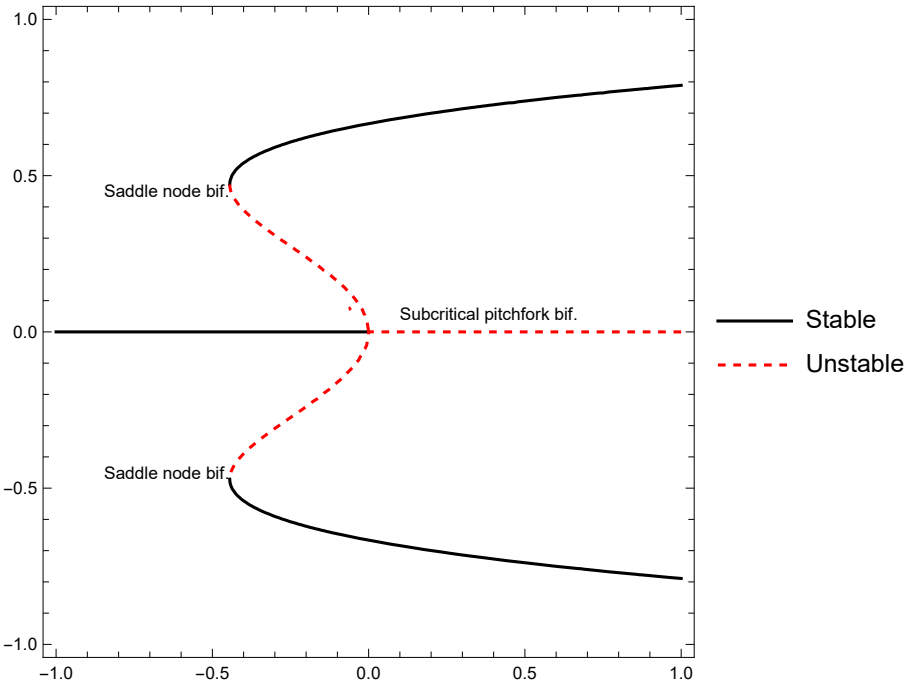
a)

In[ ]:=
```
f[x_] := r * x + 4 * x^3 - 9 * x^5;
dx = D[f[x], x];
cplot = ContourPlot[{
     ConditionalExpression[f[x], dx < 0] == 0, ConditionalExpression[f[x], dx > 0] == 0},
   {r, -1, 1}, {x, -1, 1}, ContourStyle → {{Black}, {Red, Dashed}},
   PlotLegends → {"Stable", "Unstable"}];
align[Right] = {1, 0};
align[Center] = {0, 0};
align[Left] = {-1, 0};

Text1 = Text["Saddle node bif.", {-4 / 9, 0.45}, align[Right]];
Text2 = Text["Saddle node bif.", {-4 / 9, -0.45}, align[Right]];
Text3 = Text["Subcritical pitchfork bif.", {0.1, 0.05}, align[Left]];

txt = Graphics[{Text1, Text2, Text3}];
Show[cplot, txt]
```

Out[ ]=



Plot: Options expected (instead of {r, −2, 2}) beyond position 2 in Plot[f[x], {x, −2, 2}, {r, −2, 2}]. An option must be a rule or a list of rules.

b)

```
In[ ]:= Clear[x]
        f[x_] := r * x + 4 * x^3 - 9 * x^5;
        dx;
        Solve[dx == 0, x]
```

$$\text{Out[ ]}= \left\{\left\{x \to -\frac{\sqrt{2-\sqrt{4+5\,r}}}{\sqrt{15}}\right\}, \left\{x \to \frac{\sqrt{2-\sqrt{4+5\,r}}}{\sqrt{15}}\right\}, \left\{x \to -\frac{\sqrt{2+\sqrt{4+5\,r}}}{\sqrt{15}}\right\}, \left\{x \to \frac{\sqrt{2+\sqrt{4+5\,r}}}{\sqrt{15}}\right\}\right\}$$

```
        x1 = - Sqrt[2 - Sqrt[4 + 5 * r]] / Sqrt[15];
        Solve[f[x1] == 0, r]
```

$$\text{Out[ ]}= -\frac{\sqrt{2-\sqrt{4+5\,r}}}{\sqrt{15}}$$

```
In[ ]:= {{r → 0}}
        x2 = Sqrt[2 - Sqrt[4 + 5 * r]] / Sqrt[15];
        Solve[f[x2] == 0, r]
```

```
        {{r → 0}}
        x3 = - Sqrt[2 + Sqrt[4 + 5 * r]] / Sqrt[15];
        Solve[f[x3] == 0, r];
```

$$\text{Out[ ]}= \{\{r \to 0\}\}$$

```
In[ ]:= {{r → -4/9}}
        x4 = Sqrt[2 + Sqrt[4 + 5 * r]] / Sqrt[15];
        Solve[f[x4] == 0, r]
```

$$\text{Out[ ]}= \left\{\left\{r \to -\frac{4}{9}\right\}\right\}$$