Maxime Genicion

Assignement 5

6 Global operators

1) Fourier transform

a - Match images / spectrums

b) explain [column]

Image	Transform	Rationale
1	e	Reduction, and baboon looks good.
2	j	Big central middle line could correspond to having text
3 (10)	g/b	3/10 are rotated picture, g/b are rotated transforms (seems like a quarter rotation, 90°, as Hor → Vertic and Diagonal is rotated)
4	d	Big central Horiz line could correspond to vertical zones
5	a/f	Same picture but shifted → identical-looking transform
6		
7	c	Hexagone mapping to multi-lines pattern?
8	i	Lot of squares, and the pattern in (i) looks like the square pattern of the course
9	h	3-D forms (rings) could map to the "rings-like" pattern on the transform
10 (3)	b/g	Same as 3

2) Hadamard transform

a) Compute transform

$T = G \cdot I \cdot G$

$$= \frac{1}{2}\begin{pmatrix} 1&1&1&1 \\ 1&1&-1&-1 \\ 1&-1&1&-1 \\ 1&-1&-1&1 \end{pmatrix} \cdot \begin{pmatrix} 3&7&3&7 \\ 1&3&7&3 \\ 3&1&3&1 \\ 7&3&7&1 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1&1&1&1 \\ 1&1&-1&-1 \\ 1&-1&1&-1 \\ 1&-1&-1&1 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 14&20&32&12 \\ -6&0&0&8 \\ -2&-4&4&4 \\ 6&12&0&4 \end{pmatrix} \cdot \begin{pmatrix} \cdots \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 78&-10&14&-26 \\ 2&-14&-14&2 \\ 2&-14&2&2 \\ 22&14&-10&-2 \end{pmatrix}$$

* Inv. done in software

b) Compute inverse

With $T = GIG$, we expect $I = G^{-1}GIGG^{-1} = G^{-1}TG^{-1}$

Compute G^{-1}: with $G = \frac{1}{2}\begin{pmatrix} 1&1&1&1 \\ 1&1&-1&-1 \\ 1&-1&1&-1 \\ 1&-1&-1&1 \end{pmatrix}$, $G^{-1} = \frac{2}{1} \cdot \frac{1}{4}\begin{pmatrix} 1&1&1&1 \\ 1&1&-1&-1 \\ 1&-1&1&-1 \\ 1&-1&-1&1 \end{pmatrix}^*$

$$\hat{I} = G^{-1}TG^{-1} = \frac{1}{2}\begin{pmatrix} 1&1&1&1 \\ 1&1&-1&-1 \\ 1&-1&1&-1 \\ 1&-1&-1&1 \end{pmatrix} \cdot \frac{1}{4}\begin{pmatrix} 78&-10&14&-26 \\ 2&-14&-14&2 \\ 2&-14&2&2 \\ 22&14&-10&-2 \end{pmatrix} \cdot \frac{1}{8}(H)$$

$$= \frac{1}{8}\begin{pmatrix} 104&-24&-8&-24 \\ 56&-24&8&-24 \\ 56&-24&40&-24 \\ 56&32&16&-32 \end{pmatrix} \cdot (H) \cdot \frac{1}{2}$$

$$= \frac{1}{16}\begin{pmatrix} 48&112&104&112 \\ 16&48&112&48 \\ 48&16&104&16 \\ 112&144&112&16 \end{pmatrix} = \begin{pmatrix} 3&7&3&7 \\ 1&3&7&3 \\ 3&1&3&1 \\ 7&3&7&1 \end{pmatrix}$$