TUNNELING

Quantum Phenomena 1
xx.06.2023
Lecture Nr X

Maxim Mai

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Reminder from the last lecture

- Quantum theory is probabilistic:

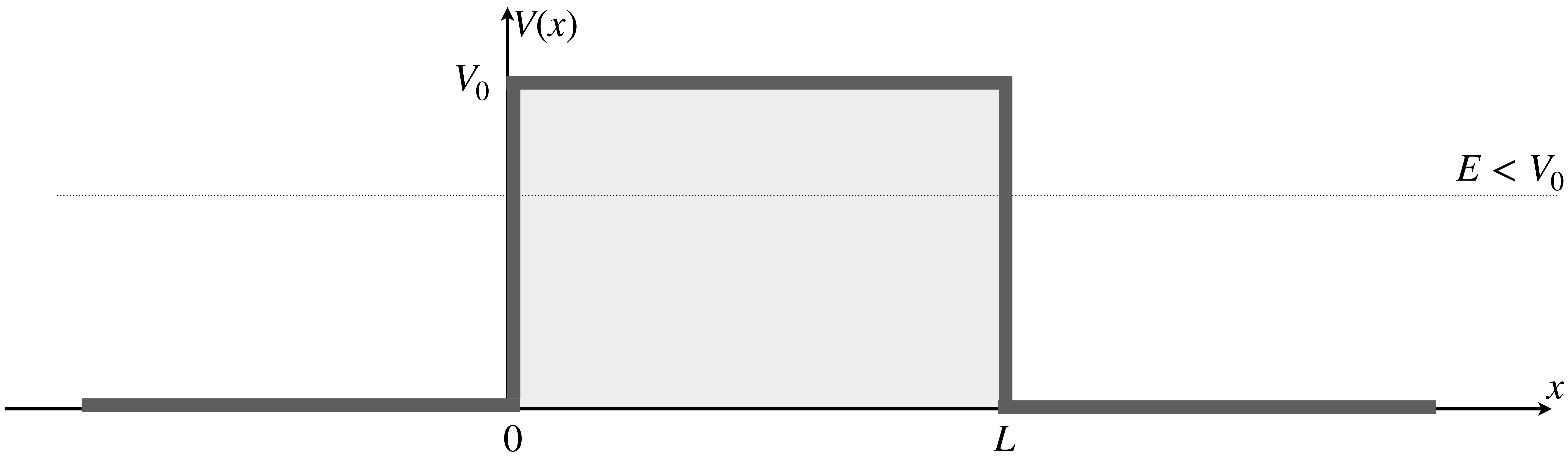
$$P(x) = |\psi(x)|^2$$

- Wave functions are smooth and continuous.

- Wave functions obey Schrödinger equation (time independent):

$$\frac{d^2}{dx^2}\psi(x) = -\frac{2m}{\hbar^2}\big(E - V(x)\big)\,\psi(x)$$

Potential barrier



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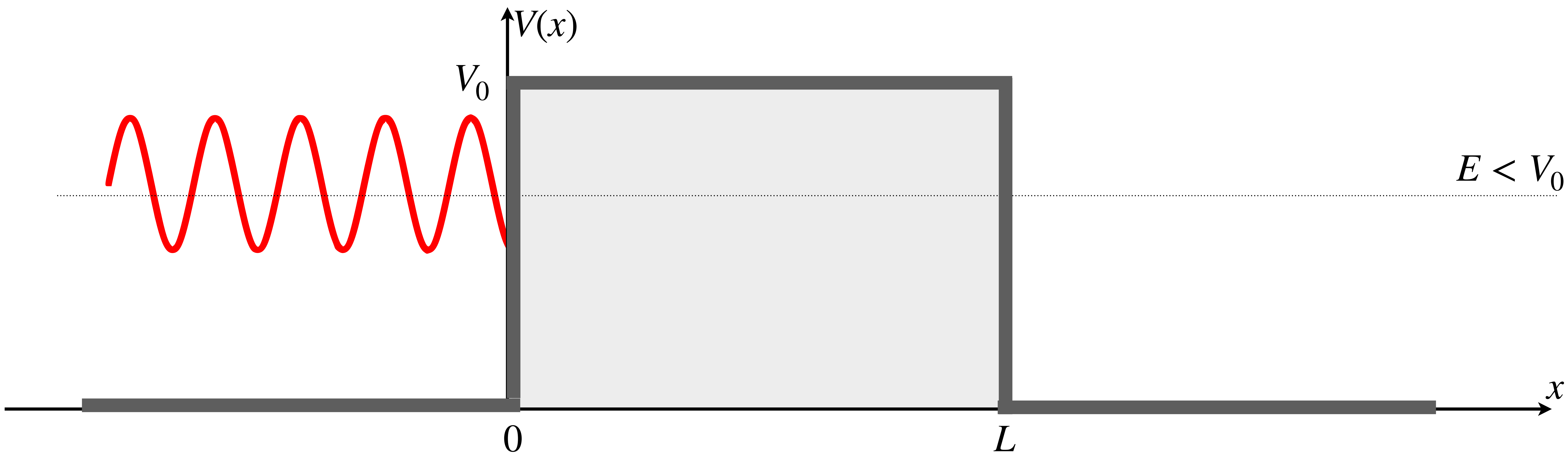

