



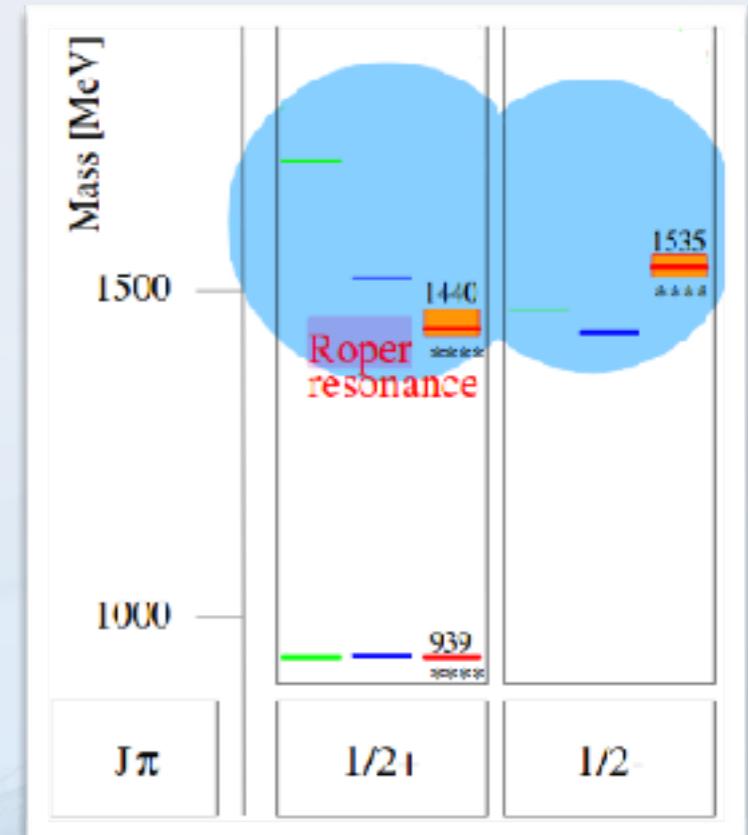
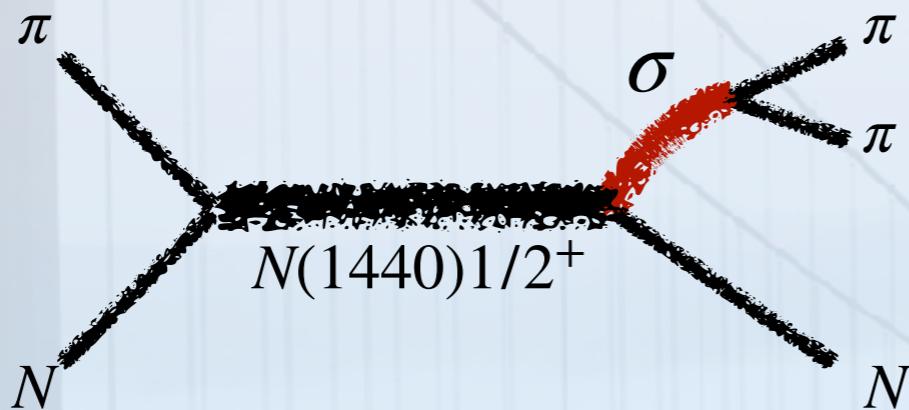
Bridging phenomenology & lattice QCD in the 3-body sector

Maxim Mai

MOTIVATION: QCD PUZZLES

◎ Roper-puzzle

- reversed mass pattern cf. constituent Quark Model
- large coupling to $\pi\pi N$ channels



Loring et al. EPJA10 (2001)

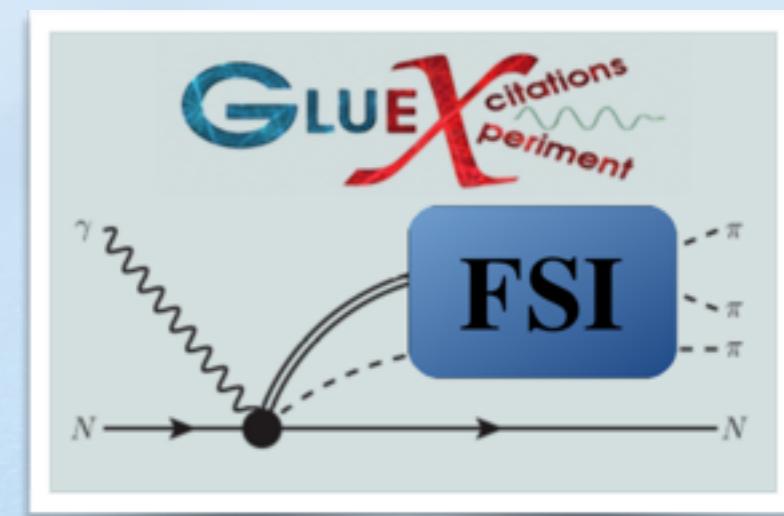
◎ $a_1(1260)$ & spin-exotics (GlueX, COMPASS, BESIII)

- indicator for the importance of gluonic d.o.f.
- cannot decay into $\pi\pi$ but only $\pi\pi\pi$ channel

◎ $X(3872)$

- decays dominantly into $D\bar{D}\pi$

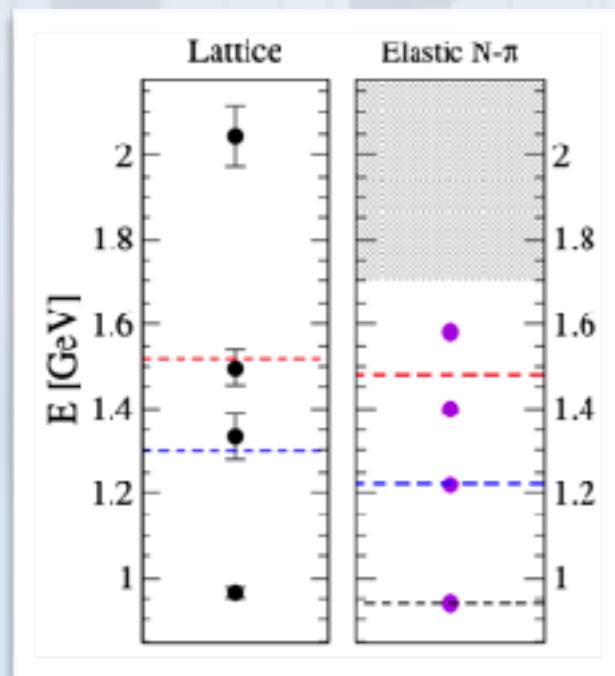
◎ ...



