

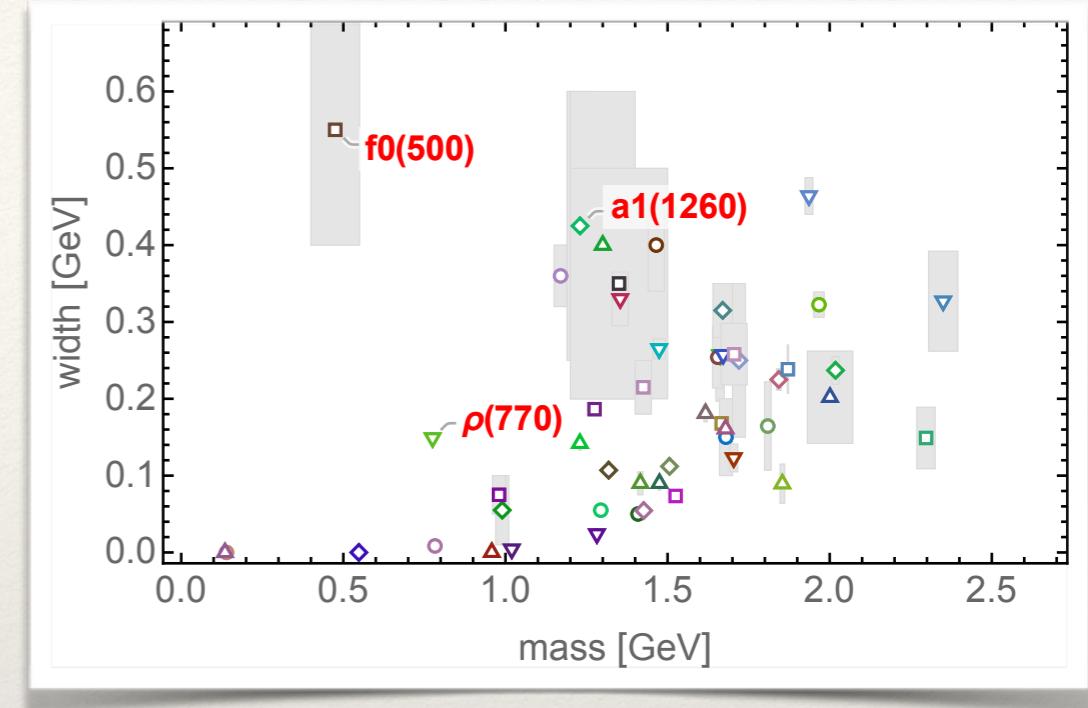


2- and 3-body hadron spectroscopy from lattice QCD

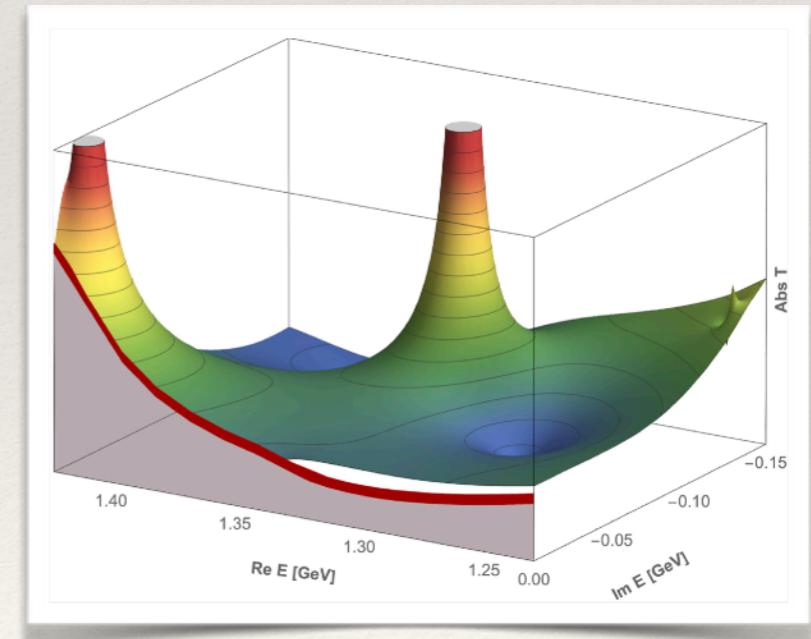
Maxim Mai
maxim-mai.github.io/exo.pdf

QCD at low energies

- ❖ Confinement
- ❖ Mass generation
- ❖ Intricate spectrum of excited states



- ❖ Universal parameters of resonances:
 - analytic properties of *scattering amplitude*
 - pole position = **(mass - i width/2)**
 - residuum \sim **(coupling constant)**



Puzzles

❖ $a_1(1260)$

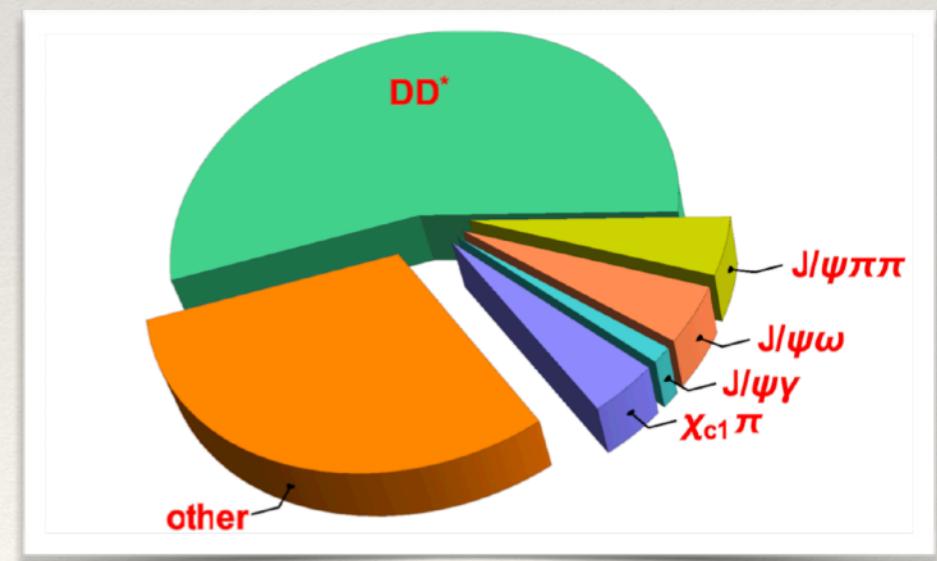
- does **not** decay into $\pi\pi$ but only $\pi\pi\pi$ channel
- test channel for the search for
spin-exotics → gluonic d.o.f. (GlueX, COMPASS)



❖ $X(3872)$

- puzzling production mechanism
- large BR to $D\bar{D}\pi$
- ...similarly for further heavy XYZ exotics

