a)

In[38]:=
```
Clear["Global`*"]
r[r_] = μ * r - r^3;
θ[r_] = ω + ν * r^2;

s = Solve[r[r] == 0, {r}]
```

Out[41]= $\{\{r \to 0\}, \{r \to -\sqrt{\mu}\}, \{r \to \sqrt{\mu}\}\}$

In[43]:=
```
{{r → 0}, {r → -√μ}, {r → √μ}}
p = (Sqrt[μ] * θ[Sqrt[μ]]) / (2 * π * Sqrt[μ])
```

Out[43]= $\{\{r \to 0\}, \{r \to -\sqrt{\mu}\}, \{r \to \sqrt{\mu}\}\}$

Out[44]= $\frac{\mu\,\nu + \omega}{2\,\pi}$

b)

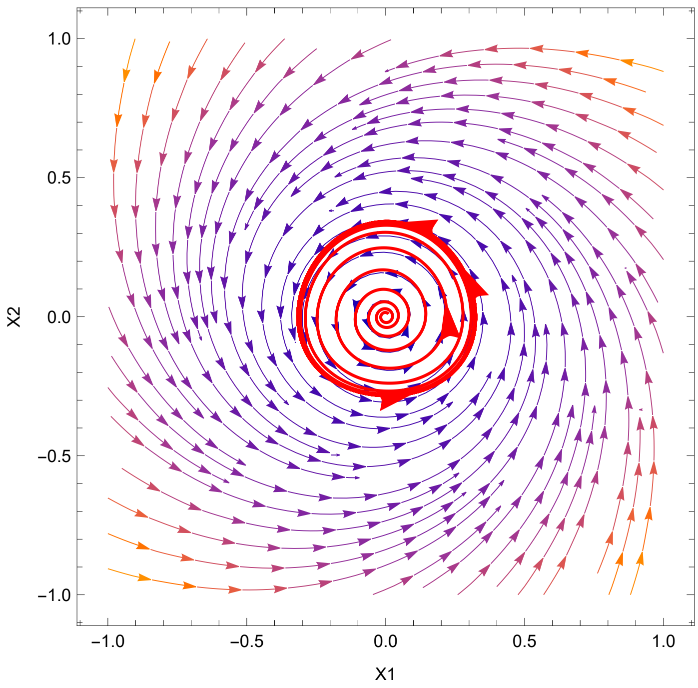
In[55]:=
```
X1dot[X1_, X2_] := 1 / 10 * X1 - X2^3 - X1 * X2^2 - X1^2 * X2 - X2 - X1^3;
X2dot[X1_, X2_] := X1 + 1 / 10 * X2 + X1 * X2^2 + X1^3 - X2^3 - X1^2 * X2;
F1 :=
  X1'[t] == 1 / 10 * X1[t] - X2[t]^3 - X1[t] * X2[t]^2 - X1[t]^2 * X2[t] - X2[t] - X1[t]^3;
F2 := X2'[t] == X1[t] + 1 / 10 X2[t] + X1[t] * X2[t] + X1[t]^3 - X2[t]^3 - X1[t]^2 * X2[t];
LC = NDSolve[{F1, F2, X1[0] == X2[0] == 0.01}, {X1, X2}, {t, 0, 100}];
traj = ParametricPlot[
    Evaluate[{X1[t], X2[t]} /. {LC}, {t, 0, 100}, PlotStyle → {Red}]] /.
  Line[x_] → {Arrowheads[{0, 0.06, 0.06, 0}], Arrow[x]}, {-0.5, 0.5};

plot = StreamPlot[{X1dot[X1, X2], X2dot[X1, X2]},
    {X1, -1, 1}, {X2, -1, 1}, FrameLabel → {"X1", "X2"}];
Show[plot, traj]
```

••• Syntax:

"traj = ParametricPlot[Evaluate[{X1[t], X2[t]} /. {LC}, {t, 0, 100}, PlotStyle → {Red}]] /. Line[x_] → {Arrowheads[{0, 0.06, 0.06, 0}], Arrow[x]}, {-0.5, 0.5};" is incomplete; more input is needed.

