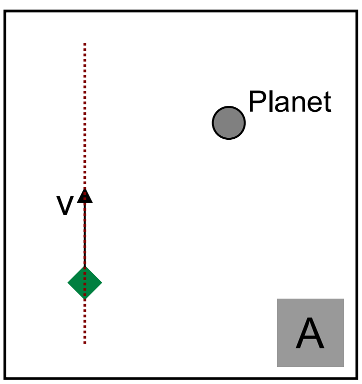
EOSC 211: Some Numerical Integration – Project Lab 1 background

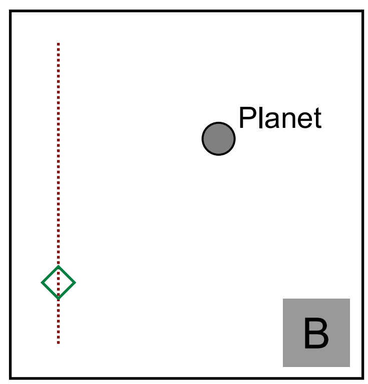
Simulations

Sketch how you think the spacecraft's flight path and speeds in the two other trajectories will compare with the one shown.

Sketching out the Problem:

1. sketch the direction of the force and acceleration on the spacecraft in A.





2. sketch the spacecraft's position, velocity and acceleration at some time t = Δt later in B.

Calculations:

We will use the following initial conditions:

s_{x0} = -3050 km

s_{y0} = -3 * R_{merc}

v_{y0} = 7 km/s in the positive y-direction

v_{x0} = 0

The values for the mass and radius of Mercury and the gravitational constant, G, are

$M_{merc} = 3.3 \times 10^{23}$ kg

R_{merc} = 2440 km

$G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

Step 1: Fill in the initial conditions – ie the x-y coordinates of speed and position of the spacecraft at time t=0 in cols 5-8 of row 1 of the table. Calculate, s the distance of the spacecraft from the planet, and s^2.

t (s)	**Δv_x (m/s)**	**Δv_y (m/s)**	**Δs_x (m)**	**Δs_y (m)**	**v_x (m/s)**	**v_y (m/s)**	**s_x (m)**	**s_y (m)**	**s^2 (m)**	**a (m/s^2)**	**a_x (m/s^2)**	**a_y (m/s^2)**
0	**XXX**	**XXX**	**XXX**	**XXX**								
60												
120												

Step 2: Calculate the magnitude of the acceleration, a, on the spacecraft at time, t=0 due to the planet and add it to the table above. Resolve the acceleration into its x- and y- coordinates and fill these in (a_x and a_y).

Step 3: If we assume the acceleration is constant over a time interval Δt, then after the time Δt there is a change in velocity due to this acceleration. This is in the direction of the acceleration vector so the easiest thing is to work in terms of the x- and y- components of the change in velocity.

Write down the equations for the x- and y- components of the *change in velocity* in terms of the x- and y- components of acceleration and the time interval Δt.

Δv_x = ____________ Δv_y = ____________

Similarly there is a change in position, which depends both on the velocity at the beginning of the time interval and on the acceleration:

$\Delta s_x = v_x \Delta t + \frac{1}{2} a_x \Delta t^2$ $\Delta s_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

Fill in these changes in velocity and position (occurring after 60 seconds) in row 2, columns 1-4 of the table.

Step 4: The new velocity after a time Δt is the initial velocity plus the change in velocity.
So in our table above we can calculate the x- and y-components of velocity at t=60 seconds using

$v_x^{t=60} = v_x^{t=0} + \Delta v_x$ $v_y^{t=60} = v_y^{t=0} + \Delta v_y$

and the x- and y- components of position at t= 60 seconds using

$s_x^{t=60} = s_x^{t=0} + \Delta s_x$ $s_y^{t=60} = s_y^{t=0} + \Delta s_y$

Calculate the new v_x, v_y, s_x, s_y and fill in columns 5-8 of the second row of the table.

Step 5: You can now see that you can essentially repeat steps 2-4 for each successive 60 seconds of the spacecraft's trajectory using Steps 2-4 above. To calculate the entire spacecraft trajectory you would repeat steps 2-4 until you have reached a time $t = t_{final}$ (given as 40 minutes in part 5 of the assignment). **Hint:** If you do this by hand (correctly!) for the first 3 rows given you can of course check your code using this table….