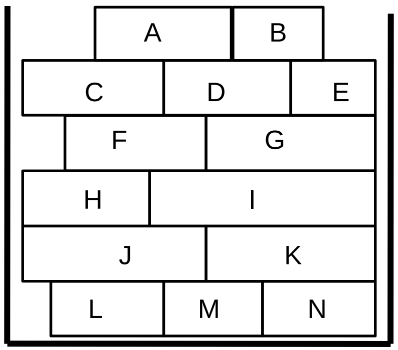
Order Relations

Order Relations
Hasse diagrams

(Defn 3.22 – 3.24)

A web-based travel agent has a number of holidays to offer. You describe the type of holiday you are looking for, and ask for recommendations. The website should display possible holidays, starting with the ones most likely to appeal to you.

A robot is to unload containers from a ship. No container can be unloaded if another is on top of it. How do we specify valid sequences for the robot?



An AI-character in a game must carry out certain tasks in order to stop an opponent. Some tasks must be completed before others begin, while other tasks will only be possible depending on the actions carried out by the user. How do we describe the task relationships to the AI character?

We need an ordering for the holidays, containers and tasks

A homogeneous relation $R \subseteq AxA$ is an order relation if and only if

(i) R is anti-symmetric

(ii) R is transitive

This is based on the relation "<" on numbers.

for any numbers x, y and z:

anti-symmetric: if $x < y$ *and* $x \neq y$, then $y \not< x$

transitive: if $x < y$ and $y < z$, then $x < z$

and so "<" is an order on the integers

Example: the "subset" relation

Let X, Y and Z be sets defined over some universal set U.

anti-symmetric: $X \subseteq Y$ and $Y \subseteq X$, then $X = Y$

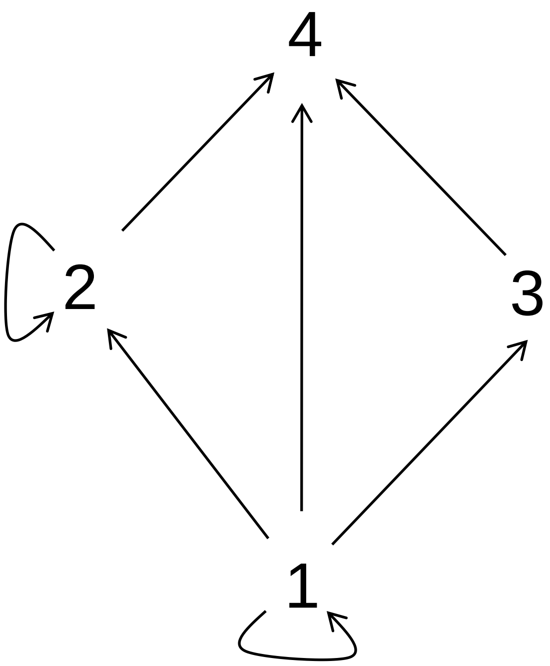
transitive: $X \subseteq Y$ and $Y \subseteq Z$, then $X \subseteq Z$

So subset is an order over a collection of sets

Exercise: Let $A = \{1,2,3,4\}$

Let $R \subseteq AxA = \{(1,1), (2,2), (1,3), (3,4),(1,2),(2,4),(1,4)\}$

Is R an order on A?



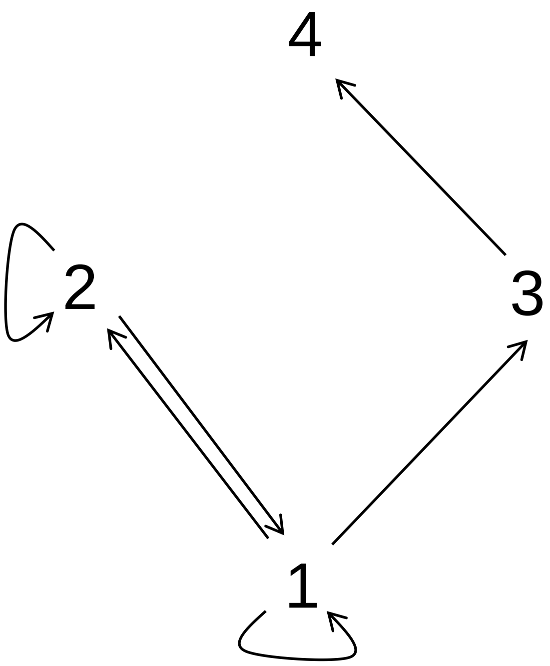
Anti-symmetric?

