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Dynamical Evolution of Supernova Remnants

Lecture Notes

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Supernova Remnants

Supernova explosions are very violent events which transfer a significant amount of energy to the interstellar medium (ISM) and are responsible for a large variety of physical processes. We will not discuss the actual explosion mechanisms here which are thought to be due to carbon deflagration of white dwarfs (type I) or the core collapse of massive stars (type II) but we will follow the dynamical evolution of the supernova remnant (SNR), i.e. the expanding cloud of hot gas in the ISM. This evolution can be divided into different phases according to the dominant physical processes, and simplified models can be made at least for the first stages, to get an idea of typical time scales, expansion velocities and sizes.

Free Expansion Phase

The shock wave created by the explosion moves outwards into the (more or less homogeneous) interstellar gas at highly supersonic speed. During the first phase of the SNR evolution the surrounding ISM has no influence on the expansion of the shock wave, i.e. the pressure of the interstellar gas is negligible. Assuming that most of the supernova energy E_{SN} (on the order of 10^{44} J, excluding the part carried away by neutrinos) is transformed into kinetic energy of the ejected gas we can estimate the ejection velocity v_{e} by using

$$E_{\mathrm{SN}} = \frac{1}{2}\, M_{\mathrm{e}}\, v_{\mathrm{e}}^2 \qquad \rightarrow \qquad v_{\mathrm{e}} = \left(\frac{2E_{\mathrm{SN}}}{M_{\mathrm{e}}}\right)^{1/2} \tag{1}$$

where M_{e} is the ejected mass. The shock radius scales as

$$R_{\mathrm{s}}(t) = v_{\mathrm{e}}\, t\,. \tag{2}$$

The schematic structure of the SNR at this phase can be described as follows: behind the strong shock front which moves outwards into the ISM compressed interstellar gas is accumulating. This interstellar material is separated from the ejected stellar material by a so-called contact discontinuity, i.e. a surface between two different materials with similar pressure and velocity but different density (unlike a shock front where all three quantities change). Behind the contact discontinuity a reverse shock is staring to form in the ejected stellar material.

After some time the accumulated mass of the ISM compressed between the forward shock and the contact discontinuity equals the ejected mass of stellar material, and it will start to affect the expansion of the SNR. By definition, this is the end of the free expansion phase, and the corresponding radius of the SNR, the so-called sweep-up radius R_{SW}, is defined by

$$M_{\mathrm{e}} = \frac{4\pi}{3} R_{\mathrm{SW}}^3\, \rho_0 \qquad \rightarrow \qquad R_{\mathrm{SW}} = \left(\frac{3M_{\mathrm{e}}}{4\pi\rho_0}\right)^{1/3} \tag{3}$$

where ρ_0 is the initial density of the interstellar medium. This radius is reached at the sweep-up time $t_{\mathrm{SW}} = R_{\mathrm{SW}}/v_{\mathrm{e}}$.

Around the sweep-up time $t_{\rm SW}$ the structure of the SNR changes due to the effects of the accumulated interstellar material. The reverse shock starts to travel inwards, heating the ejected stellar gas to high temperatures. After some time a flat pressure structure is established and the SNR enters the next phase where the expansion is driven by the thermal pressure of the hot gas.

Sedov-Taylor Phase

After the passage of the reverse shock, the interior of the SNR is so hot that the energy losses by radiation are very small (all atoms are ionized, no recombination). The following pressure-driven expansion phase can therefore be regarded as adiabatic, the cooling of the gas is only due to the expansion. Taylor (1950) and Sedov (1959) gave an exact self-similar solution for such a pressure-driven explosion but we will restrict our analysis to simple estimates based on the approximation of a geometrically thin shell (with $\Delta R/R$ much smaller than 1), and the mass of the shell being equal to the swept-up mass. Taking the gas pressure P into account, the equation of motion for the expanding shell can be formulated as

$$\frac{d}{dt}\left(\frac{4\pi}{3}R_{\rm s}^3\rho_0\dot{R}_{\rm s}\right) = 4\pi R_{\rm s}^2 P\,. \tag{4}$$

The pressure and internal energy E of an ideal gas are related by

$$P = (\gamma - 1)\frac{E}{V} \tag{5}$$

where V denotes the volume. In the case of an adiabatic expansion (no losses of energy due to radiation) we can set E equal to the explosion energy $E_{\rm SN}$, and $V = 4\pi R_{\rm s}^3/3$. Assuming $\gamma = 5/3$ we get

$$P = \frac{E_{\rm SN}}{2\pi R_{\rm s}^3} \tag{6}$$

and inserting this expression into the equation of motion we obtain

$$\frac{d}{dt}\left(\frac{1}{3}R_{\rm s}^3\rho_0\dot{R}_{\rm s}\right) = \frac{E_{\rm SN}}{2\pi R_{\rm s}}\,. \tag{7}$$

Making a power law ansatz for the shock radius

$$R_{\rm s} = A\,t^{\eta} \tag{8}$$

and inserting it into Equation (7) we obtain

$$\eta = \frac{2}{5} \qquad \text{and} \qquad A = \left(\frac{25E_{\rm SN}}{4\pi\rho_0}\right)^{1/5}, \tag{9}$$

i.e. the shock radius $R_{\rm s}$ and the corresponding velocity $v_{\rm s}$ behave like

$$R_{\rm s}(t) = \left(\frac{25E_{\rm SN}}{4\pi\rho_0}\right)^{1/5} t^{2/5} \tag{10}$$

$$v_{\rm s}(t) = \frac{2}{5}\left(\frac{25E_{\rm SN}}{4\pi\rho_0}\right)^{1/5} t^{-3/5} \tag{11}$$

The post-shock temperature of the gas can be derived using the Rankine-Hugoniot conditions (which express the basic conservation laws accross the shock front). In the limit of a strong shock the post-shock pressure and density are given by

$$P_1 = \frac{2\,\rho_0\,v_\mathrm{s}^2}{\gamma+1} \qquad \text{and} \qquad \frac{\rho_1}{\rho_0} = \frac{\gamma+1}{\gamma-1}\,. \tag{12}$$

Using these two conditions together with the ideal gas law

$$P_1 = \frac{k}{\mu\,m_u}\,\rho_1 T_1 \tag{13}$$

and assuming $\gamma = 5/3$ we obtain

$$T_1 = \frac{3\,\mu\,m_u}{16\,k}\,v_\mathrm{s}^2\,. \tag{14}$$

Inserting the expression for v_s derived above we find

$$T_\mathrm{s}(t) = T_1(t) = \frac{3\,\mu\,m_u}{100\,k}\left(\frac{25E_\mathrm{SN}}{4\pi\rho_0}\right)^{2/5} t^{-6/5} \tag{15}$$

which describes the decrease of the post-shock temperature T_s during the adiabatic, pressure-driven expansion of the Sedov-Taylor phase.

Cooling Phase and Final SNR Evolution

As the SNR expands and cools adiabatically it will reach a critical temperature of about 10^6 K. At this temperature the ionized atoms start to capture free electrons and they can lose their excitation energy by radiation. The radiative losses of energy become significant putting an end to the adiabatic expansion of the SNR. Due to the efficient radiative cooling the thermal pressure in the post-shock region decreases and the expansion slows down. The SNR is entering the so-called snow plough phase since more and more interstellar gas is accumulated until the swept-up mass is much larger than the ejected stellar material.

Finally the shell breaks up into individual clumps, probably due to a Rayleigh-Taylor instability (hot thin gas is pushing cool dense gas) and the SNR disperses into the ISM as the expansion velocity decreases to values typical of the interstellar gas.

References

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