

# PRECISION MEASUREMENTS OF $^{20}\text{F}$ BETA DECAY

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## A DISSERTATION

Submitted to  
Michigan State University  
in partial fulfillment of the requirements  
for the degree of

2019

## ABSTRACT

### PRECISION MEASUREMENTS OF $^{20}\text{F}$ BETA DECAY

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The beta decay of  $^{20}\text{F}$  is an attractive low-energy probe for new physics. It is an allowed Gamow-Teller decay, which has a parameter is sensitive to beyond the standard model tensor couplings in the weak interaction. The parameter sensitive to these interactions is the Fierz term  $b_{WM}$ . A beta spectrum shape measurement was done to measure the Fierz term in  $^{20}\text{F}$ . The shape measurement is incomplete. There is uncertainty in how to deal with the effect of the absorption of the gamma ray in the implant detector. Once this effect is resolved, the uncertainty on the measurement will be  $0.0042_{stat} \pm 0.0066_{sys}$ .

In addition to the shape measurement, a half-life measurement was done,. Previous measurements were inconsistent. The half-life was measured as  $11.0011(69)_{stat}(30)_{sys}$  s. This is the most precision measurement of the half-life of  $^{20}\text{F}$ , and this result has been confirmed by an additional measurement [5].

## **ACKNOWLEDGMENTS**

To everybody who helped with me

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# Chapter 1

## Introduction

The goal of this measurement is to test the standard model. For precision measurements, careful measurement of a parameter thought to be zero is done. If a non-zero result is found, new physics is discovered. The usual window for low energy precision measurements is through beta decay. These measurements are not the only method to look for new physics.

The most direct way to look for new physics is to look for the new particles that generate that physics. One is to look for new particles with high energy experiments. The direct detection of new particles is very general and a powerful technique. Precision measurements at high energies with colliders often done as well.

Low energy precision measurements, on the other hand, are much more specialized. Precision measurements are sensitive to only certain channels of new physics. Care must be taken that the channel of physics is not covered by collider experiments already. For nuclear physics searches, this is done by looking at beta decay, which provides a sensitive probe to several avenues of new physics.

### 1.1 Beta Decay

Beta decay is one of the processes by which unstable nuclei lose energy. The general process is

$${}^A_Z P \rightarrow {}^A_{Z\pm 1} D + e^\pm + \nu_e \quad (1.1)$$

with  ${}^A_Z P$  being the parent nucleus,  ${}^A_{Z\pm 1} D$  being the daughter nucleus,  $e^\pm$  is the outgoing electron or positron, and  $\nu$  is an outgoing neutrino. The neutrino is an anti-electron neutrino in the case of beta<sup>-</sup> decay, and an electron neutrino in the case of beta<sup>+</sup> decay. A related process is electron capture, where a proton in the nucleus captures an inner electron and turns into a neutron.

The simplest type of nuclear beta decay is allowed beta decay. There are two types of allowed beta decay, which differ in angular momentum  $J$  and isospin  $T$  selection rules. The two types are called Fermi and Gamow-Teller beta decay. For a Fermi transition, the change of the angular momentum  $J$  and the change of the isospin  $T$  is both zero. For a Gamow-Teller transition, the change in the angular moment  $J$  is 0 or  $\pm 1$  and the change of the isospin  $T$  is 0 or  $\pm 1$ . However, a Gamow-Teller transition cannot cause a transition between two states of  $J = 0$ . These are called super-allowed beta decays. There are also mixed allowed transitions, where both matrix elements of Fermi decay and Gamow Teller decay contribute. In order to see what physics beyond the standard model beta decay measurements are sensitive to, a closer look at beta decay is needed.

### 1.1.1 Microscopic View of Beta Decay

At a microscopic level, the process of beta decay involves one of the quarks inside the nucleon emitting a  $W$  particle. This quark changes flavor, and the nucleon changes as well. The microscopic view of beta decay (for the beta<sup>-</sup> case) is shown in figure 1.1.

A down quark in a neutron goes to an up quark. This process emits a  $W^-$  boson that

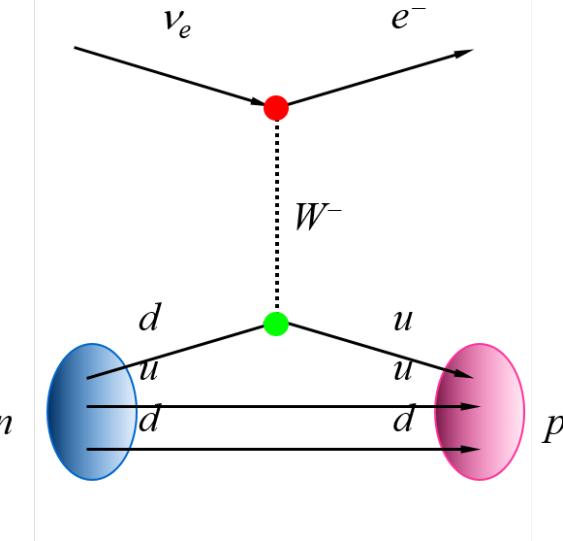


Figure 1.1: What beta<sup>-</sup> decay looks like microscopically

decays into an electron and an anti-electron neutrino. This means that any beta decay measurements are probing the weak sector. These measurements are complimentary to high energy measurements of weak interactions. For this work, a nucleon level treatment of beta decay will suffice.

### 1.1.2 Matrix Elements of Beta Decay

In order to calculate the beta decay rate, Fermi's golden rule is needed. This is

$$\lambda = \frac{2\pi}{\hbar} \|M\|^2 \rho(E) \quad (1.2)$$

The density of states,  $\rho(E)$ , will be discussed further in the thesis. This matrix element  $M$ , to first order, is [14]

$$|M|^2 = \xi [1 + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b \frac{m_e}{E_e} + \frac{<\vec{J}>}{J} \cdot (A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} + D \frac{\vec{p}_e \times \vec{p}_\nu}{E_e E_\nu})] \quad (1.3)$$

$\xi$  is written out to linear order is

$$\xi = \frac{1}{2} |M_F|^2 |C_V + C'_V|^2 (1 + |\rho|^2) \quad (1.4)$$

For equation 1.4,  $M_F$  is the Fermi matrix element,  $C_V$  and  $C'_V$  are vector coupling constants, and  $|\rho|$  is the ratio of the Gamow-Teller matrix element to the Fermi matrix element. [14]

Going back to equation 1.3,  $\langle \vec{J} \rangle$  is the average total angular momentum. The constants  $a$ ,  $b$ ,  $A$ ,  $B$ , and  $D$  can be written in terms of the coupling constants as well.  $a$  depends on the ratio of the Fermi and Gamow-Teller matrix elements  $\rho$ . This form of the observables informs the experimental design.

## 1.2 Types of Precision Measurements in Beta Decay

Given the form of equation 1.3, to get at different terms, different types of experiments are needed. A polarized nucleus is need to extract  $a$ . The nuclear spin points in one direction. Then, by angular momentum conservation,  $\vec{p}_e \cdot \vec{p}_\nu$  can be found. Other correlations between energy and momenta have sensitivity to other terms in equation 1.3. Equation 1.3 is not complete, and many other terms exist.

For an unpolarized beam where only the energy of the electron is measured, the momenta are averaged over and all terms disappear expect for  $b$ . There is also the phase space factor. There are two general kinds of unpolarized beta decay measurements. The first is where the differential energy spectrum is measured. There is a measurement of  $b$  over the entire range of values, and the  $\frac{1}{W}$  dependence of the matrix element is probed. To do this measurement, the beta decay in question must be available enough in order to get enough statistics for a good spectrum. Other requirements for a spectrum shape measurement are described in the

next chapter.

If the nucleus is rare, another way to get at the physics in equation 1.3 is to measure an  $ft$  value. The  $f$  is the integral of the decay rate. The  $t$  is the half-life of the nucleus times the branching ratio. These are both measured and multiplied together to get a quantity that is proportional to the matrix element. This can be done even if the nucleus in question is difficult to produce. For each measurement,  $b$  is measured, but modulated by an average value of the energy dependence. Multiple measurements are needed to get good constraints on  $b$ .

In both of these techniques, electromagnetic and hardonic corrections are needed. To measure the  $ft$  values, the integral of these corrections are important. For the shape measurements, the energy dependence of the corrections are important. The systematic uncertainties for both of the techniques are different. These depend on the exact details of the experiment.

The term  $b$  is known and the Fierz interference term. A measurement of this term is the ultimate goal of the experiment described in this thesis. A more detailed discussion of the term is needed.

### 1.2.1 Fierz Term

The Fierz term,  $b$ , in equation 1.3, can be rewritten in terms of effective couplings. This is

$$b = \pm \sqrt{1 - \alpha^2 Z^2} \frac{1}{1 + \rho} \text{Re} \left( \frac{C_S + C'_S}{C_V + C'_V} + |\rho|^2 \frac{C_T + C'_T}{C_A + C'_A} \right) \quad (1.5)$$

where  $\rho$  is the ratio of the Gamow-Teller matrix element to Fermi matrix element,  $\alpha$  is the QED fine structure constant [14] The subscripts of  $C$  indicate which object the coupling

corresponds to. Here,  $A$  means axial vector,  $V$  stands for polar vector,  $S$  stands for scalar, and  $T$  stands for tensor. The  $C$  coefficients correspond to parity conserving interactions, and the  $C'$  coefficients correspond to parity non-conserving coefficients [22] This means, that in a pure Fermi decay, the Fierz term is sensitive to any non-standard scalar term, while in a pure Gamow-Teller decay, the Fierz term is sensitive to any non-standard Tensor terms.

The Fermi decays have been measured using super-allowed beta decays. For most of these measurements, however, decay of interest happens very rarely. The  $ft$  value was measured.  $ft$  values give the value of the Fierz term averaged over the beta decay energy spectrum. Several measurements of different decays are needed to get a good constraint. The other part of the Fierz term is the tensor coupling. In order to get a good measurement of tensor couplings, an allowed Gamow-Teller decay must be used. The nucleus for this measurement was  $^{20}\text{F}$ . This measurement was done carefully to compete with high energy probes.

### 1.2.2 High Energy Probes of the Fierz Term

There have been probes sensitive to the Fierz term done with high energy techniques [14]. The precision of these measurements is important to keep in mind, as this is the precision goal of the beta decay measurement. Several high-energy channels are used, and the coupling charges are directly measured. To compare the results of the channels to a precision beta decay measurement, the Fierz terms needs to be rewritten. In the case of a Gamow-Teller transition, the Fierz term in equation 1.5 can be re-written to

$$b_{GT} = \pm 2\sqrt{1 - \alpha^2 Z^2} \operatorname{Re}\left(\frac{C_T + C'_T}{C_A + C'_A}\right) \quad (1.6)$$

In terms of the quark level coupling constants, this can be rewritten as [14]

$$b_{GT} = \pm 2\sqrt{1 - \alpha^2 Z^2} Re\left(\frac{8g_t\epsilon_t}{2g_A}\right) \quad (1.7)$$

with  $g_t$  being the tensor charge,  $\epsilon_t$  the tensor coupling and  $g_A$  the axial vector charge. This is assuming the only physics beyond the standard model is in  $\epsilon_t$ . The value of  $g_t = 0.987(55)$  and  $g_A = 1.278(33)$  [14]. This means, for  $^{20}\text{F}$ , the sensitivity to  $b_{GT}$  in terms of  $\epsilon_t$  is given by

$$b_{GT} = \pm 6.2 * Re(\epsilon_t) \quad (1.8)$$

This can be used to describe the ultimate sensitivity needed.

For the high energy probes, the Large Hadron Collider collides two protons and looks at the output. The experimental signature for the LHC is missing transverse energy. The energy of the output particles is compared to that of the standard model background, and the difference recorded. To probe the same physics as beta decay, one or more of the outgoing particles are electrons or positrons. For events with one electron and other particles, the constraint on  $\epsilon_t$  is  $|\epsilon_t| < 1.3 \times 10^{-3}$ . Looking at neutral currents, where the output particles include an electron and a positron, gives stronger constraints. There, the constraint is  $|\epsilon_t| < 0.6 \times 10^{-3}$ . To be competitive with the stronger constraint from high energy experiments, the Fierz term  $b_{GT}$  should be measured to better than  $3.7 \times 10^{-3}$ .

In order to achieve such a measurement, the technique used was a shape measurement technique. The next chapter will describe why this method works with a measurement of  $^{20}\text{F}$ .

# Chapter 2

## Properties of $^{20}\text{F}$ Decay

For a potential beta energy spectrum shape measurement, several criteria for a nucleus are needed. The first is that the nucleus and the decay mode must be available in sufficient quantities that a proper decay spectrum shape can be measured. This means that the nucleus needs to be relatively close to stability. The decay mode needs to be clean. If there are several competing decay modes of similar strength, the shape of the beta decay spectrum gets complicated. The same thing happens if there are several gamma rays in the decay. Having one gamma ray is useful, as a coincidence measurement can be used to exclude much of any background spectra in the measurement. In order to get a useful theoretical interpretation of the measurement of the Fierz term, a purely Fermi or Gamow-Teller transition is needed. The superallowed decays are purely Fermi, so the limit on any potential scalar couplings is strong. An allowed Gamow-Teller transition gives sensitivity to tensor couplings. A nucleus that fulfills these criteria is  $^{20}\text{F}$ .

### 2.1 $^{20}\text{F}$ Decay Characteristics

The decay scheme is given in figure 2.1. As seen in the figure,  $^{20}\text{F}$  decays 99.99% of the time to the first excited state of  $^{20}\text{Ne}$ . This decay is very clean, as there are very few contaminants from other decay branches. The  $2^+$  state seen in the decay scheme is not the isobaric analogue state to the ground state of  $^{20}\text{F}$ . That state is much higher in energy.

The beta decay therefore has an isospin change of 1. This means that the allowed Fermi matrix element is zero, and the decay is an allowed Gamow-Teller transition. The forbidden transitions only contribute to higher order. The half-life of  $^{20}\text{F}$  is about 11 seconds. This means waiting for the  $^{20}\text{F}$  to decay will take less than a minute to get good statistics.

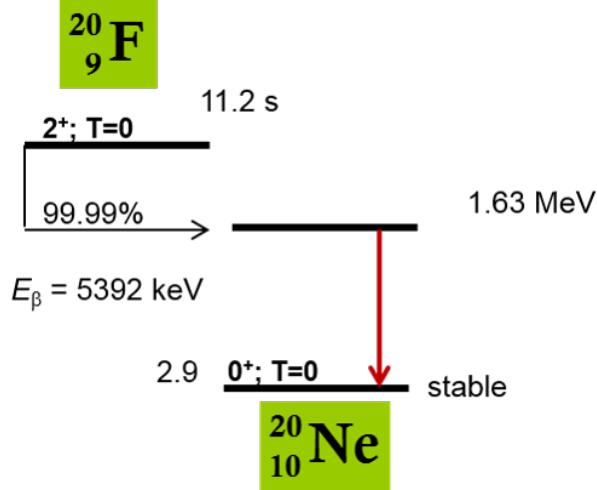


Figure 2.1: The decay scheme of  $^{20}\text{F}$ .

The 1.6 MeV gamma ray in the transition allows for a coincidence measurement to take place. One gamma ray does not distort the spectrum enough to cause any issues.  $^{20}\text{F}$  is relatively close to stability. It can be made in sufficient qualities to get a high statistics beta spectrum. Because of these characteristics, this measurement was not the first to use  $^{20}\text{F}$ .

## 2.2 Previous Measurements in $^{20}\text{F}$ Decay

Some of the earliest measurements of  $^{20}\text{F}$  were  $ft$  value measurements [33] [2]. The half-lives were measured with a plastic-scintillator in either case. These  $ft$  values were compared to the mirror nucleus  $^{20}\text{Na}$  in order to search for second class currents in beta decay.

Another, more recent measurement whose goal was putting limits on second class currents was done with a polarized  $^{20}\text{F}$  and  $^{20}\text{Na}$  beam [24]. The technique was an alignment

measurement over different energies instead of an  $ft$  value. The  $^{20}\text{F}$  was made with a polarized deuteron beam impinging on a  $^{19}\text{F}$  target. The direction of the outgoing electron was measured and recorded. The angular distribution was measured using the beta-nuclear magnetic resonance (NMR) technique. The correlations were corrected for and plotted vs energy. The outcome of that analysis was a measurement of the various form factors that correct the beta decay spectrum shape. The resulting  $^{20}\text{F}$  energy spectrum is shown in figure 2.2 In the energy spectrum in figure 2.2, there are several background spectra. Since this measurement was an asymmetry measurement, the influence of the backgrounds are reduced. Different polarization were used to eliminate systematic effects. Double ratios were taken to account for difference in solid angle and detector efficiency. The threshold was taken to be 2 MeV, as below that energy, the spectrum is distorted.

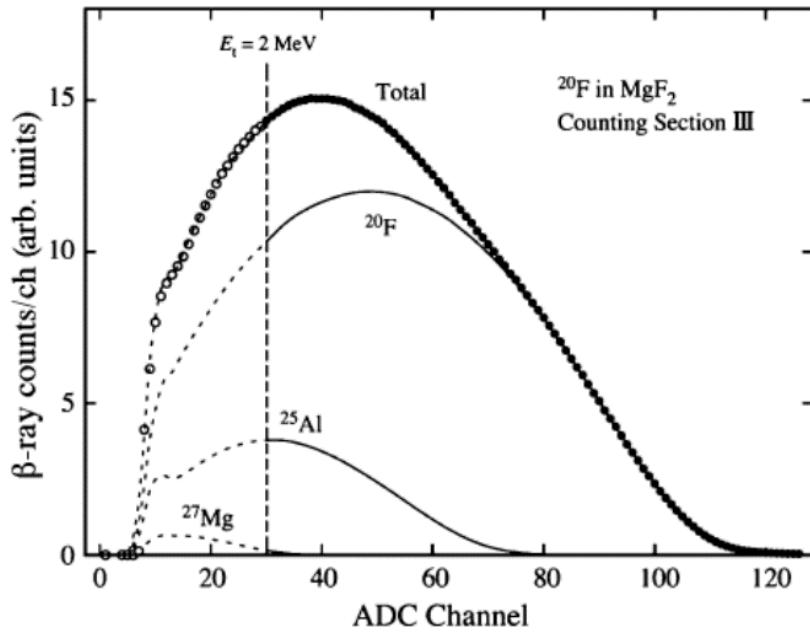


Figure 2.2: A sample beta energy spectrum using a beta-NMR technique [24]

There are several important systematic effects in this measurement. The two largest for the  $^{20}\text{F}$  measurement have to do with corrections for the solid angle and corrections for the

1.634 MeV gamma ray. The corrections for the solid angle have to do with estimates of the scattering of the electrons. This happens at the implantation target, and can happen on the walls of the spectrometer. To counteract this effect, a veto detector was included in the detector set up. This veto detector was situated near the walls of the magnet, so any scattering off of the magnet could be vetoed. The other large systematic effect has to do with gamma rays. Since the  $^{20}\text{F}$  is polarized, the direction of the emitted gamma rays is anisotropic. The gamma rays contribute and pile-up with the detected gamma spectrum in different ways. This has to be corrected for with the detector response. With all these effects, the second class current effects were found to be consistent with zero.

### 2.2.1 Shape Measurements

Several direct shape measurements have been made in  $^{20}\text{F}$ . Two measurements with different systematic effects will be discussed. Both of them looked for the linear term in the hardonic correction to the beta decay spectrum shape.

A similar measurement of the  $^{20}\text{F}$  spectrum shape was done with a spectrometer [8]. The goal was to measure the nuclear shape factor and get a measurement of the linear term. This shape factor will be described further in a later chapter. The ultimate goal was to test the conserved vector current (CVC) hypothesis. For this experiment, the direction of the outgoing electrons was not recorded.  $^{20}\text{F}$  was created with a deuteron beam on a  $^{19}\text{F}$  target, much like the previous experiment. The target was  $^6\text{LiF}$  on a carbon foil. After the target was bombarded, and the  $^{20}\text{F}$  created, the outgoing electron went through the foil and into the spectrometer. The spectrometer was tuned to accept a certain energy of electrons. Then, data on the decay of  $^{20}\text{F}$  at different energies were collected. A sample energy spectrum is shown in figure 2.3.

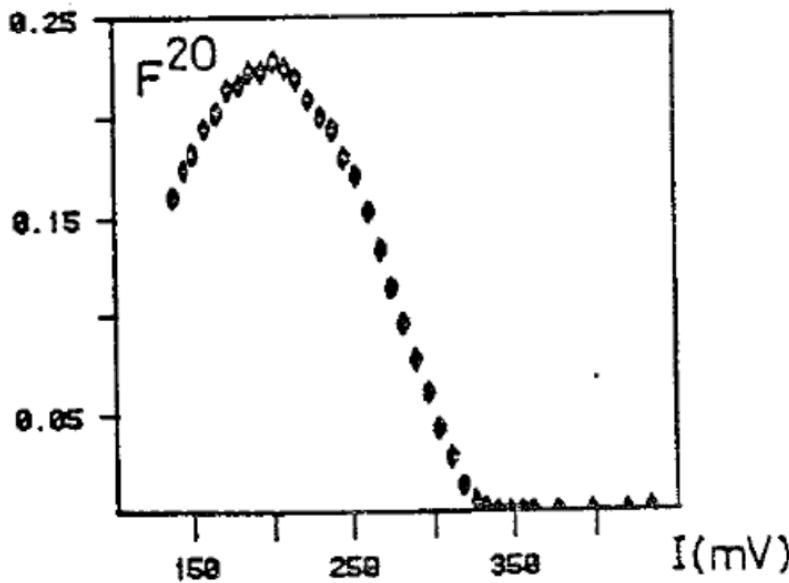


Figure 2.3: A sample beta energy spectrum using a spectrometer [8]

An important consideration is the normalization of each point. For each spectrometer setting, the amount of measurement time, the acceptance of the spectrometer, and the source activity have to be accounted for. To help with the normalization, a beta telescope near the target was used. This measures the activity of the source, and takes into account the degradation of the target. However, there are still issues with adjusting for the efficiency of the beta telescope. This efficiency changes as the spectrometer's field is changed. The location of the telescope was optimized to minimize this effect. Another way of normalization was to continuously adjust the spectrometer field so that the entire energy range is sampled over each measurement cycle. The issue there is that changing the magnetic fields induces currents, which are hard to correct for. In this measurement, adjusting the spectrometer field was used as a benchmark for the beta telescope normalization method.

An issue with this measurement is that the shape factor is not just the linear term. The terms depend on the same nuclear form factors and are not independent. The other terms

were calculated and fixed. The shape factor will be discussed in detail in the next chapter.

Ultimately the resulting measurement of the linear term matched the CVC prediction

Using a single detector would take care of the normalization automatically. A shape measurement of  $^{20}\text{F}$  was made with a high purity germanium detector [16]. The linear term in the shape factor was measured just like in the previous measurement. A spectrometer was used with a high purity germanium detector. The magnetic field was used to enhance the solid angle for the electrons compared to the gamma rays. The  $^{20}\text{F}$  was made with a deuteron beam on a natural LiF target. An implant and decay cycle was used to measure the beta decay spectrum. A calibration beta spectrum is shown in figure 2.4.