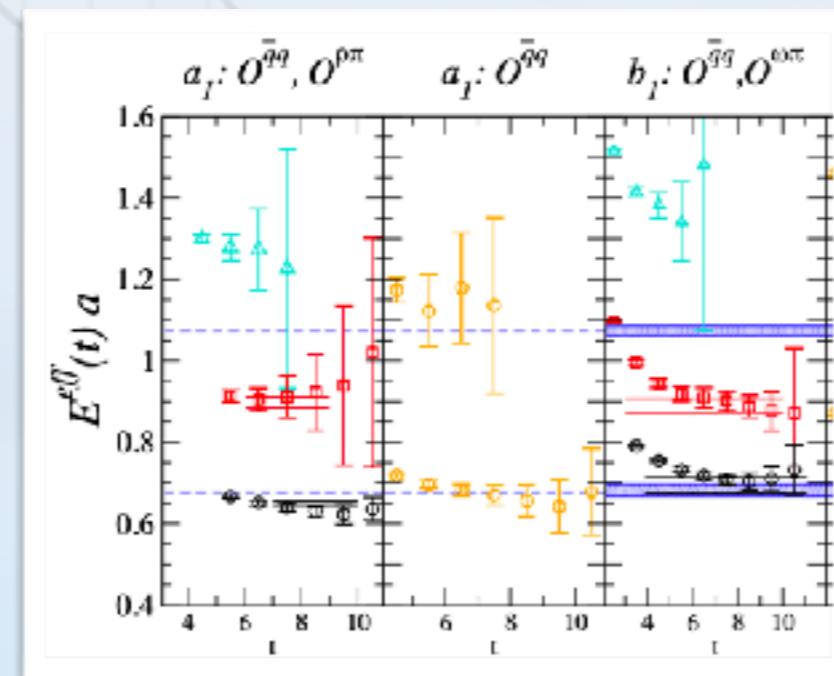
AB-INITIO APPROACH: LATTICE QCD

- first Lattice results are available

◎ $N(1440)1/2^+$

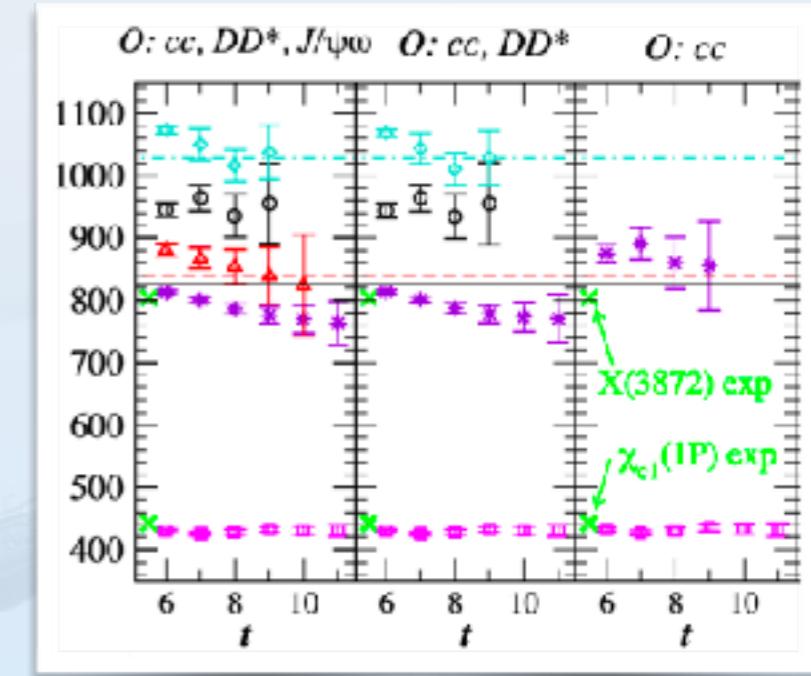


Lang et al. PRD 95(2017)



Lang et al. JHEP 1404

◎ $X(3872)$



Prelovsek, Leskovec PRL111 (2013)

◎ $I=2 \pi\rho$

Woss et al. JHEP 1807 [Hadspec]

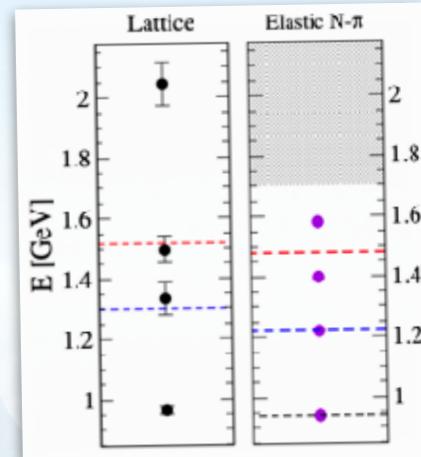
◎ $I=3 \pi\pi\pi \rightarrow$ brand new calculations

Hoerz/Hanlon (2019)

◎ ... more to come

Lattice QCD:

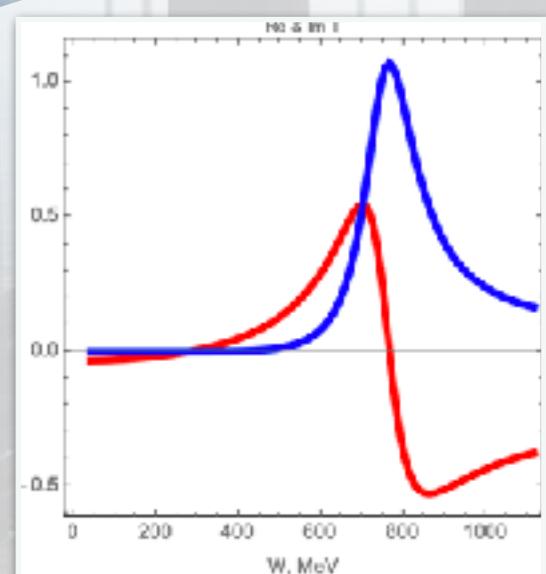
Euclidean Space-time in **finite volume**
→ results are **real and discrete!**



3-BODY QUANTIZATION CONDITION

Infinite volume world = phenomenology:

Universal properties of hadronic systems encoded in the **S-matrix**
→ **complex and continuous**



STATE OF THE ART

● 2-body case – Lüscher's method

- one-to-one mapping (also with extensions to multi-channels, spin, ...)

