Introduction to Word Embeddings

Maximilian Mozes
Department of Security and Crime Science
University College London

maximilian.mozes@ucl.ac.uk

Agenda

1. Introduction

2. word2vec

3. GloVe

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- However, a single word can have
  - different meanings (*parts of speech*)

*Death in Venice is my favorite **novel**.*

*He came up with a **novel** idea.*

  - different meanings in different contexts

*Don't use the trackpad, just use your **mouse**.*

*My cat just caught a **mouse**.*

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Language is very complex!

Representing words

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- In computational settings, information are represented as numbers
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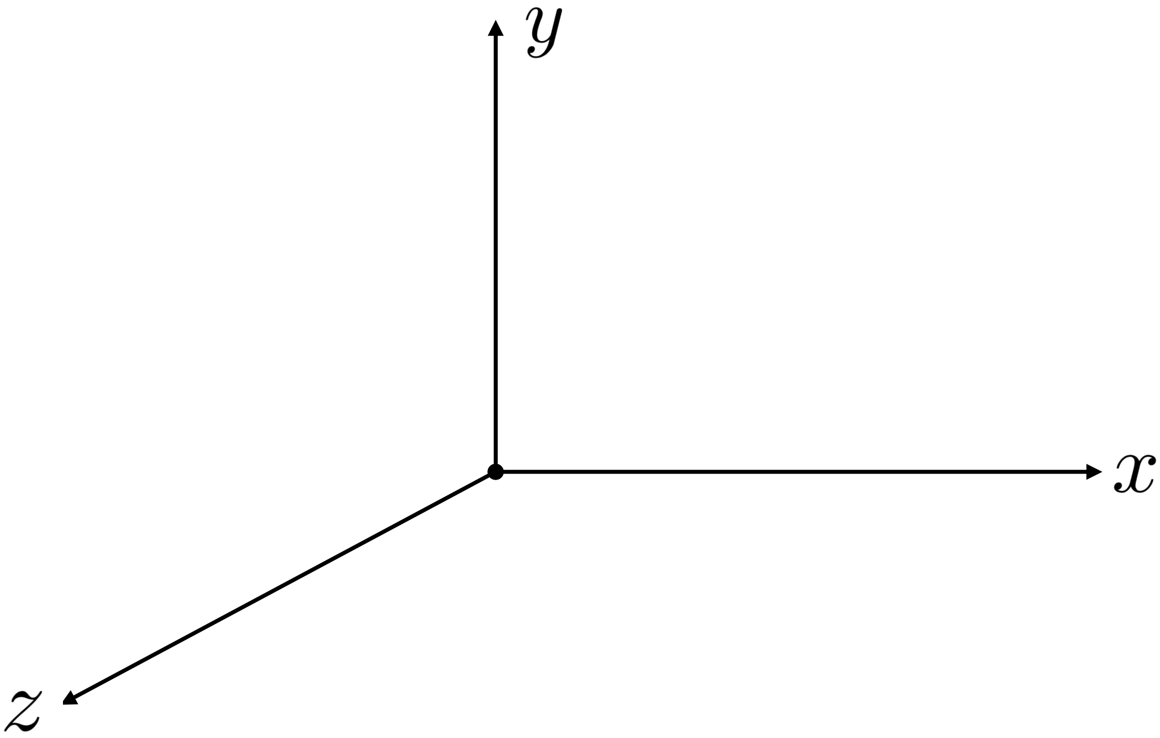
$$\text{mouse} \mapsto \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

Representing words

Word embeddings map words to lists of numbers (**vectors**)



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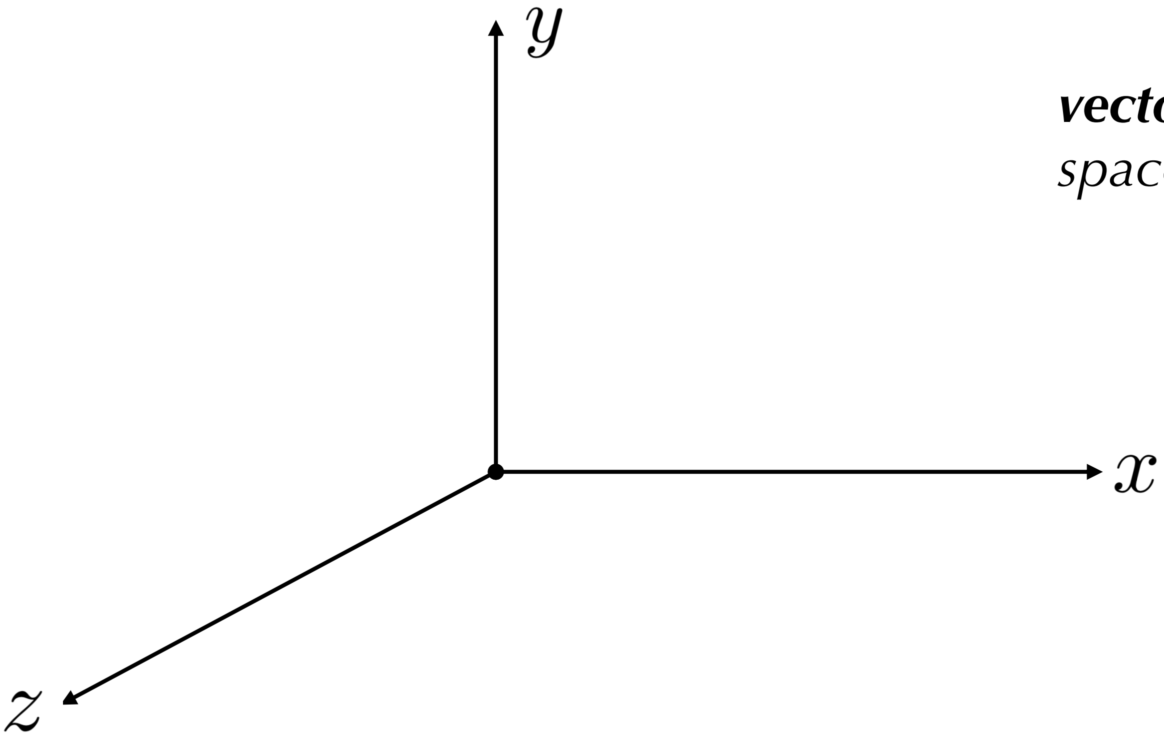

vector space:
space where vectors live

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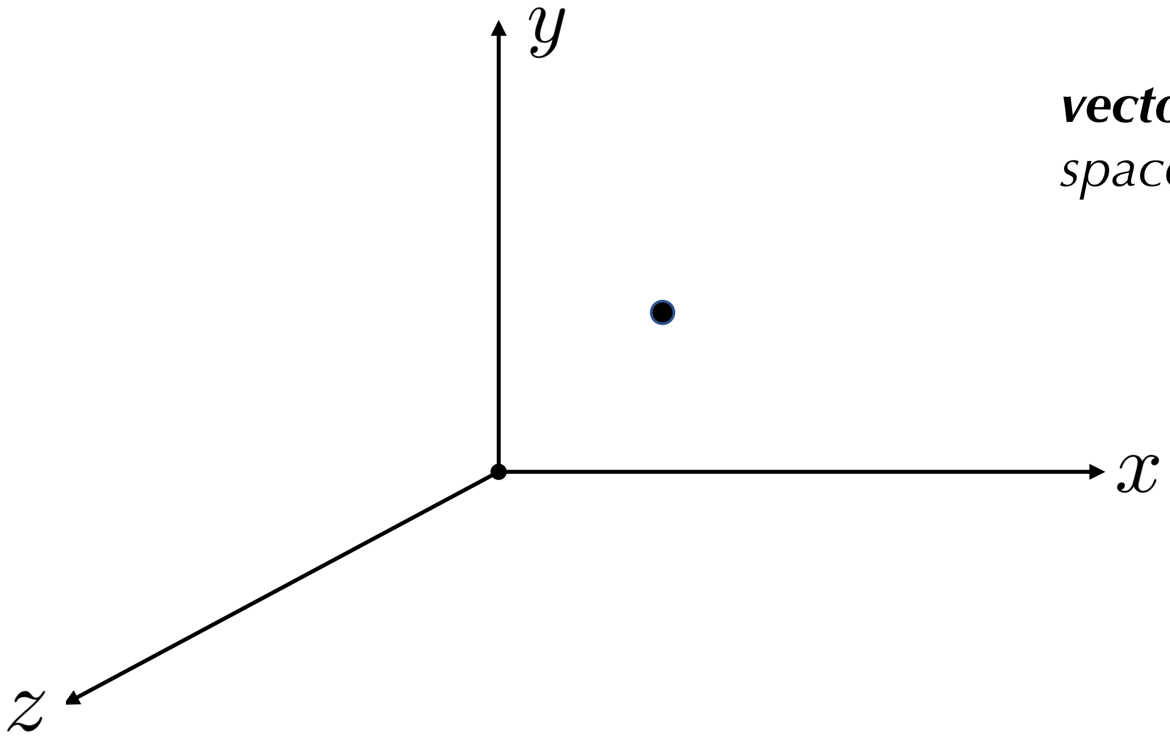

