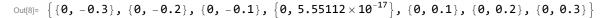
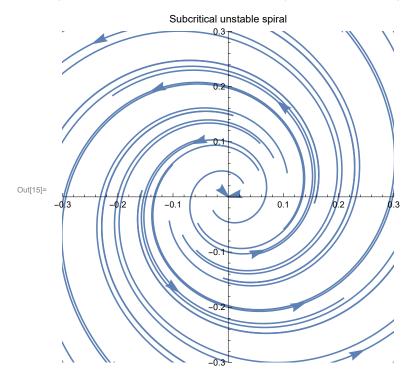
d)

```
In[16]:= Clear["Global`*"]
       xmin = -0.3;
       xmax = 0.3;
       ymin = -0.3;
       ymax = 0.3;
       solution[x0_, y0_] =
          NDSolve[\{x'[t] = \mu * x[t] - 4y[t] - x[t]^3, y'[t] = 4x[t] + \mu * y[t] + 2y[t]^3,
              x[0] = x0, y[0] = y0 /. \mu \rightarrow -1, \{x, y\}, \{t, -1, 1\}];
       IC0 = Table[\{0, y\}, \{y, ymin, ymax, 0.1\}]
       IC1 = Table[{xmin, y}, {y, ymin, ymax, 0.2}];
       IC2 = Table[{xmax, y}, {y, ymin, ymax, 0.2}];
       IC3 = Table[{x, ymin}, {x, xmin, xmax, 0.2}];
       IC4 = Table[{x, ymax}, {x, xmin, xmax, 0.2}];
       ICs = Join[IC0, IC1, IC2, IC3, IC4];
       plot =
          Table[ParametricPlot[
               Evaluate[\{x[t], y[t]\} /. solution[ICs[i, 1], ICs[i, 2]]], \{t, -1, 1\}, PlotRange \rightarrow
                \{\{xmin, xmax\}, \{ymin, ymax\}\}, PlotLabel \rightarrow "Subcritical stable spiral"] /.
             Line[x_{-}] \Rightarrow \{Arrowheads[\{0, 0.0375, 0.0375, 0\}], Arrow[x]\}, \{i, Length[ICs]\}];
       Show[{plot}]
       ••• NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.
\text{Out}[22] = \left\{ \left\{ \text{0, -0.3} \right\}, \left\{ \text{0, -0.2} \right\}, \left\{ \text{0, -0.1} \right\}, \left\{ \text{0, 5.55112} \times \text{10}^{-17} \right\}, \left\{ \text{0, 0.1} \right\}, \left\{ \text{0, 0.2} \right\}, \left\{ \text{0, 0.3} \right\} \right\}
                               Subcritical stable spiral
Out[29]=
```

```
In[1]:= Clear["Global`*"]
    Clear["Global`*"]
    xmin = -0.3;
    xmax = 0.3;
    ymin = -0.3;
    ymax = 0.3;
    solution[x0_, y0_] =
      NDSolve[\{x'[t] = \mu * x[t] - 4y[t] - x[t]^3, y'[t] = 4x[t] + \mu * y[t] + 2y[t]^3,
          x[0] = x0, y[0] = y0  /. \mu \rightarrow 1, \{x, y\}, \{t, -1, 1\}];
    IC0 = Table[\{0, y\}, \{y, ymin, ymax, 0.1\}]
    IC1 = Table[{xmin, y}, {y, ymin, ymax, 0.2}];
    IC2 = Table[{xmax, y}, {y, ymin, ymax, 0.2}];
    IC3 = Table[\{x, ymin\}, \{x, xmin, xmax, 0.2\}];
    IC4 = Table[{x, ymax}, {x, xmin, xmax, 0.2}];
    ICs = Join[IC0, IC1, IC2, IC3, IC4];
    plot =
      Table[ParametricPlot[
          Evaluate[\{x[t], y[t]\} /. solution[ICs[i, 1], ICs[i, 2]]], \{t, -1, 1\}, PlotRange \rightarrow
            \{\{xmin, xmax\}, \{ymin, ymax\}\}, PlotLabel \rightarrow "Subcritical unstable spiral"] /.
         Line[x_{-}] \Rightarrow \{Arrowheads[\{0, 0.0375, 0.0375, 0\}], Arrow[x]\}, \{i, Length[ICs]\}];
    Show[{plot}]
```