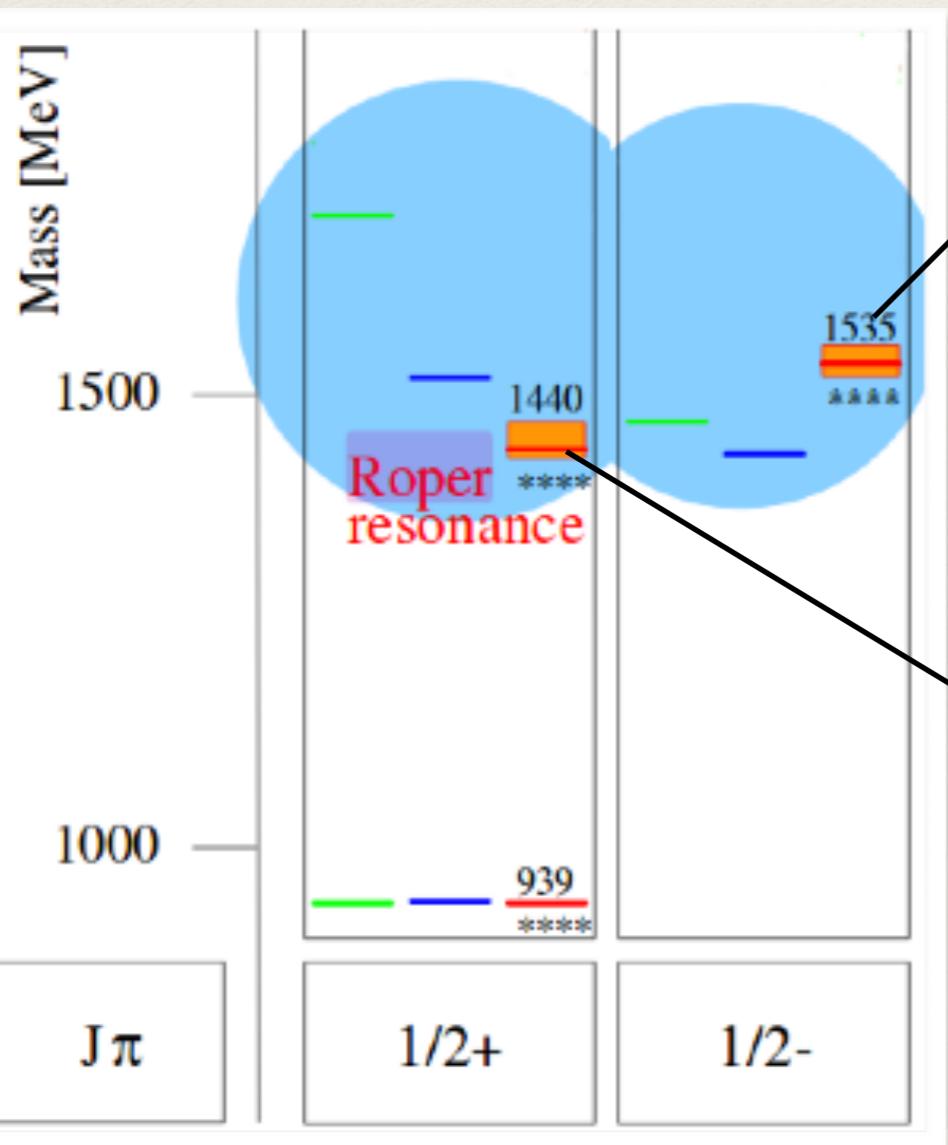
Belle 2003



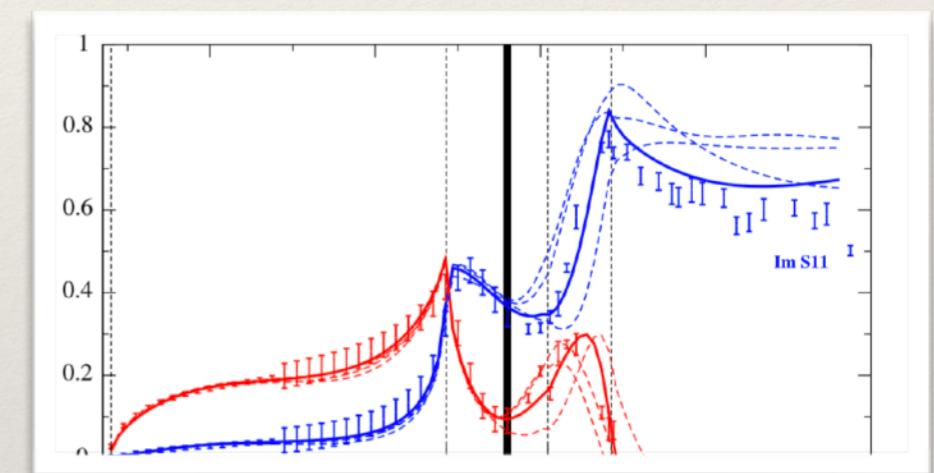
Puzzles

❖ Roper-puzzle

reversed mass pattern cf.
constituent Quark Model



$N(1535)1/2^-$ easily accessible from
dynamical approaches, e.g.,



Bruns, MM, Meissner PLB 697(2011)

...but $N(1440)1/2^+$ has large
BR to $\pi\pi N$



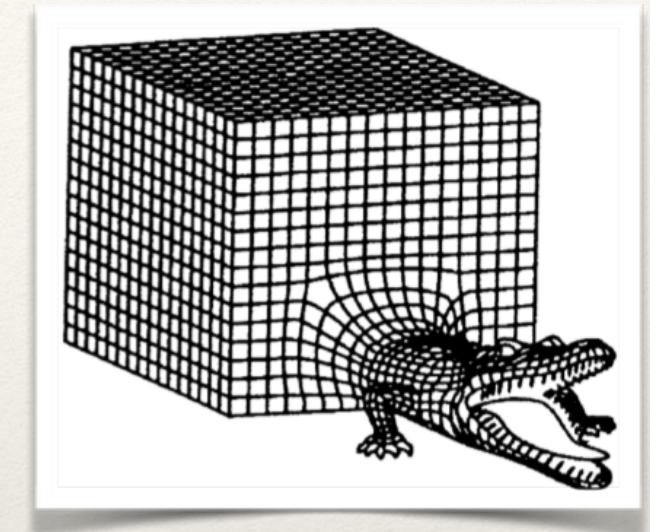
Lattice QCD

- ❖ the only systematic (non-perturbative) approach to QCD
- ❖ numerical ab-initio calculations

... with some technical hurdles:

1. discretized Euclidean space-time

- continuum limit



2. unphysical pion mass

- well established techniques from ChPT
- extensions to SU(3), baryons, resonances, ...

Gasser/Leutwyler 1984

3. finite volume

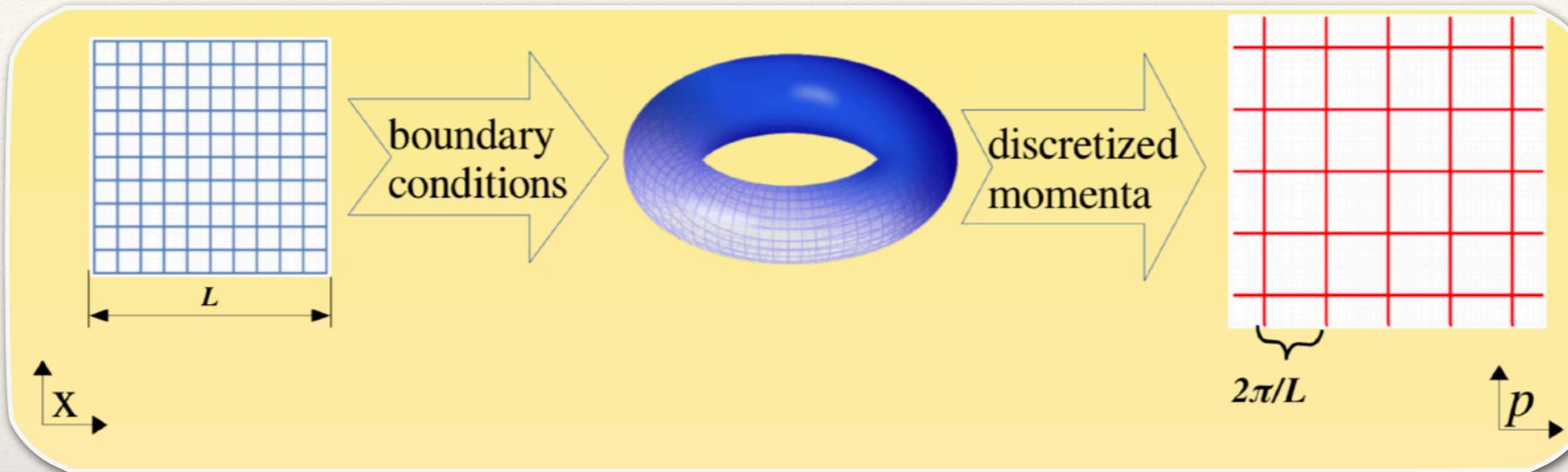
- well understood for 2 hadrons
- new developments for 3-hadron systems

Luescher 1986

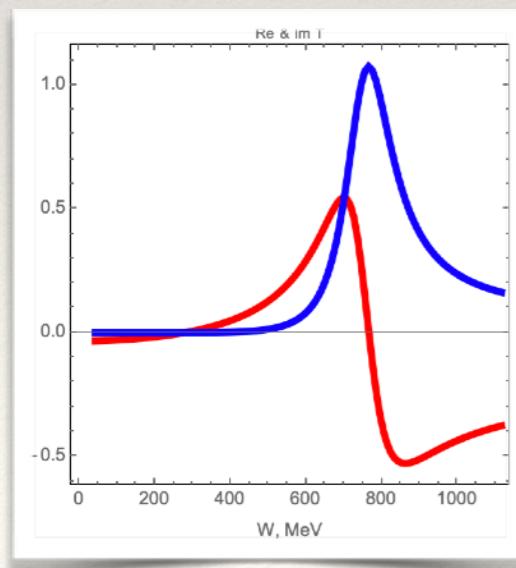
...later in this talk

Interaction spectrum in a box

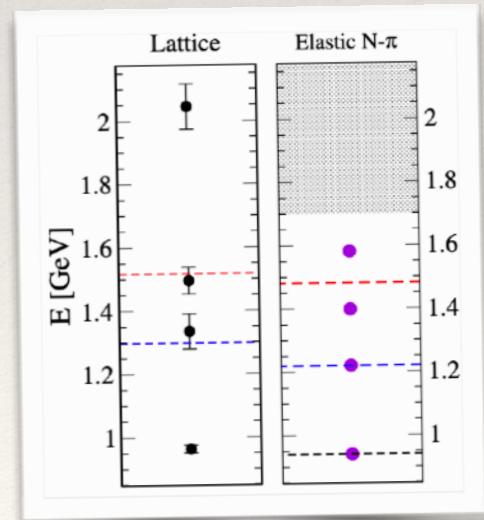
- ❖ Lattice calculations are performed in finite volume



- all momenta are discretized
- interaction spectrum is **real-valued** and **discrete**



QUANTIZATION CONDITION



State of the art

❖ 2-body case

- well established (**Luescher's method**)
- **one-to-one mapping** between phase-shifts and energy eigenvalues
- extensions for coupled-channels, spin, ...

