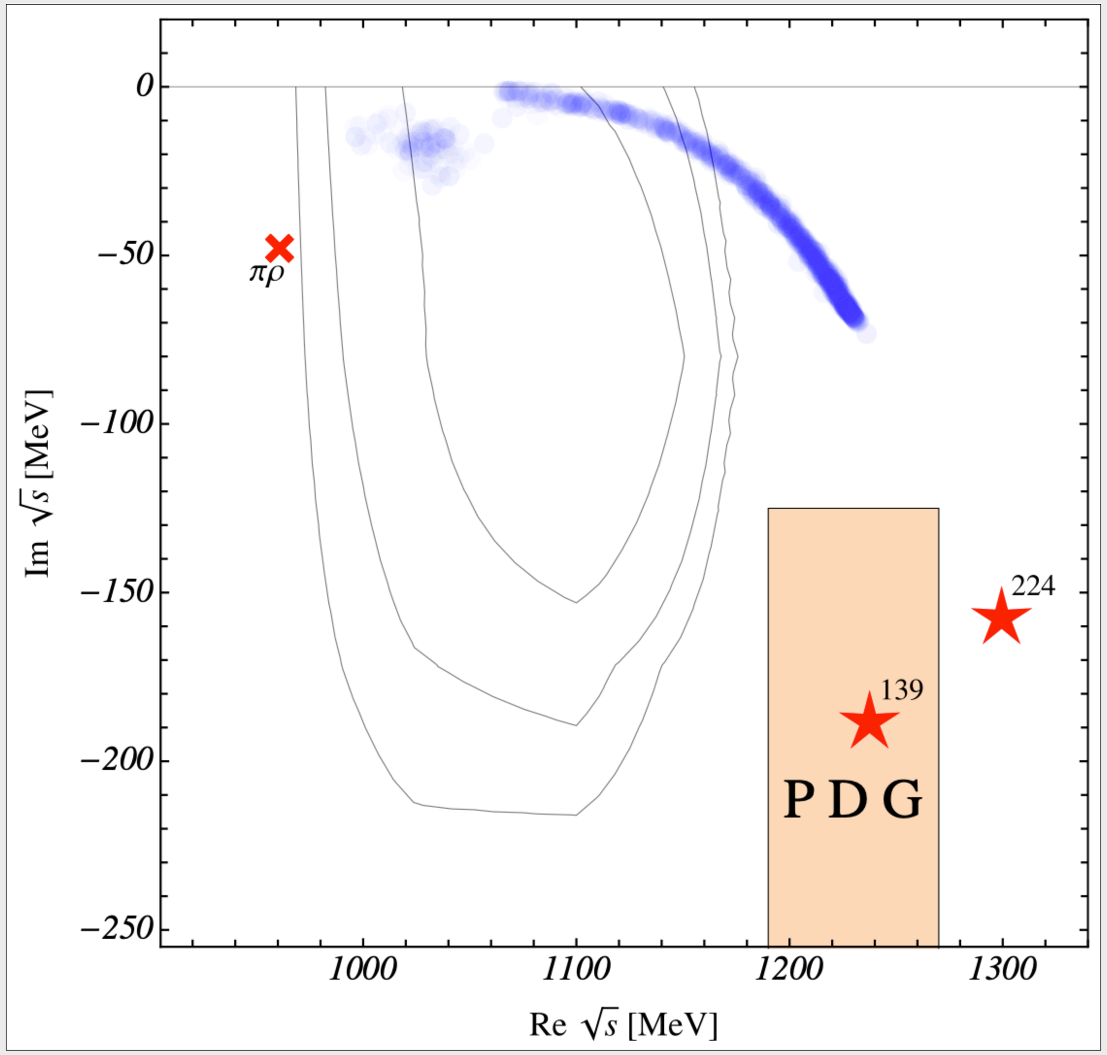
THE a1(1260)-RESONANCE FROM LATTICE QCD

2107.03973 [hep-lat]



Maxim Mai, A. Alexandru, R. Brett, C. Culver
M. Döring, F. Lee, D. Sadasivan [GWQCD]

slides

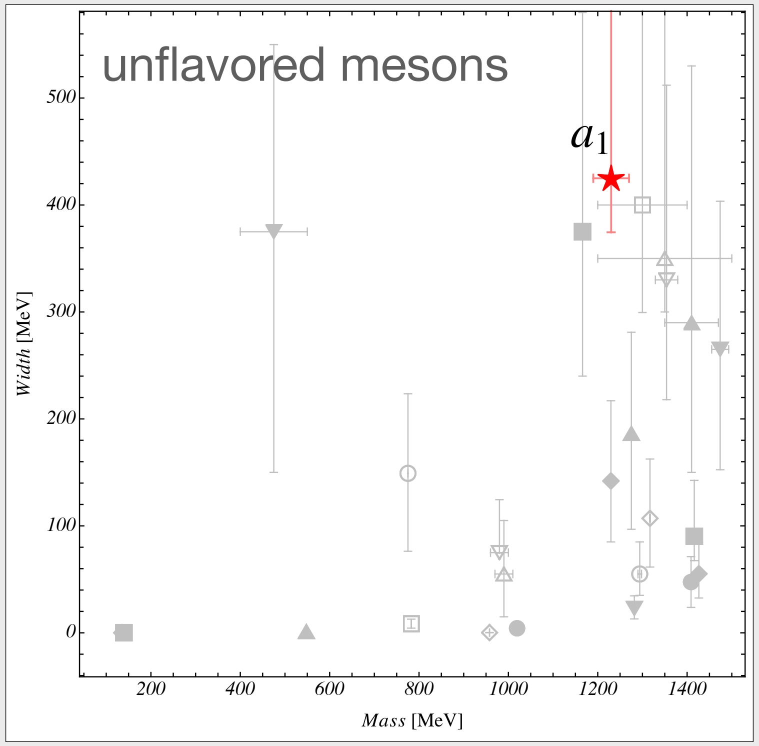
https://maxim-mai.github.io/talks/LAT21-MM.pdf

QCD SPECTRUM

Many states of QCD have large coupling to 3-body channels

- ω(782), a_1(1260)...
- exotic mesons: π_1(1600), ... **exp. searches @ COMPASS, GlueX**
- Roper resonance *N*(1440)*

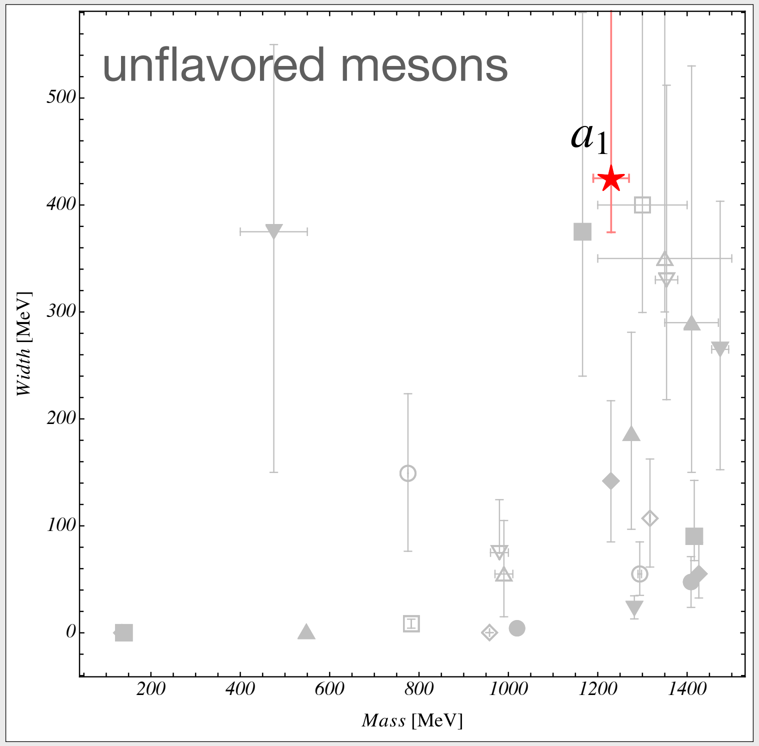
This work: ***a_1(1260)* from lattice QCD**