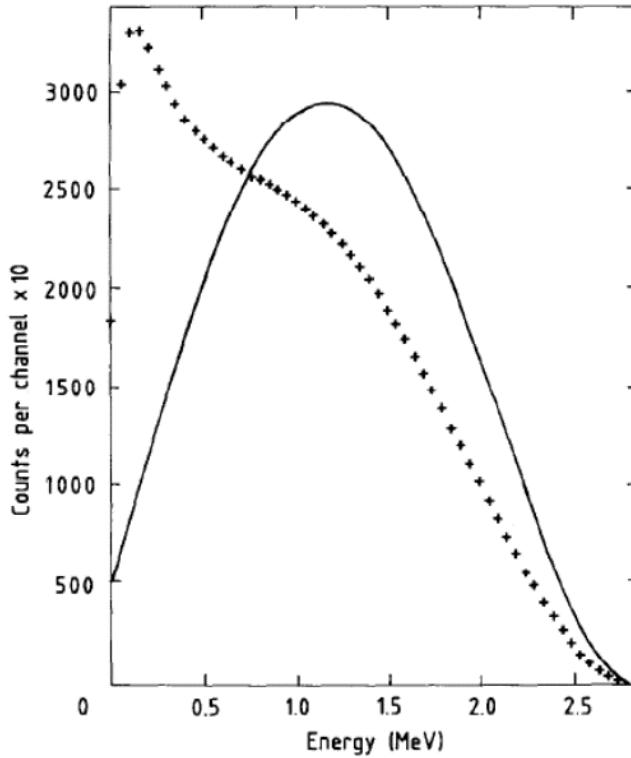


Figure 2.4: A beta spectrum using a scintillator [16]. The line shows the statistical beta spectrum. The crosses show the measured beta spectrum. This spectrum shows the back scattering effect

As seen from figure 2.4, there is a large distortion at low energies. This is due to back

scattering inside the germanium detector. What happened is that electrons entered the detector, deposited some of their energy, and bounced back out. This caused a large distortion at low energy. Another issue for this measurement was that energy calibration was non-linear. Below 1.5 MeV, there was a non-linearity up which caused a distortion up to 35 keV. This all had to be accounted for and contributed to the uncertainty. The resulting energy spectrum and fit is shown in figure 2.5.

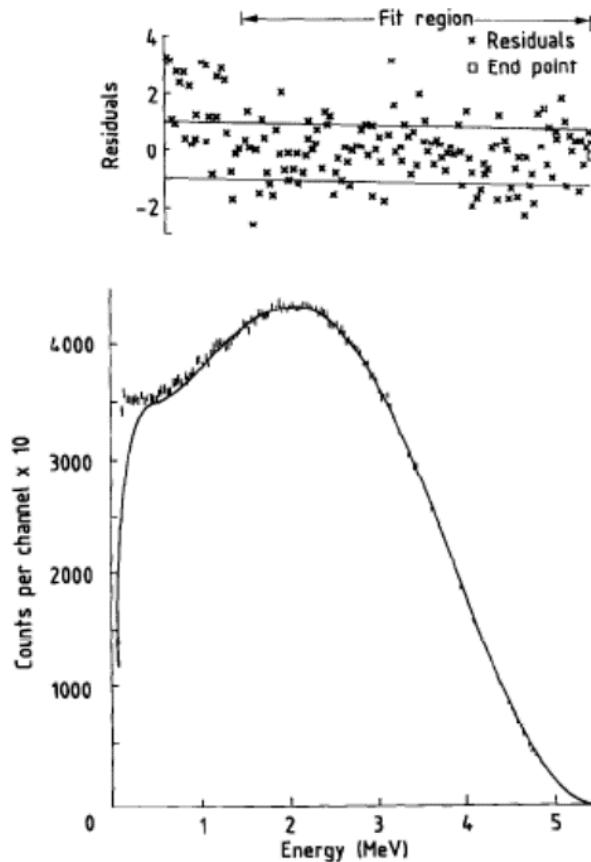


Figure 2.5: The measured and fitting  $^{20}\text{F}$  spectrum [16]

The beta spectrum is greatly distorted. Due to contamination, the effective shape measurement analysis was started at 1.5 MeV. For this measurement there were several systematic effects. The largest uncertainty was due to uncertainties in the response function. The paper refers this to "peak position." This experiment used many different sources to get a

calibration. They also had to take the bremsstrahlung of the electrons into account. That systematic effect was the limiting factor in their measurement

Much like the spectrometer measurement, the other terms of the shape factor were fixed. Here however the other terms were calculated as a systematic error. It was found that the quadratic term had more of an effect in the measurement than the  $1/W$  term. Ultimately, the measured linear term was consistent with the CVC hypothesis.

The current work is also was a shape spectrum measurement, much like the two cited above. The ultimate goal is however a determination of the Fierz term. In order to get at the Fierz term, the beta energy spectrum must be described precisely. The next chapter deals with the corrections to the beta decay spectrum.

# Chapter 3

## Theoretical Description of the Beta Energy Spectrum

To obtain a measure of the Fierz term, the beta energy spectrum must be precisely described.

The description of the beta energy spectrum is written as a series of corrections multiplying the main phase space factor. Through out this chapter,  $\hbar = c = 1$ .

The beta energy spectrum shape can be broken up in three different sections. The three terms are written out as

$$Q(E) = PS(E) * C(E) * \left(1 + b_{GT} \frac{m_e}{E}\right) \quad (3.1)$$

where  $Q(E)$  is the total beta decay energy spectrum,  $PS(E)$  is the phase space of the electron,  $C(E)$  are all the hadronic and electromagnetic corrections multiplied together, and  $b_{GT}$  the Fierz term. This chapter will describe all that goes into the determining the beta energy spectrum.

### 3.1 Experimental Inputs to Describe the Beta Spectrum

The corrections depend on several parameters of the decay. Some of them are just numbers, such as the atomic number of the daughter or mother nucleus or the mass number of the system. However, there are two important parameters that are experimental measurements.

One is the Q-value of the decay, which is defined as

$$Q = m_{20F} - (m_{20Ne} + E_{level}) \quad (3.2)$$

where  $Q$  is the q-value,  $m_{20F}$  is the nuclear mass of  $^{20}\text{F}$ ,  $m_{20Ne}$  is the nuclear mass of  $^{20}\text{Ne}$ , and  $E_{level}$  is the energy of the 1.6 MeV nuclear state in  $^{20}\text{Ne}$ . The tabulated masses for these nuclei are the atomic mass,  $M$ . The masses of the electrons have to be subtracted off. For  $^{20}\text{F}$ ,  $M_{20Ne} = 19.9924401762(17)\text{amu}$ ,  $M_{20F} = 19.999981252(31)\text{amu}$  [29], and  $E_{level} = 1.633674(15)\text{MeV}$  [27]. However, to get the total energy,  $m_e$  has to be added to the difference of atomic masses.

This mass difference is not quite the maximum electron energy. To get to the maximum electron energy,  $E_0$ , the energy of the recoiling nucleus has to be taken into account. The formula is

$$E_0 = Q \frac{1 + \frac{m_e^2}{2MQ}}{1 + \frac{Q}{2M}} \quad (3.3)$$

with  $m_e$  is the electron mass and  $M$  is the average nuclear mass [18].

The other parameter is the charge radius of the daughter nucleus. There are several ways to calculate the charge radius. In this work, the charge radius was taken from the measured

root mean square (RMS) charge radius and converted. It was assumed that the  $^{20}\text{Ne}$  nucleus was a sphere. From this, the radius was calculated using

$$r = \sqrt{\frac{5}{3}} r_{rms} \quad (3.4)$$

where  $r_{rms}$  is the root mean square charge radius, and  $r$  the charge radius. For this work,  $r_{rms} = 3.0055(21)$  [3]

With the two parameters, the q-value and the phase space, the shape of all beta decay spectra can be described.

## 3.2 Phase Space Factor

The main part of the beta energy spectrum is the phase space factor. It is

$$\frac{dN}{dE} = C * p_e W (W_0 - W)^2 \quad (3.5)$$

were  $C$  is a constant,  $p_e$  is the electron momentum,  $W$  the total electron energy, and  $W_0$  the maximum electron energy. This is derived from the density of states of the particle. The constant has factors that come from the integration This comes in from Fermi's Golden Rule, which is shown in equation 1.2.

The matrix element is shown in equation 1.3. The main effect on the spectrum shape originates from the density of states. The number of states for both the electron and the neutrino is

$$N = \frac{1}{(2\pi\hbar)^6} \int dr_e^3 \int dp_e^3 \int dr_{\nu e}^3 \int dp_{\nu e}^3 \quad (3.6)$$

where  $r$  and  $p$  correspond to the position and momentum of the electron ( $e$ ) and the neutrino ( $\nu_e$ ). Doing the integral over both  $r$ 's give two factors of the volume, which disappear when the next factor is introduced. Since the neutrinos are unmeasured, the momentum of the neutrinos is integrated over. Assuming spherical symmetry, the new integral is

$$dN = \frac{V^2}{4\pi^4 \hbar^6} p_e^2 dpe p_{\nu_e}^2 dp_{\nu_e} \quad (3.7)$$

This is simplified by approximating the neutrino as mass-less. The momentum of the neutrino is then equal to the energy of the neutrino. Then, the total energy  $E$  is written as a sum of the neutrino energy (or momentum) and the beta energy. Plugging that in gives

$$dN = \frac{V^2}{4\pi^4 \hbar^6} (E - E_e)^2 p_e^2 dpe \quad (3.8)$$

after integrating over the neutrino degrees of freedom. Then, the only thing left to do to recover equation 3.5 is to rewrite  $dp$  in terms of  $dE$ . The energy  $E_e$  has been replaced with  $W$ , which is the energy of the electron divided by the electron mass, and  $E$  has been replaced by the Q-value. Since the measurement is not an absolute one, but only concerned with the shape of the spectrum, the normalization factor is arbitrary.

This is the main factor in describing the beta decay energy spectrum.

### 3.3 Variables of the Correction Factors

For the rest of the discussion of the beta decay corrections, everything is given without units. These corrections are largely electromagnetic in origin. All of the energies are divided by the electron mass. For this work,  $m_e = 0.510998928$  MeV The other important variables are

shown in table 3.1

Table 3.1: Variables used in the corrections

Variable	Description	Equation or Value
$A$	Mass number	20
$Z$	Atomic number of daughter	10
$\alpha$	Fine structure constant	$1/137.0359$
$R$	Root mean square charge radius of daughter	3.055 fm [3]
$W$	Total energy of the electron	$E/m_e$
$p$	Electron momentum	$\sqrt{W^2 - 1}$
$W_0$	Maximum electron energy	$\frac{E_0}{m_e}$
$M_{ave}$	Average nuclear mass	$\frac{M_{20Ne} + M_{20F} - 19m_e}{2m_e}$
$\gamma$	Special relativity $\gamma$ factor	$\sqrt{1 - (\alpha Z)^2}$

## 3.4 Electromagnetic Corrections and Hardonic Corrections

A graph of the various electromagnetic corrections are shown in figure 3.1, along with the hadronic shape factor.

### 3.4.1 Fermi Function

The largest correction by over all size is the Fermi function. This accounts for the interaction of the charge of the outgoing electron and the charge of the nucleus. It is calculated by taking the Dirac equation wave functions and assuming the nucleus is a point charge of  $Ze$ . The Dirac wave functions are taken down to a nuclear radius  $R$ . This is since the wave functions diverge [34].

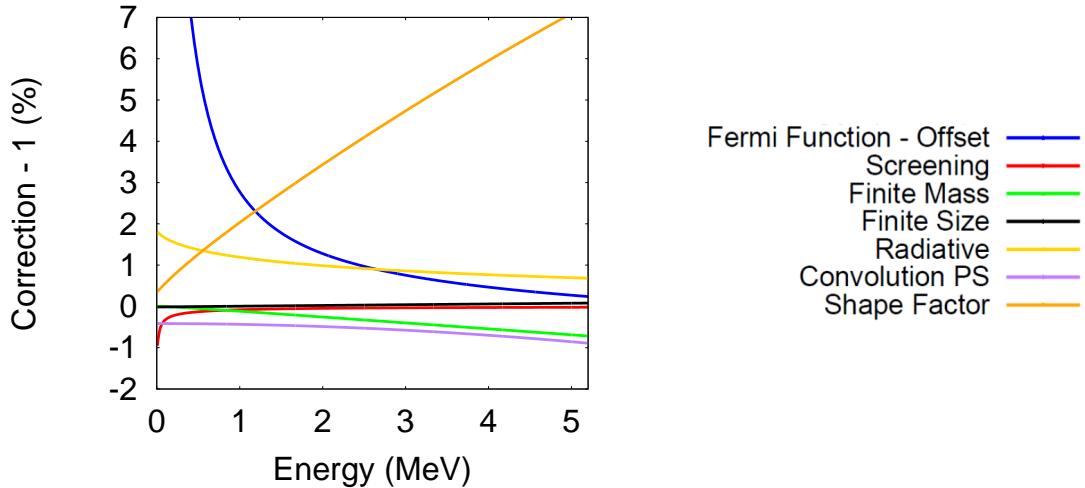


Figure 3.1: Electromagnetic and hardonic corrections for  $^{20}\text{F}$

The Fermi function is

$$F(Z, W) = 2 \frac{\gamma + 1}{\Gamma(2\gamma + 1)^2} (2pR)^{2(\gamma-1)} e^{\frac{\pi\alpha ZW}{p}} \left\| \Gamma\left(\gamma + i\frac{\alpha ZW}{p}\right) \right\|^2 \quad (3.9)$$

While this is the largest correction, it is the most understood one. There is no change of shape due to the uncertainty in  $R$  because it alters as a factor the functional form of the Fermi function.

### 3.4.2 Radiative Correction

The radiative correction is the next largest electromagnetic correction. For this measurement, the correction is only needed to first order, which is on order  $\alpha$ . The correction is a QED correction that stems from photons emitted from the beta particle. This photon can be absorbed by the nucleus of the daughter, which makes it a virtual photon. The photon can also be a real photon that propagates out off to infinity. The real photons are known as inner bremsstrahlung.

There are different descriptions of the radiative correction. The first is by Sirlin [26] and is

$$\begin{aligned} R(W, W_0) = & 1 + \frac{\alpha}{2\pi} [3 \ln(M) - \frac{3}{4} + 4(\frac{\operatorname{arctanh}(\beta)}{\beta} - 1) * (\frac{W_0 - W}{3W} - \frac{3}{2} + \ln(2(W_0 - W))) \\ & + \frac{4}{\beta} L(\frac{2\beta}{1+\beta}) + \frac{\operatorname{arctanh}(\beta)}{\beta} * (2 * (1 + \beta^2) + \frac{(W_0 - W)^2}{6W^2} - 4 \operatorname{arctanh}(\beta))] \end{aligned} \quad (3.10)$$

with  $\beta = \frac{p}{W}$ ,  $M$  the proton mass, and  $L(\frac{2\beta}{1+\beta})$  referencing to the Spence function, which is [36]

$$L(x) = \int_0^x \frac{\ln(1-t)}{t} dt \quad (3.11)$$

The Sirlin formula assumes that the inner bremsstrahlung photons are not detected at all. This is mostly true if the source of the beta decay is centered outside of a detector. However, if the source is implanted inside of the detector, such as it is in this experiment, some of the inner bremsstrahlung is absorbed.

If all the inner bremsstrahlung is absorbed, a different form of the first order radiative correction is needed. There are many equivalent forms of this, but the one that was used for this experiment was by Fayans [9]. The form of this radiative correction is

$$\begin{aligned} R(W, W_0) = & 1 + \frac{\alpha}{\pi} [(\frac{2}{\beta} \ln(\frac{2\beta}{1+\beta}) + \frac{7}{8\beta} + \frac{3\beta}{8}) \ln(\frac{1+\beta}{1-\beta}) \\ & - 2 \ln(\frac{4\beta^2}{1-\beta^2}) + \frac{4}{\beta} L(\frac{2\beta}{1+\beta}) + \frac{23}{8} + \frac{3}{2} \ln(M)] \end{aligned} \quad (3.12)$$

Unfortunately, the amount of inner bremsstrahlung absorbed depends on the geometry. A

more careful treatment of the radiative correction is needed.

### 3.4.2.1 Inner Bremsstrahlung

To first order, the energy spectrum of the inner bremsstrahlung photons is independent of  $Z$ . This is exactly like the two radiative corrections shown in equations 3.10 and 3.12. The spectrum is written out as [21]

$$\Phi(k, W_e) = \frac{\alpha p}{\pi p_e k} \left( \frac{W_e^2 + W^2}{W_e p} \right) \log(W + p) - 2 \quad (3.13)$$

where  $\Phi(k, W_e)$  is the probability density of seeing a photon of energy  $k$  from an electron of initial energy  $W_e$ . This equation was derived using outgoing waves from the Dirac equation in polar coordinates. This was calculated with the first order Born approximation. This means that at low energies, equation 3.13 is inaccurate. That can be seen, as the equation diverges as  $k$  goes to zero. If more orders of the approximation are added, this divergence can be controlled. These higher orders would correspond to emitting multiple photons. Each of these orders would have a probability reduced by a factor of  $\alpha$  compared to equation 3.13. The higher orders would be less significant, except at low energies. Since the probability density diverges, the higher orders could contribute. This would make the probability of emitting no photons finite.

However, another possibility is to add a cutoff. As long as the cutoff is high enough to be in the region where the first order Born approximation is valid, but low enough not to cut out gamma rays that are not full absorbed, equation 3.13 is usable. The cutoff used was 50 keV. This cutoff was checked using Monte Carlo simulation, and is within the region where the detector and the source geometry absorbs all gamma rays.

To quantify the effect of the inner bremsstrahlung, a GEANT4 simulation was used. Electrons were generated using the phase space and the radiative correction in equation 3.12. Then, for each electron, equation 3.13 was sampled and a photon generated. No other physics process was looked at in this simulation. The ratio consisting of the energy absorbed over the initial energy was the output of the simulation. That ratio is the effective efficiency of absorbing the inner bremsstrahlung photons. Multiplying that ratio by equation 3.12 gives the effective radiative correction. Comparing all three radiative corrections can be seen in figure 3.2

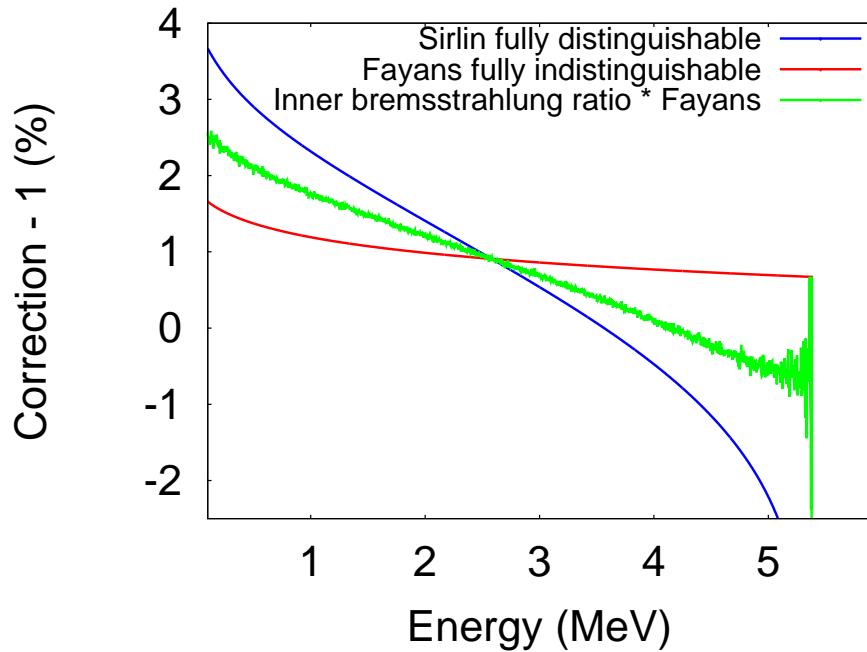


Figure 3.2: Comparison of three different radiative corrections. The green line depends strongly on the detector geometry.

The effect of the inner bremsstrahlung for a finite detector is to put the radiative correction halfway between the Sirlin and the Fayans formulas.

### 3.4.3 Screening

The screening correction is the next largest electromagnetic correction. This is a correction to the Fermi function. It corresponds to the screening of the nuclear charge due to the electron cloud of the atom. The strongest effect of the screening correction is at low electron energies and for large atomic numbers.

#### 3.4.3.1 Potentials Used in Screening Derivation

To calculate this correction, the Coulomb potential used to calculate the Fermi function is replaced with a Hulthén potential. This is potential is

$$V(r) = -\frac{\alpha Z \beta}{e^{\beta r} - 1} \quad (3.14)$$

where  $r$  is the distance away from the center and  $\beta$  is a parameter characterizing the diffuseness of the electron cloud. The ratio of this new Fermi function to the old Fermi function is the screening correction which multiplies the phase space.

The screening correction is negligible except for near the origin, while near the origin, equation 3.14 behaves as

$$V(r) = -\frac{\alpha Z}{r} + \frac{1}{2}\alpha Z \beta \quad (3.15)$$

and  $\beta$  is given by [4]

$$\beta = 2C(Z)\alpha Z^{\frac{1}{3}}m_e \quad (3.16)$$

Here, the  $Z$  is number the electrons of the daughter atom because the assumption is that

beta decay is occurring from neutral atoms. The only unknown is  $C(Z)$ .

To find the value of  $C(Z)$ , a comparison to another method of treating screening is needed. In another method of describing screening, the potential is described as a series of exponentials. This is written as [6]

$$V(r) = -\frac{\alpha Z}{r} \sum_n c_n e^{-b_n x} \quad (3.17)$$

where  $b_n$  and  $c_n$  are constants. The sum of all the  $c_n$  adds up to one. The value of  $x$  is [6]

$$x = 1.13\alpha Z^{\frac{1}{3}} r m_e \quad (3.18)$$

When equation 3.17 is expanded near the origin, the result is

$$V(r) = -\frac{\alpha Z}{r} + \alpha Z 1.13\alpha Z^{\frac{1}{3}} m_e \sum_n b_n c_n \quad (3.19)$$

By comparing equations 3.15, 3.16 and 3.19, the results obtained are

$$C(Z) = 1.13 \sum_n c_n b_n \quad (3.20)$$

For fluorine,  $n = 1$ ,  $c = 1$ , and  $b = 0.907$  [6].

### 3.4.3.2 Screening Correction Formula

From equation 3.20, the factor  $C(Z)$  in equation 3.16 is evaluated  $0.907 * 1.13 = 1.02491$ .

Equation 3.15 can then be used to calculate the screening correction. The result is [4]

$$Q(Z, W) = X \left( \frac{W'}{W} \right) \left| \frac{\Gamma(\gamma + iy')}{\Gamma(\gamma + iy)} \right|^2 \left| \frac{\Gamma(\gamma + 2i\frac{p'}{\beta})}{\Gamma(\gamma + 2i\frac{p}{\beta})} \right|^2 e^{-\pi y} \left( \frac{2p}{\beta} \right)^{2(1-\gamma)} \quad (3.21)$$

with  $y = \frac{\alpha Z W}{p}$ ,  $y' = \frac{\alpha Z W'}{p'}$ ,  $\gamma$  is in table 3.1,  $W' = W - \frac{1}{2}\alpha Z \beta$ , and  $p' = \frac{1}{2}p + \frac{1}{2}\sqrt{p^2 - 2\alpha Z W' \beta}$ .  
 $X(\frac{W'}{W})$  is

$$X = \frac{1 + \frac{W' + \gamma m}{W'} \frac{\beta^2}{p} + \frac{1}{2}\gamma^2 [1 + (1 - \frac{\alpha Z}{\beta(W+m)})^{\frac{1}{2}}]^{-2} \frac{W-m}{W'} \frac{\beta^2}{p} [1 - \frac{1-\gamma}{8\gamma} \frac{\beta^2}{p}]}{(1 + \frac{\beta^2}{4p})} \quad (3.22)$$

The factor  $X$  in equation 3.22 is very close to 1. This correction matters mostly at low energy, and is very flat above 100 keV.