Solutions of the Schrödinger equation in each region

$$\Psi_{\mathrm{I}}(x) = A\, e^{ikx} + B\, e^{-ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

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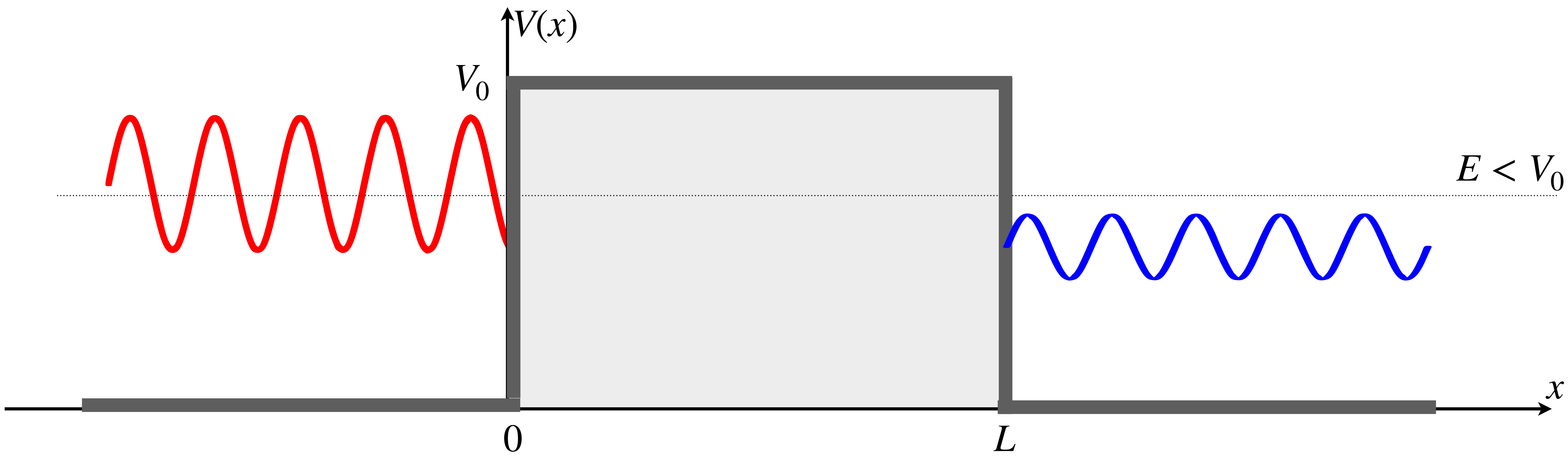

Solutions of the Schrödinger equation in each region

$$\Psi_{\mathrm{I}}(x) = A\, e^{ikx} + B\, e^{-ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\Psi_{\mathrm{III}}(x) = F\, e^{ikx} + G\, e^{-ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