QCD SPECTRUM

Many states of QCD have large coupling to 3-body channels

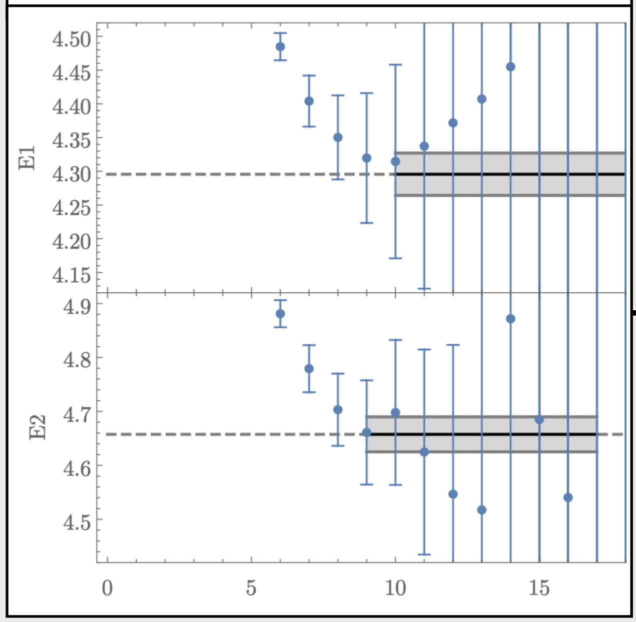
- $\omega(782)$, $a_1(1260)$...
- exotic mesons: $\pi_1(1600)$, ... **exp. searches @ COMPASS, GlueX**
- Roper resonance *N*(1440)*



This work: ***a₁(1260)* from lattice QCD**

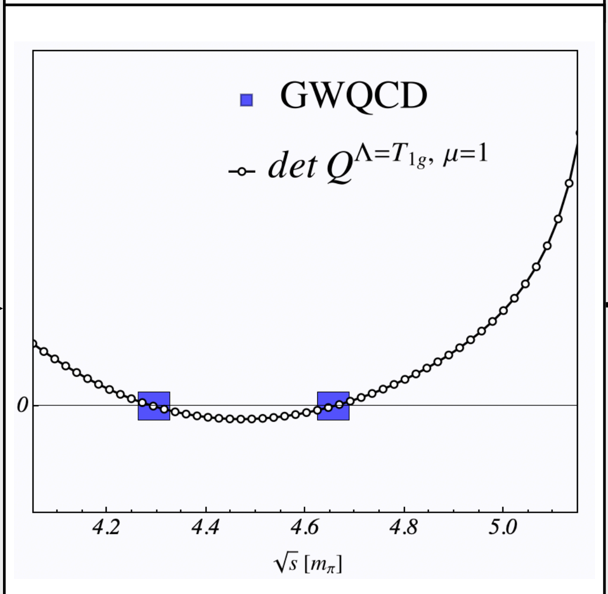
- Universal parameters from poles on the Riemann surface
- 3 step procedure:

Finite-volume Spectrum



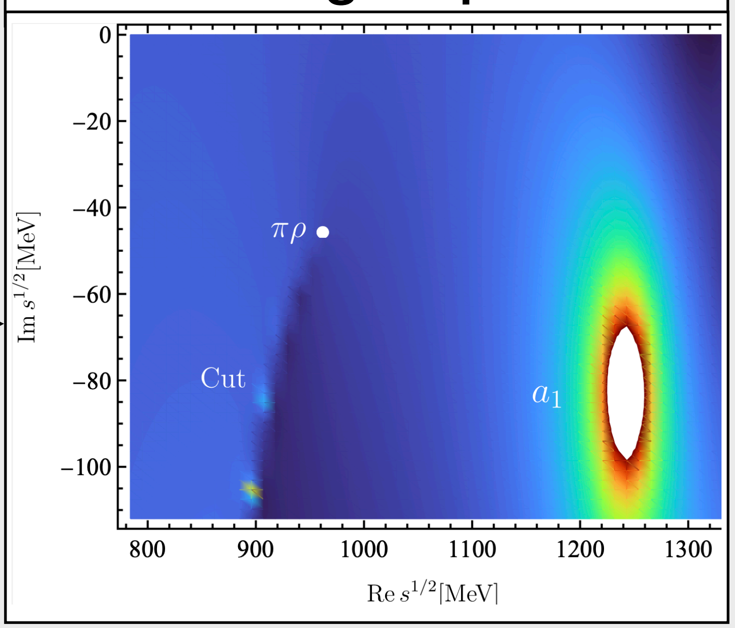
energy eigenvalues

3-body quantization conditions



volume indep. quantities

Analytic continuation scattering amplitude



https://maxim-mai.github.io/talks/LAT21-MM.pdf

FINITE-VOLUME SPECTRUM

GWQCD ensemble used for 2/3 pion calculations

Alexandru, Brett, Culver, Guo, Lee, Pelissier (2013-2020)
PRD87,PRD94,PRD98,PRD96,PRL117,PRD100

Some key details: *(more in the next talk -- Ruairí Brett)*

- N_f = 2 dynamical fermions, LapH smearing
- ***P***=(0,0,0), m_π=224 MeV, $m_\pi L$=3.3
- *GEVP with one-, two-, three-meson operators*
- *Relevant irrep(O_h) for* ***a1(1260)*** $\boldsymbol{I^G (J^{PC}) = 1^- (1^{++})}$: T_{1g}

Geometry	$\mathbf{P}$	Λ	$J^P\ (I^G = 1^-)$
Cubic	$\mathbf{P} = (0,0,0)$	T_{1g}	$1^+, 3^+, \ldots$
		A_{1u}	$0^-, 4^-, \ldots$

FINITE-VOLUME SPECTRUM

GWQCD ensemble used for 2/3 pion calculations

Alexandru, Brett, Culver, Guo, Lee, Pelissier (2013-2020)
PRD87,PRD94,PRD98,PRD96,PRL117,PRD100

Some key details: *(more in the next talk -- Ruairí Brett)*

- $N_f = 2$ dynamical fermions, LapH smearing
- ***P***=(0,0,0), m_π=224 MeV, $m_\pi L$=3.3
- *GEVP with one-, two-, three-meson operators*
- *Relevant irrep(O_h) for* ***a1(1260)*** $I^G(J^{PC}) = 1^-(1^{++})$: T_{1g}

Geometry	$\mathbf{P}$	Λ	$J^P\ (I^G = 1^-)$
Cubic	$\mathbf{P} = (0,0,0)$	T_{1g}	$1^+, 3^+, \ldots$
		A_{1u}	$0^-, 4^-, \ldots$

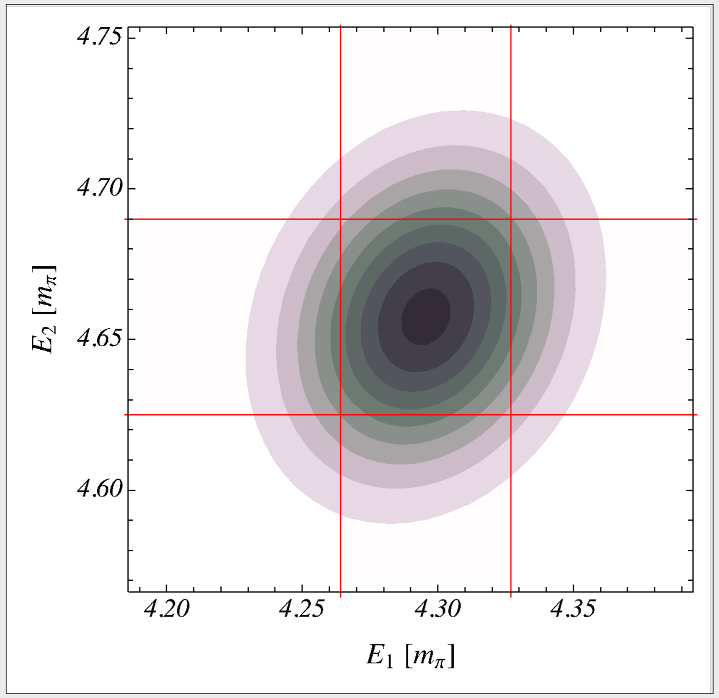
Key insights:

- three-meson operators necessary for stable extraction of the excited state

  c.f. need for $\rho\pi$ operators in pioneering two-meson a_1-calculation
  Lang et al. JHEP 04, 162 (2014)

- high-momentum states are required: $\pi(0,0,0)\pi(1,1,0)\pi(-1,-1,0)$ etc..

