## a)

In[ ]:=

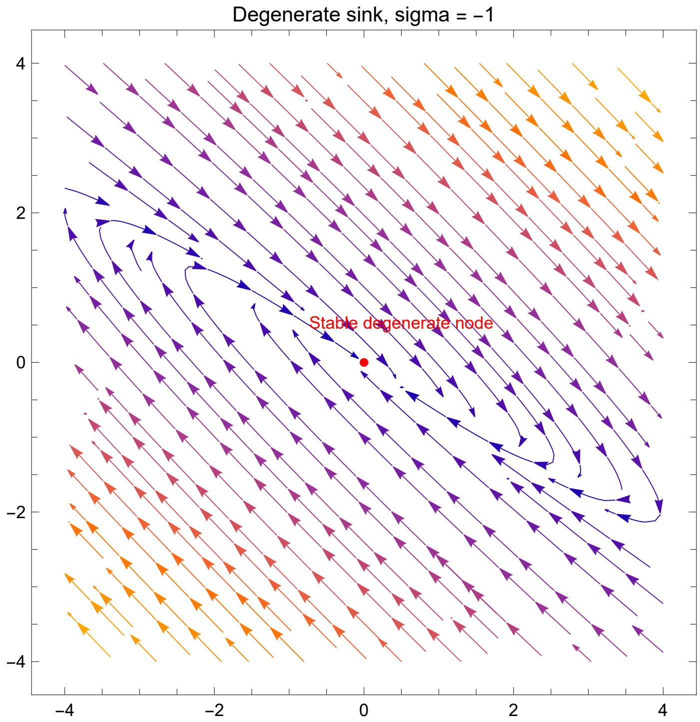
```
sigmaOne = -1;
Solve[(sigmaOne + 3) * x + 4 * y == 0 && -(9 / 4) * x + (sigmaOne - 3) * y == 0, {x, y}]

sigmaTwo = 0;
Solve[(sigmaTwo + 3) * x + 4 * y == 0 && -(9 / 4) * x + (sigmaTwo - 3) * y == 0, {x, y}]

sigmaThree = 1;
Solve[(sigmaThree + 3) * x + 4 * y == 0 && -(9 / 4) * x + (sigmaThree - 3) * y == 0, {x, y}]


plot1 = StreamPlot[{(σ + 3) x + 4 y, -(9 / 4) x + (σ - 3) y} /. {σ → -1}, {x, -4, 4},
   {y, -4, 4}, PlotLabel → "Degenerate sink, sigma = -1", Epilog → {Red,
     PointSize[Medium], Point[{0, 0}], Text["Stable degenerate node", {0.5, 0.5}]}];
plot2 =
  StreamPlot[{(σ + 3) x + 4 y, -(9 / 4) x + (σ - 3) y} /. {σ → 0}, {x, -4, 4}, {y, -4, 4},
   PlotLabel → "Uniform motion, sigma = 0", Epilog → {Red, PointSize[Medium],
     Line[{{-4, 3}, {4, -3}}], Text["Line fixed points", {-0.5, 2}]}];
plot3 = StreamPlot[{(σ + 3) x + 4 y, -(9 / 4) x + (σ - 3) y} /. {σ → 1}, {x, -4, 4},
   {y, -4, 4}, PlotLabel → "Degenerate source, sigma = 1", Epilog → {Red,
     PointSize[Medium], Point[{0, 0}], Text["Unstable degenerate node", {0.5, 0.5}]}];
Show[plot1]
Show[plot2]
Show[plot3]
```

Out[ ]= $\{\{x \to 0, y \to 0\}\}$

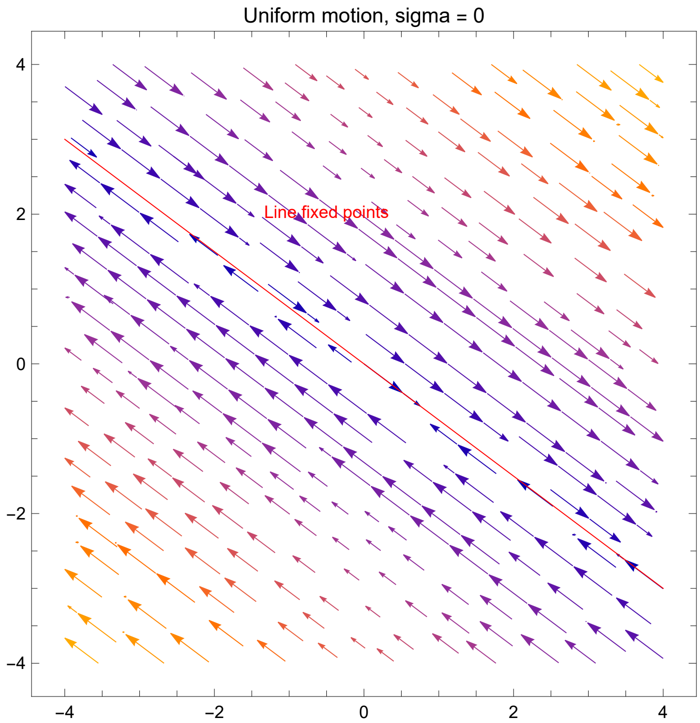
Out[ ]= $\left\{\left\{y \to -\frac{3x}{4}\right\}\right\}$

