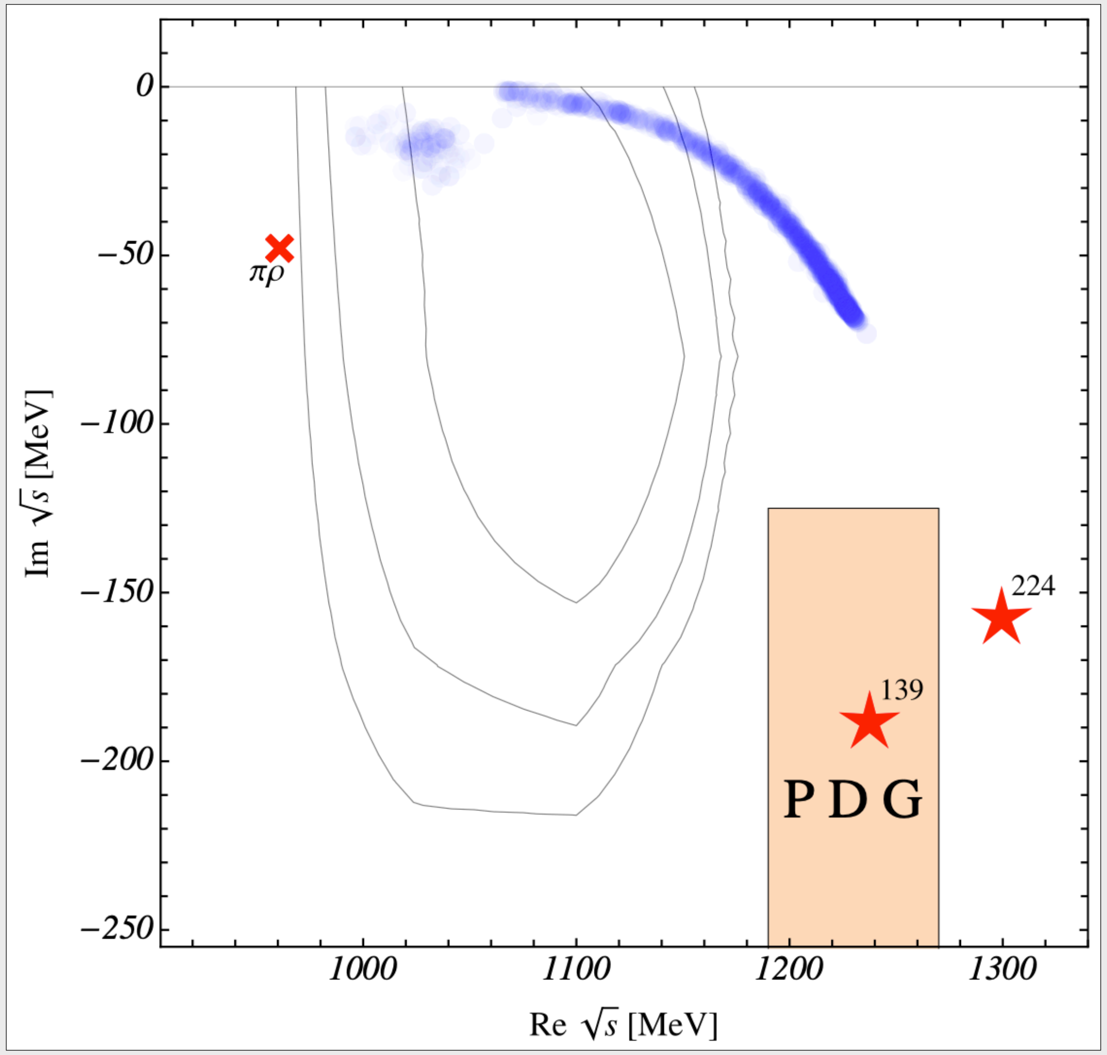
THE a1(1260)-RESONANCE FROM LATTICE QCD

2107.03973 [hep-lat]



Maxim Mai, A. Alexandru, R. Brett, C. Culver
M. Döring, F. Lee, D. Sadasivan [GWQCD]

slides

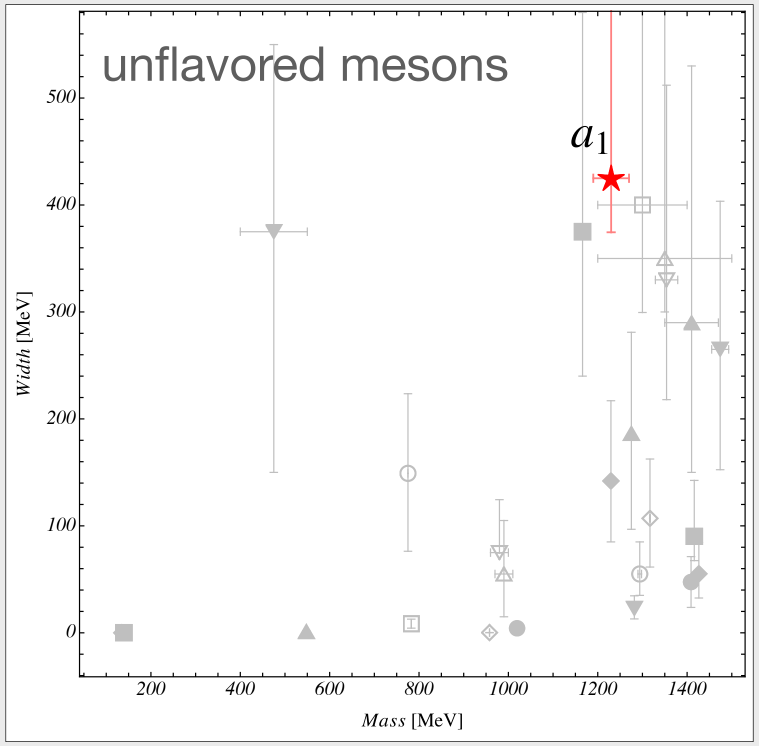
https://maxim-mai.github.io/talks/LAT21-MM.pdf

QCD SPECTRUM

Many states of QCD have large coupling to 3-body channels

- ω(782), a_1(1260)...
- exotic mesons: π_1(1600), ... **exp. searches @ COMPASS, GlueX**
- Roper resonance *N*(1440)*