- two interacting levels exists below 5π threshold



3-BODY QUANTIZATION CONDITION

Discrete, real finite-volume (lattice) spectrum → continuous complex-valued amplitudes

- established in 2-body: Lüscher's method, extensions...

Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky...

- 3-body methods matured (this session)

Bedaque, Blanton, Briceño, Davoudi, Döring, Grießhammer, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe, Wu

Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

3-BODY QUANTIZATION CONDITION

Discrete, real finite-volume (lattice) spectrum → continuous complex-valued amplitudes

- established in 2-body: Lüscher's method, extensions... **Lüscher, Gottlieb, Rummukainen, Feng, Li, Liu, Döring, Briceño, Bernard, Meißner, Rusetsky...**
- 3-body methods matured (this session) **Bedaque, Blanton, Briceño, Davoudi, Döring, Grießhammer, Guo, Hammer, Hansen, MM, Meißner, Müller, Pang, Polejaeva, Romero-López, Rusetsky, Sharpe, Wu**

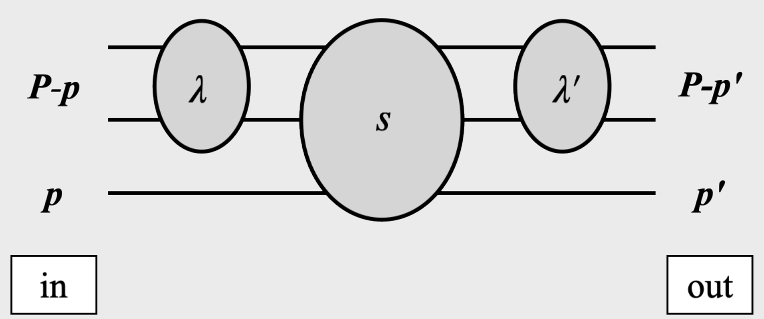
Reviews: Hansen/Sharpe(2019) MM/Döring/Rusetsky(2021)

Finite Volume Unitarity **MM, Döring EPJA (2017) PRL (2019)**

- basic idea:

(unitary three-body amplitude) $\overset{\int \frac{d^3\mathbf{k}}{(2\pi)^3} \to 1/L^3 \sum_k}{\Rightarrow}$ ***(singular ⇔ three mesons are on-shell)*** ⇔ ***(energy eigenvalues)***

$$\boxed{0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]^{\Lambda}_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}}$$



- *extended to higher spin and coupled-channels: new degree of freedom (λ)*
- *∞-dim. determinant equation in* $\mathbf{p} \in \frac{2\pi}{\mathbf{L}}\mathbf{Z}^3$ → *practical applications require truncation*

→ *common to all quantization conditions*

see discussion in e.g. MM/Döring/Rusetsky(2021)

3-BODY QUANTIZATION CONDITION

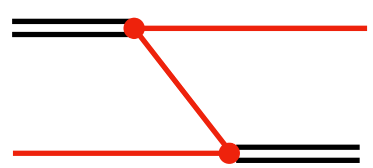
$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

3-BODY QUANTIZATION CONDITION

$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

one-particle exchange

- fixed by 3b-unitarity



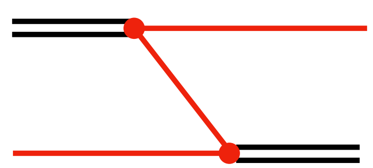
- no free parameters

3-BODY QUANTIZATION CONDITION

$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

one-particle exchange

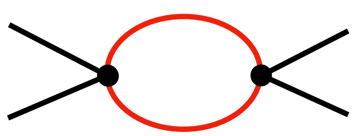
- fixed by 3b-unitarity



- no free parameters

two-body self-energy

- fixed by 2b-unitarity



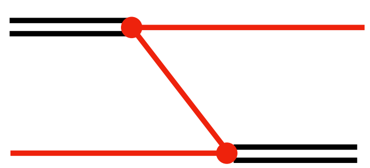
- no free parameters

3-BODY QUANTIZATION CONDITION

$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

one-particle exchange

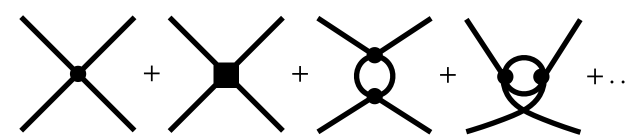
- fixed by 3b-unitarity



- no free parameters

two-body kernel

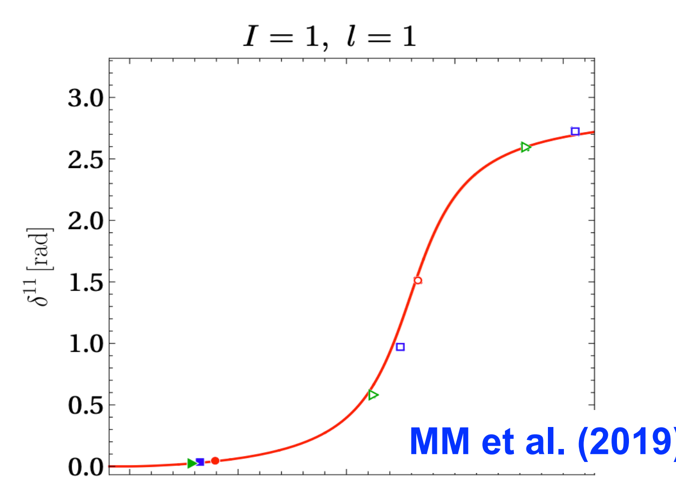
- dynamics of *I=1* $\pi\pi$ system



- regular function ⇒ polynomial

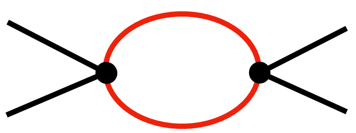
$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

- parameters *(a_0,a_1)* from cross-channel fit to $\pi\pi$ GWQCD levels



two-body self-energy

- fixed by 2b-unitarity



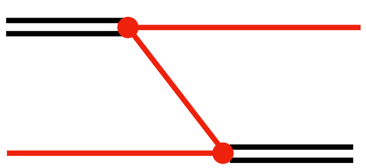
- no free parameters

3-BODY QUANTIZATION CONDITION

$$0 = \det\left[B(s) + C(s) - 2L^3 E_{\mathbf{p}}\left(\tilde{K}_2^{-1}(s) - \Sigma_2^L(s)\right)\right]_{(\lambda'\lambda)(\mathbf{p}'\mathbf{p})}$$

one-particle exchange

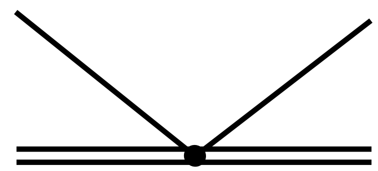
- fixed by 3b-unitarity



- no free parameters

three-body force

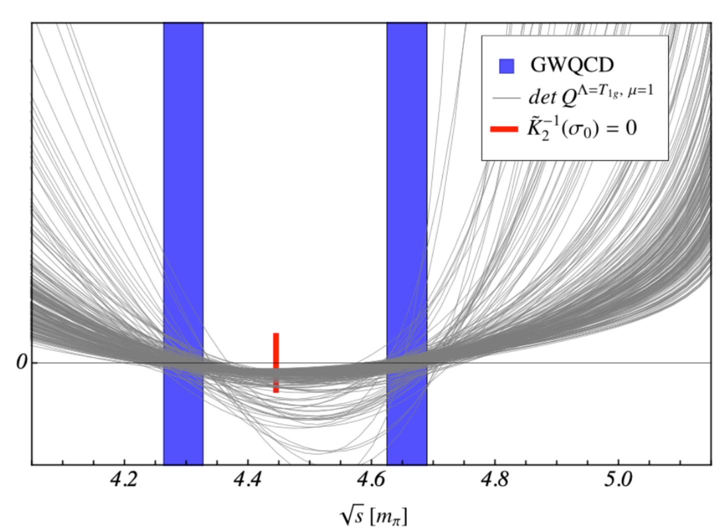
- dynamics of $\rho\pi$ system



- regular function ⇒ Laurent series

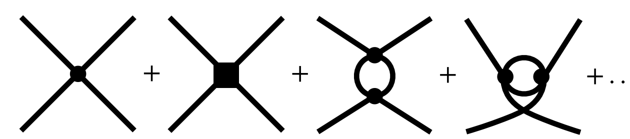
$$C_{\ell'\ell}(s,\mathbf{p}',\mathbf{p}) = \sum_{i=-1}^{\infty} c^{(i)}_{\ell'\ell}(\mathbf{p}',\mathbf{p})(\mathbf{s}-\mathbf{m}^2_{\mathbf{a_1}})^{\mathbf{i}}$$

