Question Bank 6

Question 1 Let X have the binomial distribution with parameters n and p, and show that

$$E\Big(\frac{1}{1+X}\Big) = \frac{1-(1-p)^{n+1}}{(n+1)p}.$$

Find the limit of this expression as $n \to \infty$ and $p \to 0$, the limit begin taken in such a way that $np \to \lambda$ where $0 < \lambda < \infty$. Comment.

Question 2 A coin is tossed repeatedly, and heads turns up with probability p on each toss. Let h_n be the probability of an even number of heads in the first n tosses, with the convention that 0 is an even number. Find a difference equation for the h_n and deduce the probability generating function for n.

Question 3 Show that

$$G(x,y,z,w) = \frac{1}{8}(xyzw + xy + yz + zw + zx + yw + xz + 1)$$

is the joint generating function of four variables that are pairwise and triplewise independent, but they are nevertheless not independent.