Asset Prices, Global Portfolios, and the International Financial System

Maxime Sauzet*

September 3, 2021
PRELIMINARY

I adapt the framework of Sauzet (2021a) to an international setting to characterize the global solution to the international portfolio problem in full generality, a long-standing open issue in international finance. In this two-country, two-good environment, investors have recursive preferences and a bias in consumption towards their local good. The framework highlights the role of the allocation of wealth across international investors for portfolios, asset prices, and risk sharing, an aspect that had received little emphasis in such a setting. The model allows to revisit various results in the international portfolio choice literature. In addition, I show that it can replicate a number of facts about the structure and dynamics of the international financial system, and of asset returns in that context.

Keywords: International Portfolio Choice, Asset Pricing, International Finance and Macroeconomics, International Financial System, Wealth Allocation. ***JEL codes:*** E0, F3, F4, G1.

*Boston University, Questrom School of Business. Email: msauzet@bu.edu. For invaluable support and guidance, I thank my advisor Pierre-Olivier Gourinchas, as well as Nicolae Gârleanu, Martin Lettau, Hélène Rey, and Jón Steinsson. For valuable comments and discussions, I thank Harjoat Bhamra, Markus Brunnermeier, Nicolas Coeurdacier, Xiang Fang, Christian Gollier, Tarek Hassan, Chen Lian, Matteo Maggiori, Ulrike Malmendier, Sebastian Merkel, Thomas Mertens, Emi Nakamura, Maurice Obstfeld, David Sraer, Richard Stanton, Annette Vissing-Jørgensen, Johan Walden, Fernando Zapatero, and seminar participants at Berkeley Economics, Berkeley Haas, Bocconi, Boston University Questrom, Bristol, CERGE-EI, the Foster School of Business, Toulouse, the University of Zurich, and the Wisconsin School of Business. This paper evolved from the second part of my Job Market Paper, previously circulated as "Asset Prices, Global Portfolios, and the International Financial System" (November 2, 2020). All errors are my own.

1. Introduction

> *"At the moment, we have no integrative general-equilibrium (monetary) model of international portfolio choice, although we need one."*
>
> Obstfeld (2004)

More than a decade later, the international portfolio choice problem remains a long-standing open issue in international finance to which the literature only provides a piecemeal answer. In this paper, I adapt the framework of Sauzet (2021a) to an international setting to characterize the global solution to the international portfolio choice problem in full generality. The model allows to revisit various results in the international portfolio choice literature in a unified framework. It is also a well-suited building block towards several applications and extensions. I focus on one of them and show that the model can be used to capture a number of stylized facts about the structure and dynamics of the international financial system, and of asset returns in that context.

The main economic message from the characterization is that the allocation of wealth across investors matters in a general international portfolio choice setting. This finding resonates with an emerging theme in the broader economic literature that has recently emphasized the role of the wealth distribution in determining economic outcomes in macroeconomics (e.g. Brunnermeier and Sannikov, 2014, Kaplan et al., 2018), finance (e.g. Gomez, 2017, Lettau et al., 2019, Greenwald et al., 2020), and economics more generally (e.g. Piketty and Zucman, 2014). In other words, "capital is back" in this setting too: the allocation of wealth across international investors has a prime role in driving asset prices, portfolios, and risk sharing, an aspect that had received little emphasis thus far.

To derive this result, I adapt the "222" framework (Two Investors, Two Trees, Two Goods) of Sauzet (2021a) to an international setting. The framework itself adapts recent advances in multi-agent continuous-time asset pricing models to a two-investor, two-tree, two-good economy. Investors have recursive preferences and a bias in consumption towards their local good. This allows me to overcome two main limitations in the international portfolio choice literature.

First, while a majority of contributions rely on special cases to facilitate the resolution, I allow for general recursive preferences and an arbitrary degree of substitutability across goods. The former matters because (i) recursive preferences are not log so that investors are not myopic and their portfolios feature hedging demands that have a prime role in this international context, and (ii) recursive preferences are not constant relative risk aversion (CRRA), which leads the allocation of wealth across investors to become a state variable in its own right that has an important impact beyond current fundamentals.[1] An arbitrary degree of substitutability across goods ensures, by moving away from the case of unitary elasticity of substitution, that asset returns are not perfectly correlated so that the portfolio choice between them is well-defined.[2] Throughout, the generality of the specification allows to study the impact of a number of important dimensions of preferences.

Second, while most contributions have relied on low-order local approximations, I solve the model using a global solution method. This makes it possible to fully trace out the evolution of economic variables with the state of the economy, in sharp contrast to local methods that mostly capture evolutions in a small neighborhood of a specific state.[3] This innovation is particularly valuable in situations such as here in which economic outcomes turn out to be strongly state-dependent, and in which policy functions can be very non-linear as a result of heterogeneity, or imperfect risk sharing. In addition, because increasing the order of approximation is notoriously cumbersome for the type of local methods that have been used in the literature, most contributions have focused on so-called zero-order (i.e. steady-state) portfolios. Such portfolios, which are constant, are silent on any time variation in investors' positions.

[1]Specifically, recursive preferences break the link between the elasticity of intertemporal substitution and the inverse of risk aversion. They also help in generating quantitatively more plausible risk premia while maintaining a reasonable risk-free rate. Hedging terms are absent more generally as long as the risk aversion is equal to one.

[2]The case of unitary elasticity of substitution across goods has received considerable attention in the literature since the seminal contribution of Cole and Obstfeld (1991). For instance, it is assumed in Pavlova and Rigobon (2007, 2008, 2010), Colacito and Croce (2011, 2013), Maggiori (2017), and Colacito et al. (2018), among others.

[3]Under a set of assumptions, local methods could be used to study an economy further in the state space, cf. for instance Mertens and Judd (2018). However, such methods remain difficult to use in an international portfolio context due to the portfolio indeterminacy that arises in the corresponding deterministic economy. More generally, defining the state around which to approximate the equilibrium is also non-trivial. The literature has focused on using the symmetric economy as an approximation point, but this might not be a well-defined steady state in an international context in particular in the presence of imperfect risk sharing, incomplete markets, and non-stationarity. The global method in this paper circumvents all those difficulties naturally.

Instead, the global method in this paper naturally captures their dynamics, an aspect that is not innocuous: like other outcomes, portfolios are inherently time-varying. For instance, the bias in portfolio holdings towards the home or foreign asset that emerges in equilibrium is strongly reinforced as the wealth share of an investor decreases, and the relative portfolio weights of different assets also vary substantially with the relative supply of goods in the world economy.

More generally, I augment the framework in a number of ways, e.g. by introducing labor income as a constant share of output, imperfect financial integration, or asymmetries in preferences, which allow me to analyze the international portfolio choice problem in a variety of contexts. This is made possible in part by the fact that throughout, I solve for the *decentralized* equilibrium to the economy so that I am able to study cases in which the standard planner solution (that have been popular in the literature) cannot be used.

To summarize the characterization in Sauzet (2021a) (cf. the paper for details), the allocation of wealth matters in two ways in this context: as a state variable that captures the average investor in the world economy and directly impacts economic outcomes, and as a pricing factor that is hedged by international investors. Its effect is relevant even in a baseline with symmetric calibration and perfect risk sharing, but grows tremendously as markets become imperfectly integrated, and as investors become more heterogeneous – be it through a larger home bias in consumption, the introduction of labor income, or asymmetries in preferences – to the point where it can match or surpass the impact of fundamentals. The results emphasize both (i) the state-dependence of most economic variables in this environment – e.g. portfolios vary substantially with the allocation of wealth and the relative supply of goods –, and (ii) the vital impact of the calibration of preferences – e.g. the potency of imperfect financial integration is strongly reduced with a high elasticity of intertemporal substitution. This makes the novel framework presented in that paper, which is based on a global solution method and allows for general recursive preferences including asymmetries, particularly adapted to study this economy. This reiterates the main message: capital is back in this international economy too![4]

[4] These results are also reminiscent of recent findings in the price impact literature, in which quantities, represented here by the portfolios held by each investor and captured in aggregate by the wealth share, strongly impact asset prices and risk compensations. Contributions in this spirit include Kouri (1982), Jeanne and Rose (2002), Hau and Rey (2006), and more recently Gabaix and Maggiori (2015), Camanho et al. (2018), Gabaix and Koijen (2020), and Koijen and Yogo

Taken together, those innovations make it possible to revisit a number of results in the international portfolio choice literature under a unified framework. First, the calibration of preferences can have a significant impact. For instance, the discussion of portfolios under imperfect financial integration in Section 3 confirm but qualify the findings in Bhamra et al. (2014) in this general setting with non-log preferences and home bias in consumption: imperfectly integrated markets *can* deliver portfolios consistent with the data *provided that* the elasticity of intertemporal substitution is moderate. Second, the global solution shows that a number of variables, chief among them portfolios, can be strongly state-dependent, even under perfectly integrated markets, so that relying on local approximations can be problematic. Lastly, my discussion throughout emphasizes the key role that can be played by the allocation of wealth for global portfolios and asset prices, an aspect on which that literature had put little emphasis thus far.

Because of its generality, the framework represents a versatile building block towards several applications and extensions. I then focus on one main application, in which I show that the model can reproduce a number of stylized facts about the structure and dynamics of the international financial system, and in particular the role of the United States, and of asset returns in this context.[5]

The domestic country is now taken to represent the United States, the country at the center of the international financial system, and its representative investor is assumed to display a higher tolerance for risk. This assumption, in the spirit of Caballero et al. (2008), Gourinchas et al. (2017), and Maggiori (2017), is meant to capture the greater development and depth of U.S. financial markets in the general context of this paper. Like in Gourinchas et al. (2017), and Maggiori (2017), by making the country as a whole better able and willing to carry financial risk in the world economy, this asymmetry naturally replicates its average external position (Fact 1, Gourinchas and Rey, 2007b): the United States plays the role of the world banker, by borrowing in safe securities from the rest of the world, and investing in risky assets internationally. This large negative net foreign asset position is associated with higher excess returns on the external balance sheet of the country on average, given the higher share of risky assets that pay more in expectation: this is the exorbitant privilege of the world banker (Fact 2, Gourinchas et al., 2017). Importantly, the

(2020).

[5] For an overview and additional references, those facts are summarized in Section 4.1.

economy also still features two meaningfully different equity assets and a modest degree of imperfect financial integration delivers a home bias in equity holdings broadly consistent with empirical observations (Fact 4, Coeurdacier and Rey, 2013).

The framework does not only replicate facts about external portfolios on average however, and the asymmetry in risk tolerance yields a number of predictions about the *dynamics* of the international financial system that are strongly borne out in the data. As a crisis hits, the center country is impacted particularly severely due to its high allocation to risky assets, so that it transfers a large amount of wealth to the rest of the world. This exorbitant duty is the flip side of its exorbitant privilege in normal times: the United States must become the world insurer in times of trouble (Fact 3, Gourinchas et al., 2017). In addition, by worsening the wealth position of the risk-tolerant world banker, the shock leads to a sharp increase in global risk aversion, which in turn pushes up all risk premia and Sharpe ratios worldwide. These two markers are reminiscent of some aspects of the Global Financial Cycle (Fact 5, Rey, 2013, Miranda-Agrippino and Rey, 2020), for which a general equilibrium exploration had remained elusive. Those patterns are representative of the type of global risk-off scenarios that typically occur in times of global crisis such as most recently in the Great Recession of 2008 or the Global Pandemic of 2020.

In addition, the model allows to study the evolution of portfolios as a response to those shocks, and can shed light on the process of external adjustment of the center country. For the latter, while its net foreign asset position strongly deteriorates following the shock, the sharp increase in risk premia that occurs simultaneously highlights the role of valuation effects as proposed in Gourinchas and Rey (2007a) in this situation: the higher expected returns on its global portfolio ease some of the pressure on the domestic country to balance its external position in the short term. This negative relationship between net foreign asset position and expected risk premia therefore replicates the type of predictability relationship between the two documented in Gourinchas and Rey (2007a), and extended to more recent data in Gourinchas et al. (2019) (Fact 6).[6]

From an asset pricing perspective, the model speaks to a number of facts about asset returns dynamics in this international environment. Namely, risk premia, Sharpe

[6] In the long run, the higher share of risky assets in the domestic portfolio also leads the domestic country to grow in world wealth and its net foreign asset position to become positive. This further alleviates the burden on the necessity of short-term adjustment in times of crisis, and allows the country to run a more negative net foreign asset position for a while.

ratios – and to some extent volatilities and correlations in a relevant region of the state space – are all countercyclical in the sense that they increase following the shock, consistent with a wide range of evidence notably for the United States (Fact 7, Lettau and Ludvigson, 2010, among others). Those patterns are the reflection of the type of dynamics emerging in asset pricing settings with heterogeneous agents (e.g. Weinbaum, 2009), in an economy in which there are also two goods, two assets, and a home bias in consumption. Importantly, those patterns are driven for a large part not by changes in the quantity of risk but by the evolution of the compensation for risk, captured here by the time-varying global risk aversion. This is in line with a large literature that has seen changes in the price of risk emerge as a crucial explanation behind asset return predictability more generally.

Another value of studying those questions in the general framework of this paper is that it allows to perform a number of counterfactual exercises. For instance, I show that a mild decrease in the frictions in international markets can generate the secular decrease in home bias that has been documented in recent decades (Fact 4, Coeurdacier and Rey, 2013), as well as some of the increase in the financial synchronization that has been observed throughout the world over a long-time horizon but particularly in the last three decades (Fact 8, Jordà et al., 2019). A re-interpretation of the model at a lower frequency could also be used to make sense of the secular decline in interest rate that has been observed worldwide, provided that the wealth share of the domestic risk-tolerant country decreases in the long run (Fact 9, Caballero et al., 2008, Hall, 2016). Finally, changes in the tax on foreign dividends, potentially asymmetric, could also be used to study the impact on global asset prices, portfolios, and risk sharing, of macroprudential policies aimed at curbing sudden international capital flows.

In summary, a seemingly small change in the specification of the model – the introduction of asymmetries in risk tolerance – generates a vast number of facts about the structure and dynamics of the international financial system and of asset returns, which are strongly borne out in the data. The model is also a well-suited building block for many potential extensions. The most promising among them are related to the introduction in an international setting of financial intermediaries of the type that has been discussed in the recent intermediary asset pricing literature e.g. in Daníelsson et al. (2012), He and Krishnamurthy (2013), Adrian and Shin (2014), or Adrian and Boyarchenko (2015). Illustrations are briefly discussed in Section 4.5 and Appendix B, for instance with the inclusion of a global asset manager (Sauzet, 2021d). From the perspective of extensions, solving for the decentralized equilibrium of this economy like I do in this paper will prove particularly valuable: the framework is readily set to tackle a wide range of market structures beyond imperfect risk sharing. In addition, the implementation of those extensions will likely require higher-dimensional methods such as the "projection methods via neural networks" being developed in Sauzet (2021c). I leave all these promising avenues for future research.

Related literature

This paper contributes to two main strands of literature.

First, I contribute to the literature on multi-agent asset pricing models, which has a long and distinguished history since the seminal contributions of Dumas (1989, 1992), Wang (1996), Basak and Cuoco (1998), Chan and Kogan (2002), and more recently Brunnermeier and Pedersen (2009), Weinbaum (2009), Brunnermeier and Sannikov (2014), Gârleanu and Pedersen (2011), Chabakauri (2013), Gârleanu and Panageas (2015), Drechsler et al. (2018). This literature is also related to the modern literature on heterogeneous agents in closed-economy macroeconomics such as Kaplan et al. (2018). To those contributions, I bring two goods, two assets, two countries, as well as a home bias in consumption. The home bias in consumption is particularly important because it introduces a fundamental level of heterogeneity between investors even absent asymmetries, and is responsible for most mechanisms in the economy including the rise of a substantial bias in portfolio holdings through hedging demands, the shape and comovement of risk premia, and a well-defined exchange rate. As such, this is one of the main differences with the international model of Brunnermeier and Sannikov (2015, 2019). Having two assets also fundamentally relates my paper to contributions with multiple securities but one agent e.g. Cochrane et al. (2008), Martin (2013).

Most related to my contribution are those of Pavlova and Rigobon (2007, 2008, 2010) and Stathopoulos (2017), inspired in part by Zapatero (1995), who study a pure exchange economy similar to mine, but in which preferences are log and the elasticity of intertemporal substitution across goods is equal to one. The combination of those assumptions leads the allocation of wealth to be constant, equity assets to be perfectly correlated in the absence of demand shocks, and hedging demands to be absent due to myopic portfolios. All three are important dimensions that arise in my framework once I allow for general recursive preferences and an arbitrary elasticity of substitution between goods. I therefore see my contribution has the natural continuation of this earlier research effort.

Breaking those limitations does not come without a cost however, and solving the model requires a whole new set of methods compared to those papers. In particular, the resolution of my framework is based on global projection methods, as presented in Judd (1992, 1998), the NBER Summer SI Lecture by Fernández-Villaverde and Christiano (2011), or Parra-Alvarez (2018), and as applied to multi-agent models for instance in Drechsler et al. (2018), Fang (2019), or Kargar (2019). The approximation is based on Chebyshev polynomials and orthogonal collocation, although in concurrent work, I am also developing a natural extension based on neural networks (Sauzet, 2021c, cf. Section 4.5).[7]

In addition, I also introduce asymmetries in preferences, labor income in the form of a constant share of output as in Baxter and Jermann (1997), and most importantly, imperfect financial integration. The latter is captured in a parsimonious way as a tax on foreign dividends by generalizing Bhamra et al. (2014) to a non-log environment that also features home bias, and following the seminal contribution of Basak and Gallmeyer (2003) who study a dynamic asset pricing model with asymmetric dividend taxation and a unique risky asset in a one-country one-good setting. Compared to Bhamra et al. (2014), the introduction of general preferences makes a significant difference: imperfect risk sharing has a large impact *provided that* the elasticity of intertemporal substitution is modest, a novel insight. In addition, I use a global solution instead of relying on local approximations, and am able to study the effect on the exchange rate and of hedging terms. Theoretically, the use of a tax to capture a wide range of frictions is related to the work of Gârleanu et al. (2020), who show that models with investment taxes constitute an equivalent, but substantially simpler, way to capture a rich set of impediments to financial trade.

[7] I solve for the decentralized economy throughout, but the method of Dumas et al. (2000), based on a planner, could also be used in cases in which risk sharing is perfect.

Other related papers include Cass and Pavlova (2004), Brandt et al. (2006), Martin (2011), and Maggiori (2017) that I discuss below, as well as Fang (2019) who focuses on a small open economy in which the rest of the world is taken as exogenous and in which investors do not have symmetric home bias. On the theoretical front, my paper is also related to contributions introducing recursive preferences in continuous-time e.g. Duffie and Epstein (1992), and contributions focusing on the existence and uniqueness of equilibria in the presence of multiple agents, and possibly multiple goods and incomplete markets e.g. Polemarchakis (1988), Geanakoplos and Polemarchakis (1986), Geanakoplos and Mas-Colell (1989), Geanakoplos (1990), Duffie et al. (1994), Berrada et al. (2007), Anderson and Raimondo (2008), Hugonnier et al. (2012), Ehling and Heyerdahl-Larsen (2015).

Second, I contribute to the literature on the international portfolio problem. Specifically, the advances presented above allow me to characterize the general and global solution to the international portfolio choice problem, a long-standing issue in this literature since the seminar contributions of Stulz (1983), Dumas (1989, 1992), Cole and Obstfeld (1991), Zapatero (1995), Baxter and Jermann (1997), Baxter et al. (1998), Obstfeld and Rogoff (2001), Obstfeld (2004), among many others. Obstfeld (2007) and Coeurdacier and Rey (2013) provide surveys.

To a large part of the more recent literature on the topic, such as Corsetti et al. (2008), Tille and van Wincoop (2010), Coeurdacier (2009), Devereux and Sutherland (2011), Evans and Hnatkovska (2012), Coeurdacier and Rey (2013), Coeurdacier and Gourinchas (2016), I bring (i) a solution that is global and does not rely on approximations. This allows to complete the picture and trace out the evolution of economic outcomes as we move away from the point of approximation (typically the symmetric point), which proves important in this context where variables are strongly state-dependent and potentially non-linear. I also bring (ii) general preferences, which allow to move away from special cases and study all situations under a unified framework (cf. also the discussion above of Pavlova and Rigobon, 2007, 2008, 2010, Stathopoulos, 2017). A limited number of contributions have relied on global methods in similar settings e.g. Kubler and Schmedders (2003) (one country), Stepanchuk and Tsyrennikov (2015) (one good), Rabitsch et al. (2015), and Coeurdacier et al. (2020) (one good). To those, I bring (iii) continuous-time methods, which make it possible to study portfolio drivers, in particular hedging demands, asset prices and their conditional first and second moments, as well as the determinants of wealth and state variable dynamics, in ways that are inaccessible in a discrete-time formulation and therefore make continuous-time the natural tool of choice to study this type of questions. Finally (iv), to all, in addition to labor income as in Baxter and Jermann (1997) and asymmetries in preferences, I bring imperfect financial integration, which is an important topic in international finance but had not been studied thus far in a general international portfolio choice context.[8]

My contribution is also related to those of Colacito and Croce (2011, 2013), and Colacito et al. (2018), who introduce recursive preferences in an international context. Compared to those, output does not feature long-run risk dynamics. Instead, I bring in an arbitrary elasticity of substitution across goods, which makes the two equity assets no longer perfectly correlated so that the portfolio choice is no longer indeterminate in my context. More generally, I bring (i), (ii), (iii) and (iv) above to that economy. Dou and Verdelhan (2015) deserves particular mention as well: the authors solve an international portfolio problem globally, with general preferences and endowments, portfolio constraints, and incomplete markets. However, their focus on the volatility of international capital flows is different. In addition, partly because their framework is cast in discrete time, they do not focus on describing the underlying determinant of portfolios, such as hedging demands, which are an important part of my contribution.

Finally, the main application in this paper is in the spirit of Gourinchas and Rey (2007a,b), Caballero et al. (2008), Gourinchas et al. (2017), and Maggiori (2017) that I bring to the general international portfolio choice context of my framework. Papers related to the facts that this specialization of the model replicates were discussed previously, and are also mentioned in Section 4.1 with a summary of the stylized facts. Compared to Maggiori (2017) in particular, one main difference is the presence of recursive preferences compared to log, which ensures that the wealth share across international investors is not constant even in the absence of bankers, and that portfolios feature hedging demands that are crucial to study biases in equity holdings. In addition, the elasticity of intertemporal substitution is not equal to one, so that the

[8] More general specification of labor such as a time-varying share in the spirit of Coeurdacier and Gourinchas (2016) or idiosyncratic labor income risk as in Kaplan et al. (2018) are interesting avenues for further exploration.

returns on the two equity assets are not perfectly correlated and the portfolio choice is not indeterminate. I also introduce (iv) labor income, asymmetries, and imperfect risk sharing.

The paper is organized as follows. Section 2 briefly describes the set-up of the economy, and introduces the two state variables that drive economic mechanisms: the wealth share of the domestic investor, and the relative supply of the two goods, i.e. fundamentals. Section 3 revisits various results in the international portfolio choice literature in this unified framework. Section 4 presents the main application to modeling the international financial system, as well as possible extensions. Section 5 concludes. Additional material is provided in Appendix.

2. The Economy

The setup adapts the "222" framework (Two Investors Two Trees Two Goods) introduced in Sauzet (2021a) to an international context. Investors A, B are taken to represent the domestic, and foreign (*), representative investors. Good 1 is the good produced domestically, denoted h (home), and preferred by the domestic investor in her consumption basket, while good 2 is the good produced abroad, denoted f (foreign), and preferred by the foreign investor. The rest of the setup is as in Sauzet (2021a) and the reader is referred to that paper for details.

Assumption 1 (Symmetric baseline calibration)**.** *Unless otherwise specified, the results presented in this section are obtained under the following calibration:*

- *Risk aversion:* $\gamma = \gamma^* = 15$,
- *Elasticity of intertemporal substituion:* $\psi = \psi^* = 2$,
- *Home bias in consumption:* $\alpha = \alpha^* = 0.75$, *numéraire basket:* $a = 1/2$,
- *Elasticity of substitution between goods:* $\theta = \theta^* = 2$,
- *Discount rate:* $\rho = \rho^* = 1\%$,
- *No labor income:* $\delta = \delta^* = 0$,
- *Fully integrated financial markets:* $\tau = \tau^* = 0$,
- *Output:* $\mu_Y = \mu_Y^* = 2\%, \sigma_Y = (4.1\%, 0)^T, \sigma_Y^* = (0, 4.1\%)^T$ *(no fundamental correlation).*

3. Revisiting International Portfolio Choice Results

4. Application: The International Financial System

Because of its generality, the framework in this paper represents a versatile building block towards several applications and extensions. In this section, I specialize the model to capture important dimensions of the international financial system, with a particular focus on the role of the United States, its center country. The introduction of asymmetries in the tolerance for risk of international investors naturally replicates the role of the United States as the world banker, documented in Gourinchas and Rey (2007b) and Gourinchas et al. (2017), and a modest degree of imperfect financial integration also generates a plausible home bias in equity holdings for both investors. Importantly, those additions not only allow us to match facts on the U.S. external portfolio on average, but also make it possible to study its dynamics. In particular, crisis episodes, by worsening the wealth position of the world banker, lead to a sharp increase in global risk aversion that in turn increases risk premia worldwide, in a pattern reminiscent of some aspects of the Global Financial Cycle proposed by Rey (2013) and Miranda-Agrippino and Rey (2020) but for which a general equilibrium exploration remained elusive. The framework allows to study the response of portfolios to shocks, the process of external adjustment of the center country, as well as the evolution of the (time-varying) comovement of returns. In doing so, it is able to replicate several additional facts e.g. about asset return dynamics. I also discuss a number of counterfactual experiments, as well as worthwhile extensions that pertain to the introduction of global financial intermediaries in this international context. Overall, those findings reinforce the broad message that emerged from the characterization of Sauzet (2021a) and previous sections: capital is back, and the allocation of wealth is of prime importance in this context.

4.1. Stylized facts

I start by summarizing a number of stylized facts about the international financial system and asset returns in this context, which the specialization of the model can jointly match. I focus in particular on the role of the United States, its center country.