- fit to 3-body levels



two-body kernel

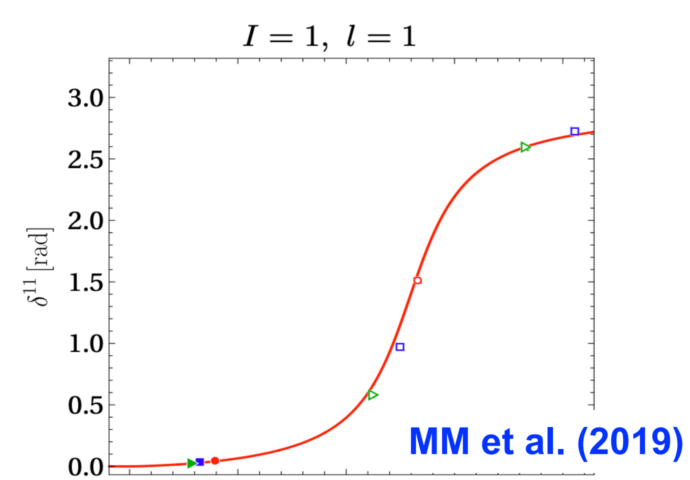
- dynamics of $I=1$ $\pi\pi$ system



- regular function ⇒ polynomial

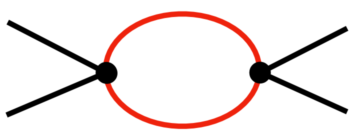
$$\tilde{K}_n^{-1}(s) = \sum_{i=0}^{n-1} a_i \cdot \sigma_p^i$$

- parameters (a_0,a_1) from cross-channel fit to $\pi\pi$ GWQCD levels



two-body self-energy

- fixed by 2b-unitarity

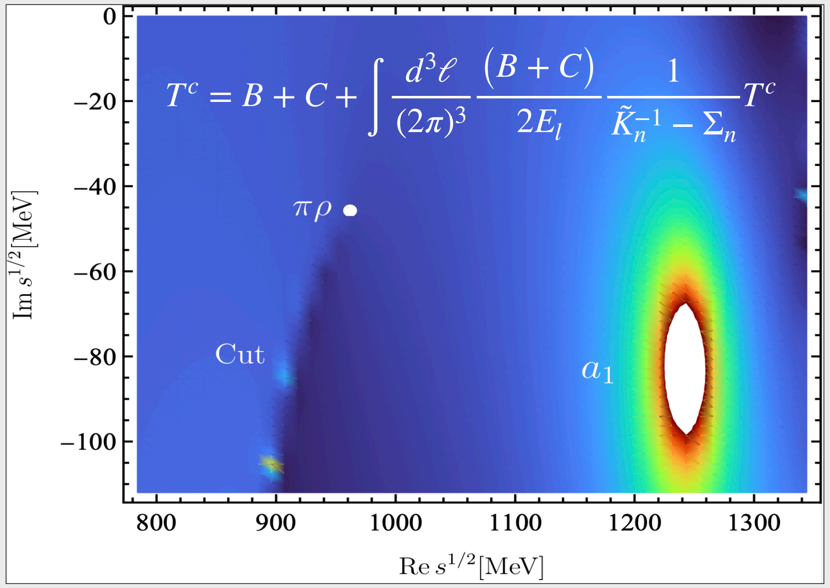


- no free parameters

RESULTS

Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta **Döring et al.(2009) Sadasivan et al. (2020)**
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via
    $$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'}\frac{1}{\mathbf{s}-\mathbf{m}^2_{\mathbf{a_1}}}\mathbf{g}_{\ell}|\mathbf{p}|^{\ell}+\mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$
    ...with large correlations



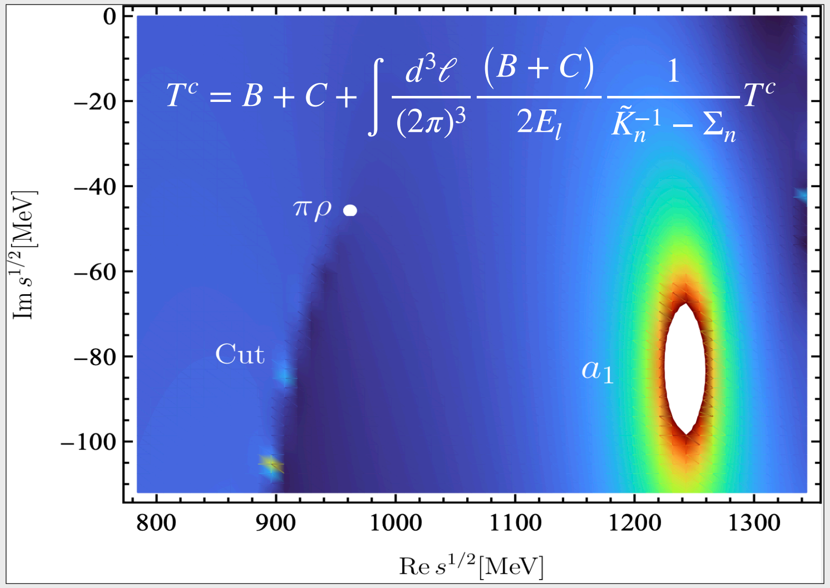
RESULTS

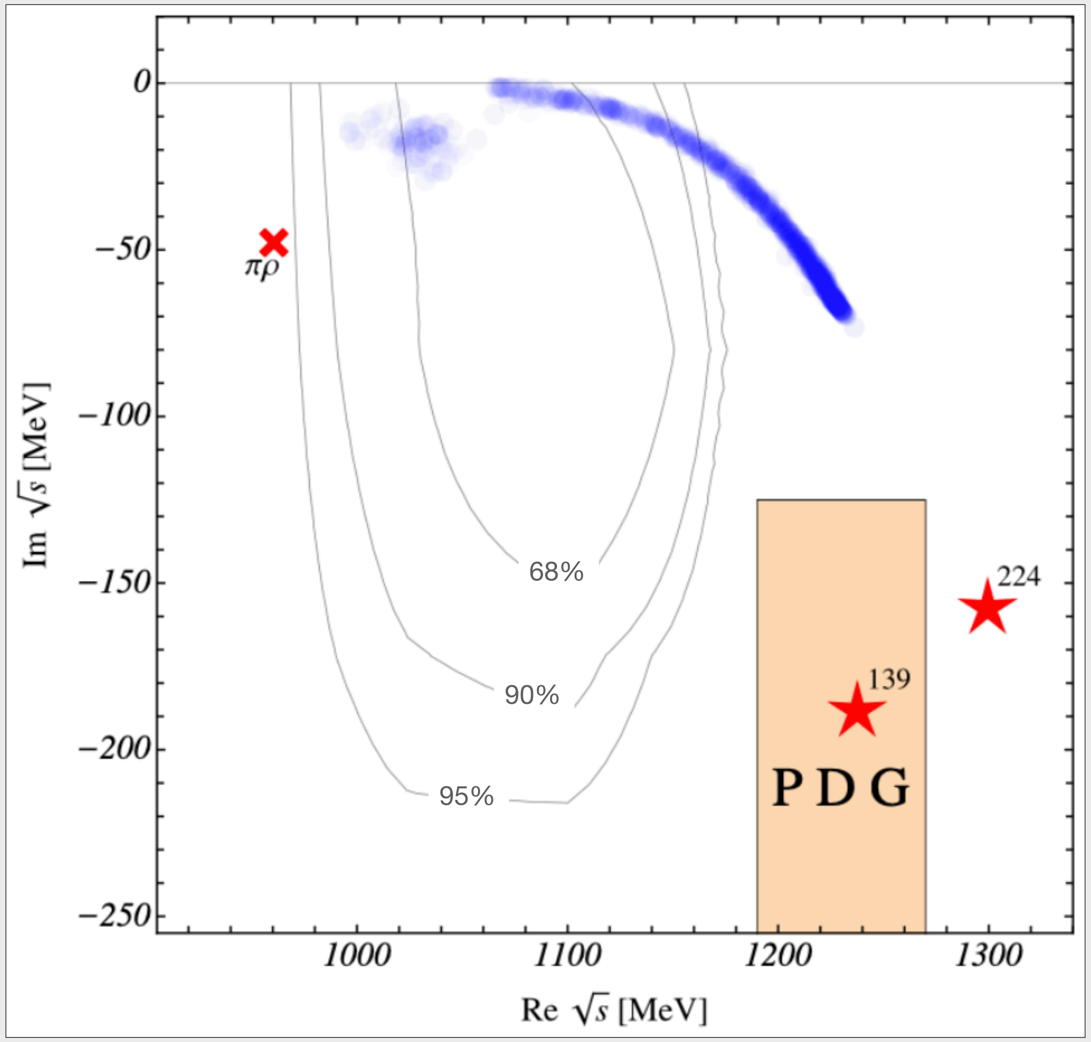
Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta **Döring et al.(2009) Sadasivan et al. (2020)**
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via