Transitive?

Exercise: Let A = {1,2,3,4}

Let R:AxA = {(1,1), (2,2), (1,3), (3,4),(1,2),(2,1)}

Is R an order on A?



Anti-symmetric?

Transitive?

An order $R \subseteq AxA$ is a strict order
if and only if R is also anti-reflexive

Consider the relation "<" on numbers.

for any numbers x, y and z:
anti-symmetric: if $x < y$ then $y \nless x$
transitive: if $x < y$ and $y < z$, then $x < z$
anti-reflexive: $x \nless x$

so "<" is a strict order over the integers

An order $R \subseteq AxA$ is a total order
if and only if for any a and b in A, either aRb or bRa or $a=b$

In other words, every pair of elements in the set is ordered with respect to each other

Example: "≤" on numbers.

for any pair of numbers x and y s.t. $x \neq y$

either $x \leq y$ or $y \leq x$

and so ≤ is a total order on the integers.

and so in the graph, every pair of elements has an arrow connecting them

An order $R \subseteq AxA$ is a partial order if it is not total

In other words, there can be two elements in the set that are not ordered with respect to each other

Example: "$\subseteq$" on sets:

for the universal set $U = \{a,b,c,d\}$

$S_1 = \{a,b,c\}$ and $S_2 = \{b,c,d\}$

$S_1 \not\subseteq S_2$ and $S_2 \not\subseteq S_1$

so "$\subseteq$" is a partial order on sets

WARNING:

some authors use slightly different definitions
for partial, total and strict orders