Fact 1 (U.S. as world banker)**.** *The United States borrows from the rest of the world in safe assets, and uses it to lever up its investment in risky assets worldwide, resulting in a negative net foreign asset position. The country therefore plays the role of the world banker. (Gourinchas and Rey, 2007b; Gourinchas et al., 2017)*

Fact 2 (Exorbitant privilege)**.** *The United States earn excess returns on average on its net foreign asset position, in particular in normal (non-crisis) times. (Gourinchas and Rey, 2007b; Gourinchas et al., 2017)*

Fact 3 (Exorbitant duty)**.** *In times of crisis, the United States plays the role of the world insurer, transferring wealth to the rest of the world. This exorbitant duty is the flip of its exorbitant privilege in normal times, and is associated with a strong deterioration in the U.S. net foreign asset position. (Gourinchas et al., 2017)*

Fact 4 (Home bias in equity holdings)**.** *The aggregate portfolio of the United States, as well as that of most countries around the world, exhibit a strong bias towards domestic equity securities. This bias has slowly decreased in recent decades. (French and Poterba, 1991; Coeurdacier and Rey, 2013)*

Fact 5 (Global Financial Cycle)**.** *Periods of global stress are characterized by risk-off scenarios in which the global risk aversion, as well as risk premia worldwide, spike up. (Some aspects of Rey, 2013; Miranda-Agrippino and Rey, 2020)*

Fact 6 (International Financial Adjustment)**.** *Periods of strong deterioration in the net foreign asset position of the United States predict higher expected risk premia on its external balance sheet in the short to medium term, consistent with valuation effects playing a key role in the process of its external adjustment. (Gourinchas and Rey, 2007a, extended to more recent data in Gourinchas et al., 2019)*

Fact 7 (Countercyclicality of asset return dynamics)**.** *The (i) risk premia, (ii) Sharpe ratios, (iii) volatilities, and (iv) correlation of risky returns are countercyclical, i.e. they increase in times of crisis. This is true in particular in times of global stress. Those evolutions are consistent with crises being periods in which not only the quantity of risk rises, but also the price of risk that is received as a compensation. (Among others, in the context of the United States: for (i), (ii), (iii), cf. Lettau and Ludvigson,* *2010; for (i), cf. Fama and French, 1989, Ferson and Harvey, 1991, Harrison and Zhang, 1999, Campbell and Diebold, 2009; for (ii), cf. Harvey, 2001; for (iii), cf. Schwert, 1989, Brandt and Kang, 2004)*

Fact 8 (Global trend in asset comovements)**.** *The comovement of equity prices worldwide has increased over the last 150 years, and particularly rapidly in the past three decades. This increase goes above and beyond the growing synchronization in real sector variables. The sharp increase in the comovement of global equity markets is driven by fluctuations in risk premia, which are themselves strongly impacted by fluctuations in global risk appetite. (Jordà et al., 2019)*

Fact 9 (Global trend in interest rates)**.** *The real rate of interest has trended down globally in recent decades, and this can be related to the relative size of the risk-tolerant world banker (the United States) decreasing in the world economy. (Caballero et al., 2008; Hall, 2016; Gourinchas et al., 2017)*

4.2. External portfolio and the exorbitant privilege

Throughout Section 4, the domestic country is taken to represent the United States, the country at the center of the international financial system. Its representative investor is assumed to display a larger appetite for risk, so that her risk aversion is now lower than that for the representative foreign investor ($\gamma = 8 < \gamma^* = 15$). This assumption, in the spirit of Caballero et al. (2008), Gourinchas et al. (2017), and Maggiori (2017), is meant to capture the greater development and depth of U.S. financial markets, making the country as a whole better able and willing to carry financial risk in the world economy. Following the discussion in Section 3 and Sauzet (2021a), I also introduce a modest degree of imperfect financial integration, $\tau = \tau^* = 15\%$, which, combined with an elasticity of intertemporal substitution of $\psi = \psi^* = 0.5$, generates plausible portfolios, as well as a realistic level for the interest rate. Other parameters are calibrated as before and summarized in Assumption 1.

The introduction of asymmetries in the tolerance for risk of the two international investors has a profound impact on the equilibrium and in particular on countries' portfolios. Specifically, the representative U.S. investor is willing to borrow from risk-averse investors in the rest of the world using the risk-free bond, so as to lever up her risky portfolio. Panel (a) of Figure 4 shows that this borrowing is large, and happens throughout the state-space. For instance, around $x_t = 30\%$, which corresponds broadly to the share of the United States in world wealth (Crédit Suisse, 2019), the country borrows about 50% ($b_t = -0.5$) of its wealth in international markets, so as to invest 150% of its wealth in risky equity securities. Those results are consistent with the findings of Gourinchas et al. (2017), who document a strongly negative net foreign position in safe securities for the United States, which uses those safe liabilities to finance its investments in risky assets worldwide.[9] In other words, the model naturally replicates Fact 1 about the role of the United States as the world banker, or more accurately, given the amount of leverage, as the world venture capitalist, as pointed out in Gourinchas and Rey (2007b). As a result, the net foreign asset position of the United States is strongly negative, like in the data. In the model, the latter is computed as the difference between the wealth invested in the foreign equity asset and in the world bond by the domestic investor, and that invested in the domestic asset by the foreign investor. As a fraction of domestic wealth, this yields

$$\frac{NFA_t}{W_t} = w_{f,t} + b_t - w^*_{h,t}\left(\frac{1 - x_t}{x_t}\right) \tag{1}$$

The results is shown in Panel (b) of Figure 4. At $x_t = 30\%$, $NFA_t/W_t = -30\%$ on average. Like the amount of borrowing, the net foreign asset position also varies strongly with the state of the economy as a result of both asymmetries and imperfect risk sharing, an aspect that I discuss when focusing on dynamics below.[10]

The modest degree of imperfect financial integration is able to generate a plausible

[9] I should note that even though the framework gets us close to several empirical measures, such as the net foreign asset position, interest rates, or portfolios, the goal of the exercise is primarily conceptual. A number of small additional extensions could be considered to make the quantitative match even closer.

[10] The international bond can also be considered to be part of the domestic country foreign *liabilities*, reflecting the fact that most countries save internationally using Treasury bonds issued by the United States. This does not change the accounting equation given that one would subtract $-b^*_t(1 - x_t)/x_t$, which is equal to b_t by market clearing. (Recall that $b_t < 0$ as the United States borrows from the rest of the world.) The measure can also be expressed as a fraction of domestic output as follows

$$\frac{NFA_t}{Y_t} = \left(w_{f,t} + b_t - w^*_{h,t}\left(\frac{1 - x_t}{x_t}\right)\right)\left(\frac{x_t}{z_t}\right)\left(\frac{(1 - \delta)p_t}{F_t}\right) \tag{2}$$

home bias in equity holdings for both investors (Fact 4). With $\tau = 15\%$, the shares in the domestic equity portfolio allocated to domestic and foreign assets for $x_t = 30\%$ are around 70% and 30% respectively, broadly in line with findings in Coeurdacier and Rey (2013) for recent years.[11] This result is of course subject to the caveats discussed in Section 3 and Sauzet (2021a). First, as I discuss below and as seen in Figure 3, portfolios again vary substantially with the state of the economy so that matching them in one given point does not guarantee matching them throughout. This is particularly true here where the impact of the allocation of wealth on portfolios is significantly larger than in the baseline of Section 3 and Sauzet (2021a). Second, for tax on foreign dividends to have a quantitatively sufficient effect, the elasticity of intertemporal substitution ψ must not be too large, and I pick a value of $\psi = 0.5$ to also be able to match the average level of the interest rate.[12] Despite those caveats, the degree of imperfect financial integration allows to generate portfolios that are broadly consistent with the data and is therefore adequate for our purposes.

Finally, as a last point in terms of average levels, the model is also able to reproduce the exorbitant privilege that the United States has benefited from as a result of its external portfolio (Fact 2). I follow the definition of Gourinchas et al. (2017), and understand exorbitant privilege to mean the excess returns on the U.S. external portfolio. This is first visible by comparing the expected returns on the *total* portfolio of the domestic investor, $\overline{\mu_{R,t}} \equiv w_{h,t}\mu_{R,t} + w_{f,t}\mu_{R^*,t} + b_t r_t$, to that of the foreign investor, $\overline{\mu_{R^*,t}} \equiv w^*_{h,t}\mu_{R,t} + w^*_{f,t}\mu_{R^*,t} + b^*_t r_t$. Panels (a) and (b) of Figure 1 plot both as a function of the wealth share. The former is larger regardless of the state of the economy, with $\overline{\mu_{R,t}} = 4.9\%$ and $\overline{\mu_{R^*,t}} = 3.9\%$ on average, reflecting the riskier position taken by the United States that is financed by borrowing internationally in the safe asset and earns a higher returns in expectation. Interestingly, this result highlights the fact the center country borrows to invest not only internationally but in its own domestic assets as well.

Second, we can also focus specifically on the returns on the *external* portfolio itself. Here, the United States earns $r^a_t = \mu_{R^*,t}$ on its external assets, which are comprised

[11] The corresponding measure, $1 - FB_t$, is of 40% on average for $x_t = 30\%$, slightly below empirical estimates, but varies substantially throughout the state space.

[12] An additional source of asymmetry that could be worthwhile to study in this context is differences in ψ. This could capture differences across countries in propensity to save, for instance between the United States, and heavy savers like Europe due to its demographic deficit, and China or southeast Asia.

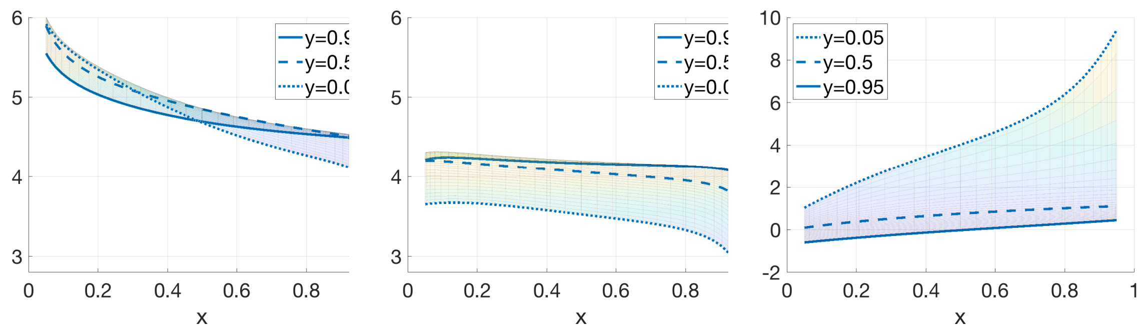
of the foreign equity asset, and pays on its external liabilities r_t^l, the weighted average of the returns on the domestic equity and on the risk-free bond. The excess returns on the external position are therefore

$$r_t^a - r_t^l \equiv \mu_{R^*,t} - \left\{ \left(\frac{w_{h,t}^*}{w_{h,t}^* + b_t^*} \right) \mu_{R,t} + \left(\frac{b_t^*}{w_{h,t}^* + b_t^*} \right) r_t \right\} \tag{3}$$

Panel (c) of Figure 1 shows the results, with $r_t^a - r_t^l = 1.4\%$ on average, and about 0.7% when $x_t = 30\%$, consistent with the lower range of estimates in Gourinchas et al. (2017).[13] The excess returns earned on its external portfolio allows the United States to finance its external deficit, and helps reduce the burden of its external adjustment process, as discussed in Gourinchas and Rey (2007a) and below.

Figure 1: Average expected returns on countries' portfolios (%)

(a) United States: $\overline{\mu_{R,t}}$ (b) Rest of the world: $\overline{\mu_{R^*,t}}$ (c) United States: $r_t^a - r_t^l$



Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

[13] In this framework, we cannot easily replicate the fact the United States not only earns a higher returns on average, but also within asset categories. Indeed, because of the home bias in equity holdings that obtains with imperfect financial integration, the returns on the foreign equity asset are in fact larger than that on the domestic equity asset for a large part of the state space, due to the fact that the foreign asset is held for a larger proportion by the foreign investor who is more risk-averse and therefore requires a higher compensation for risk on average. This is not true for the whole state space however, and one could potentially obtain a larger $\mu_{R,t}$ by reducing the impact of the home bias in equity holdings. This could also be alleviated for instance by introducing more assets, in the form of other countries/trees or several bonds. Those explorations are left for future research.

In summary, the introduction of asymmetries in risk aversion, together with a modest degree of imperfect financial integration, is able to reproduce a number of stylized facts about the average level of the external portfolio of the United States in the world economy. Specifically, those additions allow the framework to capture Facts 1, 2 and 4 of Section 4.1

4.3. Crisis, the Global Financial Cycle, and dynamic aspects

The better ability and willingness of the domestic country to bear financial risk not only naturally replicates the external portfolio of United States in levels, but also can help jointly match a number of facts about the dynamics of the international financial system and asset returns.

To see this, let us consider a negative shock to the output of the United States, the domestic country ($dZ_t < 0$). This shock, because it leads the domestic asset to do particularly poorly, brings a sharp decline in the share of wealth held by the domestic country, due to the home bias in equity holdings that results from imperfect financial integration on average. Because it affects the country most able to carry risk in the world economy, it is meant to capture a severe international crisis of the type the world experienced in the Great Recession of 2008.[14]

The decrease in the share of world wealth held by the domestic country directly captures the transfer of wealth that occurs from the United States to the rest of the world in times of international crisis. As put forward by Gourinchas et al. (2017) and summarized in Fact 3, this is the flip side of the banker role played by the United States in international markets. In good times, the world banker reaps the exorbitant

[14]The loadings of the wealth share on both shocks is shown in Figure C.20. A negative shock to the foreign output ($dZ_t^* < 0$) also leads to a decrease in the share of wealth of the domestic country in most of the state space because even though its representative investor holds more of her wealth in the domestic asset, she still holds a larger share of the foreign asset than the foreign investor. This is because she levers up her risky portfolio using the international bond. However, a shock to domestic output has a stronger effect due to the home bias in equity holdings, and is more akin to a type of world shock like the Great Recession of 2008 or the Global Pandemic of 2020. The processes of output could also be specified so as to be driven by a common component, for instance by adapting the share process of Santos and Veronesi (2006) to capture the evolution of relative supply y_t, as briefly discussed in Section 4.5. I leave this exploration for future research, but note that it could also help deliver a stationary distribution of world wealth.

privilege of its role by earning higher returns and running a large negative net foreign asset position, as we have seen. In bad times however, the country bears the exorbitant duty of insuring the rest of the world by transferring wealth to other countries. Empirically, consistent with my results, the exorbitant duty is large: Gourinchas et al. (2017) estimate that the transfer of wealth amounted to around 19% of U.S. GDP during the 2007-2009 global financial crisis. In the model, the phenomenon can also be observed clearly from the patterns of consumption, with the share of consumption (at market value) enjoyed by the United States declining monotonically with its share of world wealth, consistent with the rest of the world receiving a transfer and therefore consuming more in times of crisis.[15]

In addition and most interestingly, the global shock also gives rise to movements in asset premia that are reminiscent of some aspects of the Global Financial Cycle put to light by Rey (2013) and Miranda-Agrippino and Rey (2020), but which have so far not been captured in a general equilibrium model of the international financial system. As the share of wealth held by the United States decreases, global wealth-weighted risk aversion γ_t, which is time-varying and state-dependent in this asymmetric context, spikes up, reflecting the fact that the rest of world, financially less able and willing to carry risk, governs a larger share of world wealth following the wealth transfer.[16] This pattern captures the emergence of a risk-off scenario worldwide with the compensation for taking risk rising globally as the wealth position of the world banker deteriorates, and is one of the markers of the Global Financial Cycle summarized in Fact 5. In turn, it leads asset prices to decrease more than they would due to the sole effect of worse fundamentals, so that the risk premia and Sharpe ratios on both assets increase sharply. Similarly, the interest rate declines globally with the world becoming more risk-averse on average. This captures the global risk factor of Miranda-Agrippino and Rey (2020) and the second element of Fact 5. Figure 2 shows each of those variables as a function of the wealth share, and suggests that the impact is large. Compared to the baseline of Section 3 and Sauzet (2021a) in which the wealth share had a modest effect on risk premia, it now becomes a much more important determinant, on par

[15]The share of domestic consumption at market value is computed as

$$\frac{P_t C_t}{P_t C_t + P_t^* C_t^*} = \frac{c_t x_t}{c_t x_t + \mathcal{E}_t c_t^* (1 - x_t)} \tag{4}$$

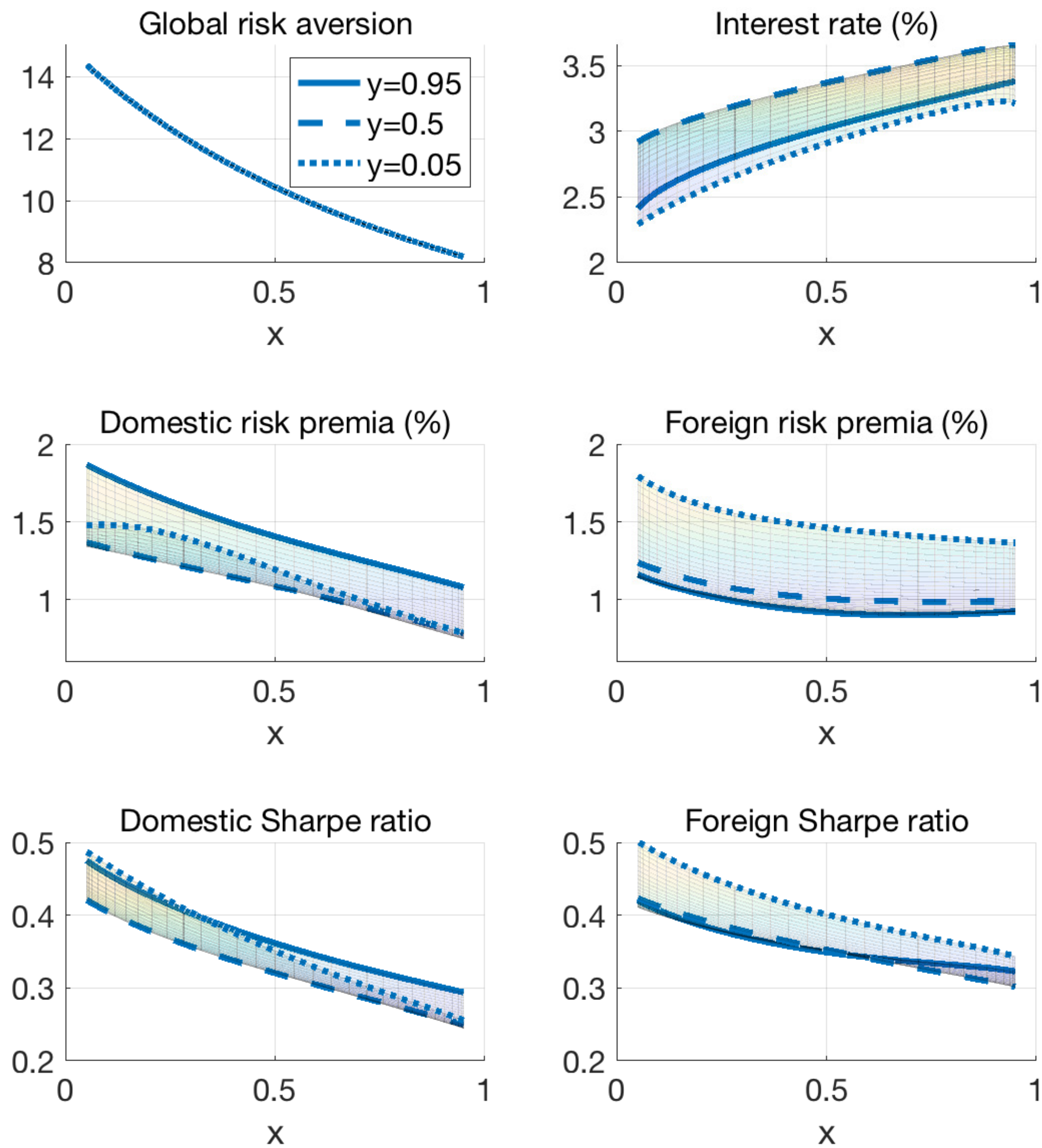
[16]As a reminder, the global wealth-weighted risk aversion is defined as $\gamma_t \equiv \left(\frac{x_t}{\gamma} + \frac{1-x_t}{\gamma^*}\right)^{-1}$.

with fundamentals captured by the relative supply of goods y_t. For instance, the interest rate decreases by 73 basis points when the domestic country goes from being dominant to small in world wealth, and risk premia increase by 25 to 60 basis points depending on the asset. Those effects are large given that the overall level of risk premia remains modest in this economy in which the only friction is a mild degree of imperfect financial integration. For instance, they represent increases in risk premia of 22 to 51%. The effect is also clear when risk premia are normalized by units of volatility: Sharpe ratios increase by 0.12 to 0.17, a 33 and 49% increase respectively. Quantitatively, this reinforced effect can also confirmed by computing elasticities with respect to each state variable.[17] The main economic message is that, in this economy that features international investors with diverging tolerance for bearing risk, the identity of the ultimate holder of most assets in the world economy, captures by x_t, matters as much or more than the fundamental underpinnings of those assets, captured by the relative supply y_t. In other words, once again, "capital is back": the allocation of wealth has a profound impact on the world economy.

To be sure, the mechanism proposed here does not cover all dimensions of the phenomenon documented by Rey (2013) and Miranda-Agrippino and Rey (2020). For one, because the model is real, there is not role for monetary policy. In addition, the type of risk-off scenario that arises at the international level could also be taking place within countries, with a shift towards safe assets for the average investor of each country. I discuss this point in the context of the reserve currency paradox below. Moreover, because there is only one international bond, which is riskless and pays the same interest for all investors, it does not capture the type of convenience yields that has been discussed as an important element in this context, e.g. by Jiang et al. (2020), and Kekre and Lenel (2020).

[17] Specifically, we can compute $|\varepsilon_{g_t,x_t}| \equiv \left|\frac{\partial \ln g_t}{\partial \ln x_t}\right|$ and $|\varepsilon_{g_t,y_t}| \equiv \left|\frac{\partial \ln g_t}{\partial \ln y_t}\right|$ where $g_t \in \{\mu_{R,t} - r_t, SR_t\}$. On average, $\overline{|\varepsilon_{\mu_{R,t}-r_t,x_t}|} = 0.41$ and $\overline{|\varepsilon_{SR_t,x_t}|} = 0.32$ on par with $\overline{|\varepsilon_{\mu_{R,t}-r_t,y_t}|} = 0.59$ and $\overline{|\varepsilon_{SR_t,y_t}|} = 0.35$. This is much larger than in the baseline in which $\overline{|\varepsilon_{\mu_{R,t}-r_t,x_t}|} = 0.02$, $\overline{|\varepsilon_{SR_t,x_t}|} = 0.03$, $\overline{|\varepsilon_{\mu_{R,t}-r_t,y_t}|} = 0.54$, $\overline{|\varepsilon_{SR_t,y_t}|} = 0.22$. The effect is present in particular in relevant parts of the state space. For instance, when $x_t = 30\%$, average elasticities are $\overline{|\varepsilon_{\mu_{R,t}-r_t,x_t}|} = 0.03$, $\overline{|\varepsilon_{SR_t,x_t}|} = 0.04$, on par or above the effect of fundamentals, $\overline{|\varepsilon_{\mu_{R,t}-r_t,y_t}|} = 0.01$, $\overline{|\varepsilon_{SR_t,y_t}|} = 0.05$.

Figure 2: Asset pricing under asymmetric risk tolerance and imperfect financial integration



Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Another relevant aspect can also be to impose portfolio constraints so that the investors of both countries, but in particular that representative of the United States, have time-varying limited risk-bearing capacity that vary with their balance sheets. Because the framework is flexible, those dimensions can be introduced and represent promising avenues for future research. I discuss some of them in Section 4.5. Still, the main economic message remains: the introduction of asymmetries in the capacity and tolerance to bear risk together with a mild degree of imperfect financial integration, which naturally replicate the aggregate portfolio of the United States on average, also give rise to dynamic aspects that had so far not been clearly elucidated and that are reminiscent of important dimensions of the evolution of the international financial system in times of severe global crisis.

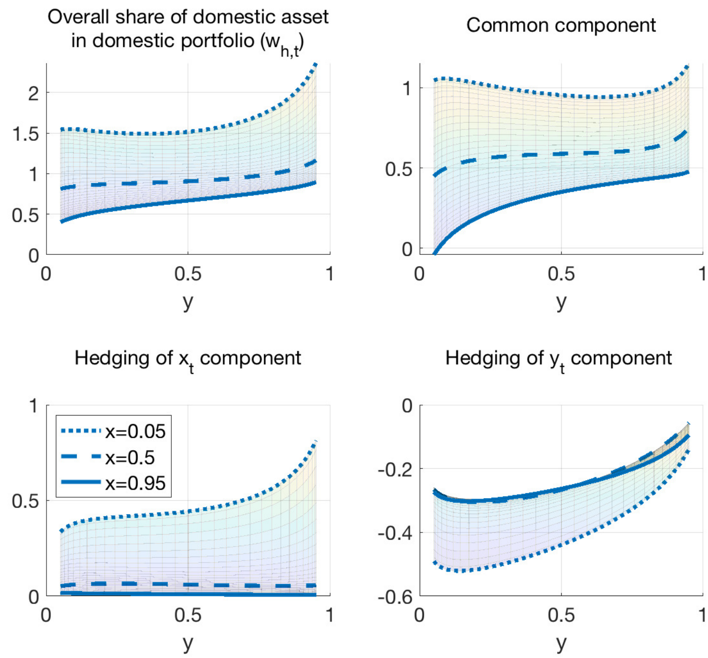
Importantly, because I study this question in a general international portfolio choice context, the framework also allows to discuss and match a number of further relevant facts.

First, portfolios can be analyzed not only on average, but also in their dynamics. In particular, even though the introduction of imperfect financial integration allows the model to match the home bias in equity holdings qualitatively, and quantitatively around $x_t = 30\%$, portfolios remain strongly state-dependent. As her wealth share decreases following the shock, the representative domestic investor tends to increase the weight of risky assets further in her portfolio, by ramping up her borrowing from the rest of the world as a share of her wealth. Her desire to do so comes both from rising risk premia worldwide in particular on the domestic asset, which impact her portfolio via its common component, and from the hedging of her wealth share risk. The latter is particularly active throughout the state space and it strongly reinforces the home bias in portfolio holdings due to the positive equilibrium relationship between wealth share and relative supply that obtains in this case, which leads the domestic asset to pay increasingly well as the domestic marginal value of wealth increases in the wealth share dimension (cf. Section 3 and Sauzet, 2021a, for details). This effect is especially strong as the wealth share of the domestic investor becomes small as seen in Figure 3. Quantitatively, the impact of the wealth share is once again significantly larger than in the baseline of Section 3 and Sauzet (2021a), due to the combination of asymmetries in preferences and imperfect financial integration. This is true both for its direct effect as a state variable that captures the average investor in the world economy – the two international agents are now very different–, and through its hedging effect – a variance decomposition similar to Section 3 and in Sauzet (2021a) suggests that the hedging of x_t now drives 39% of the changes in $w_{h,t}$ against 7% in the baseline.[18]

Portfolios also vary strongly with the relative supply of goods, due to effect of the home bias in consumption combined with asymmetries in risk tolerance and imperfectly integrated markets. As y_t decreases, which happens when the negative shock is concentrated on the domestic output, the domestic investor chooses to increase her share in the foreign asset, $w_{f,t}$, so that it locally can rise above that of the domestic asset, $w_{h,t}$. This reflects for a large part the common component of portfolios, with the share of the domestic asset in the market portfolio, z_t, being particularly small in that case, as well as the pattern of bond trading. Indeed, while the fact that the domestic investor borrows on average is driven by her higher tolerance for risk, the trading in the bond is also asymmetric: she borrows in particular as y_t decreases, driven by the introduction of imperfect risk sharing, and as discussed in Section 3 and Sauzet (2021a). When compared to the weight in the market portfolio z_t, the HB_t measure remains above 1, and in fact much so in this particular region of the state space. As the relative supply of the domestic good y_t increases however, which would happen if the foreign output is especially negatively impacted, the domestic investor borrows a lesser share of her wealth, and uses it to lever up mostly on the domestic asset whose expected returns increase particularly much, while she barely invests in the foreign asset, $w_{h,t} \gg w_{f,t}$. In this case, the weight on the foreign asset is not only small in itself, but also when compared to its weight in the market portfolio, $1 - z_t$.

