Smoothed Particle Hydrodynamics Simulations for Asteroid Deflection

Maximilian Rutz

Abstract

Fragen:
- Gliederung, SPH zu Theorie oder Numerics?
- Reihenfolge/Parallelität der Lösung von Mass/Momentum/Energy Conservation
- An wen ist eine Masterthesis gerichtet?

Contents

1 Introduction

A number of research groups [1]

2 Theory

I will start with the fluid case for SPH and then show how it can be modified for solids.

2.1 Navier-Stokes Equations

1) Conservation of Mass 2) Conservation of Momentum 3) Conservation of Energy

- we end up at Cauchy Momentum Equation

- quite new: developed in 1992 - goal was to solve Navier-Stokes Equations - Fluide die Navier-Stokes Equations gehorchen heissen newtonsche fluide - there are compressible and incompressible Navier Stokes Equations (for shockwaves) - equations describe velocity changes (in a defined region of space) over time - compressible N.S. Equations: $\rho \cdot (\frac{\partial v}{\partial t} + v \cdot \nabla v) = \rho \cdot g - \nabla p + \mu \cdot \Delta v$ - convective acceleration (schneller bei Verengung) is a vector, not the result of a dot product: $v \cdot \nabla v \equiv \begin{pmatrix} v_x \cdot \frac{\partial v_x}{\partial x} \\ v_y \cdot \frac{\partial v_y}{\partial y} \\ v_z \cdot \frac{\partial v_z}{\partial z} \end{pmatrix}$ - vector equation means three equations have to be solved independently - Rearranged long vector form of navier stokes: $\rho \cdot \begin{pmatrix} \frac{\partial v_x}{\partial t} \\ \frac{\partial v_y}{\partial t} \\ \frac{\partial v_z}{\partial t} \end{pmatrix} = -\rho \cdot \begin{pmatrix} v_x \cdot \frac{\partial v_x}{\partial x} \\ v_y \cdot \frac{\partial v_y}{\partial y} \\ v_z \cdot \frac{\partial v_z}{\partial z} \end{pmatrix} + \rho \cdot \begin{pmatrix} g_x \\ g_y \\ g_z \end{pmatrix} - \begin{pmatrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \\ \frac{\partial p}{\partial z} \end{pmatrix} + \mu \cdot \begin{pmatrix} \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \\ \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \\ \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \end{pmatrix}$ - laplacian on a vector field returns a vector, not a scalar! - pressure (is this the same in miluphcuda?): $p = k \cdot (\rho - \rho_0)$ - Mass continuity (dividing by rho also gives velocity continuity!? - yes, but only in incompressible N.S. equations - what about solids?): - $\rho \cdot (\nabla \cdot v) = 0$ - how to solve N.S. and continuity at same time? ... automatically solved in particle methods when particles are not created/destroyed - In SPH we want to know the local velocity change at each particle location - Show that: $\frac{dv_i}{dt} = \frac{\partial v}{\partial t} + v \cdot \nabla v$

- $v \cdot \nabla v$ holds for any scalar field property y that a particle traverses with velocity u $u \cdot \nabla y$ since it is the projection $u \cdot \nabla y = |u| \cdot |\nabla y| \cdot cos(\Theta)$ where Θ is the angle between u and ∇y. - Thus, the local change in the field property $\frac{dy_i}{dt}$ at the point i in the Lagrangian view corresponds to the sum of the change of the field property at a point i over time $\frac{\partial y_i}{\partial t}$ and the change $u_i \cdot \nabla y_i$ of the field property due to the movement of the particle along the field with velocity u in the Eulerian view: $\frac{dy_i}{dt} = \frac{\partial y_i}{\partial t} + u_i \cdot \nabla y_i$

This expression is called the Material Derivative $\frac{Dy}{dt}$: $\frac{Dy}{dt} \equiv \frac{\partial y}{\partial t} + u \cdot \nabla y$

It gives us the change of the field property in the reference frame of the particle (denoted by the subscript i):

In the Navier-Stokes Equations the scalar fields of interest happen to be the individual velocity components so we get: $\rho \cdot \frac{dv_i}{dt} = \rho \cdot \frac{Dv}{Dt} = \rho \cdot (\frac{\partial v}{\partial t} + v \cdot \nabla v) = \rho \cdot g - \nabla p + \mu \cdot \Delta v$

$\frac{dv_i}{dt} = g - \frac{1}{\rho_i} \cdot \nabla p + \frac{\mu}{\rho_i} \cdot \Delta v$ - This is where we change from Euler(grid methods) to Lagrangian view (particle information) ...

2.2 Cauchy Momentum equation

2.3 Constitutive equations - Deviatoric Stress Tensor

- Evolution of Deviatoric Stress Tensor - To apply the Navier-Stokes Equations to solids, a more general form is needed. - We use a subset of Navier-Stokes Equation - Constitutive equation decides whether it is a fluid or a solid

- plus continuity equation - how does ONE Navier Stokes Equation become THREE Euler Equations?

2.4 Equations of State

-

2.5 Smoothed Particle Hydrodynamics

Now that we have our compressible Navier-Stokes Equations in Lagrangian form we want to solve them.

- quite new: developed in 1992 - goal was to solve Navier-Stokes Equations - Uses Smoothing Kernels ... similar concept as basis functions in FEM - quantities are wighted average

- Terms in the Navier-Stokes Equation:

$$\frac{dv_i}{dt} = g - \frac{1}{\rho_i} \cdot \nabla p + \frac{\mu}{\rho_i} \cdot \Delta v \tag{1}$$

1) Density: $\rho_i \approx \sum_j m_j \cdot W(r - r_j, h)$

2)Pressure gradient: $\frac{\nabla p_i}{\rho_i} \approx \sum_j m_j \cdot (\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}) \cdot \nabla W(r - r_j, h)$ - why the gradient of W??

3) Viscosity term: $\frac{\mu}{\rho_i} \cdot \Delta v_i \approx \frac{\mu}{\rho_i} \cdot \sum_j m_j \cdot (\frac{v_j - v_i}{\rho_j}) \cdot \Delta W(r - r_j, h)$

Smoothing kernels: - $w = 0 for |r - r_0| > h \int_{||r' - r|| \leq h} W(r' - r, h) dr' = 1$ - concrete Kernel implementation shown in Numerics section

3 Numerics

3.1 Initial conditions

- SPH needs uniform macro structure but random micro structure

3.2 SPH - Right hand side quantities

- how are the equations from Theory section used to get the development over time - final form of Equation we used: .... - Neighborhoudsearch instead of computing Kernel over all particles is faster - Upper limit to maximum neighbor particles

3.3 Time Integration

1) Timestep algorithm for velocity update (RungeKutta 2) 2) Update particle positions from velocity and acceleration (accel from Navier Stokes) 3) Error criterium for timestep (what is used? RK45?)

4 Results

4.1 Cratering

4.2 Beta factor

5 Discussion

- beta factor on the lower end - upper limit beta ¡ 2 because of momentum conservation??

6 Conclusions

References

[1] Sabina Raducan et al. "The role of asteroid strength, porosity and internal friction in impact momentum transfer". In: *Icarus* 329 (Apr. 2019), pp. 282–295. DOI: `10.1016/j.icarus.2019.03.040`.