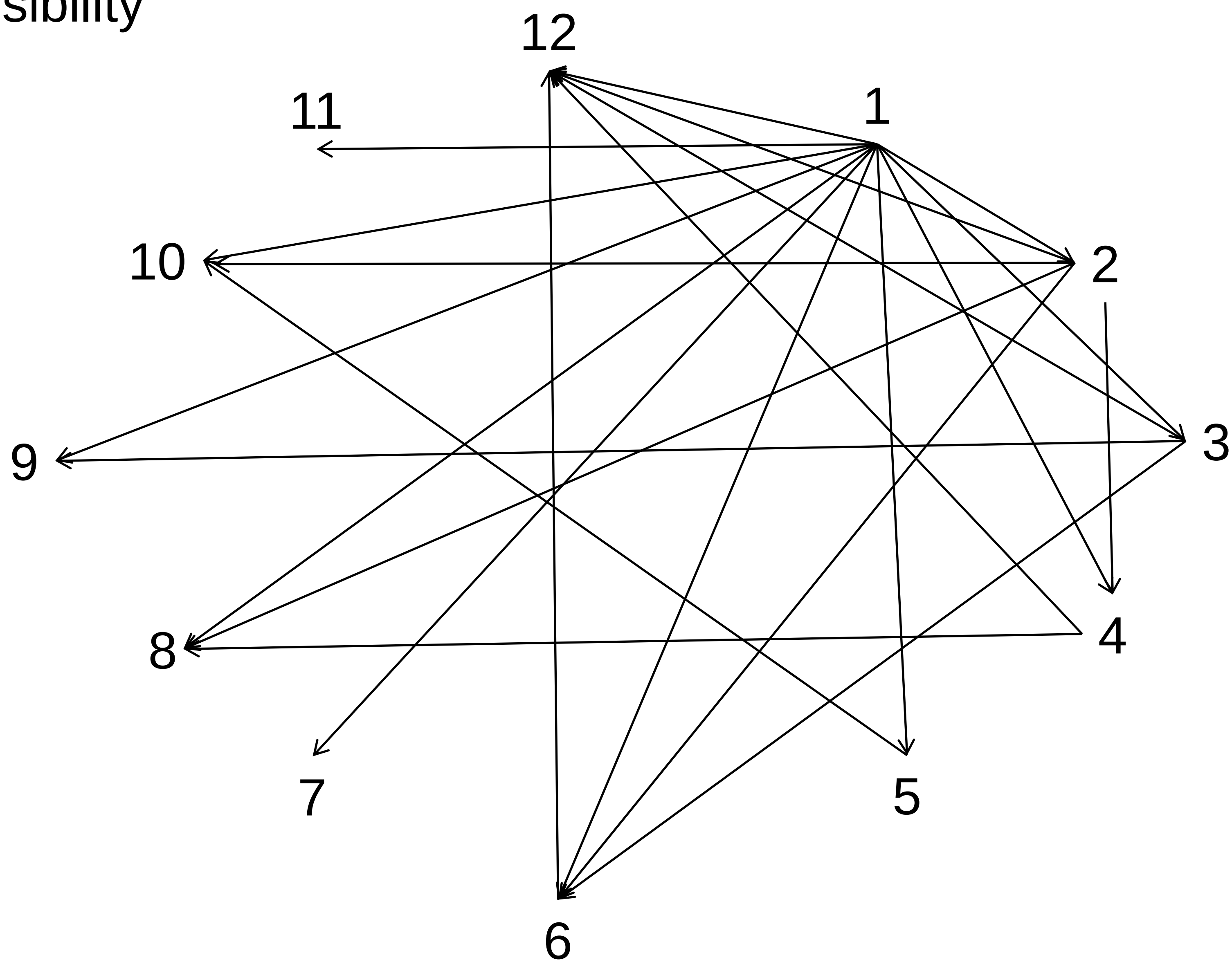
We can use the concept of the transitive closure to simplify the representation of partial orders on a set.

The Hasse diagram is a directed graph which shows the minimal subset of pairs for which the transitive closure gives the original relation.

