

Replicating Fiscal Policy Shocks In A Canonical HANK Model à la Auclert et al. (2024)

Seminar Paper
Model & Calibration Appendix

by

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Abstract

This paper presents our reasoning for model and calibration documentation in full, which can be found in the notebook to comment along on block building of the canonical HANK model by Auclert et al. (2024). We take criticism on the models calibration values and propose them to be closely matched to more empirical data, while still being able to match empirical facts.

Canonical HANK Model

The authors decided for a standard incomplete markets model with underlying Bewley-Huggett-Aiyagari characteristics - meaning incomplete insurance markets and heterogeneous households, describing ex-ante similar households, which change ex-post, because they are exposed to different idiosyncratic shocks, which they can only insure against through riskless bonds or capital as financial markets are incomplete (Kirkby, 2019). Merged with the New Keynesian Paradigm of forward-looking households and firms with market imperfections like sticky prices and wages, the model is underlying for calibration of marginal propensities for consumption, aggregate values of income and wealth as well respectively their distributions.

The Household Problem

The economies household decision making is ruled by the following consumption maximisation problem:

$$\begin{aligned} \max_{\{c_{it}\}} \quad & E_0 \left[\sum_{t=0}^{\infty} \left(\prod_{s \leq t-1} \beta_{is} \right) \{ \log(c_{it}) - v(n_{it}) \} \right] \\ \text{s.t.} \quad & c_{it} + a_{it} \leq (1 + r_t^p) a_{it-1} + (1 - \tau_i) w_t e_{it} n_{it} \\ & a_{it} \geq 0 \end{aligned} \tag{1}$$

This comprises as a model with unit mass of agents $i \in [0, 1]$ that are identical ex-ante but become heterogeneous ex-post because each individual is hit by private shocks e_{it} to labor productivity and to their discount factor β_{it} , which follow an exogenous Markov process expressing the

transitions from one state of shocks and discount factor (e, β) to another (e', β') ((expressed by the matrix $\prod(e', \beta'|e, \beta)$); the cross-sectional distribution over these states is kept at its stationary value, and average efficiency units of labor are normalized to one.

To study dynamics, the authors consider small, one-time, unexpected disturbances at date 0 (MIT shocks to aggregates like r_t, w_t and τ_t) that move the economy away from its stationary equilibrium. This corresponds to computing first-order IRF's in a fully stochastic counterpart. Each household i has time-separable preferences with $\log(u(c_{it}))$ utility in consumption and a separable disutility of labor $v(n_{it})$, supplies efficient units of labor ($e_{it}n_{it}$) according to an exogenously set hours choice nit (e.g., by working unions), pays labor income taxes at rate τ_t , and saves in a mutual fund with assets a_{it} subject to a zero-borrowing constraint $a_{it} \geq 0$, earning the realized return on the fund r_t^p .

This poses an individual problem as with given idiosyncratic income risk (e_{it} shocks) and the inability to borrow or fully insure, agents choose consumption c_{it} and next-period assets a_{it+1} to smooth consumption over time while self-insuring against income fluctuations. The combination of income risk (e_{it}), discount factor heterogeneity (β_{it}), and borrowing constraints ($a_{it} \geq 0$) generates realistic risk-averse saving behavior.

The Supply Side Set-Up

The authors assume that firm output equals the aggregate of effective labour, acting as price setters including a mark-up attached to the nominal wage over their nominal marginal resulting in a price as of $P = \mu W_t$, implying that real wage $w = \frac{1}{\mu}$.

$$Y_t = N_t, \quad N_t = E[e_{it}n_{it}]$$

Introducing a *dividend tax* equal to *labour income tax* τ_i , the post-ex dividends are $d_t = (1 - \tau)(Y_t - w_t N_t) = (1 - \tau)(1 - \frac{1}{\mu})Y_t$. With the New Keynesian paradigm comes price stickyness and the authors use the following assumptions from the literature towards price stickyness:

As of subject to maximise agents utility under constraint of adjustment cost, wages are set by workers union (Erceg et al., 2000). Unions know how to allocate all labour hours over workforce and do so uniformly (Auclert, 2019). The union sets wages to maximize the expected utility of a worker with average consumption (Hagedorn et al., 2019; Auclert, 2019). This yields a linear wage first-order **Phillips curve** for wage inflation under Calvo-Pricing in the form of

$$\pi_t^w = \kappa \left(v'(N_t) - \frac{1 - \tau_t}{\mu C_t} \right) + \bar{\beta} \pi_{t+1}^w \quad (2)$$