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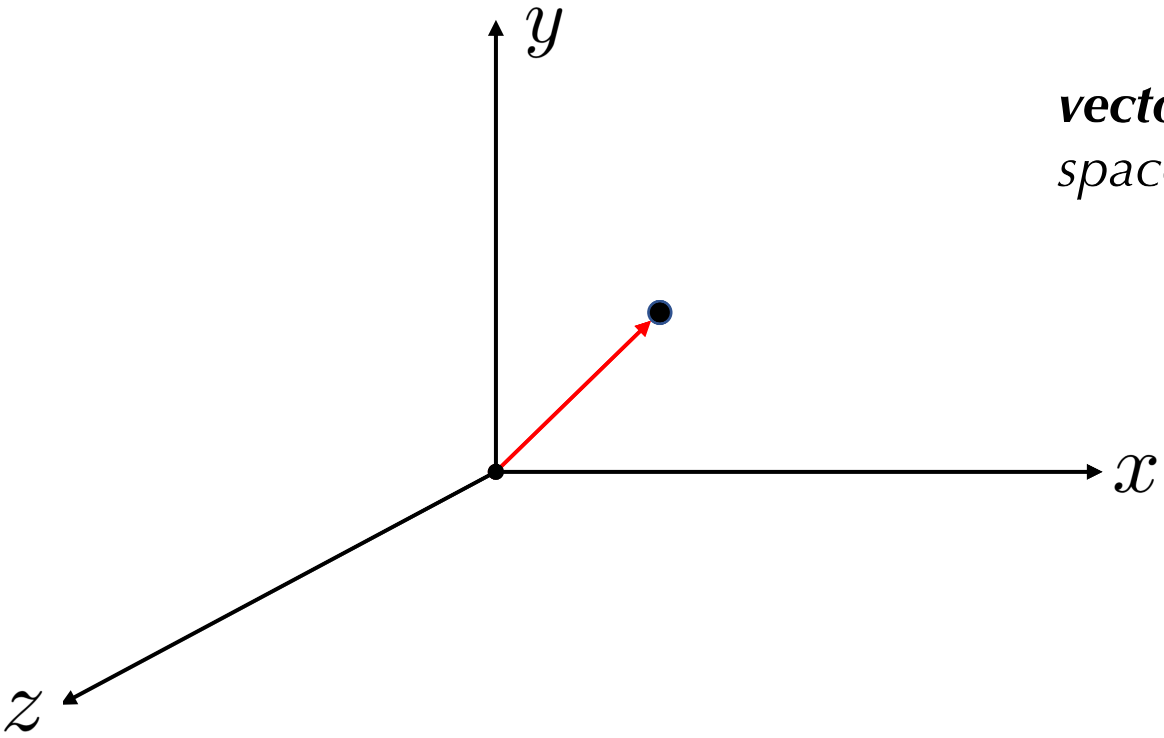

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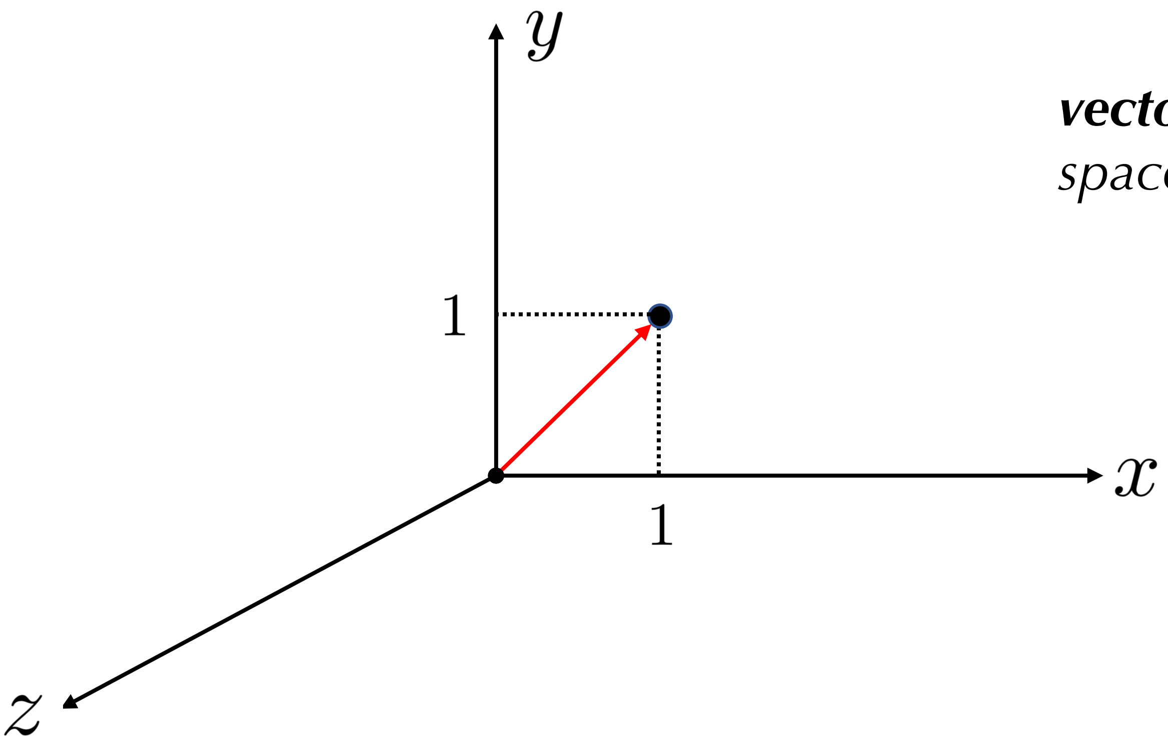

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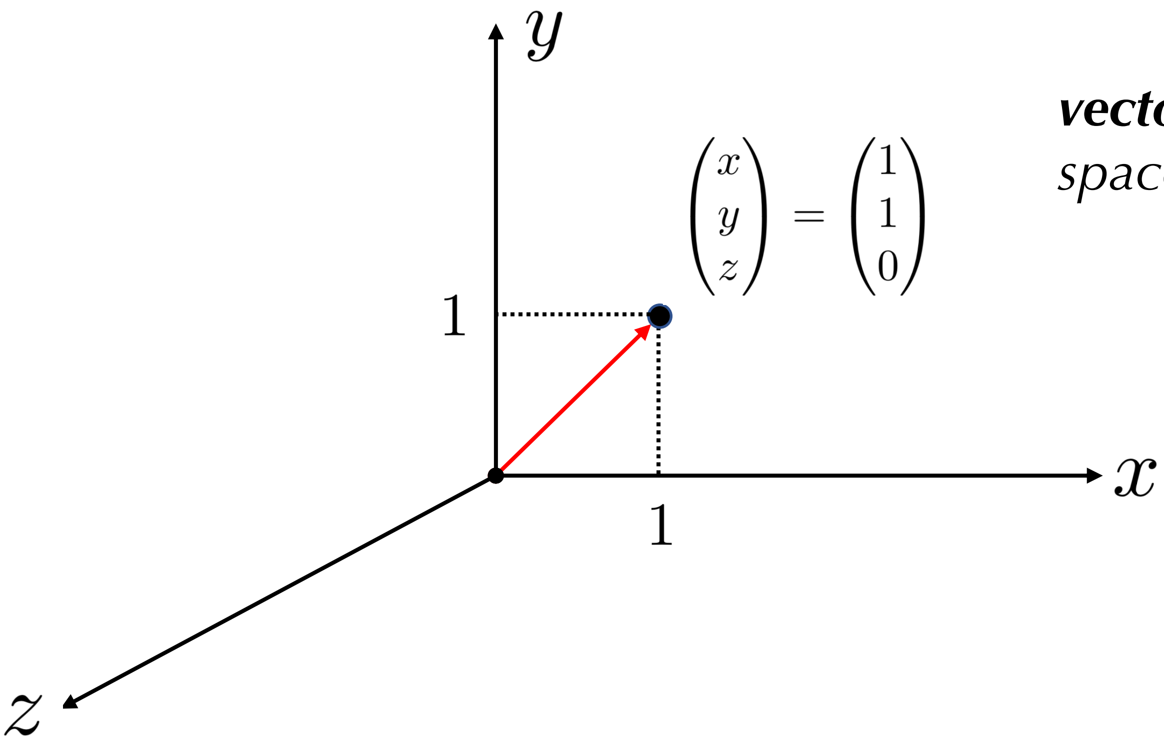

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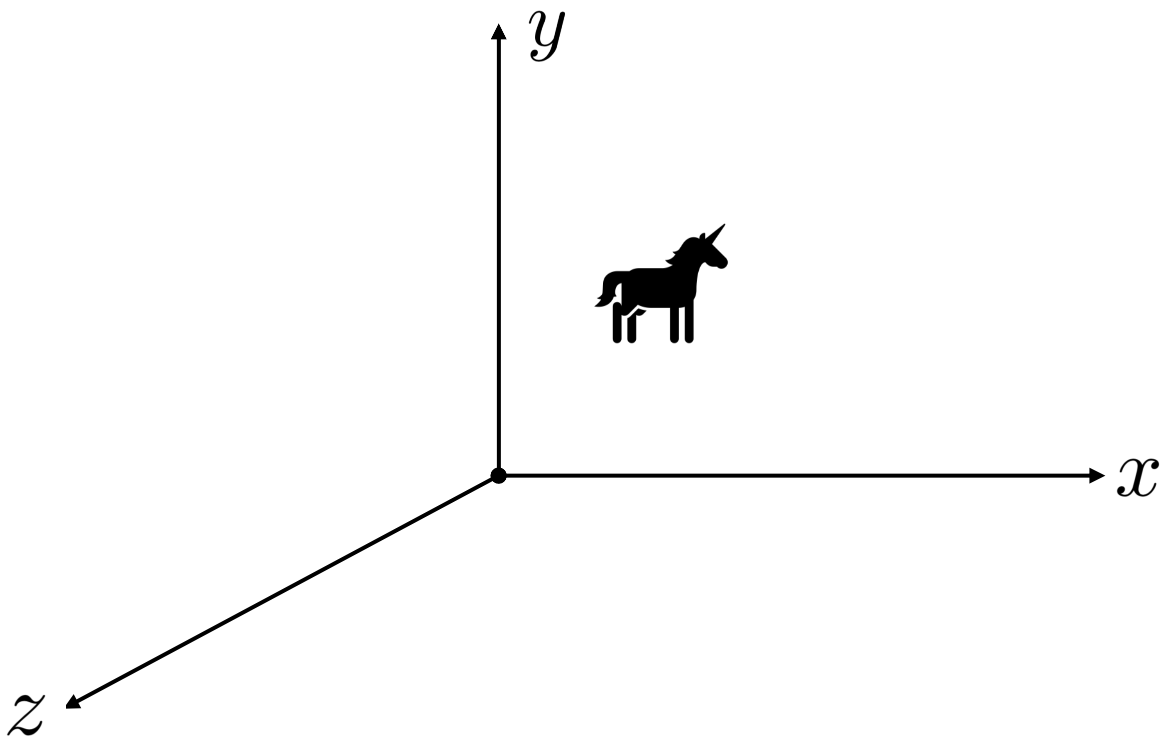

