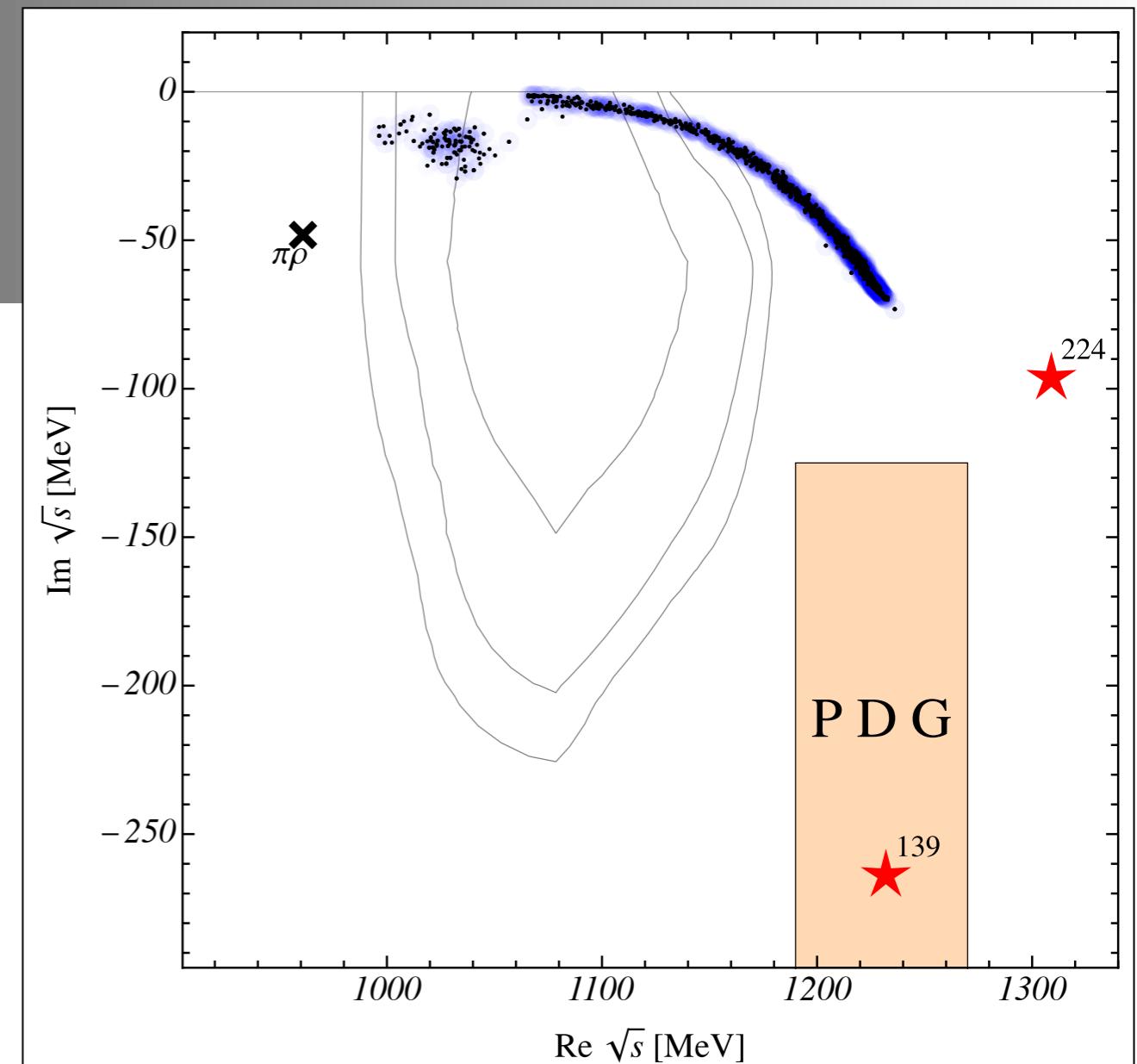


Three-body dynamics of the $a_1(1260)$ resonance from lattice QCD

2107.03973 [hep-lat]