In[•]:=



c)

```
(*Comparing coefficients one can see that μ=1/10,
ω=1, ν=1 makes the systems identical*)
```

d)

In[60]:=
```
Clear["Global`*"]
μ = 1/10; ω = 1; ν = 1; T = (2 π)/(μ ν + ω);
X1Dot[X1_, X2_] := 1/10 * X1 - X2^3 - X1 * X2^2 - X1^2 * X2 - X2 - X1^3;
X2Dot[X1_, X2_] := X1 + 1/10 * X2 + X1 * X2^2 + X1^3 - X2^3 - X1^2 * X2;
Dynsys =
  {X1'[t] == 1/10 * X1[t] - X2[t]^3 - X1[t] * X2[t]^2 - X1[t]^2 * X2[t] - X2[t] - X1[t]^3,
   X2'[t] == X1[t] + 1/10 * X2[t] + X1[t] * X2[t]^2 + X1[t]^3 - X2[t]^3 - X1[t]^2 * X2[t]};
jacobian = {{D[X1Dot[X1[t], X2[t]], X1[t]], D[X1Dot[X1[t], X2[t]], X2[t]]},
  {D[X2Dot[X1[t], X2[t]], X1[t]], D[X2Dot[X1[t], X2[t]], X2[t]]}}
```

$$\left\{\left\{\frac{1}{10} - 3\,X1[t]^2 - 2\,X1[t] \times X2[t] - X2[t]^2,\ -1 - X1[t]^2 - 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\},\ \left\{1 + 3\,X1[t]^2 - 2\,X1[t] \times X2[t] + X2[t]^2,\ \frac{1}{10} - X1[t]^2 + 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\}\right\}$$

```
r0 = Sqrt[μ];
solution = NDSolve[Join[
    {Dynsys[[1]], Dynsys[[2]], M11'[t] == jacobian[[1]][[2]] * M21[t] + jacobian[[1]][[1]] * M11[t],
     M12'[t] == jacobian[[1]][[2]] * M22[t] + jacobian[[1]][[1]] * M12[t],
     M21'[t] == jacobian[[2]][[2]] * M21[t] + jacobian[[2]][[1]] * M11[t],
     M22'[t] == jacobian[[2]][[2]] * M22[t] + jacobian[[2]][[1]] * M12[t],
     X1[0] == r0, X2[0] == 0, M11[0] == M22[0] == 1, M12[0] == M21[0] == 0}],
   {X1, X2, M11, M12, M21, M22}, {t, 0, T}];
```

$$\left\{\left\{\frac{1}{10} - 3\,X1[t]^2 - 2\,X1[t] \times X2[t] - X2[t]^2,\ -1 - X1[t]^2 - 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\},\ \left\{1 + 3\,X1[t]^2 - 2\,X1[t] \times X2[t] + X2[t]^2,\ \frac{1}{10} - X1[t]^2 + 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\}\right\}$$

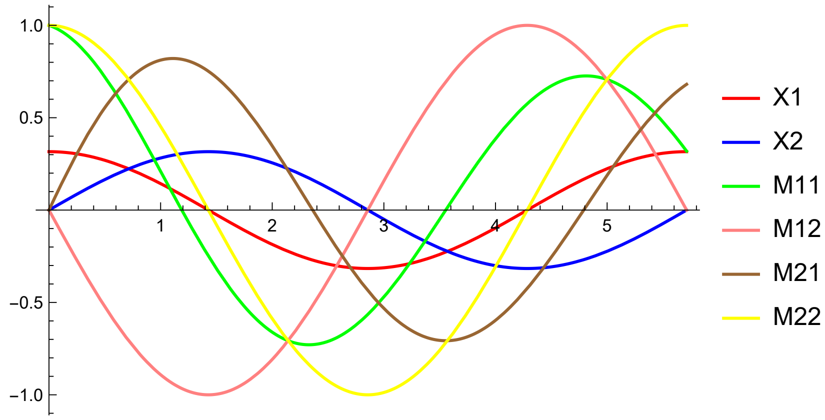
```
Plot[{X1[t] /. solution, X2[t] /. solution, M11[t] /. solution,
  M12[t] /. solution, M21[t] /. solution, M22[t] /. solution},
 {t, 0, T}, PlotLegends → {"X1", "X2", "M11", "M12", "M21", "M22"},
 PlotStyle → {Red, Blue, Green, Pink, Brown, Yellow}]
```