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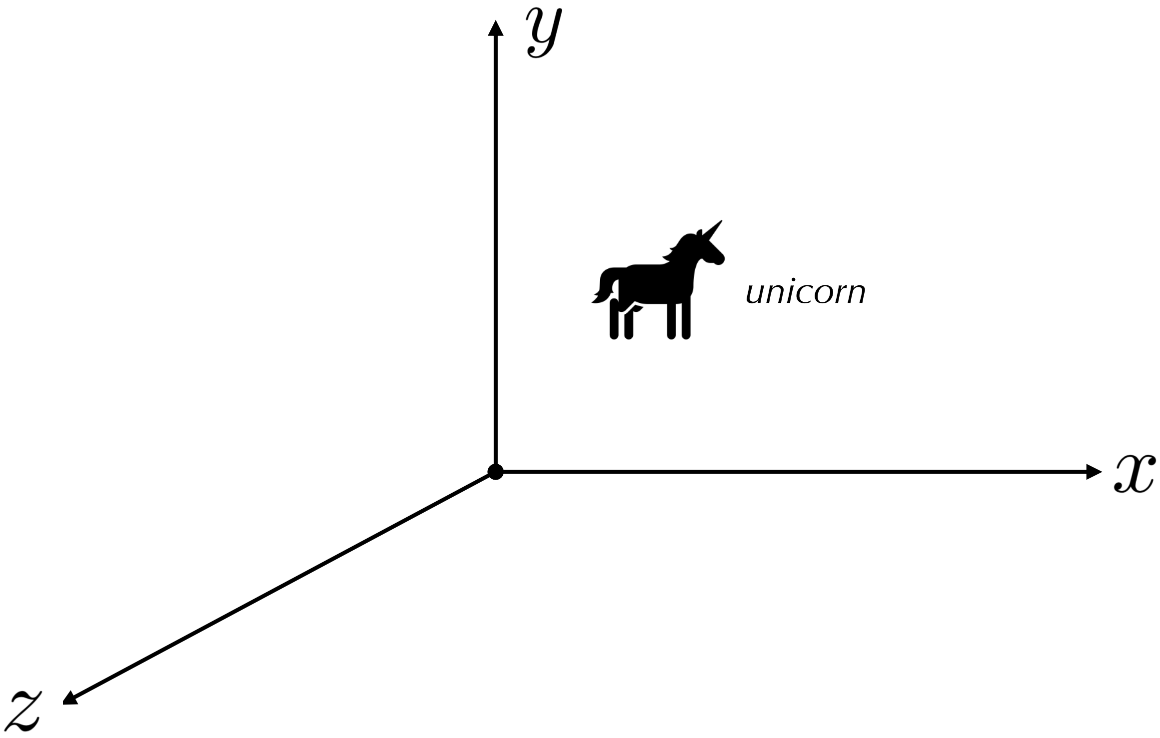

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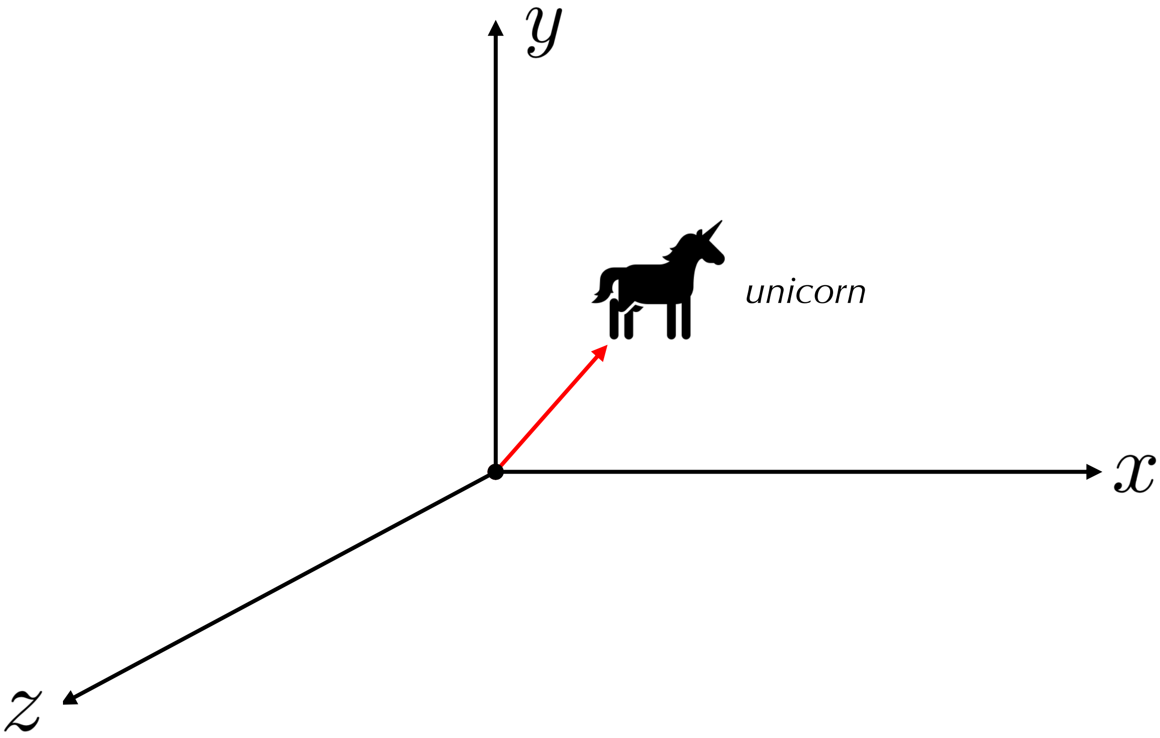

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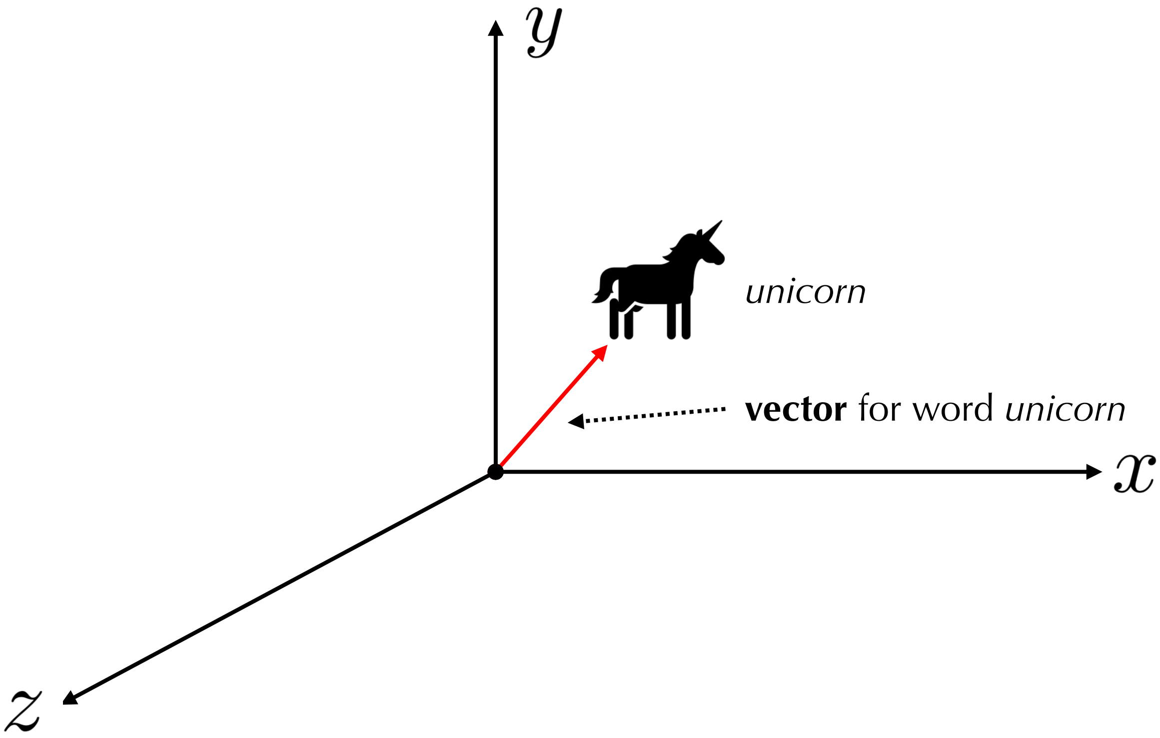

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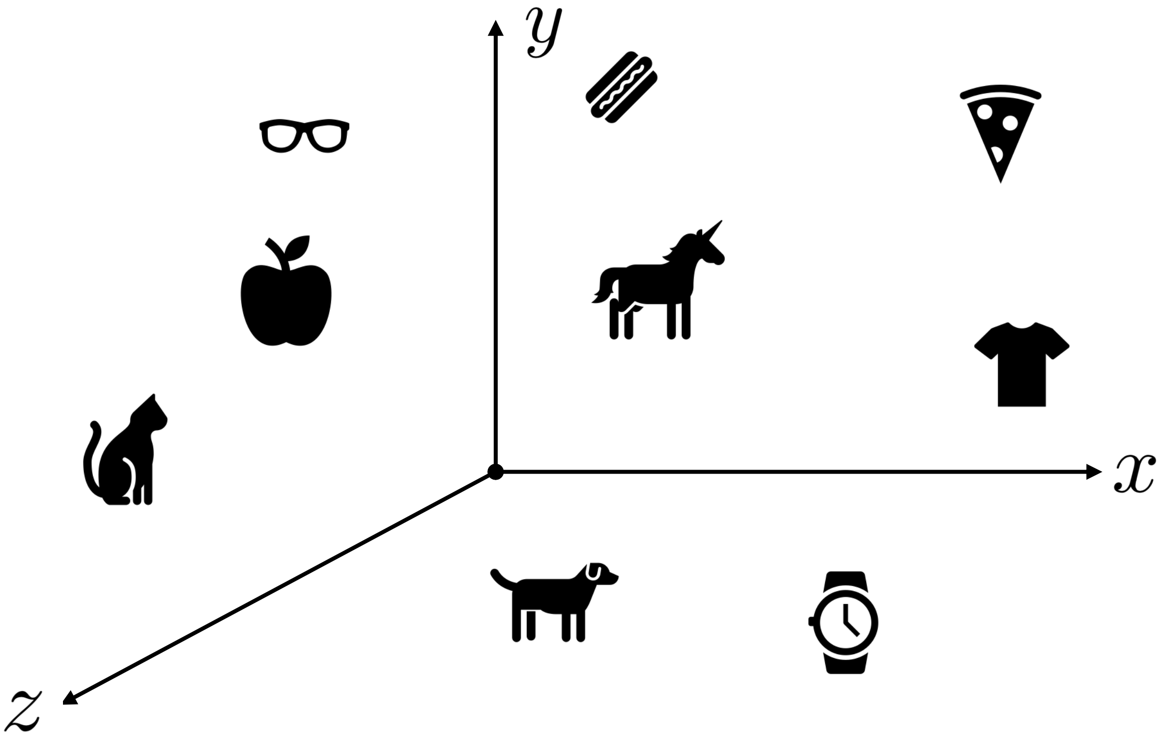

