Solutions

April 27, 2022

Solution to 1.1 (Knapsack Game)

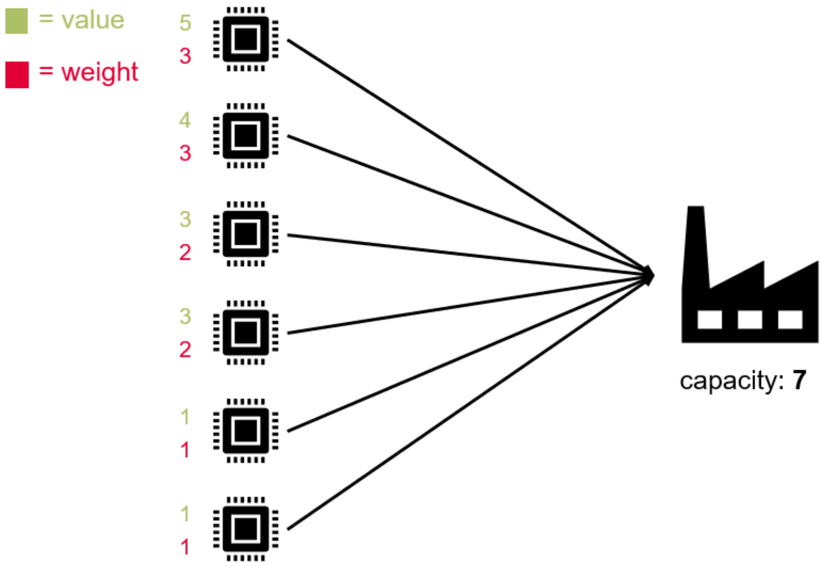

Figure 1: Knapsack instance

Any configuration that has total weight less than the capacity (7) would be acceptable as a possible configuration. The true optimum configuration has value=11 and weight=7, with selecting items 1,3, and 4 (which corresponds to (5,3),(3,2), and (3,2))

Solution to 1.2 (Weighting Factors)
The best option here is b) $A = 10$.
Recall:

$$E = A(1 - \sum_n y_n)^2 + A(\sum_i w_i x_i - \sum_n n y_n)^2 - \sum_i c_i x_i.$$

Here,

$$-\sum_i c_i x_i,$$

maximizes the value of our knapsack whereas the penalty terms,

$$A(1 - \sum_n y_n)^2 + A(\sum_i w_i x_i - \sum_n n y_n)^2,$$

ensure we have a feasible solution. Because the constant in front of

$$-\sum_i c_i x_i.$$

is 1 we need to make A larger than 1 so that it is never favorable to violate the penalty terms (which give us a feasible solution) in order to greater satisfy the value maximizing constraint. If we prioritize the value maximizing constraint over the penalty constraints we could end up with an infeasible solution. In order to ensure the penalty terms get priority we must have $A > max(c_i)$. Here the maximum value is 5, so any choice of A with $A > 5$ will work. Now, we can eliminate choice a) $A = 1$. Both b) $A = 10$ and c) $A = 10000$ are feasible choices for A. However, b) $A = 10$ is the best option because with c)$A = 10000$, the penalty constraints will be so much greater than the value maximizing constraint that the optimizer could start to be less sensitive the value maximizing constraint.

Solution to 2.1 (Variable Assignment - Pen and Paper)
$x_{0,0} = 0$
$x_{0,1} = 1$
$x_{0,2} = 0$
$x_{1,0} = 0$
$x_{1,1} = 0$
$x_{1,2} = 1$
$x_{2,0} = 1$
$x_{2,1} = 0$
$x_{2,2} = 0$

Solutions to 2.2-2.7 (QUBO Formulation Code)
See homework_tsp_solutions.ipynb for the solution code.