Gottlieb, Rummukainen, Feng, Meißner, Li, Liu, Doring, Briceno, Rusetsky, Bernard...

● 3-body case

- presumably no one-to-one mapping:
complex kinematics (8 variables)
sub-channel dynamics
- theoretical developments and pilot numerical investigation

Sharpe, Hansen, Briceno, Hammer, Rusetsky, Polejaeva, Griesshammer, Davoudi, Guo...

Sharpe, Hansen, Briceno, Hammer, Rusetsky, Polejaeva, Griesshammer, Davoudi, Guo... → MM, Doring(2017)

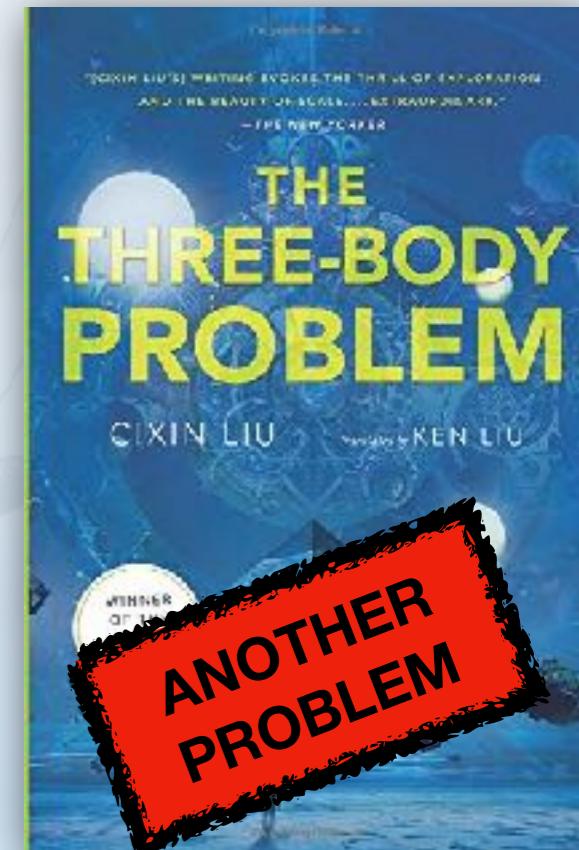
Sharpe, Hansen, Briceno, Hammer, Rusetsky, Polejaeva, Griesshammer, Davoudi, Guo... → Pang, Hammer, Rusetsky, Wu(2017)

Sharpe, Hansen, Briceno, Hammer, Rusetsky, Polejaeva, Griesshammer, Davoudi, Guo... → Hansen, Briceno, Sharpe(2018)

Doring, Hammer, MM, Pang, Rusetsky, Wu (2018)

Blanton, Romero-López, Sharpe (2018)

Pang, Wu, Hammer, Meißner, Rusetsky (2019)



- first application to Lattice QCD results :

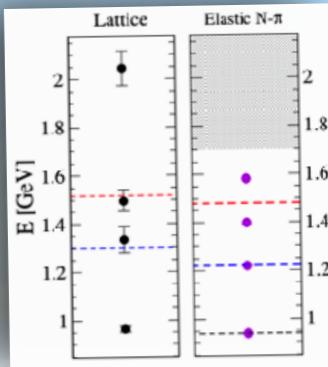
finite volume ($\pi^+\pi^+$) and ($\pi^+\pi^+\pi^+$) spectrum

MM, Doering (2018) → PRL122 (2019)

- analysis of 2019 Hoerz/Hanlon ($\pi^+\pi^+\pi^+$) spectrum (boosted systems/...)

MM, Alexandru, Culver, Doering [1909.05749] Blanton, Romero-López, Sharpe [1909.02973]

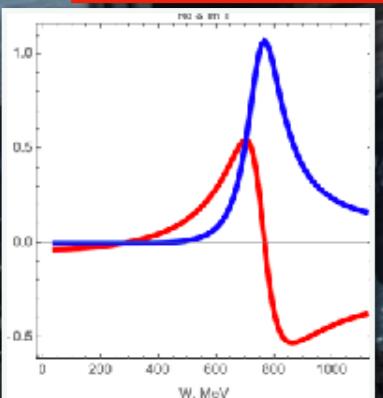
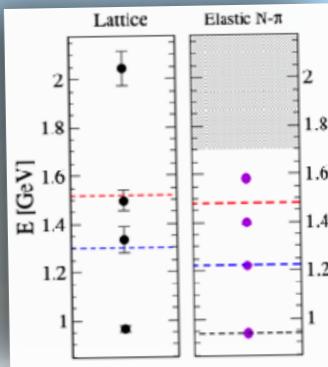
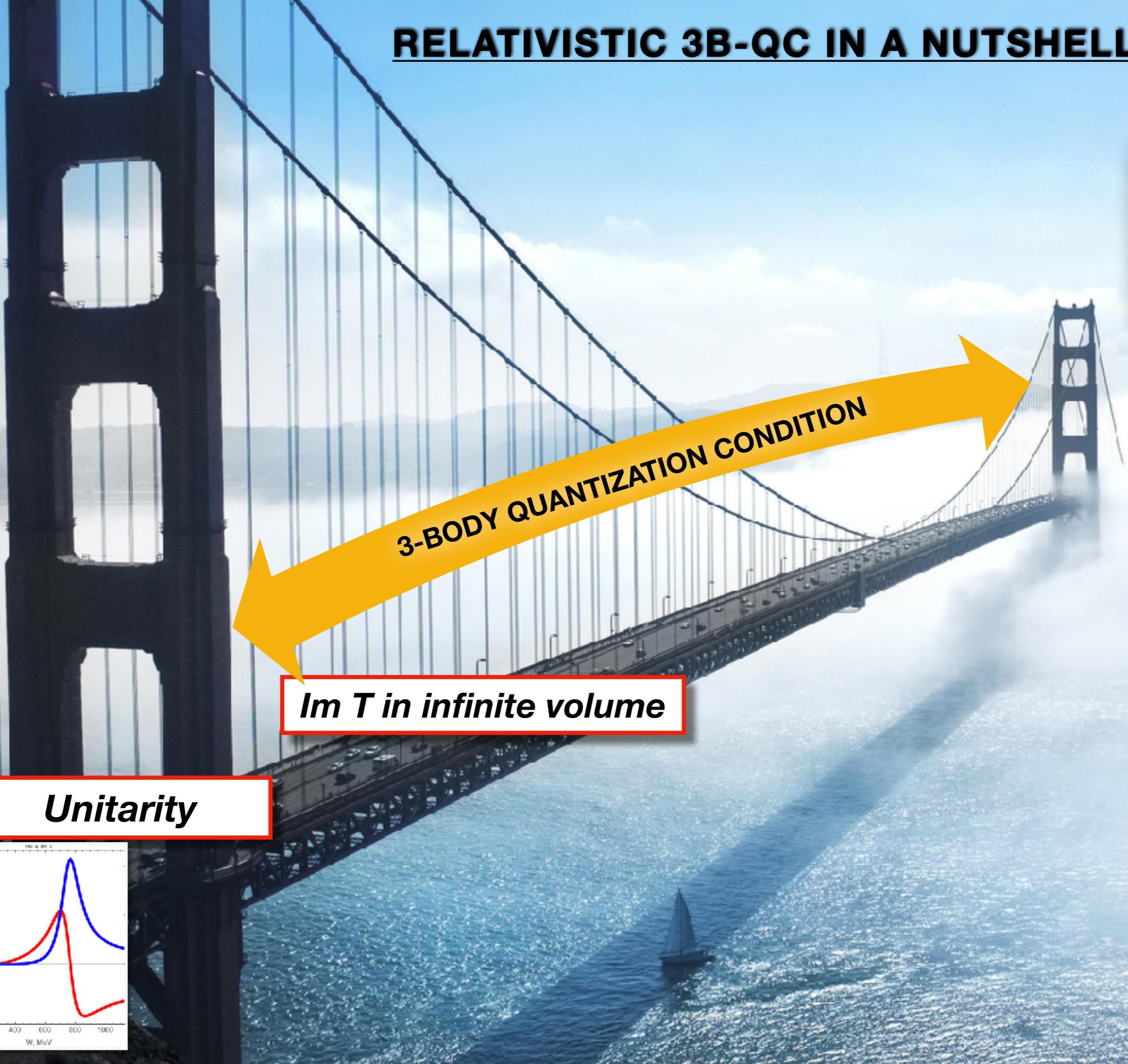
RELATIVISTIC 3B-QC IN A NUTSHELL