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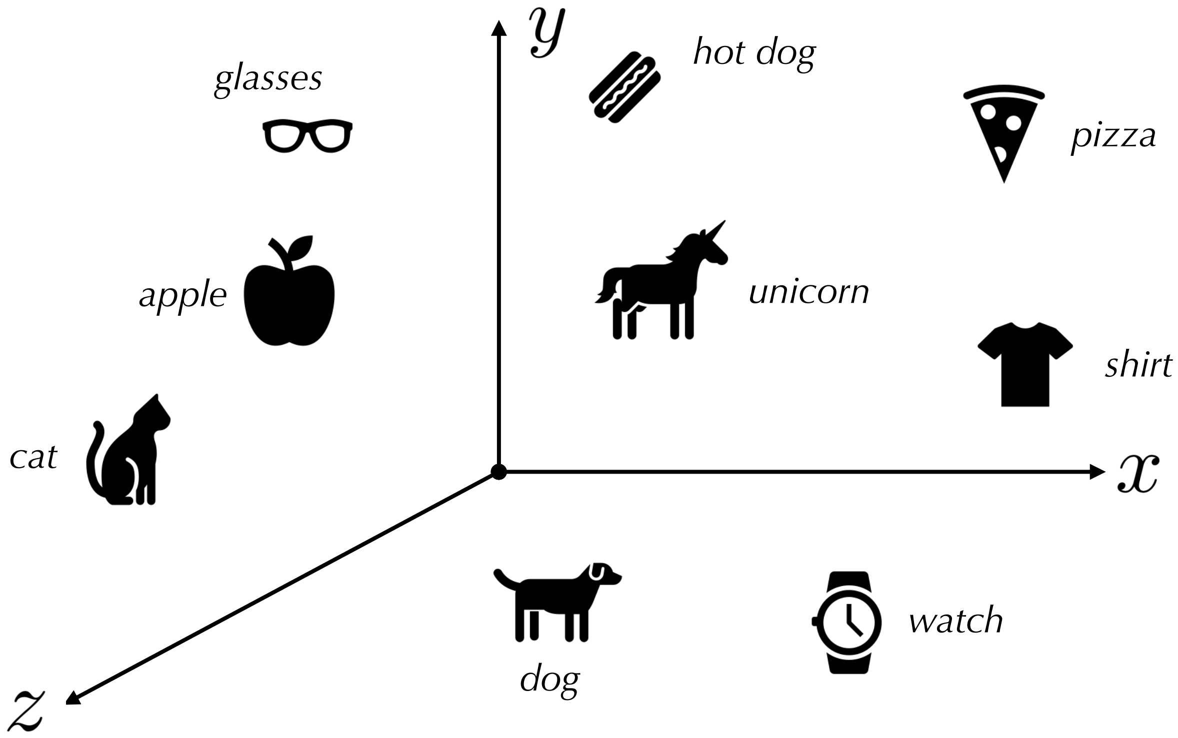
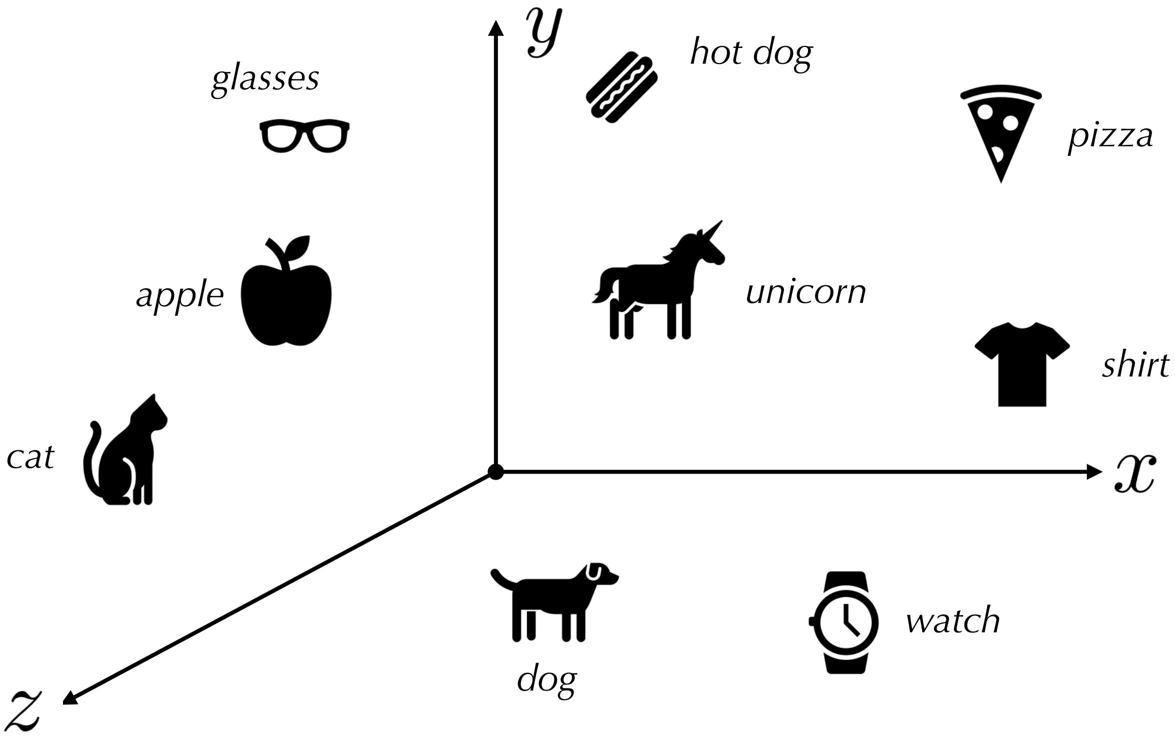


Representing words

However, different words have different semantic relations



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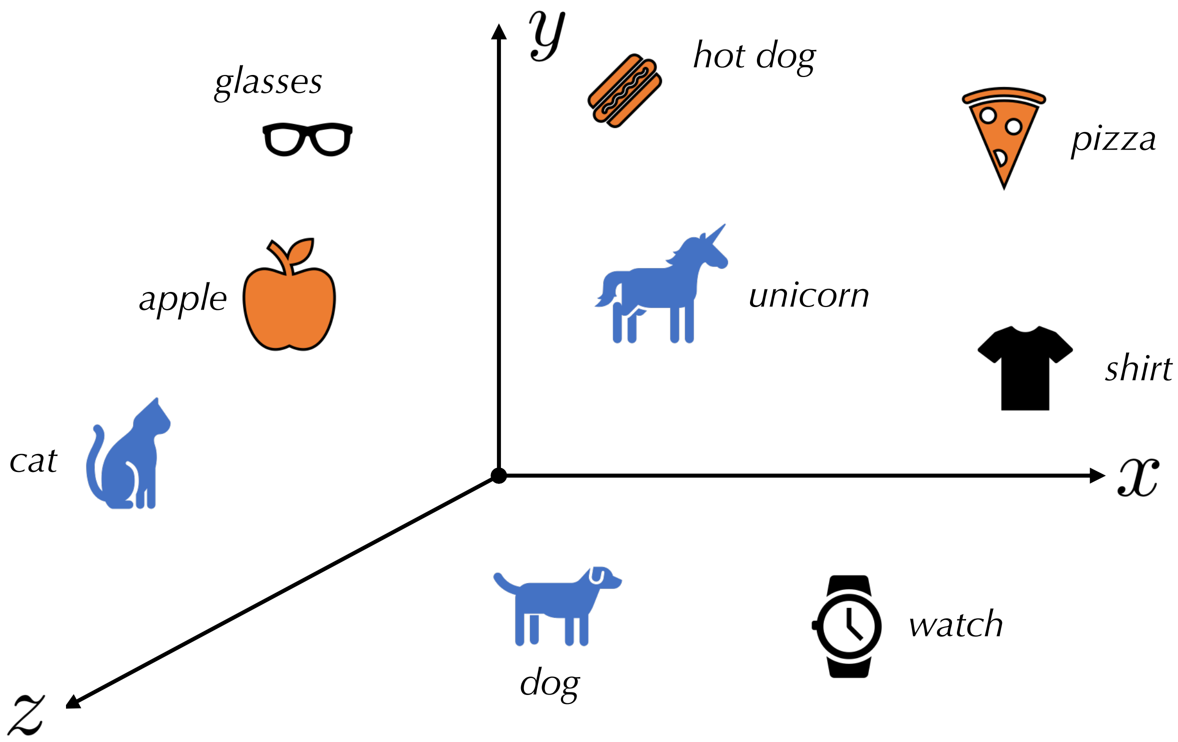

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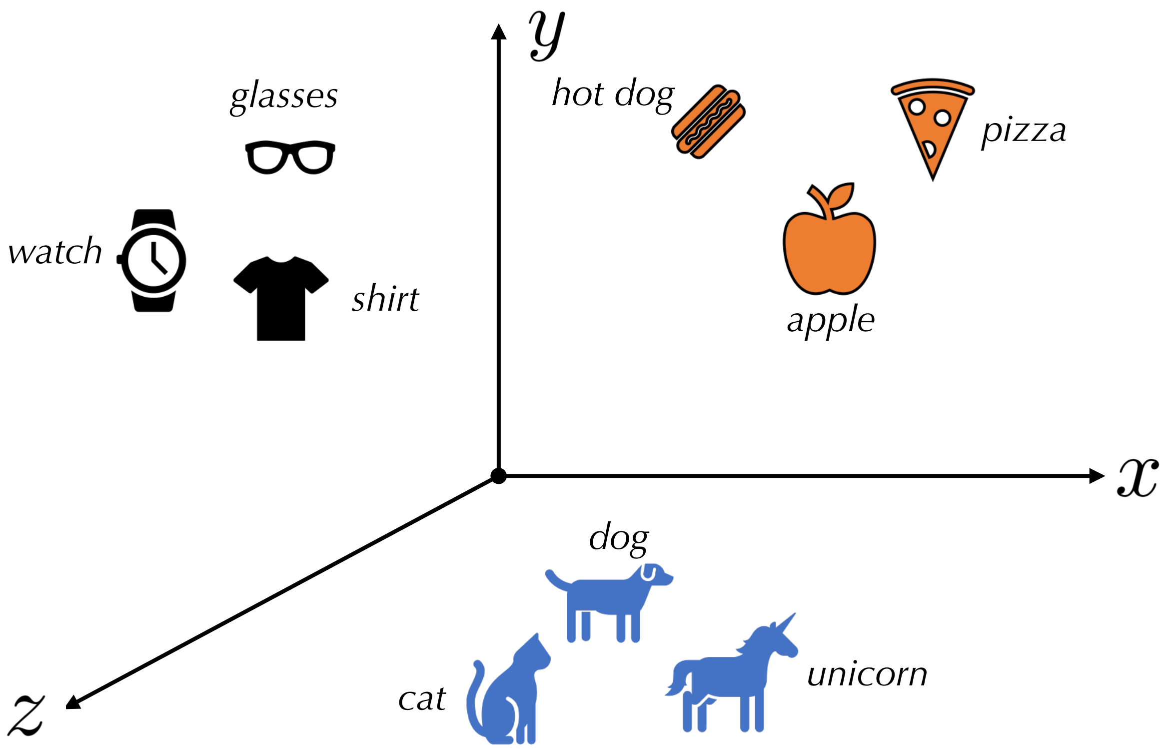
Word embeddings aim to bring related words together in this space