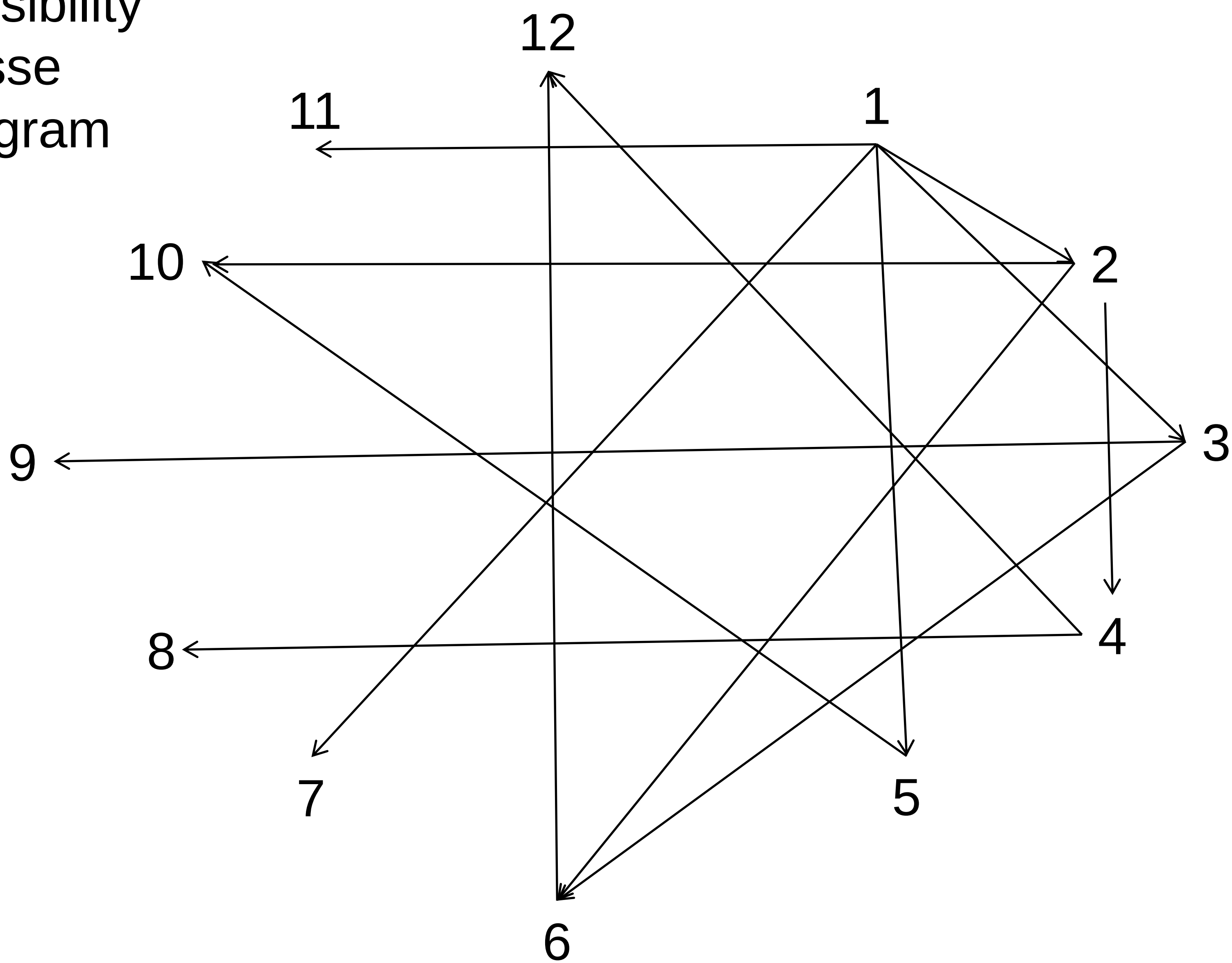
Formally, a covers b if and only if aRb and there is no element c s.t. aRc and cRb.

A Hasse diagram links every pair (a,b) s.t. a covers b.