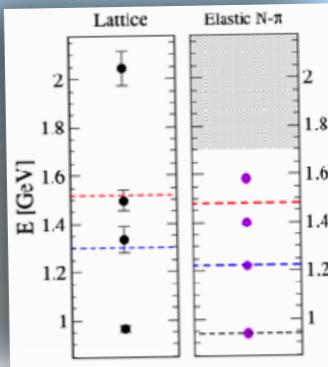
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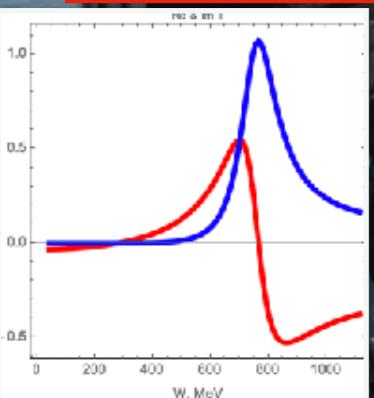


3-BODY QUANTIZATION CONDITION

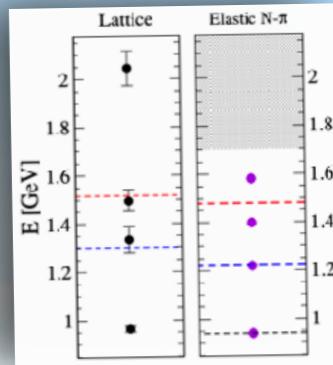
on-shell configurations

Im T in infinite volume

Unitarity



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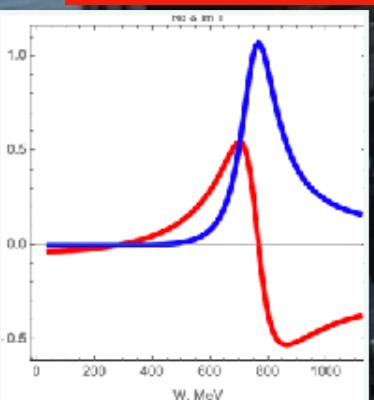
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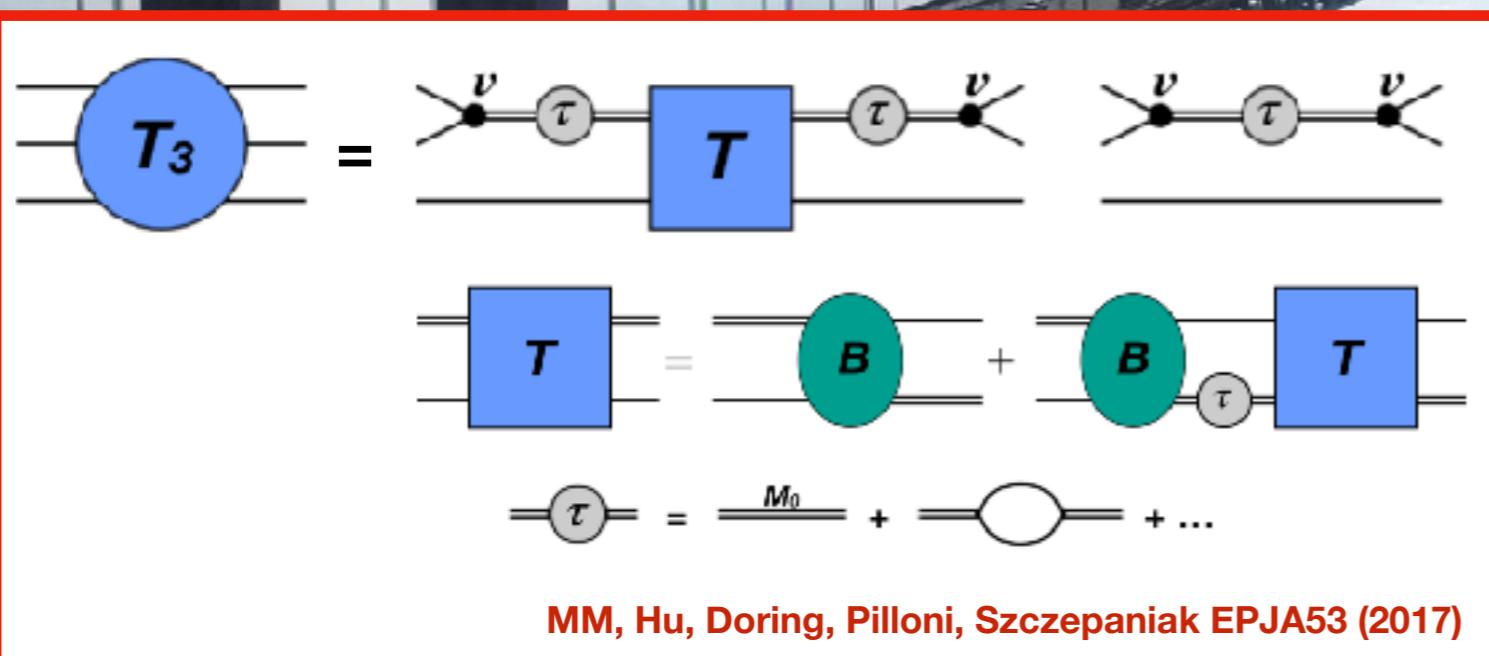
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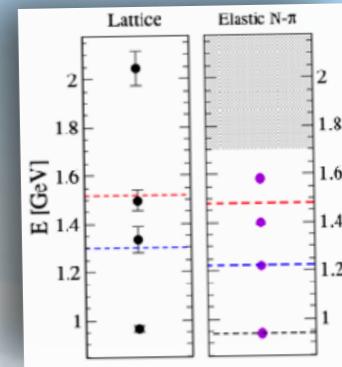
*Power-law
finite-volume effects*