**Maxim Mai, A. Alexandru, R. Brett, C. Culver,
M. Döring, F. Lee, D. Sadasivan
[GWQCD]**



DE-SC0016582
DE-FG02-95ER40907



PHY-2012289



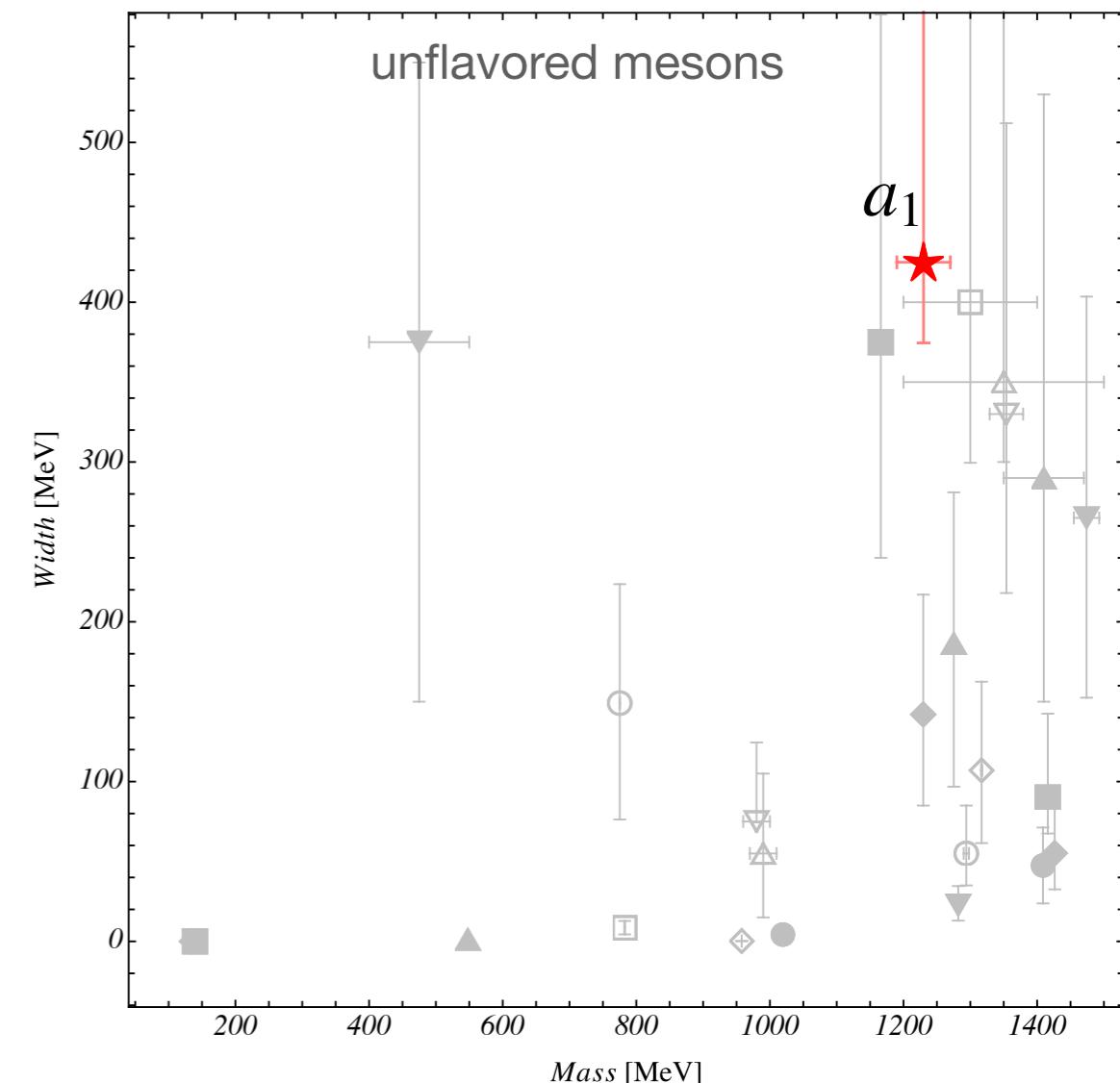
MA-7156

Pattern and production mechanisms of many excited hadrons are unresolved issues

Many questions are related to the hadronic 3-body problem

- Roper resonance $N^*(1440)$
- exotic mesons: $\pi_1(1600)$, ...
- $\omega(782)$, $a_1(1260)$, ...

experimental searches
@COMPASS, GlueX

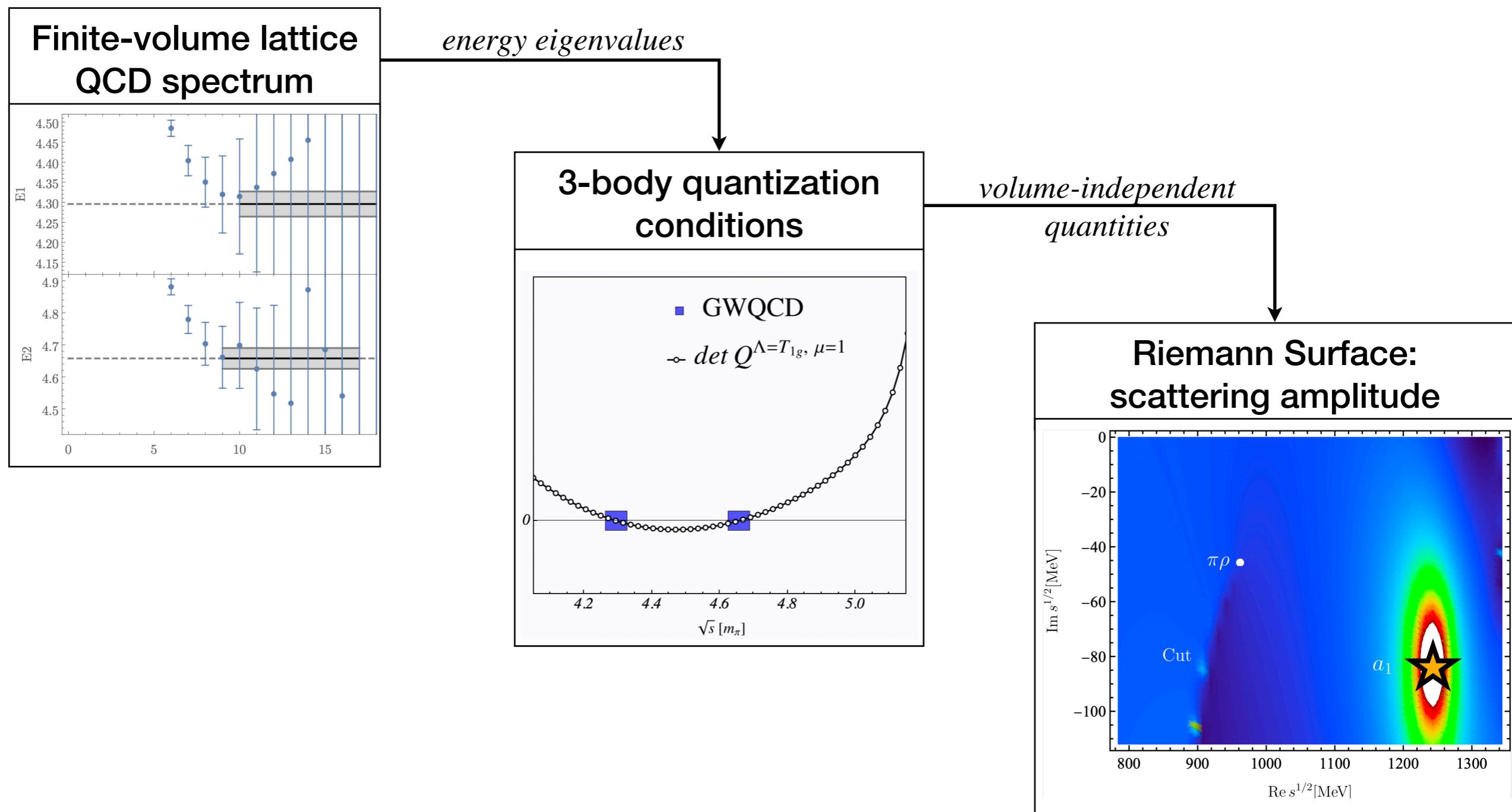


This work: $a_1(1260)$ from lattice QCD

- Goal #1: Calculate resonant three-body system in lattice QCD.
- Goal #2: Determine universal parameters of three-body resonance from QCD.

Universal parameters of resonances from poles on Riemann surface

- 3-step procedure:



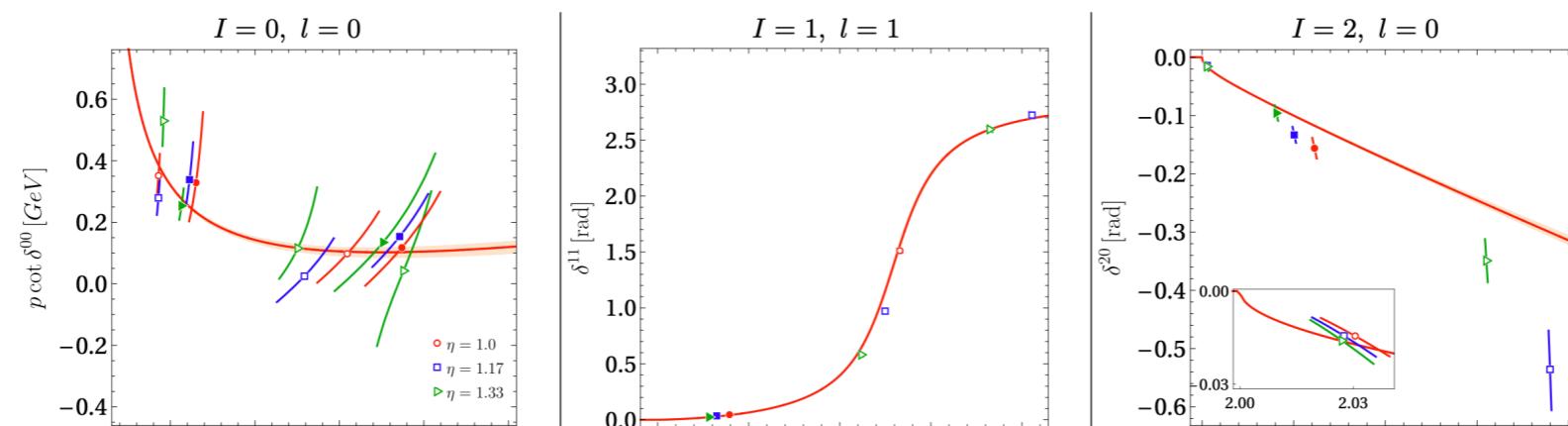
Phenomenological studies of $a_1(1260)$ from e.g. τ -decays (ALEPH@CERN)