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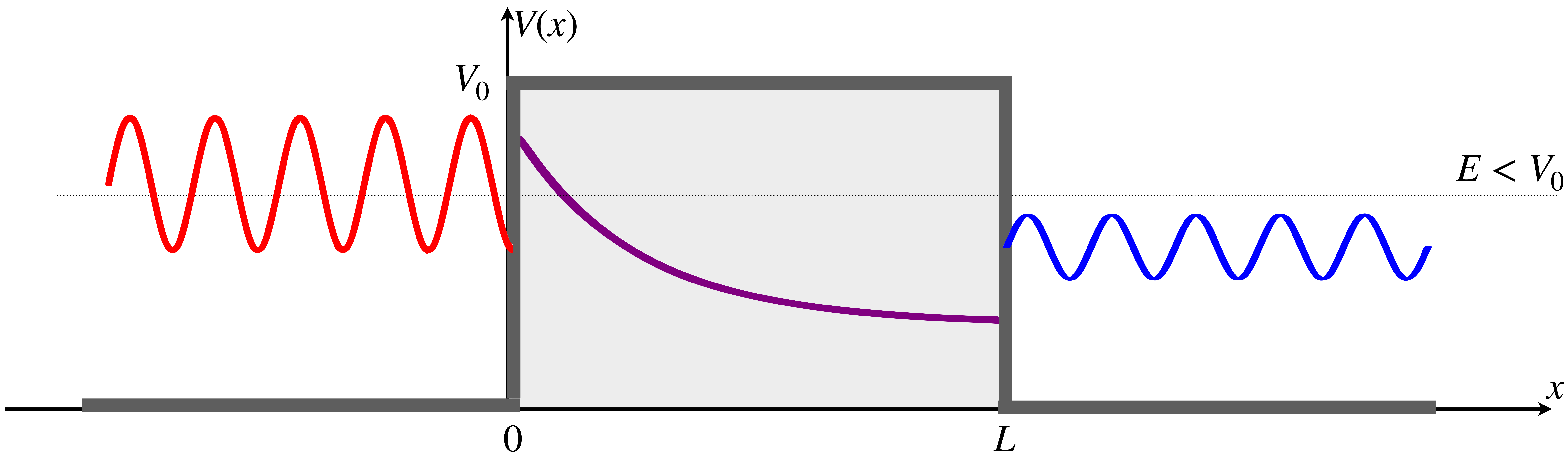

Solutions of the Schrödinger equation in each region

$$\Psi_{\mathrm{I}}(x) = A\,e^{ikx} + B\,e^{-ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

$$\Psi_{\mathrm{II}}(x) = C\,e^{\kappa x} + D\,e^{-\kappa x}$$

$$\kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

$$\Psi_{\mathrm{III}}(x) = F\,e^{ikx} + G\,e^{-ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar}$$

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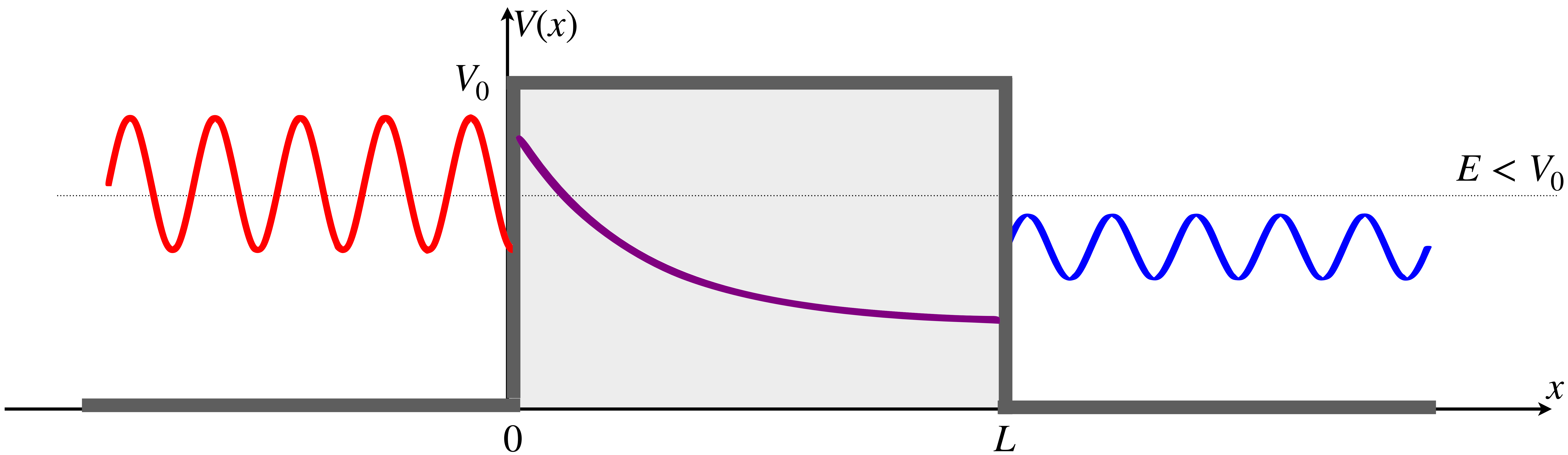