RELATIVISTIC 3B-QC IN A NUTSHELL

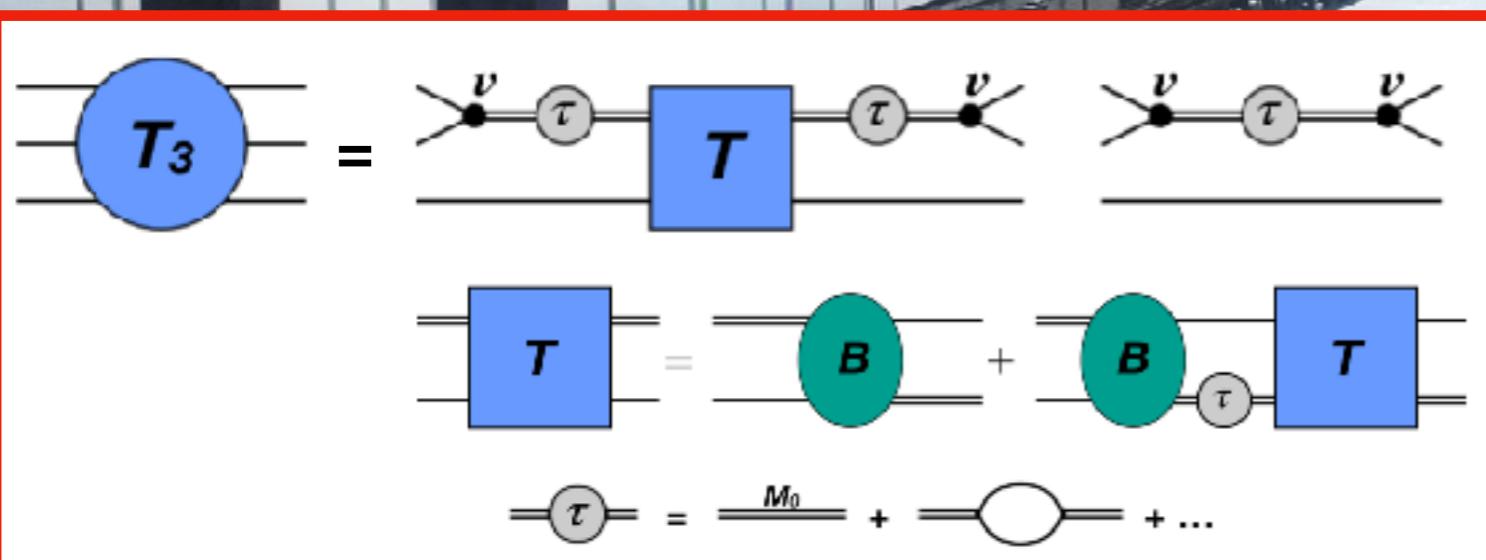


RELATIVISTIC 3B-QC IN A NUTSHELL



$$\text{Det}\left(B^{ss'}(W^2) + \tau_s^{-1}(W^2, L) \delta_{ss'}\right) = 0$$

MM, Doring EPJA53 (2017)