### 3.4.4 Finite Mass Correction

The finite mass correction corrects for the recoil motion of the  $^{20}\text{Ne}$  nucleus. The form of finite mass corrections is given as [35]

$$R(W, W_0, M) = 1 + r_0 + \frac{r_1}{W} + r_2 W + r_3 W^2 \quad (3.23)$$

with  $M$  the nuclear mass of  $^{20}\text{Ne}$ . The forms of the  $r_i$  depend on if the decay is a vector decay or an axial decay. This is due to angular momentum conservation. Since  $^{20}\text{F}$  is an axial decay, the form of the finite mass correction is

$$\begin{aligned} r_0 &= -\frac{2W_0}{M} - \frac{W_0^2}{6M^2} - \frac{77}{18M^2} \\ r_1 &= -\frac{2}{3M} + \frac{7W_0}{9M^2} \\ r_2 &= \frac{10}{3M} - \frac{28W_0}{9M^2} \\ r_3 &= \frac{88}{9M} \end{aligned} \quad (3.24)$$

This effect is different than the hadronic shape factor described later.

### 3.4.5 Finite Size Correction

The finite size correction originates from treating the nucleus as a uniformly charged sphere instead of a point particle. This sphere has a radius of  $R$ , as shown in equation 3.4. The form of the finite size correction is [35]

$$L_0(Z, W) = 1 + \frac{13(\alpha Z)^2}{60} - \alpha ZWR \frac{41 - 26\gamma}{15(2\gamma - 1)} - \alpha ZR\gamma \frac{17 - 2\gamma}{30W(2\gamma - 1)} + a_{-1} \frac{R}{W} + \sum_{n=0}^5 a_n (WR)^n + 0.41(R - 0.0164)(aZ)^{4.5} \quad (3.25)$$

Where the  $a_i$  coefficients are parameterized by

$$a = \sum_{x=1}^6 b_x (\alpha Z)^x \quad (3.26)$$

The  $b_x$  coefficients are numbers shown in table 3.2

Table 3.2: Coefficients for finite mass correction

	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$
$a_{-1}$	0.115	-1.8123	8.2498	-11.223	-14.854	32.086
$a_0$	-0.00062	0.007165	0.01841	-0.053736	1.12691	-1.5467
$a_1$	0.02482	-0.05975	4.84199	-15.3374	23.9774	-12.6534
$a_2$	-0.14038	3.64953	-38.8143	172.1368	-346.708	288.7873
$a_3$	0.008152	-1.15664	49.9663	-273.711	657.6292	-603.7033
$a_4$	1.2145	-23.9931	149.9718	-471.2985	662.1909	-305.6804
$a_5$	-1.5632	33.4192	-255.1333	938.5297	-1641.2845	1095.358

### 3.4.6 Convolution of Lepton and Nucleon Wavefunctions

This is the last of the relevant electromagnetic corrections. In figure 3.1, the name given is, "convolution PS." This correction  $C(Z, W)$  accounts for the interaction of the lepton

and nucleon wavefunctions. The radial part of the nucleon wavefunctions are modeled as a rectangle with width  $R$ . This is adequate for a shape measurement. Much like the finite size and mass corrections, this correction depends on the type of  $\beta$  decay. Since the decay of interest is an axial vector decay, the form of this correction is

$$C(Z, W) = 1 + C_0 + C_1 W + C_2 W^2 \quad (3.27)$$

where the  $C_i$  coefficients are defined as [35]

$$\begin{aligned} C_0 &= -\frac{-233(\alpha Z)^2}{630} - \frac{(W_0 R)^2}{5} + \frac{2\alpha Z RW_0}{35} \\ C_1 &= -\frac{21\alpha Z R}{35} + \frac{4W_0 R^2}{9} \\ C_2 &= -\frac{4R^2}{9} \end{aligned} \quad (3.28)$$

This correction depends on both the maximum energy of the outgoing electron and the charge radius. The shape contribution from this correction is minimal.

### 3.4.7 Hadronic Corrections

Nuclear form factors are one of the largest corrections to the beta spectrum. There are four relevant form factors that come together into a correction called here the nuclear shape factor. The form factors are described in table 3.3 [24] [7]

The shape factor is written as

$$S(E) = 1 + c_0 + c_1 + \frac{c-1}{E} + c_2 E^2 \quad (3.29)$$

Table 3.3: Nuclear form factors

Form Factor	Name	Value
$c_{1h}$	Fermi matrix element	0.253
$c_{2h}$	Matrix element momentum transfer factor	$0.755 \text{ fm}^2$
$b_{WM}$	Weak magnetism	43.4
$d$	Induced tensor term	40.5

where the  $C$  terms depend on the form factors and nuclear decay variables. The dependence of the coefficients  $c_i$  is given by[7]

$$\begin{aligned}
 c_0 &= -\frac{2E_0}{M_{ave}}\left(1 + \frac{d}{c_{1h}} + \frac{b_{WM}}{c_{1h}}\right) + \frac{2c_{2nuc}}{9c_{2nuc}}11m_e^2 \\
 c_{-1} &= -\frac{m_e^2}{3M_{ave}}\left(2 + \frac{2b_{WM}}{c_{1h}} + \frac{d}{c_{1h}}\right) - \frac{2c_{2nuc}}{9c_{1h}}2m_e^2E_0 \\
 c_1 &= \frac{2}{3M_{ave}}\left(5 + \frac{b_{WM}}{c_{1h}}\right) + \frac{2c_{2nuc}}{9c_{1h}}20E_0 \\
 c_2 &= -\frac{40c_{2nuc}}{9c_{1h}}
 \end{aligned} \tag{3.30}$$

Two of the form factors,  $b_{WM}$  and  $c_{1h}$  are calculated from experimental data.  $d$  and  $c_{2nuc}$  are calculated from nuclear theory. The calculation of  $c_{1h}$  depends on the  $ft$  value and is [24]

$$c_{1h}^2 = \frac{2ft_0}{ft_{20}} \tag{3.31}$$

where  $ft_0$  is the  $ft$  value of the Fermi superallowed  $\beta$  decays, and  $ft_{20}$  the average of the  $ft$  values of the decays of  $^{20}F$  and  $^{20}\text{Ne}$ . This gives a value of  $0.254 \pm 0.004$  [24]. The weak magnetism,  $b_{WM}$ , is of special theoretical interest.

### 3.4.7.1 The Weak Magnetism

The form factor having the largest effect is the weak magnetism form factor. It can be calculated from a  $M1$  analog gamma-ray decay strength. This decay strength is from the 10.275 MeV analog state to the 1.634 MeV first excited state in  $^{20}\text{Ne}$ . This is a  $2^+$  to  $2^+$  angular momentum transition, and a 1 to 0 isospin transition. The decay strength,  $\Gamma_{M1}$ , can be used to calculate  $b_{WM}$  as shown in equation 3.32

$$b_{WM} = \sqrt{\frac{6\Gamma_{M1}M^2}{\alpha E_\gamma^3}} \quad (3.32)$$

where  $M$  is the nuclear mass and  $E_\gamma$  the energy of that gamma ray. This gives a value of  $43.4 \pm 1.2$  [24].

Getting a measurement of this value is of interest for two reasons. One, it tests the Conserved Vector Current (CVC) hypothesis. This hypothesis states that the weak interaction possesses a universal strength and a universal  $V - A$  form [11]. This means that the quantity  $b_{WM}$  relates to difference of the magnetic moments of the neutron and the proton. In decays involving a  $T = 1$  multiplet, the equation 3.32 is valid. This means the largest of the nuclear corrections can be determined from experimental data, which has a much smaller uncertainty than any theory calculations. The second reason is more experimental. It gives a parameter with which to benchmark the measuring techniques and the analysis. If the analysis produces the expected value of  $b_{WM}$ , the same procedure should give a good value for  $b_{GT}$ .

### 3.4.7.2 The Induced Tensor Form Factor

From equation 3.30, the induced tensor form factor  $d$  contributes to the corrections much in the same way as the Fierz term. The induced tensor form factor is calculated with theory. The form factor is given by [7]

$$d = Ag_AM_{\sigma L} \quad (3.33)$$

where  $A$  is the mass number (20 in this case),  $g_A$  the axial vector coupling constant, and  $M_{\sigma L}$  a matrix element of the decay. This matrix element is

$$M_{\sigma L} = <\beta||\tau^+ i\vec{\sigma} \times \vec{l}||\alpha> \quad (3.34)$$

where  $\beta$  is the final state,  $\tau^+$  the isospin raising operator,  $\vec{\sigma}$  the spin operator,  $\vec{l}$  the angular momentum operator, and  $\alpha$  the initial state [7]. In order to get the initial and final states  $\alpha$  and  $\beta$ , shell model calculations have to be used. Different calculations have found  $d$  of 40.5 [7] and 20.5 [24]. Since  $d$  is divided by the average nuclear mass, the uncertainty in this form factor will not cause a large systematic effect.

## 3.5 Summary

Before the theoretical corrections were fed into a Monte Carlo simulation, the uncertainties from the experimentally determined quantities was checked. Theoretical spectra were built with each value plus or minus it's uncertainty. Ratios between the spectra were taken and fit with a line. The results of those ratios are seen in figure 3.3.

Since the slopes of each line are much smaller than the shape factor values, the experi-

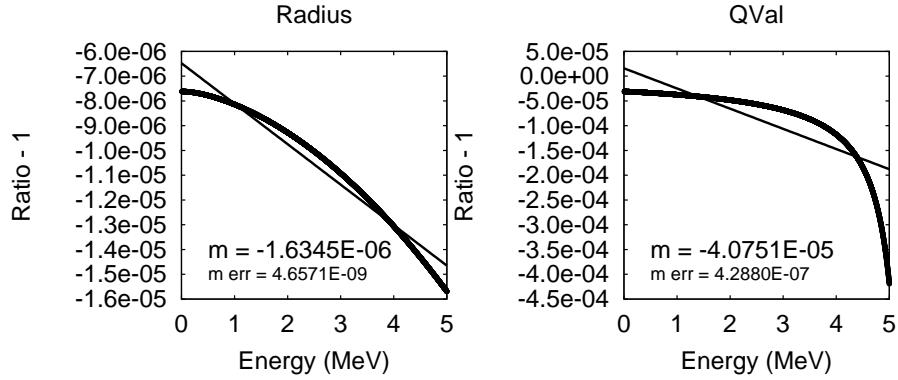


Figure 3.3: Estimate of the theory uncertainty due to the experimental parameter uncertainty. Shown are the slopes for each experimental parameter. They are orders of magnitude smaller than the  $10^{-2}/\text{MeV}$  of the  $C_1$ .

mental uncertainties are negligible. The various theoretical corrections that are needed are now discussed below. With all these corrections, the experiment can be described to the required precision. The next chapter will describe details of the experimental set up and the detector configuration.

# Chapter 4

## Experimental Description

The experiment was run at the National Superconducting Cyclotron Laboratory (NSCL) in East Lansing, Michigan from September 1, 2015 to September 7, 2015.

The experimental technique avoids many of the systematic effects that were important in previous shape measurements in  $^{20}\text{F}$ .

### 4.1 Experimental Technique

The technique used was a calorimetric technique, where the isotope of interest is implanted inside a detector. This reduced several effects that could otherwise distort the beta energy spectrum. The largest of these is backscattering. If a beta decay source is placed outside a scintillator, there is a chance that the electrons will go into the detector material and bounce back out. This happens in the surface of the detector, and therefore the electron deposits part of its energy into the detector. Backscattering causes the largest distortion at low energies. However, the description of backscattering using detector simulations varies too much to get a sufficient description for a precision shape measurement [19]. Beta decay electrons can scatter out of the sides of the detectors as well. This also distorts the beta spectrum in a similar way. Lastly if there is a dead layer at the surface of the crystal, the energy deposited in that layer will not be absorbed.

With the calorimetric technique, the radioactive nucleus is surrounded by detector ma-

terial. As long as the nuclei are implanted deep enough, the electrons will not have enough energy to escape the detector. Even if the electron scatters several times, it still deposits all its energy. This range depends on the detector material. Since any dead layers are at the surface, the decays inside the detector do not see them. A  $4\pi$  angular coverage is also obtained, as the detector material surrounds the source.

There are some caveats for a calorimetric technique. The first is that a large enough detector is needed, otherwise the electrons could escape at high energies. A large effect is that electrons moving through the detector material will emit a lot of bremsstrahlung. Using a low-Z detector material will lessen this effect. However, Monte Carlo detector simulations can describe the production and absorption of bremsstrahlung to enough precision to allow for a precision beta energy measurement. Another issue is that accelerators used to generate the isotope of interest must be able to create them at high enough energy in order to implant the isotopes deep enough into your detector. This limits what nuclei can be used for this technique. The act of implanting the nuclei gives the detector a lot of energy. This means that an implant and decay cycle is needed for this experimental technique.

For this experiment, a beam of  $^{20}\text{F}$ , was implanted into two different detectors, which were used to cross-check different systematic effects. One was a CsI(Na) scintillator detector, and the other was a EJ-200 polyvinyltoluene scintillator detector. After an amount of  $^{20}\text{F}$  was implanted, the beam was turned off by dephasing the RF that drove the cyclotrons. Then, the  $^{20}\text{F}$  was allowed to decay inside the detector. Given the half-life of  $^{20}\text{F}$  of 11 s, the measuring time was varied between 22 and 60 s.

## 4.2 The $^{20}\text{F}$ Beam

The beam of  $^{20}\text{F}$  was created at the coupled cyclotron facility at the NSCL. The primary beam of  $^{22}\text{Ne}$  was accelerated by the coupled cyclotrons to 150 MeV/A. A typical intensity of the primary beam was around  $60 * 10^{-5}$  pnA. It was impinged on a  $188 \text{ mg/cm}^2$  Be target and sent through the A1900 fragment separator. The resulting  $^{20}\text{F}$  beam was at an energy of 130 MeV/A. The intensity of the  $^{20}\text{F}$  was about  $2 * 10^{-5}$  pnA. The beam was sent to the experimental vault where the detectors were sitting.

### 4.2.1 Beam Purity

The purity measured was at 99.4%, with the main contaminates being  $^{19}\text{O}$ . To double check the beam purity, two detectors were used. One was a thin Si PIN detector that was used as a delta E detector upstream of the main detector set-up. The other detector was a small CsI(Na) detector similar to the implant detector that was used during the steering and during the beam size measurement. It had a much lower voltage than the one used for the implantation measurement. Both of these in tandem were used to build a particle identification plot. From this, it was found that the beam is very clean.

### 4.2.2 Beam Size and Location

To test the size of the beam, a parallel plate avalanche counter (PPAC) was used. The size of the detector was 10 cm by 10 cm square. It was placed 65 cm upstream of the detector. A horizontal and vertical grid was in the detector. Depending on where on each of the grids the particles hit, different charges were sent to either end of the PPAC. The signals were fed into a digital data acquisition system, and read out as an energy. The difference of the two

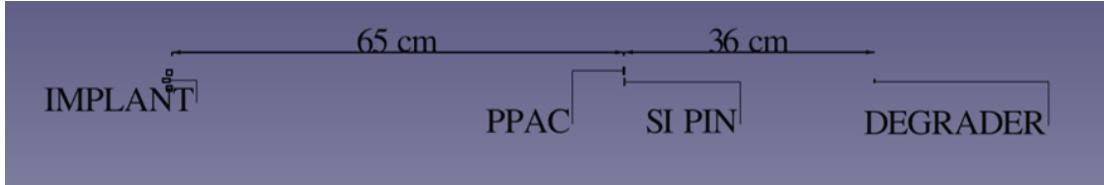


Figure 4.1: A sketch of the location of the detectors upstream of the implant detector. The detectors are to scale.

signals divided by the sum was interpreted as a position. The location of the PPAC and other detectors are shown in figure 4.1.

To calibrate the PPAC , a mask with several holes was used. This mask covered the front of the detector, and an alpha decay source in the vacuum was placed in front of the PPAC. Then, everything was left to run until an image of the mask was formed. The mask had holes in it every 1 cm. It also had a large L shaped hole so that the orientation of the mask could be seen. With this, the PPAC could be calibrated.

Before taking data for the beta energy spectrum measurement, the PPAC was inserted into the beam. After adjusting the parameters of the upstream beam optics, the final beam size at the PPAC was measured. The calibrated data of the signals was built into a 2-D histogram. There was some ringing in the PPAC, so the peak of the beam spot was fitted with a 2-D gaussian function. The average of the Gaussian and the sigma in the x and y direction was taken. From the standard deviations, the half widths at half maximum (HWHM) in both directions were calculated. The distribution of the locations were sampled at eight different points. The points are shown as the black dots in Fig: 4.2. They are one HWHM away from the center.

After averaging the 8 points, the range on the  $z$  axis was taken from the average of those points to the top. The resulting beam was measured to be 8 mm wide by 6 mm high.

This is the size of the spot at the PPAC. Ion optics simulations were used to build

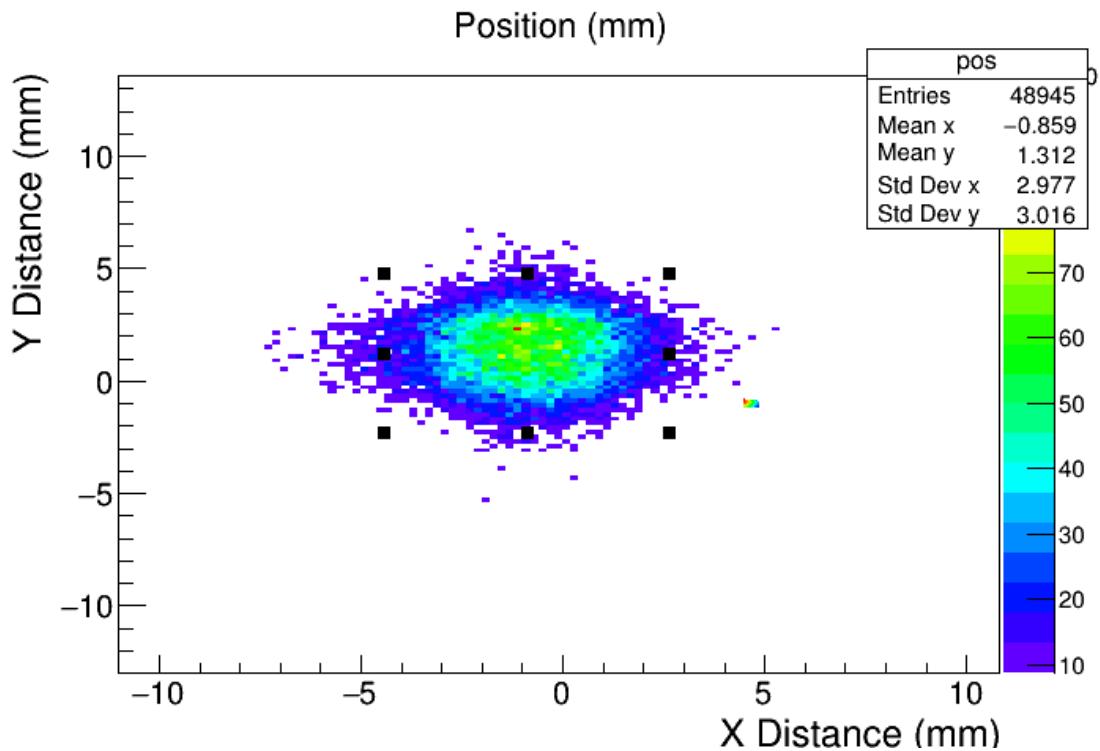


Figure 4.2: The beam spot of the calibrated PPAC spectrum. The samples of the spectrum that were averaged together are shown in the black dots. The center of the fit is not shown.

the envelope of the beam at the PPAC and at the target. Using these cross sections, the magnification between the two locations was calculated. The magnification was different for the  $x$  and  $y$  directions.

For the depth of the implantation of the beam, ion simulations were used. The LISE++ program was fed the beam magnets. It was given the energy of the beam and the detector set up. It then calculated the depth of the implantation of the detector and the range of the depth inside the detector. This gave a depth of 3.02 cm in side the PVT detector and a depth of 1.156 cm in the CsI(Na) detector. The range of the depth was 1.2 mm in the PVT detector and 0.4 mm in the CsI(Na) detector.

#### **4.2.2.1 Verification of Implantation Depth**

In order to verify the simulations for the beam implantation depth, a degrader was inserted into the beam for a beam depth measurement. The degrader was made of aluminum and had two different thicknesses: 7.89 mm and 11.38 mm. The location can be seen in figure 4.1. The beam was caught by a CsI(Na) scintillator of a similar design to the one used during the main measurement. The angle of the degrader was changed to change the effective thickness of aluminum that the beam had to travel through. By comparing the measurement to the one predicted to stop the beam out side of the detector, the depth of implantation could be verified. This was done after the main run, and the simulations proved to be correct.

#### **4.2.2.2 Spacers in Implant Detector**

The actual CsI(Na) implant detector had PTFE spacers between the reflector and the aluminum housing. A spacer would be 1.5 mm thick. It was unknown if the spacer was in the path of the beam during an implant cycle. A similar crystal was taken apart for a repair. Pictures of that detector being taken apart showed where the spacer were on the side of the detector.

This is shown in figure 4.3

To see if the spacer was located in the path of the beam, the data with the degrader in front of the beam was used. The rates inside the implant detector was compared to the effective thickness of aluminum. It was found took 14 mm of aluminum to bring the rate down to zero. This is shown in figure 4.4. If there were a PTFE spacer in the way of the beam, it would take 12 mm of aluminum to completely stop the beam. These results were calculated using LISE++ and are shown in figure 4.5.



Figure 4.3: A CsI(Na) detector being taken apart. The red circle shows the side spacers

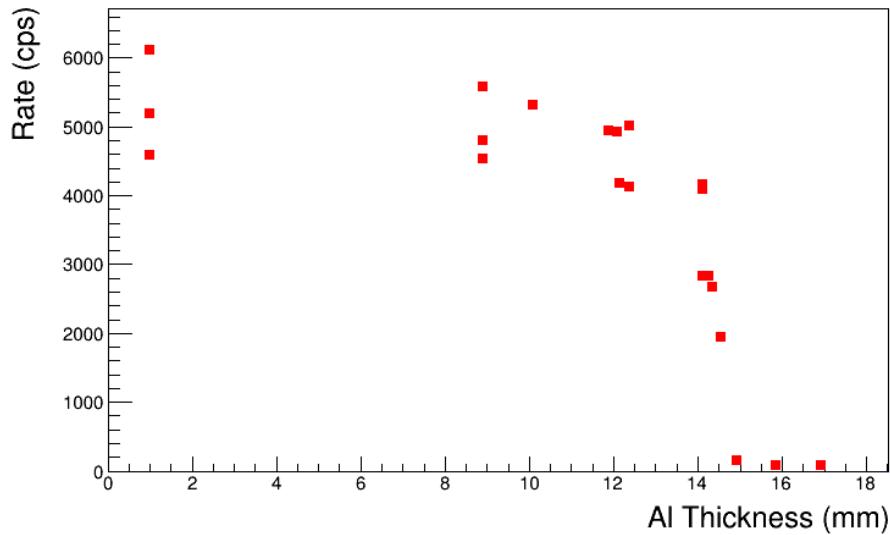


Figure 4.4: The experimental data from the degrader. All the fluorine is blocked after 14 mm of degrader

## 4.3 Detector Configuration

### 4.3.1 $^{20}\text{F}$ Decay Detectors

One implant detector was a scintillator  $3'' \times 3''$  cylinder of EJ-200 polyvinyltoluene (PVT).

