

ST 2054, 3068, 6003.

ST 2054, ST 3068, Finish 9 questions out of 10

ST 6003, 2 compulsory questions and Choose 7 out of rest 8.
From Supratik's From Liang's.

Start with notes, assignments, tutorials and past exam papers.
Less bookwork typed questions

- Axioms of prob, prob of events, union, intersection,
- conditional prob, prob of indep. events.
- Prob mass f^1 , prob density f^1 , conditional density f^2 , marginal density.
- Typical discrete dist^{ns}: Poi, Geo, Bin, Neg Bin,
" " " : Normal, exp, etc.

Know how to identify a particular distⁿ given, MGF, pdf,

- Chebyshev, convergence in prob., Continuity Theorem.

Note: no more bookwork question for this material

- Expectation, Variance, (conditional),

Be able to find $E(X|Y)$ given certain marginal or joint density.

- PCA. Be able to calculate the co-variance matrix given two vectors, and carry out the PCA.

- Moment generating f^x , MGF of compound distⁿ; MGF of joint distⁿ,
- Biased and unbiased estimator, sampling variance/mean, sampling moments,
- Jacobian (of Joint Distⁿ), non-linear transform of a Random vector, e.g. polar system.
- CIs, Hypothesis testing, ANOVA

$$(iii) \quad P[X > 0, Y > 0] = P\left[\omega > 0, Z > -\frac{\omega \rho}{\sqrt{1-\rho^2}}\right],$$

convert to polar coordinates.

$$\text{Recall: } \iint_{\mathbb{R}^2} f(x, y) dx dy = \iint_{\mathbb{R}^2} f(r \cos \theta, r \sin \theta) r dr d\theta.$$

$$\begin{aligned} \text{let } Z &= r \sin \theta, & \omega^2 + Z^2 &= r^2 > \omega^2 + \frac{\omega^2 \rho^2}{1-\rho^2} = \omega^2 \left(\frac{1+\rho^2}{1-\rho^2} \right) > 0. \\ \omega &= r \cos \theta, & \Rightarrow r &> 0 \end{aligned}$$

$$\therefore Z > -\frac{\omega \rho}{\sqrt{1-\rho^2}} \quad \frac{Z}{\omega} > -\frac{\rho}{\sqrt{1-\rho^2}} \quad \tan \theta > -\frac{\rho}{\sqrt{1-\rho^2}}$$

$$\theta > \tan^{-1}\left(-\frac{\rho}{\sqrt{1-\rho^2}}\right) = -\tan^{-1}\left(\frac{\rho}{\sqrt{1-\rho^2}}\right) = -\sin^{-1}(\rho)$$

$$\Rightarrow P\left[\omega > 0, Z > -\frac{\omega \rho}{\sqrt{1-\rho^2}}\right] = \int_0^\infty \int_{-\frac{\omega \rho}{\sqrt{1-\rho^2}}}^\infty \frac{1}{2\pi} e^{-\frac{1}{2}(\omega^2 + Z^2)} dZ d\omega$$

$$= \int_{-\sin^{-1}(\rho)}^{\frac{1}{2}\pi} \int_{r=0}^\infty \frac{1}{2\pi} e^{-\frac{1}{2}r^2} r dr d\theta = \int_{-\sin^{-1}(\rho)}^{\frac{1}{2}\pi} \frac{1}{2\pi} d\theta$$

$$= \frac{1}{2\pi} \theta \Big|_{-\sin^{-1}(\rho)}^{\frac{1}{2}\pi} = \frac{1}{4} + \frac{1}{2\pi} \sin^{-1} \rho$$