Mikhailenko et al. [JPAC] (2018)
Sadasivan/MM,..(2020)

Theoretical access from lattice QCD (only ab-initio approach)

- full quark-gluon dynamics
- technical issues
 1. Discretized Euclidean space-time >> continuum limit
 2. Unphysical pion mass >> chiral extrapolations
 3. Finite Volume >> quantization condition
- countless studies of 2-body systems

[NPLQCD], [RQCD], [ETMC], [HadSpec], ... Review: Rev.Mod.Phys. 90 (2018) 2



e.g. MM et al. [GWQCD] (2019)

- several new calculations of non-resonant 3-body systems with maximal isospin: $\pi^+\pi^+\pi^+$, $K^-K^-K^-$, ...

[NPLQCD]; Hörz/Hanlon; [GWQCD]; [HadSpec]; Blanton et al.

Key details of GWQCD lattice QCD calculation

- $N_f = 2$ dynamical fermions, LapH smearing
- $\mathbf{P} = (0, 0, 0)$, $m_\pi = 224$ MeV, $m_\pi L = 3.3$
- GEVP with one-/two-/three-meson operators
- Relevant irrep(O_h) for $a_1(1260)$ $I^G (J^P C) = 1^- (1^{++})$:

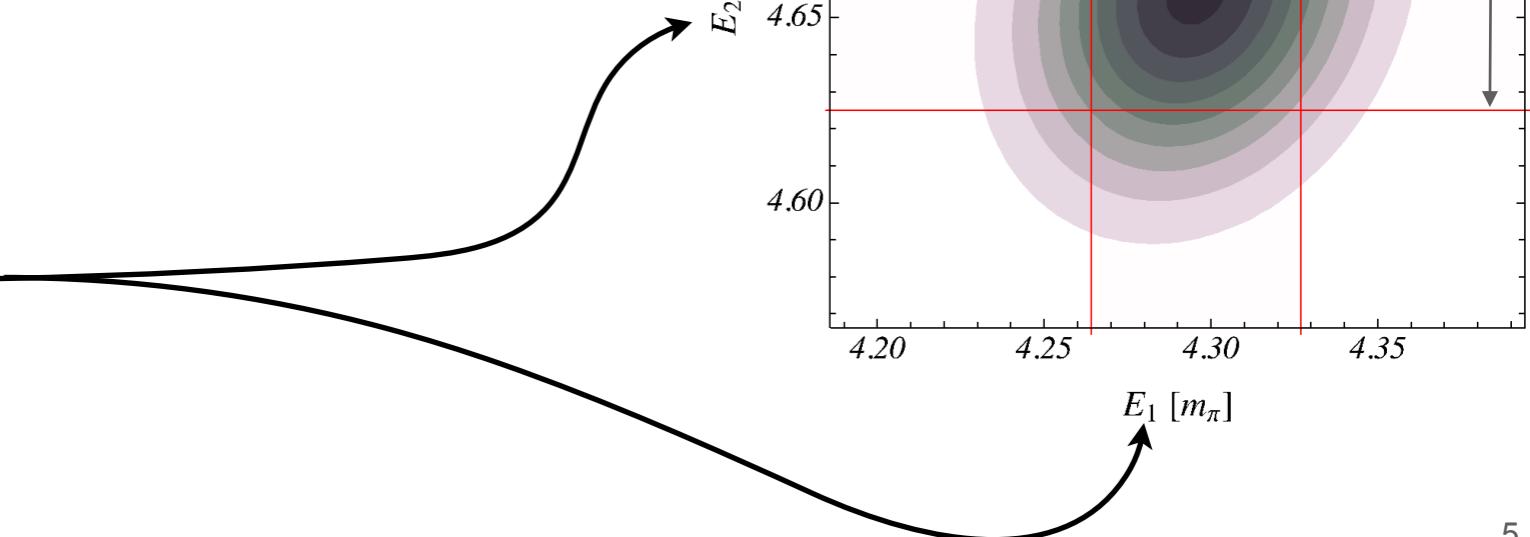
Alexandru, Brett, Culver, Guo, Lee, Pelissier (2013-2021)
 PRD87, PRD94, PRD98, PRD96, PRL117, PRD100

\mathbf{P}	Λ	$J^P (I^G = 1^-)$
$\mathbf{P} = (0, 0, 0)$	T_{1g}	$1^+, 3^+, \dots$
	A_{1u}	$0^-, 4^-, \dots$

Key results:

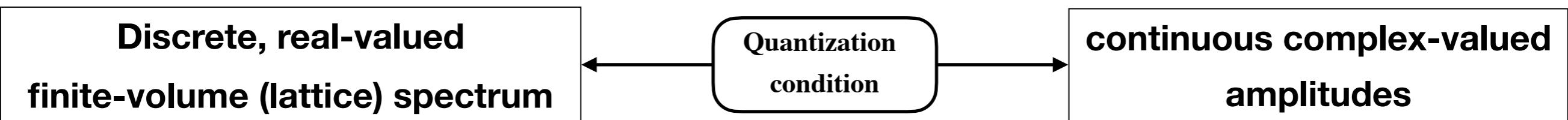
- 3π operators essential for the excited state extraction
c.f. $\rho\pi$ operators in previous 2-meson a_1 calculation
- high-momentum states are required:
 $\pi(0,0,0)\pi(+1,+1,0)\pi(-1,-1,0)$ etc..
- two levels exists below 5π threshold

Lang et al. JHEP 04, 162 (2014)



