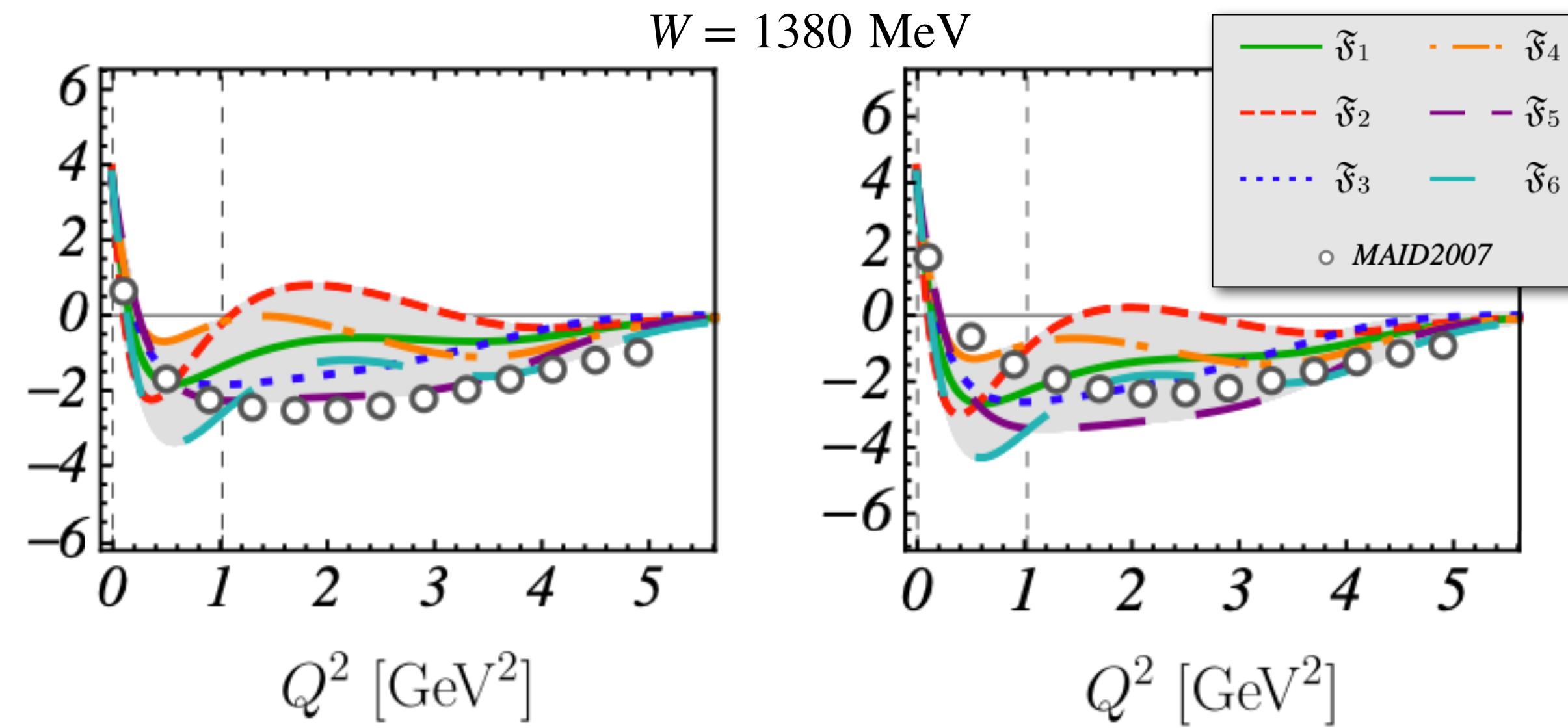


MULTIPOLES

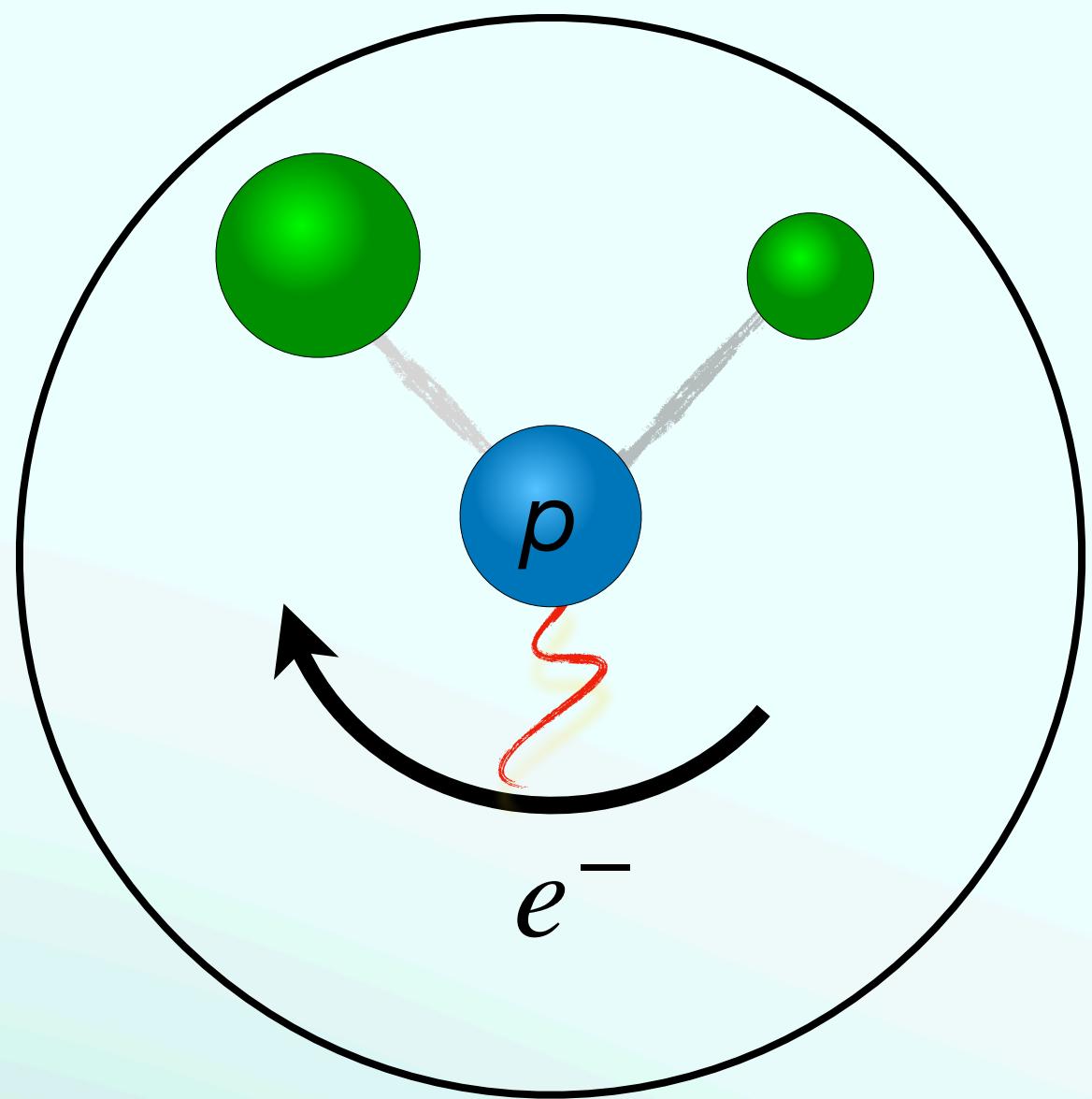


Roper:

- Non-trivial Q^2 behavior
- Zero transition



[JBW] MM et al. *Phys.Rev.C* 103 (2021) 6; *Phys.Rev.C* 106 (2022) 015201



SUMMARY

SUMMARY

Jülich-Bonn-Washington model

- new model developed
- constraints from scattering,
photoproduction data and fundamental
principles
- fits to piN/etaN data finished
- WEB INTERFACE:

<https://jbw.phys.gwu.edu>



OUTLOOK

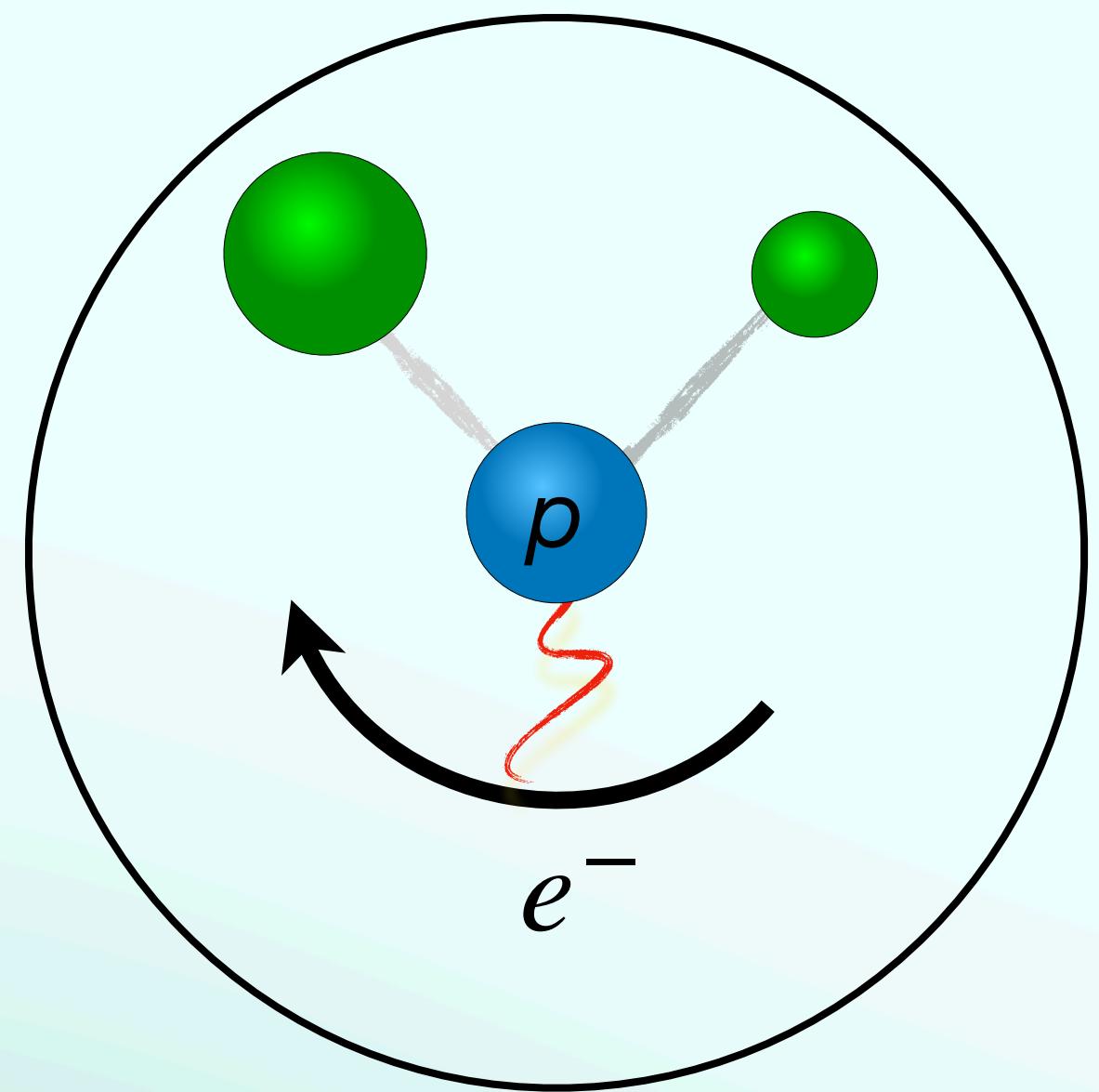
[this year] $\pi N/\eta N/K\Lambda$ fits (nearly world data)

[this decade] simultaneous fit to scattering and
photoproduction data

[this decade] statistical studies of parameter
importance (LASSO, Machine Learning, ...)

Landay et al., Phys.Rev.D (2019), 1810.00075 [nucl-th]

[this decade] energy independent analysis(?)



THANKS