[18]For $w_{h,t}$, in more details, the common, hedging of x_t, and hedging of y_t components drive 69%, 39%, and -2%, respectively. For $w_{f,t}$, the contribution of the common, hedging of x_t, and hedging of y_t components are of 19%, 17%, and 37%, with the tax component accounting for the remaining 27%. In the baseline of Section **??**, the first three figures were 70%, 7%, and 34% respectively, for both $w_{h,t}$ and $w_{f,t}$.

Figure 3: Components of the domestic portfolio



Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Taken together, those results highlights once again that portfolios are strongly time-varying in this context, in fact more so than in the baseline case of Section 3 and Sauzet (2021a). Once again, assessing whether those patterns are borne out in the data calls for additional international portfolios facts. From that perspective, the large and detailed data gathering effort undertaken for the Global Capital Allocation project of Maggiori et al. (2020) and Coppola et al. (2020) will assuredly prove invaluable. Especially important would also be the establishment of facts about portfolio rebalancing at a higher frequency and in response to shocks. If portfolios instead do not vary as much, this could be the sign of a failure to optimize in the part of international investors, in particular in times in which it is most valuable.

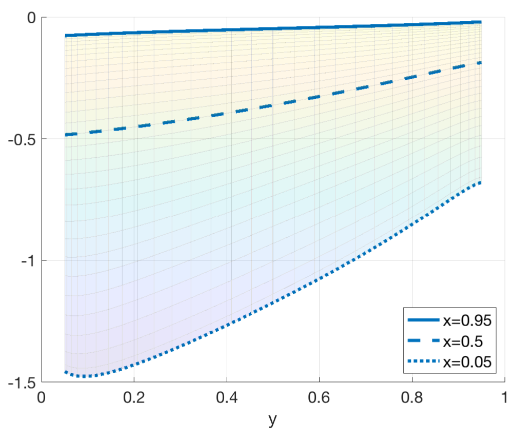
Second, the process of external adjustment of the country at the center of the international financial system can also be studied in this context. As the shock hits, not only does the share of world wealth held by the United States decrease, reflecting the wealth transfer to the rest of the world coming from its role as world insurer, but the net foreign asset position of the country also strongly deteriorates. This is shown in Panel (b) of Figure 4, and is consistent once again with the findings of Gourinchas et al. (2017) in Fact 3. In this context, the phenomenon arises because the United States borrows an increasing share of its wealth as its wealth share decreases, as shown in Panel (a), broadly in line with the increased burden of its safe liabilities to the rest of the world that results empirically in a sizable decline of the net foreign safe asset of the country further into negative territory in times of crisis. In addition, the sharp increase in expected risk premia that occurs simultaneously emphasizes the primordial role in this situation of valuation effects of the type proposed by Gourinchas and Rey (2007a) for the process of external adjustment. In my model, they take the form of time-varying risk premia: the United States invests heavily in risky assets whose expected returns are large in particular in times of crisis. Those ensures that its net foreign asset position is expected to improve in the future, thereby facilitating the process of external adjustment and allowing the country to sustain an even more negative external position when the crisis hits. In the long run, the higher returns earned on risky assets on average lead the domestic country to accumulate wealth at a faster rate so that the domestic investor progressively dominates the world economy. This is accompanied by a positive net foreign asset position, which also participates in easing the pressure of the external adjustment process even when the net foreign asset position is negative at shorter-term horizons.[19] I come back to long-run dynamics briefly in Section 4.4. Accordingly, the combination of the dynamics of the net foreign asset position of the domestic country and that of expected returns also replicates Fact 6 about the predictability regressions of Gourinchas and Rey (2007a), and extended to more recent data in Gourinchas et al. (2019). In the short run, a deterioration of the net foreign asset position of the United States is indeed associated with higher expected returns, $\mu_{R,t} - r_t, \mu_{R*,t} - r_t$, as documented previously when the shock hits. This pattern persists for medium-term horizons with risk premia remaining elevated for the time that the net foreign asset position stays below its initial level. Yet, as the wealth share and net foreign asset position continue improving back, owning to higher returns on the external portfolio on average, expected risk premia decrease

[19] In fact, here, the net foreign asset position accumulated by the domestic country is such that the country can sustain a trade deficit even in the long run.

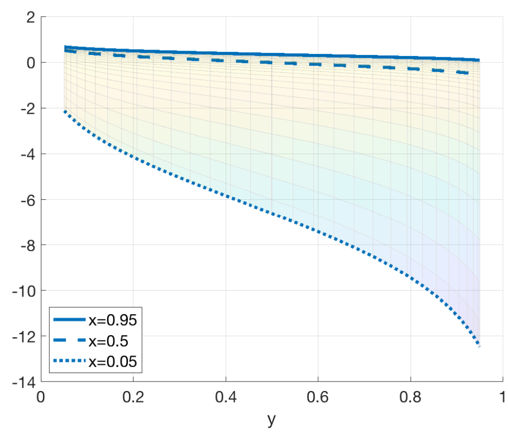
back towards their initial level or below. The negative relationship between the initial change in net foreign asset position and expected risk premia therefore dampens with the horizon, as in the work of the authors. In the long run, x_t becomes increasingly large so that risk premia are reduced further due to the reduced global risk aversion, and the relationship between the initial change in net foreign asset position with those returns becomes muted like in the data.

Figure 4: External position of the domestic country

(a) Share of bond in domestic portfolio (b_t)



(b) Net foreign asset position (NFA_t/W_t)



Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Finally, the framework also generates a number of asset pricing facts that are broadly in line with empirical observations, and that emerge here in an international context.

Firstly, the evolution of risk premia discussed previously shows that expected returns on both assets increase following a negative shock to the output of either of the country. This result is in line with countercyclical risk premia that have been documented in a wide variety of contexts (Fact 7, (i)), and emerges due to the heterogeneity between investors. In line with results in simpler one-good one-country contexts with stylized preferences e.g. in Weinbaum (2009), the share of wealth held by the risk-tolerant investor, here the investor representative of the domestic country, decreases with the shock, so that risk aversion and risk premia spike up, reflecting the fact that equity prices fall more than they would due purely to the worsening of fundamentals. Importantly, the sharp rise in risk aversion emphasizes that in this context risk premia are driven for an important part by large fluctuations in the compensation for risk, instead of solely being the result of changes in the quantity of risk. This is in line with a large literature that has seen changes in the price of risk emerge as a crucial explanation behind asset return predictability more generally.

In addition, in this general environment, the asymmetry in tolerance for risk interacts intimately with (i) the core heterogeneity that emerges due to the home bias in consumption, (ii) the fact that the two trees produce differentiated goods so that asset returns are impacted by changes in the prices of goods, and (iii) imperfect financial integration. For instance, for most values of y_t, the risk premium on the domestic asset increases faster than that on the foreign asset as the wealth share decreases. This reflects a larger change in ownership in this asset that the domestic investor tends to favor due to her home bias in equity holdings, which itself results from imperfect financial integration. As her wealth share decreases, the foreign investor has to pick up the slack, and the extra compensation for risk that the latter requires increases domestic returns comparatively more. Similarly, the effect of the relative supply on risk premia is also asymmetric. When the domestic country dominates the world economy, $\mu_{R,t} - r_t$ and $\mu_{R*,t} - r_t$ evolve broadly as expected and increase with the relative supply of the respective good underlying their payoffs. When the wealth share of the domestic country is small however, the risk premium on the domestic asset, $\mu_{R,t} - r_t$, also increases significantly when the relative supply of the *foreign* good is large, i.e. when y_t is small, reflecting in part the fact that its returns tend to locally comove negatively with the world-weighted marginal value of wealth (cf. Proposition **??**). The patterns of risk premia also reflect the evolution of the dividend yields on the two assets, F_t, F_t^*, which are inherently state-dependent in this context (Figure C.21).

Conditional Sharpe ratios are also strongly countercyclical, increasing as the wealth share decreases following a shock to the output of any of the trees. This can be seen in Figure 2, and is consistent with results in Harvey (2001) and Lettau and Ludvigson (2010) for the United States (Fact 7, (ii)). This pattern is driven mostly by the evolution of risk premia, and happens despite the volatility of the two risky assets being also mildly countercyclical for a relevant part of the state space, as discussed below. In terms of levels, the Sharpe ratio on the domestic asset is smaller for most of the state space, reflecting the fact that the volatility of the domestic asset is larger on average, following the larger impact of the change in ownership mentioned above.

Turning to second moments more specifically, which are shown in Figure C.22, volatilities are slightly larger on average than in the baseline of Section 3 and Sauzet (2021a) owning to the combination of additional heterogeneity and imperfect financial integration. They exhibit a mild countercyclicality, as observed empirically, in at least a relevant part of the state space (Fact 7, (iii))[20]. In particular, the volatilities of both assets, $\left(\sigma_{R,t}^{T}\sigma_{R,t}\right)^{-1/2}$, $\left(\sigma_{R*,t}^{T}\sigma_{R*,t}\right)^{-1/2}$ increase following the negative shock to U.S. output that has been the focus throughout the section. For the foreign asset, this occurs regardless of the wealth share of the domestic investor because the volatility of the foreign asset naturally increases following the decrease in y_t that also ensues. For the domestic asset, this occurs provided that the wealth share of the domestic country is not too small, and that the relative supply of the domestic good is not too large. Both are likely to happen following the shock considered here, and starting from a share of the United States in world wealth around $x_t = 30\%$ as in the data. More generally, Figure C.22 emphasizes that like their expectations, the second moments of returns are inherently time-varying, and non-monotonic. This is most visible beyond volatilities by looking at the conditional covariance and correlation of returns in Panels (e) and (f). Once again, following the negative shock to domestic output, and as long as the effect is large enough on y_t as well, both can increase and are therefore countercyclical in line with Fact 7, (iv).[21] In this context, this happens because the covariance and correlation increase as the relative supply of the domestic good decreases, and as the wealth share of the domestic investor decreases, provided that the latter is not too large. Note that like for volatilities, both are strongly non-monotonic. For instance, the correlation tends to have a U-shape relationship with the wealth share: it decrease with the wealth share as x_t is small, but then starts *in*creasing with the wealth share, as soon as the x_t surpasses around 40 or 50%. This is in sharp contrast to the baseline in which the correlation of returns tended to reach

[20]This is the so-called "leverage" effect and has been the subject of a large literature, starting from Schwert (1989). Cf. also Brandt and Kang (2004), as well as Lettau and Ludvigson (2010) for an exploration of the patterns of both the first and second conditional moments of returns.

[21]This also occurs for a negative shock to the foreign output, which decreases x_t for most of the state space, but increases y_t.

its maximum around the symmetric point, $x_t = 50\%$ (Figure C.32).

Taken together, those results highlight that the deep and complex interactions between the several dimensions of the model – multiple goods, home bias in consumption, asymmetries, imperfect financial integration – yield asset pricing facts that are broadly consistent with empirical observations.

In summary, the introduction of asymmetric tolerance for risk in the framework studied in Section 3 and Sauzet (2021a), combined with imperfectly integrated markets, allows to match not only the external portfolio of the United States, but also gives rise to a number of additional predictions that are strongly borne out in the data. Those results emphasize the value of being able to study those phenomena in a unified general equilibrium model of international portfolio choice. From a more theoretical perspective, here again, the impact of the wealth share not only on portfolios but also on asset prices, is strongly reinforced. This has shined through throughout the discussion, and is also reflected in the underlying drivers of the economy, with the hedging of wealth risk becoming a crucial determinant of portfolios as was the case previously in Section 3 and Sauzet (2021a). In brief, once again, "capital is back" in this international context, and its allocation is of first-order importance for economic outcomes.

4.4. Counterfactuals and long-term trend

Another value of studying those questions in the framework of this paper is that it allows to perform a number of counterfactual exercises of which I say a word here.

First, the gradual decline in the average level of home bias that has been observed worldwide can be naturally captured by a decline in the degree of imperfect financial integration τ. For instance, going from $\tau = 15\%$ to $\tau = 10\%$, leads the home bias in equity holdings at $x_t = 30\%$ to decline from $HB_t = 1.5$ to $HB_t = 1.225$ on average, consistent with (or even slightly larger than) the decline documented in Coeurdacier and Rey (2013). To get a sense of magnitudes, the shares of the domestic and foreign asset in the domestic equity portfolio, $w_{h,t}/(w_{h,t}+w_{f,t})$, $w_{f,t}/(w_{h,t}+w_{f,t})$, go from 68%

and 32%, to 55% and 45%, respectively.[22] This evolution is not only powerful in the model, but also appears realistic given that a reduction in barriers to international financial trade is likely to be one of the main determinants of the explosion in capital flows worldwide that has occurred since the 1990s.[23] Importantly, and as expected given our discussion throughout, the change in the degree of financial integration, even though it allows to broadly match the home bias in equity holdings around $x_t = 30\%$, is not without consequences for the evolution of portfolios throughout the state space. For instance, the share of the domestic portfolio invested in the domestic asset can go from large, in fact above 100% of wealth using leverage when x_t is small, to much smaller as τ decreases.[24] The change in the degree of financial integration is also reflected in the trading of the bond. Even though it remains large due to the persisting differences in tolerance for risk, bond trading becomes slightly more limited as markets become more integrated, consistent with the fact that like in Bhamra et al. (2014), an increasing share of risk sharing takes place through trading in equity assets. This prediction is consistent with the fall in bond trading that has occurred in the 1990s in G7 countries documented in Evans and Hnatkovska (2014). The share of the bond in portfolios also becomes less asymmetric as a function of the relative supply.

Better integrated markets in turn have consequences in terms of asset prices. Risk premia and Sharpe ratios decrease moderately on average, while the interest rate in-

[22]The raw shares go from $w_{h,t} = 105\%$ and $w_{f,t} = 49\%$, to $w_{h,t} = 85\%$ and $w_{f,t} = 70\%$. Contrary to the shares in the *equity* portfolio, which sum to 100%, raw shares in the total portfolio do not because the bond is used to lever up.

[23]This is not to say that this process has been smooth: an important dimension of the Global Financial Cycle, and of the Great Recession in particular, has been a strong retrenchment in capital flows. Cf. Milesi-Ferretti and Tille (2011), and Miranda-Agrippino and Rey (2020). Those are aspects that extensions of the framework, discussed in Section 4.5, could help match as well.

[24]If τ becomes too small, or even goes towards zero, the home bias in equity holdings ultimately turns back into a foreign bias as discussed in Section **??** given that goods are good substitutes, consistent with modern estimation of θ. Even though τ is likely to have decrease significantly over time however, a world with no friction whatsoever in international financial markets remains far from realistic so that this concern remains modest in practice. Still, generating additional ways of reproducing the home bias in equity holdings that obtains in the data by introducing additional channels that could make the wealth share itself move differently and more autonomously, is an important avenue for future research. As mentioned previously, this could for instance take the form of idiosyncratic labor income or financial risk, or a time-varying labor income share against which investors would want to hedge by tilting their portfolios towards the home asset in the spirit of Coeurdacier and Gourinchas (2016).

creases.[25]. Each becomes slightly less dependent on the wealth share, consistent with risk sharing between international investors becoming easier, even though x_t broadly remains as important a determinant as fundamentals due to the remaining asymmetries in risk tolerance. However, the consequence is most visible in terms of asset comovements: the correlation across asset returns worldwide in the model goes from 0.68 to 0.75 to 0.77 on average, for $\tau = 15, 5$ and 0%. This result, which arises naturally with the better integration of international markets and despite no correlation in fundamentals, goes some way in replicating the findings of Jordà et al. (2019). Indeed, the authors document a large secular increase in the synchronization of global equity markets that goes above and beyond the growing integration of real variables, and is driven in part by risk premia and changes in global risk aversion. Like for portfolios, the decrease in τ impacts not only the level but also the shape of the conditional covariance and correlation of returns. Even though $\text{corr}_t(dR_t, dR_t^*)dt^{-1}$ still tends to increase faster as the relative supply of the domestic good decreases, consistent with the remaining asymmetries in risk tolerance combined with investors having differing preferred goods, the evolution in the wealth share dimension changes significantly. While the correlation tends to increase as one of the countries become dominant in world wealth when risk sharing is imperfect, reaching a global minimum at the point of symmetry in the world economy, it tends to *in*crease when this happens under perfect risk sharing, reaching close to a global maximum at the symmetric point like in the baseline of Section 3 and Sauzet (2021a). This underlines the interaction between asset correlation, which are related to the diversification benefits provided by the assets even though it varies a lot in this context, and the extent of risk sharing.

Even though a full discussion is omitted in the interest of space, asymmetric relaxation in the degree of international market integration is also an interesting phenomenon to look into: in practice, the financial markets of the United States have become much more accessible than that of some of its major trading patterns such as China, India, and other emerging markets. This is captured in the current framework by assuming that $\tau = 15\%$, i.e. the United States faces frictions when investing in foreign equities, while $\tau^* < \tau = 15\%$, i.e. foreign investors have a much wider access to capital markets in the United States. As expected, the amount of asymmetry in risk premia, their comovements, and portfolios, is greatly reinforced.

[25]Specifically, risk premia decrease by 6.6bp (5%) on average, and Sharpe ratios by 0.0143 (4%). The interest rate increases by 9bp (3%).

More generally, introducing stochastic degrees of financial integration, for instance with time-varying taxes, or micro-founding the underlying source of frictions in international markets are interesting avenues left for future research. Changes in the tax on foreign dividends, potentially asymmetric, could also be used to study the impact on global asset prices, portfolios, and risk sharing, of macroprudential policies aimed at curbing sudden international capital flows.

The model could also be used to study a number of additional counterfactual scenarios. For instance, what happens when the financial systems of other countries become more developed so that foreign investors become more able and willing to carry risk? Here, this could be captured as a decrease in the risk aversion of the foreign investor γ^* towards that of the United States $\gamma = 5$, and the economy would get closer to that studied in Section 3 and Sauzet (2021a) with imperfect risk sharing but less asymmetries. In this context, introducing more than two trees and countries could prove worthwhile to study the type of phenomena that occur when the country at the center of the international financial system switches, e.g. when the world transitioned from the United Kingdom to the United States at its center in the 20th century. It could shed light on the likely impact of China or Europe becoming the new center country, or a second big player in international financial markets in a multi-polar world. The effect on asset pricing, portfolios, and risk sharing are likely to be large, given the diverging preferences of those countries, in terms of goods but also in their preference for saving *etc.* This could also capture possible periods of instability that can occur in the transition from one hegemon to the other in the spirit of Nurkse (1944) and Farhi and Maggiori (2018). Another dimension that appears important would be to introduce several types of investors in the economy, and in particular global financial intermediaries. I discuss some such possible extensions below, and leave those promising directions for ongoing or future research.

Finally, the framework could also be used not only to study short-term high-frequency dynamics typical of crisis situations like above, but could also be reinterpreted and used at a lower frequency. In that case, it can shed light on what happens once the share of the United States in world wealth decreases as is likely to happen with the rise of emerging countries as central players of the international financial system. This can also be captured as a decrease in the wealth share of the United States in this model, and naturally leads to a decrease in world wide interest rate of the type that has been discussed in Caballero et al. (2008) or Hall (2016).

Note that, for this application to be fully analyzed, the model would have to be modified in the following sense. As is, the asymmetric position taken in the international bond, with the domestic investor borrowing for most of the state space, sometimes aggressively, leads to the expected result that in the long run, the domestic risk-tolerant investor comes to hold the majority of the wealth in the world economy. This is reminiscent of standard results in the classical literature on multi-agent asset pricing such as Dumas (1989) and comes from the fact that risky assets, to which the domestic investor allocates a larger weight in her portfolio, pay a positive premium in expectations. In this environment, this takes the form of both the drift, $\mu_{x,t}x_t$, and diffusion terms, $\sigma_{xz,t}x_t, \sigma_{xz^*,t}x_t$, of the wealth share all being positive for most of the state space, which results in an increase in the wealth share for both domestic and foreign shocks as well as in expectations (Figures C.19 and C.20). In short, the United States dominates the economy in the long run. To reverse this result, there are several possibilities.

One of them could be to introduce the fact that the output of the foreign tree, if it is meant to represent that of emerging countries, grows faster than the output in the United States. Provided that the foreign country holds a sufficiently large share of the foreign asset to benefit from it, which happens if markets are sufficiently imperfectly integrated but the asymmetry in tolerance for risk remains moderate, the foreign investor could be made to dominate the economy in the long run as the output of the foreign tree also comes to dominate. If not sufficient, this could also be combined with a reintroduction of labor income, which, by providing the foreign investor with a guaranteed share of the output of the faster-growing tree, would once again tend to push its wealth share to increase. There could also be other ways for instance using asymmetries in other dimensions of preferences such as the elasticity of intertemporal substitution, which partly governs the propensity to save of investors. Importantly, once the foreign country starts becoming larger in the long run, the global interest rate would gradually decline as has been observed empirically, with the risk-tolerant investor in the United States becoming increasingly small, in the spirit of Hall (2016) and as also discussed in Gourinchas et al. (2017). This phenomenon would happen in the long run not withstanding the fact that short-run dynamics would still be akin to those presented in Section 4.3.

Those brief illustrations show that the framework is a versatile building block to study a wealth of real-world applications in a unified context.

4.5. Extensions

Beyond those applications, the framework can be extended to capture important additional specificities of the international financial system. A couple of such extensions have been mentioned throughout the paper such as the introduction of additional countries or assets, or a more general specification of the share of labor income or taxes as being stochastic (cf. also Sauzet, 2021a). Various ways of making the model stationary could also be interesting to explore.[26]

In addition, because I solve for the decentralized solution throughout, the framework is readily set to tackle more general market structures beyond imperfect risk sharing such as incomplete markets that would arise in the presence of idiosyncratic labor income risk as in Kaplan et al. (2018), or capital risk as in Brunnermeier and Sannikov (2014, 2015). Particularly interesting and relevant in this context will also be the addition of constraints on the portfolios of international investors, e.g. by adapting Gârleanu and Pedersen (2011), Chabakauri (2013) to my two-good international economy, which could lead to a strong reinforcement of the type of dynamics discussed in Section 4.3. Taken together, those different channels will likely lead to a strengthening of the dispersion and role of the wealth share in equilibrium.

The framework can also be extended along more ambitious dimensions. The most promising among them relate to the introduction in an international setting of the type of financial intermediaries of the type that has been discussed in the recent intermediary asset pricing literature e.g. in Daníelsson et al. (2012), He and Krishnamurthy (2013), Adrian and Shin (2014), or Adrian and Boyarchenko (2015). Those global intermediaries, which are very relevant in practice, can be involved in the dealing of foreign currencies, in the spirit of Hau and Rey (2006) and Gabaix and Maggiori

[26] This could be done e.g. by adapting the share process of Menzly et al. (2004), Santos and Veronesi (2006) to y_t so that neither of the goods and assets dominates the economy in the long run, which could also ensure the survival of both investors. Another possibility could be to adapt the overlapping-generations structure of Gârleanu and Panageas (2015) to my multi-good international context.

(2015), or can play the role of bankers as in Maggiori (2017) and Jiang et al. (2020). As an illustration, in ongoing work (Sauzet, 2021d), I explore a third possibility: the introduction of a global asset manager. This addition is briefly described in Appendix B.1 and could help capture additional aspects of the Global Financial Cycle of Rey (2013) and Miranda-Agrippino and Rey (2020), pertaining to the leverage and role of global financial intermediaries. The combination of global financial intermediaries with time-varying demand for safe assets, which could be generated by the introduction of multiple heterogeneous investors *within* each country, could also help make way towards a resolution for the so-called "reserve currency paradox" emphasized by Maggiori (2017). I briefly touch upon this question in Appendix B.2 and it is also explored in ongoing work Sauzet (2021f).

Finally, from a methodological standpoint, the number of state variables is likely to rapidly increase with those extensions. Because computationally traditional projection methods are very much subject to the curse of dimensionality, higher-dimensional methods will be required. For instance, even the addition of a third state variable, like in the global asset manager extension, renders the resolution significantly slower, and increasing the order of approximation much beyond $N = 10$ proves difficult.[27] One such method consists in naturally extending the concept of projection approaches, but to replace the Chebyshev polynomials in the approximation by neural networks, which are designed specifically for high-dimensional settings. I am developing these "projection methods via neural networks" for continuous-time models in Sauzet (2021c). I discuss them in slightly more details in Section B.3, and they should prove very useful as I pursue yet more ambitions extensions.

In summary, the framework in this paper is well-suited to handle several applications and extensions. The main application throughout Section 4 has shown that the model is able to replicate a vast number of facts about the structure and dynamics of the international financial system, and about asset returns in that context, which are strongly borne out in the data. More generally, the combination of the extensions mentioned above and higher-dimensional resolution approaches such as the "projection methods via neural networks" developed in Sauzet (2021c) provide many promising avenues for future research.

[27] Finer ways to construct the Chebyshev polynomials and corresponding grids, such as complete polynomials or Smolyak's algorithm, can help. Ultimately however, they are also limited.

5. Conclusion

In this paper, I adapt the "222" framework (Two Investors, Two Trees, Two Goods) of Sauzet (2021a) to an international setting. This allows me to characterize the global solution to the international portfolio choice problem in full generality, a long-standing open issue in international finance to which the literature had only provided a piecemeal answer.

The main economic message from the characterization is that the allocation of wealth across investors matters in a general international portfolio choice setting. This finding resonates with an emerging theme in the broader economic literature that has recently emphasized the role of the wealth distribution in determining economic outcomes in macroeconomics (e.g. Brunnermeier and Sannikov, 2014, Kaplan et al., 2018), finance (e.g. Gomez, 2017, Lettau et al., 2019, Greenwald et al., 2020), and economics more generally (e.g. Piketty and Zucman, 2014). In other words, "capital is back" in this setting too: the allocation of wealth across international investors has a prime role in driving asset prices, portfolios, and risk sharing, an aspect that had received little emphasis thus far.