3-BODY QUANTIZATION CONDITION

<https://maxim-mai.github.io/talks/PWA21-MM.pdf>



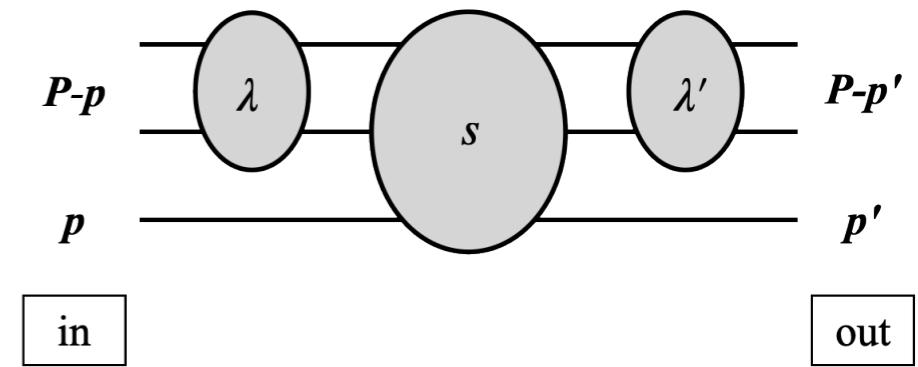
- established in 2-body: Lüscher's method, extensions... [Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky...](#)
- 3-body methods matured [Bedaque, Blanton, Briceño, Davoudi, Döring, Grießhammer, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe, Wu](#)
[Reviews: Hansen/Sharpe\(2019\) MM/Döring/Rusetsky\(2021\)](#)

Finite Volume Unitarity

[MM, Döring EPJA \(2017\) PRL \(2019\)](#)

- extended to higher spin and coupled-channels: new degree of freedom (λ)

$$0 = \det \left[B(s) + C(s) - 2L^3 E_{\mathbf{p}} \left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s) \right) \right]_{(\lambda' \lambda)(\mathbf{p}' \mathbf{p})}^{\Lambda}$$



- ∞ -dim. determinant equation in $\mathbf{p} \in \frac{2\pi}{L} \mathbb{Z}^3$ → practical applications require truncation
→ common to all quantization conditions

[see discussion in e.g. MM/Döring/Rusetsky\(2021\)](#)

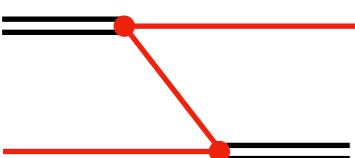
3-BODY QUANTIZATION CONDITION

<https://maxim-mai.github.io/talks/PWA21-MM.pdf>

$$0 = \det \left[B(s) + C(s) - 2L^3 E_p \left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s) \right) \right]_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}^\Lambda$$

one-particle exchange

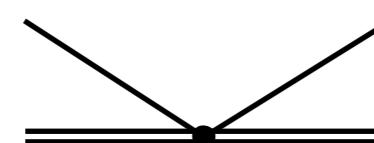
- fixed by 3b-unitarity



three-body force

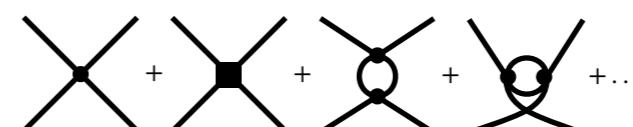
- dynamics of $\rho\pi$ system
- regular function \Rightarrow Laurent series
- fit to 3-body levels

$$C_{\ell'\ell}(s, \mathbf{p}', \mathbf{p}) = \sum_{i=-1}^{\infty} c_{\ell'\ell}^{(i)}(\mathbf{p}', \mathbf{p})(s - m_{a_1}^2)^i$$



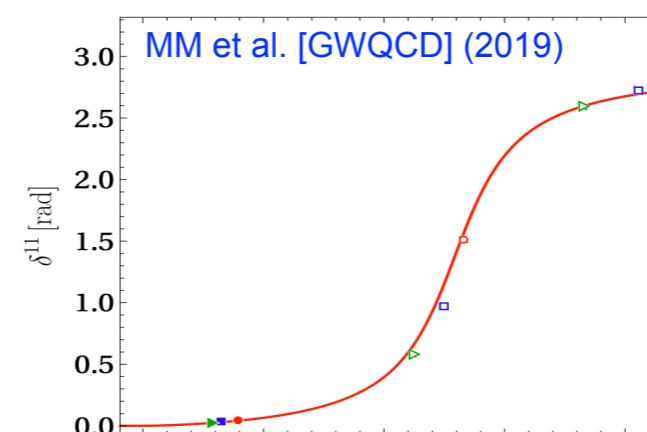
two-body kernel

- dynamics of $I=1 \pi\pi$ system



$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

- regular function \Rightarrow polynomial



two-body self-energy

- fixed by 2b-unitarity
- no free parameters



3-BODY QUANTIZATION CONDITION

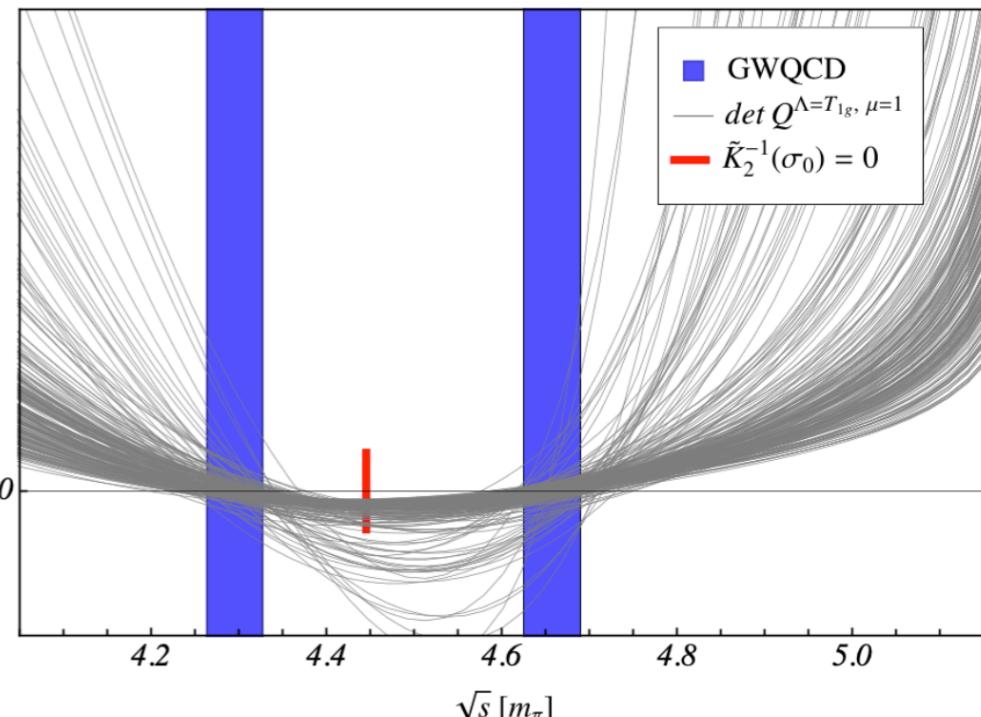
<https://maxim-mai.github.io/talks/PWA21-MM.pdf>