$$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'}\frac{1}{\mathbf{s}-\mathbf{m}_{\mathbf{a}_1}^2}\mathbf{g}_\ell|\mathbf{p}|^\ell + \mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$

...with large correlations





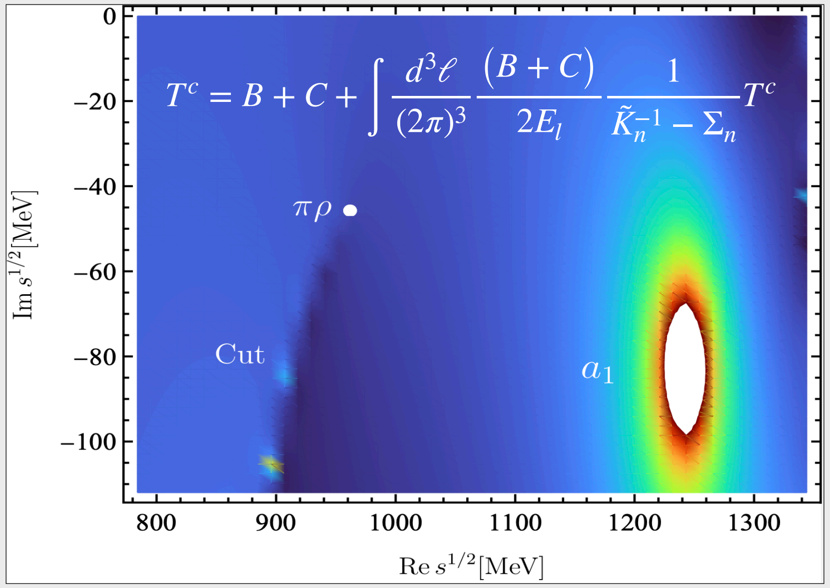
RESULTS

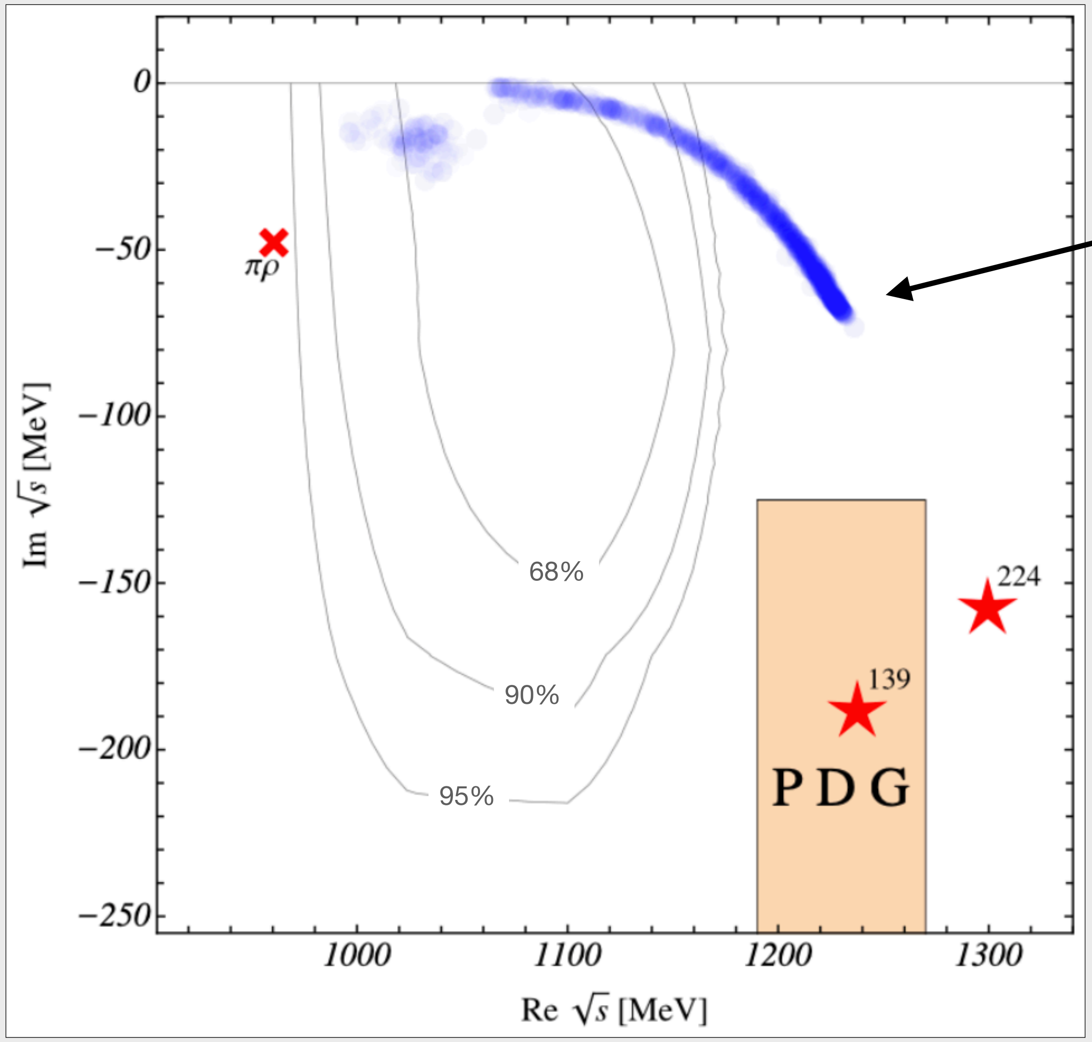
Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta Döring et al.(2009) Sadasivan et al. (2020)
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via

$$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'} \frac{1}{\mathbf{s}-\mathbf{m}^2_{\mathbf{a}_1}} \mathbf{g}_\ell |\mathbf{p}|^{\ell} + \mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$