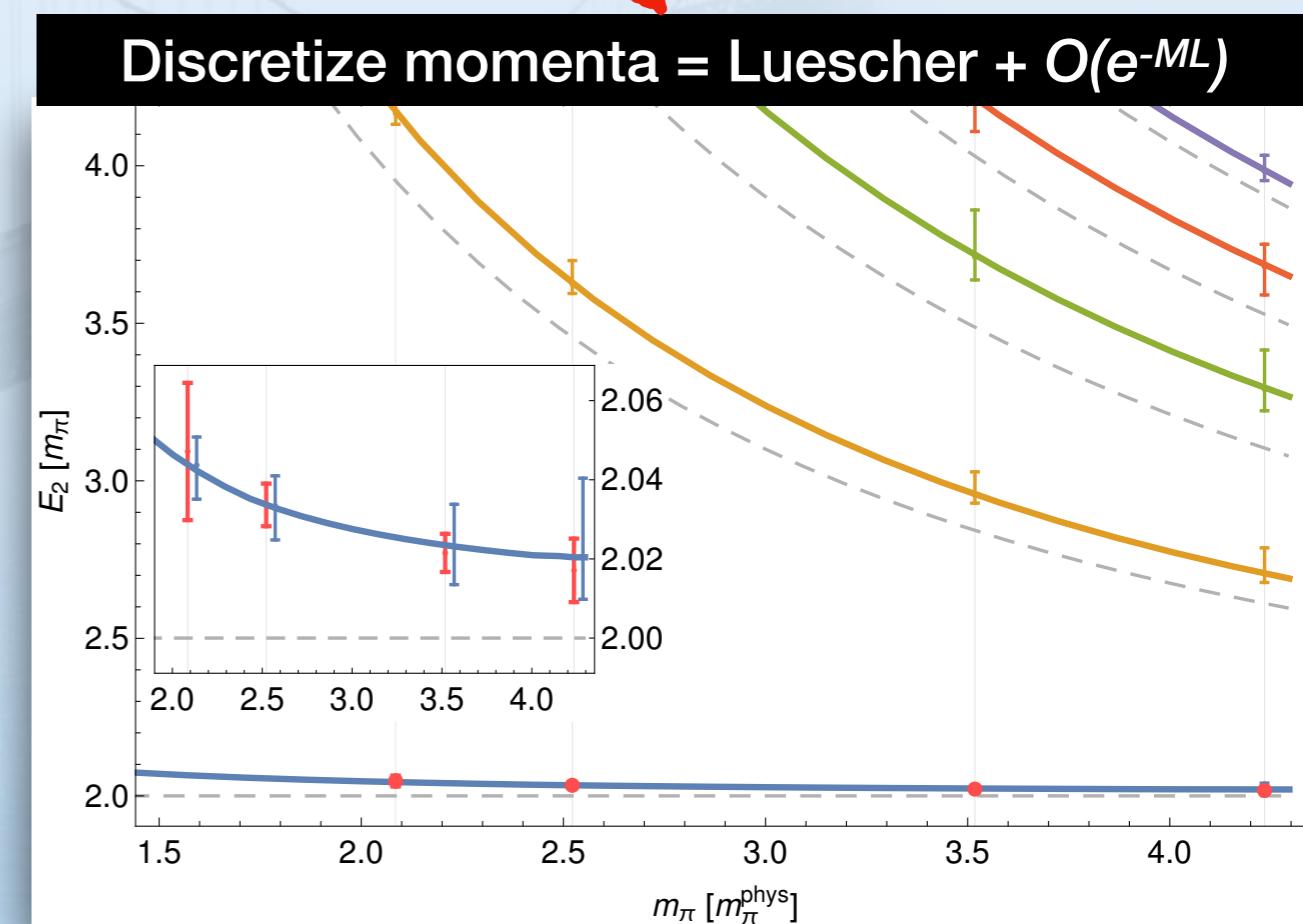
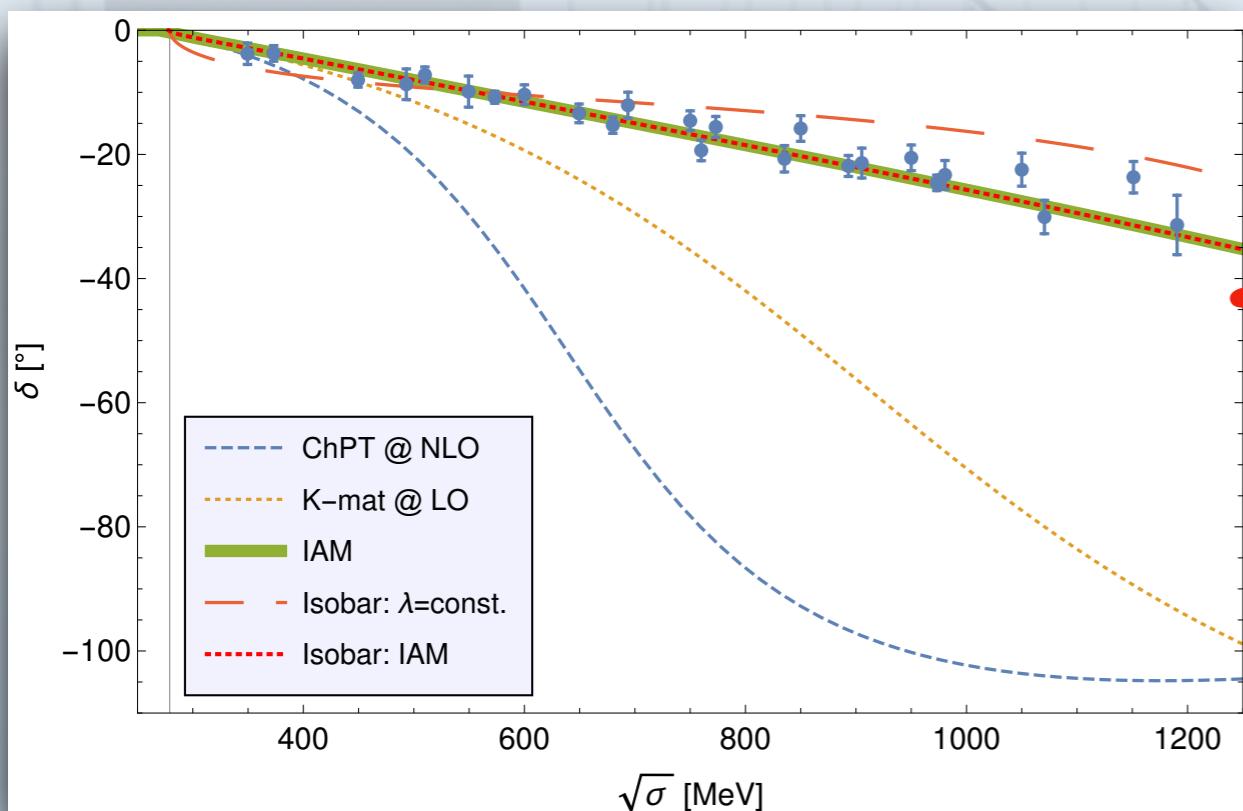
MM, Hu, Doring, Pilloni, Szczeplaniak EPJA53 (2017)

PHYSICAL APPLICATION

MM, Doring (2018) PRL 122

◎ 2-body sub-channel:

- One-channel problem – $\pi^+\pi^+$ system in S-wave
- IAM parametrized scattering amplitude



