Question Bank 3

Question 1

In a game show, you are asked to choose one of the three doors. One conceals a new car and two conceal goats. You choose, but your chosen door is not opened immediately. Instead the presenter opens another door, which reveals a goat. He approaches you and asks "would you like an opportunity to change your choice to the third door, which is unopened and unchosen so far?". Let p be the conditional probability that the third door conceals the car. The presenter's protocol is:

(i) he is determined to show you a goat, with a choice of two, he picks one at random. Show $p = \frac{2}{3}$.

(ii) he is determined to show you a goat; with a choice of two goats (named B and N) he shows you B with probability b. Show that, given you see b, the probability $p = 1/(1+b)$.

(iii) he opens a door chosen at random irrespective of what lies behind. Show $p = \frac{1}{2}$.

(iv) Show that, for $\alpha \in [\frac{1}{2}, \frac{2}{3}]$, there exists a protocol such that $p = \alpha$. Are you well advised to change your choice to the third door?

Question 2

Let F be a distribution function and r a positive integer. Show that the following are distribution functions:

(i) $F(x)^r$,

(ii) $1 - \{1 - F(x)\}^r$

(iii) $F(x) + \{1 - F(x)\} \log\{1 - F(x)\}$

(iv) $(F(x) - 1)e + \exp(1 - F(x))$.

Question 3

Let X be a random variable with distribution function F, and let $a = (a_m : -\infty < m < \infty)$ be a strictly increasing sequence of real numbers satisfying $a_{-m} \to -\infty$ and $a_m \to \infty$ as $m \to \infty$. Define $G(x) = P(X \leq a_m)$ when $a_{-m} \leq x < a_m$, so that G is the distribution function of a discrete random variable. How does the function G behave as the sequence a is chosen in such a way that $\sup_m |a_m - a_{m-1}|$ becomes smaller and smaller?

Question 4

Which of the following are density functions? Find c and the corresponding distribution function F for those that are.

(a)

$$f(x) = \begin{cases} cx^{-d}, & \text{if } x > 1 \\ 0, & \text{otherwise} \end{cases}$$

(b)

$$f(x) = ce^x(1 + e^x)^{-2}$$