...with large correlations





distribution is finite
~1230-i60 MeV

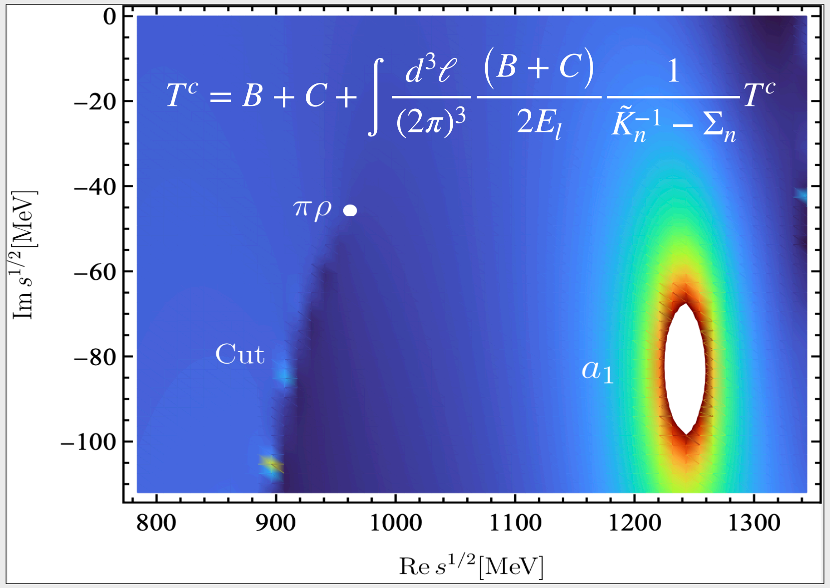
RESULTS

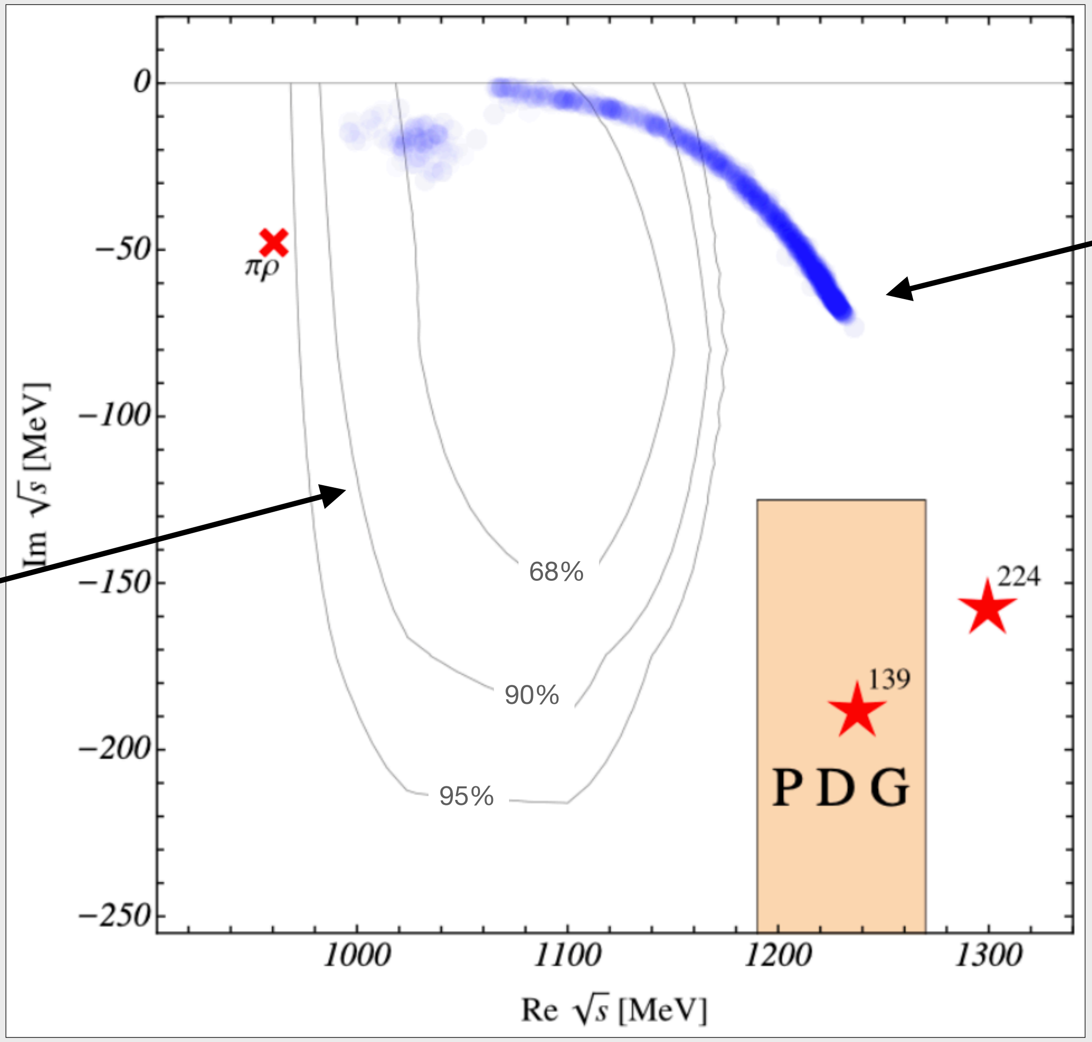
Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta   Döring et al.(2009) Sadasivan et al. (2020)
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via

$$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'}\frac{1}{\mathbf{s}-\mathbf{m}^2_{\mathbf{a}_1}}\mathbf{g}_\ell|\mathbf{p}|^{\ell}+\mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$

...with large correlations





distribution is finite
~1230-i60 MeV

Lüscher + Breit-Wigner approximation

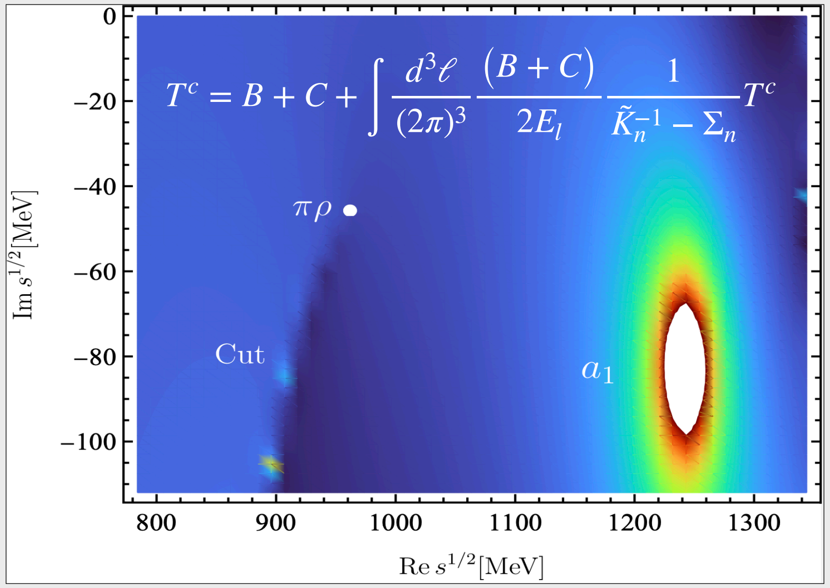
c.f. Lang et al. JHEP 04(2014)