The allocation of wealth matters both as a state variable that captures the average investor in the world economy and directly impacts economic outcomes, and as a pricing factor that is hedged by international investors. Its effect is relevant even in a baseline with symmetric calibration and perfect risk sharing, but grows tremendously as markets become imperfectly integrated, and as investors become more heterogeneous. The results also emphasize both (i) the state-dependence of most economic variables in this environment – e.g. portfolios vary substantially with the allocation of wealth –, and (ii) the vital impact of the calibration of preferences – e.g. the potency of imperfect financial integration is strongly reduced with a high elasticity of intertemporal substitution. This makes the novel framework presented in this paper, which is based on a global solution method and allows for general recursive preferences including asymmetries, particularly adapted to study this economy.

The framework is a well-suited building block towards several applications and extensions. For instance, it makes it possible to revisit various results in the international portfolio choice literature under a unified framework. I then focus on one main application, and show that the model can be used to capture a number of stylized facts about the structure and dynamics of the international financial system, and of asset returns in that context.

The introduction of asymmetries in the tolerance for risk of international investors naturally replicates the role of the United States as the world banker, documented in Gourinchas and Rey (2007b) and Gourinchas et al. (2017), and the exorbitant privilege enjoyed by the country in the form of higher excess returns. A modest degree of imperfect financial integration also generates a plausible home bias in equity holdings for both investors.

Importantly, the framework does not only replicate facts about external portfolios on average, but the asymmetry in risk tolerance also yields a number of predictions about the *dynamics* of the international financial system that are strongly borne out in the data. As a crisis hits, the center country is impacted particularly severely due to its high allocation to risky assets, so that it transfers a large amount of wealth to the rest of the world. This exorbitant duty is the flip side of its exorbitant privilege in normal times: the United States must become the world insurer in times of trouble. In addition, by worsening the wealth position of the risk-tolerant world banker, the shock leads to a sharp increase in global risk aversion, which in turn pushes up all risk premia and Sharpe ratios worldwide. These two markers are reminiscent of some aspects of the Global Financial Cycle of Rey (2013), and Miranda-Agrippino and Rey (2020), for which a general equilibrium exploration had remained elusive. Those patterns are representative of the type of global risk-off scenarios that typically occur in times of global crisis such as most recently in the Great Recession of 2008 or the Global Pandemic of 2020.

The model can also shed light on the reaction of global portfolios to shocks, and the process of external adjustment of the center country, emphasizing the primordial role played by valuation effects in this context. It also allows to run a number of counterfactual exercises.

From an asset pricing perspective, the specialization of the model also speaks to a number of facts about asset returns dynamics in this international environment. Namely, risk premia, Sharpe ratios – and to some extent volatilities and correlations in a relevant region of the state space – are all countercyclical in the sense that they increase following the shock, consistent with a wide range of evidence notably for the

United States. Importantly, those patterns are driven for a large part not by changes in the quantity of risk but by the evolution of the compensation for risk, captured here by the time-varying global risk aversion. This is in line with a large literature that has seen changes in the price of risk emerge as a crucial explanation behind asset return predictability more generally.

In summary, a seemingly small change in the specification of the model – the introduction of asymmetries in risk tolerance – generates a vast number of facts about the structure and dynamics of the international financial system, and about asset returns, which are strongly borne out in the data.

The model is also a well-suited building block for many potential extensions (cf. also Sauzet, 2021a). The most promising among them are related to the introduction in an international setting of financial intermediaries of the type that has been discussed in the recent intermediary asset pricing literature, and illustrations were briefly discussed in Section 4.5 e.g. with the inclusion of a global asset manager (Sauzet, 2021d). The implementation of those extensions will likely require higher-dimensional methods such as the "projection methods via neural networks" being developed in Sauzet (2021c). I leave all these promising avenues for future research.

References

Adrian, Tobias, and Nina Boyarchenko, 2015, Intermediary Balance Sheets, 2015 Meeting Papers 239, Society for Economic Dynamics.

Adrian, Tobias, and Hyun Song Shin, 2014, Procyclical leverage and value-at-risk, *Review of Financial Studies* 27, 373–403.

Anderson, Robert M., and Roberto C. Raimondo, 2008, Equilibrium in continuous-time financial markets: Endogenously dynamically complete markets, *Econometrica* 76, 841–907.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance* 59, 1481–1509.

Basak, Suleyman, and Domenico Cuoco, 1998, An Equilibrium Model with Restricted Stock Market Participation, *The Review of Financial Studies* 11, 309–341.

Basak, Suleyman, and Michael Gallmeyer, 2003, Capital Market Equilibrium with Differential Taxation, *Review of Finance* 7, 121–159.

Baxter, Marianne, and Urban Jermann, 1997, The international diversification puzzle is worse than you think, *American Economic Review* 87, 170–80.

Baxter, Marianne, Urban J Jermann, and Robert G King, 1998, Nontraded goods, nontraded factors, and international non-diversification, *Journal of International Economics* 44, 211 – 229.

Berrada, Tony, Julien Hugonnier, and Marcel Rindisbacher, 2007, Heterogeneous preferences and equilibrium trading volume, *Journal of Financial Economics* 83, 719 – 750.

Bhamra, Harjoat S., Nicolas Coeurdacier, and Stéphane Guibaud, 2014, A dynamic equilibrium model of imperfectly integrated financial markets, *Journal of Economic Theory* 154, 490 – 542.

Brandt, Michael W., John H. Cochrane, and Pedro Santa-Clara, 2006, International risk sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics* 53, 671 – 698.

Brandt, Michael W., and Qiang Kang, 2004, On the relationship between the conditional mean and volatility of stock returns: A latent var approach, *Journal of Financial Economics* 72, 217–257.

Brunnermeier, Markus, and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Brunnermeier, Markus K., and Lasse Heje Pedersen, 2009, Market liquidity and funding liquidity, *The Review of Financial Studies* 22, 2201–2238.

Brunnermeier, Markus K., and Yuliy Sannikov, 2015, International credit flows and pecuniary externalities, *American Economic Journal: Macroeconomics* 7, 297–338.

Brunnermeier, Markus K., and Yuliy Sannikov, 2019, International monetary theory: A risk perspective, *Working Paper* .

Caballero, Ricardo J., Emmanuel Farhi, and Pierre-Olivier Gourinchas, 2008, An equilibrium model of "global imbalances" and low interest rates, *American Economic Review* 98, 358–93.

Camanho, Nelson, Harald Hau, and Hélène Rey, 2018, Global portfolio rebalancing and exchange rates, Working paper.

Campbell, John Y., 1999, Chapter 19 asset prices, consumption, and the business cycle, volume 1 of *Handbook of Macroeconomics*, 1231 – 1303 (Elsevier).

Campbell, Sean D., and Francis X. Diebold, 2009, Stock returns and expected business conditions: Half a century of direct evidence, *Journal of Business & Economic Statistics* 27, 266–278.

Cass, David, and Anna Pavlova, 2004, On trees and logs, *Journal of Economic Theory* 116, 41 – 83.

Chabakauri, Georgy, 2013, Dynamic Equilibrium with Two Stocks, Heterogeneous Investors, and Portfolio Constraints, *The Review of Financial Studies* 26, 3104–3141.

Chan, YeungLewis, and Leonid Kogan, 2002, Catching up with the joneses: Heterogeneous preferences and the dynamics of asset prices, *Journal of Political Economy* 110, 1255–1285.

Cochrane, John H., Francis A. Longstaff, and Pedro Santa-Clara, 2008, Two Trees, *The Review of Financial Studies* 21, 347–385.

Coeurdacier, Nicolas, 2009, Do trade costs in goods market lead to home bias in equities?, *Journal of International Economics* 77, 86 – 100.

Coeurdacier, Nicolas, and Pierre-Olivier Gourinchas, 2016, When bonds matter: Home bias in goods and assets, *Journal of Monetary Economics* 82, 119 – 137.

Coeurdacier, Nicolas, and Hélène Rey, 2013, Home bias in open economy financial macroeconomics, *Journal of Economic Literature* 51, 63–115.

Coeurdacier, Nicolas, Hélène Rey, and Pablo Winant, 2020, Financial integration and growth in a risky world, *Journal of Monetary Economics* 112, 1 – 21.

Colacito, Ric, Max Croce, Steven Ho, and Philip Howard, 2018, Bkk the ez way: International long-run growth news and capital flows, *American Economic Review* 108, 3416–49.

Colacito, Riccardo, and Mariano M. Croce, 2011, Risks for the long run and the real exchange rate, *Journal of Political Economy* 119, 153–181.

Colacito, Riccardo, and Mariano M. Croce, 2013, International asset pricing with recursive preferences, *Journal of Finance* 68, 2651–2686.

Cole, Harold L., and Maurice Obstfeld, 1991, Commodity trade and international risk sharing: How much do financial markets matter?, *Journal of Monetary Economics* 28, 3 – 24.

Coppola, Antonio, Matteo Maggiori, Brent Neiman, and Jesse Schreger, 2020, Redrawing the map of global capital flows: The role of cross-border financing and tax havens, Working Paper 26855, National Bureau of Economic Research.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc, 2008, International Risk Sharing and the Transmission of Productivity Shocks, *The Review of Economic Studies* 75, 443–473.

Crédit Suisse, 2019, Global wealth report 2019, Credit suisse research institute.

Daníelsson, Jón, Hyun Song Shin, and Jean-Pierre Zigrand, 2012, Procyclical leverage and endogenous risk, Mimeo, LSE, `http://www.riskresearch.org`.

Devereux, Michael B., and Alan Sutherland, 2011, Country portfolios in open economy macro-models, *Journal of the European Economic Association* 9, 337–369.

Dou, Winston Wei, and Adrien Verdelhan, 2015, The volatility of international capital flows and foreign assets, *Working Paper* .

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl, 2018, A model of monetary policy and risk premia, *The Journal of Finance* 73, 317–373.

Duarte, Victor, 2019, Machine learning for continuous-time economics, *Available at SSRN: https://ssrn.com/abstract=3012602* .

Duffie, D., J. Geanakoplos, A. Mas-Colell, and A. McLennan, 1994, Stationary markov equilibria, *Econometrica* 62, 745–781.

Duffie, Darrell, and Larry Epstein, 1992, Stochastic differential utility, *Econometrica* 60, 353–94.

Dumas, Bernard, 1989, Two-Person Dynamic Equilibrium in the Capital Market, *The Review of Financial Studies* 2, 157–188.

Dumas, Bernard, 1992, Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World, *The Review of Financial Studies* 5, 153–180.

Dumas, Bernard, Raman Uppal, and Tan Wang, 2000, Efficient intertemporal allocations with recursive utility, *Journal of Economic Theory* 93, 240–259.

Ehling, Paul, and Christian Heyerdahl-Larsen, 2015, Complete and incomplete financial markets in multi-good economies, *Journal of Economic Theory* 160, 438 – 462.

Evans, Martin D.D., and Viktoria Hnatkovska, 2012, A method for solving general equilibrium models with incomplete markets and many financial assets, *Journal of Economic Dynamics and Control* 36, 1909 – 1930.

Evans, Martin D.D., and Viktoria V. Hnatkovska, 2014, International capital flows, returns and world financial integration, *Journal of International Economics* 92, 14–33.

Fama, Eugene F., and Kenneth R. French, 1989, Business conditions and expected returns on stocks and bonds, *Journal of Financial Economics* 25, 23 – 49.

Fang, Xiang, 2019, Intermediary leverage and currency risk premium, *Available at SSRN: https://ssrn.com/abstract=3290317.* .

Farhi, Emmanuel, and Matteo Maggiori, 2018, A Model of the International Monetary System*, *The Quarterly Journal of Economics* 133, 295–355.

Fernández-Villaverde, Jesus, and Lawrence Christiano, 2011, Nber summer institute, computational tools & macroeconomic applications, Methods lectures.

Ferson, Wayne E., and Campbell R. Harvey, 1991, The variation of economic risk premiums, *Journal of Political Economy* 99, 385–415.

French, Kenneth, and James Poterba, 1991, Investor diversification and international equity markets, *American Economic Review* 81, 222–26.

Gabaix, Xavier, and Ralph S.J. Koijen, 2020, Granular instrumental variables, *Available at SSRN: https://ssrn.com/abstract=3368612.* .

Gabaix, Xavier, and Matteo Maggiori, 2015, International liquidity and exchange rate dynamics*, *The Quarterly Journal of Economics* 130, 1369–1420.

Gârleanu, Nicolae, and Stavros Panageas, 2015, Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing, *Journal of Political Economy* 123, 670 – 685.

Gârleanu, Nicolae, Stavros Panageas, and Jianfeng Yu, 2020, Impediments to Financial Trade: Theory and Applications, *The Review of Financial Studies* 33, 2697–2727.

Gârleanu, Nicolae, and Lasse Heje Pedersen, 2011, Margin-based asset pricing and deviations from the law of one price, *The Review of Financial Studies* 24, 1980–2022.

Geanakoplos, John, 1990, An introduction to general equilibrium with incomplete asset markets, *Journal of Mathematical Economics* 19, 1 – 38.

Geanakoplos, John, and Andreu Mas-Colell, 1989, Real indeterminacy with financial assets, *Journal of Economic Theory* 47, 22 – 38.

Geanakoplos, John D, and Heraklis M Polemarchakis, 1986, Existence, regularity, and constrained suboptimality of competitive allocations, *Essays in Honor of Kenneth J. Arrow: Volume 3, Uncertainty, Information, and Communication* 3, 65.

Gomez, Matthieu, 2017, Asset prices and wealth inequality, 2017 Meeting Papers 1155, Society for Economic Dynamics.

Gourinchas, Pierre-Olivier, and Hélène Rey, 2007a, International financial adjustment, *Journal of Political Economy* 115, 665–703.

Gourinchas, Pierre-Olivier, and Hélène Rey, 2007b, From World Banker to World Venture Capitalist: U.S. External Adjustment and the Exorbitant Privilege, in *G7 Current Account Imbalances: Sustainability and Adjustment*, NBER Chapters, 11–66 (National Bureau of Economic Research, Inc).

Gourinchas, Pierre-Olivier, Hélène Rey, and Nicolas Govillot, 2017, Exorbitant Privilege and Exorbitant Duty, Working paper, University of California, Berkeley, London Business School.

Gourinchas, Pierre-Olivier, Hélène Rey, and Maxime Sauzet, 2019, The international monetary and financial system, *Annual Review of Economics* 11, 859–893.

Greenwald, Daniel L, Martin Lettau, and Sydney C Ludvigson, 2020, How the wealth was won: Factors shares as market fundamentals, Working Paper 25769, National Bureau of Economic Research.

Hall, Robert E., 1988, Intertemporal substitution in consumption, *Journal of Political Economy* 96, 339–357.

Hall, Robert E, 2016, The role of the growth of risk-averse wealth in the decline of the safe real interest rate, Working Paper 22196, National Bureau of Economic Research.

Hansen, Lars Peter, Paymon Khorrami, and Fabrice Tourre, 2018, Comparative valuation dynamics in models with financing restrictions .

Harrison, Paul, and Harold H. Zhang, 1999, An investigation of the risk and return relation at long horizons, *The Review of Economics and Statistics* 81, 399–408.

Harvey, Campbell R., 2001, The specification of conditional expectations, *Journal of Empirical Finance* 8, 573 – 637, On the Predictability of Asset Returns.

Hassan, Tarek, 2013, Country size, currency unions, and international asset returns, *The Journal of Finance* 68, 2269–2308.

Hau, Harald, and Hélène Rey, 2006, Exchange Rates, Equity Prices, and Capital Flows, *The Review of Financial Studies* 19, 273–317.

He, Zhiguo, and Arvind Krishnamurthy, 2013, Intermediary Asset Pricing, *American Economic Review* 103, 732–770.

Hugonnier, J., S. Malamud, and E. Trubowitz, 2012, Endogenous completeness of diffusion driven equilibrium markets, *Econometrica* 80, 1249–1270.

Imbs, Jean, and Isabelle Méjean, 2015, Elasticity optimism, *American Economic Journal: Macroeconomics* 7, 43–83.

Jeanne, Olivier, and Andrew K. Rose, 2002, Noise Trading and Exchange Rate Regimes*, *The Quarterly Journal of Economics* 117, 537–569.

Jiang, Zhengyang, Arvind Krishnamurthy, and Hanno Lustig, 2020, Dollar safety and the global financial cycle, Working Paper 27682, National Bureau of Economic Research.

Jordà, Òscar, Moritz Schularick, Alan Taylor, and Felix Ward, 2019, Global financial cycles and risk premiums, *IMF Economic Review* 67, 109–150.

Jordà, Òscar, Moritz Schularick, and Alan M. Taylor, 2016, *Macrofinancial History and the New Business Cycle Facts*, 213–263 (University of Chicago Press).

Judd, Kenneth, 1992, Projection methods for solving aggregate growth models, *Journal of Economic Theory* 58, 410–452.

Judd, Kenneth, 1998, *Numerical Methods in Economics*, volume 1, first edition (The MIT Press).

Kaplan, Greg, Benjamin Moll, and Giovanni L. Violante, 2018, Monetary policy according to hank, *American Economic Review* 108, 697–743.

Kargar, Mahyar, 2019, Heterogeneous intermediary asset pricing, *Available at SSRN: https://ssrn.com/abstract=3258119* .

Kekre, Rohan, and Moritz Lenel, 2020, Dollar demand and international risk sharing, Working paper.

Koijen, Ralph S. J, and Motohiro Yogo, 2020, Exchange rates and asset prices in a global demand system, Working Paper 27342, National Bureau of Economic Research.

Kouri, Pentti, 1982, Balance of Payment and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model, *Economic interdependence and Flexible Exchange Rates ed. Jagdeep Bhandari and Bluford Putnam* 11656.

Kubler, Felix, and Karl Schmedders, 2003, Stationary Equilibria in Asset-Pricing Models with Incomplete Markets and Collateral, *Econometrica* 71, 1767–1793.

Lettau, Martin, and Sydney C. Ludvigson, 2010, Chapter 11 - measuring and modeling variation in the risk-return trade-off, in YACINE AT-SAHALIA, and LARS PETER HANSEN, eds., *Handbook of Financial Econometrics: Tools and Techniques*, volume 1 of *Handbooks in Finance*, 617 – 690 (North-Holland, San Diego).

Lettau, Martin, Sydney C. Ludvigson, and Sai Ma, 2019, Capital share risk in u.s. asset pricing, *The Journal of Finance* 74, 1753–1792.

Maggiori, Matteo, 2017, Financial intermediation, international risk sharing, and reserve currencies, *American Economic Review* 107, 3038–71.

Maggiori, Matteo, Brent Neiman, and Jesse Schreger, 2020, International currencies and capital allocation, *Journal of Political Economy* 128, 2019–2066.

Martin, Ian, 2011, The forward premium puzzle in a two-country world, Working Paper 17564, National Bureau of Economic Research.

Martin, Ian, 2013, The lucas orchard, *Econometrica* 81, 55–111.

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145 – 161.

Menzly, Lior, Tano Santos, and Pietro Veronesi, 2004, Understanding predictability, *Journal of Political Economy* 112, 1–47.

Mertens, Thomas M., and Kenneth L. Judd, 2018, Solving an incomplete markets model with a large cross-section of agents, *Journal of Economic Dynamics and Control* 91, 349 – 368, Special Issue in Honour of Prof. Carl Chiarella.

Milesi-Ferretti, GianMaria, and Cédric Tille, 2011, The great retrenchment: international capital flows during the global financial crisis, *Economic Policy* 26, 285–342.

Miranda-Agrippino, Silvia, and Hélène Rey, 2020, U.S. Monetary Policy and the Global Financial Cycle, *The Review of Economic Studies* rdaa019.

Nurkse, Ragnar, 1944, *International Currency Experience: Lessons of the Inter-war Period*, Series of League of Nations Publications. 2. Economic and financial. 1944. 2 (UN).

Obstfeld, Maurice, 2004, External adjustment, *Review of World Economics (Weltwirtschaftliches Archiv)* 140, 541–568.

Obstfeld, Maurice, 2007, International risk sharing and the costs of trade, *The Ohlin Lectures* Stockholm School of Economics.

Obstfeld, Maurice, and Kenneth Rogoff, 2001, The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?, in *NBER Macroeconomics Annual 2000, Volume 15*, NBER Chapters, 339–412 (National Bureau of Economic Research, Inc).

Parra-Alvarez, Juan, 2018, A comparison of numerical methods for the solution of continuous-time dsge models, *Macroeconomic Dynamics* 22, 1555–1583.

Pavlova, Anna, and Roberto Rigobon, 2007, Asset Prices and Exchange Rates, *The Review of Financial Studies* 20, 1139–1180.

Pavlova, Anna, and Roberto Rigobon, 2008, The Role of Portfolio Constraints in the International Propagation of Shocks, *The Review of Economic Studies* 75, 1215–1256.

Pavlova, Anna, and Roberto Rigobon, 2010, An asset-pricing view of external adjustment, *Journal of International Economics* 80, 144 – 156, Special Issue: JIE Special Issue on International Macro-Finance.

Piketty, Thomas, and Gabriel Zucman, 2014, Capital is Back: Wealth-Income Ratios in Rich Countries 17002010*, *The Quarterly Journal of Economics* 129, 1255–1310.

Polemarchakis, Heraklis M, 1988, Portfolio choice, exchange rates, and indeterminacy, *Journal of Economic Theory* 46, 414–421.

Rabitsch, Katrin, Serhiy Stepanchuk, and Viktor Tsyrennikov, 2015, International portfolios: A comparison of solution methods, *Journal of International Economics* 97, 404 – 422.

Rey, Hélène, 2013, Dilemma not trilemma: the global cycle and monetary policy independence, *Proceedings - Economic Policy Symposium - Jackson Hole* 1–2.

Santos, Tano, and Pietro Veronesi, 2006, Labor Income and Predictable Stock Returns, *The Review of Financial Studies* 19, 1–44.

Sauzet, Maxime, 2021a, Two investors, two trees, two goods, Working paper.

Sauzet, Maxime, 2021c, Projection methods via neural networks for continuous-time models, Work in progress.

Sauzet, Maxime, 2021d, Global asset manager and the global financial cycle, Work in progress.

Sauzet, Maxime, 2021f, Demand for bonds, global currency dealer, and the reserve currency paradox, Work in progress.

Schorfheide, Frank, Dongho Song, and Amir Yaron, 2018, Identifying long-run risks: A bayesian mixed-frequency approach, *Econometrica* 86, 617–654.

Schwert, G. William, 1989, Why does stock market volatility change over time?, *The Journal of Finance* 44, 1115–1153.

Stathopoulos, Andreas, 2017, Asset Prices and Risk Sharing in Open Economies, *The Review of Financial Studies* 30, 363–415.

Stepanchuk, Serhiy, and Viktor Tsyrennikov, 2015, Portfolio and welfare consequences of debt market dominance, *Journal of Monetary Economics* 74, 89 – 101.

Stulz, René M., 1983, The demand for foreign bonds, *Journal of International Economics* 15, 225 – 238.

Tille, Cédric, 2001, The role of consumption substitutability in the international transmission of monetary shocks, *Journal of International Economics* 53, 421 – 444, Symposium on the Japanese economic slump of the 1990's.

Tille, Cédric, and Eric van Wincoop, 2010, International capital flows, *Journal of International Economics* 80, 157–175.

Uribe, Martin, and Stephanie Schmitt-Grohé, 2017, *Open economy macroeconomics* (Princeton University Press).

Wang, Jiang, 1996, The term structure of interest rates in a pure exchange economy with heterogeneous investors, *Journal of Financial Economics* 41, 75 – 110.

Weinbaum, David, 2009, Investor heterogeneity, asset pricing and volatility dynamics, *Journal of Economic Dynamics and Control* 33, 1379 – 1397.

Zapatero, Fernando, 1995, Equilibrium asset prices and exchange rates, *Journal of Economic Dynamics and Control* 19, 787–811.

Appendix

A. Additional equations and results

A.1. Calibration

This section provides details on the baseline symmetric calibration of Assumption 1.

At $\gamma = \gamma^* = 15$, risk aversion is a bit on the high side, although within the range of values that are common in asset pricing. This allows to generate slightly more realistic risk premia, given that the model only features mild frictions in the form of imperfect financial integration. (This is nothing but the equity premium puzzle of Mehra and Prescott (1985).) The risk aversion could be increased much further for the purpose of matching risk premia more closely to the data, given that recursive preferences decouple it from the inverse of the elasticity of intertemporal substitution. However, the focus in this paper is on the mechanisms rather than on an exact quantitative match. Moving forward, extensions of the model, some of which discussed in Section 4.5, will be the prime way to generate higher risk premia. Prominent examples include the introduction of portfolio constraints, and of non-diversifiable idiosyncratic risk.

Although the elasticity of intertemporal substitution is set at $\psi = \psi^* = 2$ in the baseline, consistent with recent estimates e.g. in Schorfheide et al. (2018) and with values around $\psi = 1.5$ that have been used in the asset pricing literature e.g. in Bansal and Yaron (2004), I discuss its effect at length in Section **??** (especiall Section **??**). I contrast the cases with $\psi = \psi^* = 0.2$ and $\psi = \psi^* = 2$, and ψ turns out to have a large impact on the potency of imperfect financial integration. For the main application of Section 4, I therefore use $\psi = \psi^* = 0.5$, which allows me to generate a plausible home bias in equity holdings while matching the broad level of the interest rate in this asymmetric context. This lower value goes some way towards the much lower estimates of the elasticity of intertemporal substitution that have been used historically in the earlier literature e.g. in Hall (1988), Campbell (1999).

The home bias in consumption $\alpha = \alpha^* = 0.75$ is consistent with the share of import in the consumption basket of the United States and other countries in recent years. The value is in line with the range of values that have been used in the literature, although slightly lower given the slight increase in world trade in recent decades. In the literature, values as high as $\alpha = 0.9$ or even $\alpha = 0.975$ are sometimes necessary from a quantitative perspective, but this is not the case in the context of this paper where I study the dynamics throughout the state space instead of local neighborhoods of a steady-state. Note that, as α increases further, portfolios and other variables become very non-linear, and the impact of the wealth share is strongly reinforced even in the baseline calibration.

The numéraire basket has a weight of $a = 1 - a = 1/2$ on each good. The value of a has no consequence on quantities and only tilts prices accordingly. I therefore stick to a symmetric numéraire basket to ease interpretation. In extensions of the model with more assets (e.g. multiple bonds), portfolio constraints, and additional sources of risk, the denomination of the numéraire could be of more interest, an aspect that I am planning to explore.