It was glued to a clear plastic disk glued to a photomultiplier tube. The plastic detector

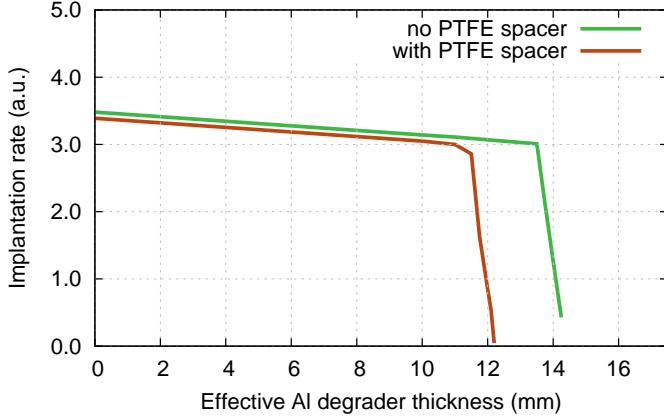


Figure 4.5: Calculating how thick an Al degrader would have to be to block the beam. A model with no spacer matches the data better.

signal is fast (around 10 ns wide) which makes pile-up a lesser concern. This detector was built by collaborators from Wittenburg University. During the experiment the voltage on the PMT of this scintillator was varied and a gain monitoring system was installed.

To monitor the gain of the plastic detector, a plastic disk containing an optical fiber was placed between the crystal and the photomultiplier tube. The other end of the fiber was fed into a box with an LED driven by a function generator, which ran at a trigger rate of 500 counts per second. The function generator produced two pulses separated by 136  $\mu$ s. These two pulses had different amplitudes, so that the gain drift could be monitored by observing the drift from two different energies. The box was made light-tight with electrical tape and black paint. An additional optical fiber fed the light to a Si PIN diode. This was to monitor the light output of the LED. A sketch of the PVT crystal and the gain monitoring system

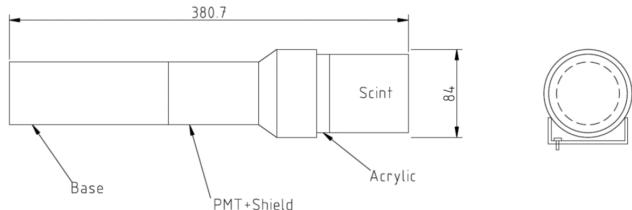


Figure 4.6: A sketch of the PVT implant detector. The plastic disk is seen along as PVT scintillator.

is shown in figure 4.6.

To help check for systematics effects, another implant detector was used. It was a 2" x 2" x 4" CsI(Na) detector. This detector was a module from the CAESAR array [32]. It does not have any gain monitoring like the PVT detector.

In order to measure the gamma ray from the  $^{20}\text{F}$  decay, a frame was built to hold four 3" x 3" x 3" CsI(Na) detectors. These were also part of the CAESAR array. The frame to hold the four detectors was designed to be able to move the four outer detectors around in various configurations. However, no other configurations were used during the experiment. A sketch of the PVT implant detector configuration is shown in figure 4.7. A sketch of the CsI Implant detector configuration is shown in figure 4.8.

To switch between different implant detectors, the central detector in the frame was removed and put on the floor. The other detector was placed in the center of the four gamma detectors. The PVT detector was supported on a metal rail, while the CsI (Na) detector was supported on pile of scrap aluminum. The rail for the PVT detector, along with the four gamma detectors, can be see in figure 4.9.

A better view of the gamma detector cross can be seen in figure 4.10.

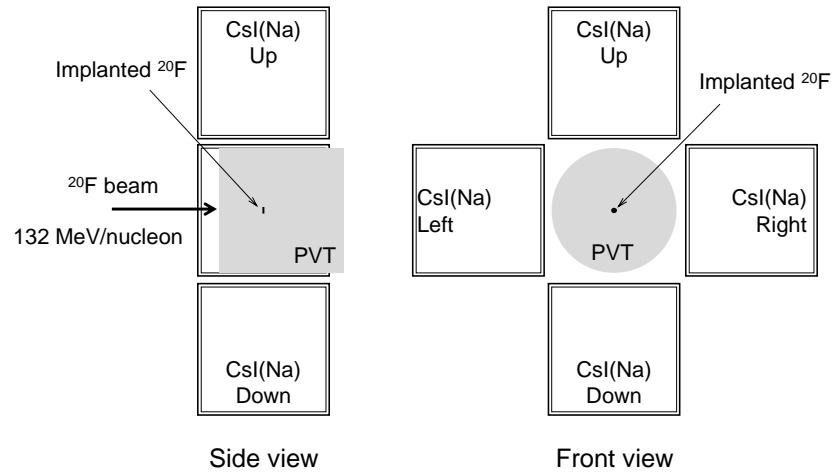


Figure 4.7: A sketch of the PVT implant detector set up. Notice that the PVT detector crystal fills up the space between the gamma detectors.

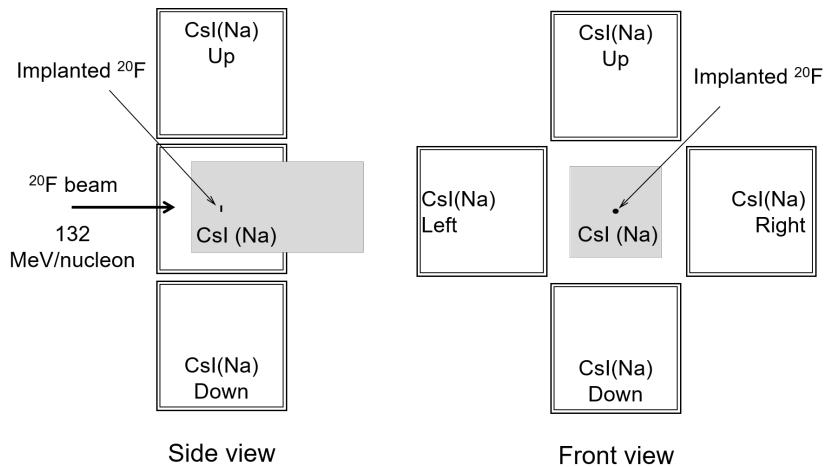


Figure 4.8: A sketch of the CsI(Na) implant detector set up. The CsI(Na) detector is recessed 1 inch.

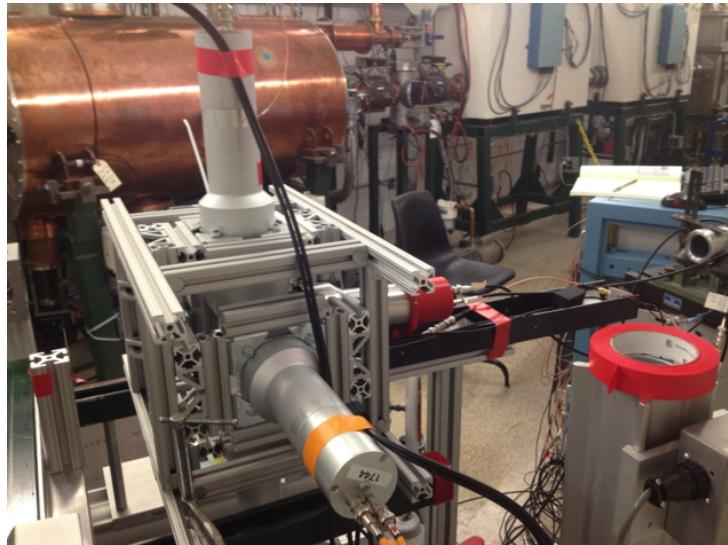


Figure 4.9: A picture of the PVT configuration. The rail the detector rested on can be seen. The optical fibers (the black cords) can be seen running into the black box with the LED.

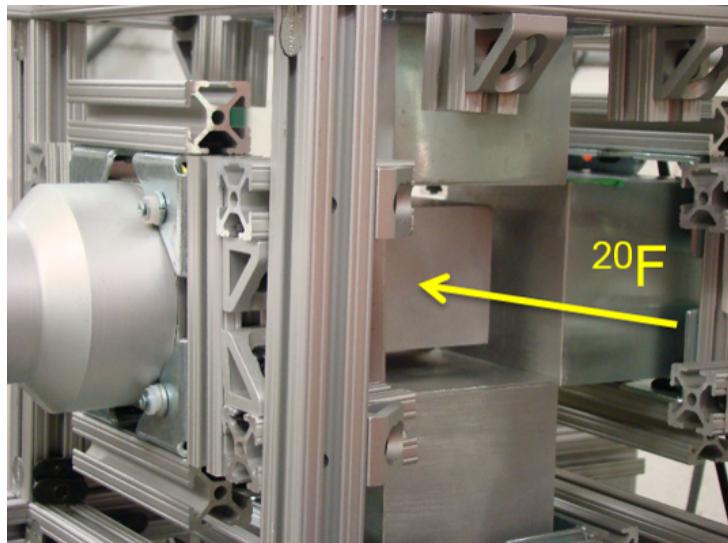


Figure 4.10: A picture of the CsI setup. The active crystal of the implant detector can be seen.

### 4.3.2 Powering the Detectors

To power the gamma and implant scintillators, three 2-channel NHQ 212M ISEG power supplies were used. The Si detector was powered by a Tennelec TC248 amplifier. The PPAC was powered by an integrated power supply.

Each detector had a different voltage. The PVT implant detector was varied in voltage over the course of the experiment. Initially, the high voltage on the PVT detector was chosen to take advantage of the dynamic range of the detector. For the PPAC, The voltage for the four gamma detectors was chosen so that they had roughly the same gain. The voltages of the detectors is shown in table 4.1.

Table 4.1: Detector power

Detector	Power Supply Setting (V)
PVT Implant Setting 1	-975
PVT Implant Setting 2	-856
PVT Implant Setting 3	-780
CsI (Na) Implant	780
CsI (Na) Gamma 1	930
CsI (Na) Gamma 2	1000
CsI (Na) Gamma 3	970
CsI (Na) Gamma 4	1015
Si Pin	20
Pin diode	15
PPAC	560

During the implantation cycle, a large amount of current was generated in the implant detectors. In order to counteract this, a limiter box was installed. The limiter box had several relays that reduced power to the PVT detector's photomultiplier tube during the beam on cycle. To accomplish this, the limiter box was given the same signal that fed the beam on/beam off for the cyclotrons. Due to concerns about the beta energy spectrum shape, the voltage on the PVT detector was changed as the experiment went on. For the

CsI(Na) implant detector, the HV supply had the current limiter enabled. The power setting was not changed for the CsI(Na) implant detector.

## 4.4 Experimental Conditions

The data was taken in runs of roughly one hour. However, many runs were cut short when the DAQ stopped recording data for one of the detectors. This was usually the up gamma detector, and was correlated with the start of a new implant cycle. In order to properly measure the decay, the beam was pulsed with an implantation time of anywhere from 1 to 2 seconds, and a decay time ranging from 22 s to 32 s. The beam was turned off since the light from the implantation of the beam would drown out any signal obtained. A run with a decay time of 60 seconds was also taken for each implant detector.

To achieve the beam pulsing, two timer boxes were used. A CAEN N1145 quad scaler module was used to control the beam off time. This module had a digital control of time down to 1 ms. Once the time finished counting down, a signal was sent to a second module. A CAEN N93B dual timer used the signal from the quad scaler as a start signal. The time period was set using a dial, which was less precise. Once the dual timer finished its time, it sent a signal back to the quad scaler to restart the cycle. The output of these two modules created a signal with one voltage level during the dual timer's time and another voltage condition during the quad scaler's time. This signal was fed into a box. The voltage level during the quad scaler's time dephased the cyclotron RF, turning the beam off. When the dual timer voltage level occurred, the RF returned to the tuned value, turning the beam on. A flowchart showing the timing circuit is shown in figure 4.11

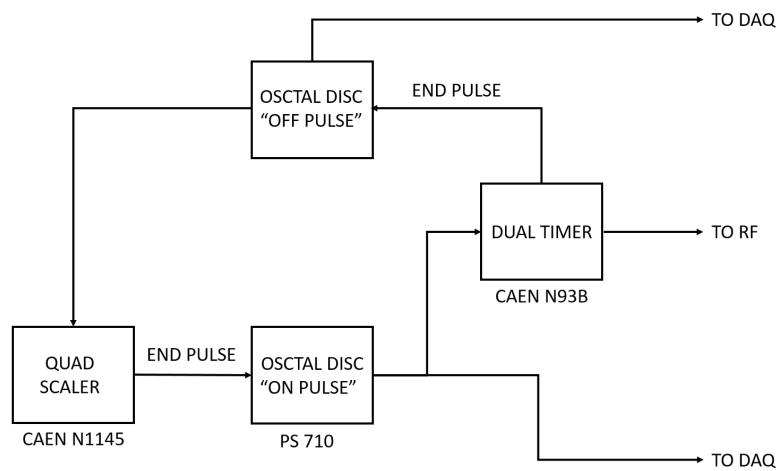


Figure 4.11: The modules used for the timing. "Desc" stands for discriminators.

# Chapter 5

## Data Processing

A digital data acquisition (DAQ) system was used to record the data for this experiment. To configure the DAQ, the detector signals were looked at using an oscilloscope. The timing of each detector signal was recorded and used to tune the data acquisition. The signals were then inserted into a digital data acquisition system.

### 5.1 Data Acquisition

All the signals for all the detectors were sent to 16 channel XIA PIXE modules. Each channel had a 250 mega-channel per second digitizer attached. The incoming signal was first digitized, and then processed by the module. The modules were run with the NSCL firmware. A list of the three modules and which channel processed the signals from which detector is seen in table 5.1. For module 2 channel 0, the digitized wave-form was recorded over 400 ns. The clock used for the digitizers was a stable EPSON SGR-8002JC programmable crystal oscillator. The first thing the PIXIE modules did was to digitize the wave forms of the signals. The digital wave forms of the signals were only saved for the PVT implant detector. The PIXIE modules' software applied a trapezoidal energy filter to the digital wave-forms. Each of the channels gave a time stamp and an energy calculated from a trapezoidal filter.

Table 5.1: DAQ cabling Scheme

Module	Channel	Detector
0	0	PVT Implant
0	1	CsI(Na) Implant
0	4	Large CsI(Na) Up
0	5	Large CsI(Na) Left
0	6	Large CsI(Na) Down
0	7	Large CsI(Na) Right
0	8	Beam On Signal
0	9	Beam Off Signal
0	10	100 Hz Pulser
0	11	PVT Gain Monitoring Pulser
1	0	PPAC Up
1	1	PPAC Down
1	2	PPAC Left
1	3	PPAC Right
1	8	Beam On Signal
1	9	Beam Off Signal
1	11	100 Hz Pulser
1	11	PVT Gain Monitoring Pulser
2	0	PVT Implant (With Digitized Wave Forms)

### 5.1.1 Trapezoidal Filter Description

After the parameters of the filter were set, the filter was taken on the digitized detector signals. First, three sums of the digitize wave-forms were taken. The first sum was over a time known as  $tPEAKING$ . Then, a second sum was taken directly after the first over a time known as  $tGAP$ . Then, a third sum was taken directly after the the second with over a period of  $tPEAKING$  again. These three sums were added weight given as

$$E = C_0 * S_0 + C_g * S_g + C_1 * S_1 \quad (5.1)$$

with  $S_0$  being the first sum over  $tPEAKING$  described above,  $S_g$  being the sum over  $tGAP$ , and  $S_1$  being the second sum over  $tPEAKING$ .  $C_0$  is described as

$$C_0 = \frac{-(1 - e^{\frac{tSPL}{\tau}})e^{\frac{tPEAKING}{\tau}}}{1 - e^{\frac{tPEAKING}{\tau}}} \quad (5.2)$$

where  $tPEAKING$  is the time described above,  $tSPL$  is the 8 ns sampling time, and  $\tau$  an additional parameter.

$C_g$  is

$$C_g = 1 - e^{\frac{tSPL}{\tau}} \quad (5.3)$$

with the same variables. Finally,  $C_1$  is

$$C_1 = \frac{1 - e^{\frac{tSPL}{\tau}}}{1 - e^{\frac{tPEAKING}{\tau}}} \quad (5.4)$$

With the same variables. This filter averaged the background before a pulse in  $C_0$  over  $tPEAKING$ . It took a sample of the pulse, averaged over  $tPEAKING$  again in  $C_1$ . Then, it subtracted off  $C_0$  from  $C_1$ , after adjusting for the exponential decay of the pulse. The time constant of that decay is the  $\tau$  parameter. When tuning the filter, the  $\tau$  parameter had the largest effect.

For this experiment, the various filter parameters are summarized in table 5.2

Table 5.2: Energy filter parameters

Detector	$\tau$	$tPEAKING$	$tGAP$
PVT implant	60 ns	208 ns	128 ns
All CsI (Na) Detectors	900 ns	480 ns	48 ns
PPAC Channels	300000 ns	1200 ns	96 ns

There was also a trigger filter in the PIXIE system. The sum in equation 5.1 is the same, but the parameters are shorter. The parameters in the trigger filter are half of the parameters in the trigger filter. There was a threshold that the output of the trigger filter

had to be above in order to be recorded. These values are shown in table 5.3

Table 5.3: Trigger filter parameters

Detector	$tPEAKING$	$tGAP$	Threshold
PVT implant	104 ns	104 ns	30
All CsI (Na) Detectors	200 ns	72 ns	20
PPAC Channels	904	104	200

The peak of the trigger filter was used to calculate the time stamps of the events.

### 5.1.2 Data Acquisition Software

In order to set the parameters of the various filters, a program called NSCOPE was used [10]. The program GUI was used to set the rise time, the gap, and the threshold for both the energy filter and the trigger filter. These parameters were set differently for each detector type. The CsI(Na) detectors (the implant and the 4 gamma detectors) shared the rise time and gap time, while other detectors had different parameters. Sample spectra were taken with NSCOPE, and the parameters saved to a file.

This file was loaded with a program called ReadOut, which ran the data taking. The ReadOut program used a ringbuffer in order to record the output of the data. The program recorded data as an .evt binary file if the record option was selected.

The ringbuffer was also fed to a program called scalers. This gave the rates of the events coming into each channel. It gave an input rate to each channel, the recorded rate, and the total over each run. This was to check how each channel in each module was doing.

For online analysis, a program called SpecTcl was used. It took the evt files and processed them into histograms. These histograms could be put in coincidence. Displays of the implant detector energy and rate were displayed. The energy of the implant vs time was also displayed. Additionally, the implant energy vs time since last beam on was plotted as

a 2-D spectra. The decay curves were also plotted. These were created by histogram the time in the detector minus the time since the last beam on. These histograms were used as diagnostics. The program allowed for gates to be drawn.

## 5.2 Data Taking and Processing

To take data, the ReadOutGUI program was activated. Data was taken in one hour intervals called runs, unless one of the detectors stopped counting early. If a detector stopped early, it was the up detector. When that happened, the run was ended before one hour was up. The events built by Readout had a time interval of 8000 ns.

Each run generated one or more .evt files. The resulting .evt files of ReadOut were processed using a program called DDAS dumper. This converted the .evt files to ROOT files. One .evt was dumped to one .root file. These ROOT files contained a TTree that had the energy and time stamps of each detector. The information was saved event by event and sorted by time. It was formatted in a C++ class called DDASEvent. This event class contained data for each channel from the data acquisition. The energy was given out in ADC units and was not calibrated. These units had a range from 0 to 32767. The time was given counts of DAQ clock. The digitized wave forms were also saved in this event.

From the ROOT files from the ddasdumper, another ROOT file was built. All the ROOT files from one run was built into one new ROOT file. These files were processed with a modified version of a program called scan. The original version was written by Stan Paulauskas. The TTree was looped over event by event. For each event, each channel was looped over. For the first event and first channel, the time integer was recorded. This first time was subtracted from all subsequent time stamps. The time differences were multiplied 8

ns to turn them into time in seconds. Then, the various energies and times for each detector were built into histograms for each channel. During looping through the events, the energy and times for the implant detectors, four gamma detectors, beam on signal, and beam off signals were saved to a variable. This information was used to build new events for a new TTree.

Each event started with a non-zero energy reading in any of the five detectors. Then, a 400 ns long time gate opened, during which all energies and times were saved into the event. The time signatures of the last beam on and beam off signals were also saved into each event. The energies of the beam on or beam off signals were 0 or 1.

# Chapter 6

## Half Life Measurement

### 6.1 Motivation

During the experiment, it was noticed that there was a disagreement between different values of the half-life. As a side project and as a first chance to thoroughly investigate the data, a half-life analysis was made. These results have been published. The previous measurements of the half-life of  $^{20}\text{F}$  are shown in figure 6.1.

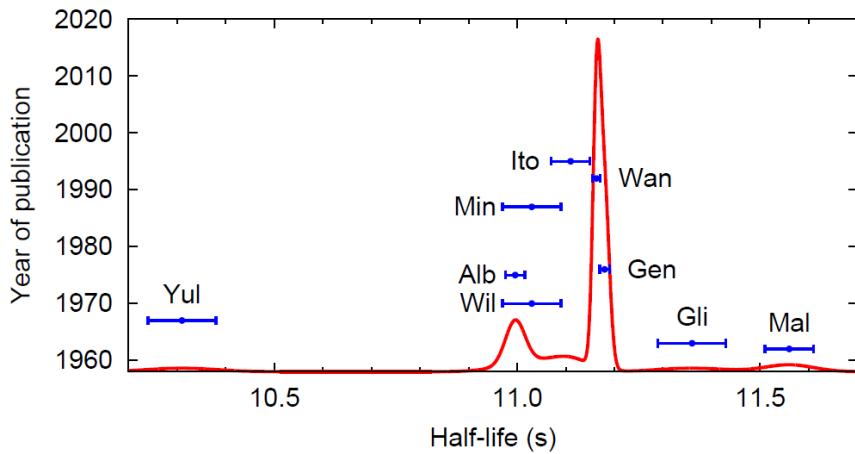


Figure 6.1: Previous measurements of the  $^{20}\text{F}$  half-life. The labels correspond to: Mal [23], Gli [13], Yul [37], Wil [33], Alb [2], Gen [12], Min [25], Wan [30] and Ito [20].

This ideogram has a red line which is a sum of Gaussian functions. The centroids of the Gaussians are the central values of the measurements. The sigmas are the errors, and each Gaussian is weighted by  $\frac{1}{\sigma^2}$ . This is to see that there are two different values that the half-lives are converging to.

## 6.2 Half life Data Analysis

First, the data was processed as described in the previous chapter.

### 6.2.1 Data Selection

Only the data from the PVT implantation set was used for the half-life analysis. The PVT data was separated into 7 sets. A summary of the set conditions is shown in table 6.1 As shown in the table, the major conditions were the PVT high voltage, the inhibitor box, and the beam current.