Continuity and smoothness

$$\Psi_{\mathrm{I}}(x) = A\,e^{ikx} + B\,e^{-ikx}$$

$$\Psi_{\mathrm{II}}(x) = C\,e^{\kappa x} + D\,e^{-\kappa x}$$

$$\Psi_{\mathrm{III}}(x) = F\,e^{ikx} + G\,e^{-ikx}$$

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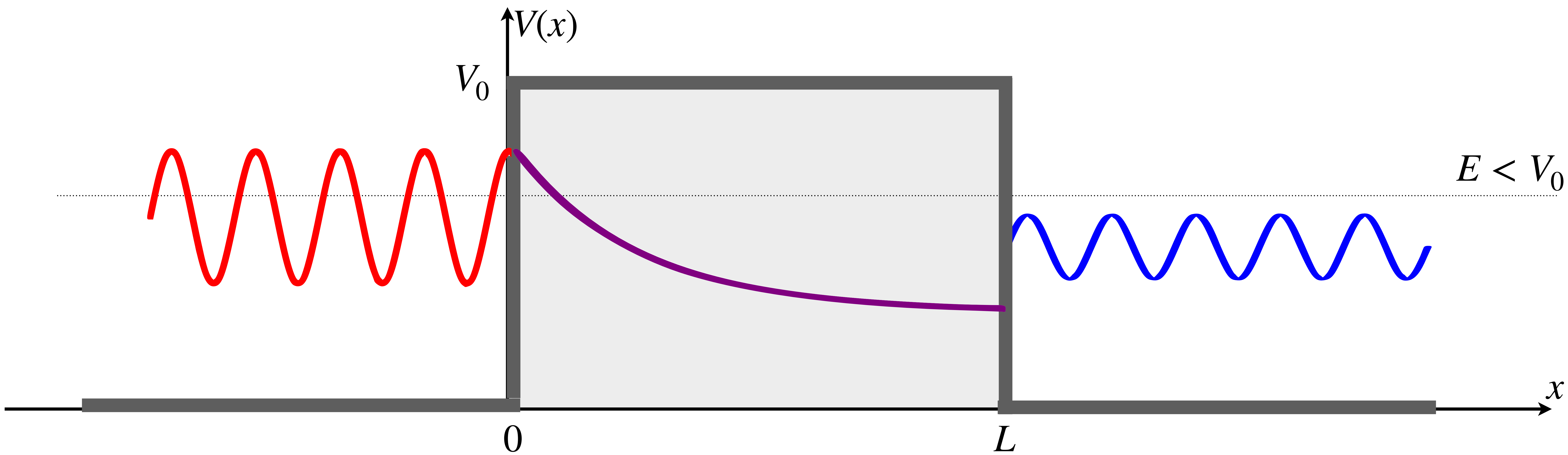

Continuity and smoothness

$$\Psi_{\mathrm{I}}(x) = A\, e^{ikx} + B\, e^{-ikx}$$

$$\Psi_{\mathrm{II}}(x) = C\, e^{\kappa x} + D\, e^{-\kappa x}$$

$$\Psi_{\mathrm{III}}(x) = F\, e^{ikx} + G\, e^{-ikx}$$

$$\left.\begin{array}{l} \Psi_{\mathrm{I}}(x) = \Psi_{\mathrm{II}}(x) \\ \Psi'_{\mathrm{I}}(x) = \Psi'_{\mathrm{II}}(x) \end{array}\right|_{x=0}$$

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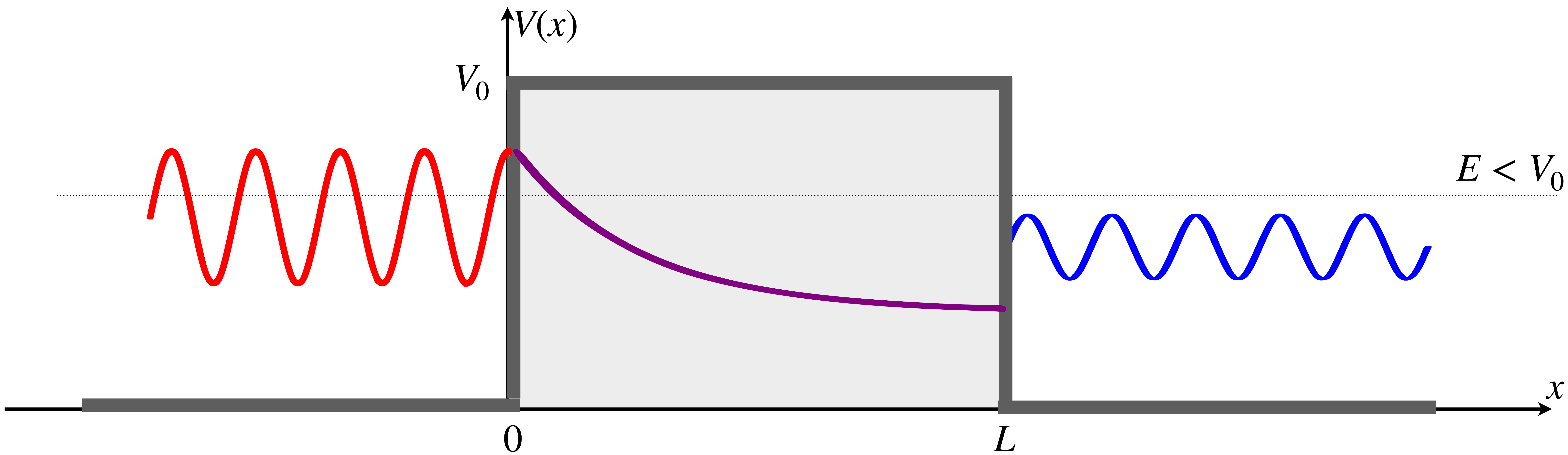