Out[ ]= $\{\{x \to 0, y \to 0\}\}$

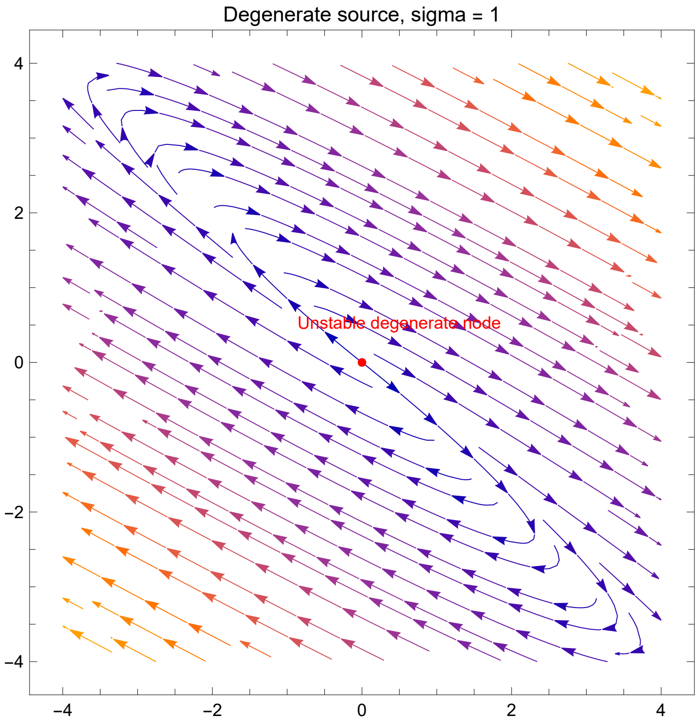
Out[ ]=


Degenerate sink, sigma = −1
Stable degenerate node
4
2
0
−2
−4
−4
−2
0
2
4

Out[ ]=


Uniform motion, sigma = 0
Line fixed points
4
2
0
−2
−4
−4
−2
0
2
4

Out[•]=




## b),c),d)

Normalize the eigenvectors and put x-component positive (-1*[x,y]) for d)

In[36]:=
```
m = {{σ + 3, 4}, {-9 / 4, σ - 3}};
Eigenvalues[m]
ev = Eigenvectors[m];
ev = ev[[1]];
ev = Normalize[ev];
ev = -1 * ev
Inverse[m]
```

$\{\sigma, \sigma\}$

$\left\{\frac{4}{5}, -\frac{3}{5}\right\}$

$\left\{\left\{\frac{-3+\sigma}{\sigma^2}, -\frac{4}{\sigma^2}\right\}, \left\{\frac{9}{4\,\sigma^2}, \frac{3+\sigma}{\sigma^2}\right\}\right\}$

## e)

The value of sigma for which M_sigma is singular is 0 since dividing by zero gives a singularity and inverse of M_sigma doesn’t exist for this value of sigma.

## f)

In[ ]:=
```
Solve[-c*d == 3 && d^2 == 4 && -c^2 == -9/4 && c*d == -3, {c, d}]
```

$$\left\{\left\{c \to -\frac{3}{2}, d \to 2\right\}, \left\{c \to \frac{3}{2}, d \to -2\right\}\right\}$$

## g)

In[3]:=
```
M = {{σ - c*d, d^2}, {-c^2, σ + c*d}};
V = Eigenvalues[M]
```

{σ, σ}

## h)

In[ ]:=
```
sigma0 = 0;
Mv = {{sigma0 - c*d, d^2}, {-c^2, sigma0 + c*d}};
Eigenvectors[Mv];
v = {d/c, 1};
v = Normalize[v]
```

$$\left\{\frac{d}{c\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}, \frac{1}{\sqrt{1+\text{Abs}\left[\frac{d}{c}\right]^2}}\right\}$$