Wage inflation arises whenever the marginal disutility of working an extra hour, $v'(N_t)$, is higher than the marginal gain to a worker with average consumption. That gain is given by the after-tax real wage $\frac{1 - \tau_t}{\mu}$ times the marginal utility of average consumption, $\frac{1}{C_t}$, evaluated over current and future periods. Hence, the Philipps Curve describes the behaviour of price inflation, keeping in mind that prices where set as with a constant markup above the nominal wage, which equalises the terms of price and wage inflation $\pi_t^w = \pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$

Government Endowments

The model governments budget consists of it's spending G_t and debt in the current state B_t which equals it's collection of labor and corporate taxes as a sum of tax revenue Y_t added by the returns from government bonds it possesses from a period before the constraint is

$$G_t + B_t = (1 + r_{t+1})B_{t-1} \quad (3)$$

According to a given intertemporal budget constraint, the defined government set-up is supposed to have a strategy in setting G_t and τ_t accordingly as well as for bonds B_t and spending G_t , however, with a still budget constraint respected tax rate.

Further assumptions on the capital market are They assume are a simple asset market and policy environment. As it includes mutual fund collection of household savings a_{it} and thus two-folded investment either in government bonds or the stock market for which the No-Arbitrage condition must hold

$$1 + r_t = \frac{p_{t+1} + d_{t+1}}{p_t} \quad (4)$$

while total household assets exactly equal the value of equity plus public debt

$$A_t = p_t + B_t \quad (5)$$

Because mutual funds are perfectly competitive, the fund just passes through the bond return to households, and its initial return is the portfolio-weighted average of the stock and bond returns:

$$(1 + r_0^p)A = p_0 + d_0 + (1 + r)B \quad (6)$$

Monetary policy sets the real interest rate on government bonds via a standard interest-rate rule with $1 + i_t = (1 + r_t)(1 + \pi_{t+1})$ and a competitive equilibrium is then defined as prices, quantities, and household decisions such that households optimize and both asset and goods markets clear such that consumption and government spending equal the economies output

$$C_t + G_t = Y_t \quad (7)$$

Once two of equations (5) to (7) hold, the third one will be satisfied automatically - by Walras's law, if all markets but one clear, the last one must clear too.

Agent Modelling

Representative-agent model

The model imposes that a representative agent solves the consumption maximisation problem in (1) with no risk on idiosyncratic income or the discount factor, while they are further able to borrow without constraints, making it a special case as mentioned by the authors. Thus, household consumption for this agent follows a textbook-like **Euler Equation** $C_t^{-1} = \beta(1 + r_t)C_{t+1}^{-1}$ in this special case.

Two Agent Model

The Bilbiie (2008) and Bilbiie (2019) and Ascari et al. (2016) based two-agent model define a share $(1 - \lambda)$ of agents whose behaviour is described by the prelemenary defined Euler equation, indexed for an unconstrained agent and a share λ of agents who do not have access to capital markets, constraining them in that their net-income labor income will always equal their consumption. Overall it is assumed that both all agents have the same productivity e_{it} and join the labour force ($n_{it} = N_t$). For constrained agents it means that consumption equals the after tax labor income and the economies aggregate consumption is the sum of unconstrained and constrained agents' consumption $C_t = (1 - \lambda)C_t^u + \lambda C_t^c$.

Calibration

The following section defines the author's calibration for the steady state of the economy. In a further static analysis, we will change parameters to check for different outcomes under our assumptions.

Under quarterly frequency, the authors calibration approximates time by values at quarterly points so that it fits the model's quarterly timing, modifying the original continuous-time income process e_{it} based on Kaplan et al., 2018. Their main goal is to find a good approximation of the wealth level and distribution for most recent U.S. economy data. We comment on the authors calibration in the following paragraph.