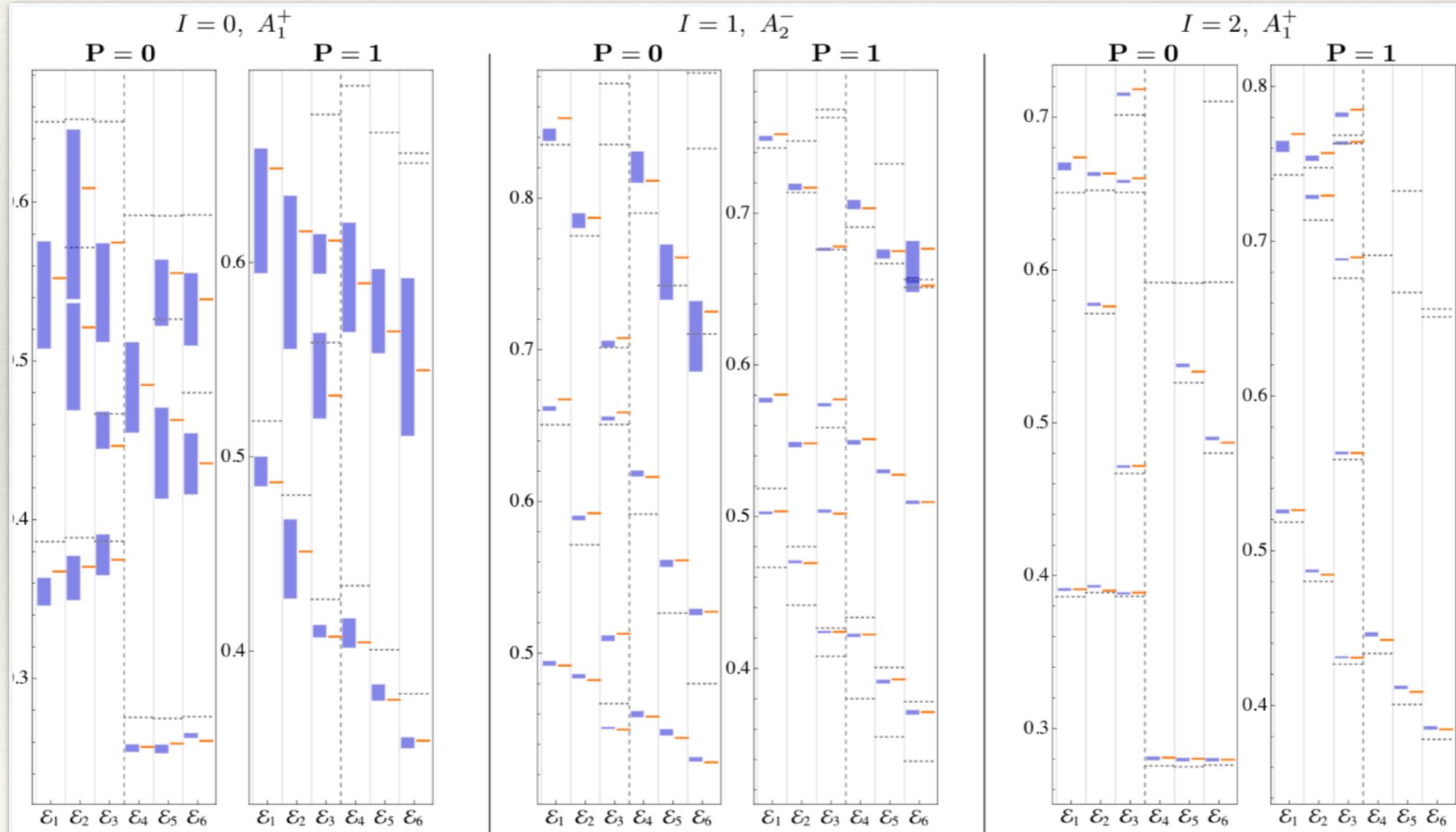
Lüscher (1986)

**Gottlieb, Rummukainen, Feng, Li, Liu, Doring,
Briceno, Rusetsky, Bernard, Meissner...**

Example: $\pi\pi$ scattering

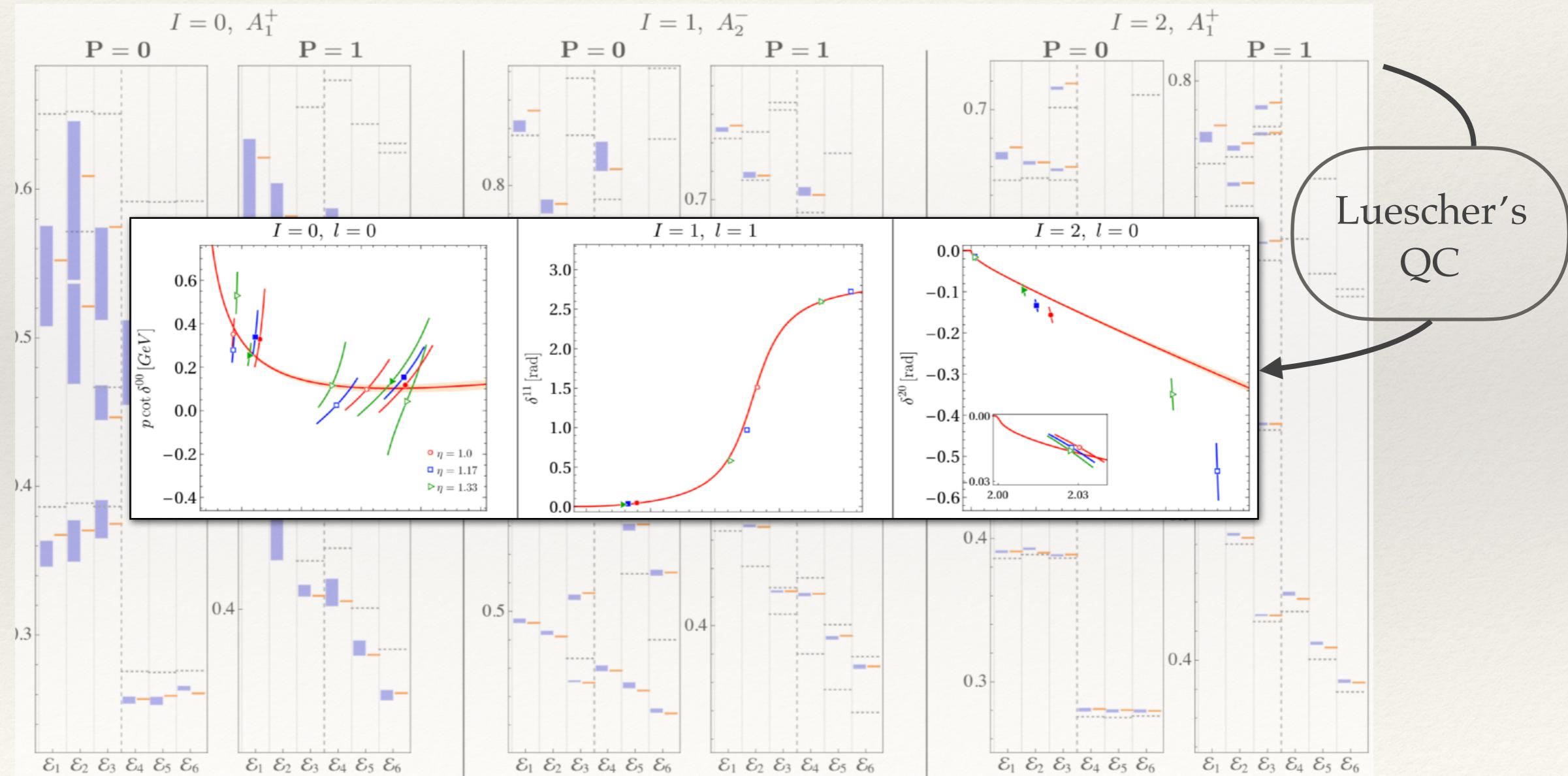
- numerous calculations of the phase-shift for all three channels
NPLQCD; HadSpec; ETMC; GW-lattice; CP-PACS;....
- simultaneous description of all $\pi\pi$ interaction channels

GW-Lattice: Guo et al. (2016) Guo et al. (2018) Culver et al. (2019) MM et al.(2019)



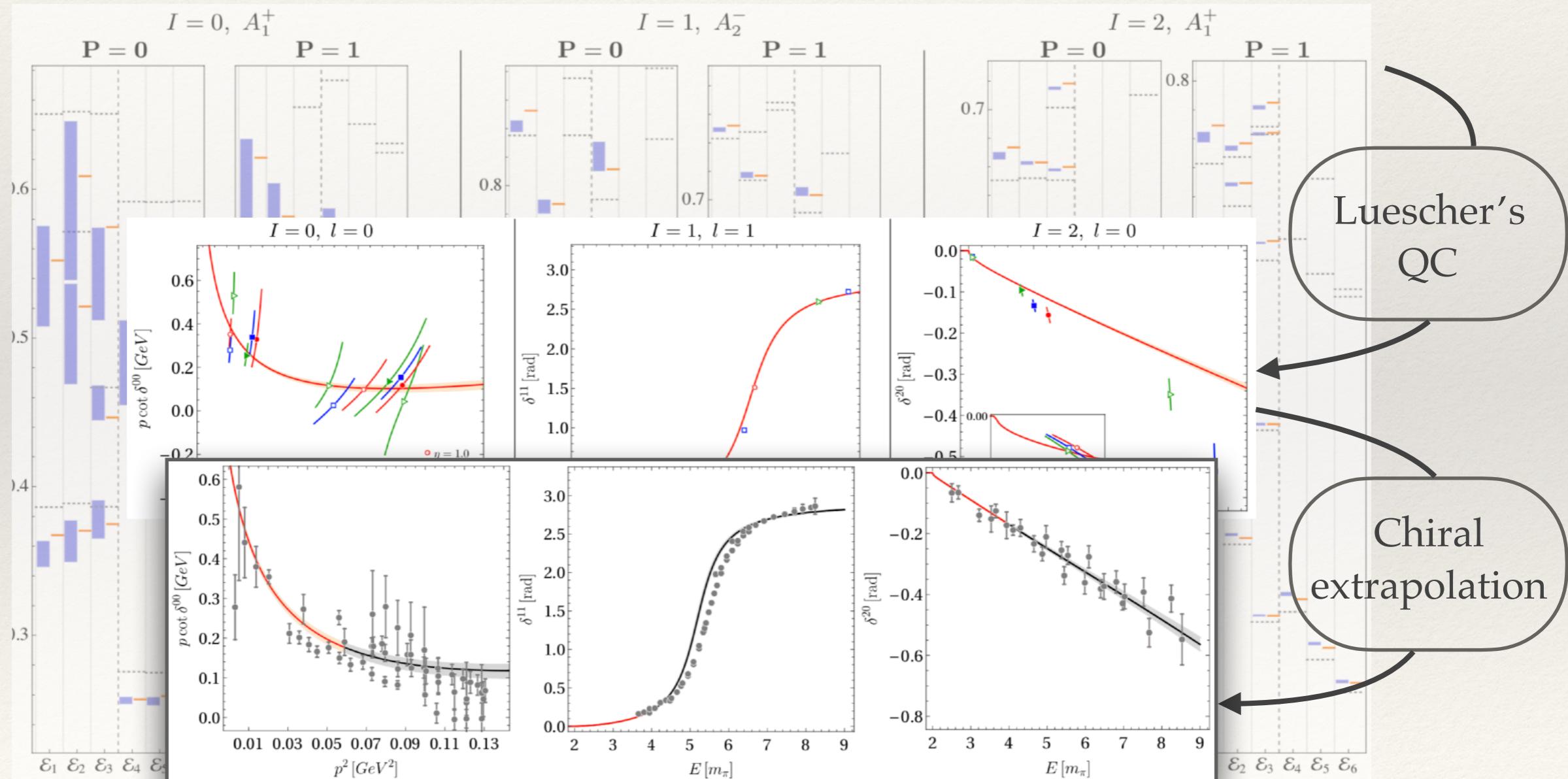
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State of the art

- ❖ 3-body case
 - complex kinematics (*8 variables*)
 - complex interaction structure (*sub-channels*)
 - theoretical developments & numerical investigations

Rusetsky, Polejaeva, Sharpe, Meissner, Davoudi, Hansen, Guo, Briceno, MM,
Doring, Hammer, Wu, Blanton, Griesshammer, Romero-López, Pang, ...

