a)

```
In[ ]:= M = {{σ + 1, 3}, {-2, σ - 1}};
        λ = Eigenvalues[M]
```

$$\text{In[ ]:=}\ \left\{-i\sqrt{5}+\sigma,\ i\sqrt{5}+\sigma\right\}$$

b)

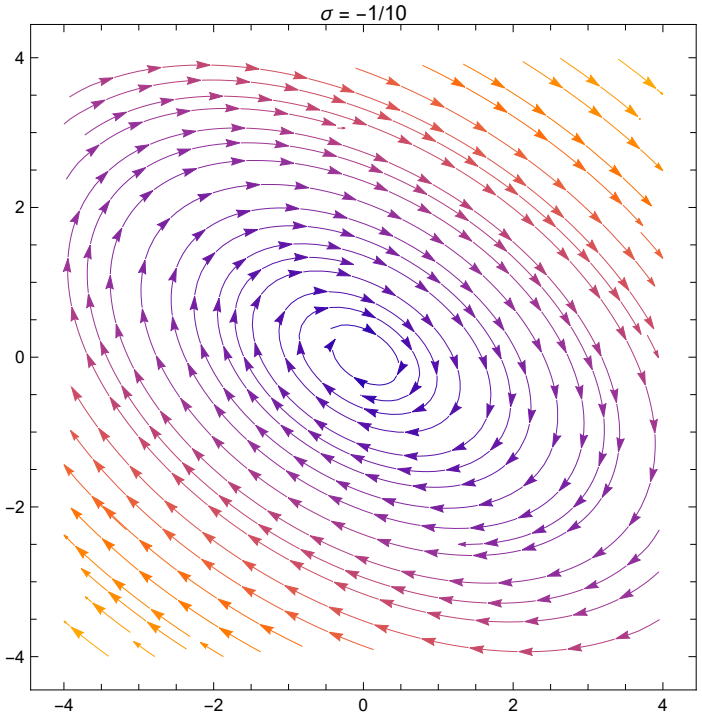
```
X = .; y = .;
X[t_] = {x[t], y[t]};
Dynsystem = X'[t] == M.X[t];
sol = DSolve[Dynsystem, {x, y}, t];
X[t_, x0_, y0_, σ_] = {x[t], y[t]} /. sol /. {C[1] → u, C[2] → v}
```

$$\left\{\left\{\frac{3\,e^{t\sigma}\,v\,\text{Sin}\left[\sqrt{5}\,t\right]}{\sqrt{5}}+\frac{1}{5}e^{t\sigma}u\left(5\,\text{Cos}\left[\sqrt{5}\,t\right]+\sqrt{5}\,\text{Sin}\left[\sqrt{5}\,t\right]\right),\right.\right.$$
$$\left.\left.-\frac{2\,e^{t\sigma}\,u\,\text{Sin}\left[\sqrt{5}\,t\right]}{\sqrt{5}}+\frac{1}{5}e^{t\sigma}v\left(5\,\text{Cos}\left[\sqrt{5}\,t\right]-\sqrt{5}\,\text{Sin}\left[\sqrt{5}\,t\right]\right)\right\}\right\}$$