This work: ***a_1(1260)* from lattice QCD**



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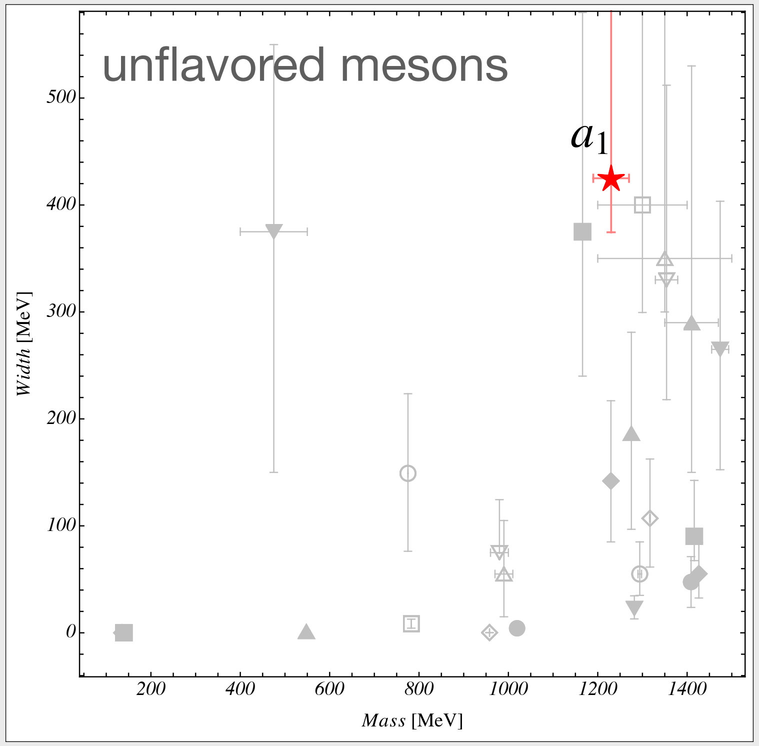
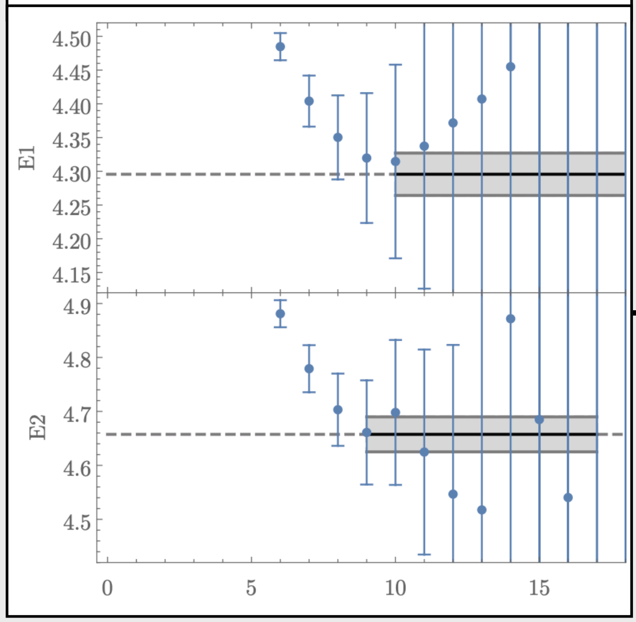


This work: ***a₁(1260)* from lattice QCD**

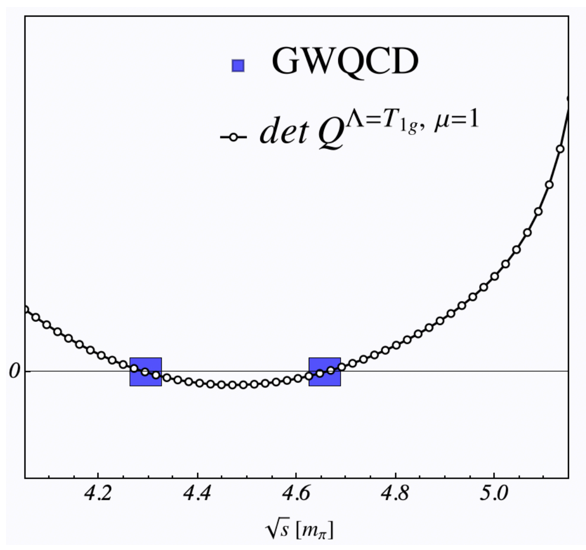
- Universal parameters from poles on the Riemann surface
- 3 step procedure:

Finite-volume Spectrum



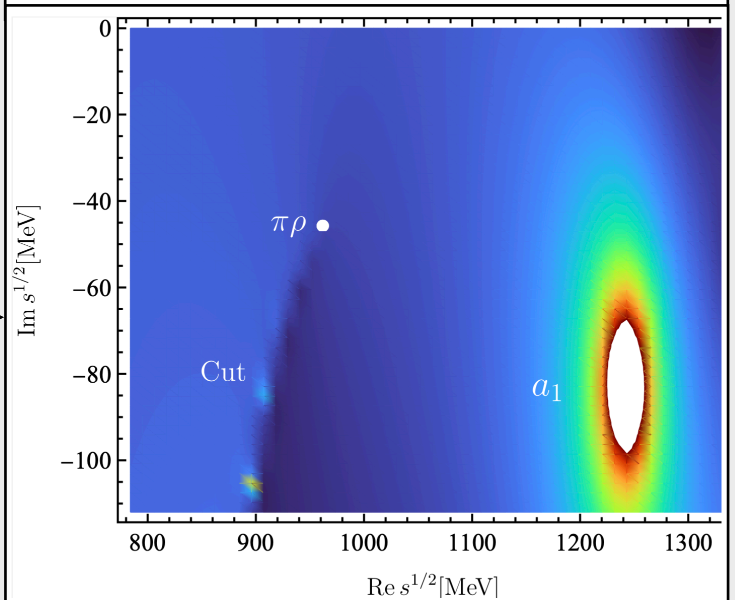
energy eigenvalues →

3-body quantization conditions



volume indep. quantities →

Analytic continuation scattering amplitude



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FINITE-VOLUME SPECTRUM

GWQCD ensemble used for 2/3 pion calculations Alexandru, Brett, Culver, Guo, Lee, Pelissier (2013-2020) PRD87,PRD94,PRD98,PRD96,PRL117,PRD100

Some key details: *(more in the next talk -- Ruairí Brett)*

- $N_f = 2$ dynamical fermions, LapH smearing
- $\boldsymbol{P}=(0,0,0)$, $m_\pi=224$ MeV, $m_\pi L=3.3$
- *GEVP with one-, two-, three-meson operators*
- *Relevant irrep(O_h) for* ***a1(1260)*** $I^G\,(J^{PC}) = 1^-\,(1^{++})$: T_{1g}

Geometry	$\mathbf{P}$	Λ	$J^P\ (I^G = 1^-)$
Cubic	$\mathbf{P} = (0,0,0)$	T_{1g}	$1^+, 3^+, \ldots$
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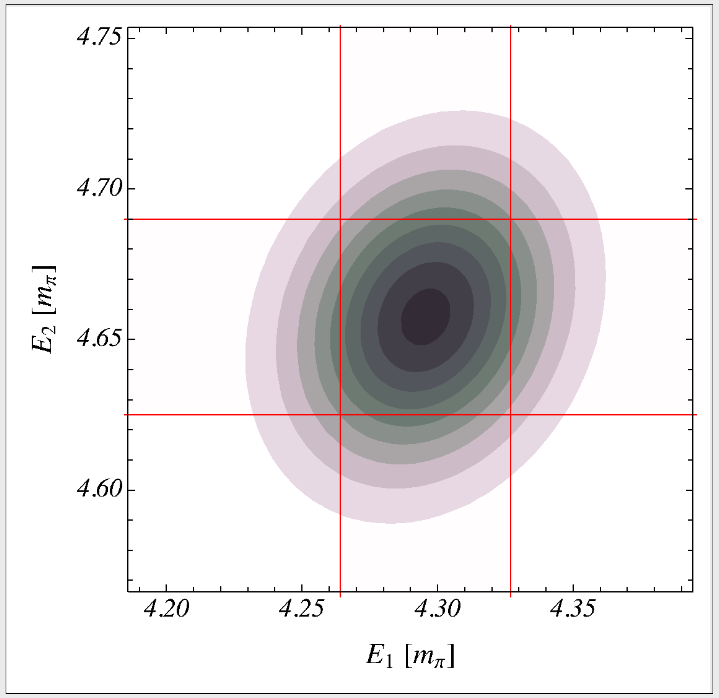
Key insights:

- 3-meson operators stabilize the excited state extraction

  c.f. need for $\rho\pi$ operators in pioneering 2-meson a_1-calculation

  Lang et al. JHEP 04, 162 (2014)

- high-momentum states are required: $\pi(0,0,0)\pi(1,1,0)\pi(-1,-1,0)$ etc..
- two interacting levels exists below 5π threshold



3-BODY QUANTIZATION CONDITION

Discrete, real finite-volume (lattice) spectrum → continuous complex-valued amplitudes

- established in 2-body: Lüscher's method, extensions...
- 3-body methods matured (this session)

Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky...