The elasticity of substitution between goods $\theta = \theta^* = 2$ is in line with modern standard estimates e.g. in Imbs and Méjean (2015), and as used in the literature. Cf. among others Tille (2001), Corsetti et al. (2008), Coeurdacier (2009), Obstfeld (2007), Bhamra et al. (2014) for a discussion. I take a value slightly lower than Imbs and Méjean (2015)'s preferred range of $[4, 6]$, as a compromise towards the lower values that had been used in the earlier literature. From an economic standpoint, most relevant is that this elasticity is above one, a point whose impact I discuss at length throughout Section **??**, and in particular in Section **??** on portfolios.

The discount rate is standard at $\rho = \rho^* = 1\%$, and allows to match the broad level of the interest rate.

In the main text of the paper, labor income is inactive: $\delta = \delta^* = 0\%$. I briefly cover the impact of labor income, which has been discussed in the literature, in Appendix A.2. In that case, I use $\delta = \delta^* = 62.5\%$, in line with the average labor share in the United States over the last 50 years.

The tax on foreign dividends, which captures imperfect financial integration, is set to $\tau = \tau^* = 0\%$ in the baseline. Its effect is discussed at length in Section **??**, and some more in Section 4.

Output processes have a growth rate in annual terms of $\mu_Y = \mu_{Y*} = 2\%$, and a volatility of $\sigma_{Yz} = \sigma_{Y*z*} = 4.1\%$. This is in line with typical values used in the literature, and broadly consistent with world averages e.g. in Uribe and Schmitt-Grohé (2017), in International Monetary Fund or World Bank data, or in longer-run series in Jordà et al. (2016). Asymmetries in output growth rates and volatilities could be an interesting exploration from the perspective of studying the integration of developed slower-growing countries, with emerging faster-growing economies. Importantly, the fundamental correlation between the output of each tree is assumed to be zero. This is not meant to capture empirical correlations, but allows to focus on the correlation between asset returns and goods prices that emerge purely endogenously.

A.2. Impact of labor income

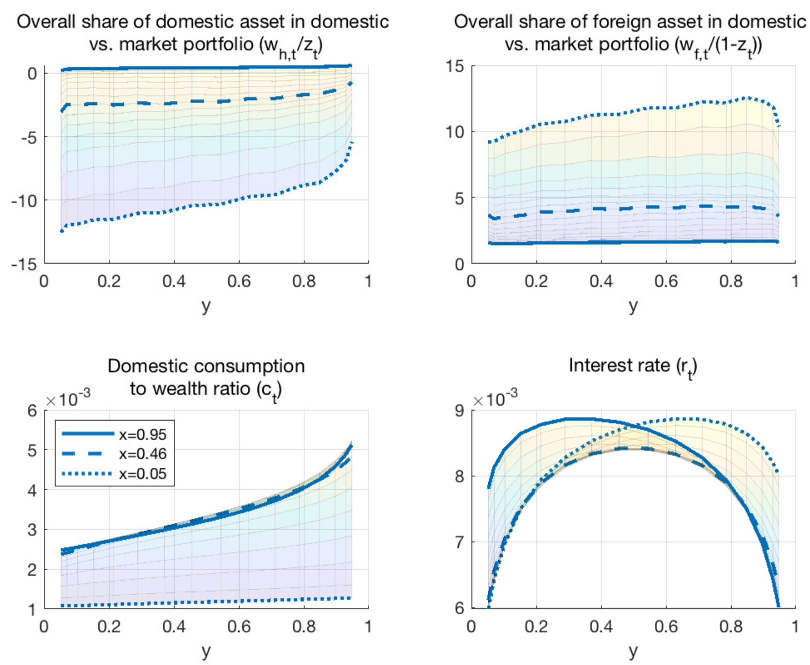
(Back to main text: Section 2.)

The heterogeneity of investors is another factor that strongly reinforces the influence of the wealth share on the equilibrium, not only conceptually but also quantitatively. This was already apparent in the analysis of the baseline calibration studied so far. As we have seen, in Section 3 and Sauzet (2021a), an increase in the home bias in consumption, which constitutes the fundamental heterogeneity in the economy, increases the impact of the wealth share significantly.

Here, I briefly study the impact of heterogeneity further by staying in a symmetric calibration but introducing labor income. Heterogeneity is also partly the focus of the application of Section 4, albeit of a different kind, as the investors will exhibit asymmetries in tolerance for risk.

As a reminder, labor income is modeled as a constant share ($\delta = \delta^*$) of the output of each tree being paid to the local investor. By making the budget constraint of each investor more dependent on the local output, labor income also increases the heterogeneity between investors in the world economy. While its effect on risk premia and Sharpe ratios is somewhat modest, labor income significantly affects portfolios, marginal values of wealth, consumptions, and the interest rate. Those are shown in Figure A.1 for a labor share δ of 62.5%, roughly in line with the average labor share in the United States over the last 50 years. In addition, its effect is once again going hand-in-hand with a bolstered importance for the wealth share.

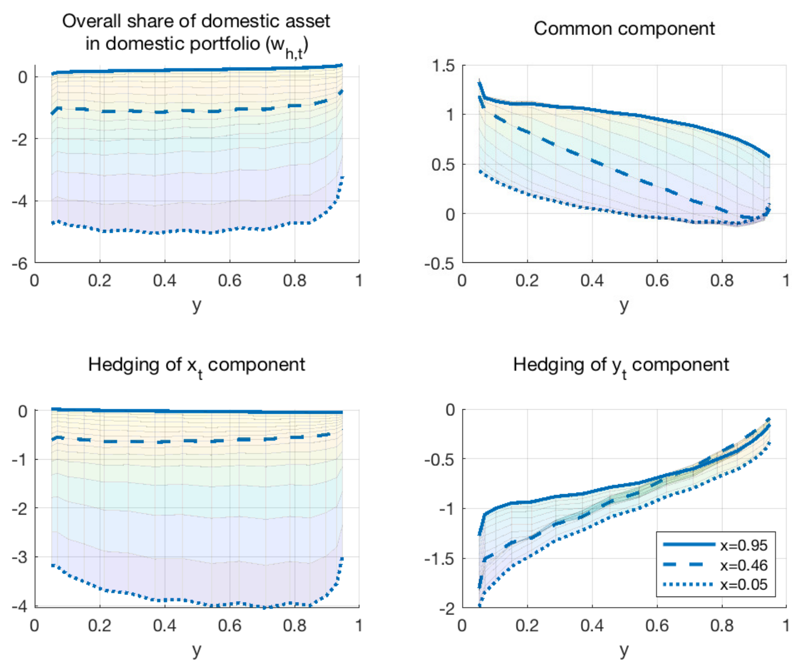
Figure A.1: Impact of the wealth share in the presence of labor income ($\delta = 62.5\%$)



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1, except that $\tau = 62.5\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

The top two panels of Figure A.1 show portfolio weights as they compare to the market portfolio, HB_t and FB_t. Because labor income is perfectly correlated with the payoff of the local asset, it renders each asset yet more unattractive to the local investor, therefore reinforcing the foreign bias in equity holdings on average. This is in line with Baxter and Jermann (1997), who argue that "The International Diversification Puzzle Is Worse Than You Think" when labor takes this form. In terms of magnitude, the impact is substantial, with the measure of home bias now varying from -12.5 to 1 as the wealth share increases, an effect of much larger magnitude than that of fundamentals. In addition, portfolios change not only on average but also inherently in a state-dependent fashion, with the foreign bias reinforced in particular as an investor holds an increasingly smaller share of world wealth. Take the domestic investor for instance: as her wealth share decreases towards zero, labor income represents an increasingly larger share of her revenues, making hedging the labor income risk increasingly important. Due to the perfect correlation between domestic labor income and the payoff to the domestic asset, this pushes the domestic investor to tilt her portfolio away from the domestic asset some more.

Figure A.2: Components of the domestic portfolio in the presence of labor income ($\delta = 62.5\%$)



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Portfolios are not only affected in their overall shape, but also in their underlying drivers. This can be observed visually in Figure A.2, which reports the weight of the domestic asset in the domestic portfolio as well as its components, and is confirmed by computing the corresponding variance decomposition of $w_{h,t}$ like before. From both, we observe that the share of $w_{h,t}$ explained by the hedging of x_t increases tremendously, going from 7.1% in the baseline without labor income, to a whooping 70.2% for $\delta = 62.5\%$. On the contrary, the common and y_t-hedging components now explain a mere 19.6% and 10.3%, instead of 69.7% and 23.1% in the baseline. In short: the hedging of wealth share risk becomes the main driver of the shape of portfolios.

Labor income also has a significant impact on marginal values of wealth, and therefore on consumptions, both becoming more dependent on the wealth share than in the baseline in which x_t affected them only modestly. For instance, the marginal value of wealth for the domestic investor decreases more markedly as the wealth share gets smaller, due to the fact that domestic labor income represents an increasing amount in comparison to domestic wealth, ensuring that the domestic investor has comparatively more resources to fund its consumption and portfolios. As a result, while the average level of consumption to wealth is broadly unchanged, domestic consumption significantly decreases as a fraction of wealth when $x_t \to 0$, as shown in the bottom left panel of Figure A.1. Interestingly, this pattern is reversed and domestic consumption increases as $x_t \to 0$, when the elasticity of intertemporal substitution ψ is small, emphasizing the impact of ψ on the relative importance of substitution and income effects.[28] When ψ is large, in particular above 1, the substitution effect is strong so that an investor ends up saving a large part of the extra labor income (as a fraction of wealth), resulting in a lower consumption as a fraction of wealth when their wealth share decreases. Conversely, as ψ is small, in particular below 1, the income effect dominates so that an investor ends up spending most of the extra labor income (as a function of wealth) on increased consumption as their wealth share decreases. This phenomenon points once again to the importance of being able to study these mechanisms in a context with general preferences, solved globally throughout the state space.

The pattern for the interest rate mirrors those for the marginal values of wealth and consumptions.[29] On average, r_t slightly decreases compared to the baseline, by

[28] I use the terms "substitution effect" and "income effect" liberally, in contrast to their more usual and restricted use that relates to the impact of the interest rate.

[29] This is also true for the pattern of the domestic and foreign dividend yields, F_t and F_t^*, which appear in the budget constraints once we divide labor income by wealth: $\delta F_t z_t/((1-\delta)x_t)$ for the domestic investor, and $\delta^* F_t^*(1-z_t)/((1-\delta^*)(1-x_t))$ for the foreign investor. Cf. Sauzet

about 21 basis points throughout the state space, reflecting the fact that an addition risk, the labor income, needs to be hedged in this economy[30], but more noticeable is the impact on the shape. The interest rate becomes more asymmetric as a function of relative output, going e.g. from around 0.6% to 0.8% depending on whether $y_t \to 0$ or $y_t \to 1$ when the domestic investor holds a small share of world wealth. This represents a reinforcement of the driver of r_t in the baseline combined with a larger investor heterogeneity. In addition, the evolution of r_t as a function of the wealth share is also worth pointing out: as x_t gets small, the interest rate noticeably increases, which has to happen in equilibrium for the domestic investor to be willing to significantly cut down on consumption. Like before, this pattern is also reversed for small values of the elasticity of intertemporal substitution, with the interest rate *de*creasing as the wealth share gets close to zero or one in that case.

Lastly, the introduction of labor income has non-linear effects on the equilibrium distribution of state variables, as shown in Figure C.5. While the dispersion of the wealth share first decreases with δ, consistent with labor income tightening the wealth distribution by ensuring a minimum level of revenues for each investor, dispersion increases back for large values of δ. In addition, as δ increases, the steepness of the relationship between x_t and y_t increases. Those effects are the results of the interplay between the several components of the drift and diffusion of the wealth share, shown in Figure XX. Note also that the second effect, with dispersion increasing back with δ, tends to occur faster for lower level of the elasticity of intertemporal substitution ψ.

Overall, labor income has a significant impact on the equilibrium and its underpinnings due to the resulting increased heterogeneity that reinforces the impact of the wealth share. The way those patterns change when considering a more general and realistic specification for labor income could prove an interesting exploration. One particular specification could be to construct labor income as a time-varying share of the output of each country, as explored for instance in Coeurdacier and Gourinchas (2016). As the authors suggest, the correlation of labor income with output, once computed with the proper conditioning, could in fact turn out to be negative,

(2021a).

[30] This effect is limited because of the perfect correlation between labor income and the payoff of the local asset.

providing a natural way to generate a home bias in equity holdings. If the share is itself stochastic, it could also provide an additional hedging motive that could prove relevant in practice also as it introduces a natural degree of market incompleteness. Labor income could also take a more general form, for instance as a separate source of idiosyncratic risk in the spirit of the recent heterogeneous-agent macroeconomic literature like Kaplan et al. (2018), or by introducing a distribution of investors in each country by generalizing the overlapping generation structure of Gârleanu and Panageas (2015) to a two-good, two-country setting. The latter hints at how labor income could help both (types of) investors survive in equilibrium.[31] I leave these promising avenues for future research.

[31]One difficulty is that this might generate a stationary distribution between investors within a country, as a constant share of them is assumed so switch between different groups of investors, but it would not be sufficient *per se* to ensure a stationary distribution of wealth between *international* investors, except by assuming that individual investors can switch between countries. The ability of labor income to ensure the survival of different types of agents is also used in He and Krishnamurthy (2013).

B. Extensions

B.1. Extension 1: global asset manager and the Global Financial Cycle (Sauzet, 2021d)

From the perspective of modeling the international financial system, an aspect that is increasingly being recognized as primordial is the role of global financial intermediaries. Those global intermediaries can be involved in the dealing of foreign currencies, in the spirit of Hau and Rey (2006) and Gabaix and Maggiori (2015), can play the role of bankers as in Maggiori (2017) and Jiang et al. (2020), or can play the role of global asset managers, like below. The main intuition is that because of their different preferences and limited risk-bearing capacity, the capitalization of those financial intermediaries is a prime determinant of asset prices, interest rates, exchange rates, and other economic outcomes worldwide. The presence of such global intermediaries is not only relevant from the perspective of realism, but could introduce a mechanism through which to capture additional aspects of the Global Financial Cycle of Rey (2013) and Miranda-Agrippino and Rey (2020), pertaining to the leverage and role of intermediaries. By way of an example, I briefly present one, the addition of a global asset manager, that I am exploring in ongoing work Sauzet (2021d). Figure C.2 summarizes the set-up.

The global asset manager constitutes a third type of investor, whose preferences, albeit still recursive and over the two goods, have the following specificities: (i) because she is a global citizen, the global asset manager has no particular bias towards any of the goods, and (ii) she is significantly more risk-tolerant than the consumer-investor of each country. The last point is in the spirit of the intermediary asset pricing literature, which typically models bankers as agents with lower risk aversion. Even though the current version of this work does not feature them, the limited risk-bearing capacity of the global asset manager, in the form for instance of portfolio constraints, will be an important addition.

The equilibrium can be represented as a function of three state variables, $X_t \equiv (x_t, y_t, u_t)'$. x_t is the wealth share of the domestic investor and is defined as before with the caveat that now, $W_t + W_t^*$ does not sum up to total world wealth, which is $W_t + W_t^* + W_t^{glam}$ and includes the wealth of the global asset manager W_t^{glam}. y_t still captures the relative supply of the goods. u_t, the new state variable, captures the share of world wealth held by the global asset manager.[32] In summary:

$$x_t \equiv \frac{W_t}{W_t + W_t^*} \quad ; \quad y_t \equiv \frac{Y_t}{Y_t + Y_t^*} \quad ; \quad u_t \equiv \frac{W_t^{glam}}{W_t + W_t^* + W_t^{glam}} \tag{B.1}$$

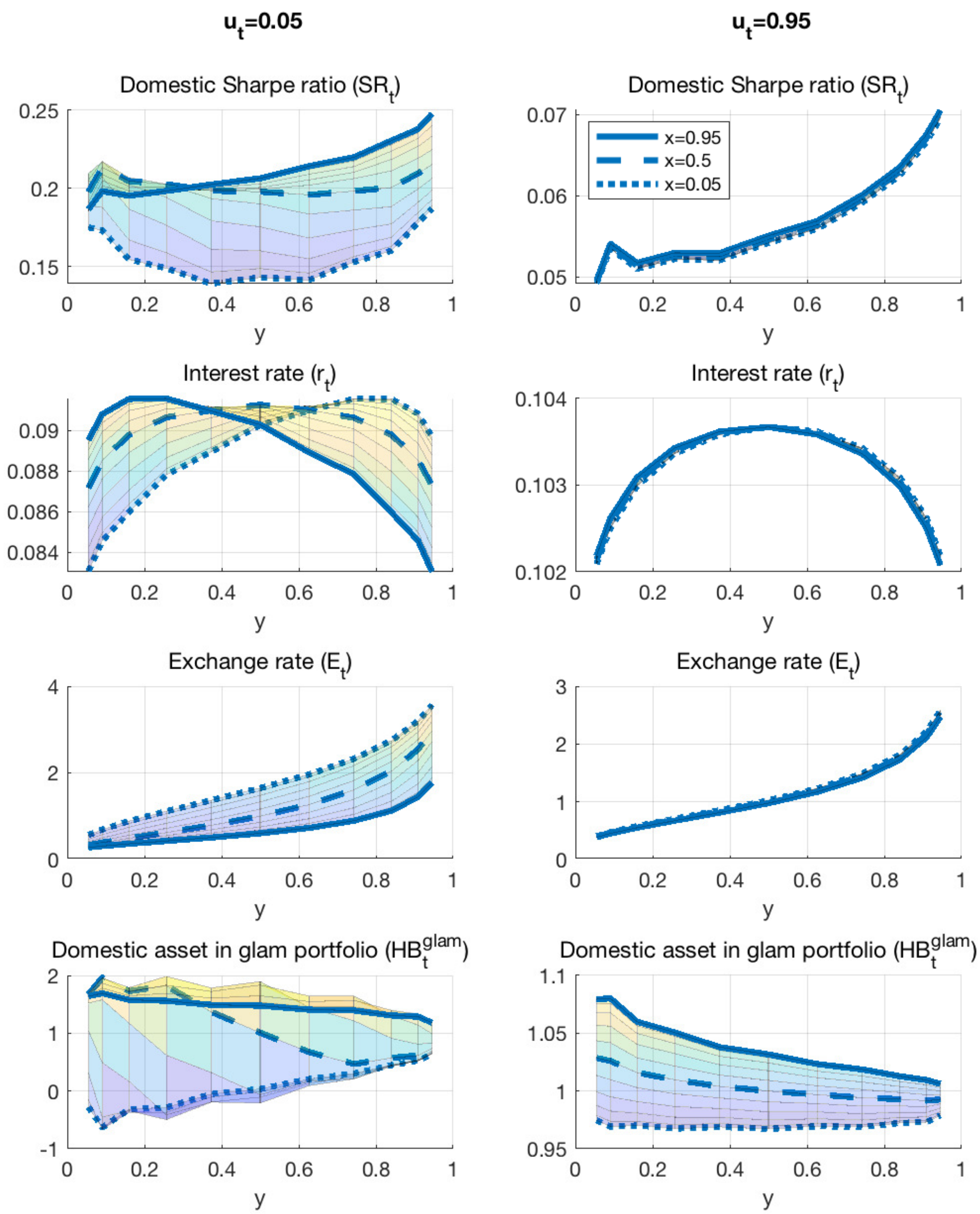
Equations are presented in Sauzet (2021d), and Figure B.1 shows the results. The preference heterogeneity of the global asset manager, coupled with that of the investor of each country, is able to generate rich patterns in global asset prices, interest rates, goods prices, and portfolios, even without portfolio constraints. For instance, the Sharpe ratio on the domestic asset is much larger when the global asset manager is poorly capitalized (u_t small), reflecting the higher compensation for risk required by the domestic and foreign consumer-investors to hold the domestic equity asset. This is also true for foreign equity, and points to the fact that a poorly capitalized global asset manager, a proxy more generally for the global financial system, leads to increased risk premia throughout the world, in a pattern reminiscent of a Global Financial Cycle. This mechanism could complement the one stemming from the role of the domestic country as world banker discussed in the main application in Section 4.3, by introducing financial intermediaries in the picture. When this happens, the risk premia on equity assets are also more dependent on the repartition of wealth across the remaining investors, captured by x_t, consistent with a crisis situation in which the identity of the average holder of an asset matters more and assets rapidly changing hands are accompanied by large swings in returns. The capitalization of the global asset manager also matters for interest rate, which tends to decrease as u_t gets small, reflecting a lower average risk tolerance in the economy, which corresponds with a higher demand for the safe asset (the international riskless bond). Goods prices are also affected, with the exchange rate depending significantly more on the allocation of wealth across consumer-investor. Note also the impact on portfolios: not only is the portfolio of the global asset manager getting further from the market portfolio as u_t decreases, but it is also increasingly affected by the allocation of wealth among the remaining consumer-investors. This reflects the fact that because she is not biased

[32] The share of the domestic and foreign investor in world wealth are now obtained as $x_t(1-u_t)$ and $x_t u_t$.

towards any particular asset, the global asset manager is here to pick up the opposite side of the trades for the other two investors, and this leads to wild changes in her portfolios especially as she gets less well-capitalized.

This brief illustration shows the promise of introducing global financial intermediaries in the framework of this paper, and highlights how it can complement the mechanisms discussed previously in the main application.

Figure B.1: Equilibrium in the presence of a global asset manager



Notes: Calibration: $\gamma^{glam} = 2 < \gamma = \gamma^* = 8, \psi = 0.2, \alpha = 0.85, \rho = 1\%$. x_t is the wealth share of the domestic investor as a fraction of $W_t + W_t^*$. y_t is the relative supply of the domestic good, which captures fundamentals. u_t is the share of world wealth held by the global asset manager.

B.2. Extension 2: towards a solution to the reserve currency paradox (Sauzet, 2021f)

In addition to global financial intermediaries, further extensions of the framework could help make way towards resolving the so-called "reserve currency paradox" emphasized by Maggiori (2017) and to which the reader is referred for details. The paradox appears as follows in the framework of my paper. Consider again that the domestic country represents the United States, the risk-tolerant country at the center of the international financial system. As we have seen, and consistent with Gourinchas et al. (2017): in normal times, the country enjoys an exorbitant privilege by earnings higher returns on average due to its riskier position, but in crisis times, it bears the exorbitant duty of insuring the rest of the world through a wealth transfer. In turn, because of the home bias in consumption, this wealth transfer towards the rest of the world tends to increase the price of foreign goods, which pushes up the price of the foreign basket and lead the domestic currency, the US dollar in this case, to depreciate. The reserve currency paradox resides in the fact that this is clearly counterfactual: empirically, the US dollar tends to *ap*preciate in crisis, which is one of the main reasons why it is the world's major reserve currency in the first place. As discussed in Maggiori (2017), this paradox does not depend on the specifics of the underlying model – for instance, the framework in this paper is quite different from his. Instead, it is deeply rooted in the presence of the home bias in consumption, an aspect that goes back all the way to the classical "transfer problem" of Keynes and Ohlin discussed previously.

Maggiori (2017) presents a potential resolution based on trade costs depending negatively on the capitalization of financial intermediaries. Another part of the story, that I plan to implement in the current framework, relies on the importance of trade in bonds. Specifically, times of crisis are periods in which the demand for safe assets usually skyrockets ("risk-off" episodes). Because the United States is the main provider of safe asset worldwide, this sudden increase in the demand for US Treasuries goes hand-in-hand with a strong upward pressure on the currency in which they are denominated. This, in my view, is one of the main ultimate drivers of US dollar appreciation in times of crisis. To introduce such channels in the framework developed in this paper, I plan to include the following elements in future extensions

(Sauzet (2021f), ongoing). First, the demand for safe asset must be meaningfully time-varying, which I plan to generate from time-varying risk aversion in the form of heterogeneous investors with varying degrees of risk aversion *within countries*. A risk-off episode would therefore correspond to an event in which the risk-tolerant investor of a country is poorly capitalized. Second, the bond of the center country should be particularly attractive in difficult times[33], which could come from an ad-hoc feature or potentially by assuming that the size of the center country is larger so that its bond ensures against a larger share of world shocks, in the spirit of Hassan (2013).[34] Third, for this "trade in assets" channel to matter enough for exchange rates so as to reverse the reserve currency paradox driven by the trade in goods, the introduction of global financial intermediaries will be important quantitatively. They could take the form of global asset managers as presented above, intermediating trade in assets, or of global foreign currency dealers in the spirit of Hau and Rey (2006) and Gabaix and Maggiori (2015). Their role would be to ensure that, like in practice, the increased demand for bonds is met with limited capacity, which ultimately leads to an upward pressure on the price of the US currency. Finally, the introduction of portfolio constraints, for both global intermediaries and for the different investors within each country, as well as other sources of market incompleteness, will also prove important for the mechanism to have bite quantitatively.

[33] A related and subtle point is to disentangle the extent to which the upward pressure on US Treasuries is itself driven by the safety of the US dollar in times of crisis.

[34] To do this, reformulating the output share y_t by adapting the share process of Menzly et al. (2004); Santos and Veronesi (2006) as mentioned previously could be particularly useful.

B.3. Extension 3: projection methods via neural networks (Sauzet, 2021c)

The extensions above make clear that the number of state variables is likely to rapidly increase with additions to the framework. Projection methods are conceptually well-suited to contexts with multiple state variables, and are typically better able to handle a larger number of them than other approaches like finite-difference methods, which become rapidly computationally too costly.[35] As a result, they are well-adapted to the environment in this paper. To be sure however, computationally, traditional projection methods also are very much subject to the curse of dimensionality, and scaling the number of state variables further up will prove limited using standard Chebyshev polynomials. For instance, even the addition of a third state variable, like in the global asset manager extension above, renders the resolution significantly slower, and increasing the order of approximation much beyond $N = 10$ proves difficult. More refined ways to construct the Chebyshev polynomials and corresponding grids, such as complete polynomials or Smolyak's algorithm, could help. Ultimately however, they are also limited and methods able to handle higher-dimensional cases will be required.

One such method consists in naturally extending the concept of projection approaches, but to replace the Chebyshev polynomials in the approximation by neural networks. In ongoing work (Sauzet, 2021c), I am developing these "projection methods via neural networks" to be applied to continuous-time problems like the one in this paper. The use not only of neural networks, but of the whole eco-system of related packages, proves of tremendous importance. First, those packages and environments, like Tensor Flow on which my implementation is based, are specifically designed for very high-dimensional contexts such as computer vision or other artificial-intelligence-type problems. As such, they are able to handle billions of observations and multiple millions of parameters. Even in the framework of this paper, this would allow me to focus on a much finer grid than do Chebyshev polynomials. Second, provided that one is judicious in the choice of the specification of the neural networks (typically in the choice of activation functions), they are naturally able to handle very non-linear

[35] The method currently developed in Hansen et al. (2018) could potentially help from that perspective.

functions. This aspect will prove particularly important when introducing portfolio constraints, which typically lead to sharp non-linearities, and are not necessarily handled well by Chebyshev polynomials especially of low order. Third, fitting neural networks conceptually in a projection framework is also particularly useful. Contrary to other methods based on neural networks that are more akin to value function iteration, e.g. Duarte (2019), a method expressed in a projection approach framework is able to naturally handle even cases for which value function iteration is difficult to adapt. For instance, economies with multiple agents and incomplete markets, for which there are several value functions as well as other unknown functions, would be difficult to cast in a value function iteration framework, but pose no particular problem for projection methods via neural networks.

