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Assignment 7

Q1 (a)

We have $m_k = E[X^k]$

$$E[e^{\theta X}] = \sum_{k=0}^{\infty} \frac{1}{k!} m_k(X)\, \theta^k$$

$$= 1 + \underbrace{\sum_{k=1}^{\infty} \frac{1}{k!} m_k(X)\, \theta^k}_{= T(\theta)}$$

$$\log(1 + T(\theta))$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \cdot T(\theta)^n = k_X(\theta)$$

We have :

① $k_X(\theta) = \sum_{n=1}^{\infty} \frac{1}{n!} k_n\, \theta^n$

and

(2) $k_X(\theta) = \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} \cdot T(\theta)^r$

$$= \sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r} \left(\sum_{k=1}^{\infty} \frac{1}{k!} m_k\, \theta^k \right)^r$$

We use polynomial coefficient matching to match coefficients of θ in above expressions