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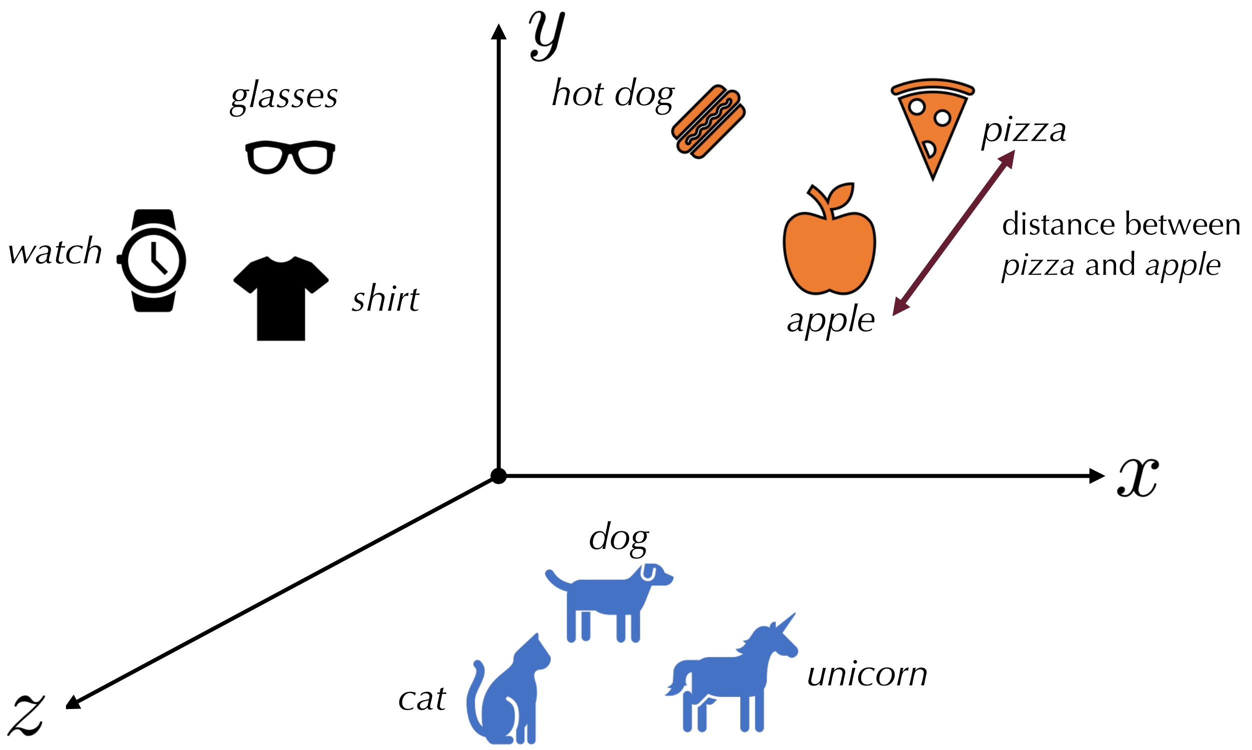

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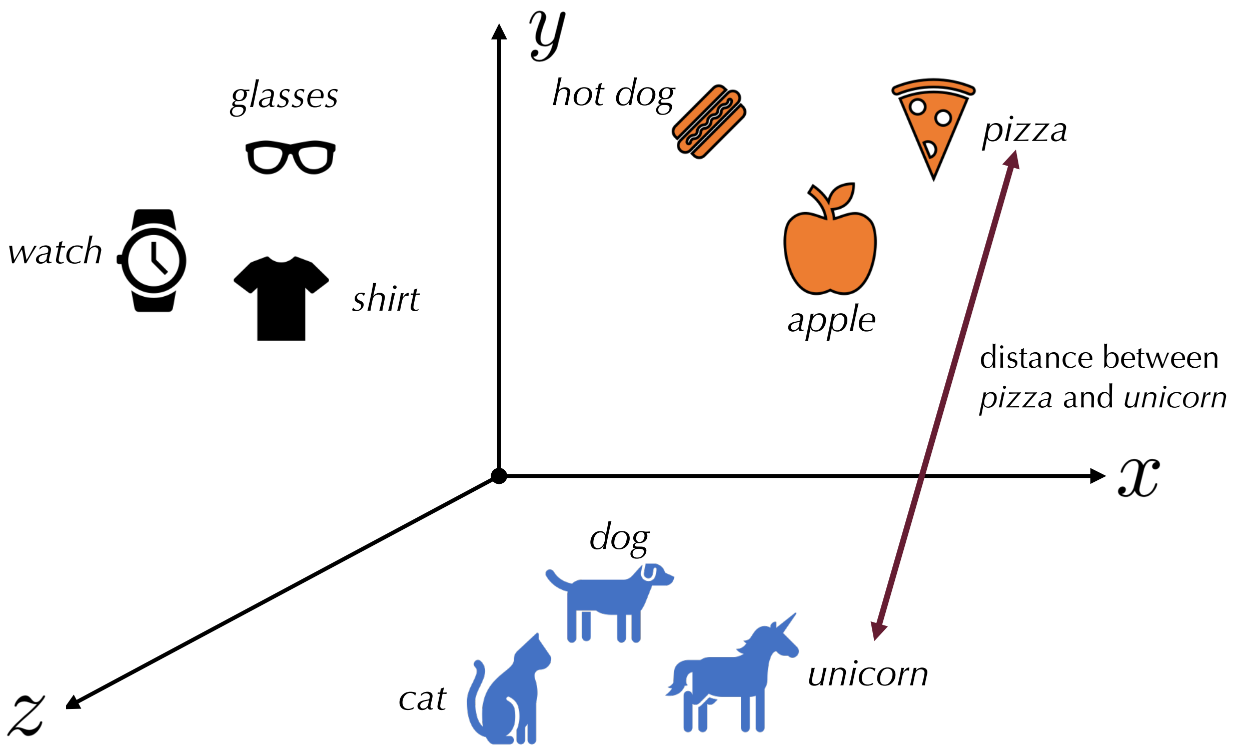

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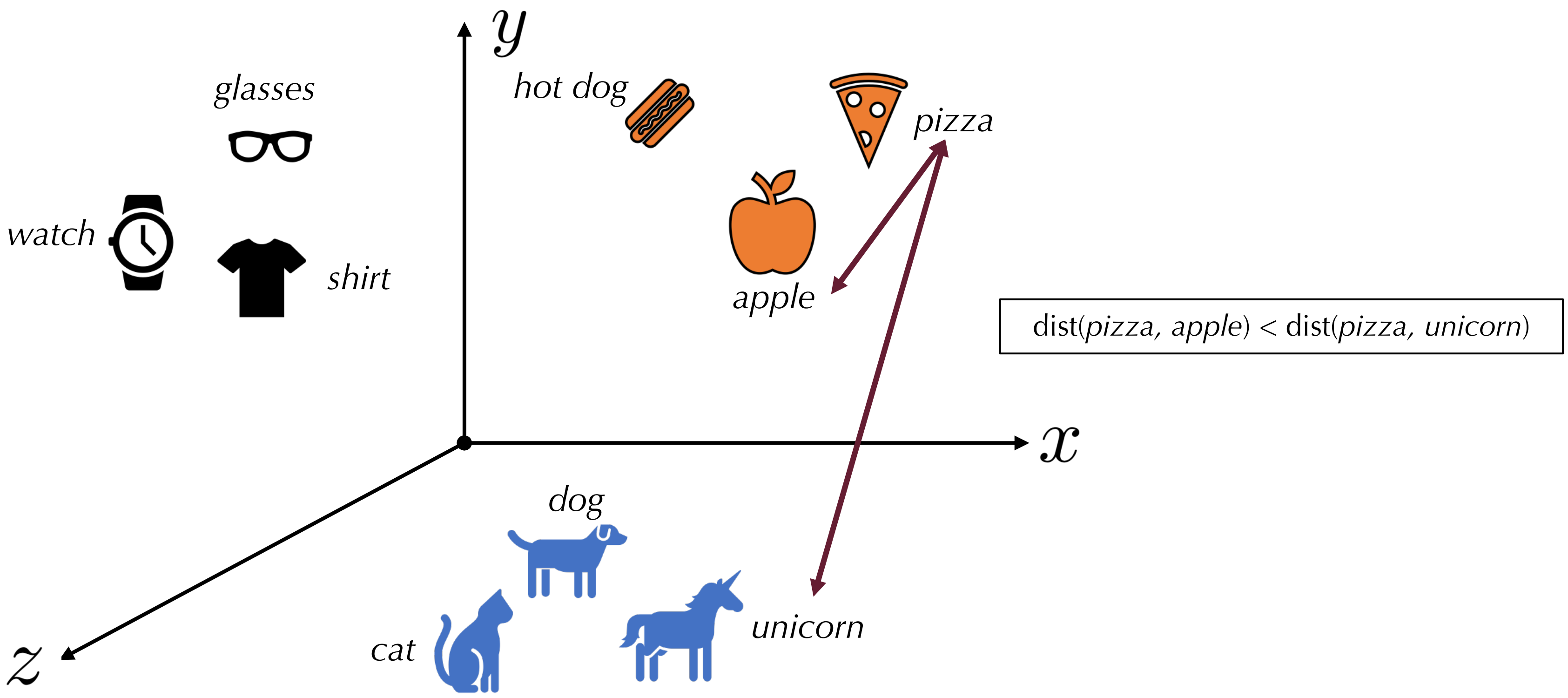