Continuity and smoothness

$\Psi_{\mathrm{I}}(x) = A\, e^{ikx} + B\, e^{-ikx}$ | $\Psi_{\mathrm{II}}(x) = C\, e^{\kappa x} + D\, e^{-\kappa x}$ | $\Psi_{\mathrm{III}}(x) = F\, e^{ikx} + G\, e^{-ikx}$

$$\left. \begin{aligned} \Psi_{\mathrm{I}}(x) &= \Psi_{\mathrm{II}}(x) \\ \Psi'_{\mathrm{I}}(x) &= \Psi'_{\mathrm{II}}(x) \end{aligned} \right|_{x=0}$$

$$\Downarrow$$

$$\begin{pmatrix} A + B = C + D \\ ik(A - B) = \kappa(C - D) \end{pmatrix}$$

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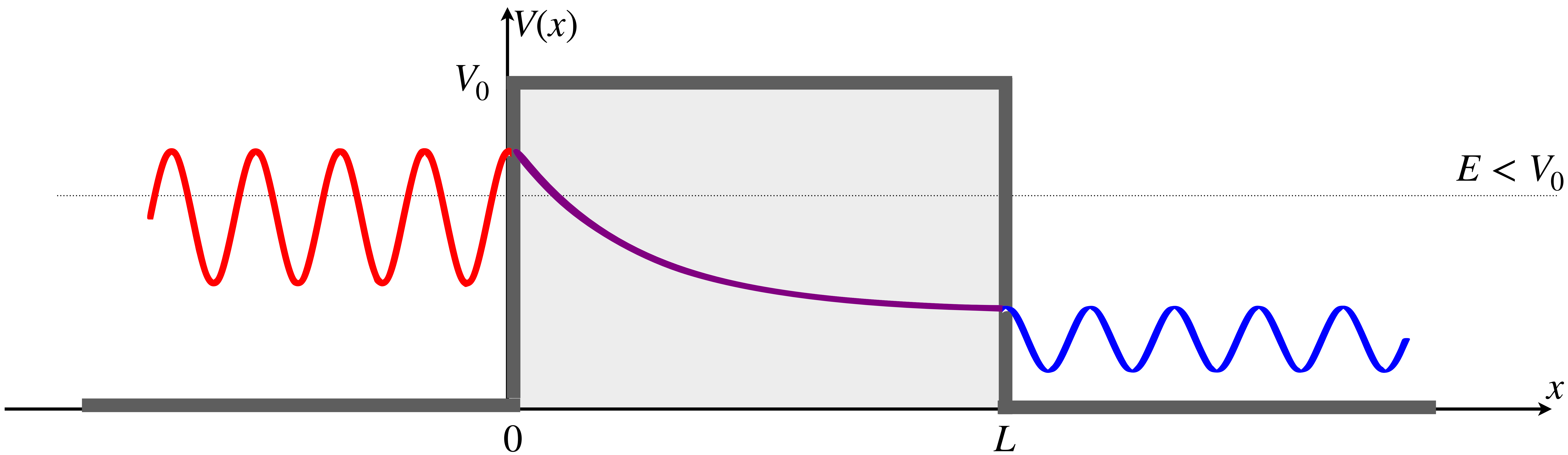