Review: Hansen,Sharpe (2019)

- first application to Lattice QCD results

MM, Doring (2018)

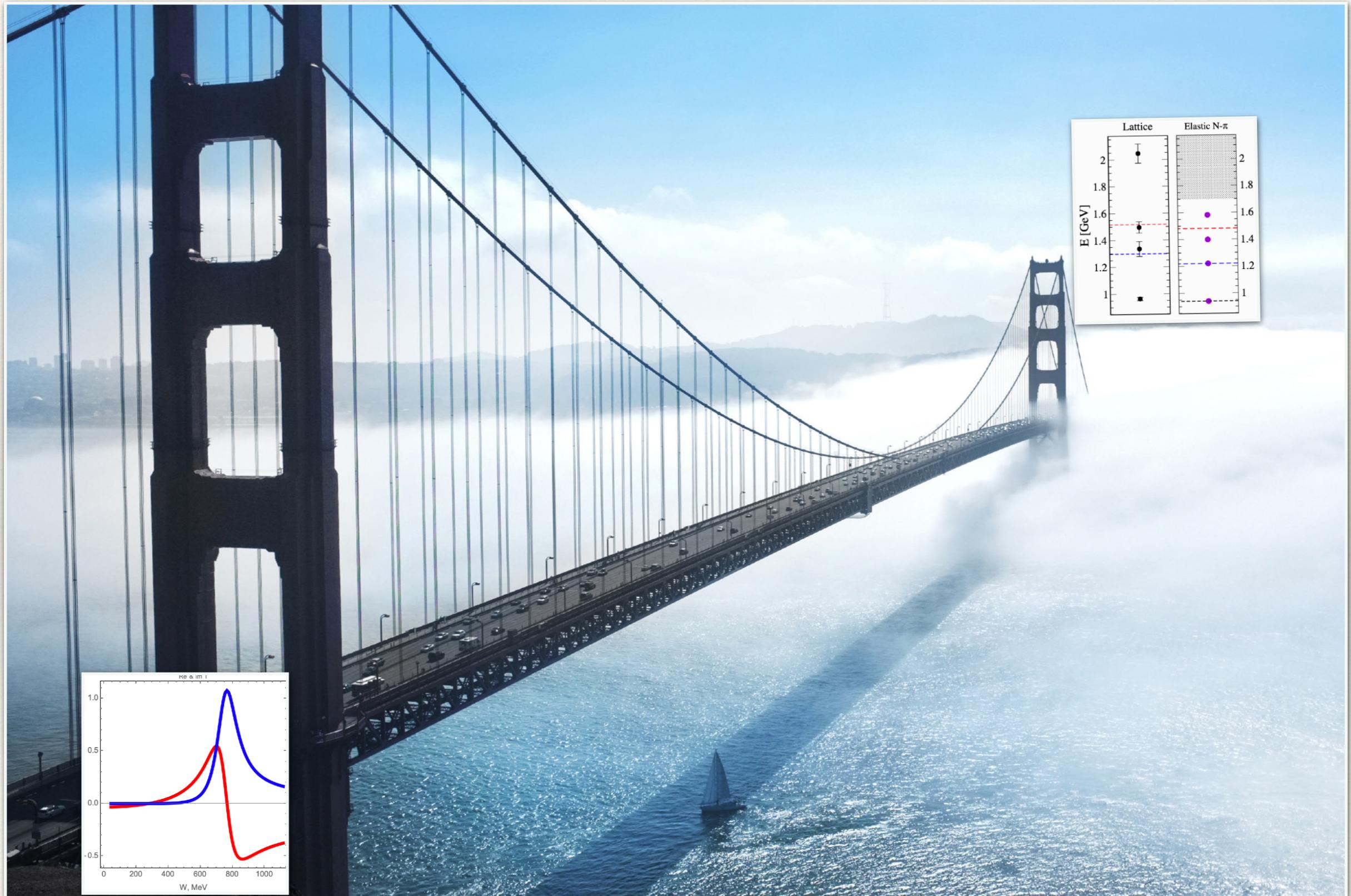
Blanton, Romero-López, Sharpe (2019)

MM, Alexandru, Culver, Doering (2019)

Culver, MM, Brett, Doring, Alexandru (2019)