Terminology

- A **word** w is a concatenation of characters
- A **document** $\mathcal{D} = (w_1, \ldots, w_n)$ is a sequence of words
- The **vocabulary** V is the set of all words of consideration
- N is the **size of our vocabulary**, i.e. $|V| = N$
- A **word vector** $\mathbf{v}_w \in \mathbb{R}^D$ for a word w is an ordered list of D real numbers
- D is the **dimension** of a word vector $\mathbf{v}_w$

Representing words as vectors

- Assume we have a finite set of words $w_i \in V, i \in [N]$
- We assign each word to an N- dimensional "zero-vector" with a non-zero entry at index i

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- Example: for word w_7 , we define a vector $\mathbf{v}_{w_7}$ such that

$$\mathbf{v}_{w_7} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} 1-6 \\ \\ \left.\vphantom{\begin{matrix}0\\ \vdots\\ 0\end{matrix}}\right\} 8-N \end{matrix}$$

Representing documents as vectors

- If we now have an entire document $\mathcal{D} = (w_1, \ldots, w_n)$, we can compute a frequency vector $\mathbf{v}_{\mathcal{D}}$ of this document by adding the word vectors for each w_i

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$V = \{\text{what}, \text{do}, \text{vectors}, \text{have}, \text{to}, \text{with}, \text{words}\}, |V| = 7$

$$\{\text{what},\ \text{do},\ \text{vectors},\ \text{have},\ \text{to},\ \text{with},\ \text{words}\}$$

1 2 3 4 5 6 7

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$$\mathbf{v}_\mathcal{D} = \sum_{i=1}^{8} \mathbf{v}_{w_i} = \begin{pmatrix}1\\0\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\1\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\0\\1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\\0\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\0\\0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}1\\2\\1\\1\\1\\1\\1\end{pmatrix}$$

Representing documents as vectors

- This is called *bag-of-words* approach
- $\mathbf{v}_{\mathcal{D}}$ now represents a term-frequency vector that can be used as a feature (e.g. for a text classifier)

Our task seems to be solved! We can efficiently model entire documents with a single vector.

There are problems…

Representing words as vectors

Problems with this approach:

1. For large N, the individual vectors become very large and inefficient
2. The word vectors are semantically **unrelated** to each other

Unrelated here means, that we cannot say anything about the semantic relationships between words by looking at their vectors.

Similarity between vectors

- In Euclidean N- space, we can measure the similarity between different vectors
- This can be done by considering the dot product $\langle \mathbf{v}, \mathbf{w} \rangle$ between two vectors $\mathbf{v}, \mathbf{w} \in \mathbb{R}^N$, defined as

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- Geometric interpretation:

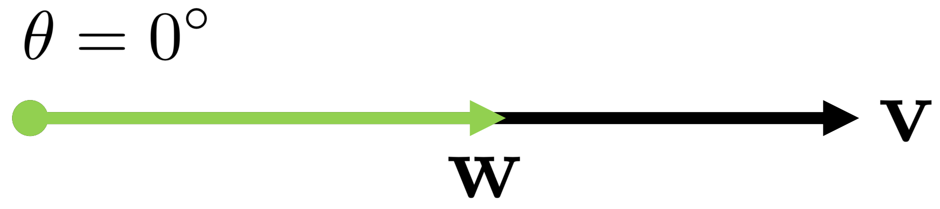
$$\cos(\theta) = \frac{\langle \mathbf{v}, \mathbf{w} \rangle}{||\mathbf{v}|| \cdot ||\mathbf{w}||} \qquad \text{hence} \qquad \langle \mathbf{v}, \mathbf{w} \rangle = \cos(\theta) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}||$$

Similarity between vectors

$$\langle \mathbf{v}, \mathbf{w} \rangle = \cos(\theta) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}||$$

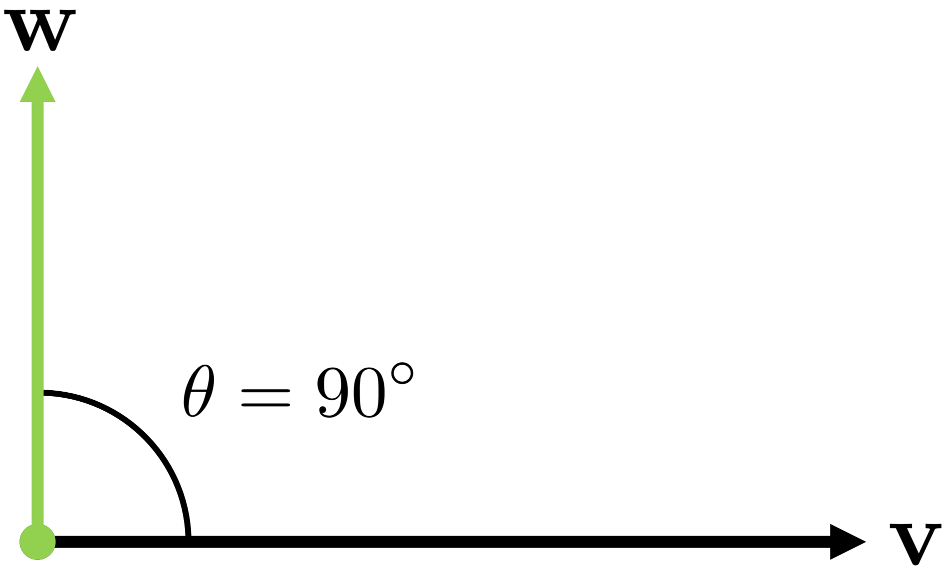
Note: $||\mathbf{v}|| = \sqrt{\sum_{i=1}^{N} v_i^2} \geq 0$

Similarity between vectors



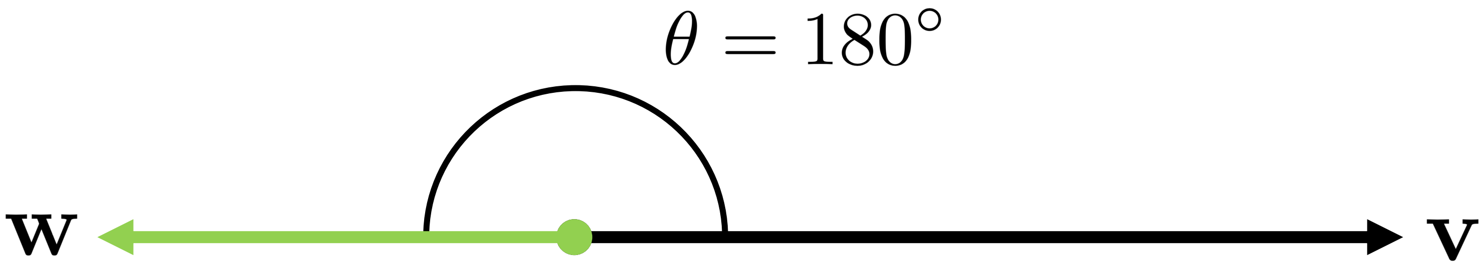
$$\langle \mathbf{v}, \mathbf{w} \rangle = \cos(\theta) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| = \cos(0^\circ) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| = 1 \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| \geq 0$$

Similarity between vectors



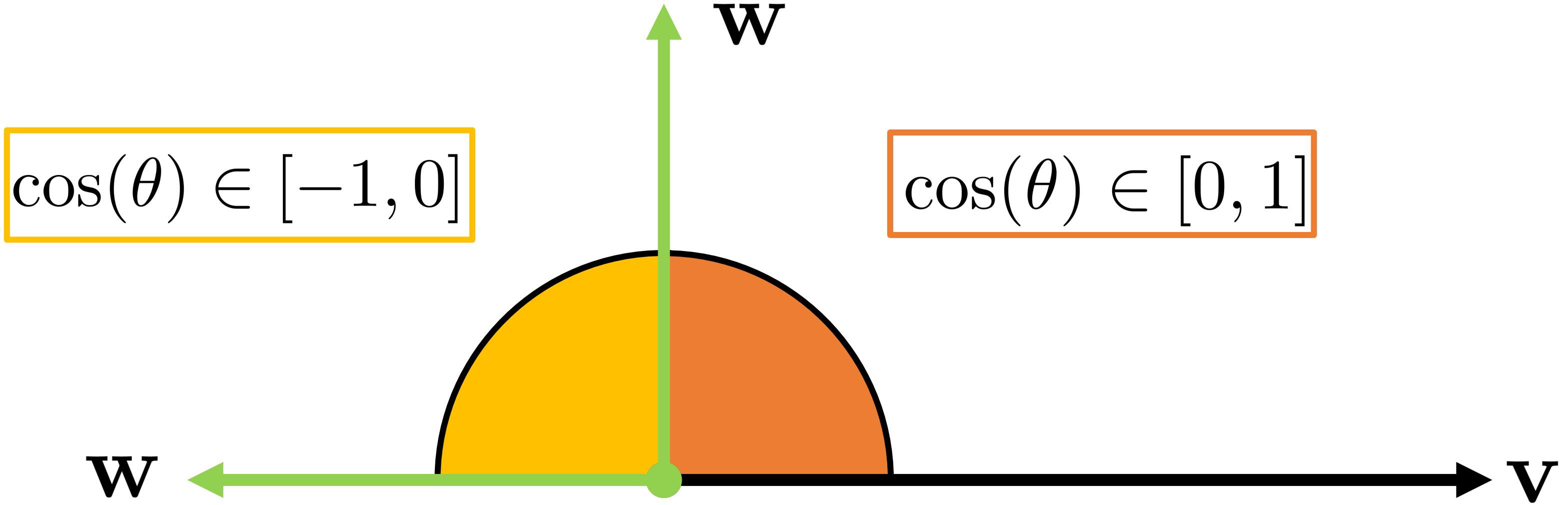
$$\langle \mathbf{v}, \mathbf{w} \rangle = \cos(\theta) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| = \cos(90^\circ) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| = 0 \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| = 0$$

Similarity between vectors



$$\langle \mathbf{v}, \mathbf{w} \rangle = \cos(\theta) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| = \cos(180^\circ) \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| = -1 \cdot ||\mathbf{v}|| \cdot ||\mathbf{w}|| \leq 0$$

Similarity between vectors



Similarity between vectors

$$\cos(\theta) \in \begin{cases} [0,1] & \text{if } 0^\circ \leq \theta \leq 90^\circ \\ [-1,0) & \text{if } 90^\circ < \theta \leq 180^\circ \\ (-1,0] & \text{if } 180^\circ < \theta \leq 270^\circ \\ (0,1] & \text{if } 270^\circ < \theta \leq 360^\circ \end{cases}$$

Similarity between vectors

The **distributional hypothesis** states that words occurring in the same context tend to have similar meanings (Harris, 1954).