RESULTS

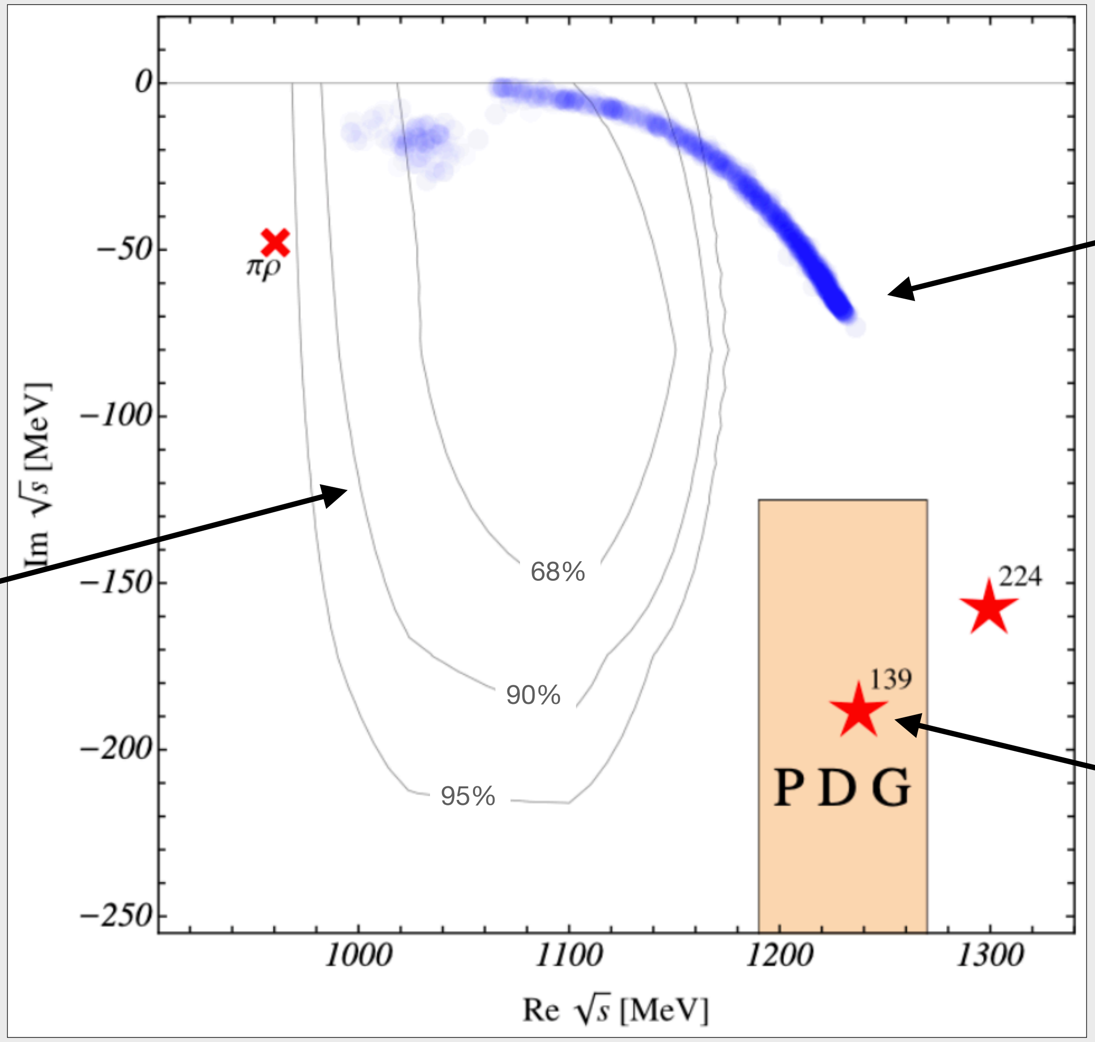
Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta Döring et al.(2009) Sadasivan et al. (2020)
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via $$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'}\frac{1}{\mathbf{s}-\mathbf{m}_{\mathbf{a}_1}^2}\mathbf{g}_\ell|\mathbf{p}|^{\ell}+\mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$

    ...with large correlations

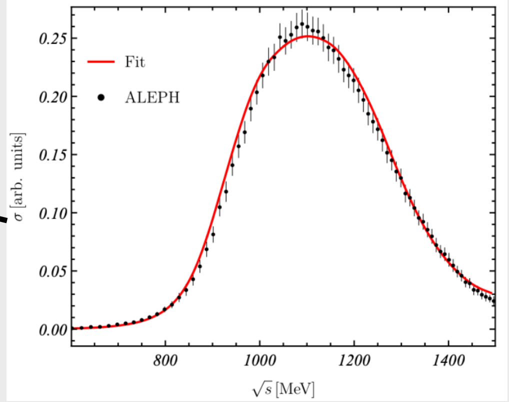


distribution is finite
~1230-i60 MeV



Lüscher + Breit-Wigner approximation

c.f. Lang et al. JHEP 04(2014)



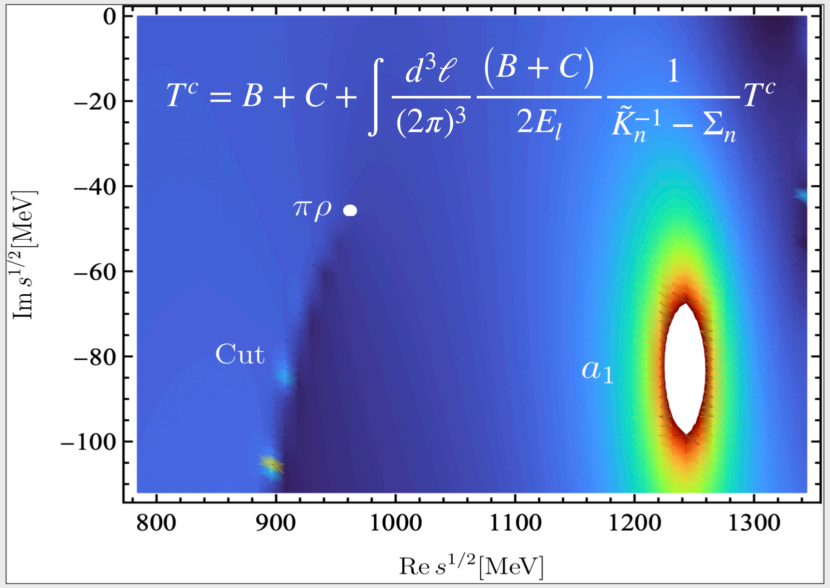
RESULTS

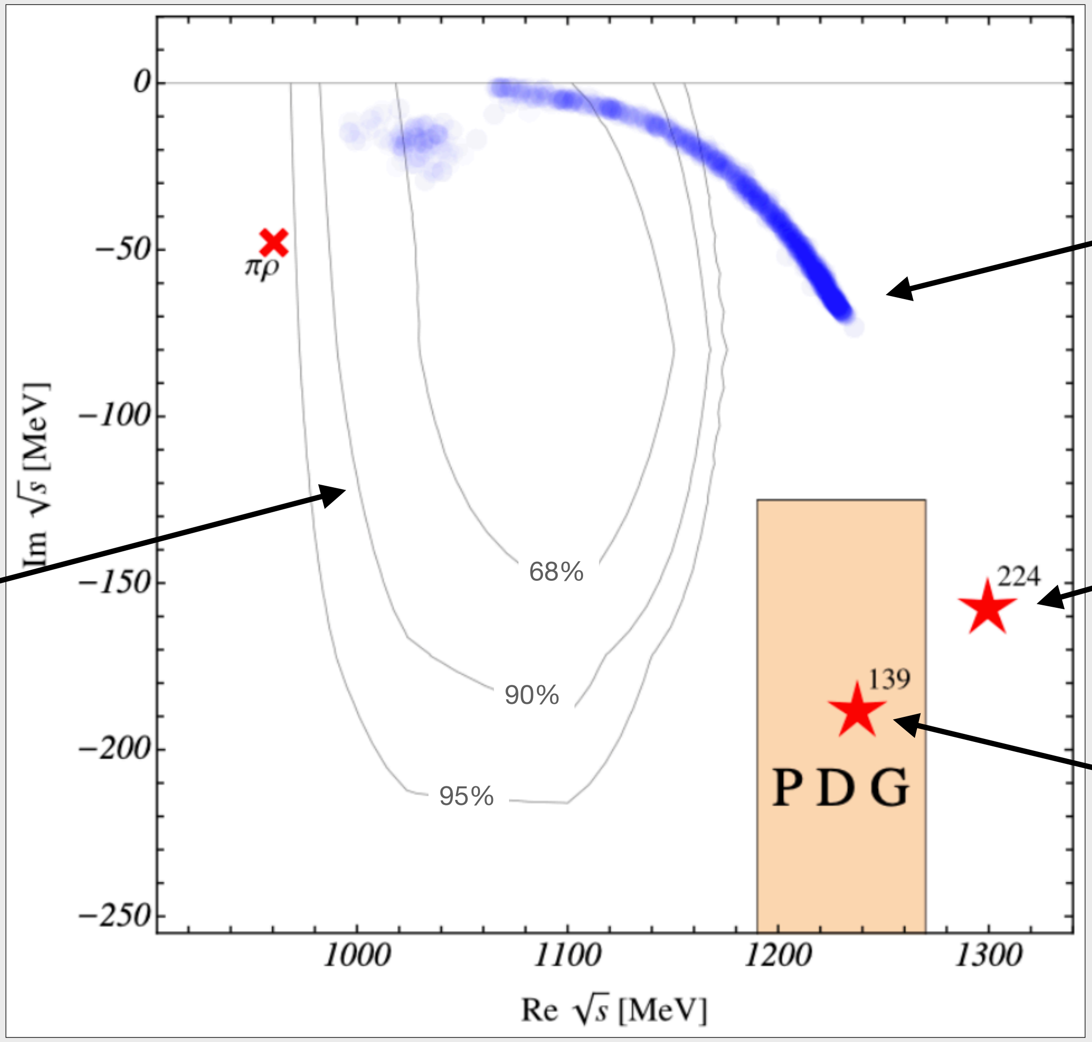
Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta **Döring et al.(2009) Sadasivan et al. (2020)**
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via

$$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'} \frac{1}{\mathbf{s}-\mathbf{m}_{\mathbf{a_1}}^2} \mathbf{g}_\ell |\mathbf{p}|^\ell + \mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$