Table 6.1: Settings for the PVT runs

Set	Beam on Time [s]	Measuring Time [s]	PVT HV [v]	HV Inhibit Installed	Beam intensity [nA]	Runs
1	1.67	30	-975	No	30	9
2	1.67	30	-975	No	93	2
3	1.67	30	-975	Yes	30	9
4	1.67	30	-975	Yes	93	11
5	1.67	30	-856	Yes	30	93
6	1.00	60	-800	Yes	93	1
7	1.10	20	-780	Yes	93	10

The inhibitor box was not installed until the third set of data. The inhibitor box reduced the high-voltage by about 100 V during the beam on period. Without the box, the current on the PMT was saturated during the beam on period. This caused a gain shift over time as the power supply recovered. This time-dependent gain shift caused large systematic errors when the beta cuts were moved. Thus, sets 1 and 2 where not used in the final analysis, but are listed here for completeness's sake. These gain shifts can be seen in figure 6.2.

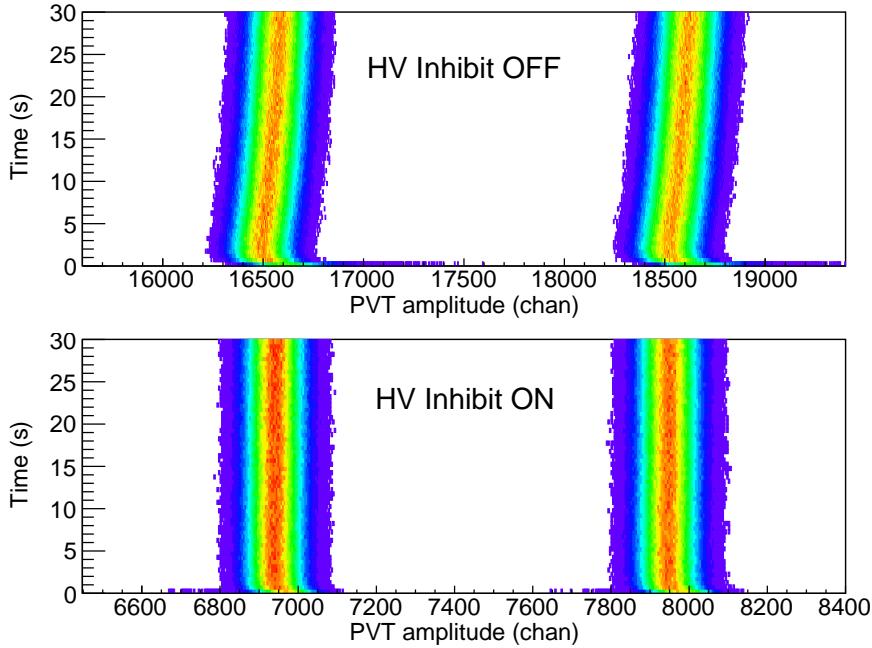


Figure 6.2: The light pulse as a function of time for set 1. The gain changes smoothly over time. This causes a large change in the half-life as the beta cuts are moved.

### 6.2.2 Cut Selection

In order to do the half-life analysis, software coincidences were imposed. Three conditions were set in total. Two conditions were on the energy in the implantation detector and one of the four CsI(Na) detectors. The cuts are summarized in table 6.2. An additional condition on the time difference between the events recorded in the two detectors. A sample spectrum of the gamma and beta energies is shown in figure 6.3

Table 6.2: Energy cuts for the gamma detectors

Detector	Lower Cut [ADC units]	Upper Cut [ADC units]
Up	1350	1650
Left	1280	1600
Down	1440	1800
Right	1450	1800

Fixed energy gates were used to select in the beta window for the PVT run sets. Each different data set had a different energy window due to the gain shifts. A sample spectrum

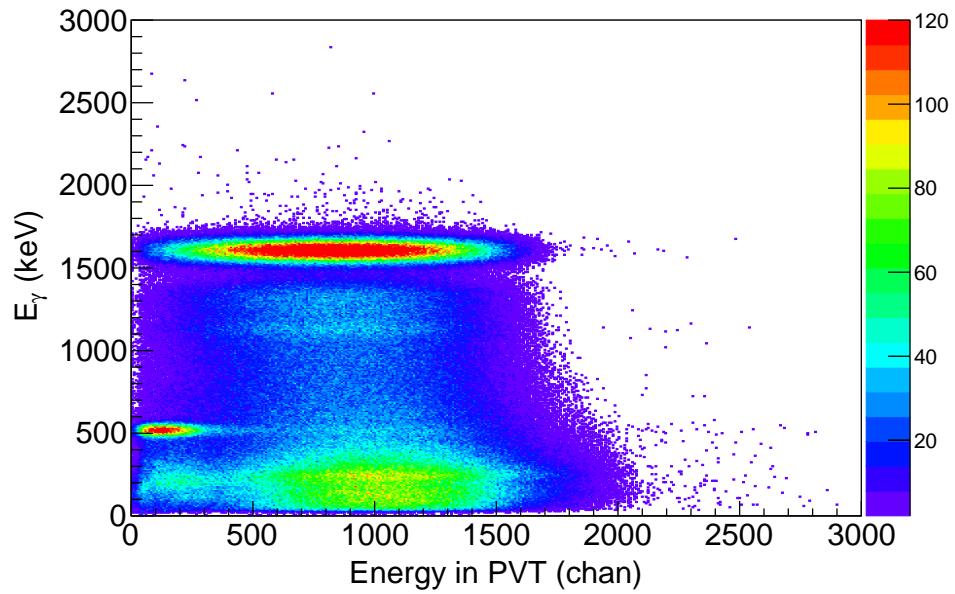


Figure 6.3: The  $y$  axis shows the gamma detector energy. The  $x$  axis is the implant detector energy. The  $^{20}\text{F}$  coincidence region can be seen towards the top of the spectrum. The spectrum at 511 keV can be seen closer to the bottom of the figure.

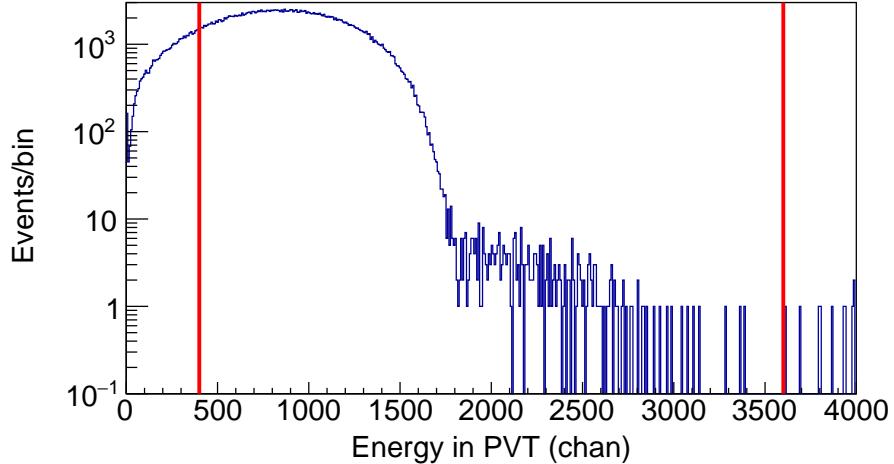


Figure 6.4: A beta spectrum built by applying the gamma cuts and time difference cuts. The beta cuts for the half-life analysis are shown as the red lines. The lower beta cut was selected to be above the 511 region. The upper beta cut was selected to include all the pile up

with the gates can be seen in figure 6.4. These cuts varied from set to set. A summary of the PVT energy cuts can be seen in table 6.3. This spectrum was put in coincidence with

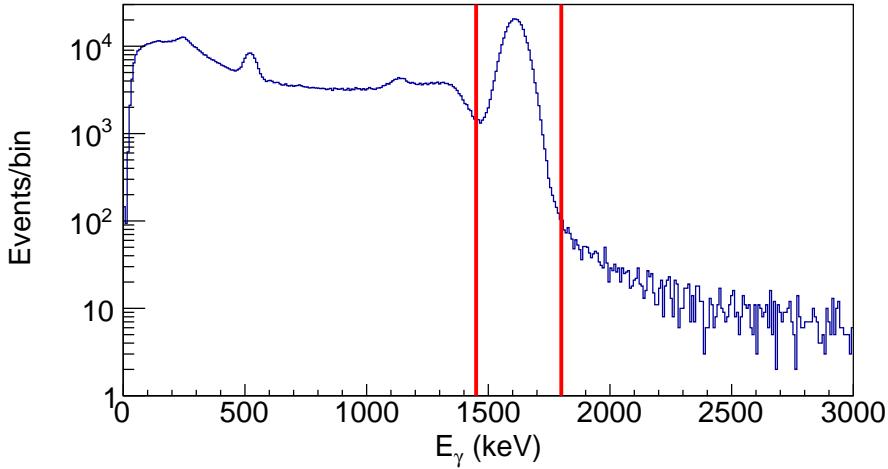


Figure 6.5: The wide cuts are to reduce the effect of the rate dependent gain.

the gamma spectra. The lower beta cut was selected to cut out the 511 beta spectrum as seen in figure 6.3. When gated around that energies, the resulting gammas measured by the large CsI(Na) detectors are consistent with  $^{10}\text{C}$  and  $^{11}\text{C}$ . This comes from the break-up of the  $^{12}\text{C}$  in the plastic scintillator.

Table 6.3: Energy cuts for the PVT detector

Set	Lower Cut [ADC units]	Upper Cut [ADC units]
1-2		2000
3		1500
4		1700
5		800
6		480
7		400

As described above, only the runs after the installation of the limiter box were used for the half-life analysis. This would be sets 3-7.

A time difference condition was set by building a spectrum of the time differences between the implant detector's event and one of the four gamma detector's events. The peak of this time difference spectrum was found, and an interval of  $\pm 24$  ns was used for this gate. A sample time difference spectrum can be seen in figure 6.6

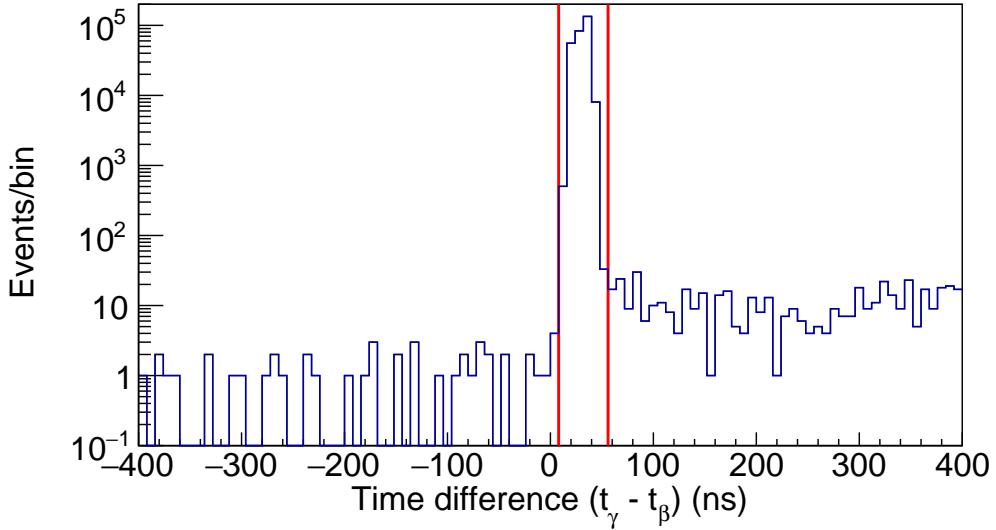


Figure 6.6: The time difference between the up detector and the PVT implant detector. The tail on the right side of the large peak is due to pile up. The narrow time cuts are shown with a dotted line. The wide time cuts are shown with a solid line.

The time difference spectrum shows several features. First is the prompt peak, which are correlated gamma and beta events. To the right of the prompt peak, there is a pedestal of events. These come from pile-up events. What happens is that a  $^{20}\text{F}$  event decays and the time stamp of the emitted beta is recorded. The corresponding gamma ray is not detected. Then, within the pile-up window, a second  $^{20}\text{F}$  decays. The implant detector is dead and does not record the second time stamp. Due to pile-up, the time stamp of the second beta event is not recorded. The gamma ray is then detected at a later time compared to the first event. An cartoon sketch of this effect is shown in figure 6.7. The pedestal can be seen in full in figure 6.8.

The other events to the left of the prompt peak and to the right of the pile-up pedestal are accidental coincidences. These include the background coincidences.

The time signals for the outer four gamma detectors were used for the half-life analysis. This gave 4 independent measurements of the half-life per run. Only full decay cycles were

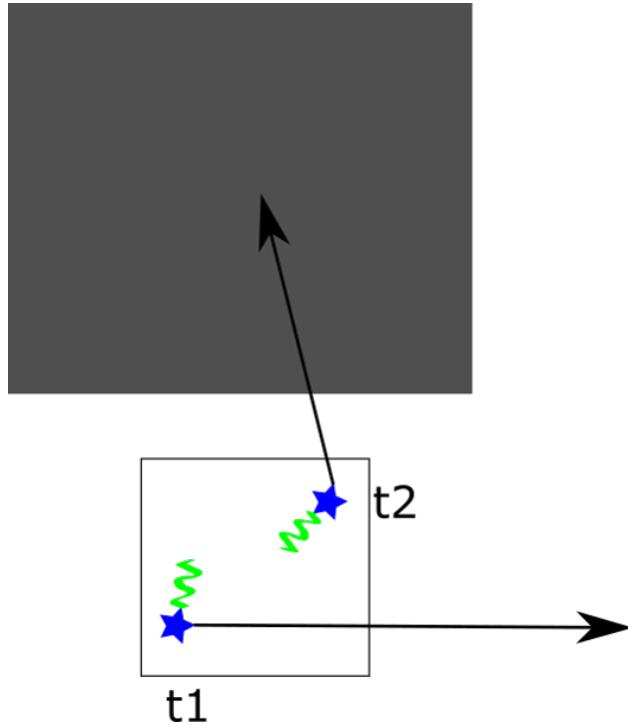


Figure 6.7: A sketch of the pile up events. A  $^{20}\text{F}$  decay happens at time =  $t_1$ . The green electron is detected in the implant detector. The black arrow (the gamma raw) is not. Then, later at time =  $t_2$ , another  $^{20}\text{F}$  decays.  $t_2$  is within the pile-up window of  $t_1$ . Both electrons energies pile-up and are added together. However, the time stamp recorded by the DAQ is still  $t_1$ . The gamma ray from event  $t_2$  can be detected in a gamma detector at time  $t_2$ . This creates the uncorrelated event pedestal in the time difference graph.

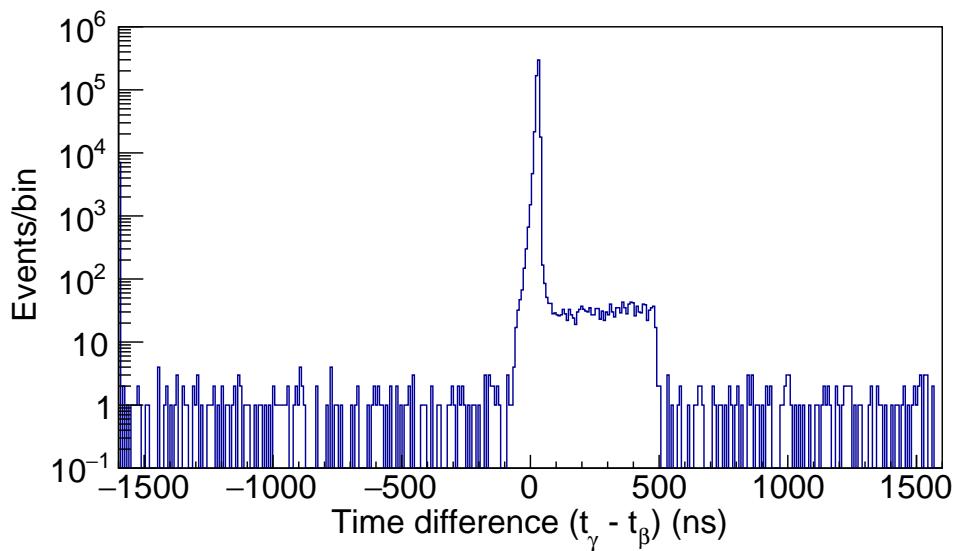


Figure 6.8: The time difference spectrum zoomed out. This figure is built from energy coincidences with energy cuts.

used. This was done by finding the last beam on and putting a condition on the run time of the events.

The dead time was corrected using measured rates and a dead time of 464 ns for the implant detector and 656 ns for the gamma detectors. The dead time was measured by building a spectrum of the time difference between consecutive time stamps. The lowest time difference was taken as the dead time. This was checked using the light pulser in the PVT detector. For each gamma detector, a histogram of the energy-filtered rate was built. The unfiltered implantation detector rate and gamma rate was built. Using the uncorrected rates as an input, the gamma detector rate was corrected for bin by bin. The correction used is

$$r_{coincidence}^c = \frac{1}{1 - r_\beta \tau_\beta} \frac{1}{1 - r_\gamma \tau_\gamma} r_{coincidence}^m \quad (6.1)$$

where  $r_{coincidence}^c$  is the corrected gamma-beta coincidence rate,  $r_\beta$  the raw measured implant rate,  $\tau_\beta$  the dead time of the implant detector,  $r_\gamma$  and  $\tau_\gamma$  the raw measured rate and dead time for the CsI(Na) detector, and  $r_{coincidence}^m$  the measured coincidence rate. Then, after corrected for the dead time, each cycle was added together relative to the last beam off. These stacked cycles were used to find the half life.

For high beam intensity runs (i.e. sets 4,5 and 7), the dead-time corrections had a size of 31 to 36 ms. For low beam intensity runs, (i.e. set 3) the dead-time correction changed the half-life by 6 to 8 ms. The values shown in this chapter are those with the dead-time correction.

Once the decay spectra were put in coincidence and stacked up, it was fit with the function

$$f(t) = a \exp(-t * \ln 2 / T_{\frac{1}{2}}) \quad (6.2)$$

where  $a$  and  $T_{\frac{1}{2}}$  are free parameters.  $a$  is the initial rate and  $T_{\frac{1}{2}}$  the half-life. The decay curves were fit from 1.5 seconds past the beam on time to 1.5 second from the end of the decay time. The fitting method used was the log likelihood method. The 60 second decay run up detector result is shown in figure 6.9. From this run, it is seen that the spectra does not decay back down to the background. The lack of background parameter in equation 6.2 comes from this observation.

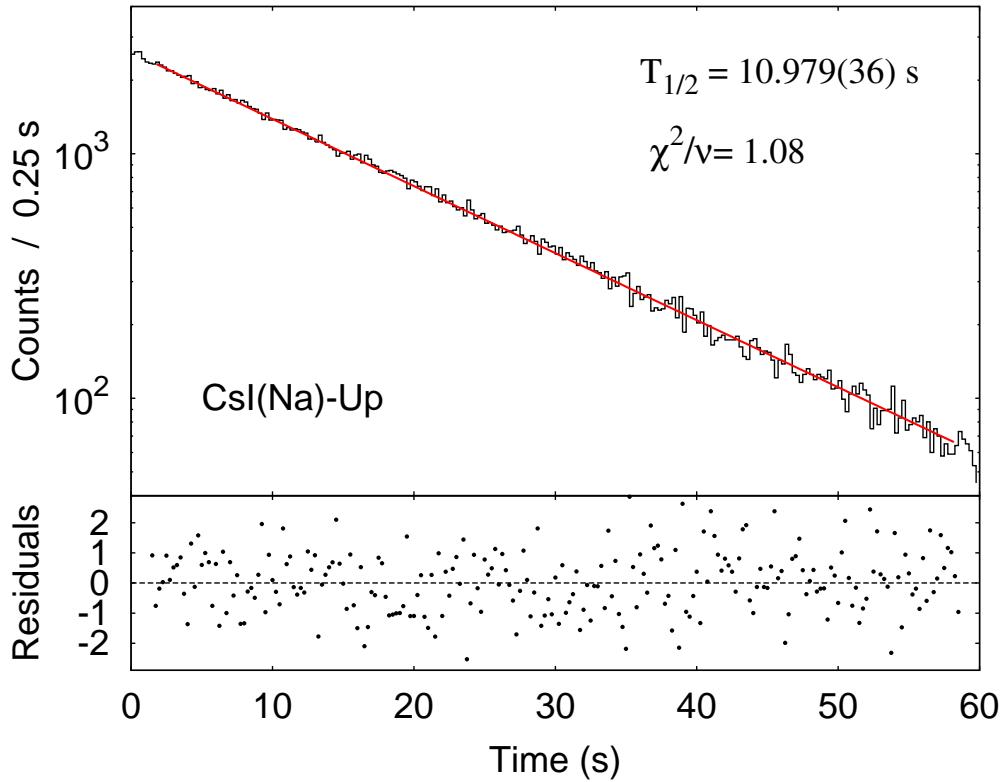


Figure 6.9: The decay spectrum from the up gamma detector is shown on the top graph. The red line is the exponential fit. The bottom graph shows the residuals from the fit.

The decay spectra were built run by run, and the resulting fitting results were averaged together. Runs with a p-value less than 0.05 were considered statistically insignificant and thrown out. After all the significant half-lives were collected, the average of them all was taken.

## 6.3 Systematic Effects

Several systematic effects were looked at. The dead time correction was applied before any other systematics were investigated.

### 6.3.1 Dead Time

The timing resolution of the clock is 8 ns. Due to this, there is an uncertainty on the dead times of at least 8 ns. The dead times were varied  $\pm 4$  ns and the half lives calculated. Half the difference of those half lives is the systematic uncertainty.

### 6.3.2 Pile Up Effects

The dead time is over-corrected. Due to pile up, some of the pile up events are not totally lost. Some of the events thought to be missing just got shuffled around. This is a problem due to the fact that energy gates are imposed. The pile-up events also have a different time structure than the regular events. Due to the fact that the pile-up goes as the singles rate squared, the half-life of the pile-up events is half that of the non-piled-up events. This creates a background with a different time signature, and causes a change in half-life.

The 48 ns time cut around the peak, shown in figure 6.6, was compared to several other, larger time cuts. The narrow time cut still has some uncorrelated background and pile-up events in them.

To account for this effect, several other time cuts were taken. These cuts were plotted vs half-life in figure 6.10. . The values were fit with a quadratic function. This function was extrapolated to zero, which was the correction due to the pile-up.

The extrapolation line is shown. The half-life was corrected to the zero value of the

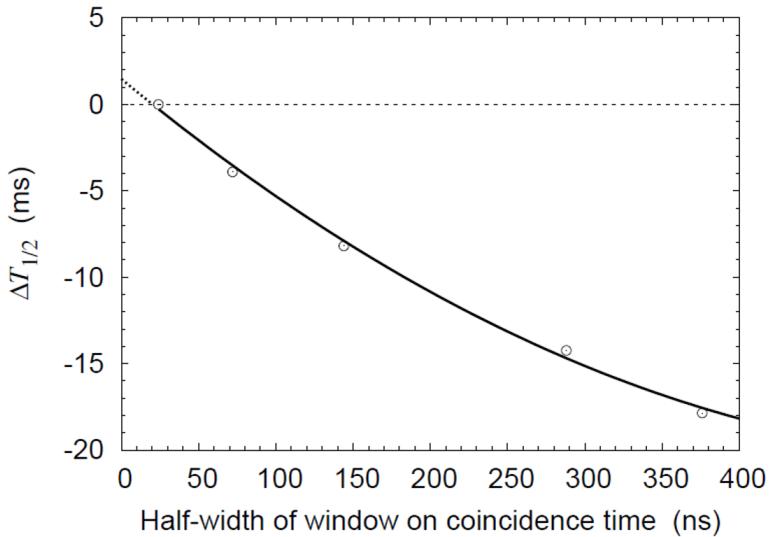


Figure 6.10: Time difference vs the resulting half-life. The line is a quadratic fit which was extrapolated to zero.

window. The difference between this extrapolated value and the previously measured value was taken as a correction and as an error bar.