Last unknown (volume-independent) quantity: 3-body term $\textcolor{red}{C}$

- pole is generated with/without explicit pole-term
- best description (with large correlations)

$$\textcolor{red}{C}_{\ell'\ell} = \textcolor{red}{g}_{\ell'} |\mathbf{p}'|^{\ell'} \frac{1}{s - \mathbf{m}_{\mathbf{a}_1}^2} \mathbf{g}_{\ell} |\mathbf{p}|^{\ell} + \textcolor{red}{c} \delta_{\ell'0} \delta_{\ell0}$$

- full resampling... accessing statistical errors

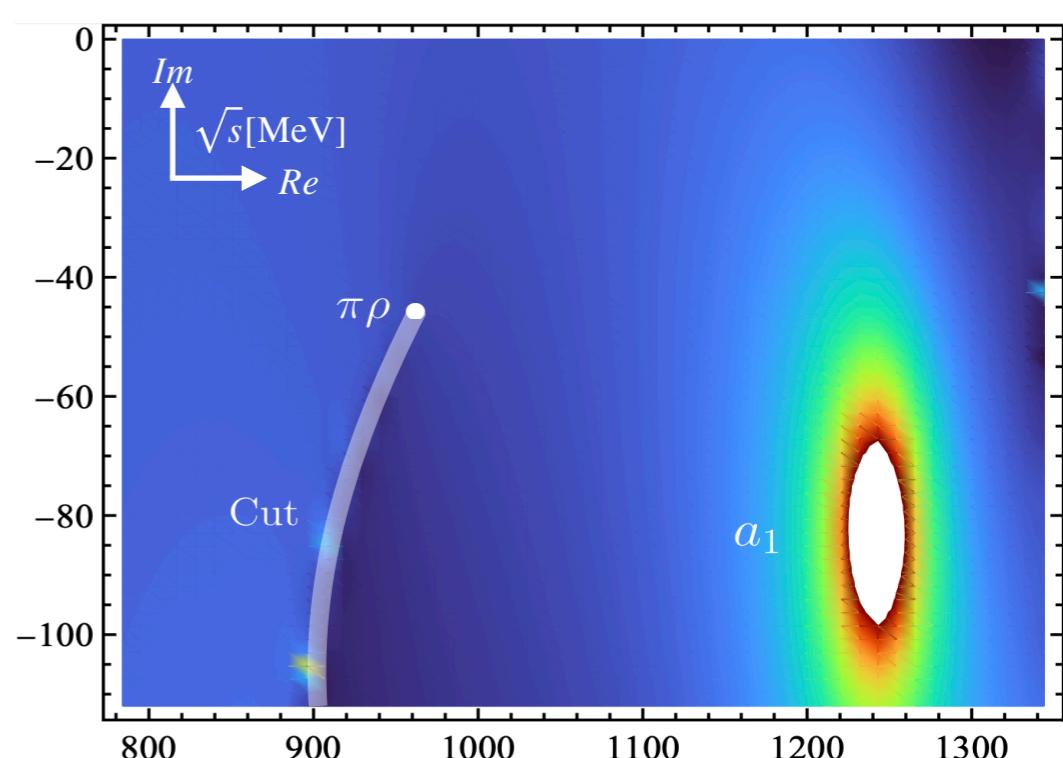


Infinite-volume scattering amplitude

- integral equation with all inputs fixed from lattice QCD results

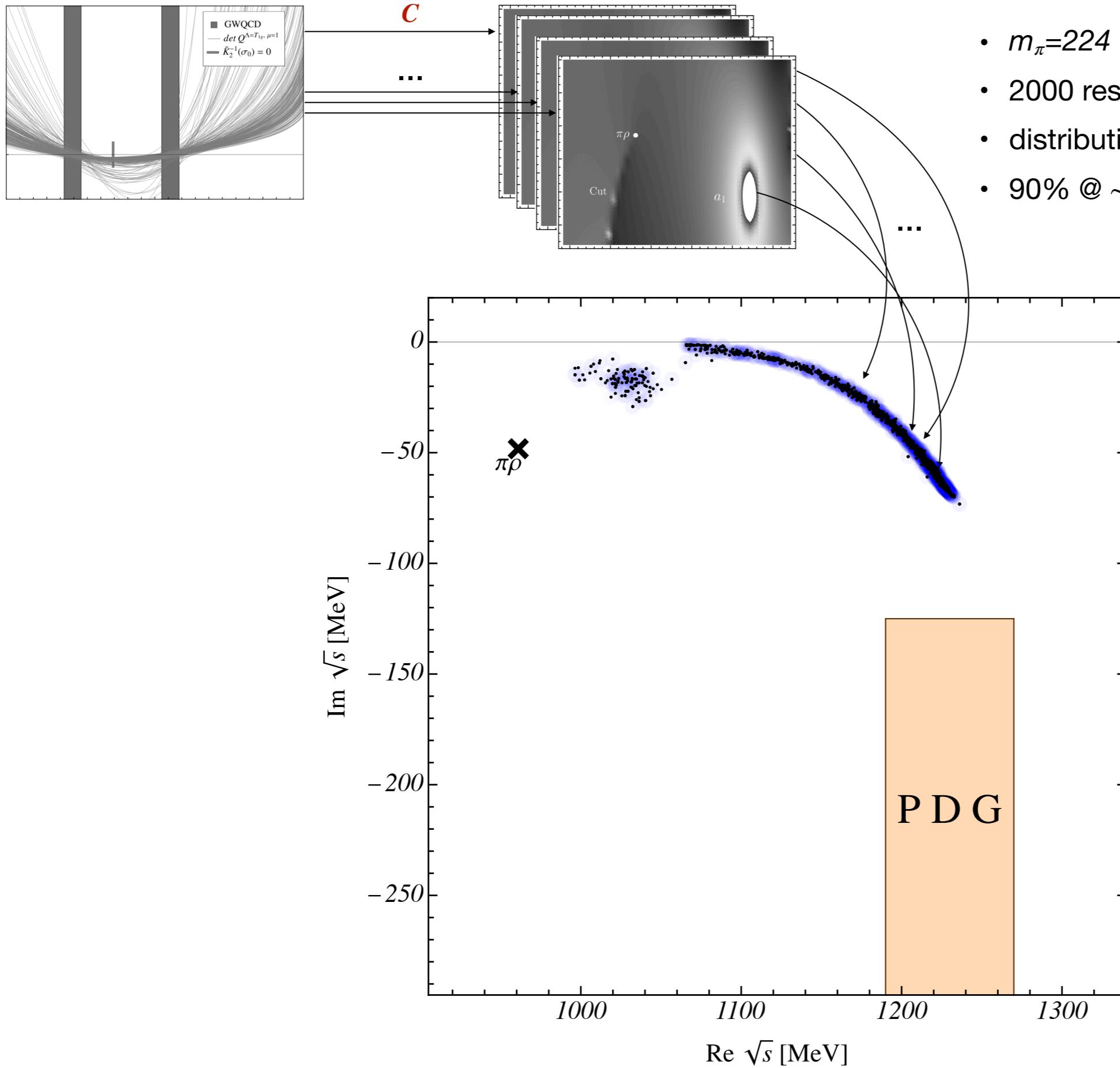
$$T^c = \textcolor{blue}{B} + \textcolor{red}{C} + \int \frac{d^3\ell}{(2\pi)^3} \frac{(\textcolor{blue}{B} + \textcolor{red}{C})}{2E_l} \frac{1}{\tilde{\mathbf{K}}_{\textcolor{green}{n}}^{-1} - \Sigma_{\textcolor{violet}{n}}} T^c$$