...with large correlations



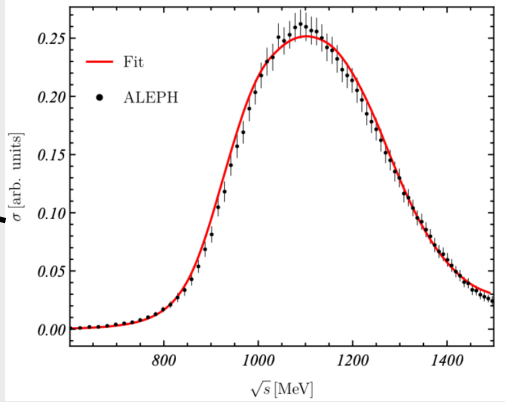


distribution is finite
~1230-i60 MeV

(naive) chiral extrapolation
confirms expectation

Lüscher + Breit-Wigner
approximation

c.f. Lang et al. JHEP 04(2014)



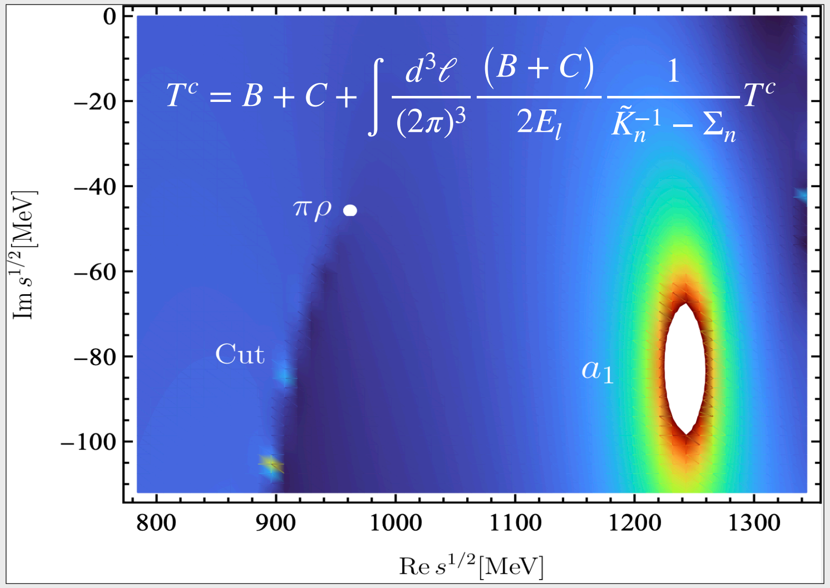
RESULTS

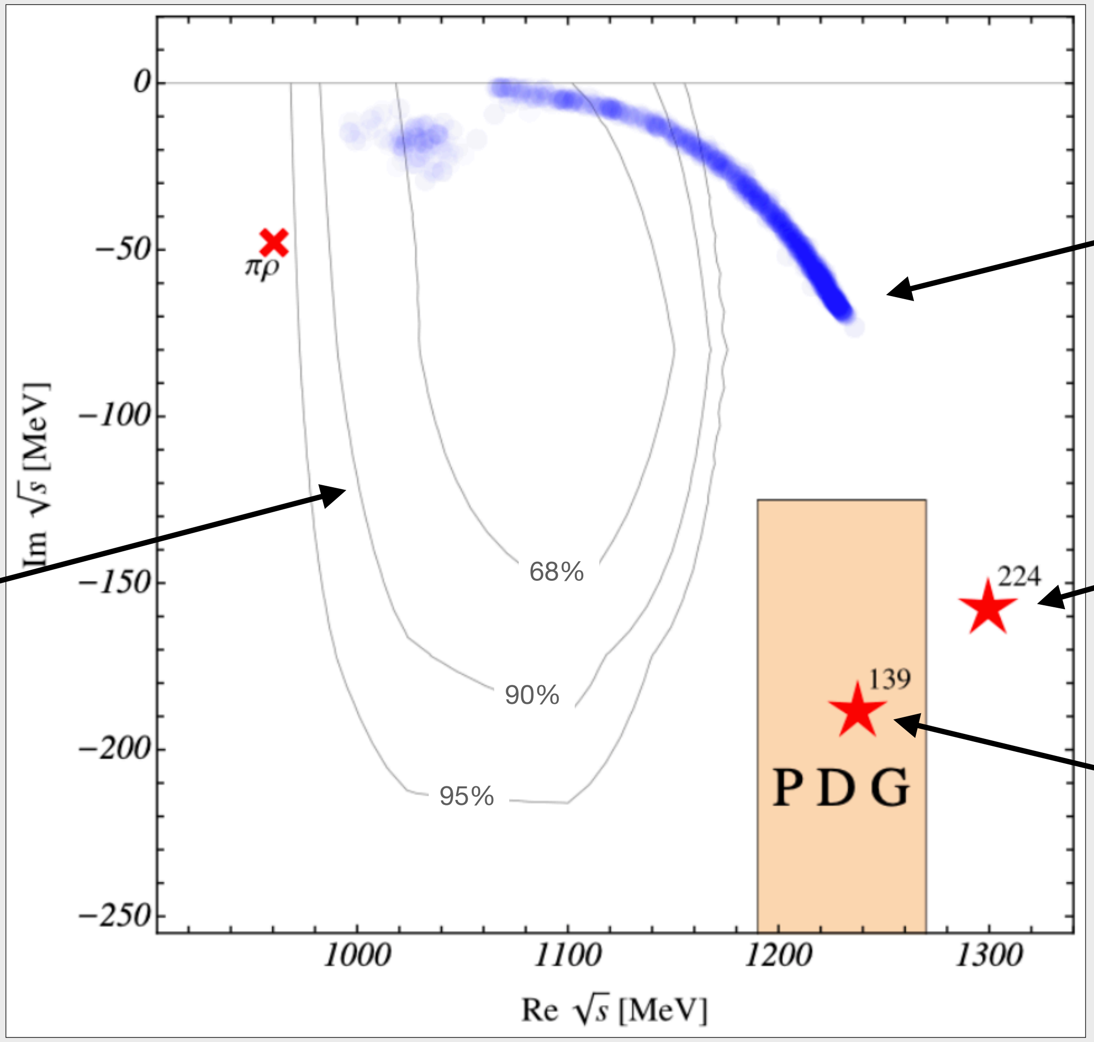
Resonance poles

- ∞-vol. scattering equation via contour deformation of spectator momenta Döring et al.(2009) Sadasivan et al. (2020)
- various forms of the 3-body term C tested:
  - pole is generated with or without explicit pole-term
  - best description via

$$C_{\ell'\ell} = g_{\ell'}|\mathbf{p}'|^{\ell'} \frac{1}{\mathbf{s} - \mathbf{m}_{\mathbf{a}_1}^2} \mathbf{g}_\ell |\mathbf{p}|^{\ell} + \mathbf{c}\,\delta_{\ell'0}\delta_{\ell 0}$$

...with large correlations



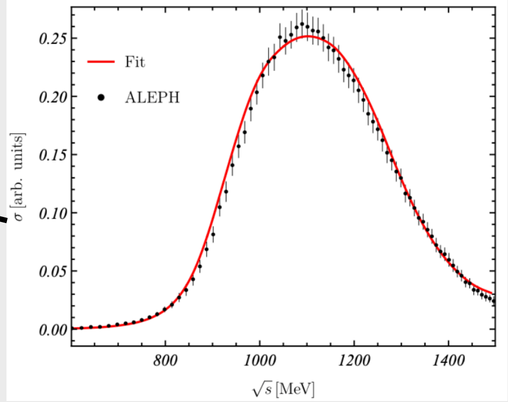


distribution is finite
~1230-i60 MeV

Lüscher + Breit-Wigner approximation

c.f. Lang et al. JHEP 04(2014)

(naive) chiral extrapolation confirms expectation



THANK YOU