Bedaque, Blanton, Briceño, Davoudi, Döring, Grießhammer, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe, Wu

Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

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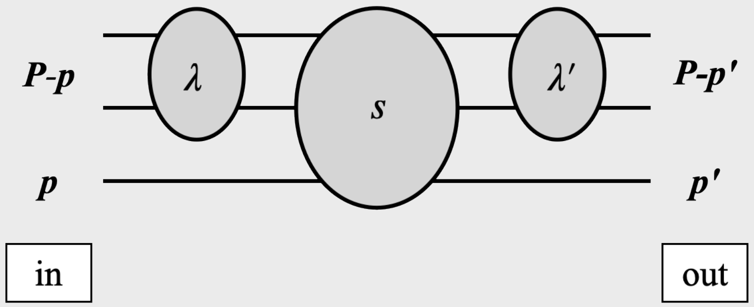
Finite Volume Unitarity MM, Döring EPJA (2017) PRL (2019)

- basic idea:

(unitary three-body amplitude) $\overset{\int \frac{d^3\mathbf{k}}{(2\pi)^3} \to 1/L^3 \sum_k}{\Rightarrow}$ ***(singular ⇔ three mesons are on-shell)*** ⇔ ***(energy eigenvalues)***

$$\boxed{0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]^{\Lambda}_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}}$$



- *extended to higher spin and coupled-channels: new degree of freedom (λ)*
- *∞-dim. determinant equation in* $\mathbf{p} \in \frac{2\pi}{\mathbf{L}}\mathbf{Z}^3$ → *practical applications require truncation*

→ *common to all quantization conditions*

see discussion in e.g. MM/Döring/Rusetsky(2021)

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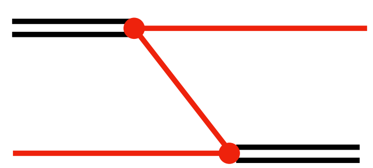
one-particle exchange

- fixed by 3b-unitarity



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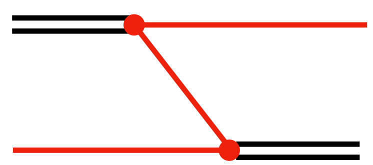

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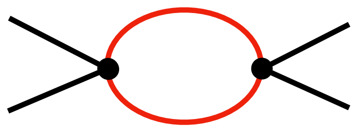
two-body self-energy

- fixed by 2b-unitarity



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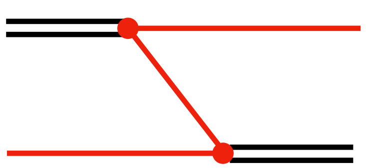
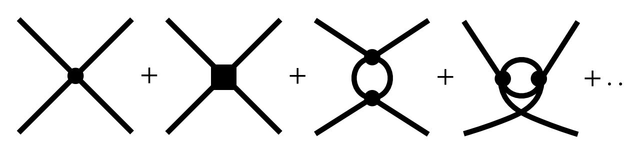


- no free parameters

two-body kernel

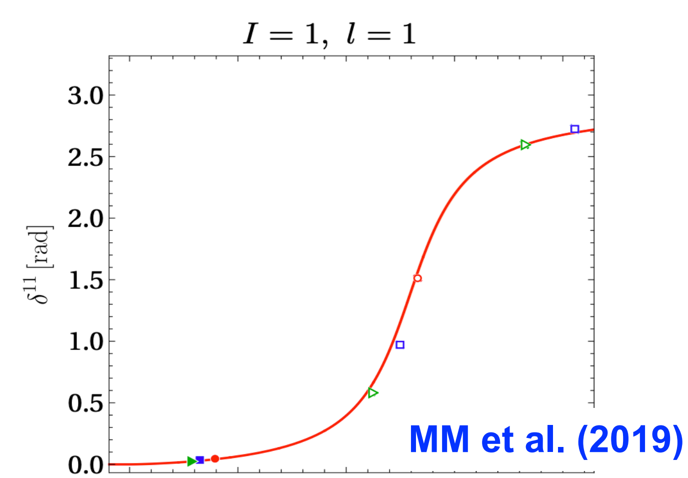
- dynamics of *I=1* $\pi\pi$ system



- regular function ⇒ polynomial

$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

- parameters *(a_0,a_1)* from cross-channel fit to $\pi\pi$ GWQCD levels



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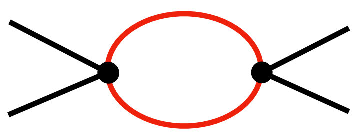

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3-BODY QUANTIZATION CONDITION

$$0 = \det\left[{\color{blue}B(s)} + {\color{red}C(s)} - 2L^3E_{\mathbf{p}}\left({\color{green}\tilde{K}_2^{-1}(s)} - {\color{purple}\Sigma_2^L(s)}\right)\right]_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

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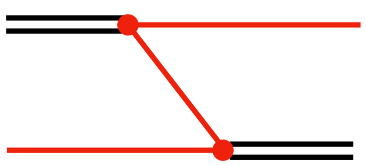
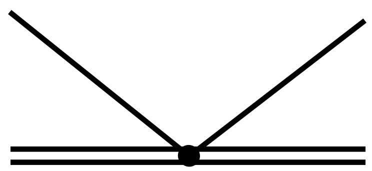


- no free parameters

<u>three-body force</u>

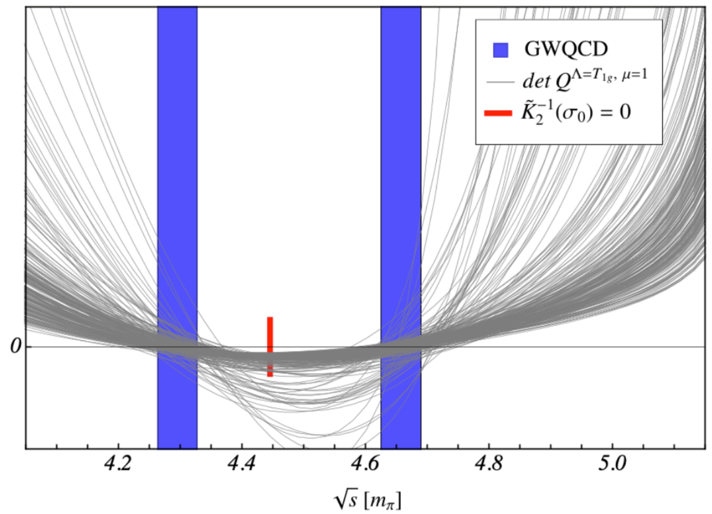
- dynamics of $\rho\pi$ system



- regular function ⇒ Laurent series

$$C_{\ell'\ell}(s,\mathbf{p}',\mathbf{p}) = \sum_{i=-1}^{\infty} c^{(i)}_{\ell'\ell}(\mathbf{p}',\mathbf{p})(\mathbf{s}-\mathbf{m}^2_{\mathbf{a_1}})^{\mathbf{i}}$$

- fit to 3-body levels



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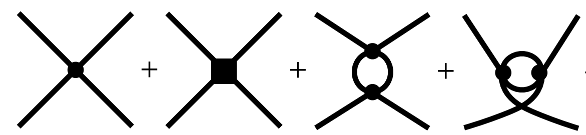