Idea: use word vectors that capture the semantic similarity between words

One-hot vectors fail at this task, since the dot product for two different words is always zero!

Similarity between vectors

If we use the dot product for two different one-hot vectors, the result will always be 0!

$$\langle \mathbf{v}_{\text{vectors}}, \mathbf{v}_{\text{words}} \rangle = \left\langle \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle = 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 0 + 0 \cdot 1 = 0$$

We need a different representation!

Word embeddings

- Enter word embeddings!
- Instead of using one-hot vectors, assign each word in the vocabulary to a dense vector representation
- If two words are semantically similar, their dot product should be high

How do we find those embeddings?

word2vec

word2vec

- Set of embedding models introduced by Mikolov et al. (2013a, 2013b)
- Given a large corpus of text, we compute a dense word embedding for each word by looking at the context in which it appears

- To do so, word2vec provides two methods:

1. **Skip-gram**: given a specific word, estimate probability of context words
2. **Continuous-bag-of-words**: given a set of context words, predict the center word

Skip-gram

Moscow is the capital of Russia, Paris is the capital of France

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w_t

Skip-gram

w_{t-3} w_{t-1} w_{t+1} w_{t+3}

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Maximise probability $p(\text{the, capital, of, Paris, is, the} \,|\, \text{Russia})$

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Maximise probability $p(w_{t-3}, w_{t-2}, w_{t-1}, w_{t+1}, w_{t+2}, w_{t+3} \mid w_t)$

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Skip-gram

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Moscow is the capital of Russia, Paris is the capital of France

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For all words in all texts (let them be indexed by $t = 1, \ldots, T$):

Maximise objective $$\prod_{t=1}^{T} \prod_{-c \leq k \leq c, k \neq 0} p(w_{t+k} \mid w_t)$$

Skip-gram

We denote our objective with

$$J(\mathbf{W}) = \prod_{t=1}^{T} \prod_{-c \leq k \leq c, k \neq 0} p(w_{t+k} \mid w_t; \mathbf{W})$$

where $\mathbf{W}$ is a matrix of the embeddings for all words in the vocabulary.

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Since we are interested in finding the optimal values for these embeddings, we instead maximise

$$\log J(\mathbf{W}) = \sum_{t=1}^{T} \sum_{-c \le k \le c, k \ne 0} \log p(w_{t+k} \mid w_t; \mathbf{W})$$

The embedding matrix $\mathbf{W}$

- $\mathbf{W}$ is a matrix of dimensions $N \times D$ (vocab size $\times$ embedding dim)

$$\mathbf{W} = \begin{pmatrix} -18.123 & 0.981 & 0.000787 & \cdots & 1.324 \\ -0.005323 & 64.35 & 0.9013 & \cdots & 7.7534 \\ -0.002321 & 8.544 & 5.23 & \cdots & 65.75665 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 3.000434 & 4.34232 & \cdots & \cdots & 0.0002132 \end{pmatrix}$$

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\end{pmatrix} \quad \textit{capital}
$$

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Retrieving a vector from $\mathbf{W}$

- Recall our N- dimensional one-hot vector
- For *Russia*, this would be $(1, 0, \ldots, 0)^\top \in \{0, 1\}^N$

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- Hence

$$
\begin{aligned}
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&= \begin{pmatrix} -18.123 \\ 0.981 \\ 0.000787 \\ \vdots \\ 1.324 \end{pmatrix}
\end{aligned}
$$

Retrieving a vector from $\mathbf{W}$

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- Hence

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$$= \begin{pmatrix} -18.123 \\ 0.981 \\ 0.000787 \\ \vdots \\ 1.324 \end{pmatrix}$$

This is our word embedding for *Russia*

Modelling probabilities

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- For words w_t, w_{t+k}, vectors are denoted with $\mathbf{v}_{w_{t+k}}, \mathbf{v}'_{w_t}$
- Then

$$p(w_{t+k} \mid w_t; \mathbf{W}) = \frac{\exp(\langle \mathbf{v}_{w_{t+k}}, \mathbf{v}'_{w_t} \rangle)}{\sum_{j=1}^{N} \exp(\langle \mathbf{v}_{w_j}, \mathbf{v}'_{w_t} \rangle)}$$

- Note: computationally very expensive (for large N)

Modelling probabilities

- Initially, the system assigns each word in the vocabulary a random vector
- For words w_t, w_{t+k}, vectors are denoted with $\mathbf{v}_{w_{t+k}}, \mathbf{v}'_{w_t}$
- Then

$$p(w_{t+k} \mid w_t; \mathbf{W}) = \frac{\exp(\langle \mathbf{v}_{w_{t+k}}, \mathbf{v}'_{w_t} \rangle)}{\sum_{j=1}^{N} \exp(\langle \mathbf{v}_{w_j}, \mathbf{v}'_{w_t} \rangle)}$$

softmax function

- Note: computationally very expensive (for large N)

Putting it all together

- We aim to maximise

$$\log J(\mathbf{W}) = \sum_{t=1}^{T} \sum_{-c \leq k \leq c, k \neq 0} \log \left(\frac{\exp(\langle \mathbf{v}_{w_{t+k}}, \mathbf{v}'_{w_t} \rangle)}{\sum_{j=1}^{N} \exp(\langle \mathbf{v}_{w_j}, \mathbf{v}'_{w_t} \rangle)} \right)$$

- Since this quickly becomes intractable, this optimisation does not find many applications

How wo we train word2vec?

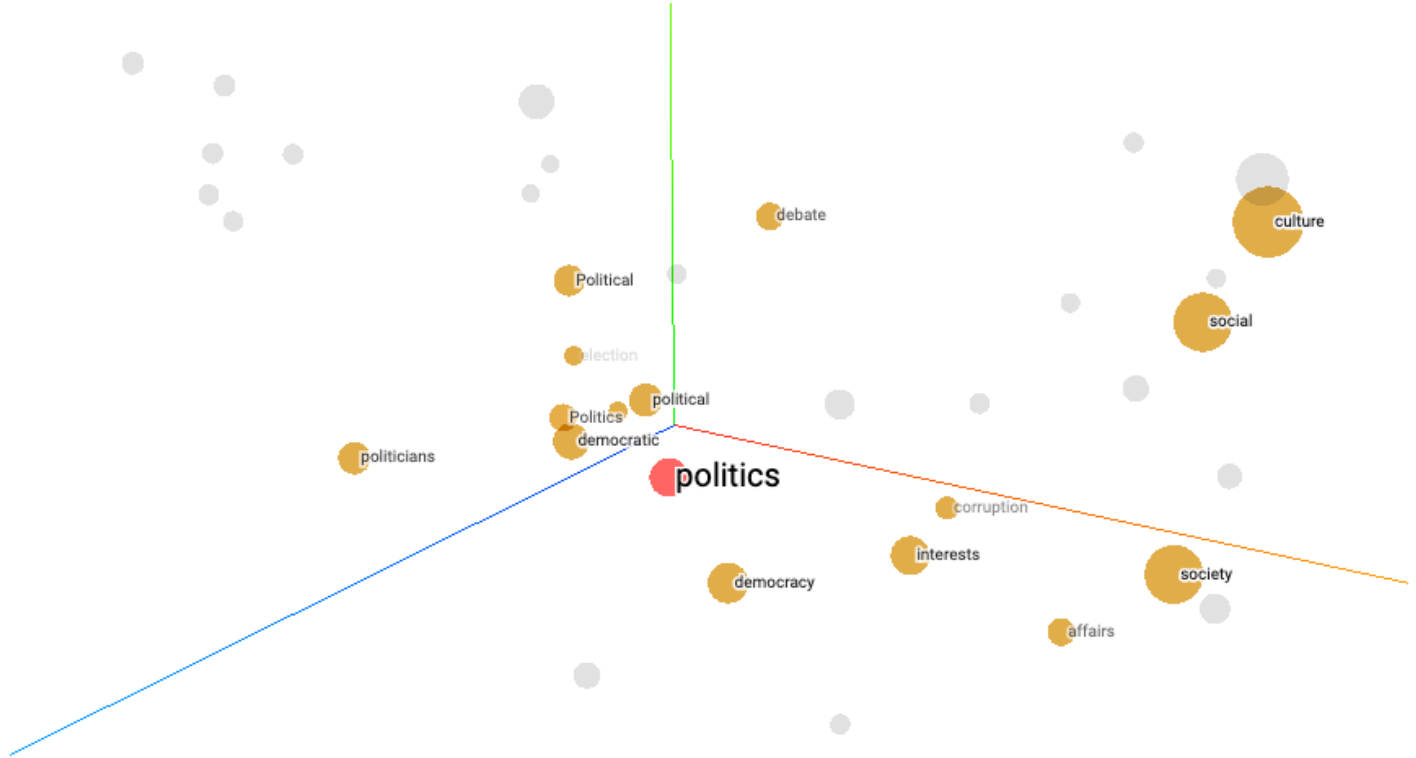
Training wor2vec models

- Mikolov et al. (2013b) propose two approximative training methods for word2vec

1. **Hierarchical Softmax**: computes an approximative softmax instead of the full one
2. **Negative Sampling**: approximation of the real objective with a logistic regression model to differentiate target words from noise words

(We will not discuss these techniques in detail here.)