Continuity and smoothness

$$\Psi_{\mathrm{I}}(x) = A\,e^{ikx} + B\,e^{-ikx} \qquad \Psi_{\mathrm{II}}(x) = C\,e^{\kappa x} + D\,e^{-\kappa x} \qquad \Psi_{\mathrm{III}}(x) = F\,e^{ikx} + G\,e^{-ikx}$$

$$\left.\begin{aligned}\Psi_{\mathrm{I}}(x) &= \Psi_{\mathrm{II}}(x)\\ \Psi'_{\mathrm{I}}(x) &= \Psi'_{\mathrm{II}}(x)\end{aligned}\right|_{x=0} \qquad \left.\begin{aligned}\Psi_{\mathrm{II}}(x) &= \Psi_{\mathrm{III}}(x)\\ \Psi'_{\mathrm{II}}(x) &= \Psi'_{\mathrm{III}}(x)\end{aligned}\right|_{x=L}$$

$$\Downarrow$$

$$\begin{pmatrix} A + B = C + D \\ ik(A - B) = \kappa(C - D) \end{pmatrix}$$

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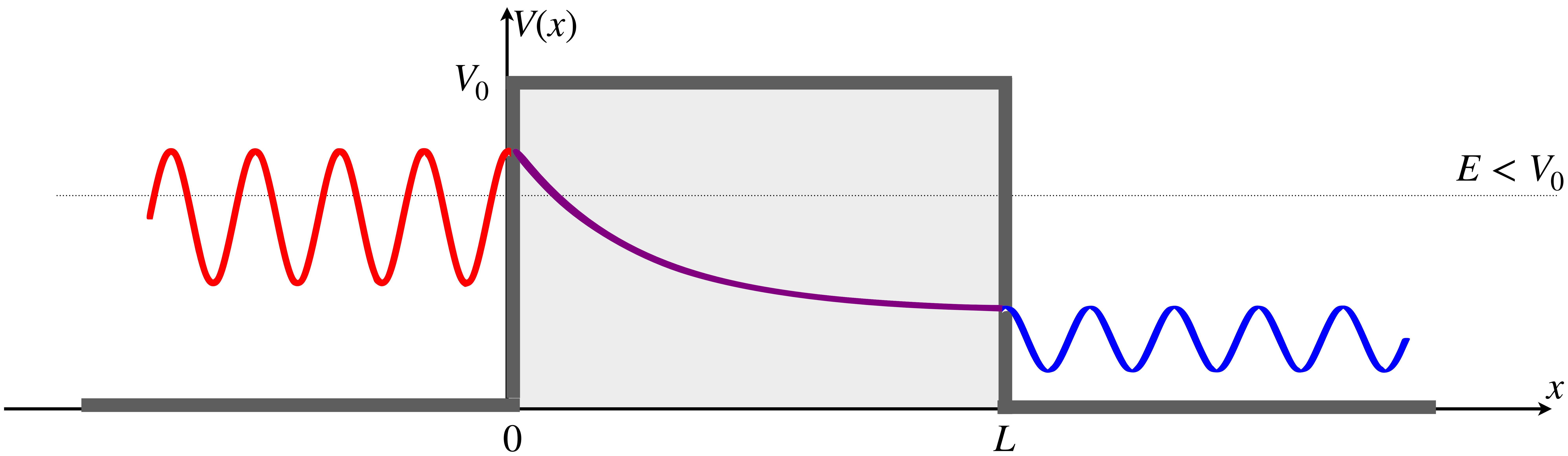

Continuity and smoothness

$$\Psi_{\mathrm{I}}(x) = A\,e^{ikx} + B\,e^{-ikx} \qquad \Psi_{\mathrm{II}}(x) = C\,e^{\kappa x} + D\,e^{-\kappa x} \qquad \Psi_{\mathrm{III}}(x) = F\,e^{ikx} + G\,e^{-ikx}$$

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$$\Downarrow \qquad\qquad \Downarrow$$

$$\begin{pmatrix} A + B = C + D \\ ik(A - B) = \kappa(C - D) \end{pmatrix} \qquad \begin{pmatrix} ..=.. \\ ..=.. \end{pmatrix}$$