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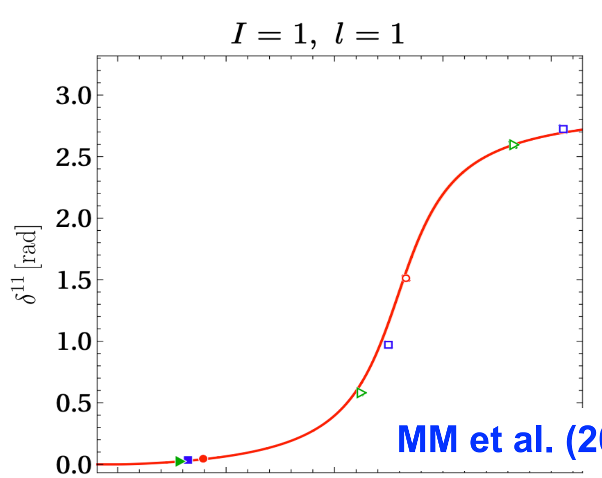

MM et al. (2019)

<u>two-body self-energy</u>

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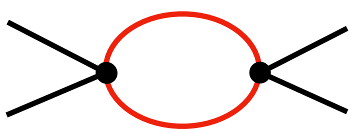

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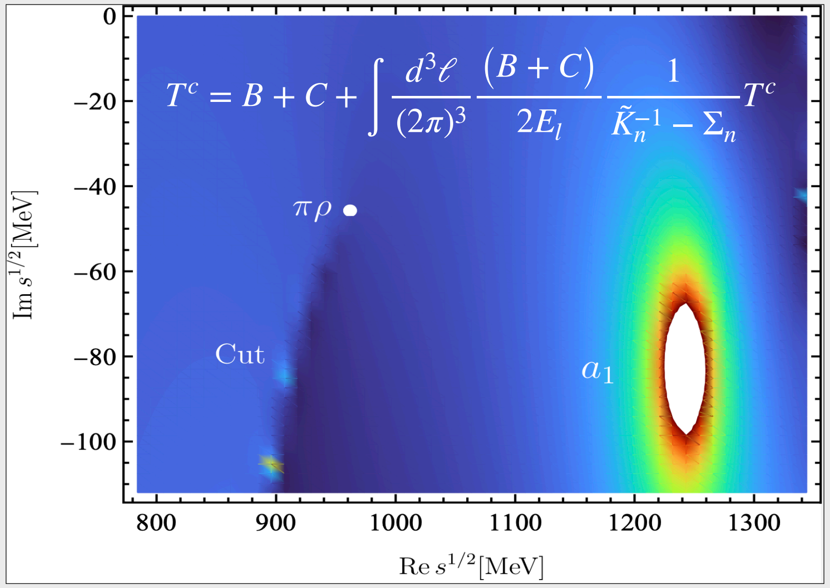
RESULTS

Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta Döring et al.(2009) Sadasivan et al. (2020)
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via $$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'}\frac{1}{\mathbf{s}-\mathbf{m}_{\mathbf{a_1}}^2}\mathbf{g}_\ell|\mathbf{p}|^\ell + \mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$

    ...with large correlations



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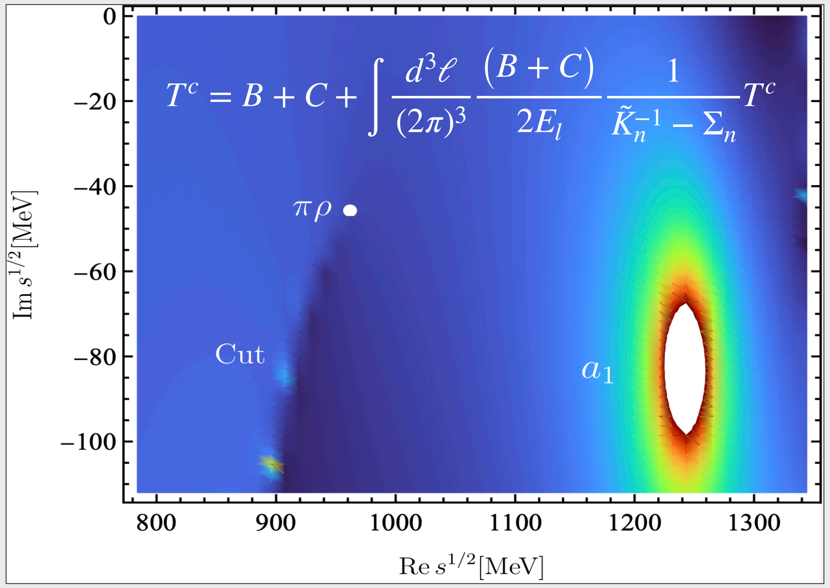

