Replicating Fiscal Policy Shocks In A Canonical HANK Model à la Auclert et al. (2024)

Seminar Paper
Model & Calibration Appendix

by

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Abstract

This appendix presents our reasoning for model and calibration documentation in full, which can be found in the notebook to comment along on block building of the canonical HANK model by Auclert et al. (2024).

Canonical HANK Model

The authors decided for a standard incomplete markets model with underlying Bewley Huggett-Aiyagari characteristics - meaning incomplete insurance markets and heterogeneous households, describing ex-ante similiar households, which change ex-post, because they are exposed to different idyiosyncratic shocks, which they can only insure againts through riskless bonds or capital as financial markets are incomplete (Kirkby, 2019). Merged with the New Keynesian Paradigm of forward-looking households and firms with market imperfections like sticky prices and wages, the model is underlying for calibration of marginal propensities for consumption, aggregate values of income and wealth as well respectively their distributions.

The Household Problem

The economies household decision making is ruled by the following consumption maximisation problem:

$$max_{\{c_{it}\}} \quad E_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s \leq t-1} \beta_{is} \right) \{log(c_{it}) - v(n_{it})\} \right]$$

$$\text{s.t.} \quad c_{it} + a_{it} \leq (1 + r_t^p) a_{it-1} + (1 - \tau_i) w_t e_{it} n_{it} \tag{1}$$

$$a_{it} \geq 0$$

This comprises as a model with unit mass of agents $i \in [0, 1]$ that are identical ex-ante but become heterogeneous ex-post because each individual is hit by private shocks e_{it} to labor productivity and to their discount factor β_{it}, which follow an exogenous Markov process expressing the transitions from one state of shocks and discount factor (e, β) to another (e', β') ((expressed by the matrix $\prod(e', \beta'|e, \beta)$; the cross-sectional distribution over these states is kept at its stationary value, and average efficiency units of labor are normalized to one.

To study dynamics, the authors consider small, one-time, unexpected disturbances at date 0 (MIT shocks to aggregates like r_t, w_t and τ_t) that move the economy away from its stationary equilibrium. This corresponds to computing first-order IRF's in a fully stochastic counterpart. Each household i has time-separable preferences with $log(u(c_{it}))$ utility in consumption and a separable disutility of labor $v(n_{it})$, supplies efficient units of labor $(e_{it}n_{it})$ according to an exogenously set hours choice nit (e.g., by working unions), pays labor income taxes at rate τ_t, and saves in a mutual fund with assets ait subject to a zero-borrowing constraint $a_{it} \geq 0$, earning the realized return on the fund r_t^p.

This poses an individual problem as with given idiosyncratic income risk (e_{it} shocks) and the inability to borrow or fully insure, agents choose consumption c_{it} and next-period assets a_{it+1} to smooth consumption over time while self-insuring against income fluctuations. The combination of income risk (e_{it}), discount factor heterogeneity (β_{it}), and borrowing constraints $(a_{it} \geq 0)$ generates realistic risk-averse saving behavior.

The Supply Side Set-Up

The authors assume that firm output equals the aggregate of effective labour, acting as price setters including a mark-up attached to the nominal wage over their nominal marginal resulting in a price as of $P = \mu W_t$, implying that real wage $w = \frac{1}{\mu}$.

$$Y_t = N_t, \qquad N_t = \mathrm{E}[e_{it}n_{it}]$$

Introducing a *dividend tax* equal to *labour income tax* τ_i, the post-ex dividends are $d_t = (1-\tau)(Y_t - w_t N_t) = (1-\tau)(1-\frac{1}{\mu})Y_t$. With the New Keynesian paradigm comes price stickyness and the authors use the following assumptions from the literature towards price stickyness:

As of subject to maximise agents utility under constraint of adjustment cost, wages are set by workers union (Erceg et al., 2000). Unions know how to allocate all labour hours over workforce and do so uniformly (Auclert, 2019). The union sets wages to maximize the expected utility of a worker with average consumption (Hagedorn et al., 2019; Auclert, 2019). This yields a linear wage first-order **Phillips curve** for wage inflation under Calvo-Pricing in the form of

$$\pi_t^w = \kappa \left(v'(N_t) - \frac{1-\tau_t}{\mu C_t} \right) + \bar{\beta}\pi_{t+1}^w \tag{2}$$