The authors build their calibration using U.S. macro aggregates (wealth-, debt-, spending- and tax-to-GDP ratios, and a long-run real interest rate) from national accounts and fiscal statistics. Their assets-to-GDP calibration A is supported by U.S. Bureau of Economic Analysis (2025) data. However, their bonds-to-GDP B value is lower than what U.S. Office of Management and Budget and FRED St. Louis (2025) data from 2020 on would suggest, while their value for government spending to GDP value G rather matches the U.S. level from 1950's, being constantly within a range

Variable	Description	Value	Variable	Description	Value
r	Real interest rate (annual)	2%	μ	Markups	1.11
A	Assets to GDP (annual)	500%	(β^L, β^H)	Discount factors (quarterly)	(0.91, 1.00)
B	Bonds to GDP (annual)	100%	ω	Share of patient	49%
M_{00}	Income-weighted MPC (quarterly)	0.2	q	Prob of new β draw (quarterly)	1%
G	Government spending to GDP	20%	T	Taxes to GDP	22%

Table 1: Calibration of the baseline HA model (Aucleter et al. (2024))

of 30% - 40% according to IMF (2025) Public Finance data. With their beforehand mentioned calibrations, they compute a tax-to-GDP ratio T/Y of 22% ($\frac{G}{Y} + r \frac{B}{Y}$) which would not quite match IMF (2025) having been on its highest level (13%) in 2000 lowest level (7.9%) within a 1973 - 2023 period. They then select preference heterogeneity parameters ($\beta^L = 0.91, \beta^H = 1$ and ω) to match the Board of Governors of the Federal Reserve System (2019) distribution after assuming that the discount factor β follows a two-state Markov chain, so that agents switch between ‘patient’ and ‘impatient’ types. These parameters are disciplined by matching the Board of Governors of the Federal Reserve System (2019) wealth distribution and a target for the income-weighted MPC. The markup is inferred residually to produce a plausible equity-to-GDP ratio within the model.

The **Phillips-curve** is treated secondary by the authors as they mainly care about real quantities, which they can compute without pinning down inflation in detail. They only specify Phillips-curve parameters to be able to solve for inflation paths when needed, not because these parameters drive their core calibration. Thus simple and conventional values for the nominal block (Frisch elasticity $v = 1$, a small Phillips-curve slope with normalization to $Y = 1$) are chosen, because we identify their research question to be about heterogeneity and real allocations, not about fine-tuning inflation dynamics.

Intertemporal MPC’s

Household behavior in general equilibrium is summarized by intertemporal marginal propensities to consume (iMPCs), which describe how aggregate consumption responds over time to shocks to labor income, capital gains, and interest rates. In our replication, we follow Aucleter et al. (2024) and use these iMPCs, which the authors computed from the underlying incomplete markets structure as sufficient statistics to build the sequence-space solution and construct the impulse responses in Figure 2. As we are not further interested in the computation and construction reasoning of the Intertemporal MPCs and its Lorenz curve, we do not focus further on this subsection.

From IMPC’s to General Equilibrium

The authors show how partial-equilibrium iMPCs can be used as sufficient statistics in general equilibrium. They first express post-tax labor income as $Z_t = (1 - \tau_t)w_t N_t = \frac{1}{\mu}(Y_t - T_t)$, so that changes in Z_t are proportional to changes in output net of taxes, $dZ = \frac{1}{\mu}(dY - dT)$. Next, they write the value of aggregate assets $p_0 + d_0$ as the discounted value of dividends d_t , which themselves depend on $(Y_t - T_t)$. Differentiating the asset-pricing equation yields how asset values react to interest-rate changes and to $dY - dT$.

In a final step, the authors ‘close’ the model by expressing the general-equilibrium response of aggregate consumption as a function of policy-relevant objects. According to that, consumption changes dC reacts toward to channels. First, changes in the interest rate dr come from **monetary policy channel**, while changes in post tax income $dY - dT$ result from a **fiscal policy channel**. Using equilibrium relationships for wages, dividends, and asset prices, they derive

$$dC = \bar{M}^r dr + \bar{M} (dY - dT),$$

where \bar{M} is our main matrix of interest for the analysis of Figure 2(a) and (b) as it averages labor-income and capital-gains iMPCs (using wage vs. dividend income shares), and M_r summarizes the direct and asset-price-mediated effects of interest-rate changes on consumption. This means that once iMPCs and income shares are known, one can characterize the general-equilibrium response of consumption to fiscal policy and interest-rate paths.

Fiscal Policy

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