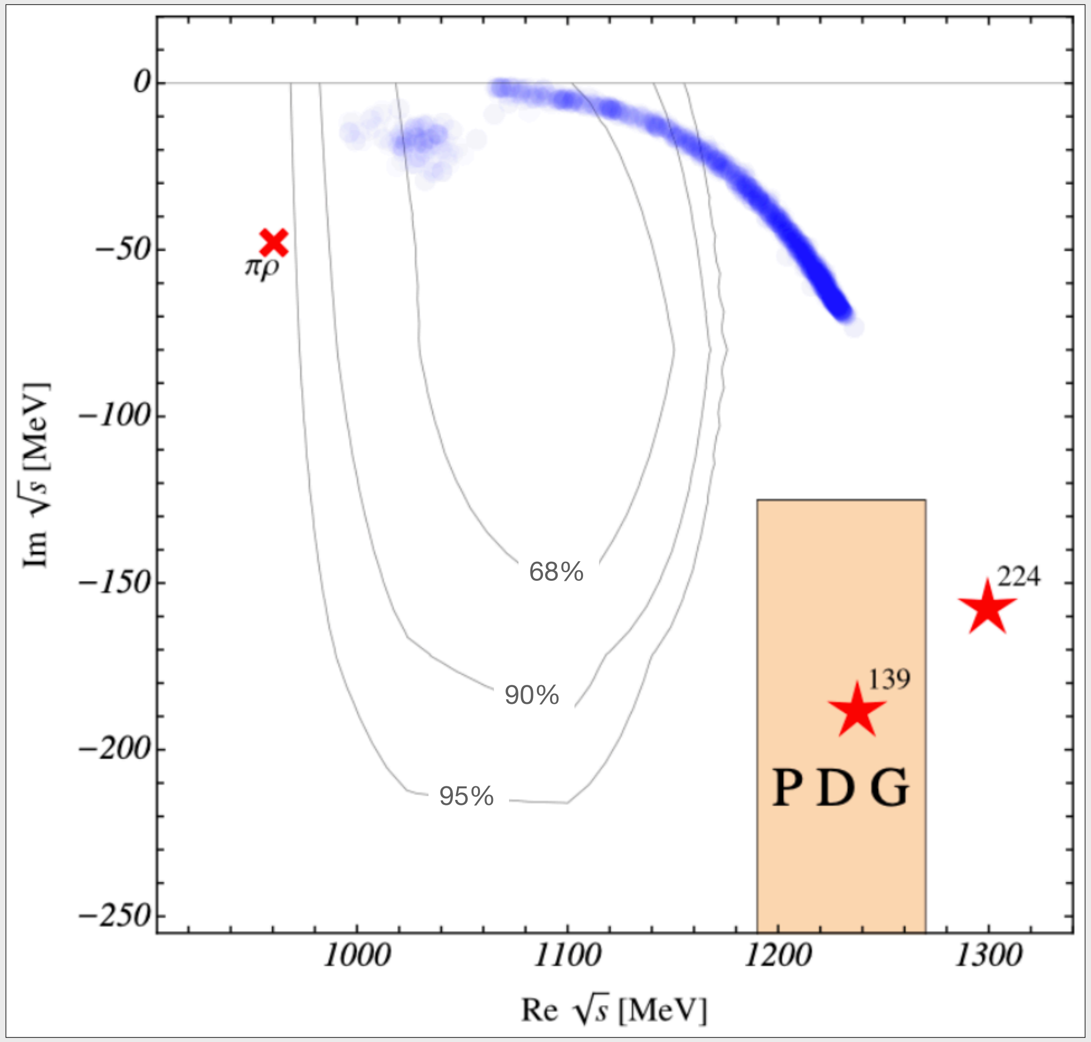

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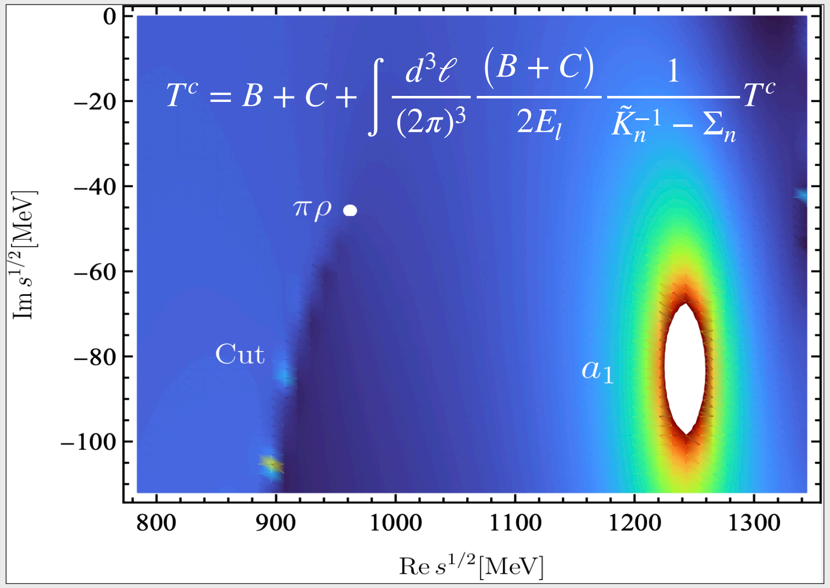

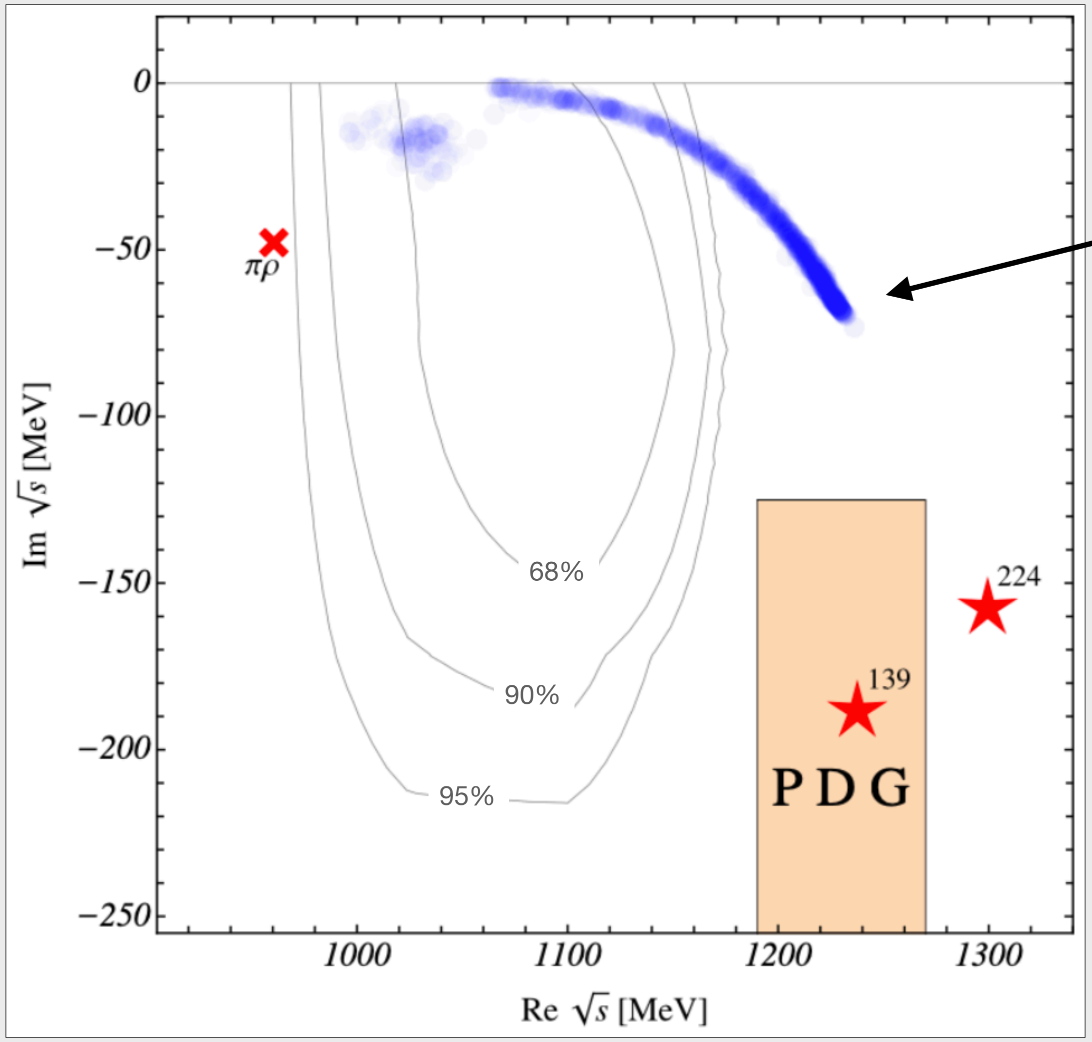

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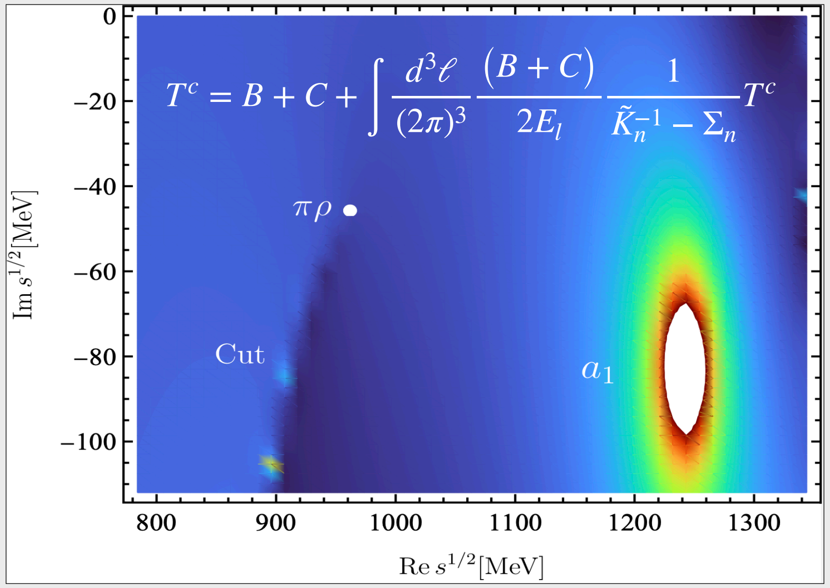