Divisibility



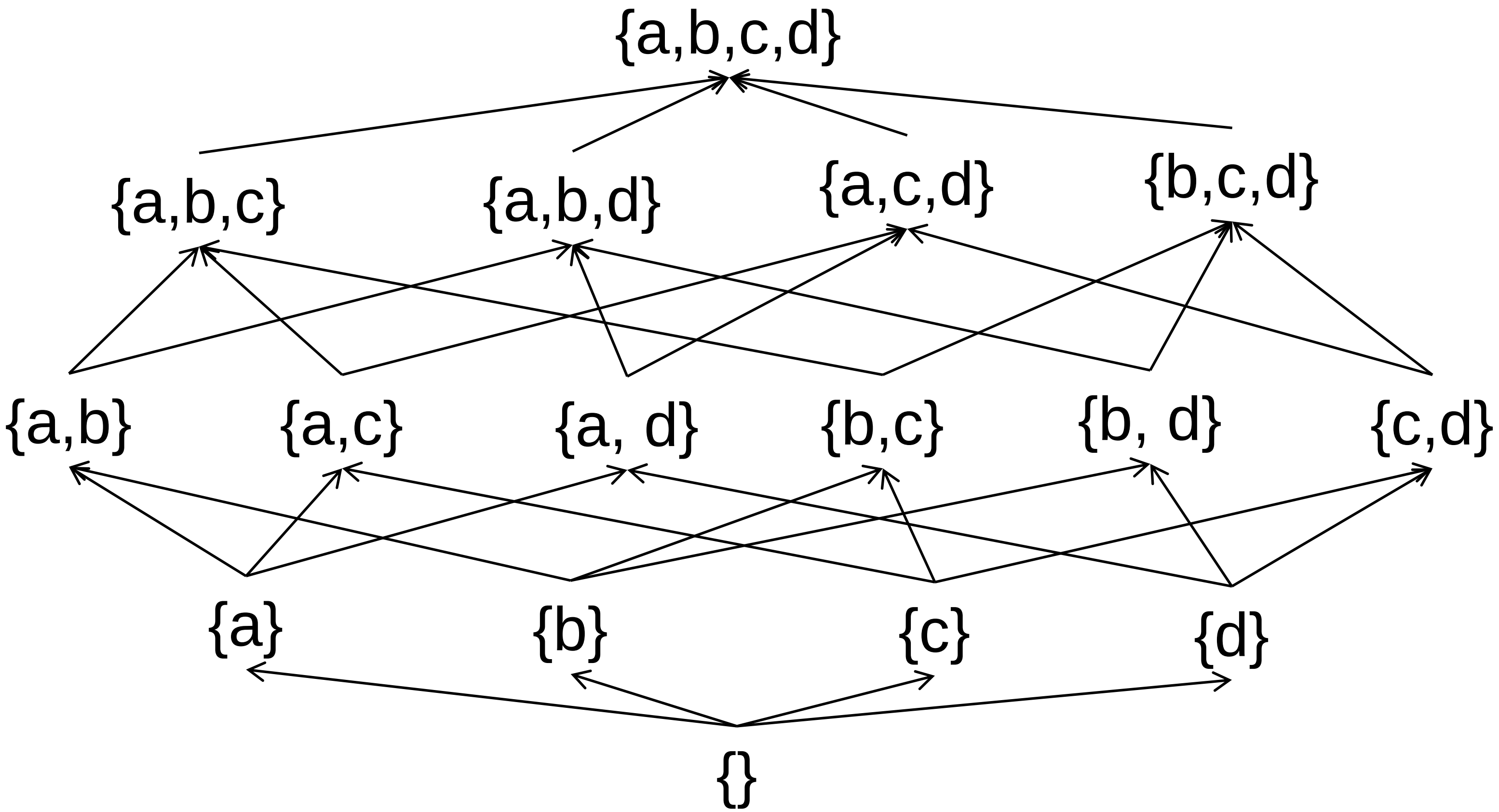
Divisibility
Hasse
Diagram



A = {a,b,c,d}

$\mathbb{P}$(A) = {{},{a},{b},{c},{d},{a,b},{a,c},{a,d},{b,c},{b,d},{c,d},
{a,b,c},{a,b,d},{a,c,d},{b,c,d},{a,b,c,d}}

Hasse diagram of $\subseteq$: $\mathbb{P}$(A)x$\mathbb{P}$(A)



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Precedence graph

From Wikipedia, the free encyclopedia

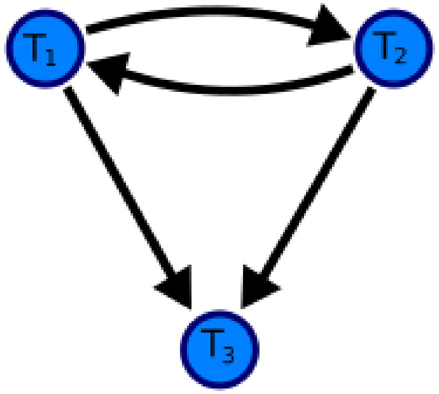
A **precedence graph**, also named **conflict graph** and **serializability graph**, is used in the context of concurrency control in databases.

The precedence graph for a schedule S contains:

- A node for each committed transaction in S
- An arc from T_i to T_j if an action of T_i precedes and conflicts with one of T_j's actions.

