Assignment 1 (15%)

You will need very few and simple calculation in order to solve the problems of this assignment.

Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 & 4 \\ 0 & -5 & 7 & 2 \\ 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

a) State which of the statements below are true and which are false (notice: every incorrect answer cancels a correct one). No answer will count as an incorrect answer.

   a. A is not invertible

   b. A is in echelon form

   c. $Nullity\ A = 1$

   d. $Rank\ A = 3$

   e. $Nullity\ A + rank\ A = 6$

   f. The number 0 is an eigenvalue of A

   g. A is in reduced echelon form

   h. There exists a vector $\bar{x} \in \mathbb{R}^4$ such that $A\bar{x} = \bar{b}$ is not consistent

   i. $\det A = 0$

b) Let W be a subspace of $\mathbb{R}^6$ having dimension 4. What is $\dim(W^{\perp})$?

c) Explain why a 3×3 matrix with eigenvalues 1, 2, and -3 is both invertible and diagonalizable.

Assignment 2 (10%)

Let

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) State the eigenvalues of A and determine a basis for each of the corresponding eigenspaces.

b) Specify whether A is diagonalisable. If so, find matrices P and D such that D is diagonal, P is invertible, and $A = PDP^{-1}$.

Assignment 3 (20%)

Consider the following system

$$3x - 4z = 7 - 5y$$

$$1 + 4z = 2y + 3x$$

$$y = -4 - 6x + 8z$$

a) Write the system in the matrix form $A\bar{x} = \bar{b}$ for $\bar{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
b) Write out the augmented matrix for this system and state its *echelon* form
c) Write out the complete set of solutions (if they exist) in parametric vector form.
d) Calculate the inverse of the coefficient matrix A you found in part (a) using the identity matrix, if it exists, or show that A^{-1} doesn't exist.

Assignment 4 (10%)

Show that for arbitrary real numbers a, b, c and d, the determinant of the following matrix is always zero.

$$\begin{bmatrix} a & 0 & d & c \\ b & 0 & -c & d \\ 0 & c & -b & a \\ 0 & d & a & b \end{bmatrix}$$

Assignment 5 (15%)

Let

$$A = \begin{bmatrix} a & 3 & -1 & 0 & -2 \\ 0 & 2 & -4 & -1 & -6 \\ -2 & -6 & 2 & 3 & 9 \\ 3 & 7 & -3 & 8 & -7 \\ 3 & 5 & 5 & 2 & 7 \end{bmatrix}$$

a) Find a such that $\det(A) = 24$.
b) Assuming, that you found $a = 1$ in (a), show that the columns of A form a basis for $\mathbb{R}^5$.
c) Show that the columns of A do not form an orthogonal basis, and use the Gram Schmidt process to obtain such an orthogonal basis.

Assignment 6 (15%)

As part of their final project, two ICT students are working on a data warehouse support system. The major workload is the warehouse orders. Thus, the key business metric is identified as number of order lines. The students want to find a method to predict CPU utilization based on the number of order lines entered into the system and have collected 31 samples of CPU utilization and number of order line entries (the data is also uploaded as 'ALI2018_6.mat'):

Sample #	CPU Utilisation	Order lines per day
1	27.01	16483
2	32.43	13142
3	21.74	12015
4	20.56	11986
5	2.85	1119
6	1.41	0
7	1.45	0
8	46.38	12259
9	21.95	6531
10	29.55	14086
11	30.04	12797
12	28.08	13141
13	3.26	454
14	1.62	1
15	29.41	5971
16	40.02	10901
17	29.86	14271
18	28.34	13728
19	34.82	12938
20	3.22	1158
21	1.43	0
22	34.22	11450
23	23.58	5311
24	33.66	17073
25	23.36	11336
26	26.76	7340
27	4.31	11330
28	2.62	0
29	33.44	10679
30	29.19	12803
31	28.11	12827

The students believe that the relationship can be modelled by either a linear function or a quadratic function:

$y_1(x) = \beta_0 + \beta_1 x$ or $y_2(x) = \delta_0 + \delta_1 x + \delta_2 x^2$

a) By creating design matrices, find the parameters of both models and state the regression functions
b) Supply a substantiated answer to which model is the best fit for the measured data
c) Use the best fitted model to predict the CPU Utilisation at 20000 order lines per day.

Assignment 7 (15%)

Consider the following matrix

$$A = \begin{bmatrix} -18 & 13 & -4 \\ 2 & 19 & -4 \\ -14 & 11 & -12 \\ -2 & 21 & 4 \end{bmatrix}$$

a) Compute a full singular value decomposition of A.
b) Use this decomposition of A, with no calculations, to write a basis for $Col\ A$, and use the values of the singular values to determine the rank of A.