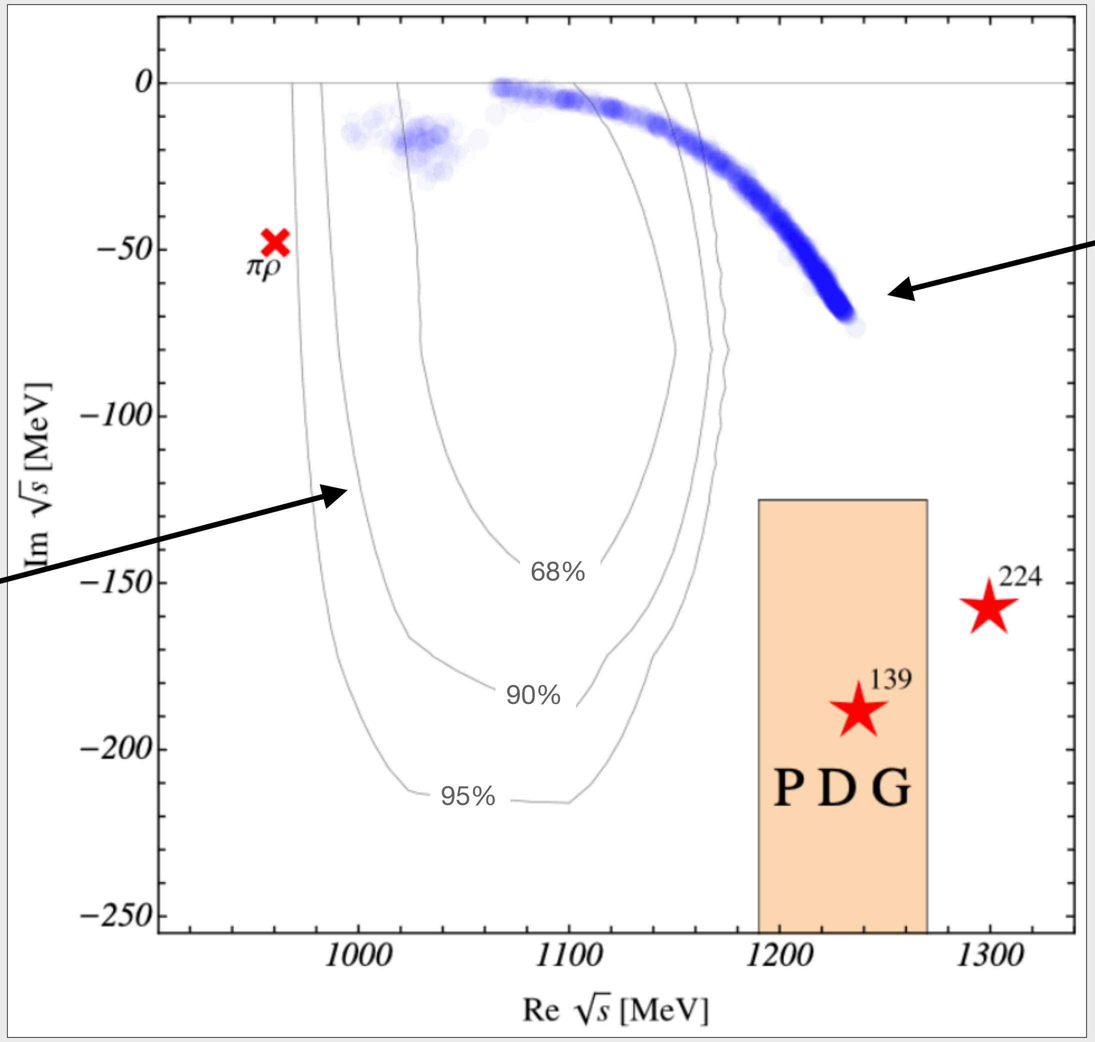

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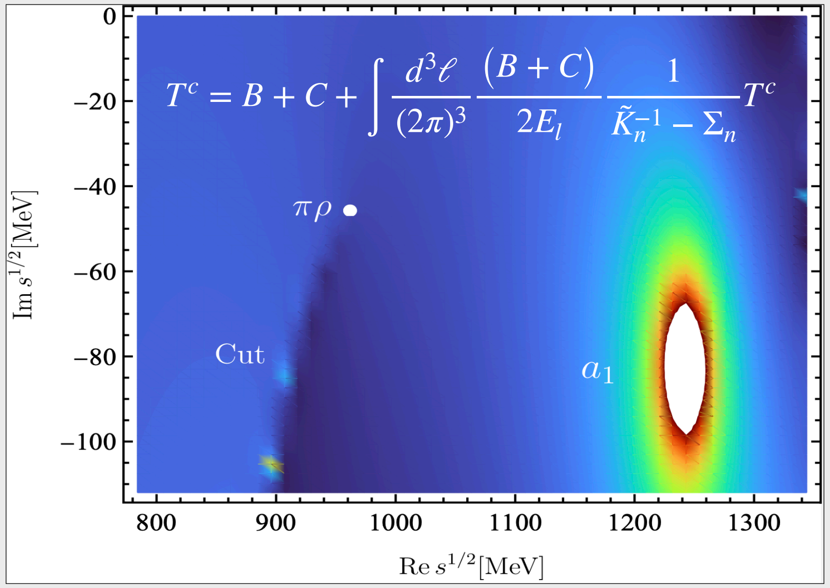

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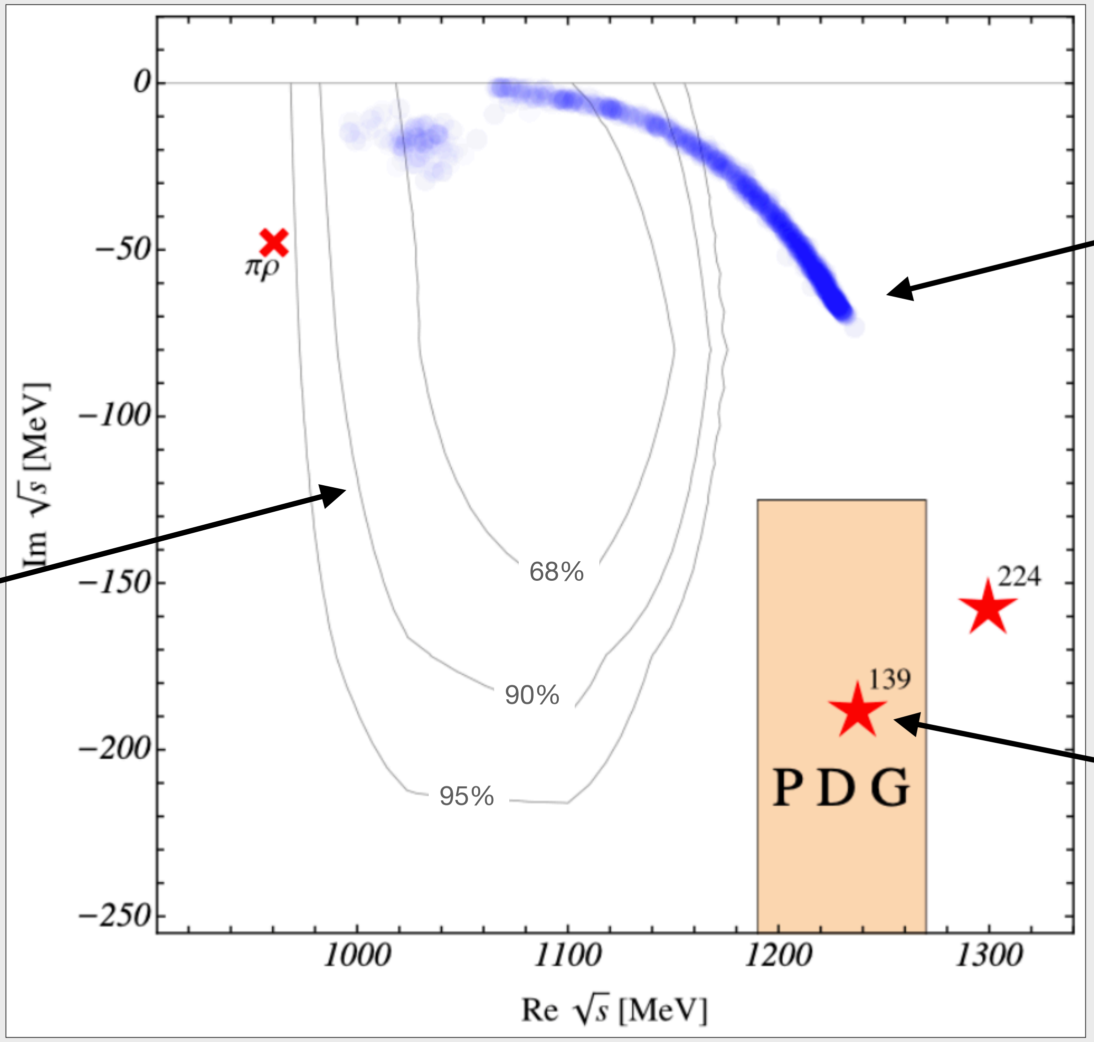

