Q1.

(i) $P[A=5, C=1, B=4]$

$$= 0.5^5 \cdot 0.2 \cdot 0.3^4 \times \frac{10!}{5! \times 1! \times 4!} = 0.06379.$$

(ii) $P[A+B=10] + P[A+C=10] + P[B+C=10]$

$$P[A+B=10] = \sum_{a+b=10} p_1^a p_2^{10-a} \cdot {}^{10}C_a$$

$$= (p_1+p_2)^{10} - p_1^{10} - p_2^{10}$$

$$= 0.8^{10} - 0.5^{10} - 0.3^{10} = 0.1064.$$

$P(A+C=10) = 0.7^{10} - 0.5^{10} - 0.2^{10} = 0.02727.$

$P(B+C=10) = 0.5^{10} - 0.3^{10} - 0.2^{10} = 0.00097.$

$P[\text{Exactly 2 types}] = 0.1346$

(iii) $P[A=3 \cap B=3 \cap C=4]$

$$= \frac{10!}{3!3!4!} \cdot 0.5^3 \times 0.3^3 \times 0.2^4 = 0.02268$$

$$P[A=3 \cap C=3 \cap B=4] = \frac{10!}{3!3!4!} \times 0.5^3 \times 0.2^3 \times 0.3^4$$

$$= 0.03402$$

$$P[B=C=3, A=4] = \frac{10!}{3!3!4!} \cdot 0.3^3 \times 0.2^3 \times 0.5^4$$

$$= 0.0567$$

$$P = 0.02268 + 0.03402 + 0.0567 = 0.1134$$

(iv) $P[\text{Three types}] = 1 - P(A=10) - P(B=10) - P(C=10)$

$$- P[\text{Two Types}]$$

$$= 1 - 0.5^{10} - 0.3^{10} - 0.2^{10} - 0.1346$$

$$= 0.8644$$

Q2.

Define F = event that a Robot is faulty.

$P(F) = p$

T = event that a Robot is tested faulty.

$P[T \mid F] = \varphi, \quad P[T \mid F'] = 0$

$$P[T] = P[T \mid F] \cdot P(F) + P[T \mid F'] \cdot P(F')$$
$$= p \cdot \varphi$$

$$P[F \mid T'] = \frac{P(T' \mid F) \cdot P(F)}{P(T')} = \frac{p(1-\varphi)}{1-p\varphi}$$

$$\Rightarrow E(X \mid Y) = Y + (n-y) \times P[F \mid T']$$
$$= Y + (n-y) \times \frac{p(1-\varphi)}{1-p\varphi}$$
$$= \frac{Y(1-p\varphi) + (n-y)p(1-\varphi)}{1-p\varphi} = \frac{np(1-\varphi) + Y(1-p)}{1-p\varphi}$$

Q3.

(a) $f_X(x) = \int_x^\infty \lambda^2 e^{-\lambda y}\, dy = \lambda e^{-\lambda x},$

$\Rightarrow f_{Y|X}(y|x) = \dfrac{f(x,y)}{f_X(x)} = \dfrac{\lambda^2 e^{-\lambda y}}{\lambda e^{-\lambda x}} = \lambda e^{-\lambda(y-x)}$

$E[Y|X] = \int_x^\infty y\, f_{Y|X}(y|x)\, dy = \int_x^\infty \lambda y e^{-\lambda(y-x)} dy$

$= e^{\lambda x} \int_x^\infty y \lambda e^{-\lambda y}\, dy$

$= e^{\lambda x} \cdot \int_x^\infty -y\, d e^{-\lambda y}$

$= e^{\lambda x} [-y e^{-\lambda y} \Big|_x^\infty + \int_x^\infty e^{-\lambda y}\, dy]$

$= e^{\lambda x} [x e^{-\lambda x} + \frac{1}{\lambda} e^{-\lambda x}]$

$= x + \frac{1}{\lambda}.$

(b) $f_X(x) = \int_0^\infty x e^{-x(y+1)}\, dy = e^{-x}, \quad 0 \le x < \infty$

$\Rightarrow f_{Y|X}(y|x) = x e^{-xy}, \quad 0 \le y < \infty$

Q 4.

premium $= d^{n-1} a$, $\Rightarrow$ no claim is made during the first $n-1$ yrs.

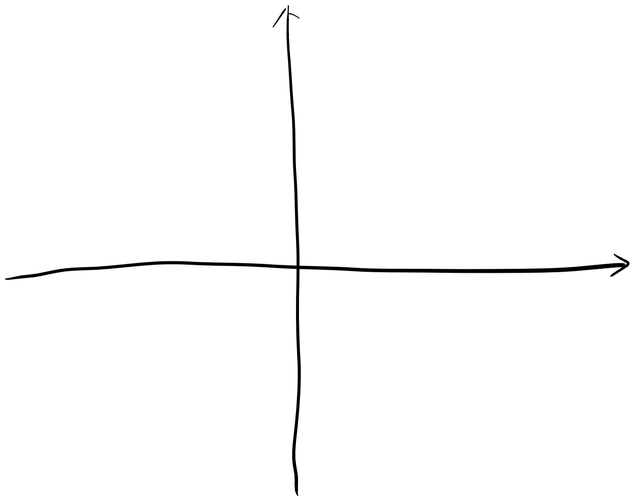
$\Rightarrow P[\quad] = q^{n-1}$.

premium $= d^{n-j-1} a$, the value of premium in yr n depends on the yr j that last claim is reported.

Thus: $P[\quad] = \underbrace{(1-q)}_{\text{at yr } j} q^{n-j-1}$.

no claim for the rest of $n-j-1$ yrs before yr n.

Q5.



Denote as X_i the # of jumps for the i^{th} direction.

The cordinates is then $(X_1+X_2,\ X_3+X_4)$

radius $= (X_1+X_2)^2+(X_3+X_4)^2$

$S = \pi r^2 = \pi[(X_1+X_2)^2+(X_3+X_4)^2]$,

$$E[S] = \sum_{\substack{X_1+X_2\\+X_3+X_4\\=n}} \pi[(X_1+X_2)^2+(X_3+X_4)^2]\cdot f(X_1, X_2, X_3, X_4),$$

where $f(X_1, \dots, X_4)$ is a joint pmf of Multi-nomial Dist^{n}.

and $f(X_1,X_2,X_3,X_4) = \dfrac{n!}{X_1!\,X_2!\,X_3!\,X_4!}\, p_1^{X_1} p_2^{X_2} p_3^{X_3} p_4^{X_4}$.

Q6.

$$\mathrm{cov}(X+Y, X-Y) = \mathrm{cov}(X,X) - \cancel{\mathrm{cov}(X,Y)} + \cancel{\mathrm{cov}(X,Y)} - \mathrm{cov}(Y,Y)$$

$$= \mathrm{Var}(X) - \mathrm{Var}(Y)$$

$$= 0.$$