Precedence graph example [edit]

Time	T_1	T_2	T_3
1	read(A)		
2		write(A)	
3		Commit	
4	write(A)		
5	Commit		
6			write(A)
7			Commit



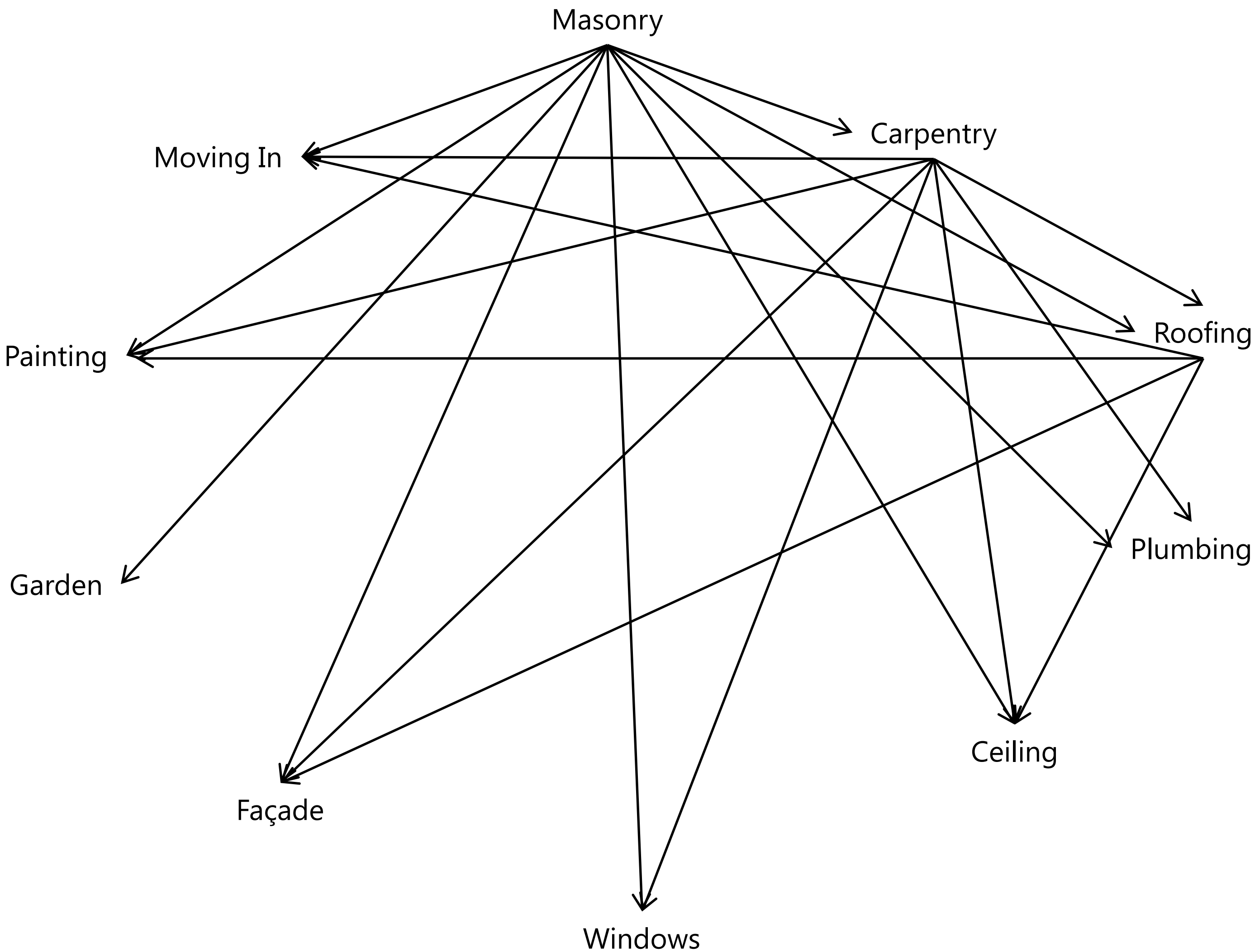
A precedence graph of 3 transactions. As there is a cycle, this schedule (history) is *not* Conflict serializable.