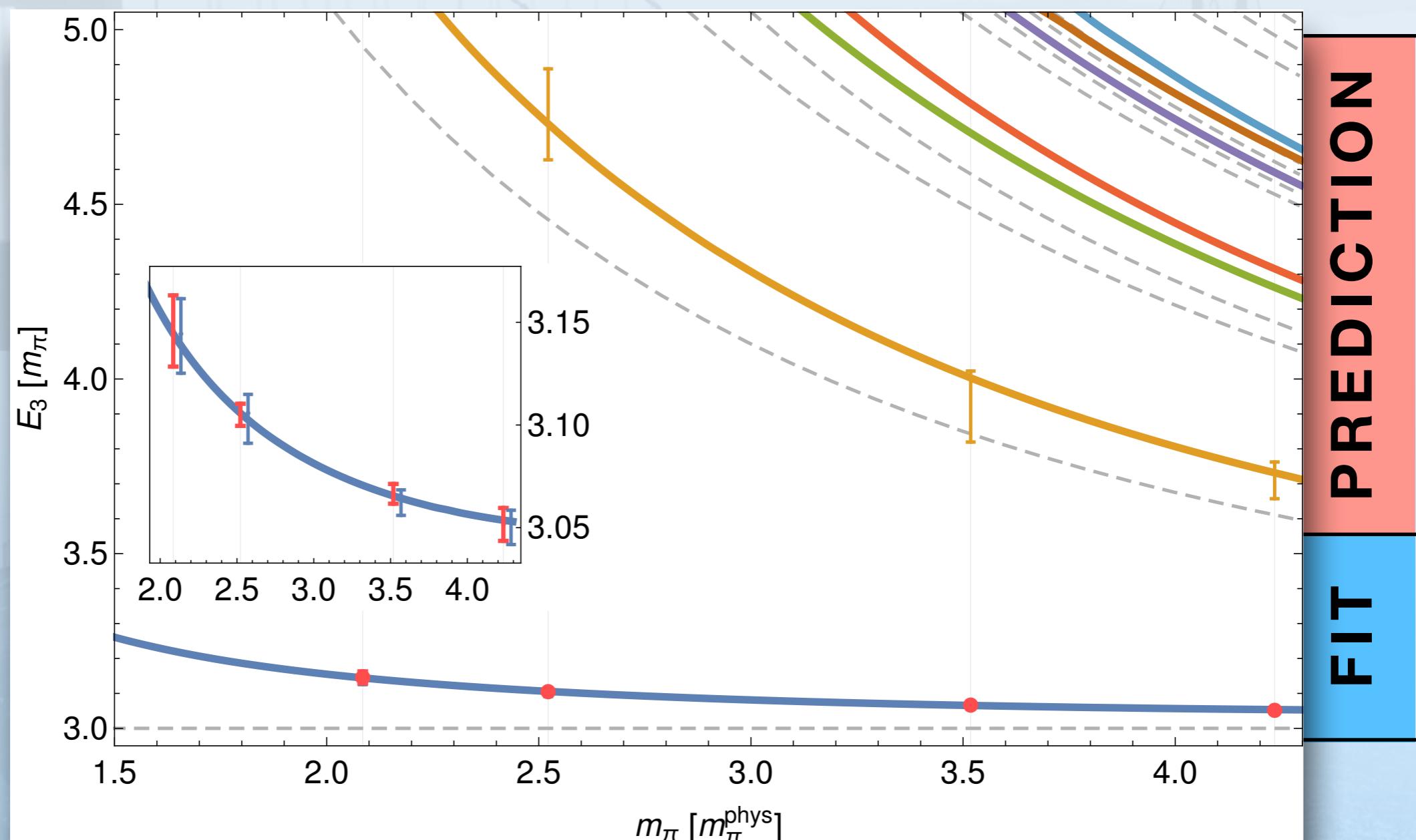
PHYSICAL APPLICATION

MM, Doring (2018) PRL 122

● 3-body spectrum

- ansatz: $\mathbf{C} = \mathbf{c} \delta^{(3)}(\mathbf{p}-\mathbf{q}) \rightarrow \mathbf{c} = 0.2 \pm 1.5 \cdot 10^{-10}$

$$\text{Det}\left(B^{ss'}(W^2) + \tau_s^{-1}(W^2, L) \delta_{ss'}\right) = 0$$



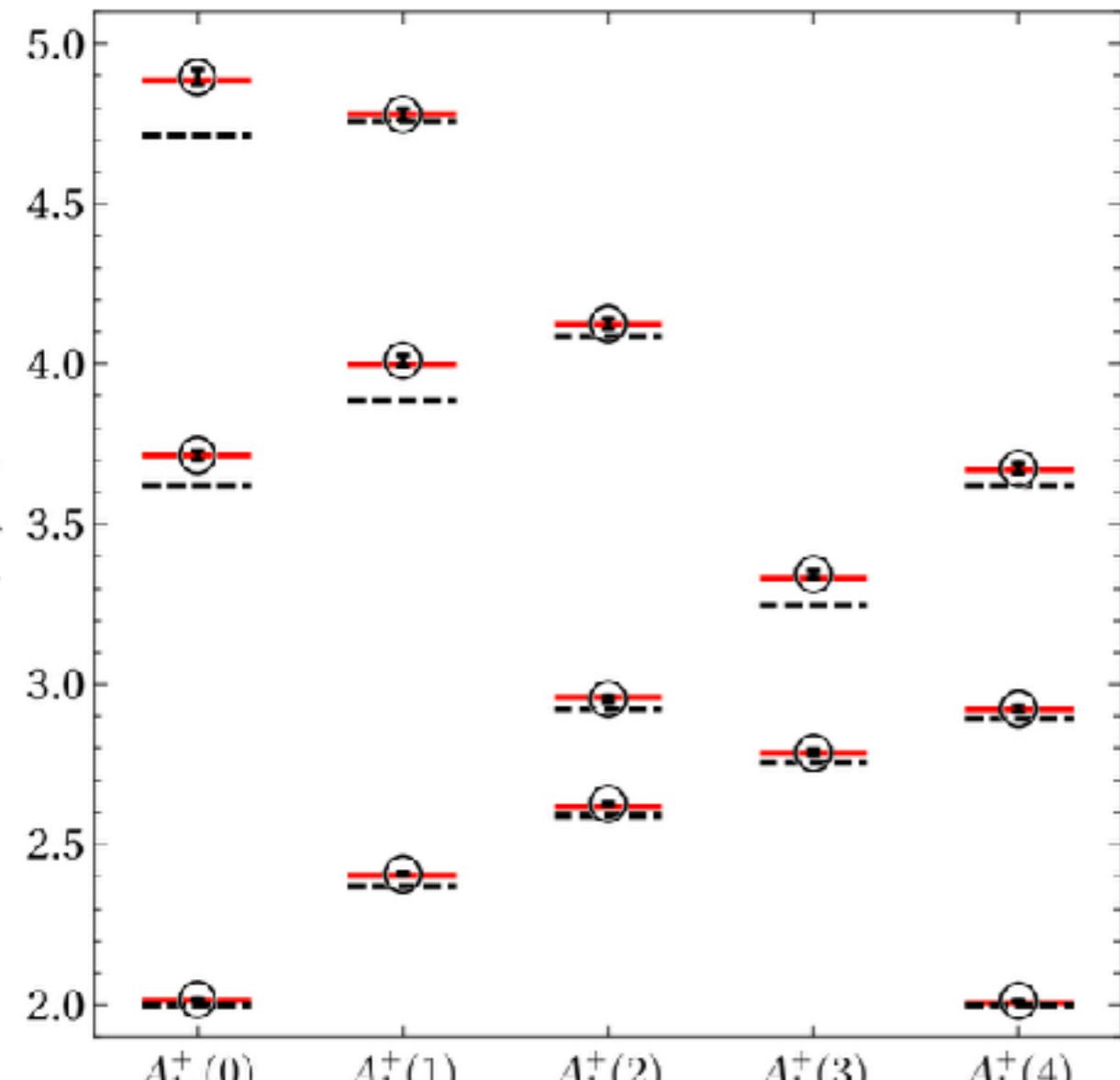
BOOSTED SYSTEMS ETC..