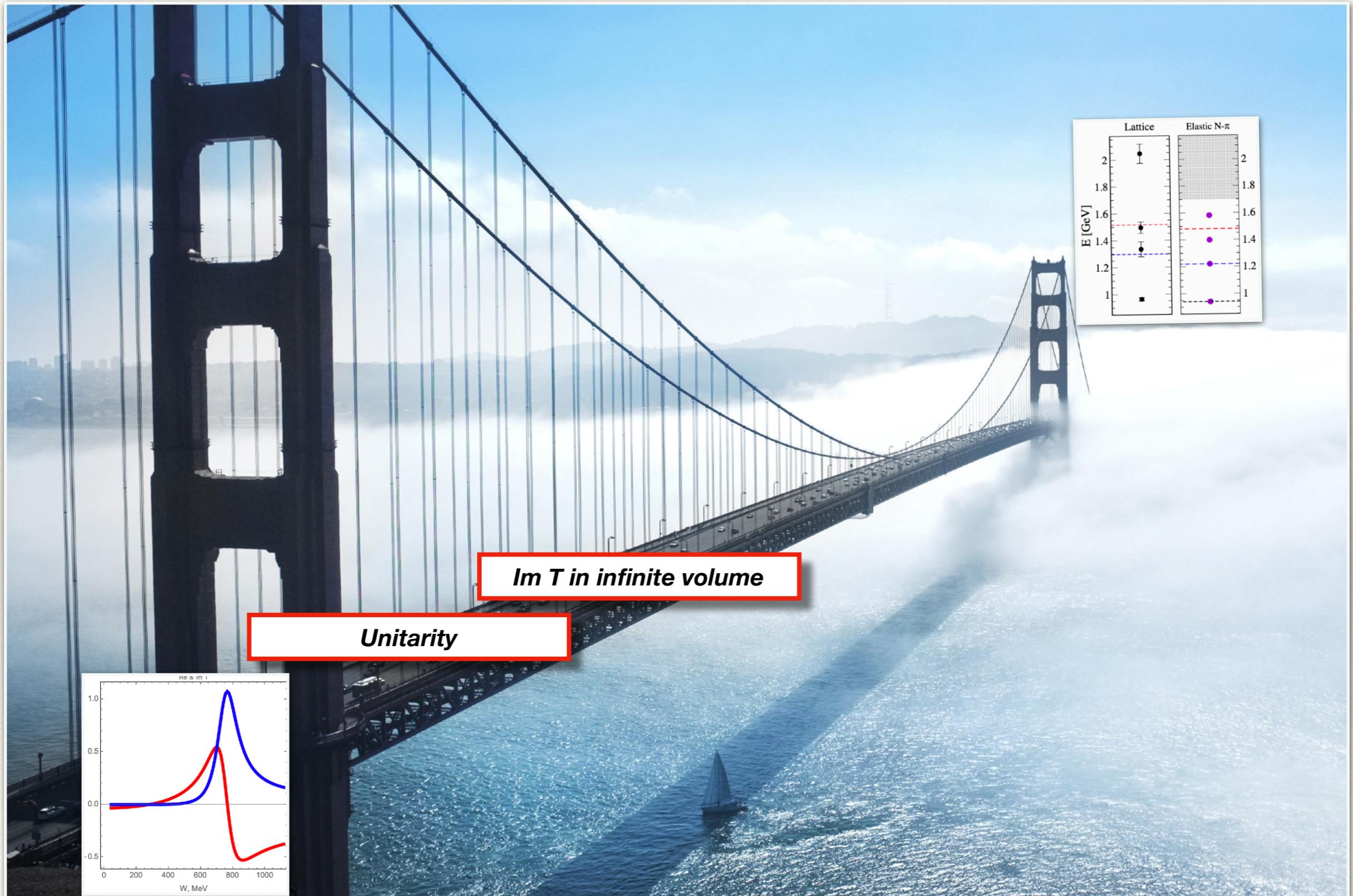
Quantization condition in a nutshell



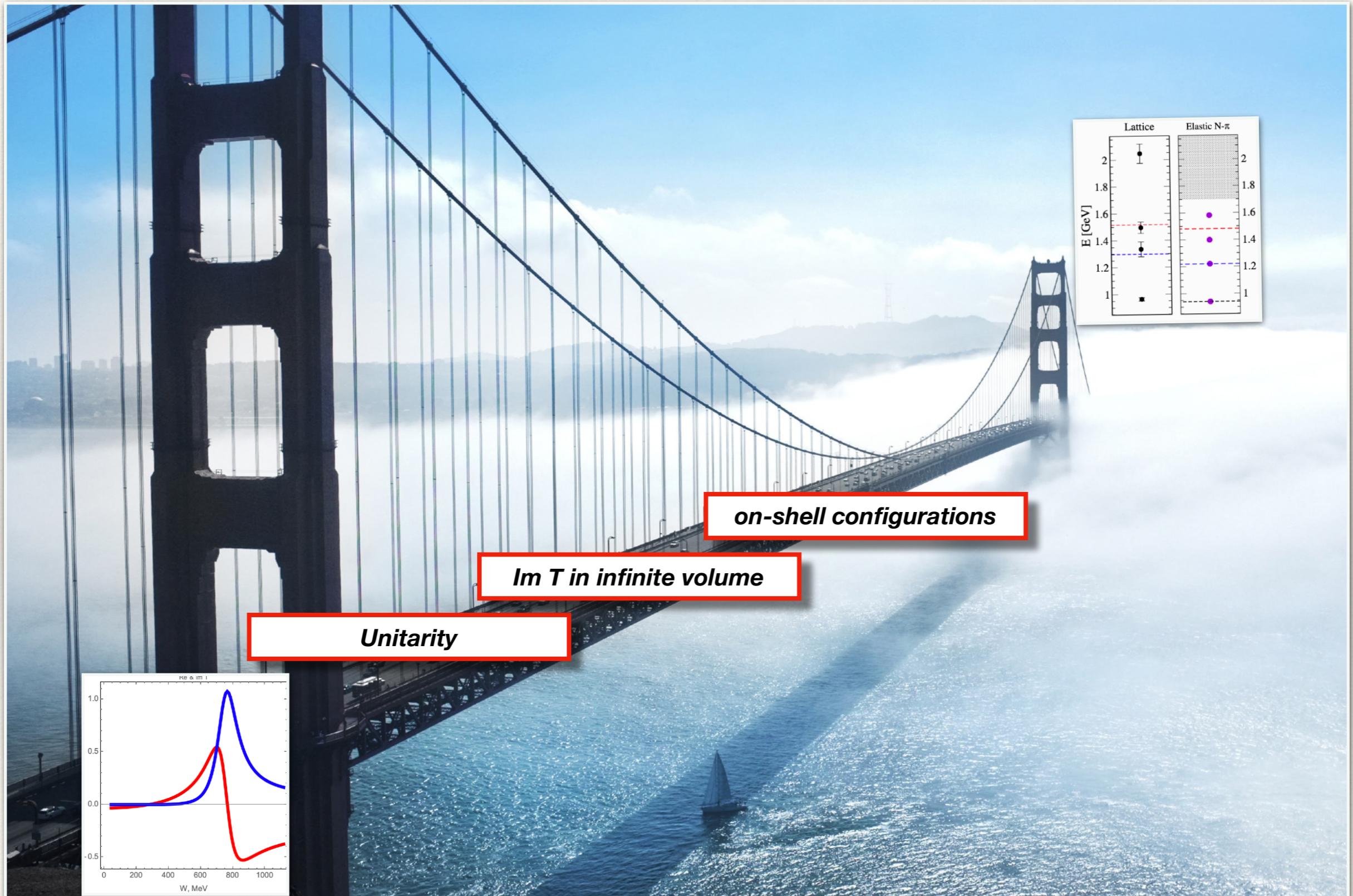
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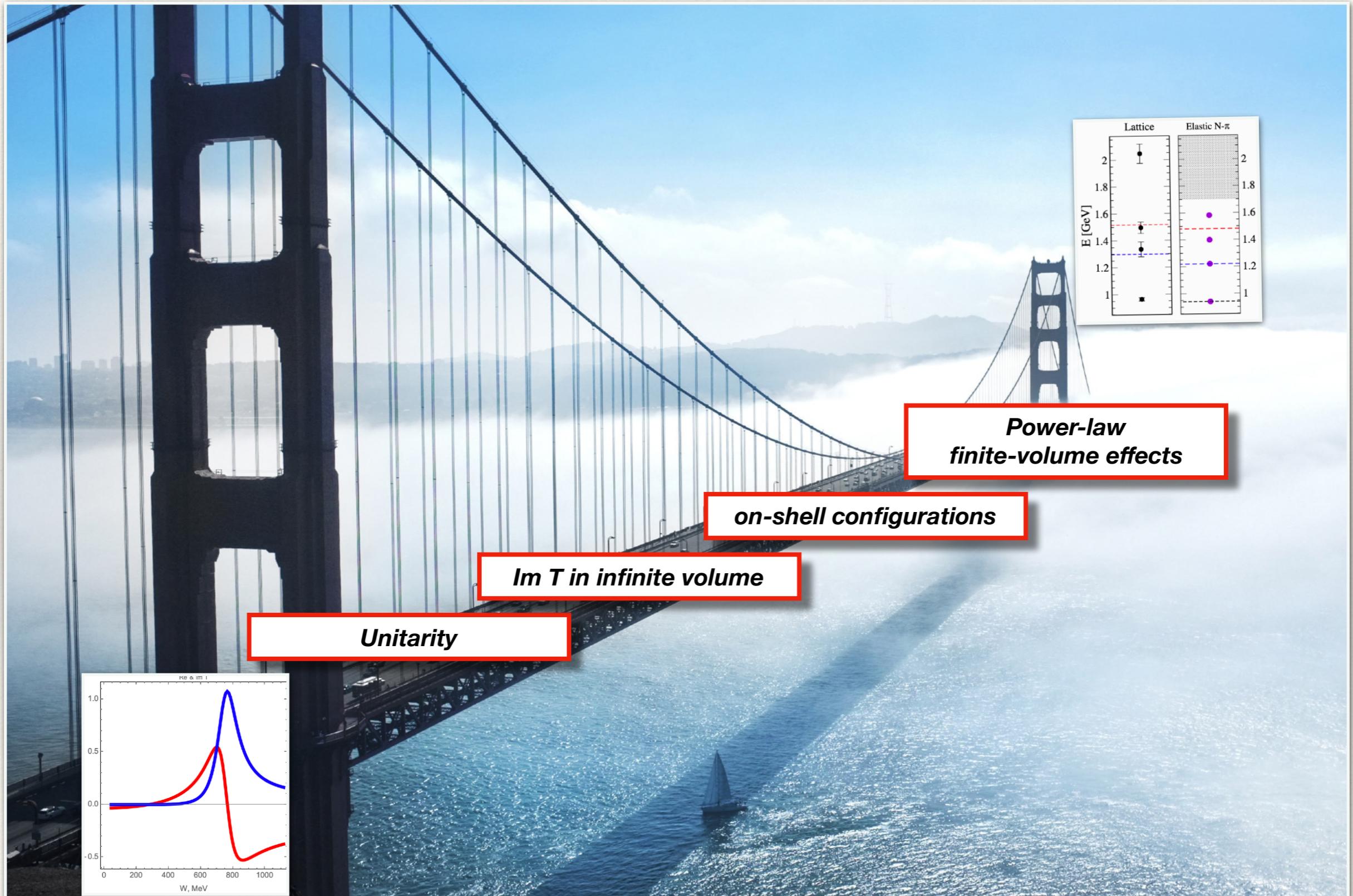
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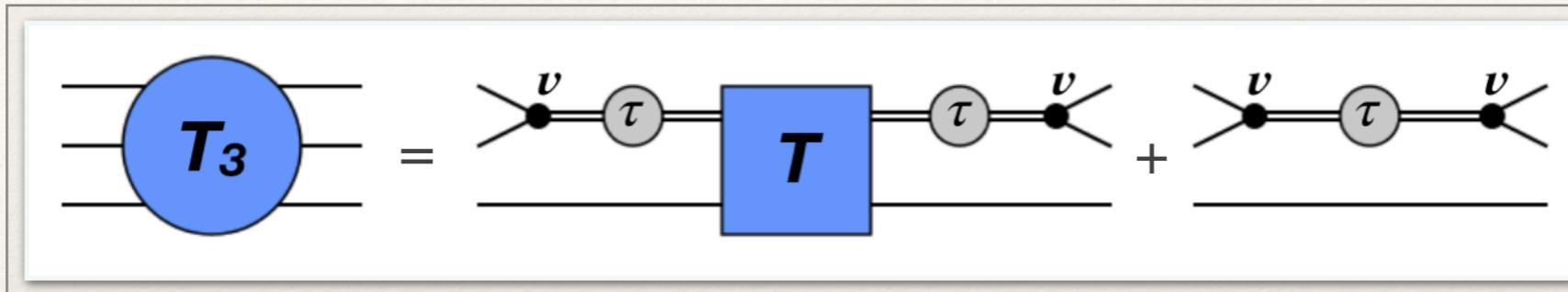


Quantization condition in a nutshell



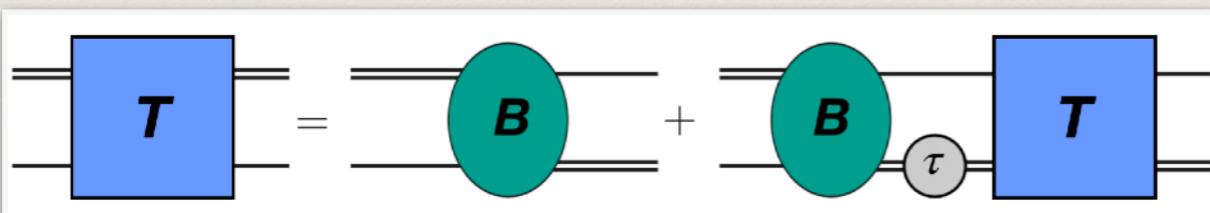
Relativistic unitary scattering amplitude