Construction scheduling problem

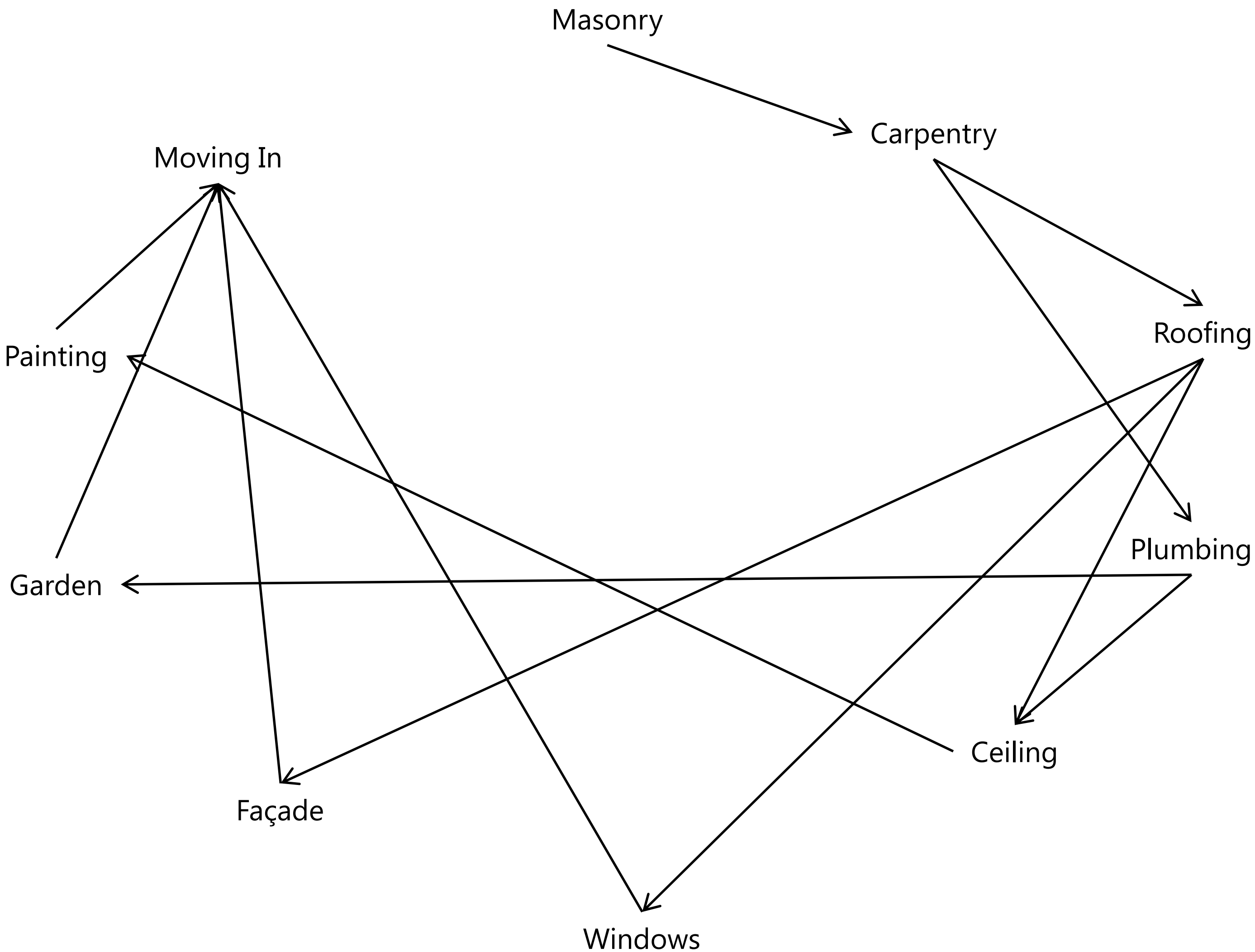
A set of tasks to be completed when building a house:

{Masonry, Carpentry, Roofing, Plumbing, Ceiling, Windows, Facade, Garden, Painting, Moving In}

Some tasks must be completed before others are finished. What are these relationships? Taken together, do they specify a partial order? If so, can we find a sequence of tasks that obeys the order? What is the best sequence?


Masonry
Carpentry
Moving In
Roofing
Painting
Plumbing
Garden
Ceiling
Façade
Windows

and so on ...