Overall, the method has promise. For instance, I solve a "Ten Trees" equivalent to Cochrane et al. (2008)'s "Two Trees" without particular difficulty, a fit that would prove impossible for Chebyshev polynomials, and even less so for finite-different methods.[36]

[36] On this problem, Martin (2013) proposes an alternative method that proves promising even with five or six trees, and possibly more. The method also allows for jumps.

C. Additional Figures

C.1. Economic set-up

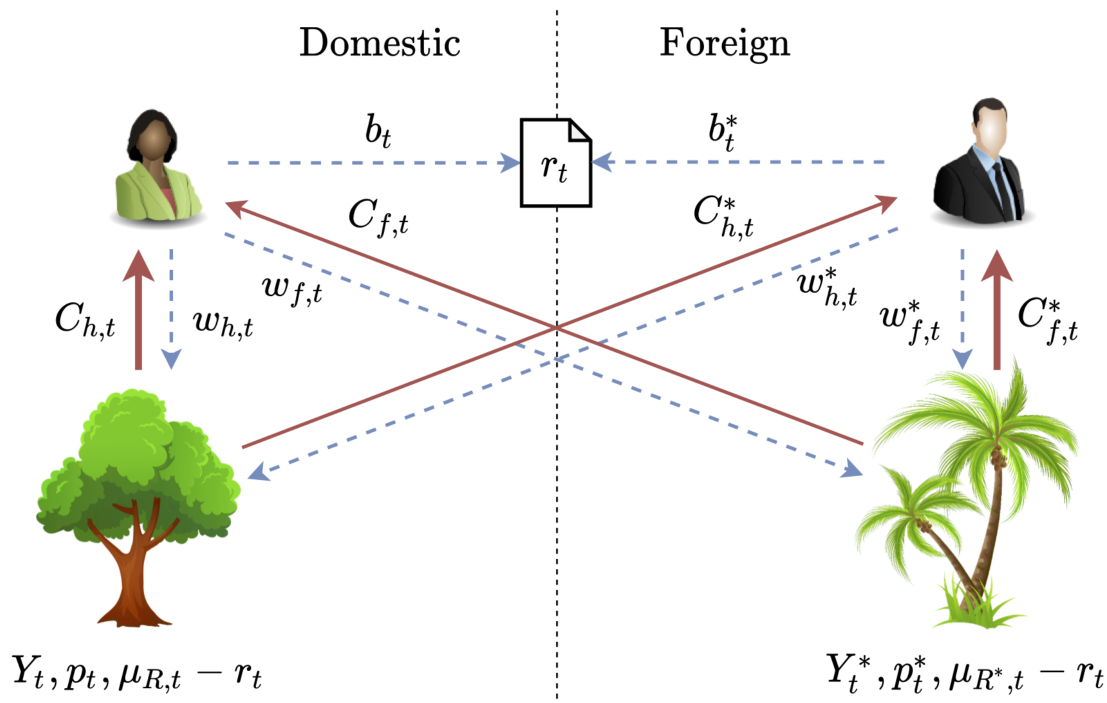


Figure C.1: Baseline international economy

Notes: Back to main text: Section 2.

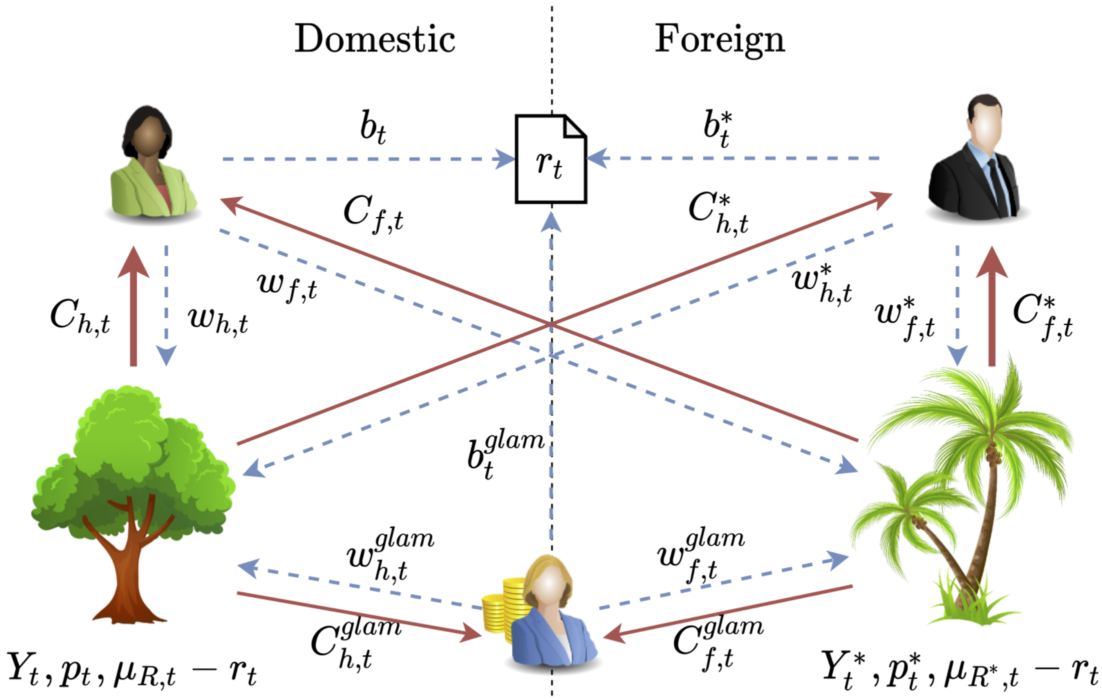


Figure C.2: International economy in the presence of a global asset manager

Notes: Back to main text: Section 4.5, back to Appendix: Section B.1.

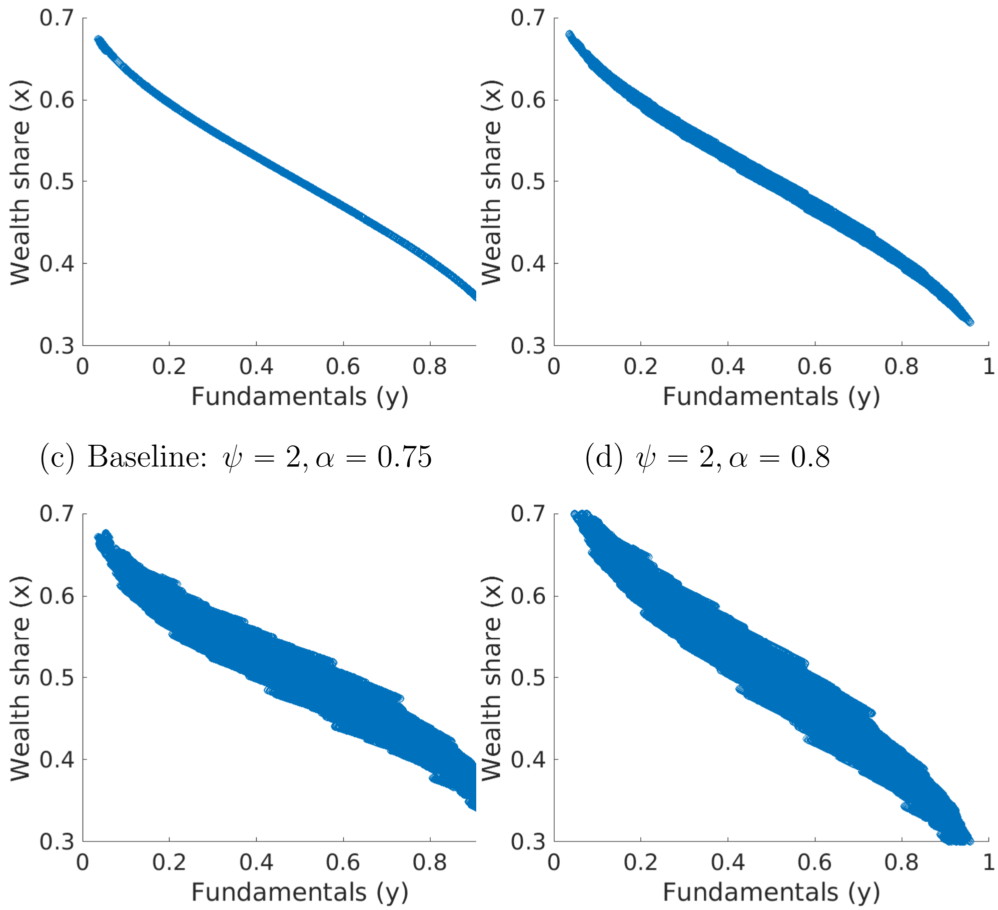
C.2. Distributions

All distributions, unless otherwise specified are obtained from $nsim = 1,000$ paths of length $T = 250$ years, with $dt = 0.01$ (biweekly frequency), starting from $X_0 = (1/2, 1/2)$. The distributions are shown from the top, and for visibility each point visited during the simulation is shown with the same intensity.

Figure C.3: Distribution of the state variables in the baseline calibration

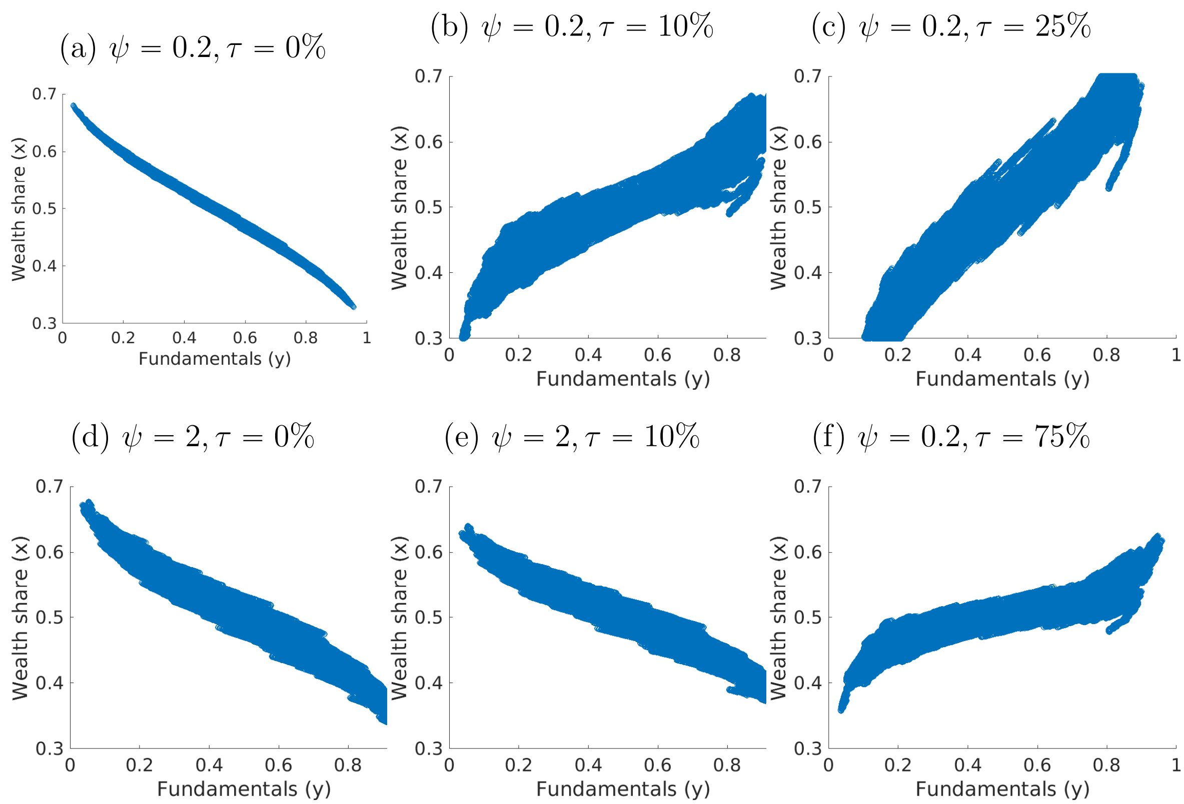
(a) CRRA: $\psi = 1/\gamma, \alpha = 0.75$ (b) $\psi = 0.2, \alpha = 0.75$

(c) Baseline: $\psi = 2, \alpha = 0.75$ (d) $\psi = 2, \alpha = 0.8$



Notes: x_t, the wealth share, which captures the share of worldwide wealth held by the domestic investor, is shown on the vertical axis. y_t, the domestic output share, which captures fundamentals, is shown on horizontal axis. Distribution seen from the top, and obtained from $nsim = 1,000$ paths of length $T = 250$, with $dt = 0.01$, starting from $X_0 = (1/2, 1/2)$.

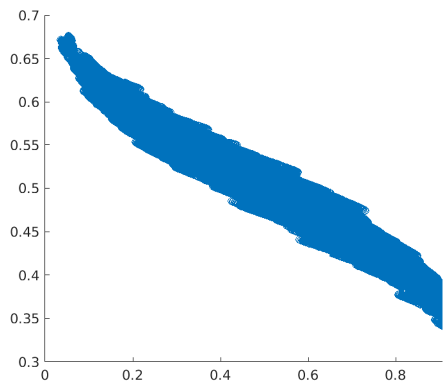
Figure C.4: Distribution of the state variables under imperfect financial integration



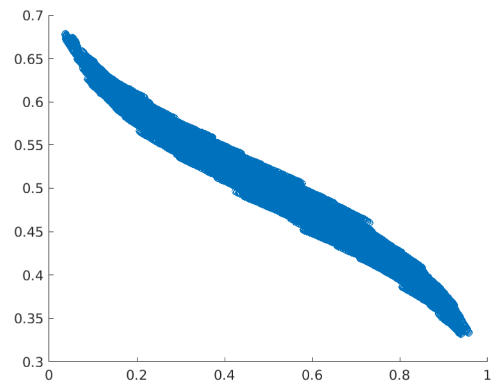
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1 (specifically $\psi = 2$), except for labor income μ. x_t, the wealth share, which captures the share of worldwide wealth held by the domestic investor, is shown on the vertical axis. y_t, the domestic output share, which captures fundamentals, is shown on horizontal axis. Distribution seen from the top, and obtained from $nsim = 1,000$ paths of length $T = 250$, with $dt = 0.01$, starting from $X_0 = (1/2, 1/2)$.

Figure C.5: Distribution of the state variables in the presence of labor income (δ)

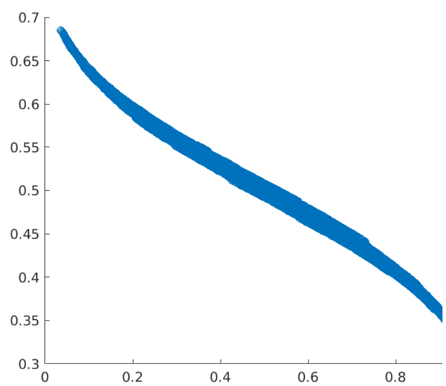
(a) Baseline: $\delta = 0\%$



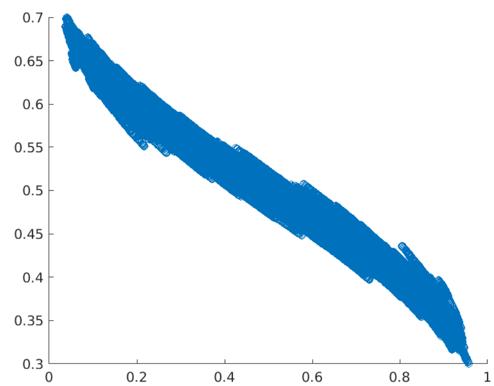
(b) $\delta = 10\%$



(c) $\delta = 25\%$



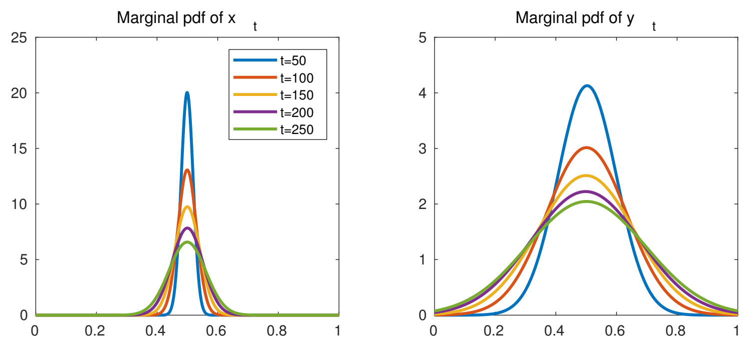
(d) $\delta = 62.5\%$



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1 (specifically $\psi = 2$), except for labor income δ. x_t, the wealth share, which captures the share of worldwide wealth held by the domestic investor, is shown on the vertical axis. y_t, the domestic output share, which captures fundamentals, is shown on horizontal axis. Distribution seen from the top, and obtained from $nsim = 1,000$ paths of length $T = 250$, with $dt = 0.01$, starting from $X_0 = (1/2, 1/2)$.

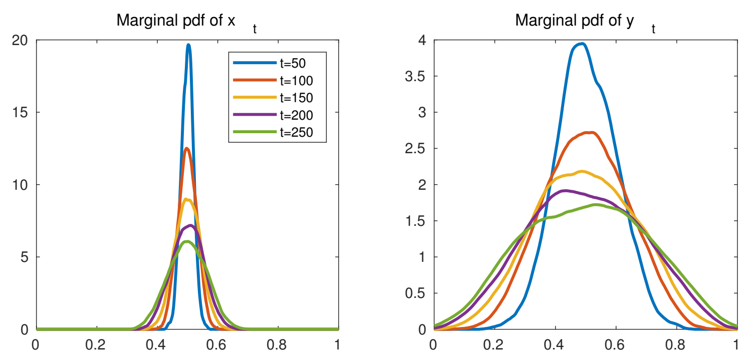
C.3. Evolution of the distribution of X_t over time

Figure C.6: Marginal distributions for x_t and y_t over time (Normal kernel, baseline calibration)



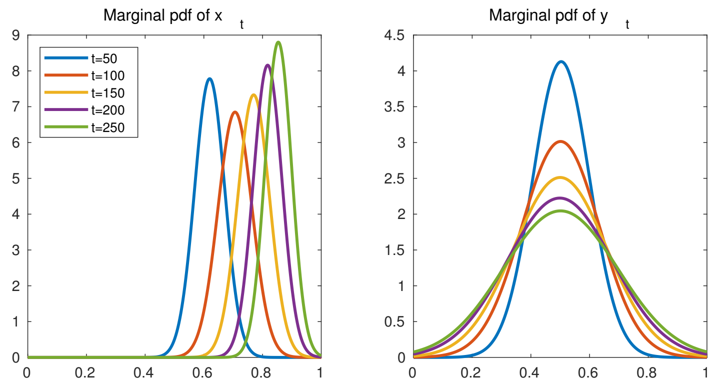
Notes: x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Distribution obtained from $nsim = 1,000$ paths of length $T = 250$, with $dt = 0.01$, starting from $X_0 = (1/2, 1/2)$.

Figure C.7: Marginal distributions for x_t and y_t over time (Epanechnikov kernel, baseline calibration)



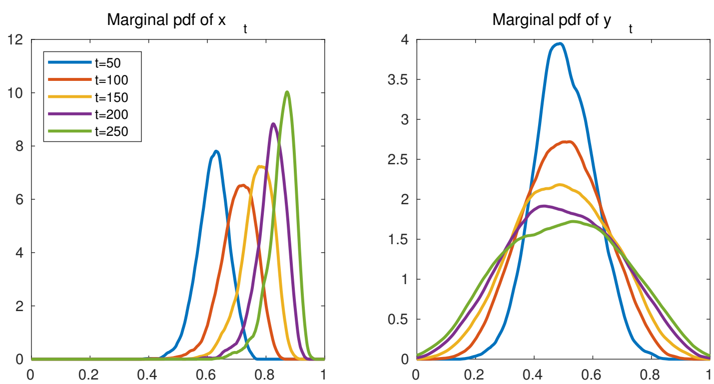
Notes: x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Distribution obtained from $nsim = 1,000$ paths of length $T = 250$, with $dt = 0.01$, starting from $X_0 = (1/2, 1/2)$.

Figure C.8: Marginal distributions for x_t and y_t over time (Normal kernel, $\gamma = 7.5 < \gamma^* = 15$)



Notes: x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Distribution obtained from $nsim = 1,000$ paths of length $T = 250$, with $dt = 0.01$, starting from $X_0 = (1/2, 1/2)$.

Figure C.9: Marginal distributions for x_t and y_t over time (Epanechnikov kernel, $\gamma = 7.5 < \gamma^* = 15$)



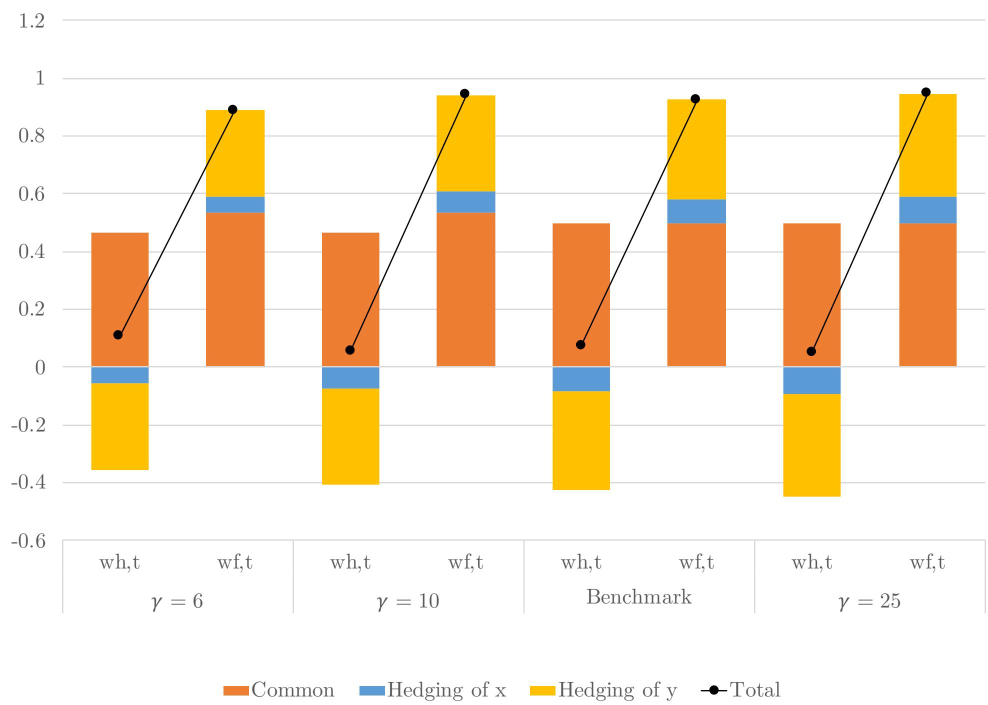
Notes: x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Distribution obtained from $nsim = 1,000$ paths of length $T = 250$, with $dt = 0.01$, starting from $X_0 = (1/2, 1/2)$.

C.4. Portfolios at the symmetric point

Figure C.10: Equity portfolio at $X_t = (1/2, 1/2)$ and home bias in consumption α

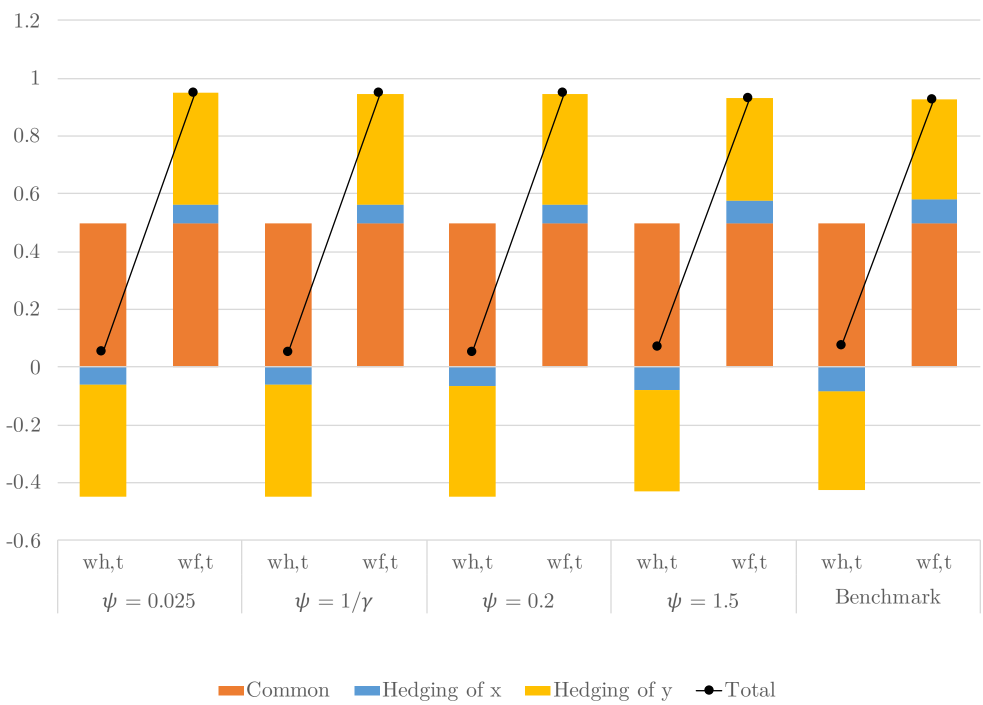
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1, except α. The figure shows portfolios when both the allocation of wealth (x_t) and the relative supply (y_t) are symmetric, $X_t = (1/2, 1/2)$.

Figure C.11: Equity portfolio at $X_t = (1/2, 1/2)$ and risk aversion γ



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1, except γ. The figure shows portfolios when both the allocation of wealth (x_t) and the relative supply (y_t) are symmetric, $X_t = (1/2, 1/2)$.