MM, Hu, Doring, Pilloni, Szczeplaniak Eur.Phys.J. A53 (2017)



connected part

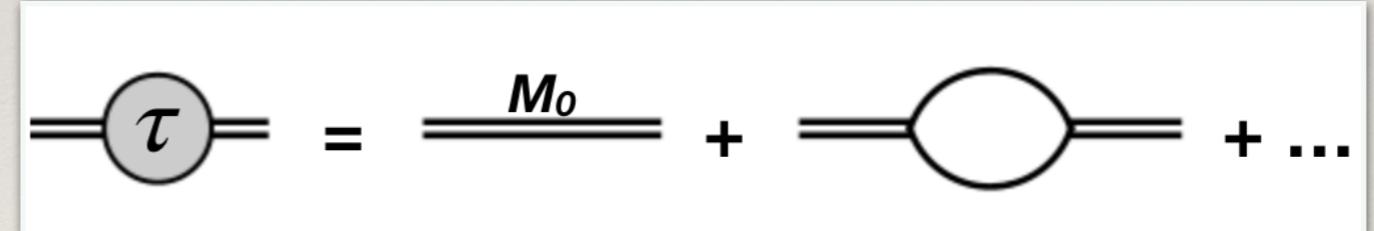
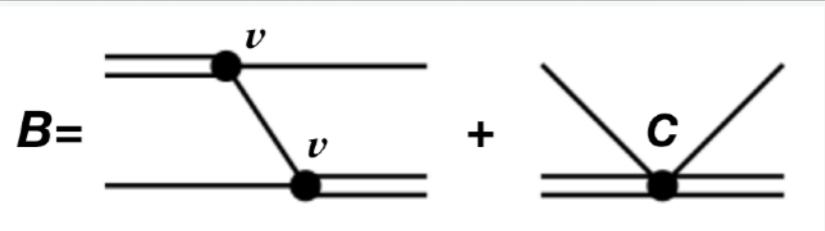
- general 4d BSE-like equation w.r.t kernel B



disconnected part

- spectator + tower of functions $\tau(Minv)$ with correct right-hand-singularities
- coupling $v(q,p)$ is cut-free

B & τ from 3-body unitarity



- analytic properties and generalization

Jackura et al. [JPAC] EPJ C79 (2019)

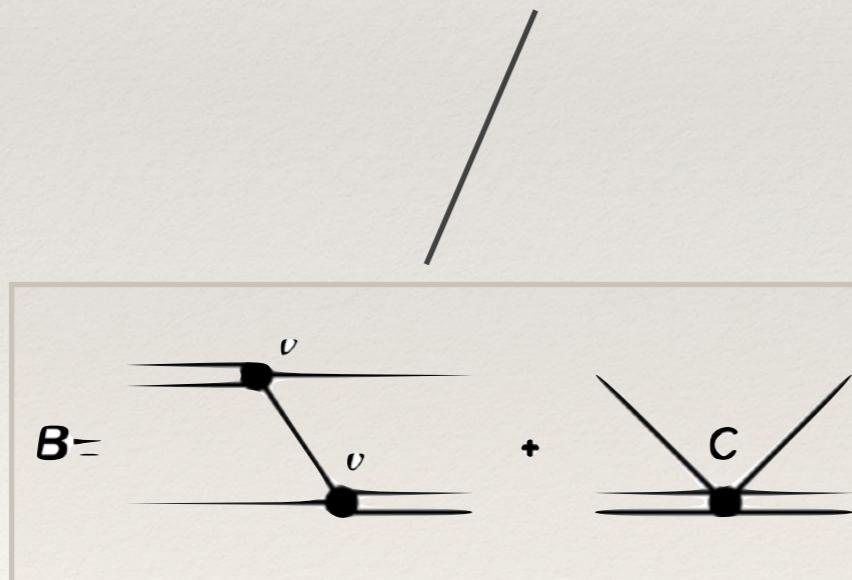
Quantization condition in a nutshell



Relativistic 3b QC

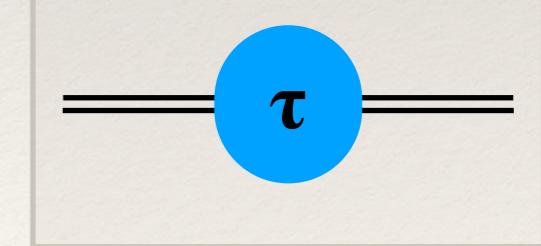
- ❖ In “finite volume world” momenta are discrete → replace integrals by sums
 - ⇒ scattering amplitude is **real-valued** and **singular matrix equation**
 - ⇒ singularities, iff total energy (E^*) is such that 3 pions are **on-shell**
 - ⇒ **relativistic 3-body quantization condition**

$$\left\langle v(\mathbf{p}_1, \mathbf{p}_2) \left[\mathbf{B}(E^*) + E_L \tau_{LP}^{-1}(E^*) \right]_{\mathbf{p}_3, \mathbf{q}_3}^{-1} v(\mathbf{q}_1, \mathbf{q}_2) \right\rangle_{\mathbf{p}_i, \mathbf{q}_j} = \infty$$



~ OPE and 3-body force

$$[E_L]_{\mathbf{p}\mathbf{q}} = \delta_{\mathbf{p}\mathbf{q}} 2L^3 \sqrt{m_\pi^2 + \mathbf{p}^2}$$



~ 2-b partial wave

Lattice studies of 3b systems

❖ $I=3 \pi\pi\pi$

Beane et al. [NPLQCD] PRL100 (2008)

❖ $X(2872)$

Prelovsek, Leskovec PRL111 (2013)

❖ $a_1(1260)$

Lang et al. JHEP 1404

❖ $N(1440)1/2^+$

Lang et al. PRD 95(2017)

❖ $I=2 \pi\eta$

Woss et al. JHEP 1807 [Hadspec]

❖ $\pi\omega/\pi\varphi$

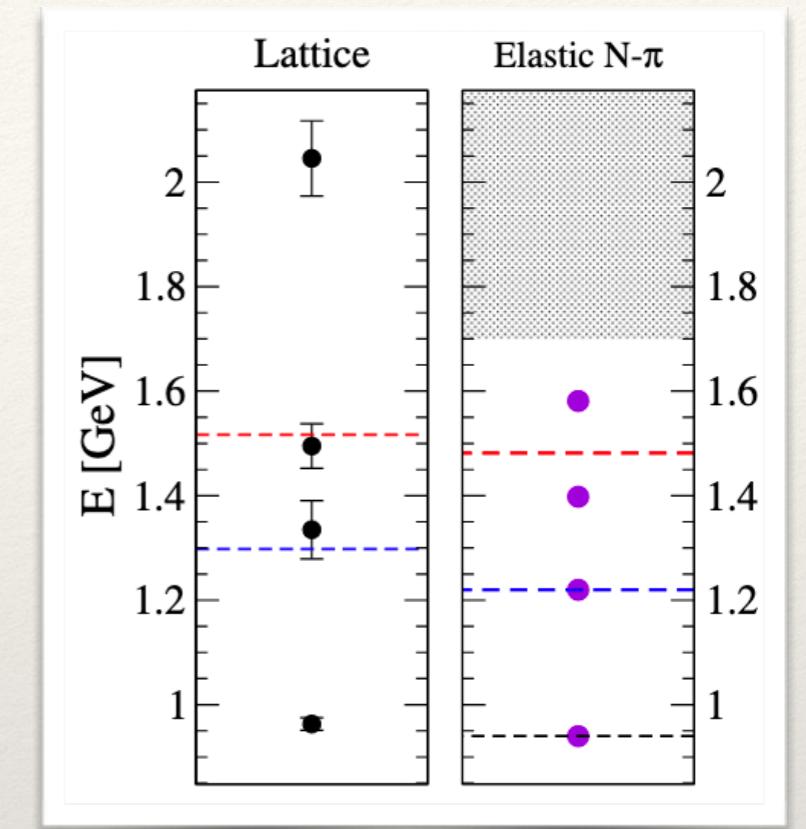
Woss et al (2019)

❖ $I=3 \pi\pi\pi$

Hörz/Hanlon PRL123 (2019)

Culver/MM/Brett/Doering/Alexandru (2019)

... more to expect



Lattice studies of 3b systems

- ❖ $I=3 \pi\pi\pi$
 - ❖ $X(2872)$
 - ❖ $a_1(1260)$
 - ❖ $N(1440)1/2^+$
 - ❖ $I=2 \pi\eta$
 - ❖ $\pi\omega / \pi\varphi$
 - ❖ $I=3 \pi\pi\pi$
- ... more to expect

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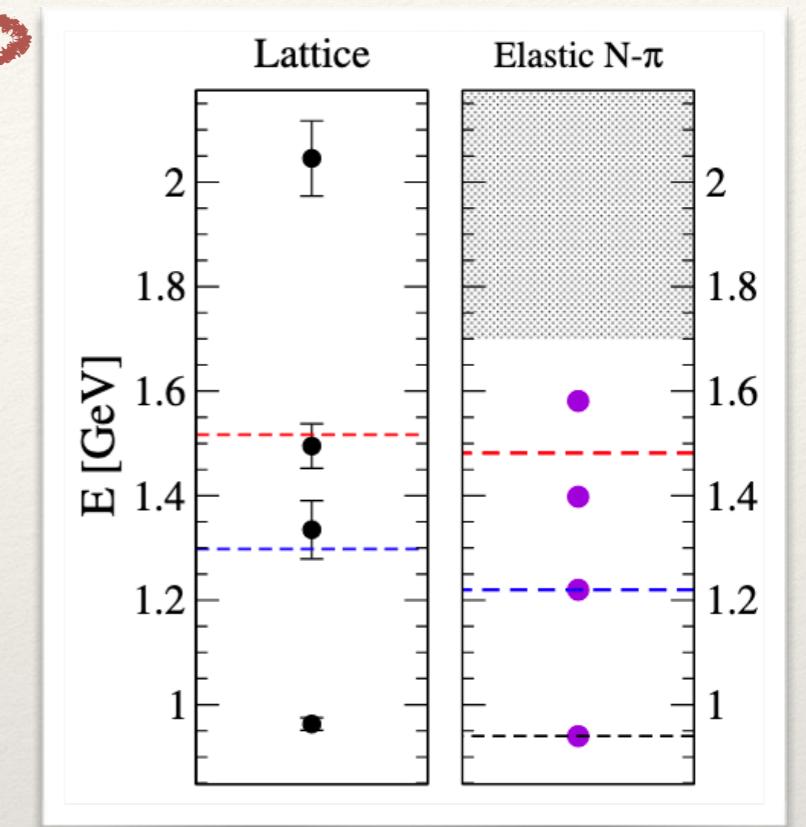
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Lang et al.(2017)

