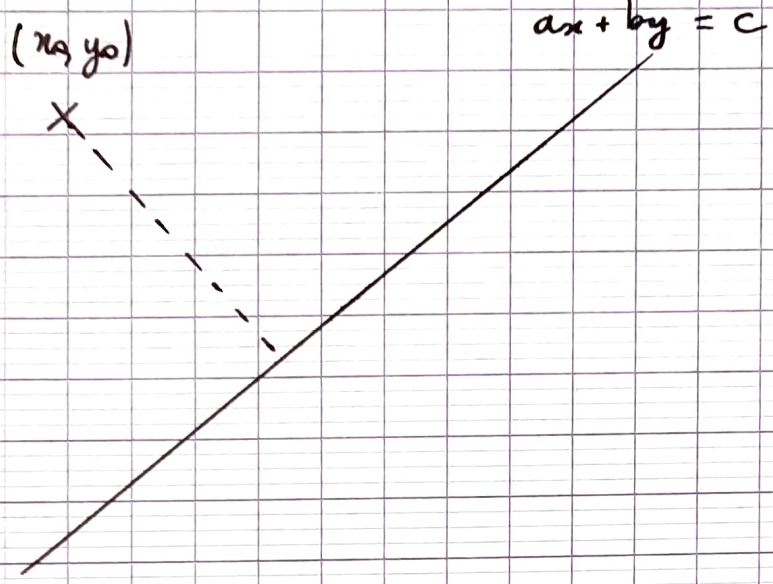
- Projecting onto a straight line -



$$\min (x - x_0)^2 + (y - y_0)^2$$

$$\text{s.t. } ax + by - c = 0$$

Lagrange multiplier

$$\mathcal{L}(x, y, \lambda) = (x - x_0)^2 + (y - y_0)^2 + \lambda(ax + by - c)$$

$$\text{So } x^* = x_0 - \frac{\lambda}{2}a \quad \text{and} \quad y^* = y_0 - \frac{\lambda}{2}b$$

$$\text{And } ax^* + by^* - c = 0 \text{ so } \boxed{\lambda = \frac{2(ax_0 + by_0 - c)}{a^2 + b^2}}$$