Homework

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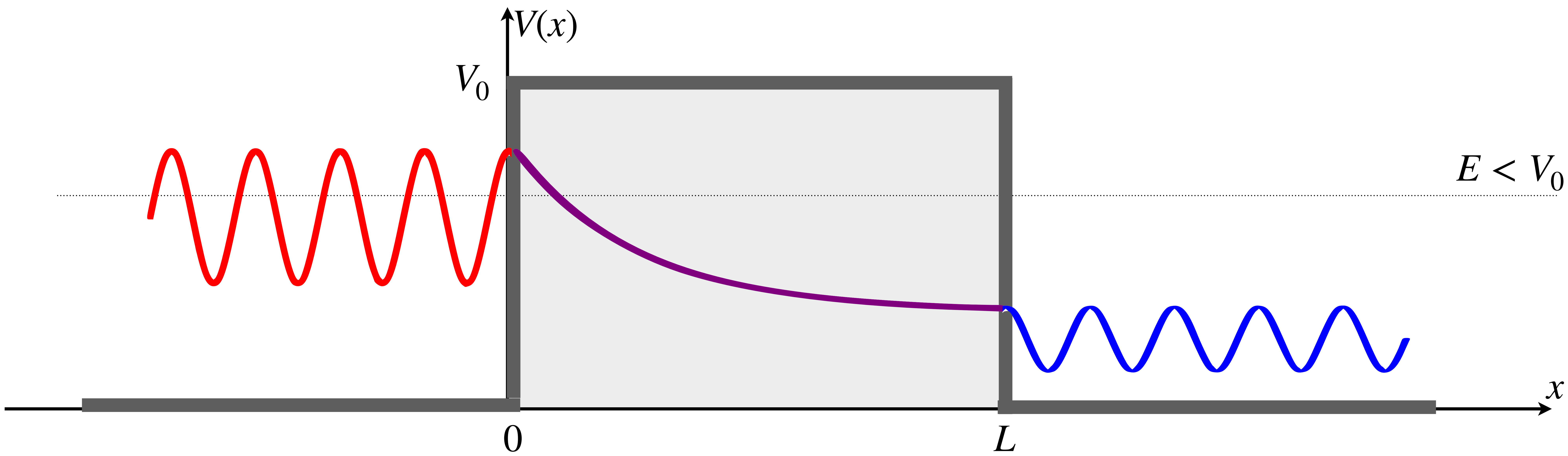

Discussion (non-classical effects)

- Probability for finding particle in the wall

$$P_\Psi(x) = |Ce^{\kappa x} + De^{-\kappa x}|^2 > 0 \quad \text{for } x \in [0,L)$$

- Probability for "*tunneling*"

$$P_\Psi(x) \sim e^{-2\kappa L} > 0 \quad \text{for } x \in [L,\infty)$$

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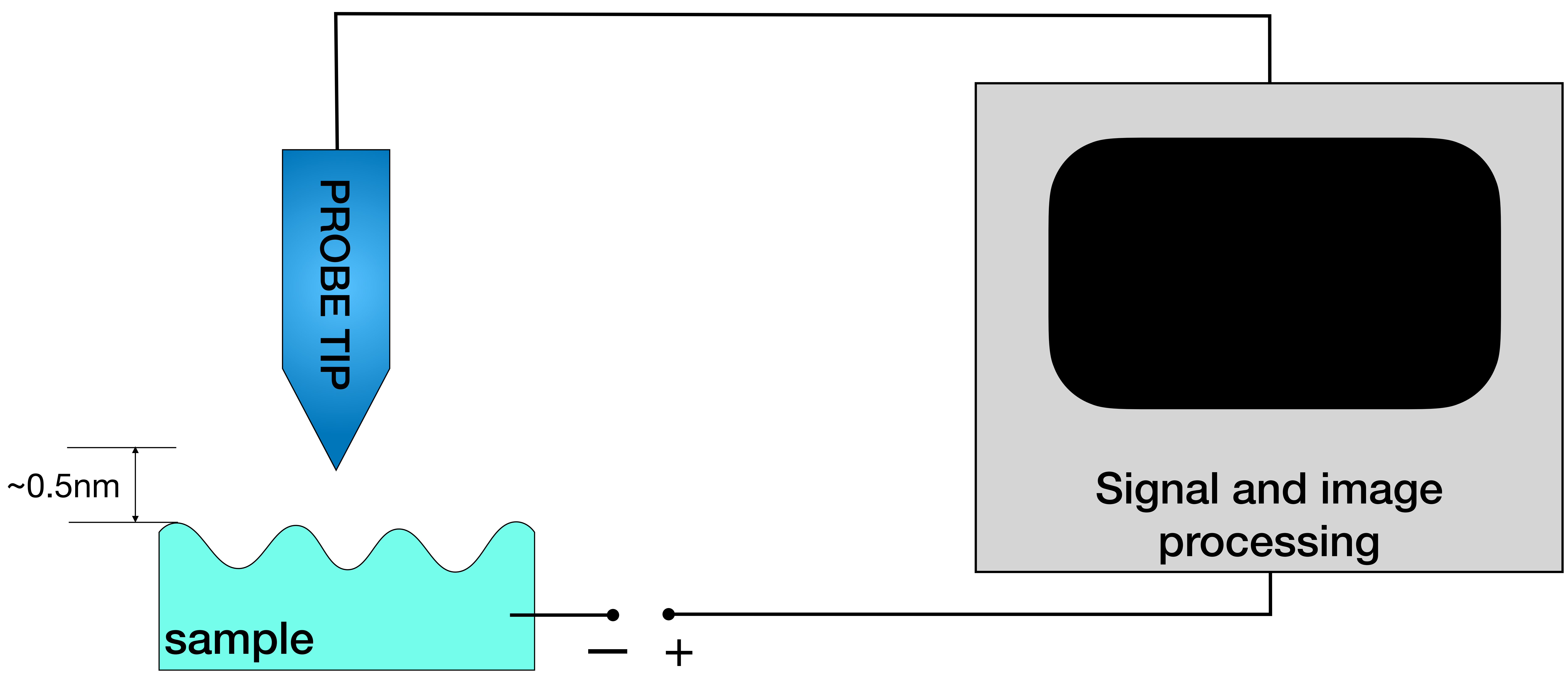
APPLICATIONS