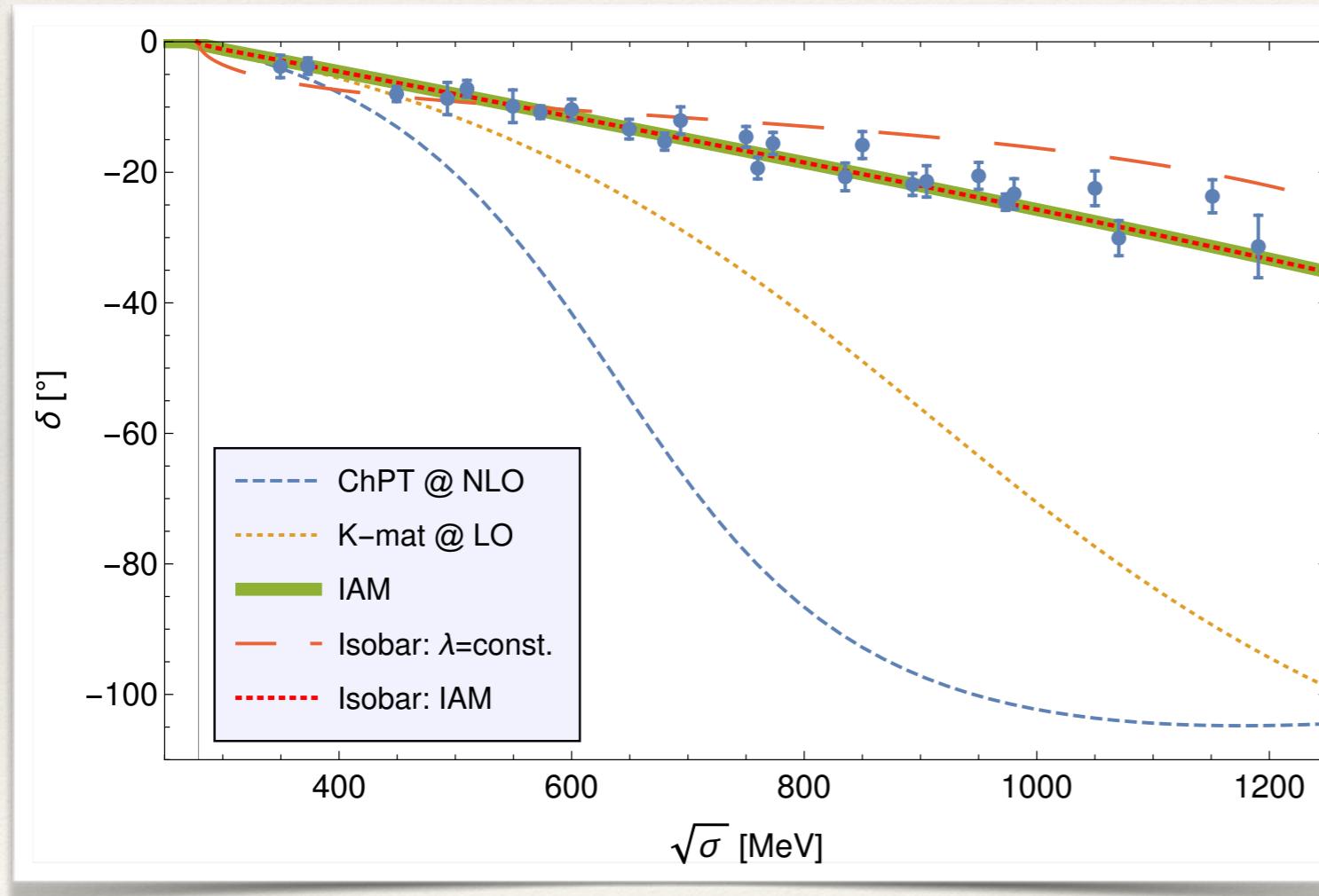
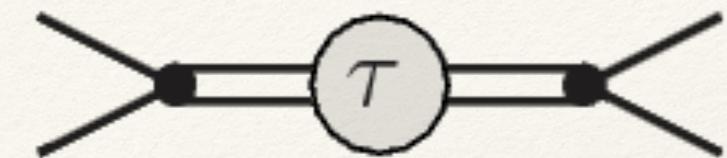


= fin vol. analysis accomplished

Three positive pions

MM, Doring PRL 122 (2018)

- ❖ 2-body sub-channel
 - One-channel problem: $\pi+\pi+$ system in S-wave
 - How to parametrize the scattering amplitude?



1) ~~$T_2 = \frac{-\lambda^2/(32\pi)}{\sigma - M_0^2 - \sum_{\pm} \int \frac{d^3 k}{(2\pi)^3} \frac{\lambda^2}{4E_k \sqrt{\sigma} (\sqrt{\sigma} \pm 2E_k)}$~~

- incorrect $m\pi$ behavior!

2) ~~$T_2 = \frac{1}{K^{-1} - ip_{cms}}$~~

- works badly for high energies

3) \checkmark $T_2 = \frac{T_{\text{LO}}^2}{T_{\text{LO}} - T_{\text{NLO}}}$ Truong(1988)

- correct σ and m_π behavior

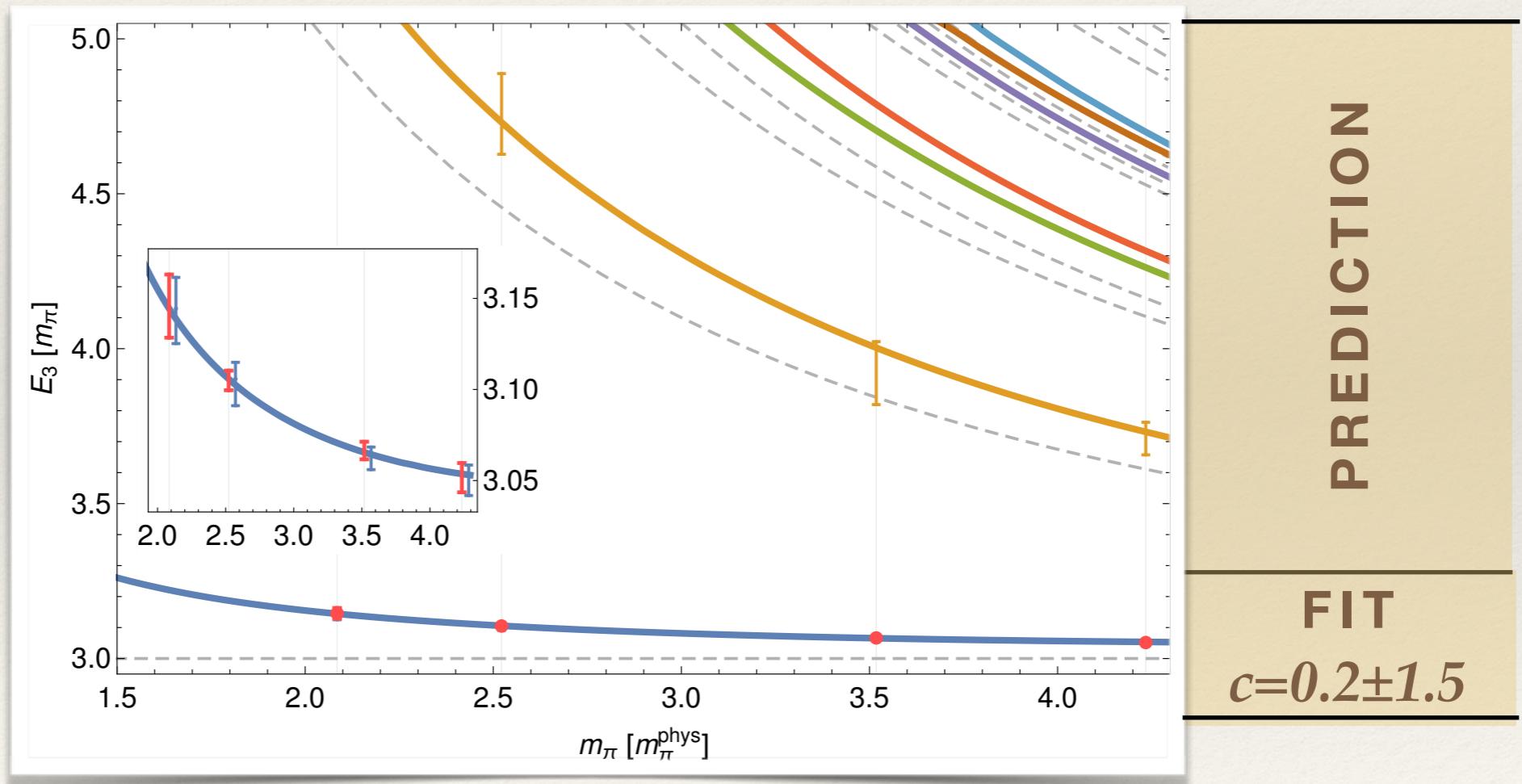
Three positive pions

MM, Doring PRL 122 (2018)

❖ 3-body spectrum

- genuine 3-body force C is not known
- momenta dependent function

$$\left\langle \mathbf{v} \frac{1}{B(E^*) + E_L \tau_{LP}^{-1}(E^*)} \mathbf{v} \right\rangle = \infty$$

