Assignment 1 (15%)

You will need very few and simple calculation in order to solve the problems of this assignment.

Consider the matrix

$$A = \begin{bmatrix} 3 & 3 & 1 & 4 & 1 \\ 0 & -15 & 7 & -20 & -5 \\ 0 & 0 & -34 & 50 & -25 \\ 0 & 0 & 0 & -60 & -140 \\ 0 & 0 & 0 & 0 & 910 \end{bmatrix}$$

a) State which of the statements below are true and which are false (notice: every incorrect answer cancels a correct one). Non-answers will count as an incorrect answer.

1. A is not invertible
2. A is in echelon form
3. $Nullity\ A\ =\ 1$
4. $Rank\ A\ =\ 5$
5. $Nullity\ A\ +\ rank\ A\ =\ 6$
6. The number -15 is an eigenvalue of A
7. A is in reduced echelon form
8. There exists a vector $\bar{b} \in \mathbb{R}^5$ such that $A\bar{x} = \bar{b}$ is not consistent
9. A is diagonalizable.

b) Three vectors $\bar{u}, \bar{v}, \bar{z} \in \mathbb{R}^7$ are linearly independent. Let $H = span\{\bar{u}, \bar{v}, \bar{z}\}$. Identify the true statement below.

1. $Dim\ H = 7$
2. $Dim\ H = 3$
3. H can be described as a line in $\mathbb{R}^7$

c) Explain why a 4×4 matrix with eigenvalues 1, 2, 3 and -3 is both invertible and diagonalizable.

d) Let V be the subspace of $\mathbb{R}^4$ given by all solutions to the equation $2x_1 - x_2 + 3x_3 = 0$. What is the dimension of V?

Assignment 2 (20%)

Let

$$A = \begin{bmatrix} -1 & 3 & 2 \\ 1 & 0 & 1 \\ 3 & 3 & a \end{bmatrix}$$

and consider the following matrix equation

$$A\bar{x} = \begin{bmatrix} -8 \\ 2 \\ b \end{bmatrix}, \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3$$

where $a, b \in \mathbb{R}$.

a) Determine the determinant of A using cofactor expansion on the second row.
b) For which values of a, b does the matrix equation have exactly one solution?
c) For which values of a, b does the matrix equation have no solution?
d) For which values of a, b does the matrix equation have an infinite number of solutions?

Assignment 3 (20%)

Consider the following system

$$x - y = 1 - 3z$$

$$y = -2x + 5$$

$$9z - x - 5y + 7 = 0$$

a) Write the system in the matrix form $A\bar{x} = \bar{b}$ for $\bar{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
b) Write out the augmented matrix for this system and state its *echelon* form
c) Write out the complete set of solutions (if they exist) in parametric vector form.
d) Calculate the inverse of the coefficient matrix A you found in part (a) using the identity matrix, if it exists, or show that A^{-1} doesn't exist.

Assignment 4 (15%)

Let A be given by

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

a) Test A for diagonalisability, and if A is diagonalizable, find an invertible matrix Q and a diagonal matrix D such that $Q^{-1}AQ = D$
b) Next, find an expression for A^n, where n is an arbitrary positive integer.

Assignment 5 (10%)

Let

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} \right\}$$

be a basis for the subspace $W \subseteq \mathbb{R}^4$

a) By using the Gram Schmidt process, find an orthogonal basis for W.
b) Now determine an orthonormal basis for W.

Assignment 6 (10%)

The file 'TV_Viewing.mat' provides sample data on the number of hours of TV viewing per week for different adults. The first column displays the age of the viewer and the second column displays the hours spent viewing TV per week. The TV executives would like to build a model for estimating TV viewing time as a function of age. They believe that the relationship can be modelled by either a linear function or a quadratic function:

$f_1(x) = \beta_0 + \beta_1 x$ or $f_2(x) = \delta_0 + \delta_1 x + \delta_2 x^2$

a) By creating design matrices, find the parameters of both models and state the regression functions
b) Supply a substantiated answer to which model is the best fit for the measured data

Assignment 7 (10%)

Compute a full singular value decomposition of A:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$$