Masonry
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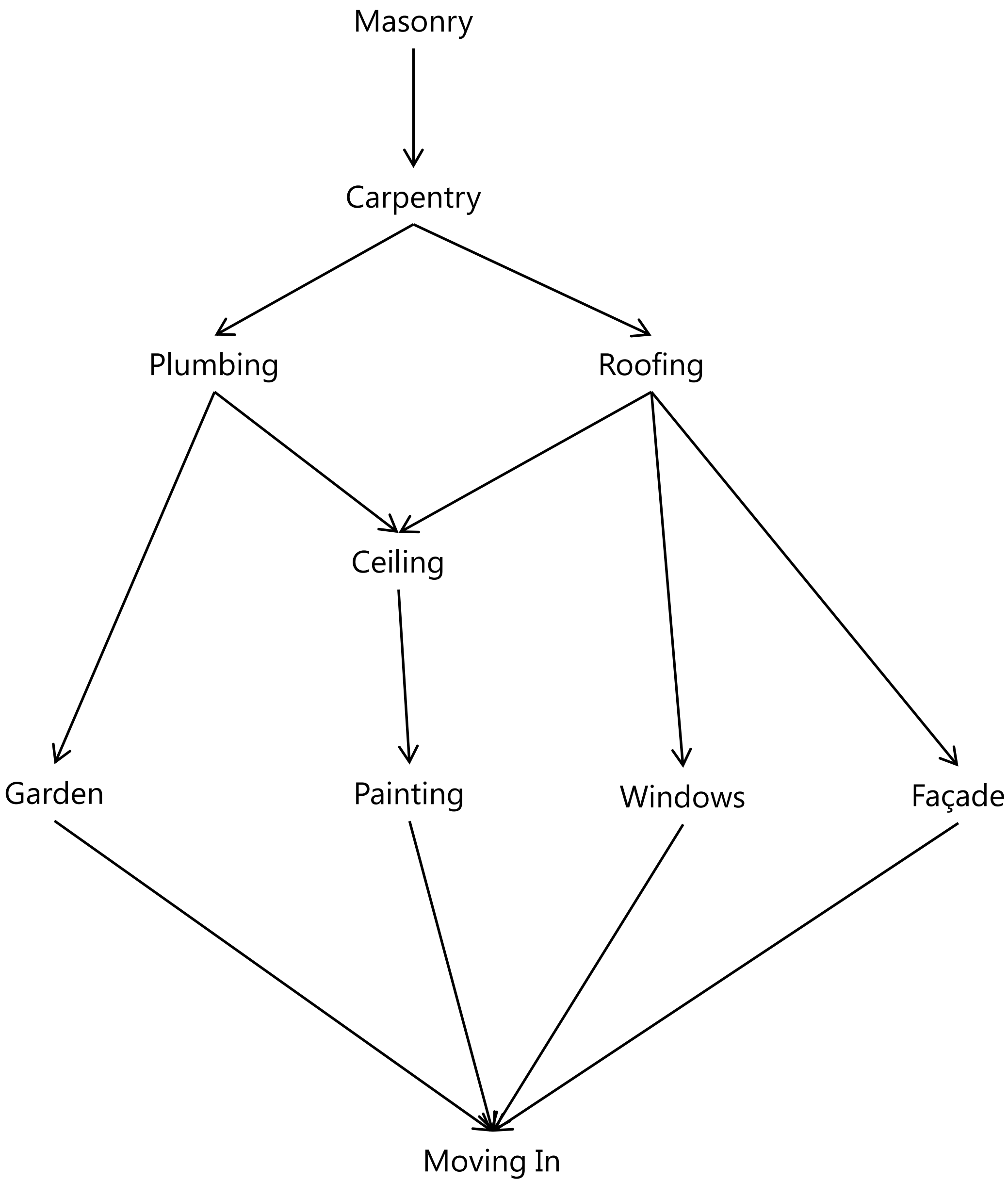
Masonry
Carpentry
Plumbing
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Garden
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Windows
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Next lecture ...

Introduction to Logic