- contour deformation of spectator momenta
- analytic continuation to 2.Riemann sheet



PARAMETERS OF $a_1(1260)$

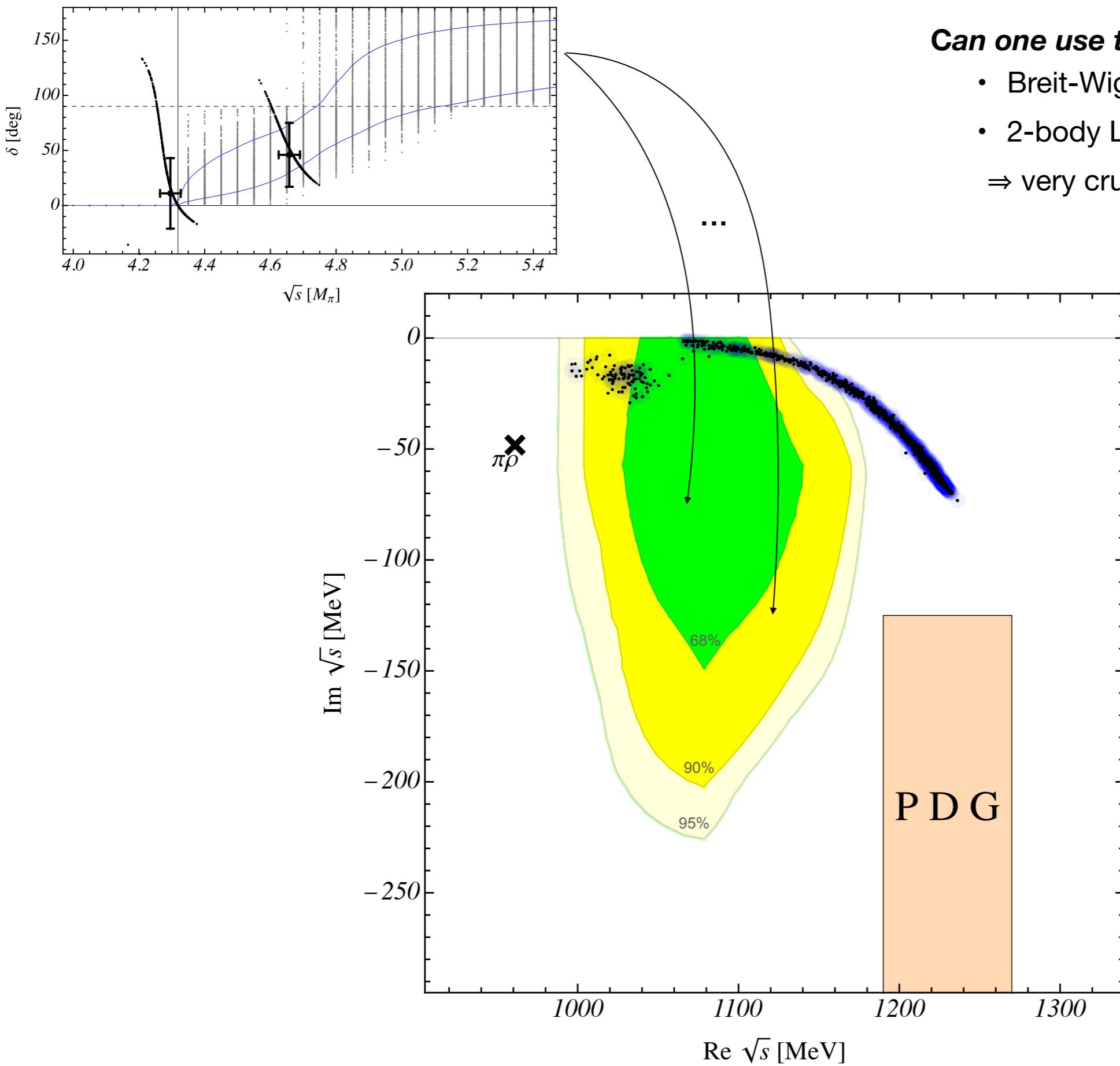
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- $m_\pi = 224 \text{ MeV}$
- 2000 resampling sets
- distribution of poles is finite
- 90% @ $\sim((1110..1210) - (20..70)i) \text{ MeV}$

PARAMETERS OF $a_1(1260)$ - CHECKS

<https://maxim-mai.github.io/talks/PWA21-MM.pdf>

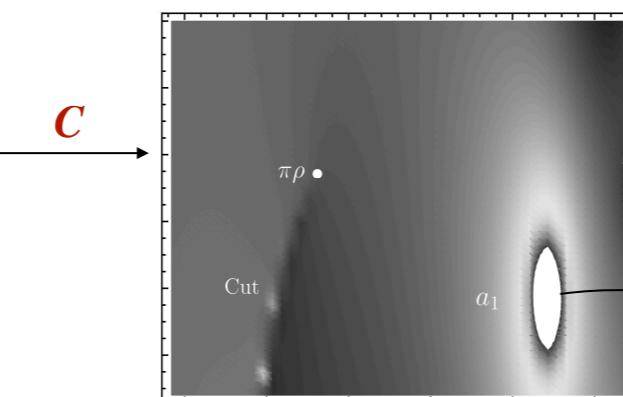
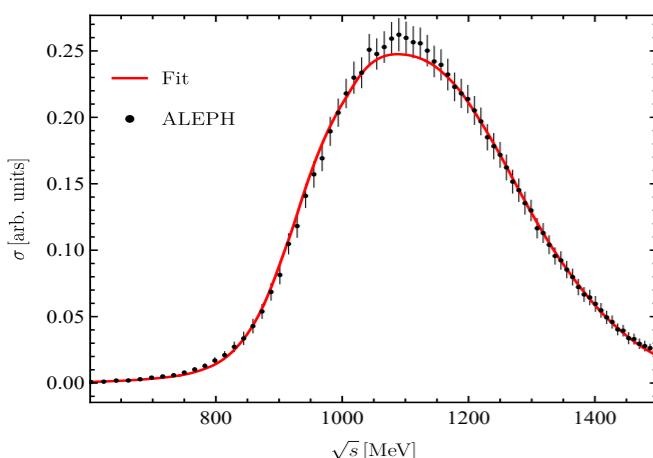


Can one use two-body approximation?

- Breit-Wigner parametrization
 - 2-body Lüscher quantization condition
- ⇒ very crude approximation!