### 6.3.3 Background

The first systematic effect is the effect of background. Due to the low decay time, directly fitting a background is impossible. There is no background region for the fitting function to anchor to, which induces a large correlation between the background and the half life. Several techniques were used to estimate the background.

#### 6.3.3.1 Simultaneous vs Separate Fitting

The first attempt was by simultaneously fitting each run with a different fitting function. The equation used was

$$f(t) = a(\exp -t * \ln 2 / T_{\frac{1}{2}} + b) \quad (6.3)$$

with  $b$  the relative background level. The half-life of the four detectors was set to be a common parameter. The relative background level and initial rate, however, were assumed to be independent for each spectrum. The resulting half life was compared to the value found with fitting each detector separately without a background.

The effect of adding a background was investigated using Monte Carlo. Exponential functions were generated and an amount of background added. The resulting function was fit with a decay curve without a background parameter. The amount of background added was varied, and the resulting half lives plotted. It was found that increasing the background decreased the value of the half life, and that the background level and the half life value were strongly correlated.

A similar Monte Carlo was written to check the simultaneous fit method. Four decay curves were simultaneously fitted the same way was the data. They were also fitted separately with only an exponential decay. The background was varied and the difference between the two values was taken. The method shows a trend similar to the one seen in the previous Monte Carlo.

In the data, it was discovered that the results of the simultaneous vs separate fitting depended on the size of the dead time correction. If the dead time was over-corrected, it induced an effect that was similar to having a larger background. If more dead time was imposed, the effect was as if there was a negative background. A dead time correction was added to the Monte Carlo, and a similar trend was seen. As long as the dead time is known exactly, the background can be extracted with this technique.

### 6.3.3.2 Spectra Arguments

The other way to try and gauge the background size is to look at the spectra. Looking at figure 6.6, it can be seen that on the left side of the large peak, there are very few counts. It can also be seen that the prompt peak in the center contains most of the counts, while there is a long tail on the right side of the peak. This tail is due to pile up. An event in the beta window piles up with another event later in time. The second event is read by a gamma detector, but is correlated with the first event, causing the large time difference. The time spectrum changes if the beta spectrum is cut in different ways.

In the gamma beta 2D plot (figure 6.3), it can be seen that there is no background events aside from the 511 region. This is due to the presence of  $^{10}\text{C}$  and  $^{11}\text{C}$ . When the detectors are put into triple and quadruple coincidence, the gamma rays of those isotopes appear. There are no gamma rays in the 1620 keV window which was filtered on.. The only coincidence possible is if there is pile up into that window along with a beta event. The estimate of those events is shown on the left side of the peak in the time difference figure shown in figure 6.6 These events are accounted for when extrapolating the different time difference cuts, as shown in figure 6.10

### 6.3.4 Cut Sensitivity

For each pair of detectors, there were four conditions: two for each the implant and gamma detectors. For the gamma cuts, the edges of the gates were varied  $\pm 5$  keV in each direction for each cut. The results is insensitive to the upper beta cut. The lower beta cut was scanned with 6 different values going evenly from the initial lower beta cut to the peak of the spectrum. Moving the lower beta cut also effects the dead time correction. In order

to disentangle the two, the lower beta cut was varied with two different time difference conditions. The location where the difference between the two calculations blows up is where the effect of the lower beta cut starts being swamped by the effect of the pile up. This is limit of where the lower beta cut was scanned. All four of these conditions were varied independently, and the procedure of generation the spectrum and fitting the decay curves was done. Half the resulting range of half-life values was taken to be a systematic uncertainty.

The fitting range of the decay spectra was varied. The start of the fit was varied bin by bin up to 6.5 seconds into the decay spectrum. The end of the fit was varied the other way to cut out the end of the decay spectrum. Since there was no noticeable systematic effect, there was no increase in error associated with this effect.

### 6.3.5 Oscillator Stability

The oscillator of the PIXE board has a stability of  $\pm 5 * 10^{-5}$ . All times in the analysis were stretched by this value, and the half life calculated. Have the difference between the stretched value and the original value is another systematic error.

### 6.3.6 Binning and Fitting

In order to check the sensitivity of the result to binning, the decay curves were rebinned by a factor of two. Half the difference between the rebinned half life and the original half life is considered the correction and the uncertainty.

For the fitting method, log-likelihood estimators were used to fit the summed data. This was done with two different packages which gave identical results. This was compared to

analytic results, which gave the same results, so the half life is insensitive to the fitting method.

## 6.4 Result and Discussion

The half-lives obtained for all runs and all four detectors are shown in figure 6.11

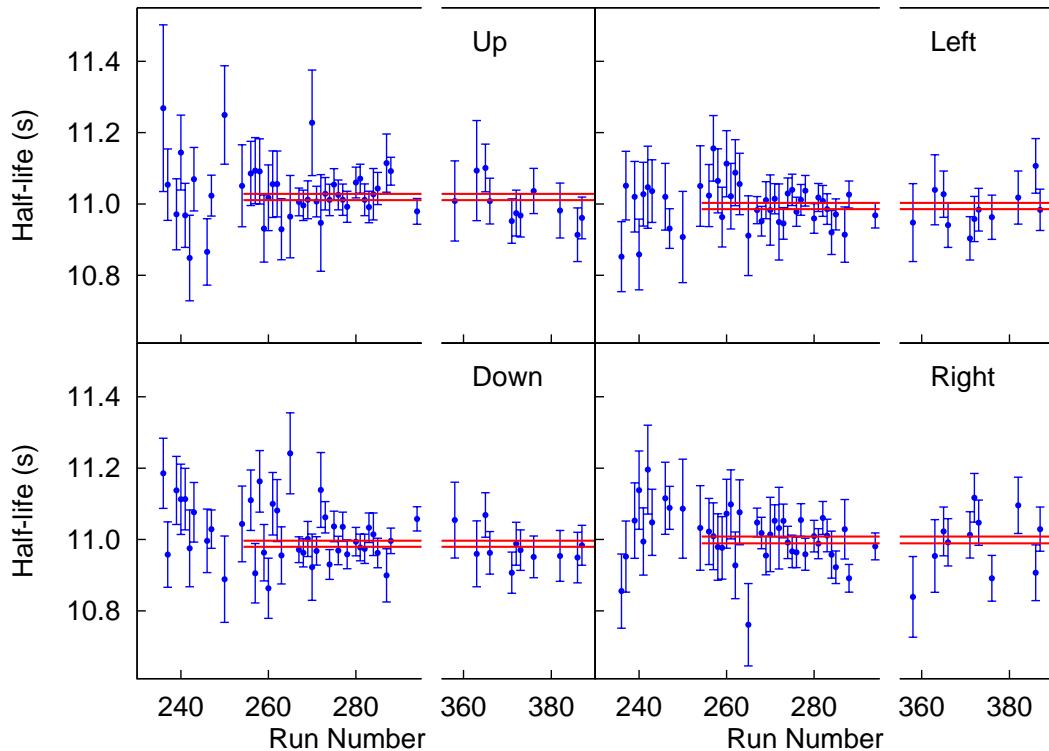


Figure 6.11: The red lines show the results of the fits over the runs that were used. The additional half-lives shown are excluded to reasons discussed previously.

The results are shown in table 6.4.

The size of the systematic errors investigated are shown in table 6.5

After the dead time correction, the value of the half-life was found to be  $11.0011 \pm 0.0069$  (stat)  $\pm 0.0030$  (syst) s.

Table 6.4: Half-life results

Detector	Half-Life	Error
Up	11.0195	0.0088
Left	10.9944	0.0087
Down	10.9980	0.0085
Right	10.9987	0.0095
Mean	10.9999	0.0044

Table 6.5: Systematics

Source	Correction [ms]	Uncertainty [ms]
Dead-time	0.00	0.24
Oscillator stability	0.00	0.80
Lower PVT cut	0.00	2.32
Lower gamma cut	0.00	0.15
Upper gamma cut	0.00	0.05
Uncorrelated events	1.47	1.47
Binning	-0.30	0.30
Total systematic	1.17	2.89

## 6.5 Conclusion

The half-life measured is most consistent with some previous measurements of a half-life of about 11 s. This can be seen in figure 6.12. This measurement disagrees with the most recent measurements.

Motivated by this measurement, another measure of the  $^{20}\text{F}$  half-life has been done. The outcome of the measurements was a half-life of  $11.0160\ (41)_{stat}(155)_{sys}$  s [5]. This confirms the result obtained in this measurement.

Several things about the data set were discovered. It was learned that there were contaminants coming from the fragmentation of the  $^{12}\text{C}$  in the plastic scintillator. It was also discovered that the rate dependent gain effect in the CsI(Na) implant detector is significant. Before doing this measurement, it was assumed that the low rates would make that effect

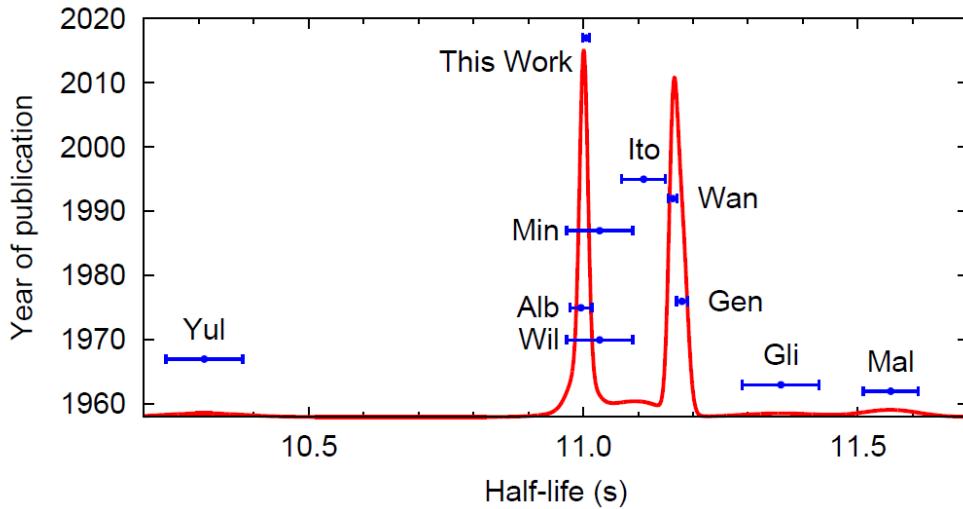


Figure 6.12: A scatter plot of previous values with this work added. The labels correspond to: Mal [23], Gli [13], Yul [37], Wil [33], Alb [2], Gen [12], Min [25], Wan [30] and Ito [20].

negligible. It was learned that after the coincidence, the amount of background left in the spectrum is negligible, and that we have no contaminates in the beam.

# Chapter 7

## GEANT4 Monte Carlo

After the half-life measurement was completed, the next step was to fit the energy spectrum for the Fierz term. The corrections to the phase space were the first part of describing the energy spectrum. The effect of bremsstrahlung and the efficiency of the detectors needed to be accounted for. The absorption of the 1.6 MeV gamma ray needed to be accounted for. The way this was done was with a GEANT4 simulation [1]. The most updated copy of the code can be found at <https://github.com/maximilian29631/Geant4for20F/>.

### 7.1 GEANT4 Monte Carlo

The phase space energy spectrum multiplied by the corrections was fed as input to a Monte Carlo. The program used to model the detectors was GEANT4 version 10.04.02. In order to use the program, the detector geometries where needed.

#### 7.1.1 Detector Geometry

A source of the detector geometry were the technical drawings of detectors. A drawing of the CsI(Na) implant detector is shown in figure 7.1 This drawing was the basis of the detector geometry fed into the simulation.

The geometry of the detectors was programmed into the simulation. The implant detector

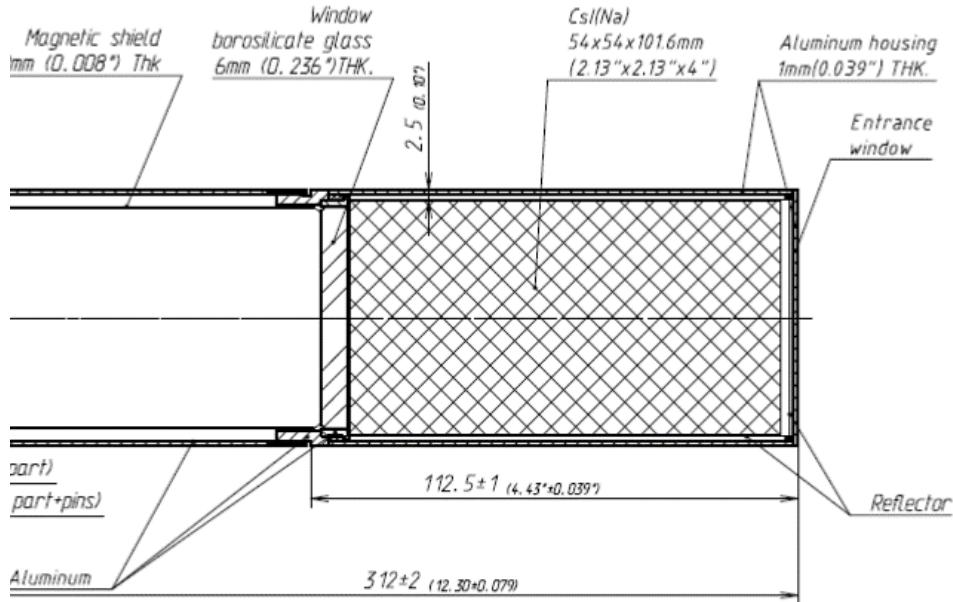


Figure 7.1: A technical drawing of the CsI(Na) implant detector

was modeled as a square prism of CsI. The square prism was 54 x 54 x 101.6 mm. Around the square prism there was an empty layer. Along the sides, the empty layer was 1.5 mm thick. On the front and back of the detector, the empty layer was 3.64 mm thick. After the empty layer, a 1 mm aluminum can surrounded the detector on all sides. The center of this detector was placed at the center of the simulation.

The four large gamma detectors also square prisms. The active volume was 79.5 x 79.5 x 76.2 mm. It was also made of CsI. An empty layer was also added to the gamma detectors. The empty layer on the sides was also 1.5 mm thick, just like the implant detector. On the front, however, the empty layer was 2.92 mm thick. On top of this, the can of aluminum, 1 mm thick on all sides of the detector was added. The four detectors were arranged in a cross. The detectors were oriented in such a way that the square face of the CsI crystal faced

the implant detector. The edge of each of the large gamma detectors was placed 25.4 mm away from the front edge of the implant detector. This modeled how the implant detector was recessed in the experiment.

A visualization of the CsI crystals in GEANT4 is shown in figure 7.2

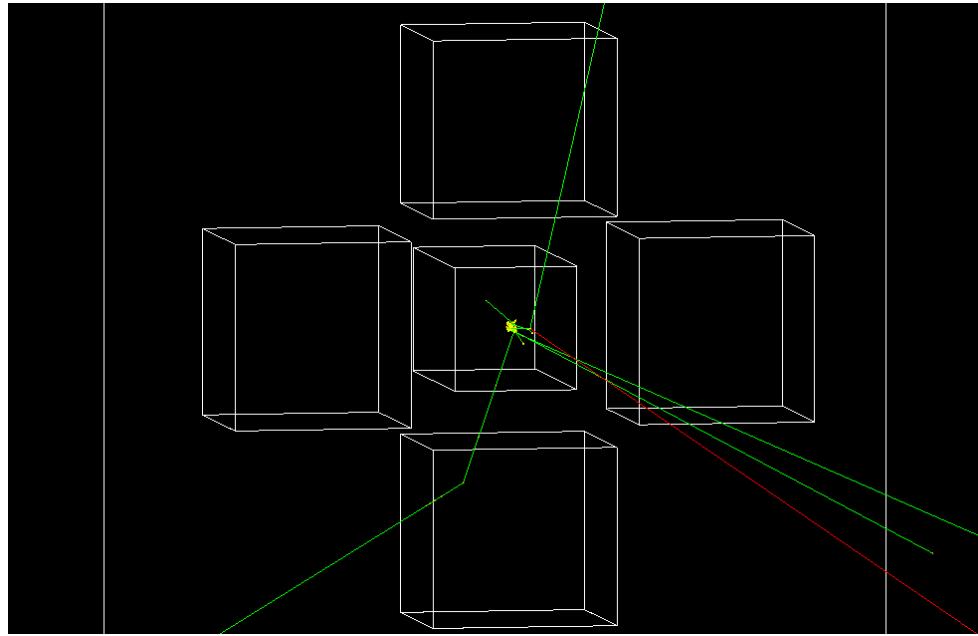


Figure 7.2: The detector geometry inside GEANT4

### 7.1.2 Source Definition

The next step was to define a region inside the implant detector. This region was where the gamma and beta particles originated from. The depth of the region was calculated using LISE++, a ion optics code. The vertical and horizontal size of the region was calculated by using the PPAC measurement and an ion optics simulation. This size was  $0.4 \times 3.5 \times 3.6$  mm. The source was implanted 11.56 mm into the detector. This measures from the front edge of the detector including the dead layer.

### 7.1.3 Primary Particle Definitions

There were three primary particles generated. Two were photons and one was an electron.

All three particles had an isotropic angular distribution.

The first particle was an electron. This selected the point in the source region. In order to generate the correct energy for this electron, the following process was used. First, all the corrections described in chapter 3 were multiplied with equation 3.5. This resulting function was evaluated at 1024 evenly space points from 8 keV to the end point energy. This list of 1024 points was fed into GEANT4 as a histogram, as GEANT4 can only take a histogram of up to 1024 bins. Linear interpolation was used to sample this histogram. The interpolation was tested and was found to reproduce the beta spectrum.

The next particle was the 1.6336 MeV photon from the  $^{20}\text{F}$  decay. This initial photon is monochromatic and isotropic. The photon was generated from the same vertex as where the electron was generated.

The last photon was to account for the inner bremsstrahlung. The radiative correction used to generate the electron energy was the formula that assumes all real photon energy is absorbed (equation 3.12). After the electron energy is generated, the formula describing the energy spectrum of the inner bremsstrahlung photons is generated. This spectrum is written out in equation 3.13. Further discussion of this formula is found in the theory chapter. A cutoff of 50 keV is imposed to the formula, as it has a singularity at zero photon energy. In addition, with this geometry, all photon energies below 100 keV are absorbed. Then, the formula is numerical integrated from 50 keV to the electron energy over 1000 steps using the trapezoidal rule. This is the total probability that an electron emits a KUB photon. This number is compared to a random number from 0 to 1. If the random number is below

the integral, the inner bremsstrahlung energy spectrum is sampled. The algorithm used for this sampling is the van Neumann method [28]. The sampled energy is given to the third primary particle. If the random number is more than the integral, the third primary particle is given an energy of 0 keV. The energy of the electron is reduced accordingly.

The two photons had their initial points moved to match that of the electrons. All three particles were then free to propagate through the detectors.

## 7.2 Particle Propagation

GEANT4 propagates the particles in steps. During or after each step, different physics processes are carried out. The physics process may create secondary particles. If these secondary particles cannot make it the length of the range cut, they will not be created. These cuts can be set differently for different particles. The range cuts correspond to a threshold of 15 keV in aluminum. These steps happen until the primary particle runs out of energy.

## 7.3 MC Output

The particles were tracked and the energy deposited in each detector summed up. The energies deposited into the implant detector was summed up into two categories. The energy deposited from the initial gamma ray was one category. The other was the energy deposited from the initial electron. After the energies of the particles reached a certain threshold, the simulation of one decay was finished. All the energies deposited into each of the detectors was summed up and saved as an event in a ROOT TTree. Then, the process was repeated. A new location inside the region was generated and another decay generated.

For the fit, four different simulations had to be run. Each of them needed  $1.4 * 10^{10}$  events in order to have 100 times the total in the data. To test the systematic effects, only  $2 * 10^9$  events were run. In order to decrease the time it took to run, the range cuts on the gamma rays were changed. The range cuts were changed from  $5 \mu\text{m}$  to  $5 \text{ mm}$  for all gamma rays. The range cuts for other particles stayed at  $5 \mu\text{m}$ .

## 7.4 Simulation Results Processing

In order to use simulations results for fitting histograms, further process had to be done in order to build the histograms for the fitting of the data. The TTrees in the output of the simulation was used. The energy absorbed in the implant detector was filtered by the energy absorbed in the outer four gamma detectors in the simulation. The energy gate was adjusted to match the energy gate of the data. No time information was recorded from the GEANT4 simulation, so no time difference condition was imposed on the simulation.

The two different categories of energy absorbed in the implant detector were built into two histograms. One histogram had a condition that the energy absorbed by the initial gamma ray had to be zero. This histogram was the energy absorbed without any summing. The second histogram was the energy absorbed where there was a contribution from the gamma ray in the implant detector. This histogram was known as the beta-gamma sum spectrum. These two histograms can be seen in figure 7.3.

These two histograms were built for each of the four different inputs that were needed for the proper shape fit. This structure was informed by testing the fit method. The details of the fit of the beta spectrum shape is in the next chapter.

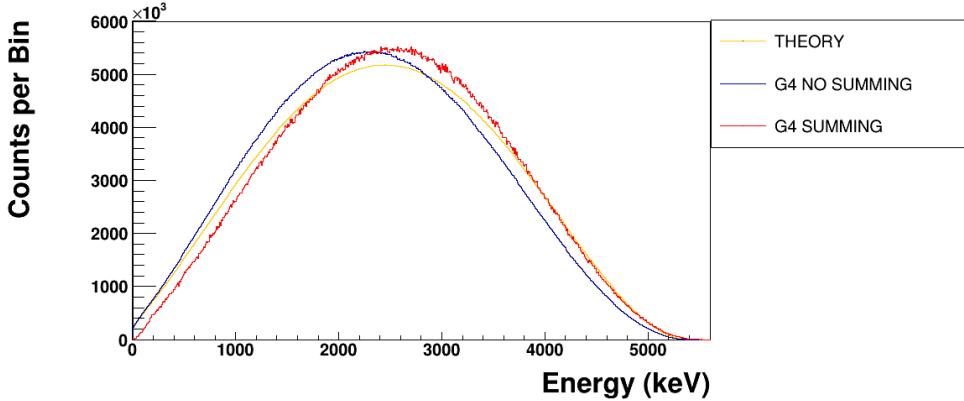


Figure 7.3: The shapes of the different histograms from the output of the GEANT4 simulation. The input beta spectrum is also plotted. The histograms are scaled to the same maximum.

## 7.5 Simulation Development

The GEANT4 simulation is based on the example TestEm5. The first changes were to add the proper detector geometry, as described above. The next changes were to add a second primary particle, which was the gamma ray from the decay curve. The data structure that recorded all the data absorbed event by event was added, so that the gamma cuts could be changed after the fact.