Wage inflation arises whenever the marginal disutility of working an extra hour, $v'(N_t)$, is higher than the marginal gain to a worker with average consumption. That gain is given by the after-tax real wage $\frac{1-\tau_t}{\mu}$ times the marginal utility of average consumption, $\frac{1}{C_t}$, evaluated over current and future periods. Hence, the Philipps Curve describes the behaviour of price inflation, keeping in mind that prices where set as with a constant markup above the nominal wage, which equalises the terms of price and wage inflation $\pi_t^w = \pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

Government Endowments

The model governments budget consists of it's spending G_t and debt in the current state B_t which equals it's collection of labor and corporate taxes as a sum of tax revenue Y_t added by the returns from government bonds it possesses from a period before the constraint is

$$G_t + B_t = (1 + r_{t+1})B_{t-1} \tag{3}$$

According to a given intertemporal budget constraint, the defined government set-up is supposed to have a strategy in setting G_t and τ_t accordingly as well as for bonds B_t and spending G_t, however, with a still budget constraint respected tax rate.

Further Assumptions on the capital market are They assume are a simple asset market and policy environment. As it includes mutual fund collection of household savings a_it and thus two-folded investment either in government bonds or the stock market for which the No-Arbitrage condition must hold

$$1 + r_t = \frac{p_{t+1} + d_{t+1}}{p_t} \tag{4}$$

while total household assets exactly equal the value of equity plus public debt

$$A_t = p_t + B_t \tag{5}$$

Because mutual funds are perfectly competitive, the fund just passes through the bond return to households, and its initial return is the portfolio-weighted average of the stock and bond returns:

$$(1 + r_0^p)A = p_0 + d_0 + (1 + r)B \tag{6}$$

Monetary policy sets the real interest rate on government bonds via a standard interest-rate rule with $1 + i_t = (1 + r_t)(1 + \pi_{t+1})$ and a competitive equilibrium is then defined as prices, quantities, and household decisions such that households optimize and both asset and goods markets clear such that consumption and government spending equal the economies output

$$C_t + G_t = Y_t \tag{7}$$

Once two of equations (5) to (7) hold, the third one will be satisfied automatically - by Walras's law, if all markets but one clear, the last one must clear too.

Agent Modelling

Representative-agent model

The model imposes that a representative agent solves the consumption maximisation problem in (1) with no risk on idiosyncratic income or the discount factor, while they are further able to borrow without constraints, making it a special case as mentioned by the authors. Thus, household consumption for this agent follows a textbook-like **Euler Equation** $C_t^{-1} = \beta(1 + r_t)C_{t+1}^{-1}$ in this special case.

Two Agent Model

The Bilbiie (2008) and Bilbiie (2019) and Ascari et al. (2016) based two-agent model define a share $(1 - \lambda)$ of agents whose behaviour is described by the prelemenary defined Euler equation, indexed for an unconstrained agent and a share λ of agents who do not have access to capital markets, constraining them in that their net-income labor income will always equal their consumption. Overall it is assumed that both all agents have the same productivity e_{it} and join the labour force $(n_{it} = N_t)$. For constrained agents it means that consumption equals the after tax labor income and the economies aggregate consumption is the sum of unconstrained and constrained agents' consumption $C_t = (1 - \lambda)C_t^u + \lambda C_t^c$.

Calibration

tbc Wednesday

Intertemporal MPC's

Household behavior in general equilibrium is summarized by intertemporal marginal propensities to consume (iMPCs), which describe how aggregate consumption responds over time to shocks to labor income, capital gains, and interest rates. In our replication, we follow Auclert et al. (2024) and use these iMPCs, which the authors computed from the underlying incomplete markets structure as sufficient statistics to build the sequence-space solution and construct the impulse responses in Figure 2.

Toward general equilibrium.

tbc Wednesday

Fiscal Policy

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