Visualisation



Nearest neighbours for word **politics** (more visualisations in practical session)

Qualitative results

Nearest neighbours

Word	1	2	3	4	5
dog	*dogs*	*puppy*	*pit_bull*	*pooch*	*cat*
cat	*cats*	*dog*	*kitten*	*feline*	*beagle*
computer	*computers*	*laptop*	*laptop_computer*	*Computer*	*com_puter*
russia	*russians*	*russian*	*korea*	*germany*	*ukraine*
france	*spain*	*french*	*germany*	*europe*	*italy*
paris	*heidi*	*london*	*france*	*dubai*	*samuel*
moscow	*russian*	*russia*	*norway*	*iranian*	*munich*

Continuous-bag-of-words (CBOW)

- Similar approach, but predict center word instead, i.e.

$$p(w_t \,|\, w_{t-3}, w_{t-2}, w_{t-1}, w_{t+1}, w_{t+2}, w_{t+3})$$

- Order of context words irrelevant
- Computationally more efficient

Remarks

- word2vec learns two vector representations for each word (one for center, one for context)
- Multiple options to obtain single embedding for each word:
    1. Just use center word representation
    2. Add both embedding representations up
    3. Concatenate both vectors

- Model parameters:
    - dimension of embedding
    - context size
    - Initialisation of embedding vectors

GloVe

GloVe

- Word embedding model proposed by Pennington et al. (2014)
- Makes use of word co-occurrences in a given corpus
- Recall that we can represent documents with word frequency vectors
- Likewise, we can represent word co-occurrences with word co-occurrence matrices
- Each row and column corresponds to a word in the vocabulary
- Each cell entry i, j denotes how often word i co-occurs with word j

Word co-occurrences

- Let N denote the vocabulary size
- We define a co-occurrence matrix $\mathbf{M}_c \in (\mathbb{N} \cup \{0\})^{N \times N}$ such that

$$\mathbf{M}_c[i, j] = \text{number of occurrences of word } j \text{ in the context } c \text{ of word } i$$

- Context c is a hyperparameter of the model

Word co-occurrences

- Example:

$$\mathcal{D} = (\text{what}, \text{do}, \text{vectors}, \text{have}, \text{to}, \text{do}, \text{with}, \text{words})$$

$$V = \{\text{what}, \text{do}, \text{vectors}, \text{have}, \text{to}, \text{with}, \text{words}\}, |V| = 7$$

$$\mathbf{M}_3 = \begin{array}{c} \begin{array}{ccccccc} \textit{what} & \textit{do} & \textit{vectors} & \textit{have} & \textit{to} & \textit{with} & \textit{words} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 2 & 2 & 2 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array} \begin{array}{l} \\ \textit{what} \\ \textit{do} \\ \textit{vectors} \\ \textit{have} \\ \textit{to} \\ \textit{with} \\ \textit{words} \end{array}$$

GloVe

- Idea: align semantic similarity of words with their observed co-occurrence frequencies
- Compare all words in vocabulary with each other
- Minimise difference between their dot product and their co-occurrence

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- Objective then is

$$\langle \mathbf{v}_i, \mathbf{v}'_j \rangle \qquad\qquad - \log\left(\mathbf{M}_c[i, j]\right)$$

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$$\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + b_i + b'_j - \log\left(\mathbf{M}_c[i,j]\right)$$

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- Compare all words in vocabulary with each other
- Minimise difference between their dot product and their co-occurrence
- Objective then is

$$\left(\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + b_i + b'_j - \log\left(\mathbf{M}_c[i,j]\right)\right)^2$$

GloVe

- Idea: align semantic similarity of words with their observed co-occurrence frequencies
- Compare all words in vocabulary with each other
- Minimise difference between their dot product and their co-occurrence
- Objective then is

$$f(\mathbf{M}_c[i,j])\left(\langle \mathbf{v}_i, \mathbf{v}'_j\rangle + b_i + b'_j - \log\left(\mathbf{M}_c[i,j]\right)\right)^2$$

GloVe

- Idea: align semantic similarity of words with their observed co-occurrence frequencies
- Compare all words in vocabulary with each other
- Minimise difference between their dot product and their co-occurrence
- Objective then is

$$\sum_{j=1}^{N} f(\mathbf{M}_c[i,j]) \left(\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + b_i + b'_j - \log\left(\mathbf{M}_c[i,j]\right)\right)^2$$

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- Compare all words in vocabulary with each other
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$$J(\mathbf{W}) = \sum_{i=1}^{N} \sum_{j=1}^{N} f(\mathbf{M}_c[i,j]) \left(\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + b_i + b'_j - \log\left(\mathbf{M}_c[i,j]\right) \right)^2$$

GloVe

- Idea: align semantic similarity of words with their observed co-occurrence frequencies
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$$J(\mathbf{W}) = \sum_{i=1}^{N} \sum_{j=1}^{N} \boxed{f(\mathbf{M}_c[i,j])} \left(\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + b_i + b'_j - \log\left(\mathbf{M}_c[i,j]\right) \right)^2$$

Weighting function

GloVe

- Idea: align semantic similarity of words with their observed co-occurrence frequencies
- Compare all words in vocabulary with each other
- Minimise difference between their dot product and their co-occurrence
- Objective then is

$$J(\mathbf{W}) = \sum_{i=1}^{N} \sum_{j=1}^{N} f(\mathbf{M}_c[i,j]) \left(\boxed{\langle \mathbf{v}_i, \mathbf{v}'_j \rangle} + b_i + b'_j - \log\left(\mathbf{M}_c[i,j]\right)\right)^2$$

Dot product

GloVe

- Idea: align semantic similarity of words with their observed co-occurrence frequencies
- Compare all words in vocabulary with each other
- Minimise difference between their dot product and their co-occurrence
- Objective then is

$$J(\mathbf{W}) = \sum_{i=1}^{N}\sum_{j=1}^{N} f(\mathbf{M}_c[i,j]) \left(\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + \underbrace{\boxed{b_i + b'_j}}_{\text{Bias terms}} - \log\left(\mathbf{M}_c[i,j]\right)\right)^2$$

GloVe

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- Compare all words in vocabulary with each other
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- Objective then is

$$J(\mathbf{W}) = \sum_{i=1}^{N} \sum_{j=1}^{N} f(\mathbf{M}_c[i,j]) \left(\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + b_i + b'_j - \boxed{\log\left(\mathbf{M}_c[i,j]\right)} \right)^2$$

Word co-occurrence frequency

GloVe

- Idea: align semantic similarity of words with their observed co-occurrence frequencies
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$$J(\mathbf{W}) = \sum_{i=1}^{N} \sum_{j=1}^{N} f(\mathbf{M}_c[i,j]) \left(\langle \mathbf{v}_i, \mathbf{v}'_j \rangle + b_i + b'_j - \boxed{\log\left(\mathbf{M}_c[i,j]\right)} \right)^2$$

Note: not defined for $\mathbf{M}_c[i,j] = 0\,!$

In practice, shift $\log(\mathbf{M}_c[i,j]) \rightarrow \log(1 + \mathbf{M}_c[i,j])$

Weighting function $f(\cdot)$

Pennington et al. (2014) propose the following characteristics for $f(\cdot)$:

1. $f(0) = 0$, i.e. if two words do not co-occur, we neglect them
2. The function should be non-decreasing
3. The function should not overweight frequent co-occurrences

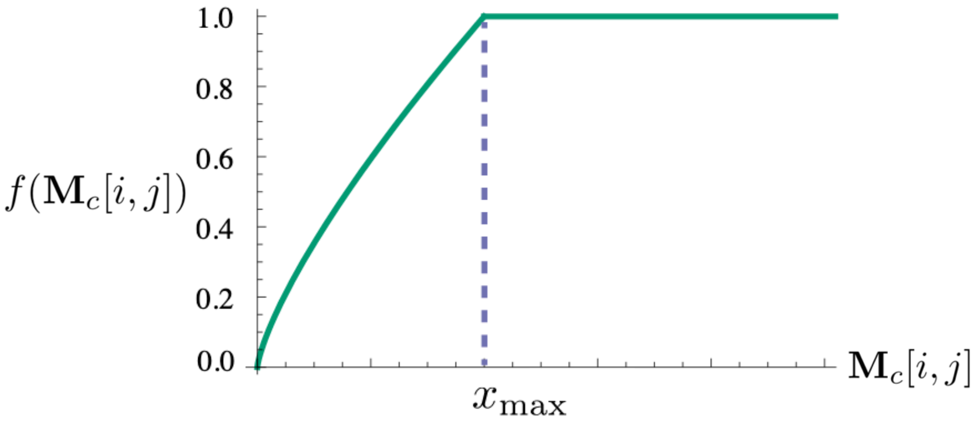
Weighting function $f(\cdot)$

Pennington et al. (2014) propose the following characteristics for $f(\cdot)$:

1. $f(0) = 0$, i.e. if two words do not co-occur, we neglect them
2. The function should be non-decreasing
3. The function should not overweight frequent co-occurrences

Author's choice:

$$f(x) = \begin{cases} (x/x_{max})^{\alpha} & \text{if } x < x_{max} \\ 1 & \text{otherwise} \end{cases}$$



Source: Pennington et al. (2014)

Qualitative results

Nearest neighbours

Word	1	2	3	4	5
dog	*dogs*	*puppy*	*pet*	*cat*	*puppies*
cat	*cats*	*kitten*	*dog*	*kitty*	*pet*
computer	*computers*	*Computer*	*laptop*	*software*	*desktop*
russia	*ukraine*	*russian*	*moscow*	*poland*	*soviet*
france	*paris*	*europe*	*italy*	*spain*	*germany*
paris	*france*	*hilton*	*montreal*	*lyon*	*prague*
moscow	*russia*	*prague*	*ukraine*	*poland*	*kiev*

Remarks

- GloVe also computes two vectors per word
- The final representation can be obtained similar to word2vec (addition, concatenation)

- Model parameters:
  - dimension of embedding
  - context size
  - initialisation of embedding vectors
  - (…)

word2vec vs. GloVe

word2vec	GloVe
• Aims to capture local contexts many different times • Computationally intractable (requires approximative methods)	• Aims to capture global context once • Computationally feasible

References

- Harris, Z.S., 1954. Distributional structure. Word, 10(2-3), pp.146-162.
- Mikolov, T., Chen, K., Corrado, G. and Dean, J., 2013a. Efficient estimation of word representations in vector space. arXiv preprint arXiv:1301.3781.
- Mikolov, T., Sutskever, I., Chen, K., Corrado, G.S. and Dean, J., 2013b. Distributed representations of words and phrases and their compositionality. In *Advances in neural information processing systems* (pp. 3111-3119).
- Pennington, J., Socher, R. and Manning, C., 2014. Glove: Global vectors for word representation. In Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP) (pp. 1532-1543).

Recommended resources

- Lecture video "Word Vector Representations: word2vec" by Christopher Manning. Available at **`https://www.youtube.com/watch?v=ERibwqs9p38`**.
- Lecture video "GloVe: Global Vectors for Word Representation" by Richard Socher. Available at **`https://www.youtube.com/watch?v=ASn7ExxLZws`**.
- Textbook "Speech and Language Processing" by Dan Jurafsky. Available at **`https://web.stanford.edu/~jurafsky/slp3/`**.

These slides are inspired by the above resources.

Thank you

Any questions?

Next up: *Applications and Limitations of Word Embeddings in CSS*

(Laura, Bennett)