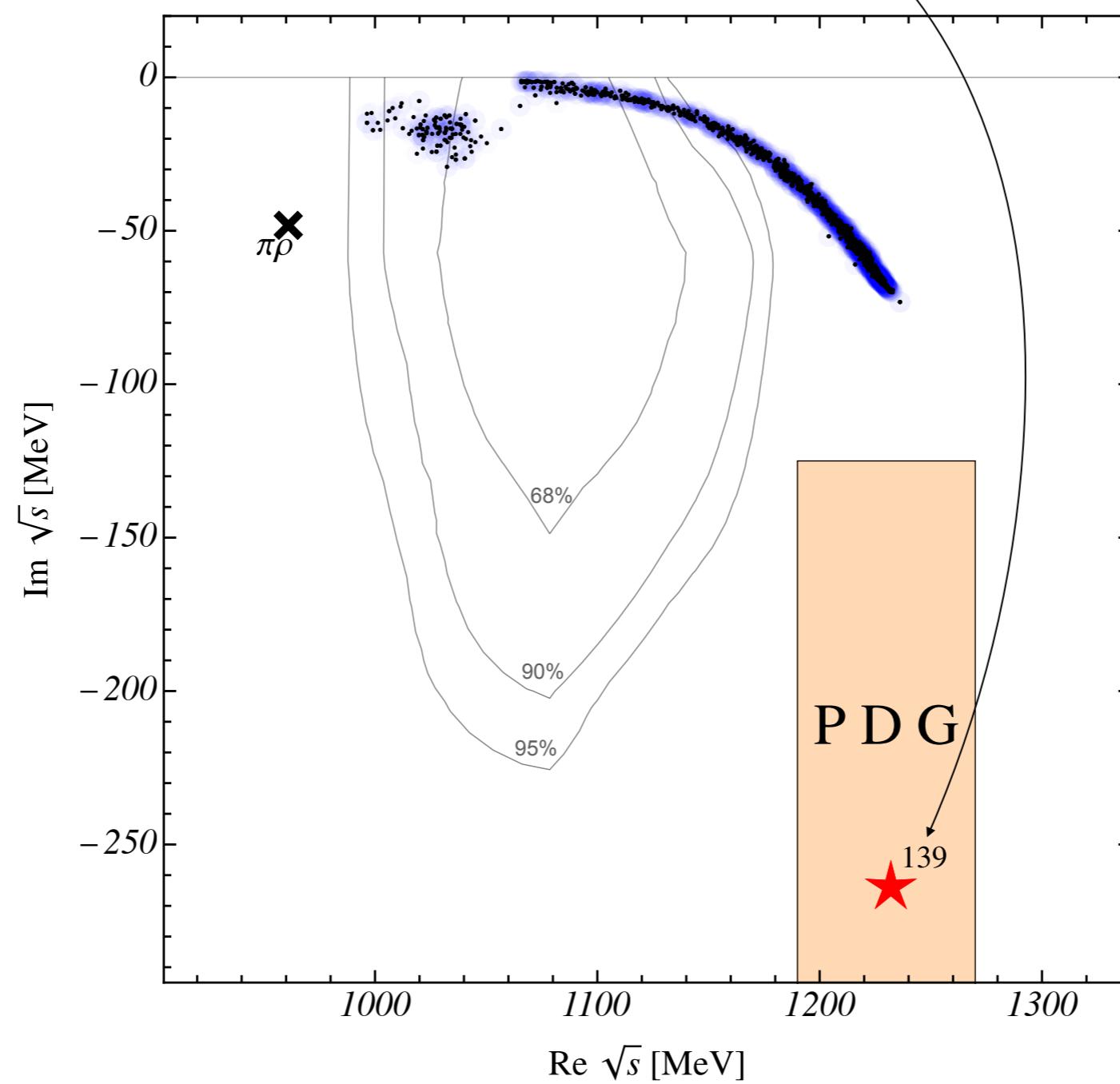
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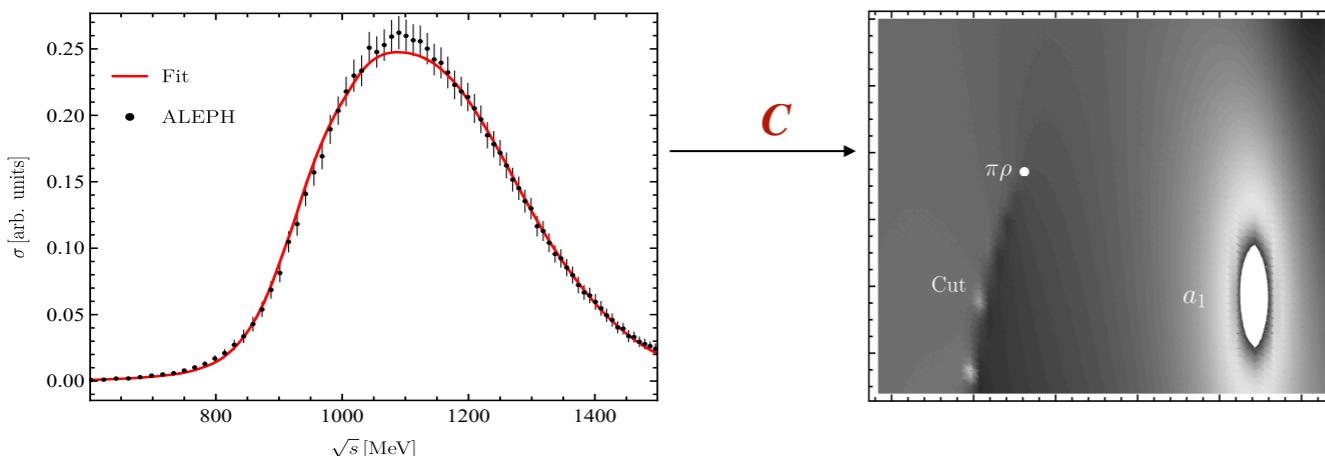
What does phenomenology says?