Figure C.12: Equity portfolio at $X_t = (1/2, 1/2)$ and elasticity of intertemp. substitution ψ

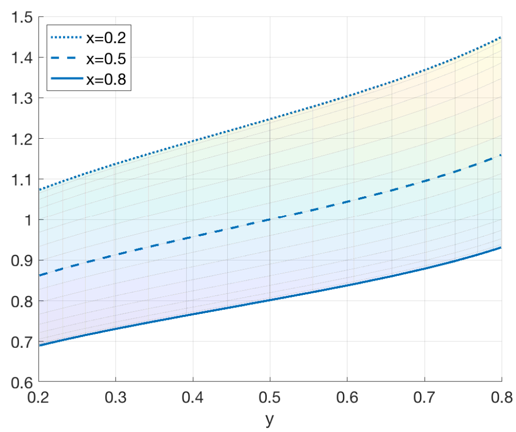


Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1, except ψ. The figure shows portfolios when both the allocation of wealth (x_t) and the relative supply (y_t) are symmetric, $X_t = (1/2, 1/2)$.

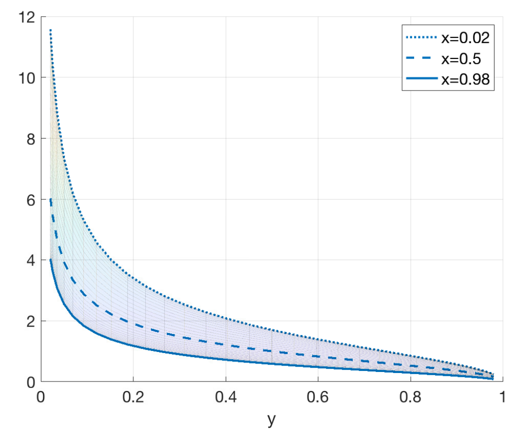
C.5. Representations as a function of both state variables

Figure C.13: Relative dividends: $p_t^* Y_t^* / (p_t Y_t)$

(a) $\theta = 0.9^* < 1$

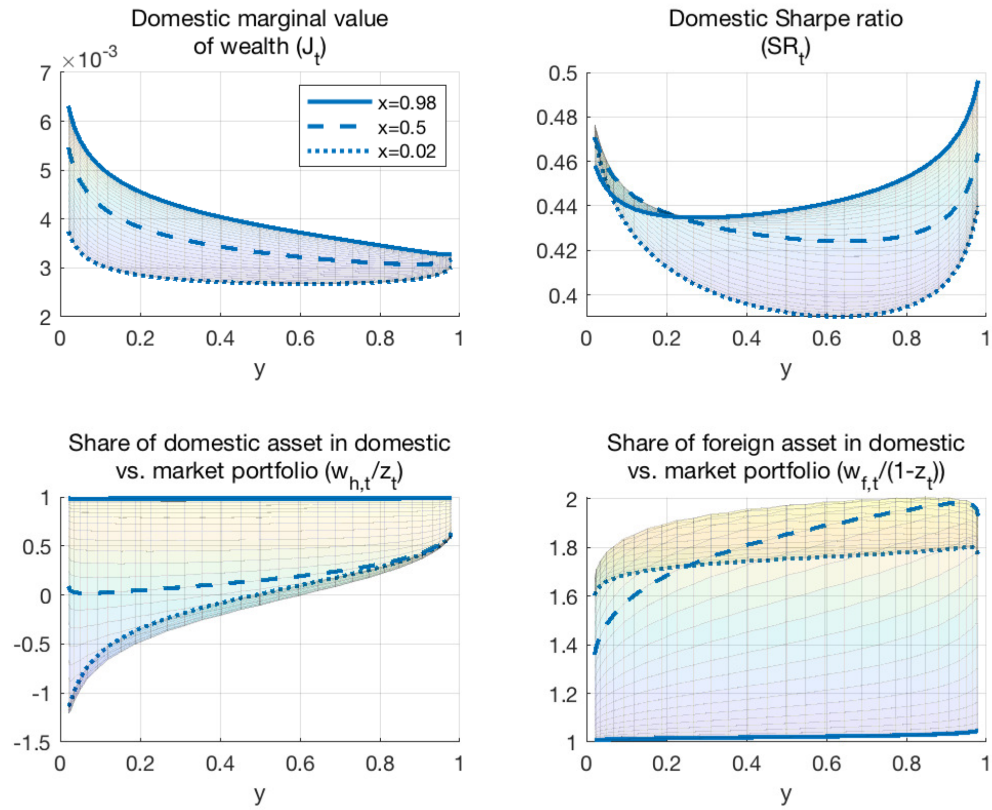


(b) $\theta = 2 > 1$



Notes: Based on the symmetric calibration of Assumption 1, except for the elasticity of substitution across goods, θ. * For Panel (a), $\gamma = 15, \psi = 1/\gamma, \alpha = 0.58$ (final calibration ongoing). x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

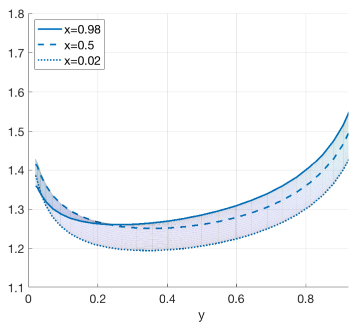
Figure C.14: Direct impact of the wealth share



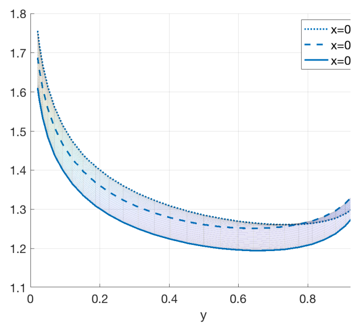
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Corresponding three-dimensional representations: Figure C.33.

Figure C.15: Expected risk premia, Sharpe ratios, and interest rate

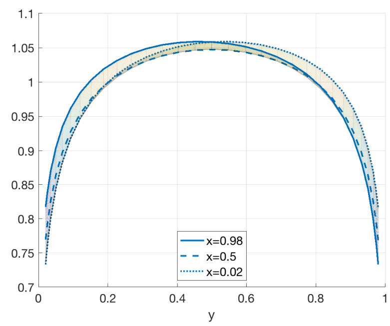
(a) Domestic ($\mu_{R,t} - r_t, \%$)



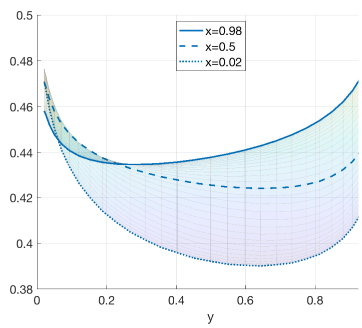
(b) Foreign ($\mu_{R*,t} - r_t, \%$)



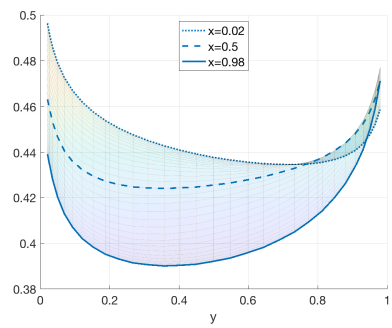
(c) Interest rate ($r_t, \%$)



(d) Domestic (SR_t)



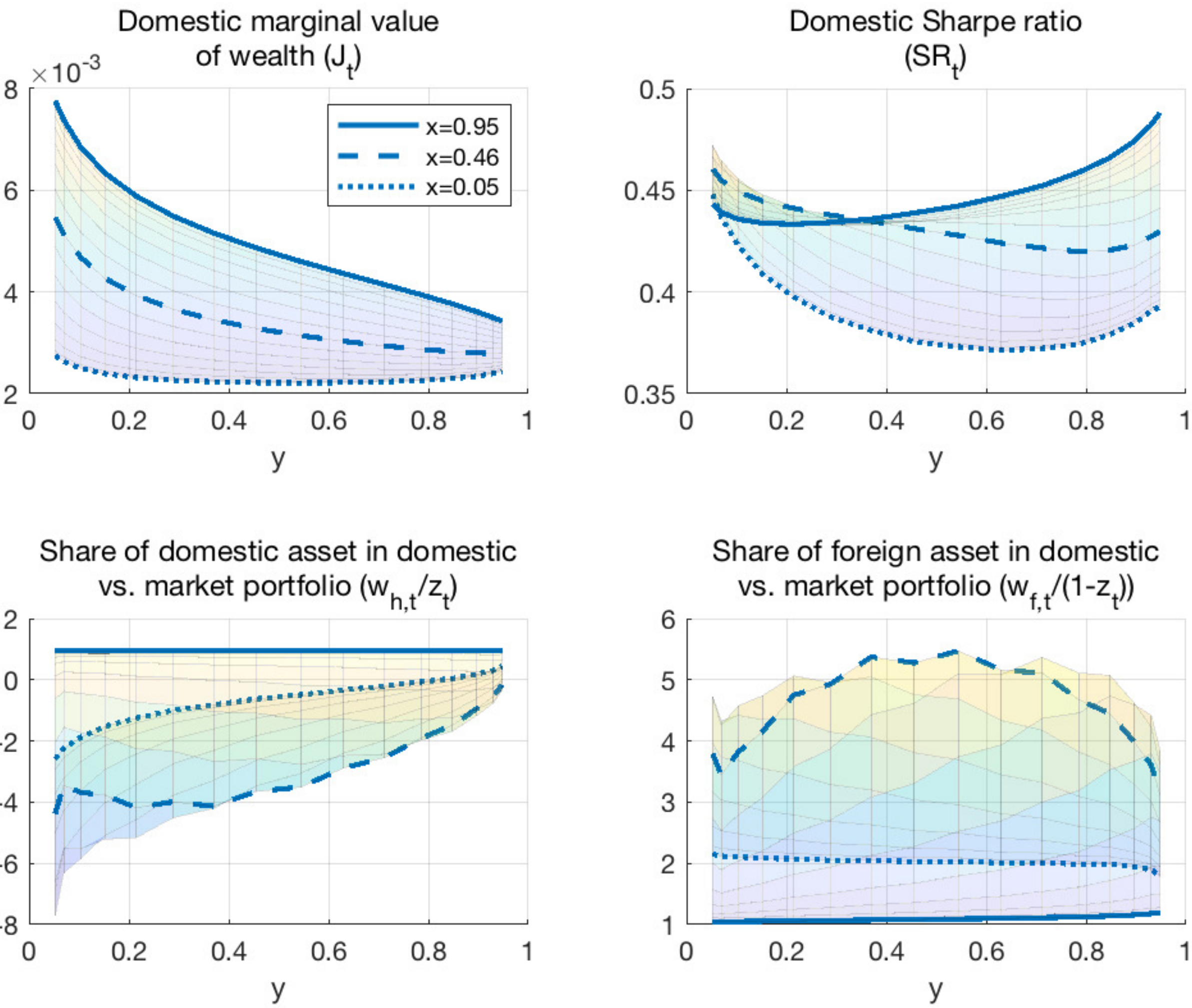
(e) Foreign (SR_t^*)



Notes: Based on the symmetric calibration of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Corresponding representation when $x_t = 1/2$: Figure **??**.

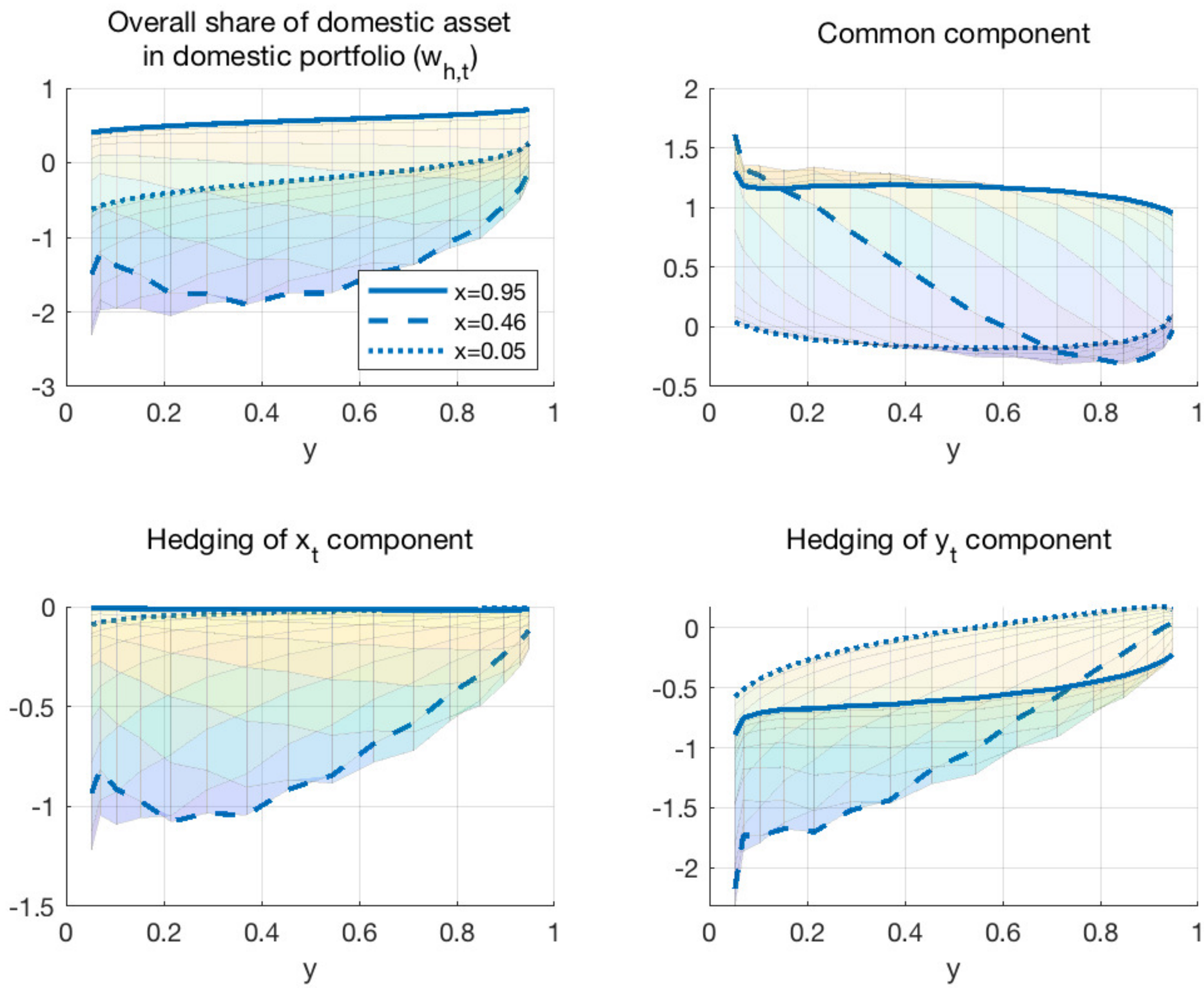
C.6. Effect of the home bias in consumption

Figure C.16: Direct impact of the wealth share for $\alpha = 0.85$



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1, except that $\alpha = 0.85$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

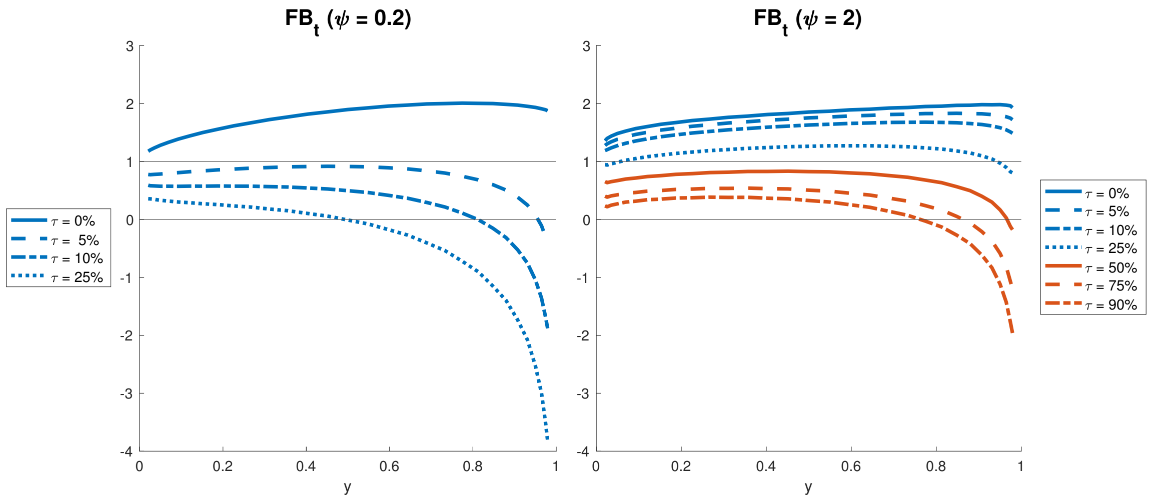
Figure C.17: Components of the domestic portfolio with $\alpha = 0.85$



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1, except that $\alpha = 0.85$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

C.7. Effect of imperfect financial integration

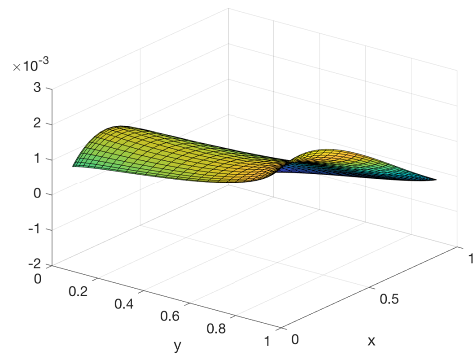
Figure C.18: Domestic equity portfolio vs. market portfolio



Notes: Based on the symmetric calibration of Assumption 1, except for ψ and τ. The figure shows a cut in which the allocation of wealth is symmetric ($x_t = 1/2$). y_t is the relative supply of the domestic good, which captures fundamentals. Effect on the home bias measure HB_t: Figure **??**.

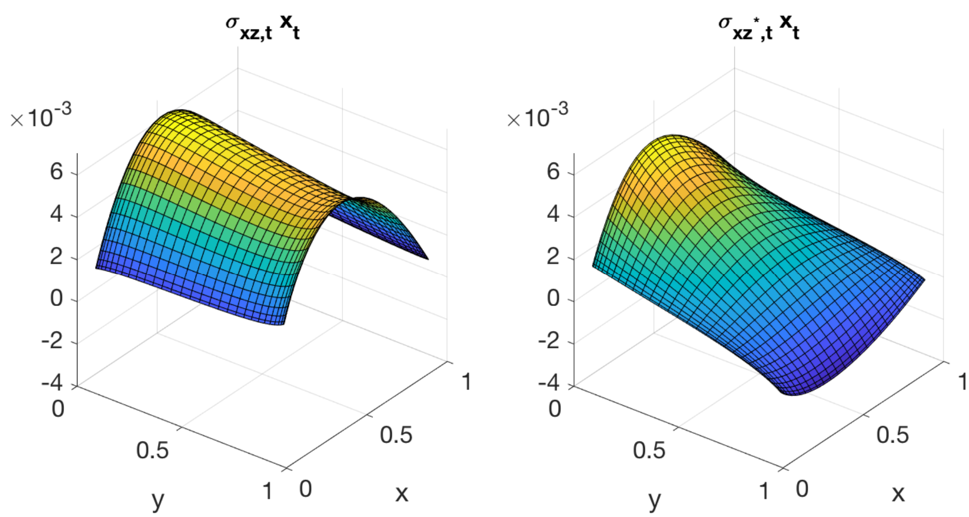
C.8. Application: The International Financial System

Figure C.19: Drift of the wealth share ($\mu_{x,t}x_t$)



Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

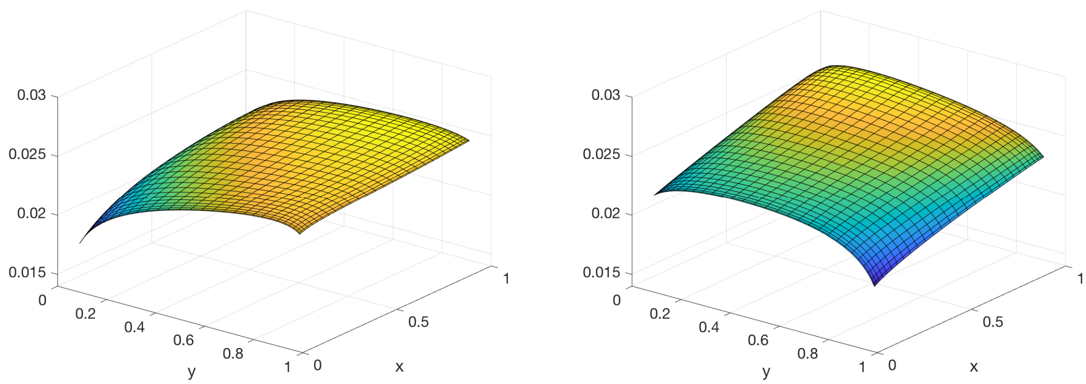
Figure C.20: Diffusion of the wealth share ($\sigma_{x,t}x_t$)



Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.21: Dividend yields in the application of Section 4

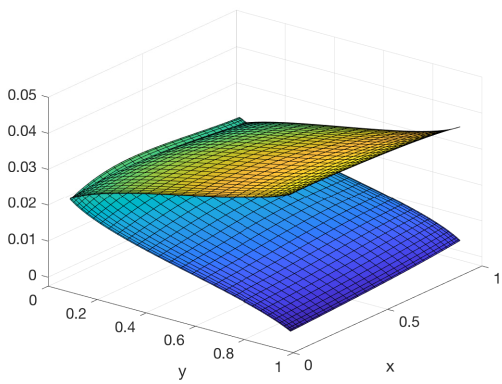
(a) Domestic equity asset: F_t (b) Foreign equity asset: F_t^*



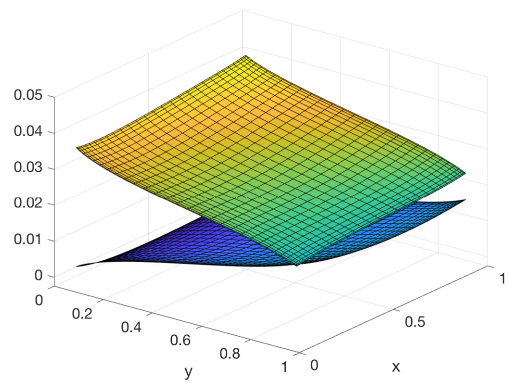
Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.22: Second moments of returns in the application of Section 4

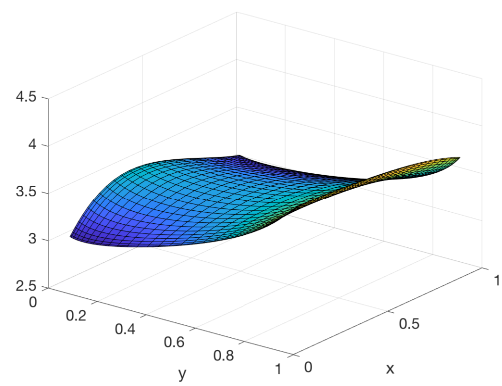
(a) Diffusion of domestic returns: $\sigma_{R,t}$



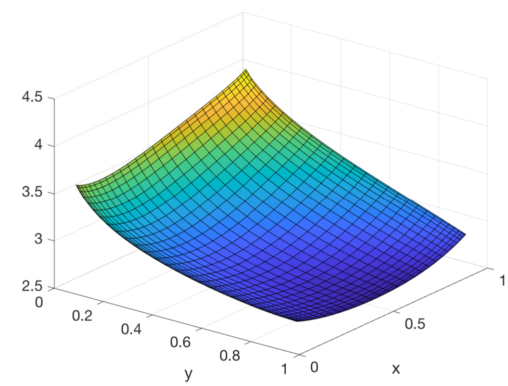
(b) Diffusion of foreign returns: $\sigma_{R*,t}$



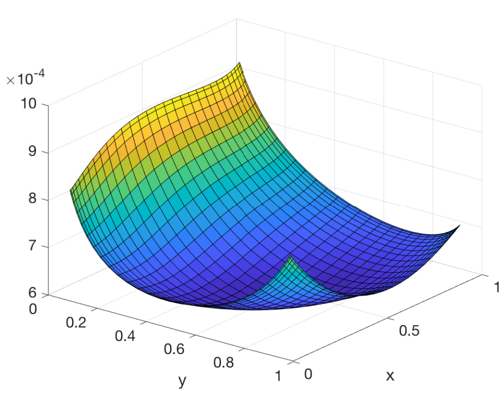
(c) Domestic volatility (%): $\left(\sigma_{R,t}^T \sigma_{R,t}\right)^{-1/2}$



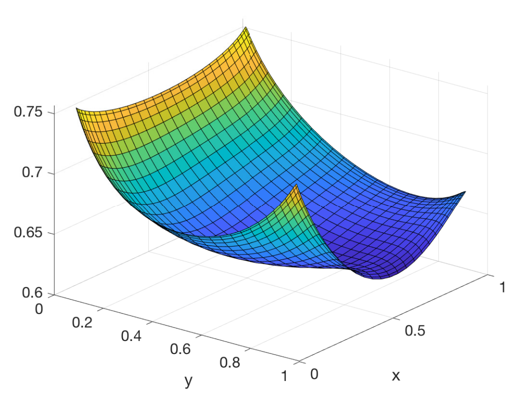
(c) Foreign volaility (%): $\left(\sigma_{R*,t}^T \sigma_{R*,t}\right)^{-1/2}$



(e) Conditional cov.: $\text{cov}_t(dR_t, dR_t^*)dt^{-1}$



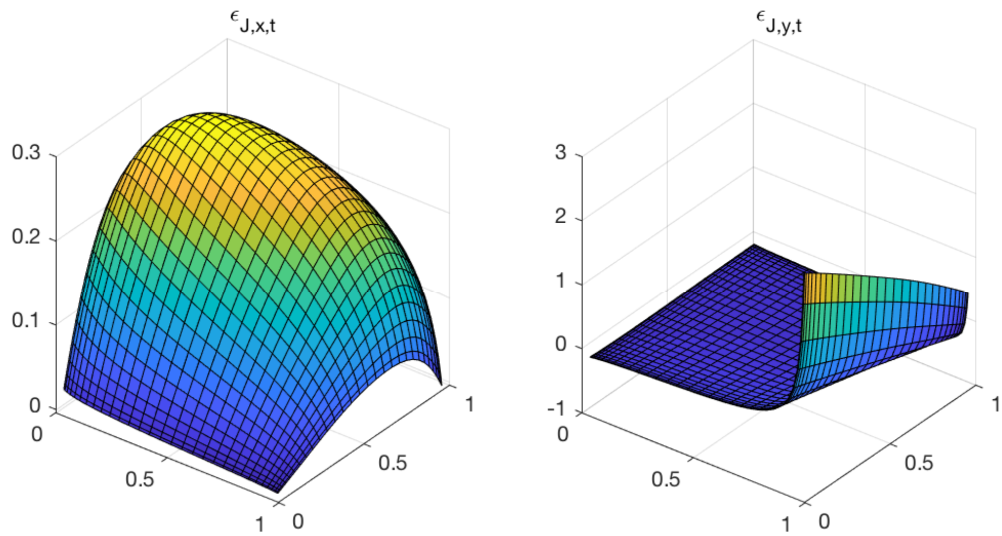
(f) Conditional corr.: $\text{corr}_t(dR_t, dR_t^*)dt^{-1}$