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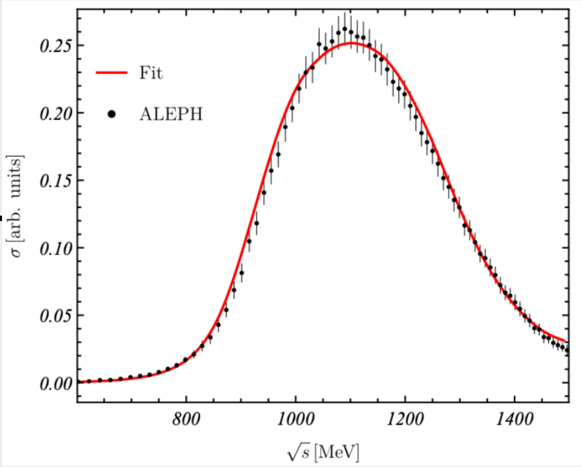

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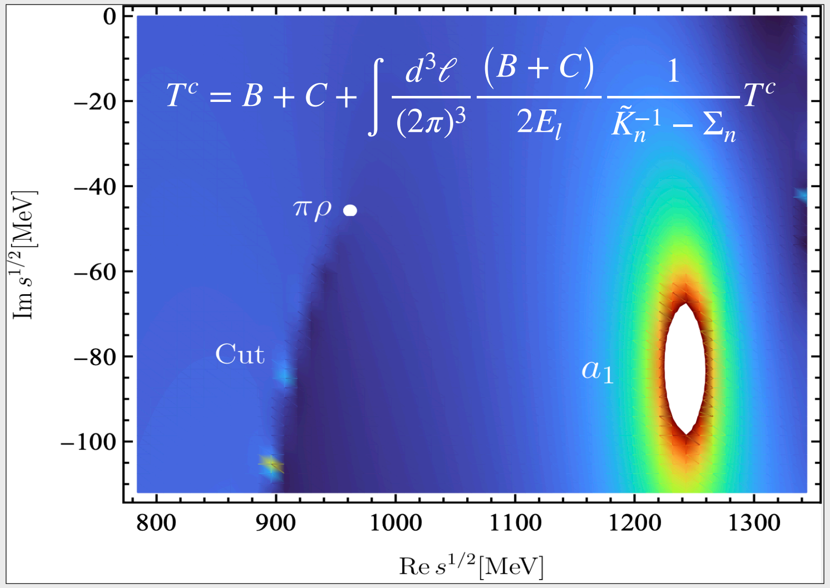