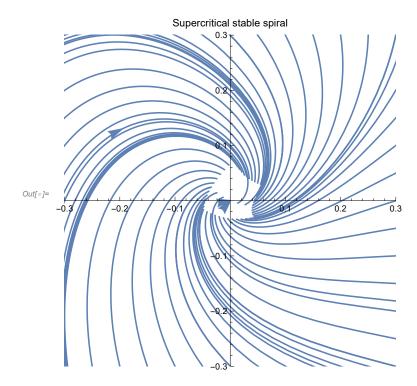
••• NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.





```
In[*]:= Clear["Global`*"]
     xmin = -0.3;
     xmax = 0.3;
     ymin = -0.3;
     ymax = 0.3;
     solution[x0_, y0_] =
        NDSolve[\{x'[t] = \mu * x[t] + y[t] - x[t]^2, y'[t] = -x[t] + \mu * y[t] + 2x[t]^2,
            x[0] = x0, y[0] = y0 /. \mu \rightarrow -1, \{x, y\}, \{t, -2, 2\}];
     IC0 = Table[{0, y}, {y, ymin, ymax, 0.1}]
     IC1 = Table[{xmin, y}, {y, ymin, ymax, 0.05}];
     IC2 = Table[{xmax, y}, {y, ymin, ymax, 0.05}];
     IC3 = Table[{x, ymin}, {x, xmin, xmax, 0.05}];
     IC4 = Table[{x, ymax}, {x, xmin, xmax, 0.05}];
     ICs = Join[IC0, IC1, IC2, IC3, IC4];
     plot =
        Table[ParametricPlot[
            Evaluate[\{x[t], y[t]\} /. solution[ICs[i, 1]], ICs[i, 2]]], \{t, -2, 2\}, PlotRange \rightarrow
             {{xmin, xmax}, {ymin, ymax}}, PlotLabel → "Supercritical stable spiral"] /.
          Line[x_{-}] \Rightarrow \{Arrowheads[\{0, 0.0375, 0.0375, 0\}], Arrow[x]\}, \{i, Length[ICs]\}];
     Show[{plot}]
     ••• NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.
Out[*]=\left\{\{0,-0.3\},\{0,-0.2\},\{0,-0.1\},\{0,5.55112\times10^{-17}\},\{0,0.1\},\{0,0.2\},\{0,0.3\}\right\}
     ••• NDSolve: At t == -1.57494, step size is effectively zero; singularity or stiff system suspected.
```

- ... Interpolating Function: Input value (-1.99992) lies outside the range of data in the interpolating function. Extrapolation will
- ••• Interpolating Function: Input value {-1.99992} lies outside the range of data in the interpolating function. Extrapolation will be used.
- ••• NDSolve: At t == -1.80563, step size is effectively zero; singularity or stiff system suspected.
- ... Interpolating Function: Input value (-1.99992) lies outside the range of data in the interpolating function. Extrapolation will
- ... General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.
- ••• NDSolve: At t == -1.75185, step size is effectively zero; singularity or stiff system suspected.
- ••• General: Further output of NDSolve::ndsz will be suppressed during this calculation.



```
In[*]:= Clear["Global`*"]
     xmin = -0.3;
    xmax = 0.3;
    ymin = -0.3;
    ymax = 0.3;
     solution[x0_, y0_] =
       NDSolve[\{x'[t] = \mu * x[t] + y[t] - x[t]^2, y'[t] = -x[t] + \mu * y[t] + 2x[t]^2,
           x[0] = x0, y[0] = y0 /. \mu \rightarrow 1, \{x, y\}, \{t, -2, 2\}];
    IC0 = Table[\{0, y\}, \{y, ymin, ymax, 0.1\}]
     IC1 = Table[{xmin, y}, {y, ymin, ymax, 0.1}];
    IC2 = Table[{xmax, y}, {y, ymin, ymax, 0.1}];
    IC3 = Table[{x, ymin}, {x, xmin, xmax, 0.1}];
    IC4 = Table[{x, ymax}, {x, xmin, xmax, 0.1}];
    ICs = Join[IC0, IC1, IC2, IC3, IC4];
     plot =
       Table[ParametricPlot[
           Evaluate[\{x[t], y[t]\} /. solution[ICs[i, 1], ICs[i, 2]]], \{t, -2, 2\}, PlotRange \rightarrow
            {{xmin, xmax}, {ymin, ymax}}, PlotLabel → "Supercritical unstable spiral"] /.
         Line[x_{-}] \Rightarrow \{Arrowheads[\{0, 0.0375, 0.0375, 0\}], Arrow[x]\}, \{i, Length[ICs]\}];
     Show[{plot}]
```

••• NDSolve: Initial condition x0 is not a number or a rectangular array of numbers.

$$\textit{Out[s]} = \left\{ \{0, -0.3\}, \{0, -0.2\}, \{0, -0.1\}, \left\{0, 5.55112 \times 10^{-17}\right\}, \{0, 0.1\}, \{0, 0.2\}, \{0, 0.3\} \right\}$$