- New data is now available for higher boosts and irreps

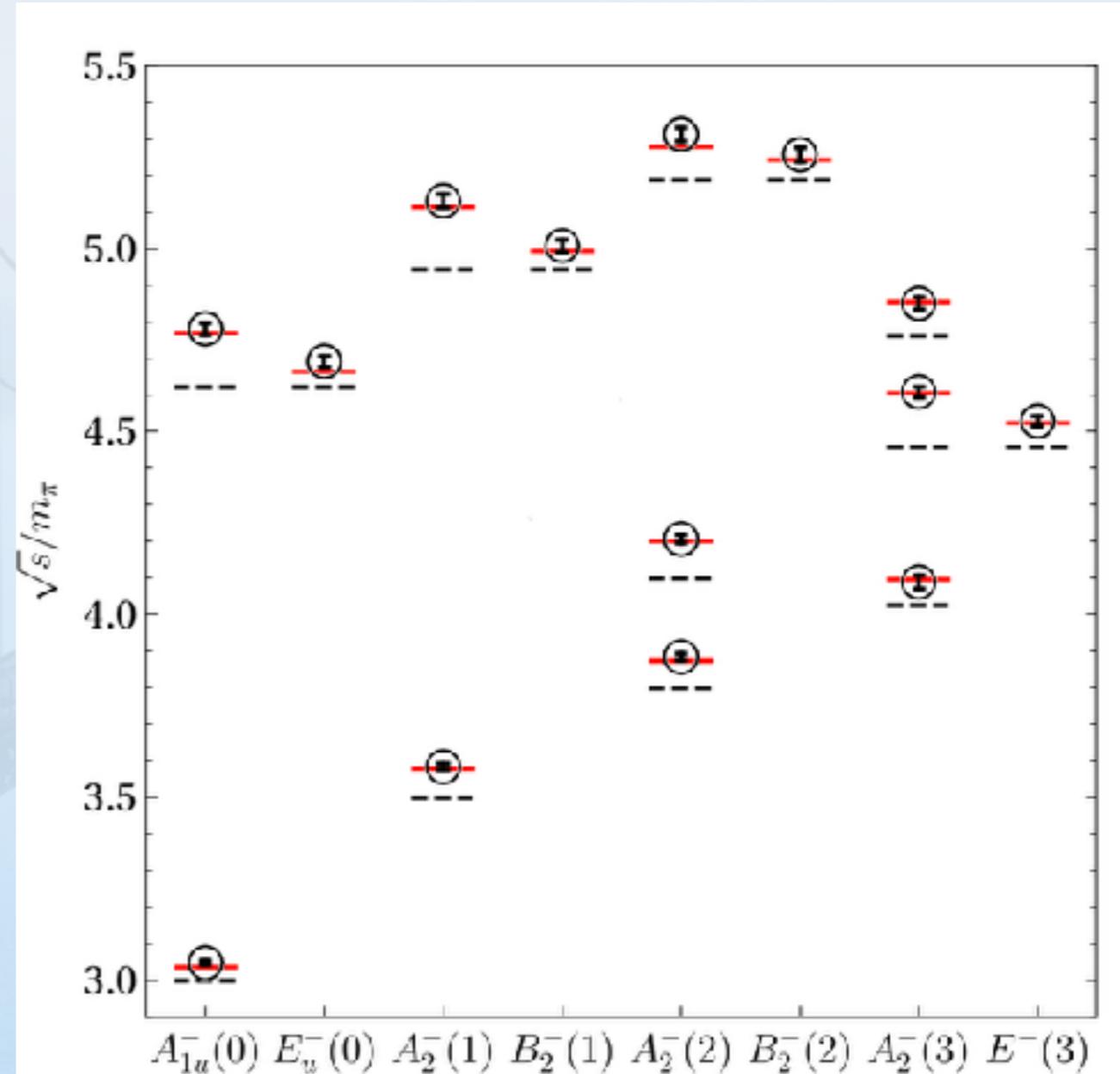
Hoerz Hanlon PRL 123 (2019)

- compare with the 3b-QC predictions:

MM, Alexandru, Culver, Doering [1909.05749]



2-body spectrum



3-body spectrum

SUMMARY

- ★ *Discretization & Projection to irreps of O_h leads to a **relativistic 3body QC***
- ★ *Excited spectrum of $\pi^+\pi^+$ & $\pi^+\pi^+\pi^+$*
- ★ **NPLQCD(2009) and Hoerz/Hanlon(2019) results ($2\pi^+$ & $3\pi^+$) analyzed**
- ★ *Predictions at physical pion mass*

EPJA53 (2017) PRD97 (2018) PRL 122 (2019) arXiv:1909.05749

★ *Parametrization via 2-b. sub-channel amplitudes*

★ *Relativistic integral equation* **EPJA53 (2017)**

this and other related talks are on
<https://maxim-mai.github.io/>

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**THANK
YOU**