Out[65]=
$$\left\{\left\{\frac{1}{10} - 3\,X1[t]^2 - 2\,X1[t] \times X2[t] - X2[t]^2,\ -1 - X1[t]^2 - 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\},\ \left\{1 + 3\,X1[t]^2 - 2\,X1[t] \times X2[t] + X2[t]^2,\ \frac{1}{10} - X1[t]^2 + 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\}\right\}$$

Out[66]=
$$\left\{\left\{\frac{1}{10} - 3\,X1[t]^2 - 2\,X1[t] \times X2[t] - X2[t]^2,\ -1 - X1[t]^2 - 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\},\ \left\{1 + 3\,X1[t]^2 - 2\,X1[t] \times X2[t] + X2[t]^2,\ \frac{1}{10} - X1[t]^2 + 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\}\right\}$$

Out[69]=
$$\left\{\left\{\frac{1}{10} - 3\,X1[t]^2 - 2\,X1[t] \times X2[t] - X2[t]^2,\ -1 - X1[t]^2 - 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\},\ \left\{1 + 3\,X1[t]^2 - 2\,X1[t] \times X2[t] + X2[t]^2,\ \frac{1}{10} - X1[t]^2 + 2\,X1[t] \times X2[t] - 3\,X2[t]^2\right\}\right\}$$

Out[70]=



e)

In[82]:=
```
M11 = M11[T] /. solution⟦1⟧;
M12  = M12[T] /. solution⟦1⟧;
M21 = M21[T] /. solution⟦1⟧;
M22 = M22[T] /. solution⟦1⟧;
M = {{M11, M12}, {M21, M22}}
```

Out[86]= $\{\{0.319053, 2.12317 \times 10^{-8}\}, \{0.680947, 1.\}\}$

f)

In[88]:=
```
Log[Eigenvalues[M]] / T
```

Out[88]= $\{5.78753 \times 10^{-9}, -0.2\}$

g,h)

```
Clear["Global`*"];

(*Define the parameters*)
mu = 1 / 10;
omega = 1;
nu = 1;

(*Calculate T*)
T = (2 Pi) / (mu * nu + omega);

(*Define the functions r and theta*)
r[rr_] := mu * rr - rr^3;
theta[rr_] := omega + nu * rr^2;

(*Define the Jacobian matrix J*)
J[rr_] := D[{r[rr], theta[rr]}, {rr, theta[rr]}];

(*Define the matrix M*)
M[rr_] := IdentityMatrix[2].MatrixExp[J[rr] * t];

(*Define the derivative matrix der*)
jacpol[X1_, X2_] := D[{Sqrt[X1^2 + X2^2], ArcTan[X1, X2]}, {X1, X2}] /.
   {X1 → r[rr] * Cos[theta[rr]], X2 → r[rr] * Sin[theta[rr]]};

(*Simplify der and substitute values for r and theta*)
jacpol = Simplify[[X1, X2], Assumptions → rr > 0] /. {rr → Sqrt[mu], theta[rr] → 0};
dinv[r_] := {{D[r * Cos[θ], r], D[r * Cos[θ], θ]}, {D[r * Sin[θ], r], D[r * Sin[θ], θ]}};
dinv = dinv[r];
cartesian = dinv.M.jacpol;
cartesian = Simplify[cartesian] /. {r → Sqrt[μ], θ → 0, t → T}
λ = 1 / T * Log[Eigenvalues[cartesian] /. r → Sqrt[μ]]
```

Out[284]= $\left\{\left\{e^{-4\pi/11}, 0\right\}, \left\{1 - e^{-4\pi/11}, 1\right\}\right\}$

Out[285]= $\left\{0, -\frac{1}{5}\right\}$