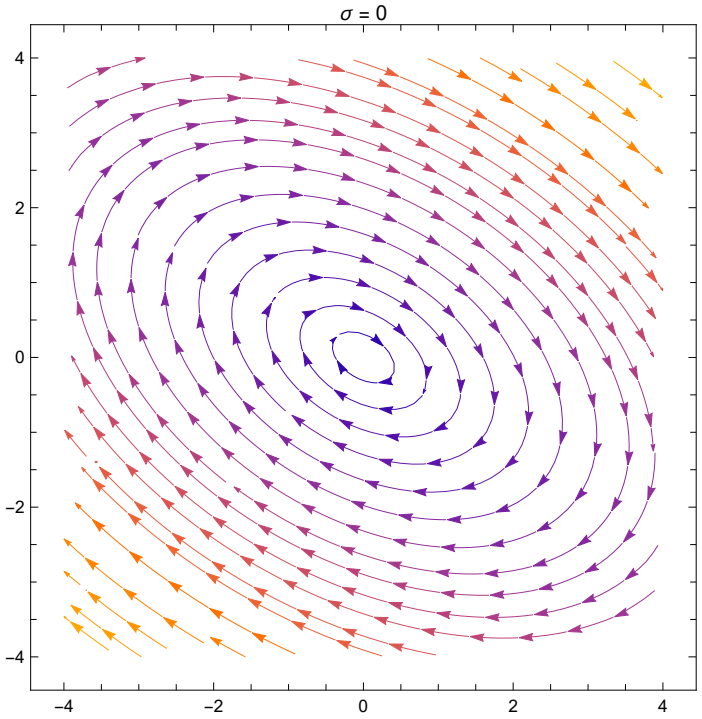
c)

```
In[25]:= plot1 = StreamPlot[{(σ + 1) x + 3 y, -2 x + (σ - 1) y} /. {σ → -1 / 10},
            {x, -4, 4}, {y, -4, 4}, PlotLabel → "σ = -1/10"];
         plot2 = StreamPlot[{(σ + 1) x + 3 y, -2 x + (σ - 1) y} /. {σ → 0},
            {x, -4, 4}, {y, -4, 4}, PlotLabel → "σ = 0"];
         plot3 = StreamPlot[{(σ + 1) x + 3 y, -2 x + (σ - 1) y} /. {σ → 1 / 10},
            {x, -4, 4}, {y, -4, 4}, PlotLabel → "σ = 1/10"];
         Show[plot1]
         Show[plot2]
         Show[plot3]
```

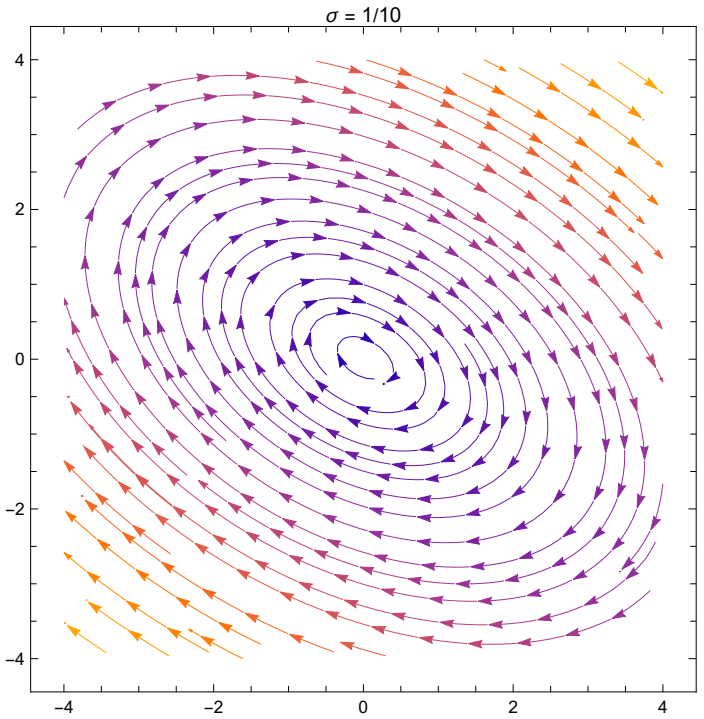
Out[28]=



Out[29]=



Out[30]=



d)

In[ ]:=
```
Solve[X[0, u, v, 0] == X[t, u, v, 0], t]
```

$$\left\{\left\{t \to \frac{2 \pi c_1}{\sqrt{5}} \text{ if } c_1 \in \mathbb{Z}\right\}\right\}$$

e)

In[ ]:=
```
solution = {3 e^(t σ) v Sin[Sqrt[5] t]/Sqrt[5] + 1/5 e^(t σ) u (5 Cos[Sqrt[5] t] + Sqrt[5] Sin[Sqrt[5] t]),
  -2 e^(t σ) u Sin[Sqrt[5] t]/Sqrt[5] + 1/5 e^(t σ) v (5 Cos[Sqrt[5] t] - Sqrt[5] Sin[Sqrt[5] t])};
major = FindMaximum[Norm[solution] /. {u → 1, v → 1, σ → 0}, t];
minor = FindMinimum[Norm[solution] /. {u → 1, v → 1, σ → 0}, t];
ratio = major / minor
```

$$\left\{1.6180339887498942`, \left\{\frac{t \to 0.6376910210893485`}{t \to 1.3401724846165075`}\right\}\right\}$$

f)

```
In[ ]:= direction = solution /. {u → 1, v → 1, σ → 0, major[[2]][[1]]};
        Normalize[direction]
Out[ ]= {0.850651 – 0.525731}
```