- $\tau \rightarrow (\pi\pi\pi)\nu_\tau$ from ALEPH@CERN
- fit to line shape to fix **C**



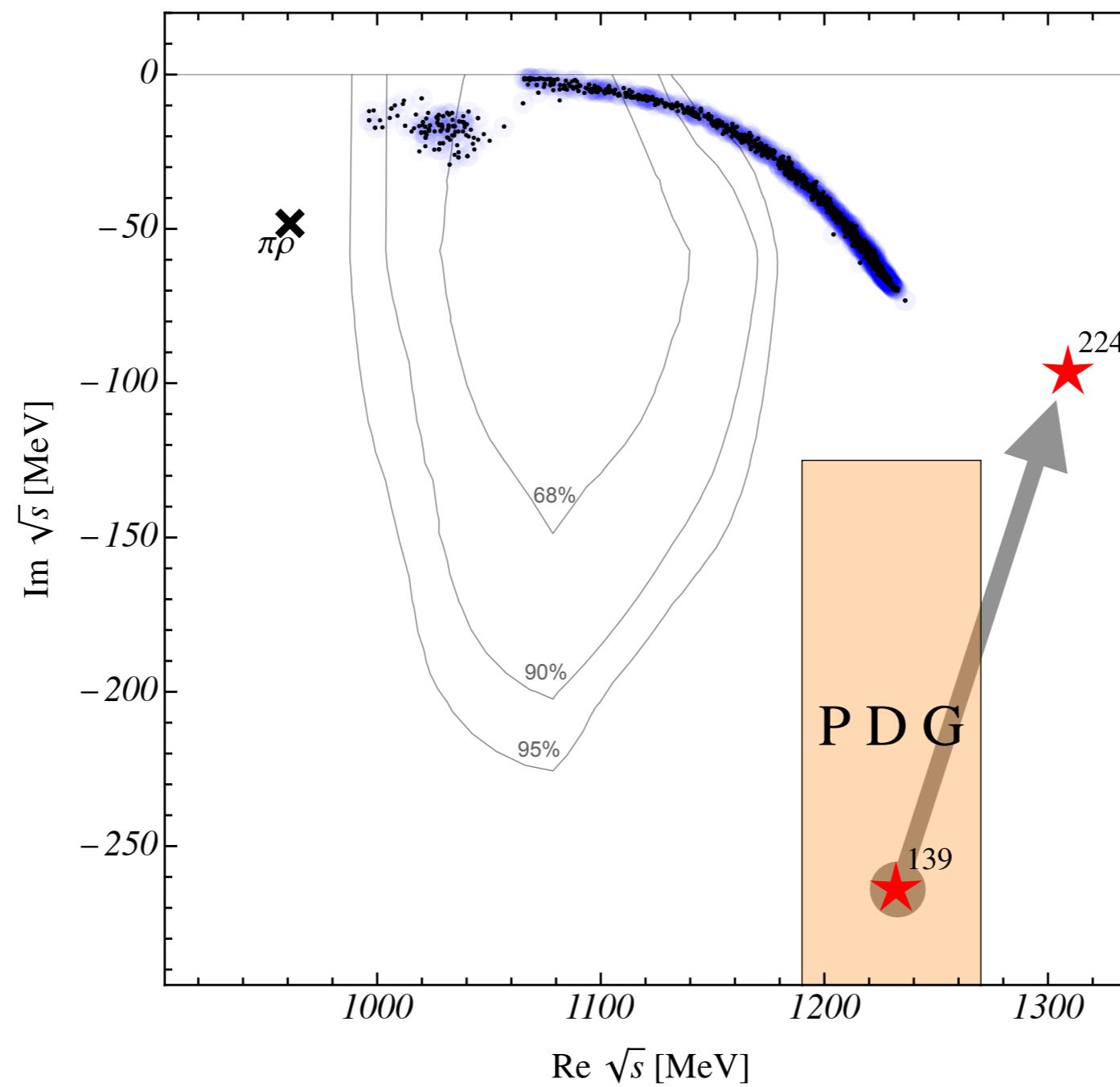
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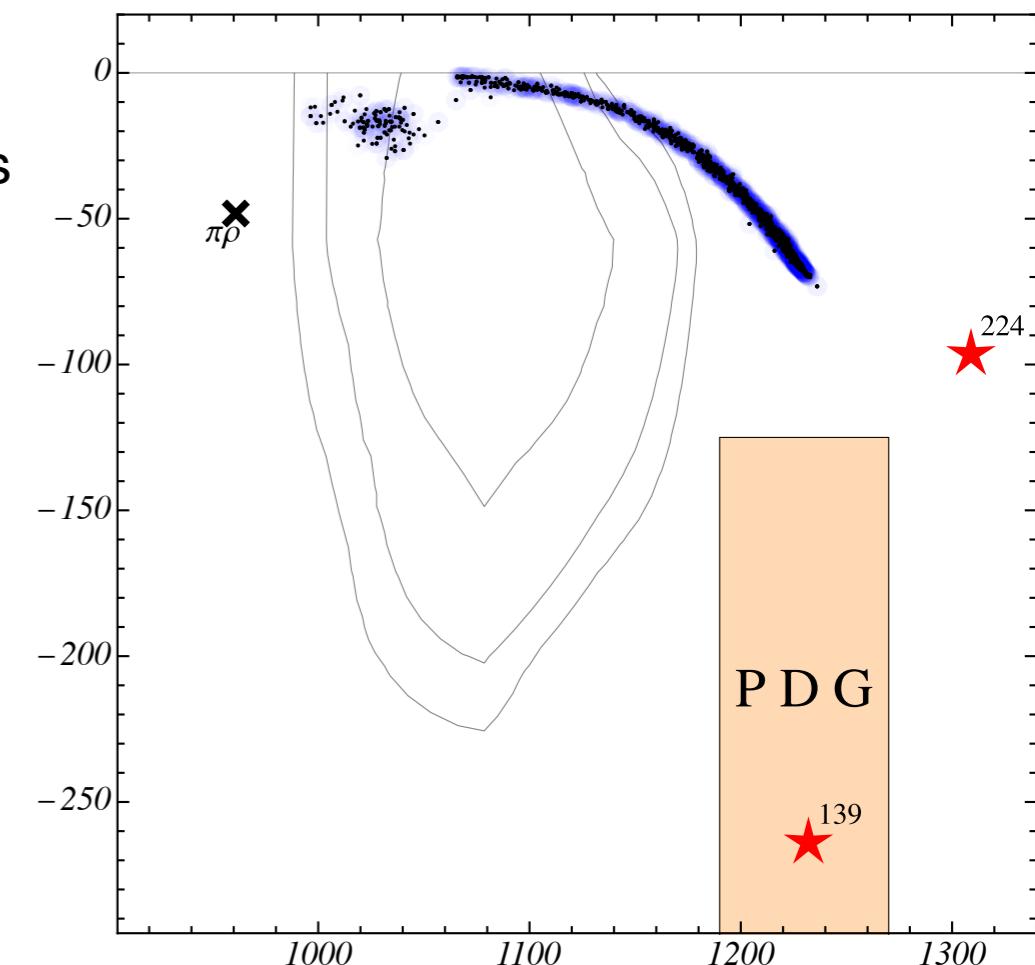


(naive) chiral extrapolation
confirms expectations:

- a_1 becomes heavier
- ... and narrower

Lattice QCD driven determination of $a_1(1260)$ parameters

- Lattice QCD levels with 1/2/3-meson operators calculated
- 3-body quantization condition extended to spin-1 systems
- unitary infinite-volume amplitude solved with complex integration contour and for complex energies
- Pole positions and couplings are determined



Outlook

- More lattice levels (another setup: m_π, L):
 - precise determination of a_1 parameters
 - importance of the $\sigma\pi$ channels
 - chiral trajectory of three-body term **C**

THANK YOU