Notes: Based on the symmetric calibration of Assumption 1, except that $\gamma = 8 < \gamma^* = 15$, $\psi = 0.5$, and $\tau = 15\%$. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

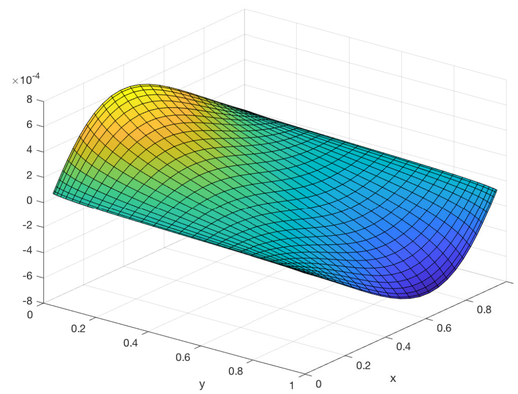
C.9. Other three-dimensional figures

Figure C.23: Conditional elasticities of the domestic marginal value of wealth (J_t)



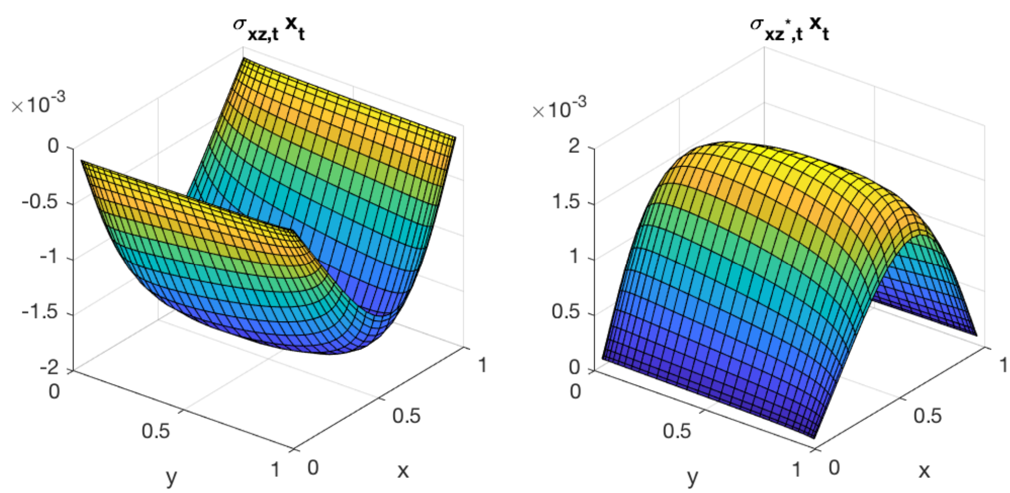
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.24: Drift of the wealth share ($\mu_{x,t}x_t$)



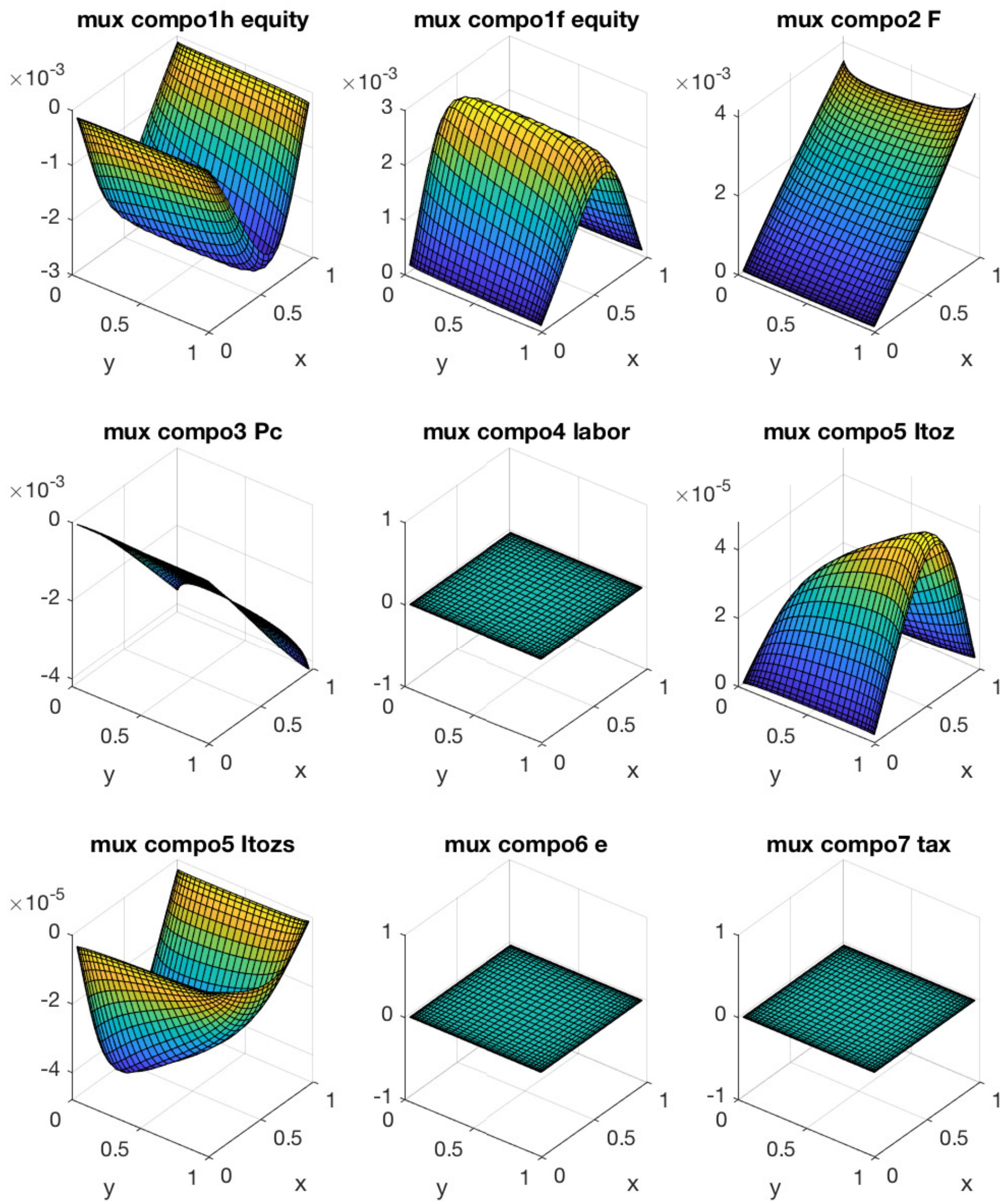
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.25: Diffusion of the wealth share ($\sigma_{x,t}x_t$)



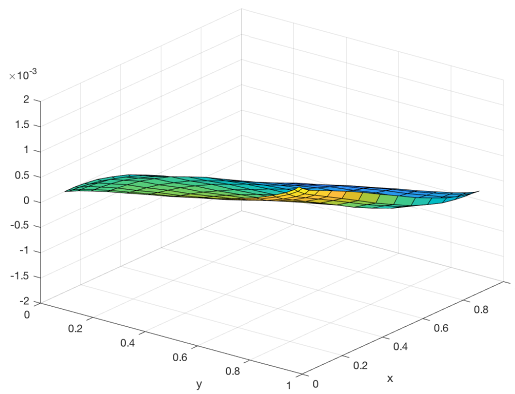
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Corresponding two-dimensional representation: Figure **??**.

Figure C.26: Components of the drift of the wealth share ($\mu_{x,t}x_t$)



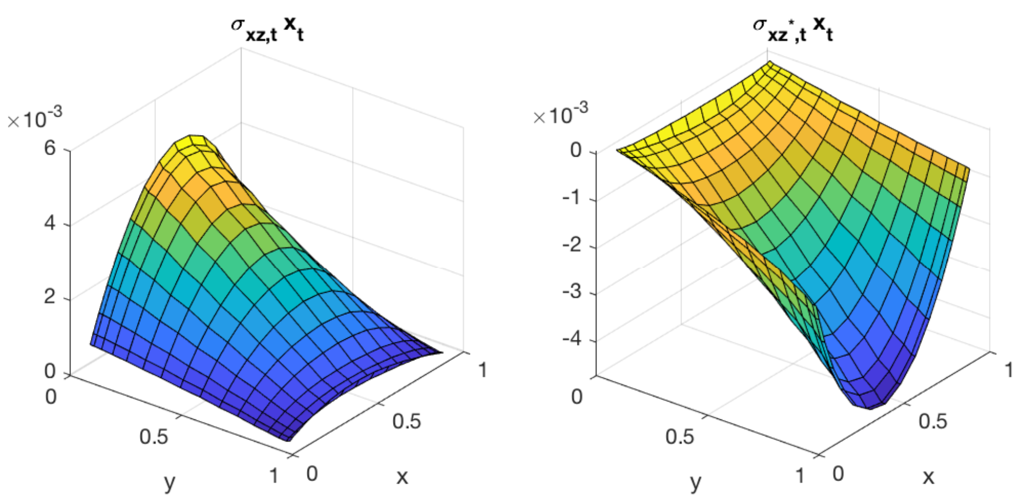
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.27: Drift of the wealth share ($\mu_{x,t}x_t$) under imperfect financial integration ($\psi = 0.2, \tau = 10\%$)



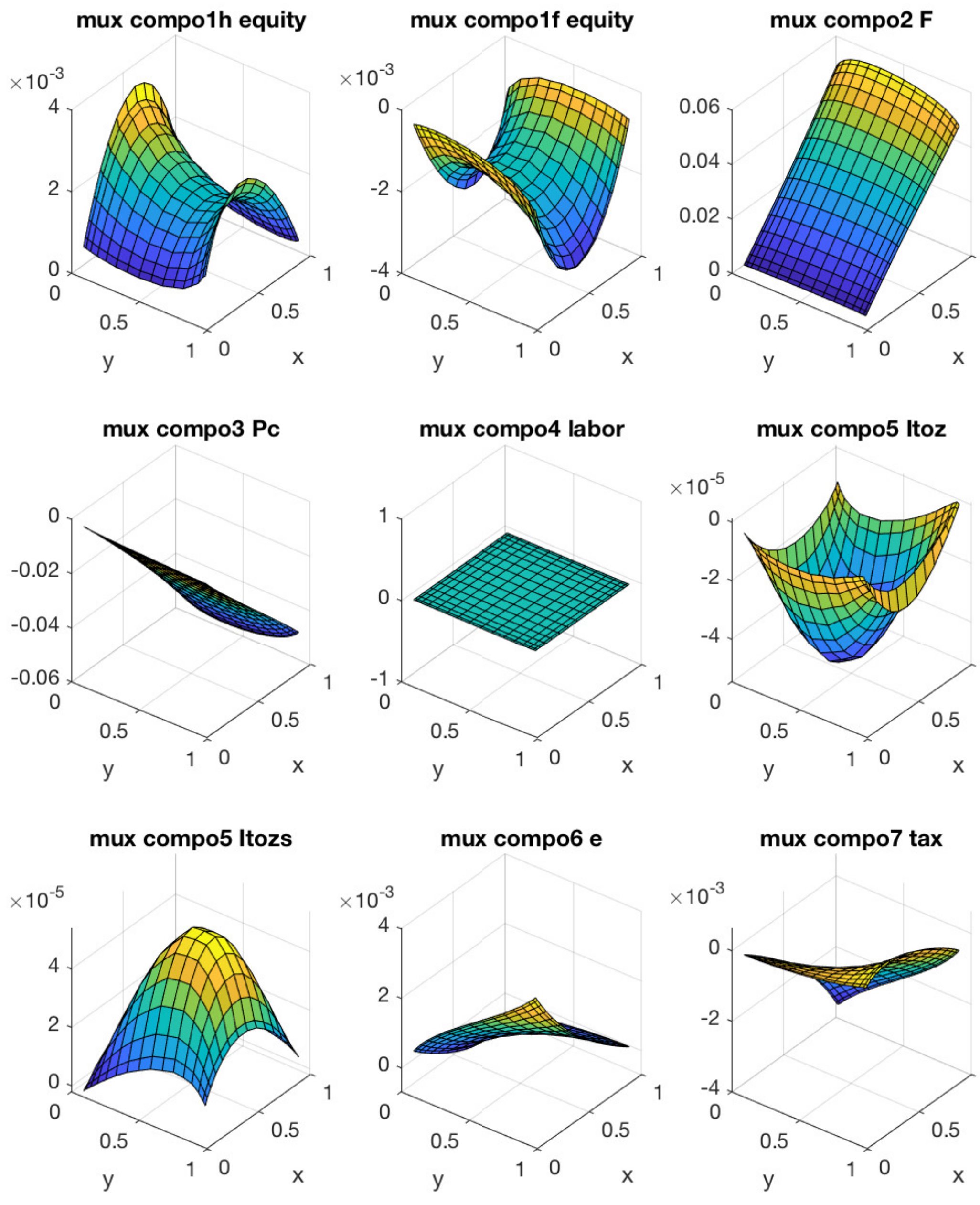
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.28: Diffusion of the wealth share ($\sigma_{x,t}x_t$) under imperfect financial integration ($\psi = 0.2, \tau = 10\%$)



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

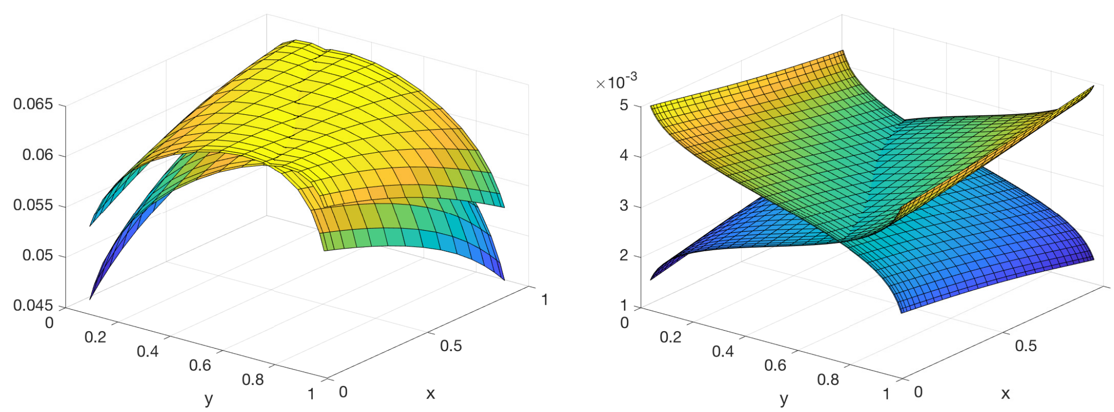
Figure C.29: Components of the drift of the wealth share ($\mu_{x,t}x_t$) under imperfect financial integration ($\psi = 0.2, \tau = 10\%$)



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

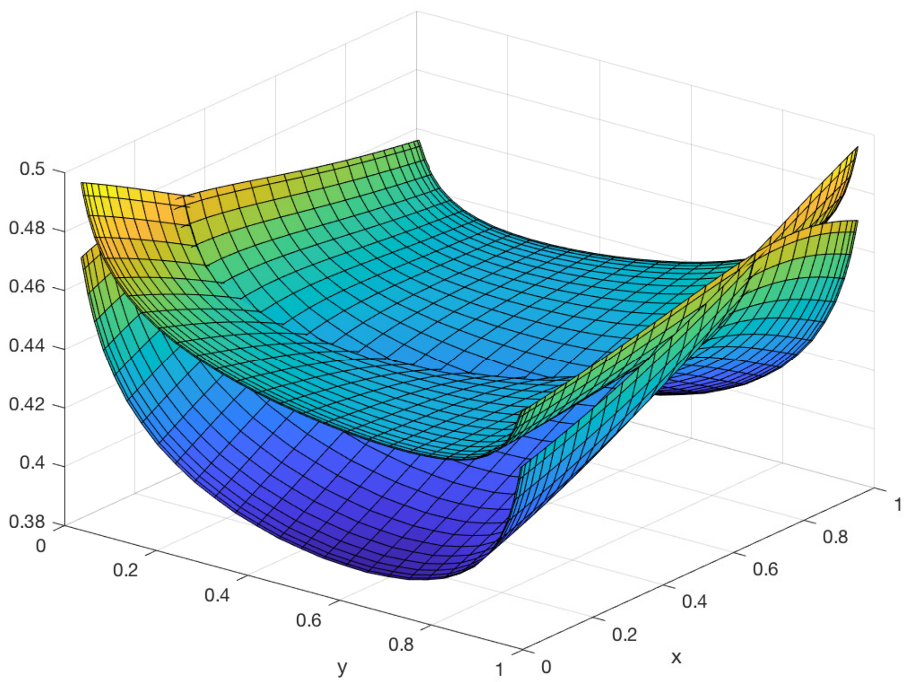
Figure C.30: Dividend yields

(a) $\psi = 0.2$ (b) $\psi = 2$



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.31: Sharpe ratios

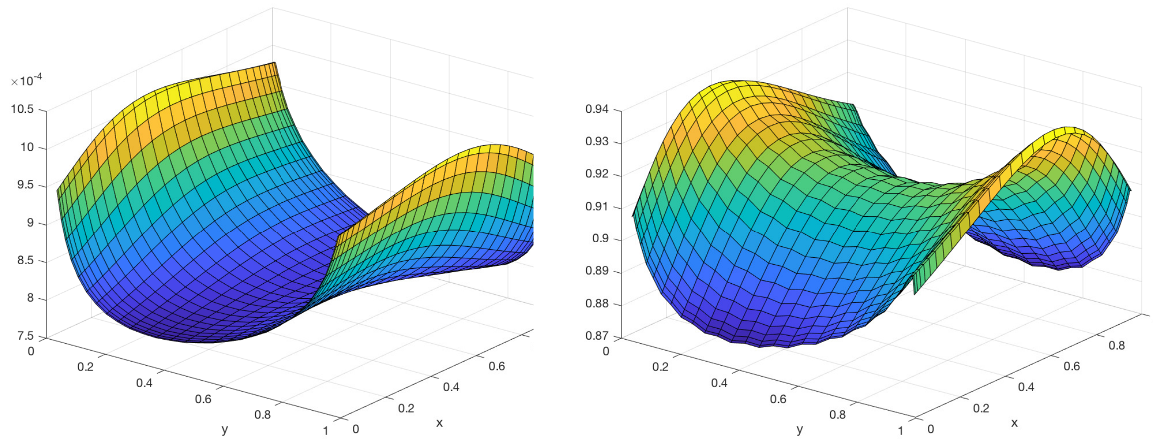


Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals.

Figure C.32: Comovement of returns

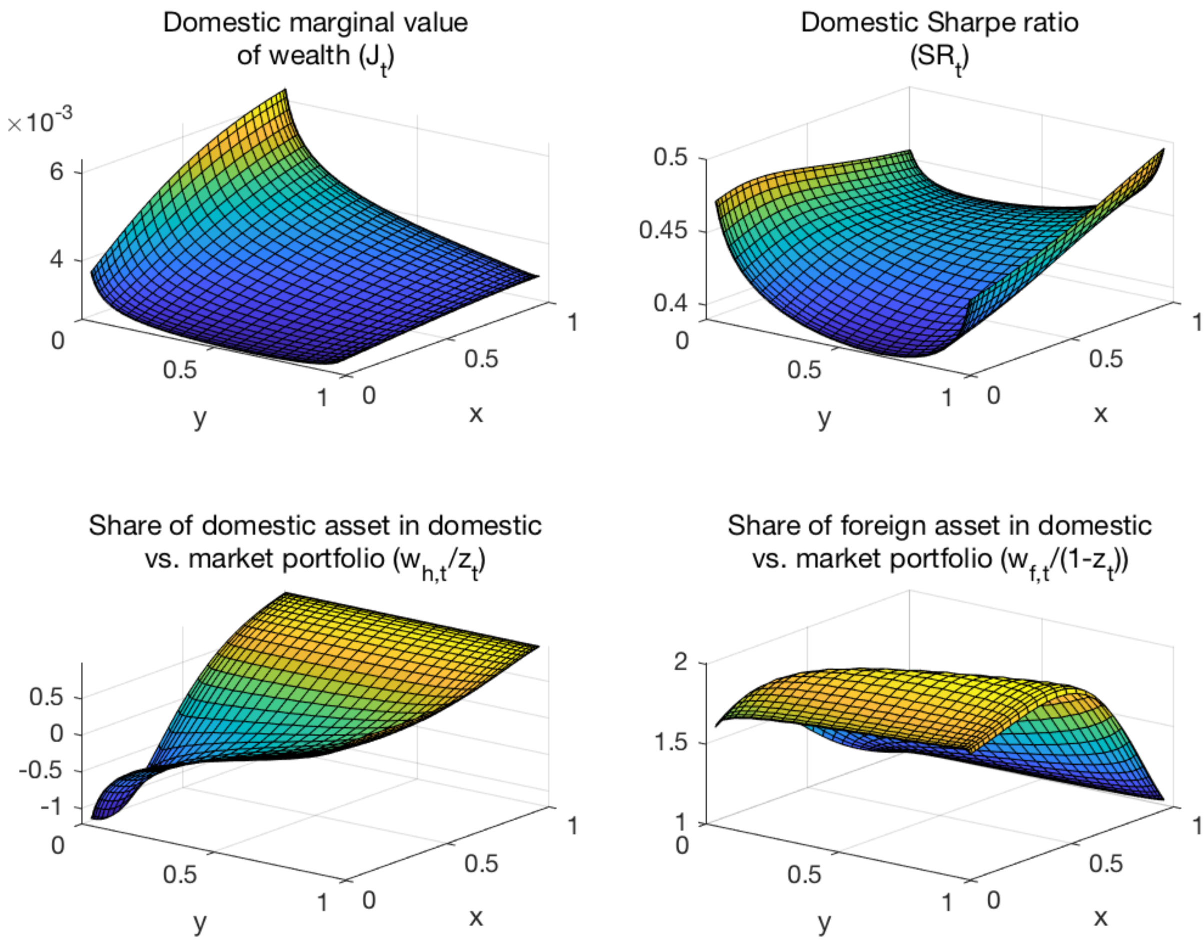
Conditional covariance: $\mathrm{cov}_t(dR_t, dR_t^*)dt^{-1}$

Conditional correlation: $\mathrm{corr}_t(dR_t, dR_t^*)dt^{-1}$



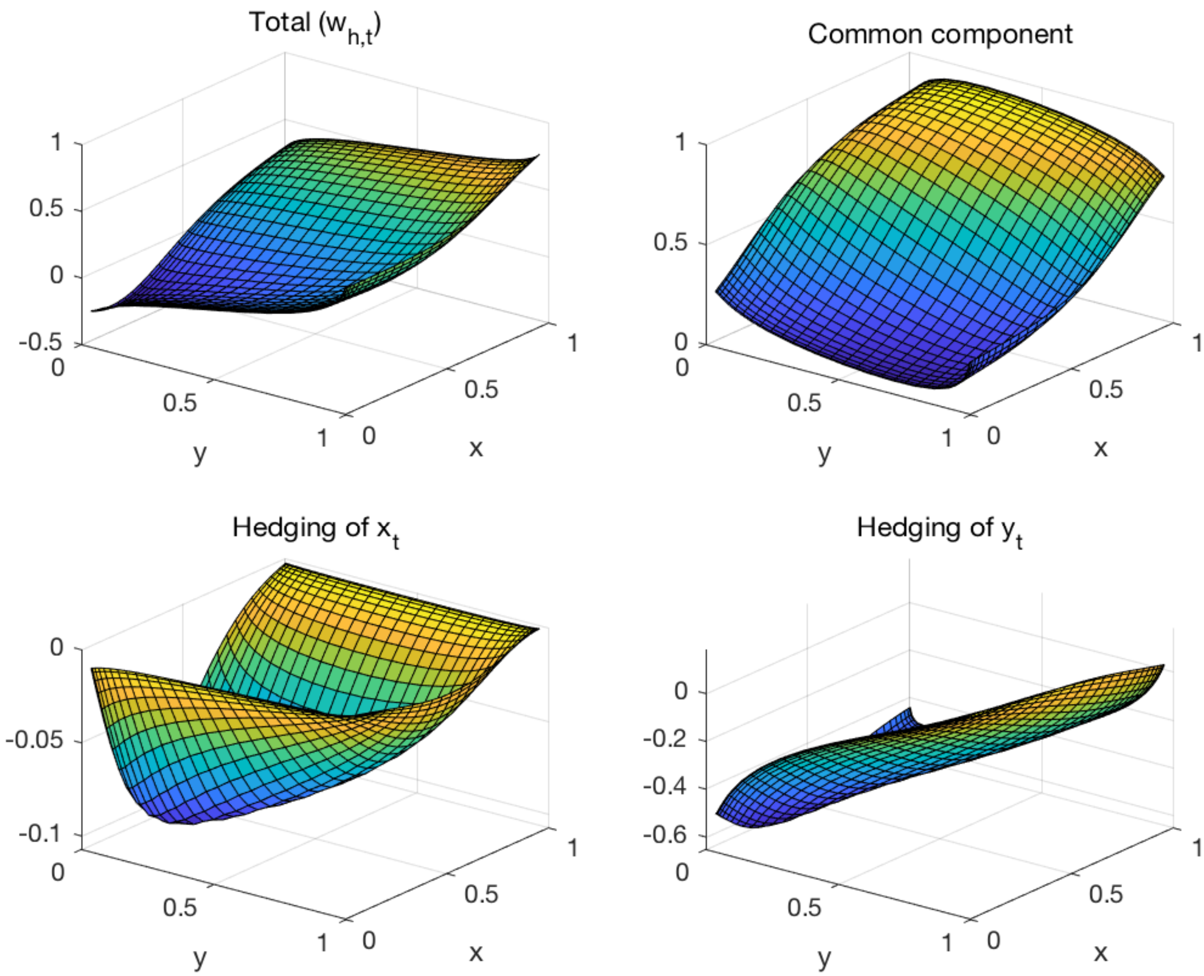
Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Corresponding two-dimensional representation: Figure **??**.

Figure C.33: Direct impact of the wealth share



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Corresponding two-dimensional representation: Figure C.14.

Figure C.34: Components of the domestic portfolio (as compared to the market portfolio)



Notes: Based on the symmetric calibration under perfect risk sharing of Assumption 1. x_t is the wealth share, which captures the share of worldwide wealth held by the domestic investor. y_t is the relative supply of the domestic good, which captures fundamentals. Corresponding two-dimensional representation: Figure **??**.