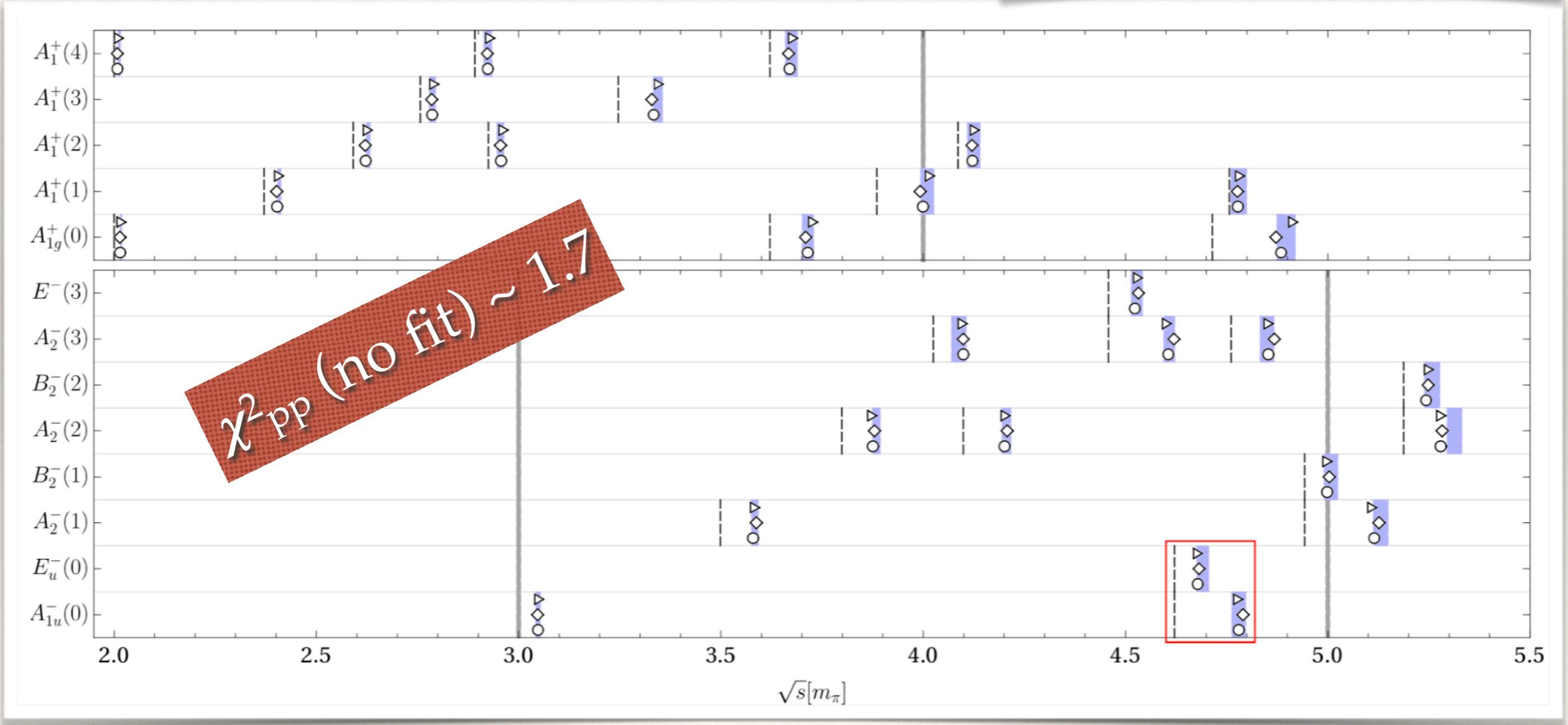
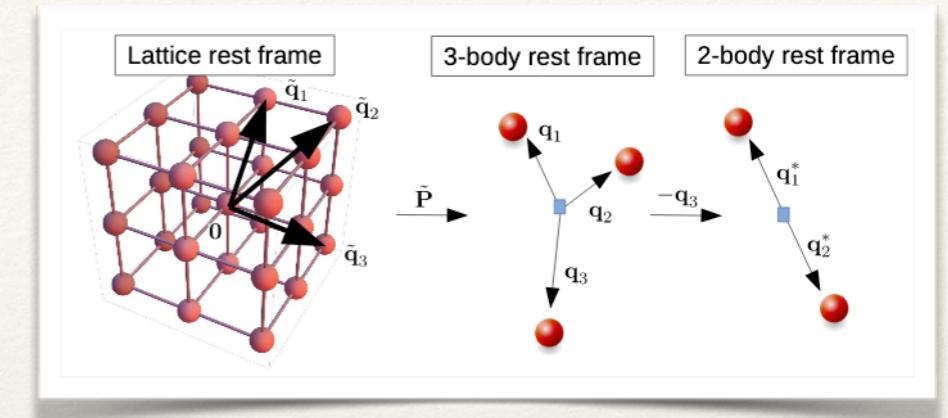


Three positive pions

MM et al. (2019)

- ❖ New data is available
 - Extend to arbitrary irreps & boosts
 - Use the same parametric input

Hörz/Hanlon PRL123 (2019)



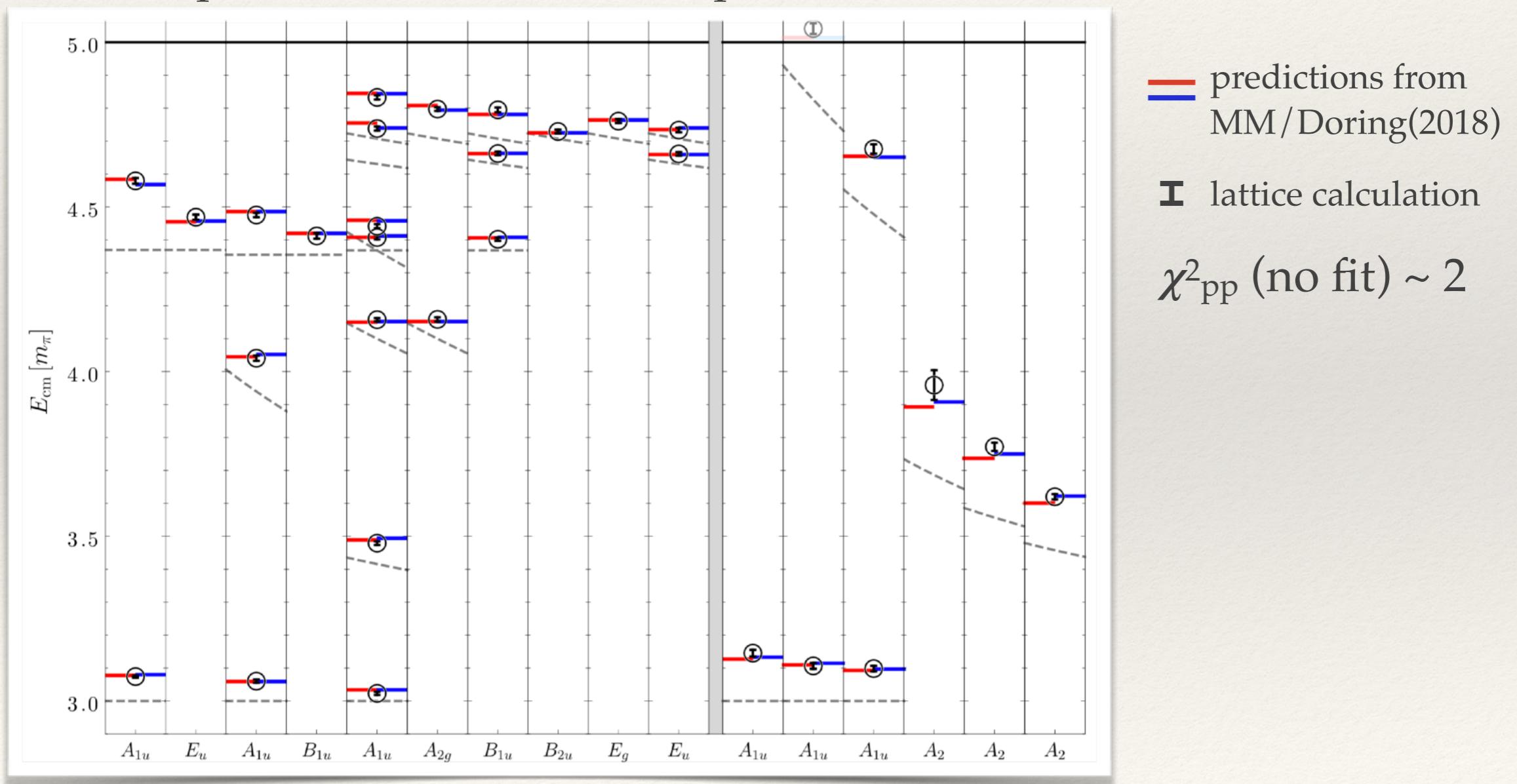
Three positive pions

- ❖ Newer data is available

Culver, MM, Brett, Alexandru, Doring (2019)

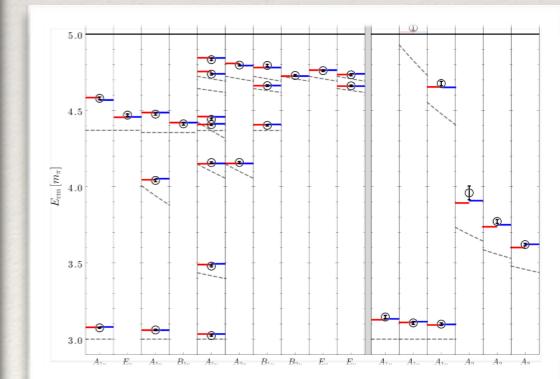
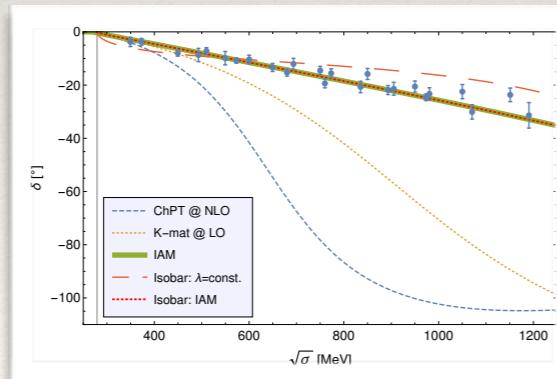
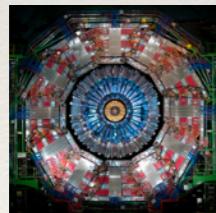
arXiv:1911.09047

- very dense spectrum from elongated boxes
- different pion masses (chiral extrapolations?)



SUMMARY

- ❖ **2-body spectroscopy from Lattice is highly evolved:**
 - new high-precision information on properties of resonances
 - complementary insights to experimental studies
 - e.g. m_π dependence, extreme quantum numbers,...
- ❖ **3-body spectroscopy from Lattice is maturing quickly**
 - many theoretical tools have been developed
 - first data-driven studies are performed (details/statistics important)



QCD