Initially, all energy absorbed by the implant was recorded, regardless of its origin. This was found not to be the correct way to fit the data. To counteract that, the class G4VUserTrackInformation was implemented. This gave each track the information about the primary particle. To split the energies from different detectors, the initial energy and charge of each primary particle was checked. Since the gamma rays are mono-chromatic, the energy of the primary particle was compared to 1.6336 MeV. If the primary particle had charge or it was a photon without an initial energy of 1.6336 MeV, it was put in the category of energy from the initial electron. This took care of most cases, unless a sampled inner bremsstrahlung photon had exactly that energy, which was rare enough to be negligible.

The radiative correction used was changed. Initially, it was assumed that all the real photons generated by the radiative correction would be absorbed. It was also assumed that the higher energy inner bremsstrahlung would not come in large enough numbers to have an appreciable effect. This would make the radiative correction in equation 3.12 adequate. However, after explicitly generating the inner bremsstrahlung photons, this was found not to be true. Therefore, the entire simulation had to be rerun with the inner bremsstrahlung photons generated explicitly.

Then, it was discovered that only running one simulation was not the correct way of fitting the data. Initially, the corrected phase space without the shape factor was fed into the GEANT4 Monte Carlo. Then, during the fitting, the shape factor multiplied the output. This was found not to be correct, so four simulations were needed. The simulations were the phase space times the pieces of the shape factor. The first version of the code took a week to run  $2 * 10^9$  events. This was an issue if four simulations would be needed. To speed up the code, the code was modified to use GEANT4 MPI parallelization. The code was then run on the fireside cluster at the NSCL. After modification, it only took 6 hours to run  $2 * 10^9$  events.

### 7.5.1 Detector Geometry Changes

The detector geometry itself was adjusted several times. Initially, the implant detector was just left as a CsI crystal without any dead layer or cans. This crystal was 50 x 50 x 97.6 mm. This size was given from the group that uses the detectors for  $\gamma$  spectroscopy. The four gamma detectors initially had CsI crystals that were 75.5 x 75.5 x 72.2 mm in size. These crystals were surrounded by 2 mm of vacuum, which was supposed to model a dead layer. After the dead layer, the crystal was surrounded by 1.5 mm of MgO on the sides. The two

sides ends of the crystal the short dimension was left open. A 1 mm Al can surrounded the crystal, and was also open on the sides. Both the can and the MgO layer were offset.

After that geometry, the implant's geometry was updated to match the gamma detectors' geometries more closely. A 2 mm gap was added, and then surrounded by 1.5 mm of MgO. On top of that, a 1 mm layer of Al was added. However, the top and bottom of the detector was left bare in the MgO powder and the Al layer. This was fixed, and the can and the MgO completely surrounded the detector.

The detector geometry with the MgO is not what the actual detector geometry is. This geometry was tuned in order to match the efficiency of the data better. With this tuning, the efficiency is off about 4% of the measured value.

It was found that the detector geometry in the simulation has a large effect on the results of the fit. The final detector geometry was that from technical drawings of the detectors. This geometry ultimately used is the one described above.

### 7.5.2 Data Output Changes

Another change that was made is in how the simulation results were built. Initially, all the energy absorbed in each detector was put into a TTree. This meant that  $2 \times 10^9$  events generated 60 GB of output files. To reduce this space requirement, first only events where any gamma detector had a non-zero input were recorded. This cut down the space required by 1/3 to 20 GB. This was not enough, however, as each fit needed four histograms. A requirement that only one detector fired in order to get the event recorded in the TTree was imposed. This cut down the space to 16 GB for  $2 \times 10^9$  events. As the final simulation needed  $1.4 \times 10^{10}$  events, this still was not enough. The lesser amount of events was used to test the fit method and calculate systematic effects.

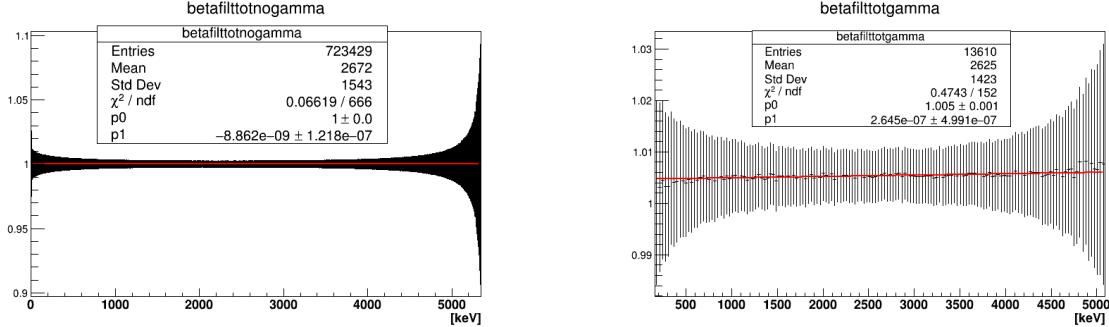


Figure 7.4: Ratio of processing data two different ways. Left shows the beta only spectrum ratio. Right shows the sum spectrum.

Since there is a requirement for only one gamma detector to absorb energy, the gamma-beta coincidences are independent detector to detector. To save space, it is then possible to build 2-D histograms for each of the four gamma detectors. The energy of the gamma ray was plotted on the x-axis, and the energy absorbed in the implant detector on the y-axis. For each detector, two of these histograms were built. One had the beta-only information. The other had the information about the beta-gamma sum spectra. Then, when it came time to apply the beta cuts, the 2-D histogram was projected onto the x-axis and the spectra built. To check if the binning applied to the histogram distorted the output spectra at all, ratios were of the two output spectra were taken. The results of the ratios can be seen in figure 7.4. The difference is very small. This method of processing the data was used to test the sensitivity of the fit to the charge radius and the beta end point.

# Chapter 8

## Fitting Beta Spectrum

The output of the GEANT4 simulations was used to fit the beta spectrum for  $b_{GT}$ . The simulations had to be processed further first.

### 8.1 Fitting Method

Four GEANT4 simulations were used for the fit. The phase space, corrections, shape factor, and Fierz term can be written as

$$\frac{dN}{dE} = PS(E) * C(E)(1 + c_0 + c_1 E + c_2 E^2 + \frac{c_{-1}}{E}) * (1 + b_{GT} \frac{m_e}{E}) \quad (8.1)$$

with  $PS(E)$  being the phase space,  $C(E)$  the corrections, the  $c_i$  the terms of the shape factor, and  $E$  the total electron energy. The  $PS(E) * C(E)$  can be distributed to the shape factor, and four terms appear. Since the  $c_i$  and  $b_{GT}$  are small,  $b_{GT} * m_e$  can be, to first order, added to  $c_{-1}$ . These four terms are the inputs to the GEANT4 simulation. These are summarized in table 8.1

Table 8.1: The four histograms used for the fit

Name	Formula	Normalization (when $E$ is in keV)
$f(E)$	$PS(E) * C(E)$	1
$g(E)$	$PS(E) * E * C(E)$	3.35e-4
$h(E)$	$PS(E) * E * E * C(E)$	9.89e-8
$j(E)$	$PS(E)/E * C(E)$	2494

Each of these pieces was generated with the same number of events. Due to the different shapes of the spectra, a relative normalization was required. The normalization was determined by taking the ratio of each the input spectrum of each piece compared to  $f(E)$  and fitting it with the relevant power of  $E$ . Each of them was filtered with respect to the energy absorbed in the gamma detectors as described in the previous chapter. Then, the detector resolution was applied to each piece.

### 8.1.1 Detector Resolution

The next step was the apply the detector resolution to this histogram. This was done by applying a convolution function to the histograms built from the GEANT4 output files. The convolution function was

$$\sigma = A\sqrt{E} \quad (8.2)$$

where  $\sigma$  was the energy resolution and  $A$  a constant found through a calibration. This was done by looping through each of the histograms' bins and reading the number of counts in each. Then, a Gaussian distribution was built for each bin. The center of the Gaussian was the center of each bin in keV. The  $\sigma$  of the Gaussian was calculated by using equation 8.2. This Gaussian distribution was sampled as many times as there were counts in the bin. These samples were filled into a new histogram. This was repeated until all bins were distributed. This was done separately for each spectrum.

### 8.1.1.1 Determining Detector Resolution

In order to properly fit the energy spectrum, a calibration of the CsI(Na) detector had to be done. The calibration was used to find what the detector resolution was. Several different energy lines were used to find the calibration and the energy resolution function. The lines were the 1.173228 MeV and 1.332492 MeV gammas from a  $^{60}\text{Co}$  source, the 0.661657 MeV gamma ray from a  $^{137}\text{Cs}$  source, and the 0.511 MeV and 1.274527 MeV gamma rays from a  $^{22}\text{Na}$  source. These gamma rays were fitting with a Gaussian and a background. The background varied from no background, a linear background, a quadratic background, and an error function background. For the  $^{60}\text{Co}$ , both peaks were fit at once. These different background gave slightly different results for the centroids and the widths of the Gaussian. The range of the centroids and widths was taken as a systematic error. From the centroids, the calibration for each detector was calculated with

$$C = G * E + b \quad (8.3)$$

where  $C$  is the location of the peak in ADC channels,  $G$  the gain in channels/keV, and  $b$  the offset in channels. This calibration curve was only used for the resolution determination. The widths of the peaks were calibrated with the gain and plotted vs energy. Then, equation 8.2 was fit to the results in order to determine  $A$ . The value of  $A$  used for this procedure was 1.112 with an uncertainty of 0.008.

After applying the detector resolution, the pile-up in the detector had to be modeled.

### 8.1.2 Pile-up Modeling

In order to model the pile-up properly, a model of the detector pulses was needed. For the CsI(Na) signals, a linear rise and then an exponential decay was used. The linear part went from 0 to 1 over 100 ns. The exponential piece started at 100 ns at 1 and decayed with a decay constant of 760 ns. This analytic equation was scaled up or down to whatever energy was sampled. It was then fed into a model of trapezoidal filter of PIXIE. The sums shown in equation 5.1 were done and the energy calculated.

This model was tuned on calibration data. A spectrum was taken of  $^{137}\text{Cs}$  at 25000 counts per second. A  $^{137}\text{Cs}$  spectrum taken at 2000 counts per second was used as a background and subtracted off. Then, samples of the background-subtracted spectrum were taken up to the end of the 661 keV peak. Monte Carlo methods were used to model the time difference between two samples. If the two samples fell within the pile-up window, then the piled-up signal models were fed into the trapezoidal filter. The calculated pile-up energy was saved to a histogram. If the two samples did not fall within the pile-up window, then they were not fed into the filter model and just filled into the histogram. The parameters were adjusted until the generated pile-up matched the measured pile-up in the spectrum. The results of the tuning is seen in figure 8.1

For the pile-up model, the resulting times are shown in table 8.2. In addition, the parameters of the actual energy for the CsI(Na) implant detector are shown.

Table 8.2: Pile-up model parameters compared to the energy filter parameters

Parameter	Pile-up model value	Energy filter value
$\tau$	880 ns	900 ns
$t_{PEAKING}$	880 ns	480 ns
$t_{GAP}$	72 ns	48 ns

The  $\tau$  value in the pile-up tuning is very similar to that of the energy filter. The other

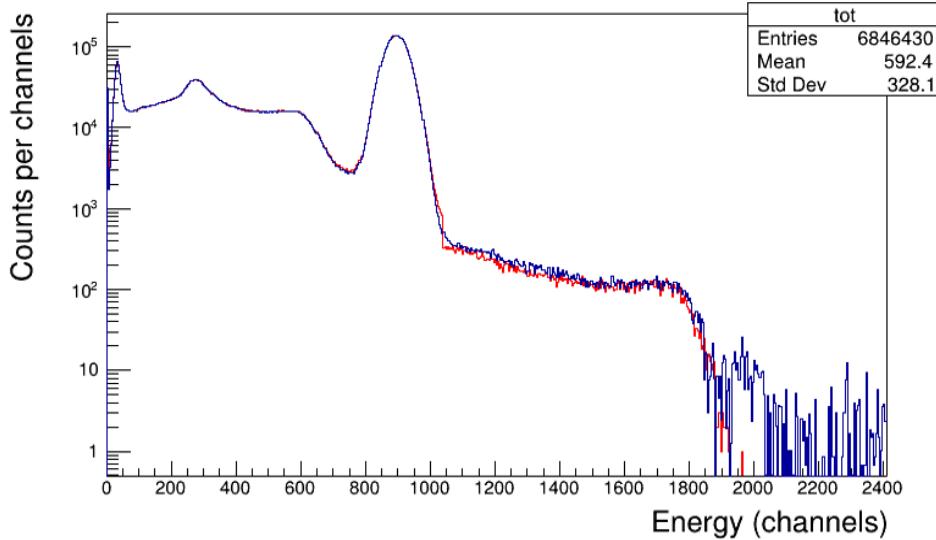


Figure 8.1: The tuning of the pile-up. The input spectrum is in blue. It was sampled up to 1000 channels. The generated spectrum is in red.

parameters are roughly double in the pile-up model compared to that in the PIXIE energy filter.

The pile-up model was applied to the simulation after the detector convolution was applied. Of the four different simulations needed for a fit, only  $f(E)$  was fed into the pile-up model. The amount of pile-up is small. Applying the pile-up to the other terms of the shape factor gives a negligible contribution. The beta only spectrum and the beta-gamma sum spectrum of this simulation was added together and fed through the pile-up model. With all the pieces in place, the fit function can be described.

### 8.1.3 Fit Function

For the fit function, the convoluted histograms and pile-up were rebinned to a bin width of 64 keV. Then, each of the histograms were turned into a spline. The five splines were fed into a fit function. The fit function was

$$H(E) = A * ((1 + c_0)f(E) + c_1g(E) + c_2h(E) + (c_{-1} + b_{GT}m_e)j(E)) + B * pu(E) \quad (8.4)$$

where  $A$  is the normalization,  $B$  the level of pile-up,  $pu(E)$  is the generated pileup spectrum, , and  $f(E), g(E), h(E)$ , and  $j(E)$  the different simulation results as shown in table 8.1 after the convolution was applied. In order to change the units of the x-axis in the data to energy in keV, a calibration was needed. For this calibration, the offset was fixed in the fit function. However, the gain of the calibration was left as a free parameter. This was to account for gain shifts, such as an apparent rate-dependent gain shift due to the afterglow in the crystal.

In the fit, each of the  $C_s$  were written out in terms of the nuclear form factors. In these form factors,  $b_{WM}$  was left as a potential free parameter. If  $b_{WM}$  was fit with the fit function,  $b_{GT}$  was fixed to be 0. If  $b_{GT}$  was fit, then  $b_{WM}$  was fixed to be 43.4. In total, the fit function had four free parameters:  $A$ ,  $B$ , the gain, and either  $b_{WM}$  or  $b_{GT}$ .

In order to benchmark this fitting method, pseudo-data was generated and fit.

#### 8.1.4 Fit Method Characterization

Different methods of the fit function were tested. The first is the so-called "hybrid method", where only one simulation is needed. The GEANT4 with only the phase space and corrections is generated. Then, it is multiplied by the shape factor. This function was

$$H(E) = A * f(E) * S(E) + B * pu(E) \quad (8.5)$$

with  $A$  being a normalization constant and  $S(E)$  the shape factor. This method of fitting is not completely correct, as the  $E$  in  $f(E)$  is not the same as the initial energy in the shape factor  $S(E)$ .

The first test was done just with a linear term in the psuedo-data. An older version of the GEANT4 code had a phase space times corrections times a linear shape factor fed into it. This psuedo-data was fit with the hybrid model. Initially, the shape factor was applied to total energy inside the implant detector. This was found to induce a shift of the measured linear term. Applying the shape factor only to the beta only part of the spectrum solved this issue.

To further test the fitting method, two more spectra were generated. One had had the nuclear shape factor with  $b_{WM} = 43.4$ , which was the new pseudo-data spectrum. The other had no shape factor. After applying the coincidence condition to both of these simulations, the function with no shape factor was used to fit the function with the shape factor. The gain in these fits was left as a free parameter, and the offset was fixed to a value of 0. To manually check the statistical uncertainty, the pseudo-data spectrum was fluctuated. Each bin had its content and error read. That bin error and content was used to build a Gaussian random number distribution. This random number distribution was sampled, and a new bin center created. After doing this to the entire spectrum, the new spectrum was fit. The resulting  $b_{WM}$  was recorded. Then,  $b_{WM}$  was set to 43.4, and the Fierz term fit. These fitted terms were saved to a histogram. This was done 1000 times to get a spread of values. From this, it was discovered that spline interpolation was the best way to fit the spectra.

The same method was used to see if the fitting method induced a lower beta cut effect or not. 1000 spectra were generated by scrambling one spectrum over and over. These spectra were fit from a lower beta cut up to the end point minus 100 keV. The lower beta cut was

varied from 200 keV up to 2 MeV. From this it was found that this fitting method induced an effect above 1200 keV. This effect was very large in the measurement of  $b_{GT}$ .

Doing the same thing with the four-histogram fitting method shown above showed that there is not such a large lower beta cut effect. The four histogram fit is the one described above. More pseudo-data was generated, along with four histograms to be used for fitting. This was largely done to get a handle of the normalization of the histograms. The normalization was obtained by taking the ratios of the histograms and fitting them with the functional forms. After the normalization, the fit was done. The output  $b_{GT}$  and  $b_{WM}$  obtained of the pseudo-data was the one put in.

## 8.2 Experimental Data Processing

After the fitting method was tested and verified, the data was ready to be fit. For the fit, the experimental data had to be processed as well. The energy spectrum of the implant detector was put in coincidence with the 4 gamma detectors. The energy cuts of the gamma detectors were the same as in the life-time measurement, as seen in table 6.2. An additional condition was that only one of the gamma detectors could have fired for each event, much like the processing for the GEANT4 events.

An additional condition in the data was a time difference condition. For this measurement, the time difference between a beta event and a gamma event had to be between -300 and 24 ns. This captures the entire spectrum, as the time stamps show a large energy dependence. Then, a 2-D histogram was built, with energy on one axis and time since last beam on on the other. The decay cycles were 22 seconds long for the CsI(Na) implant data. The decay cycles were split up into 3 segments with equal number of counts. To do this,

it was assumed that the rate depended on the time as an exponential with a half-life of 11 seconds. The time interval was split in such a way that the integral of the exponential was equal over 3 different segments. The total time interval was from 1.5 to 22 seconds. These splits worked out to be at 5.89 s and at 11.98 seconds. This 2-D histogram was projected onto the time axis between the splits in order to build the energy spectrum of the fit.

After processing the data, the fit was done run by run. First a prefit for each section was done to calibrate the energy from 500 channels to 6000 channels. This was done to obtain a calibration of the data. Then, two endpoints fixed in keV were used for the actual fit. The fitting function with a fixed offset was fit using a maximum likelihood method. The resulting  $b_{WM}$  was recorded. For the Fierz term fit, the  $b_{WM}$  of  $^{20}\text{F}$  decay (43.4) was applied to the shape factor. The fit was done again and the  $b_{GT}$  was recorded.

The offset used for the fit was varied. The effect of the offset will be discussed in the next section. The calibrated start and end of the fits were 250 keV to 7500 keV. A sample fit is shown in figure 8.2, where the offset was fixed to 0.

### 8.3 Systematic Uncertainties

After testing the fit and fitting the data, the systematic effects were investigated. The systematic uncertainties can be divided into three categories. The first set are the uncertainties that have to do with the inputs to the simulation. The second set has to do with the inputs to the fit. The final set has to do with the uncertainties of the nuclear form factors in the shape factor. One of the largest systematic uncertainty has to do with the were the fit is started.

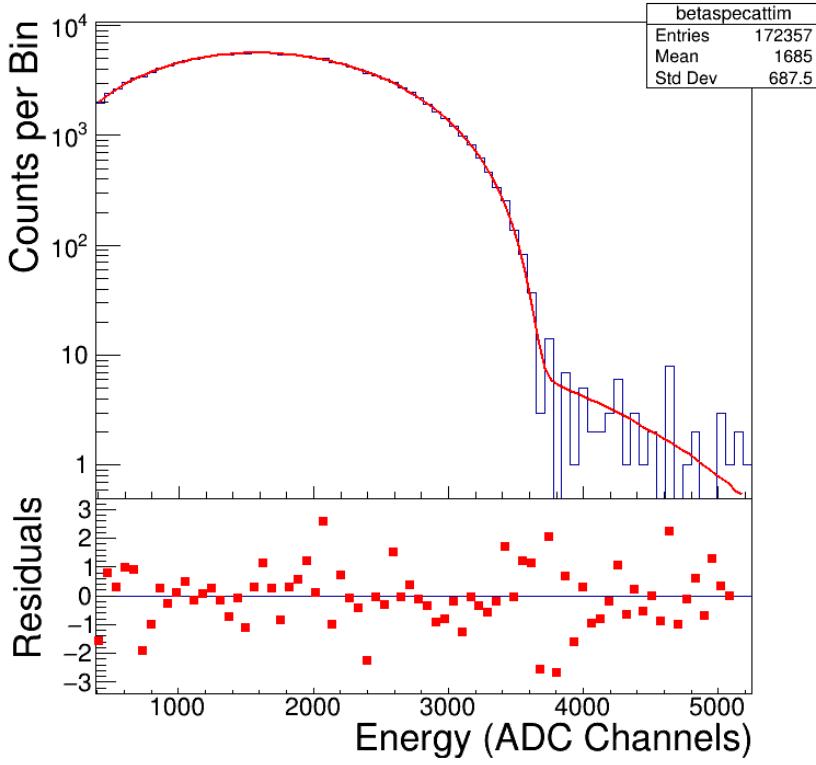


Figure 8.2: A sample fit of the beta spectrum. The residuals of the fit are shown below the graph

### 8.3.1 Lower Beta Cut Effect

The lower beta cut effect is the systematic shift of both the  $b_{WM}$  and  $b_{GT}$  as a function of where the fit started. There are several different input factors that can induce such an effect.

#### 8.3.1.1 The Offset

Initially, the offset used was the offset from the calibration in equation 8.3. The offset used was 10.05 ADC units. This calibration was determined using photons instead of electrons. It also doesn't probe the same location as where the beta electrons are.

When applying an offset of 10.05 in a fit of the data, it was found that the output varied greatly as a function of where the fit was started. For these graphs, the end of the fit (or upper beta cut) was set to 7500 keV. The detector is known to be non-linear below 200

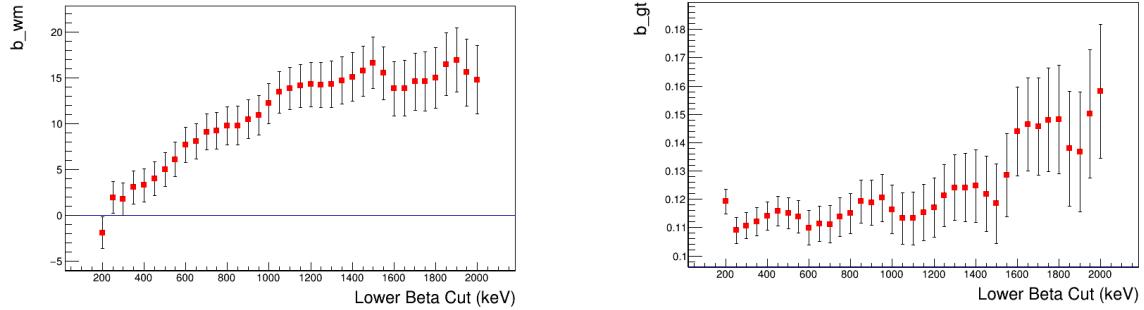


Figure 8.3: The dependence of the parameters to the start of the fit. The offset here is 10.05.