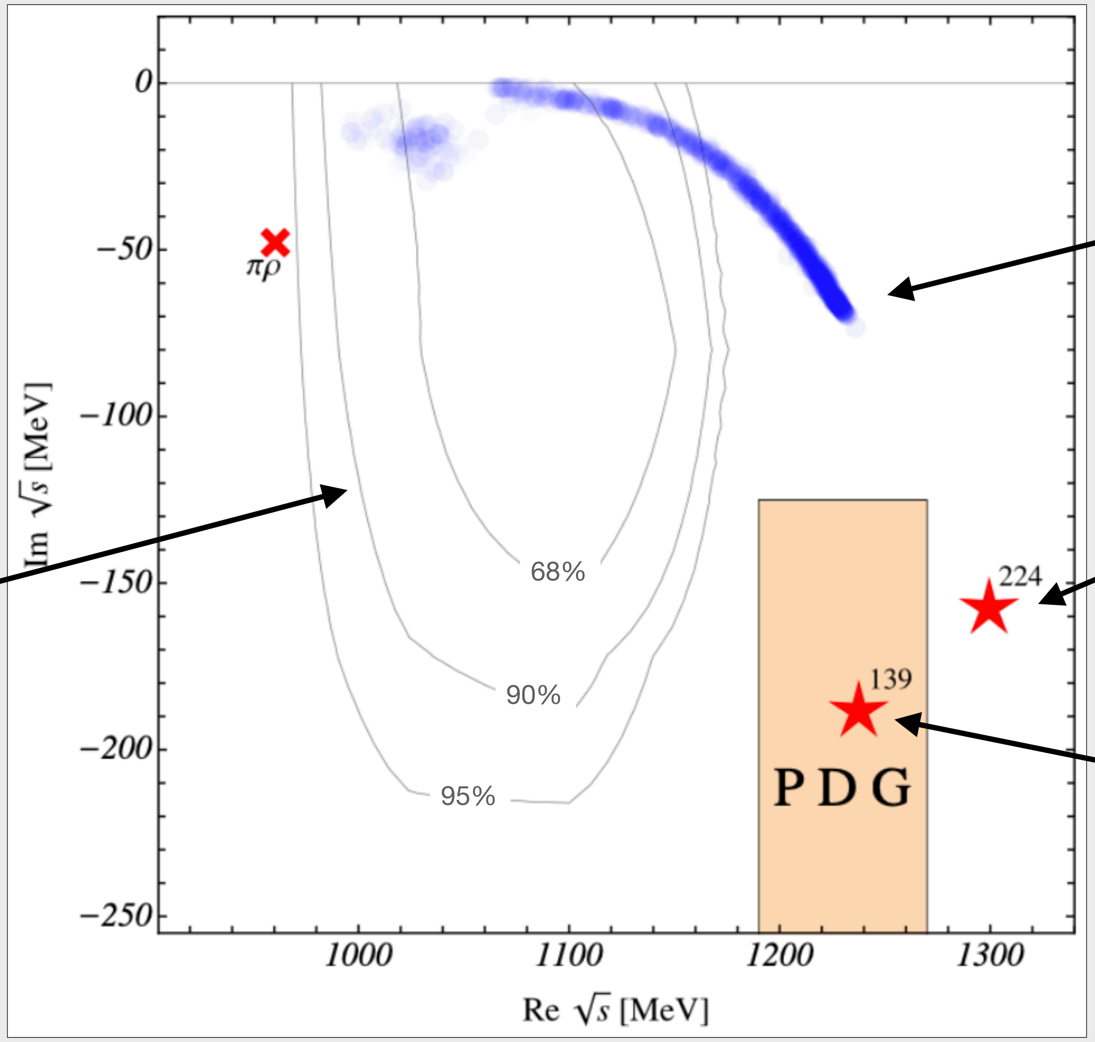

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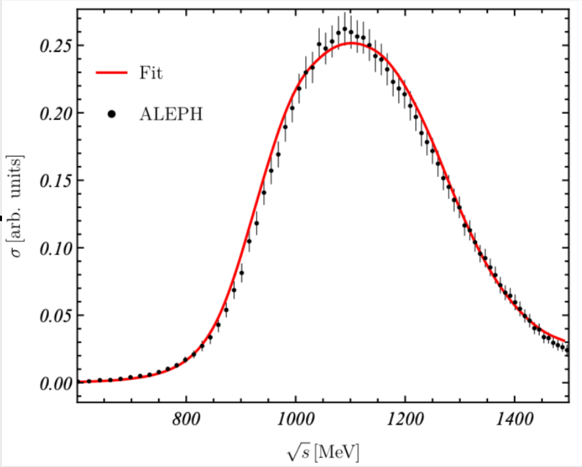

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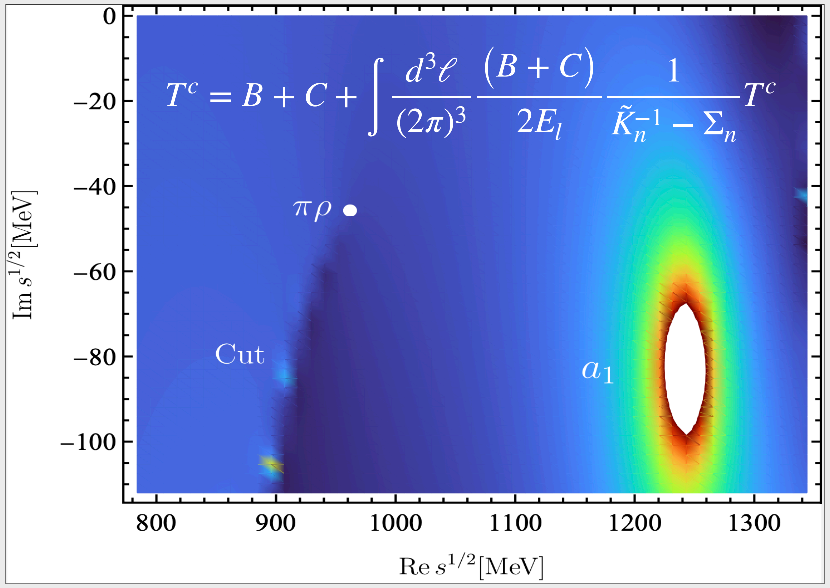

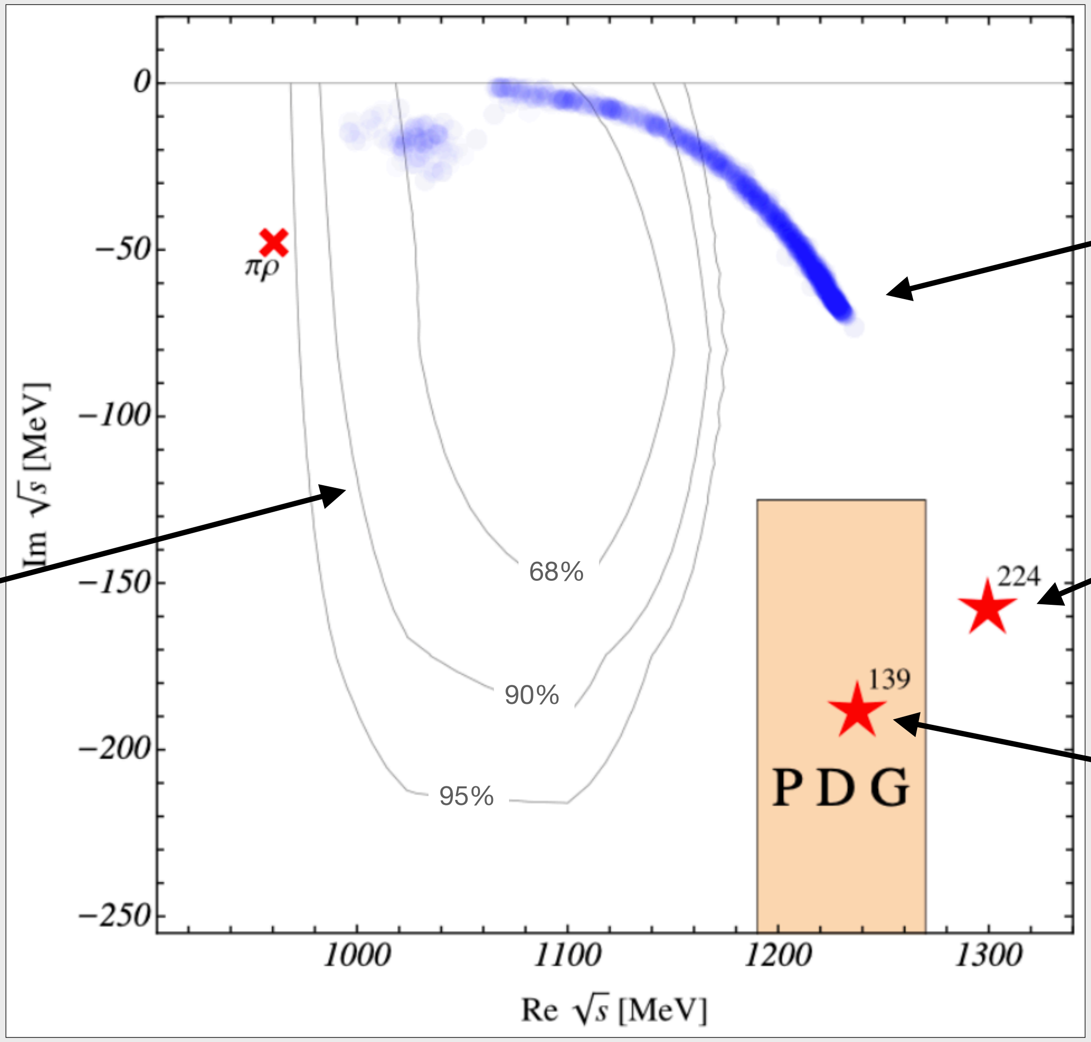

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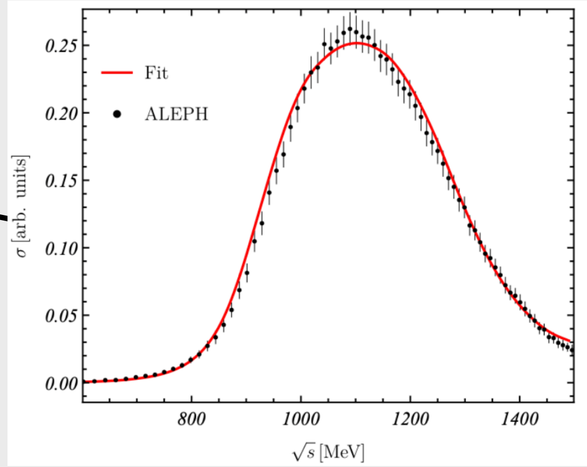

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