- α-decay/electronic circuit components/…
- Scanning tunneling microscope (STM)

Binning and Rohrer, Nobel Prize in Physics in 1986



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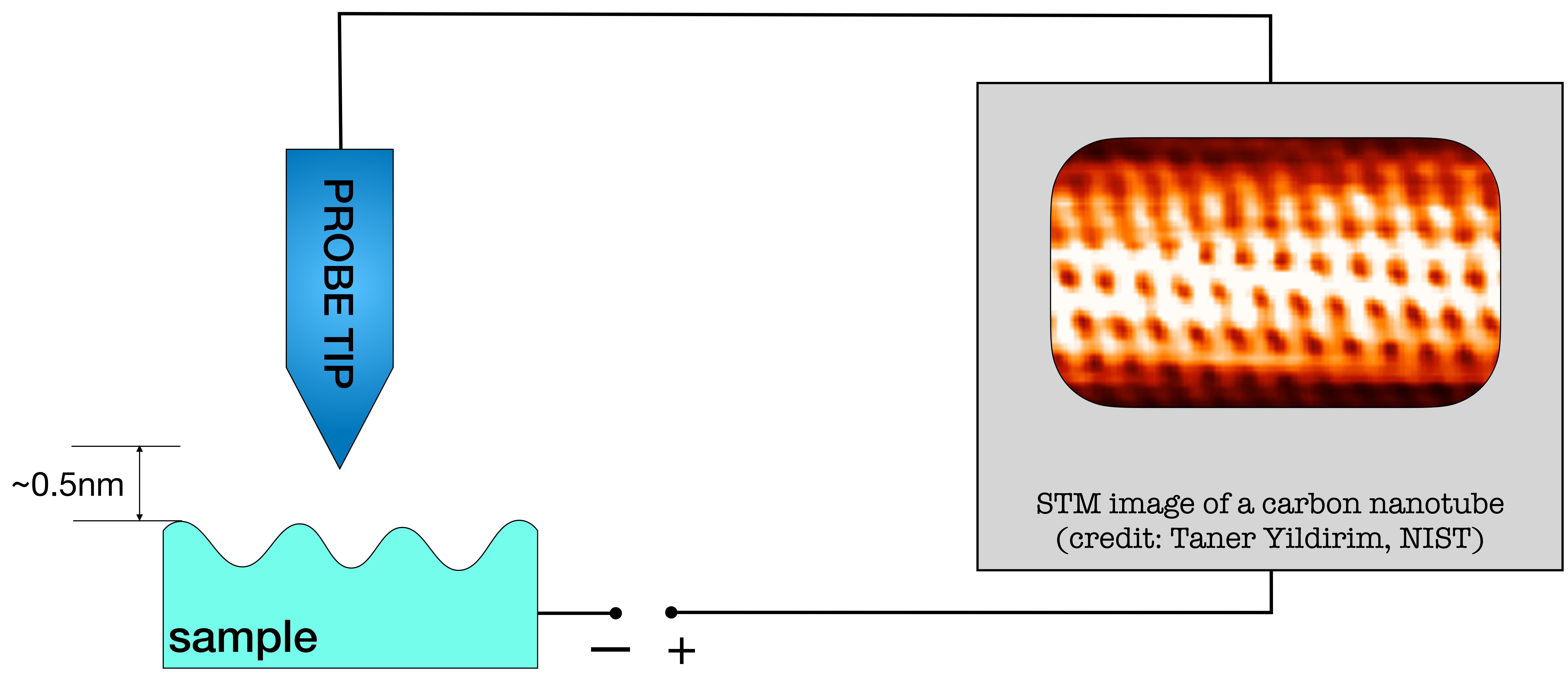

STM image of a carbon nanotube (credit: Taner Yildirim, NIST)

Further examples: https://en.wikipedia.org/wiki/Scanning_tunneling_microscope