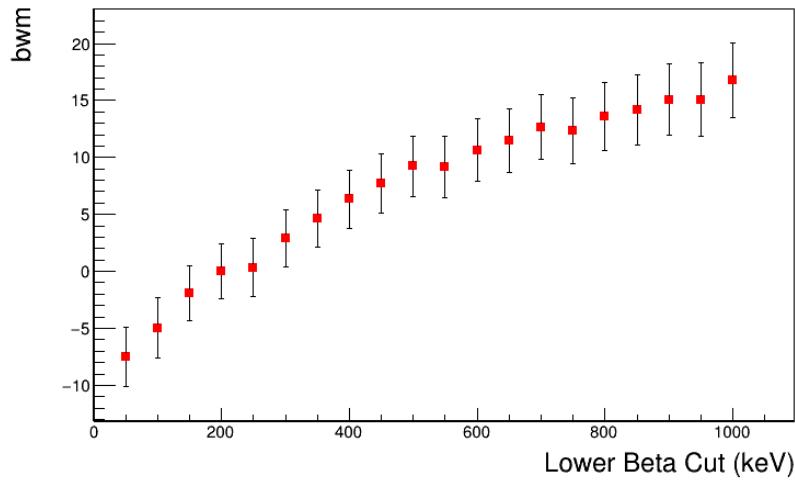


Figure 8.4: The effect of the lower beta cut in a simulation when the applied offset is 10 units higher than the real offset.

keV, so any fit starting below that point is suspect. After 2000 keV, the shape of the beta spectrum makes getting a good fit unlikely. The lower beta cut was varied between these two end points. The effect in  $b_{WM}$  and  $b_{GT}$  is shown in figure 8.3.

To test the effect of the offset, a Monte Carlo was written. A sample histogram with only the phase space and the shape factor was generated. These histograms were generated with an offset of 10 ADC units. A random gain was applied to the psuedo-data histogram. Then, the fit function had a different offset applied to it. For example, fitting the histogram generated with an offset of 10, and fit with an offset of 20 gives an effect as seen in 8.4.

This effect can be exploited to determine an offset using only the data. The size of the

slope induced is linear compared to the offset. By adjusting the offset and noting the slope of the  $b_{WM}$  vs the lower beta cut and plotting against the offset, the location of zero slope can be calculated. This graph is shown in figure 8.5. The resulting  $b_{WM}$  and  $b_{GT}$  curves

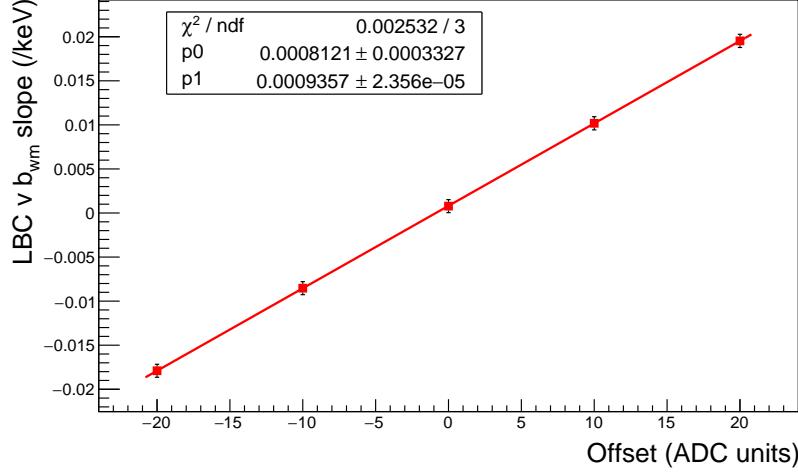


Figure 8.5: How the slope of the  $b_{WM}$  v LBC curve changes with offset. A slope of 0 corresponds to an offset of -0.87

are seen in figure 8.6.

To estimate the uncertainty on the offset, the statistical uncertainty of the slope of the lines fit to the  $b_{GT}$  was taken. The offset was varied and how the slope of the line changed as the offset changed was recorded. Then, the uncertainty of the slope was changed into an uncertainty on the offset using this information. This gave an uncertainty of 0.8 on the

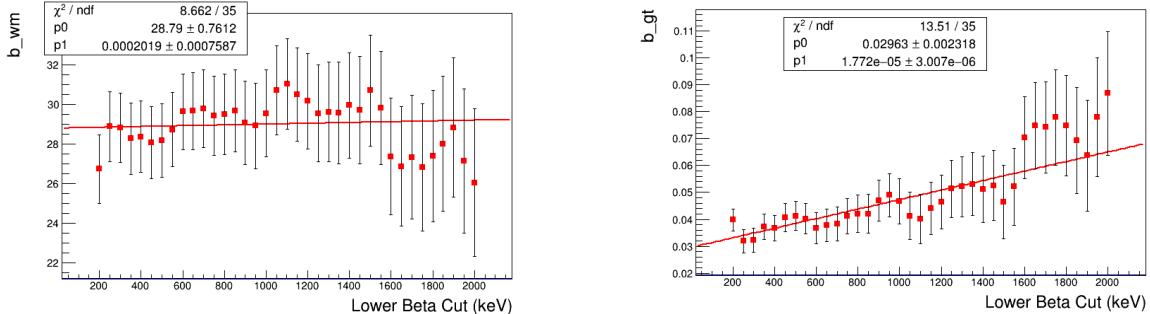


Figure 8.6: The dependence of the parameters to the start of the fit. The offset here is 0. The  $b_{WM}$  is independent of the start of the fit, but the  $b_{GT}$  still has a large effect

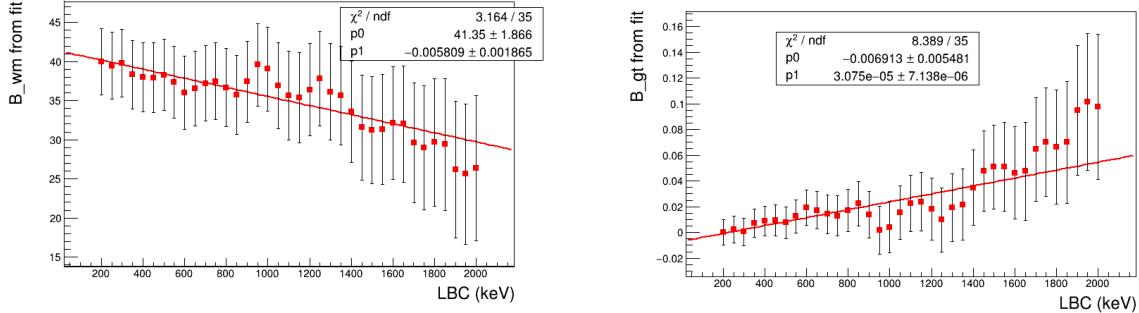


Figure 8.7: Monte Carlo data with a LBC effect due to the beta-gamma sum spectrum

offset. As seen in figure 8.6, the  $b_{GT}$  still depends greatly on where the fit starts. Other effects are important.

### 8.3.1.2 The Relative Normalization of the Beta-gamma Sum Spectrum

The output of the GEANT4 Monte Carlo was processed into two different parts. The two parts were summed with a factor of 1 in front of the beta-gamma sum spectrum in figure 8.6. This relative normalization is given from GEANT4. Using the same Monte Carlo, this factor was adjusted in the pseudo-data and the fitting functions. It was found that an effect as seen in the data can be produced by multiplying the beta-gamma sum spectrum in both the data and the fit function induced a shift very similar to that in figure 8.6. This is seen in figure 8.7.

Here both the pseudo-data and the fitting functions had their background multiplied by five. The input values of the  $b_{WM}$  and  $b_{GT}$  can be recovered by this fit. However, to do this, the offset in the fit function is set to 8.6 ADC units. The sum spectra in each of the 4 histograms is increased to 3.8 times that of output of the Monte Carlo. These two parameters are not the input parameters. The input offset was 0 and the input background level was 5.

Adjusting the level of the beta-gamma sum spectrum affects the  $b_{GT}$  more than the

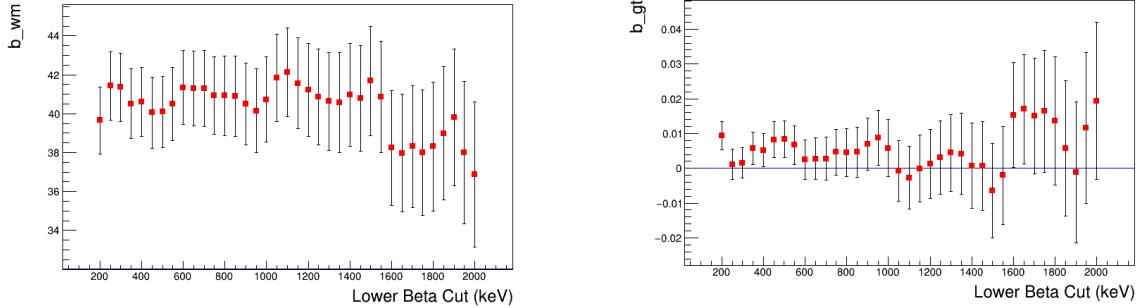


Figure 8.8: Adjusting the beta-gamma sum spectrum level to flatten out the  $b_{GT}$

$b_{WM}$ . This parameter can be adjusted to make the  $b_{GT}$  independent of the start of the fit in figure 8.6. This gives the results in figure 8.8.

However, to obtain these results, the level of the beta-gamma sum spectrum must be set to 0.096. It is unlikely that the GEANT4 Monte Carlo is that far off, and that this number corresponds to anything physical. It is unclear if this procedure is a valid one to take. This has to be investigated further and solved.

### 8.3.1.3 The Convolution

Another potential effect is if the detector resolution was applied incorrectly. To test that, a Monte Carlo simulation was written. The space space in equation 3.5 was multiplied by the shape factor in equation 3.29. This function was sampled  $10^9$  times twice to build two histograms. Both histograms had a detector resolution applied as described in section 8.1.1. One histogram was sampled a further  $10^6$  times 10 times to build a sample data set. The other convoluted histogram was used to fit the generated data set. The first histogram's  $A$  was fixed, while the second histogram's  $A$  was adjusted. When the sample data's  $A$  matched the  $A$  that were used, no dependence of  $b_{WM}$  on the start of the fit. When the  $A$  of the data sample and the  $A$ , the first thing that happened was that the  $b_{WM}$  changed. If the  $A$  differ by a factor of 2 or more, a shift in  $b_{WM}$  appears. However, this is much larger than any

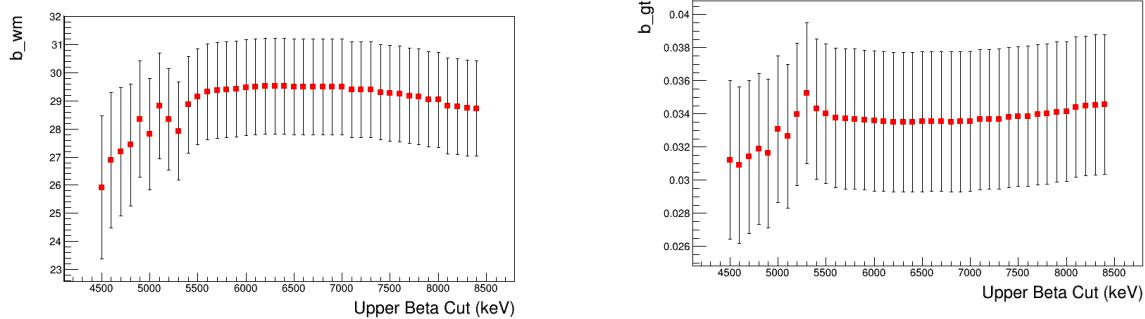


Figure 8.9: The effect of changing the upper beta cut

uncertainty in detector resolution. The effect of a wrong detector resolution is negligible.

### 8.3.2 Upper Beta Cut Effect

Changing where the fit ends has effectively no effect on the value either parameter. An issue is that at small enough upper beta cut, there is nothing to fix the pile-up parameter on. This means that there could be correlations induced. To avoid this, the pre-fit was done, and the level of pile-up recorded. The level of pile-up was fixed for the other fits across all upper beta cuts. The lower beta cut was fixed to 250 keV in the fits. The effect is shown in figure 8.9. Both parameters are flat after 5500 keV.

### 8.3.3 Other Systematic Uncertainties

The lower beta cut effect is the largest of the systematic uncertainties. Many of the other systematic uncertainties are independent of the absolute value of  $b_{GT}$ . To calculate these uncertainties, the lower beta cut was fixed to 250 keV. The offset was set to 0, and the level of the beta-gamma sum spectrum was set to 1. These parameters give the conditions in figure 8.6. With these values, the value of  $b_{GT}$  is  $0.0350(42)_{stat}$ . The value of  $b_{GT}$  is  $28.3(1.7)_{stat}$ .

For the calculation of the systematic uncertainties, one parameter at a time was adjusted. The parameter was increased by one unit of the error on that parameter. The fit was redone with the different parameter values. The output  $b_{WM}$  and  $b_{GT}$  for the changes in the parameters were recorded. The parameter then was then decreased by one unit of the error. Half the difference of the output  $b_{WM}$  and  $b_{GT}$  were the systematic error.

As stated above, the different systematic uncertainties can be divided into three different categories. There are the systematic uncertainties due to the simulation inputs, which are summarized in table 8.3 These were all calculated by running the four histograms with  $2*10^9$  events each.

Table 8.3: Systematic uncertainties due to simulation inputs

Parameter Name	Parameter Value	Parameter Uncertainty	Systematic Uncertainty $b_{WM}$	Systematic Uncertainty $b_{GT}$
Electron end point energy	5900.864 keV	0.081 keV	0.2	0.0012
Charge radius	3.0055 fm	0.0021 fm	0.2	0.0003
Convolution	1.112	0.008	0.3	0.0004

The uncertainty due to the average mass  $M_{ave}$  was ignored, as it is very small. The value of the average mass used is  $18621497.033 \pm 0.040\text{keV}$ . Since this value appears in the denominator, and the uncertainty is so small, the effect of the uncertainty due to this average mass value is negligible. On the other hand, an uncertainty in the end point can have a larger effect. The endpoint energy of the electron is only around 5 MeV.

The systematic effects due to the fit inputs are shown in table 8.4. The systematic effect on the offset is linked to the lower beta cut effect and not listed yet. The normalization of the beta-gamma sum spectrum is linked and not listed. They still need to be investigated. There is no systematic upper beta cut effect.

Table 8.4: Systematic uncertainties due to fit inputs

Parameter Name	Parameter Value	Parameter Range	Systematic Uncertainty on $b_{WM}$	Systematic Uncertainty on $b_{GT}$
Lower beta cut	250 keV	200 to 2000 keV	N/A	0.0283
Offset	0	-0.8 to 0.8	2.0	0.0055
Binning of data histogram	32	16 to 64	0.6	0.0003
Binning of fitting histograms	32	16 to 64	0.2	0.0007

Lastly, there are the systematic uncertainties due to the form factors inside the shape factor. These are summarized in table 8.5

Table 8.5: Systematic uncertainties due to nuclear form factors

Parameter Name	Parameter Variable	Parameter Value	Parameter Uncertainty	Systematic Uncertainty on $b_{WM}$	Systematic Uncertainty on $b_{GT}$
Weak magnetism	$b_{WM}$	43.4	1.2 [24]	N/A	0.0028
Induced tensor form factor	$d$	40.5	3.7 [24]	0.2	0.0004
Fermi matrix element	$c_{1nuc}$	0.253	0.004 [24]	0.5	0.0017
Second forbidden matrix element	$c_{2nuc}$	$0.755 \text{ fm}^2$	$0.257 \text{ fm}^2$ [8]	0.2	0.0011

The largest uncertainty is due to the lower beta cut effect. The second largest uncertainty is due to the uncertainty of the offset.

# Chapter 9

## Conclusion

Due to uncertainties in how to deal with the gamma-beta sum spectrum, the central value of the Fierz term is not ready to be stated. However, most of the systematic effects can still be calculated. The effect with the lower beta cut is likely due to the normalization of the beta-gamma sum spectrum. The issue is a solvable one.

### 9.1 Discussion of Results

The statistical uncertainty of  $b_{GT}$  is 0.0042. This is with 7 million events that pass the gamma coincidence. If the lower beta cut uncertainty is excluded, the total systematic uncertainty is 0.0066. This also excludes any uncertainty on the normalization of the gamma-beta sum spectrum. This is dominated by the uncertainty on the weak magnetism. If the lower beta cut uncertainty can be resolved, the total systematic uncertainty should only be a touch higher than 0.0066. This is very close to the statistical uncertainty.

To lower the systematic uncertainty, the weak magnetism should be known to a more precise value. The uncertainty of the weak magnetism comes from how it is calculated. The parameters need to calculate the weak magnetism are shown in equation 3.32 . Most of the uncertainty comes from the gamma decay strength [24]. A better measurement of this gamma decay strength would reduce the effect of the weak magnetism.

## 9.2 Results Compared to Other Measurements of the Fierz Term

A similar measurement was done with ultra-cold neutrons [17]. The beta energy spectrum was measured using a magnetic spectrometer. The neutrons were confined in the center with a magnetic field, and allowed to decay. A superratio was used to extract the Fierz term. With the systematic effects, the final results of the Fierz term was  $0.067 \pm 0.005_{stat}^{+0.090}_{-0.061} sys.$ . Even with the lower beta cut effect, the systematic uncertainty of the Fierz term in the  ${}^{20}F$  measurement is much smaller. The statistical uncertainty is comparable. Additionally, since the decay of a neutron is a mixed decay, this Fierz term is sensitive to both the tensor couplings and the scalar couplings. The beta spectrum is shown in figure 9.1.

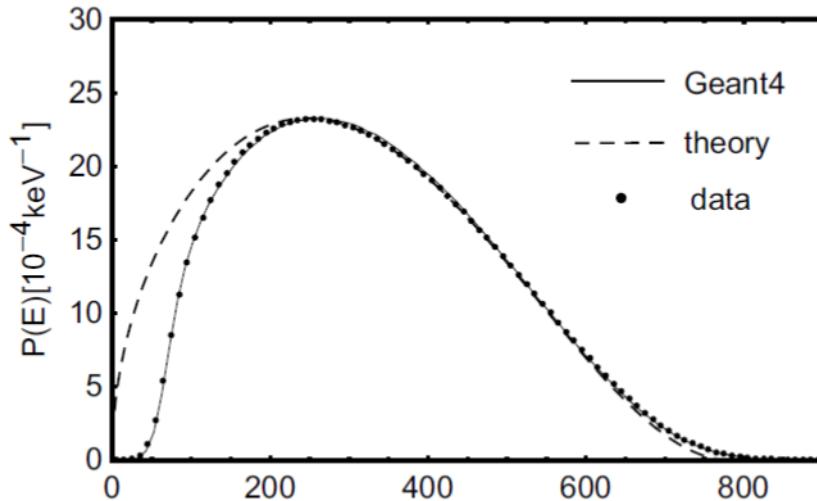


Figure 9.1: The energy spectrum of the ultra-cold neutron measurement [17]

There are large distortions at low energy due to back-scattering effects. The systematic uncertainty quoted due to backscattering effects is  $\pm 0.005$ , which is larger the statistical uncertainty of this  ${}^{20}F$  measurement. The largest systematic uncertainty in the neutron measurement was due to uncertainties in the detector calibration. The fluorine measurement

avoids this, as the gain is left as a free parameter.

Another method to get the Fierz term is by measuring  $ft$  values. From the superallowed beta decays, the result for the Fierz term is  $-0.0028 \pm 0.0026$  [15]. It was obtained with  $ft$  value measurements of 14 different nuclei. The uncertainty is the statistical uncertainty of the fit after the  $ft$  values were corrected. However, this Fierz term is sensitive to scalar couplings, and not tensor coupling like in the  $^{20}\text{F}$  measurement. It had different systematic effects, and it took 14 different measurements to come up with this number. Even with all this, the potential sensitivity to the implant technique is similar to that of 14  $ft$  value measurements.

A different measurement with sensitivity to tensor couplings was made with  $^{60}\text{Co}$  [31]. This measurement measured the beta asymmetry. Then, this asymmetry was compared to the standard model value. This was done with a large magnet. The largest uncertainty was due to the size and location of the source. The limit on the tensor couplings was between  $-0.089$  and  $0.013$ . Assuming that  $C'_A = 0$ , this corresponds to a similar error bar in the Fierz term. This uncertainty is larger than the uncertainty due to the lower beta cut effect.

In order to compare this to high energy techniques, equation 1.8 is used [14]. If the lower beta cut effect is resolved and the systematic effects described above accurate, then the total uncertainty on  $b_{GT}$  is  $0.0055$ . This corresponds to an uncertainty on  $\text{Re}(\epsilon_t)$  of  $0.0009$ . This is twice the uncertainty from the neutral current channel at the LHC. However, to reach this uncertainty, all that would be needed would be to run the experiment again with higher statistics. Then, the statistical uncertainty would be negligible, and the systematic uncertainty would be the limiting factor.

### 9.3 Further Refinements to Technique

The first thing to do to improve the experiment is to run more events. Most of the events ran for this experiment were with the PVT detector. However, the shape of beta spectrum in that detector had an odd distortion in it. If that could be investigated and solved, using a plastic scintillator would be an ideal candidate for this measurement. Less of the gamma ray would be absorbed, and there would be less bremmstrahlung in the detector.

The largest systematic effect right now is the lower beta cut effect. This can probably be solved by adjusting the normalization of the gamma-beta sum spectrum compared to the implant detector. This has to be seen that it works properly with Monte Carlo. Ultimately, the resulting offset and sum spectrum level probably are not the real, physical level. Tuning them on the stability of the results relative to the lower beta cut does give reasonable answers. Then, an estimate of the uncertainty to the offset and the level of the gamma-beta sum spectrum would have to be done. The offset is likely known to 0.5 and the level likely to 10%. This would likely be the largest systematic uncertainties. As these uncertainties depend on the slopes of the lower beta cut curves, these should get more precise as more events are built. The slope can be seen more easily as there is the error bars get smaller.

The gamma-beta sum spectrum is a trickier. If the slope of the  $b_{GT}$  is used, then at higher statistics, the same reduction could occur. Another method would be to compare the output of GEANT4 to that of another simulation code. This was done with EGSnrc, and agreement was found to  $\text{\texttt{!AGREEMENT VALUE!}}$ . Other packages and processes would be good.

In order to increase efficiency for the 1.6 MeV gamma ray, the implant detector could be more completely encased with gamma detectors. If these gamma detectors had a higher

resolution, that would mean that the signal in the implant would be much cleaner. If that were coupled with a plastic detector, then the beta spectrum of the implant detector would be much cleaner. This would hopefully eliminate the amount of gamma-beta sum spectrum.

If these systematic effects are reduced, the next largest one has to do with the uncertainty on the measurement of  $b_{WM}$ . There, a new and better measurement of the transition strength in  $^{20}\text{Ne}$  would have to be measured. Increasing the precision on the energy of the gamma rays would help both with the electron maximum energy and with the  $b_{WM}$ . This would require a different type of experiment.

## 9.4 Conclusion

With all this in place, this measurement of the Fierz term can be one of the more precise ones, if the issue with the lower beta cut can be figured out. Many of the systematic effects would be present in other such measurements. This measurement can be competitive with high energy measurements, but it took much less equipment.

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## BIBLIOGRAPHY

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