ST2054 (and ST3068, ST6003) Problem Set 1 Due 4pm 27th of September 2019

Question 1

Which of the following are identically true? For those that are not, say when they are true.

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$;

(b) $A \cap (B \cap C) = (A \cap B) \cap C$;

(c) $(A \cup B) \cap C = A \cup (B \cap C)$;

(d) $A \backslash (B \cap C) = (A \backslash B) \cup (A \backslash C)$.

Question 2

Six individuals (known as Masters) are chosen to attend the next Holy Grail War. Each Master is required to summon a Servant from three candidate classes: Saber, Lancer, and Caster. Masters are equally grouped into three teams, denoted as SS, LL and CC, who want to summon two Sabers, two Lancers and two Casters respectively. Masters in the same team are in favor of Servants from the same class. If the result of the summoning is random, find the probability that at least one master is successfully matched to his/her favorite Servant.

Question 3

Let A_r, $r \geq 1$, be events such that $P(A_r) = 1$ for all r. Show that $P\left\{ \bigcap_{r=1}^{\infty} A_r \right\} = 1$.

Question 4

You are given that at least one of the events A_r, $1 \leq r \leq n$, is certain to occur, but certainly no more than two occurs. If $P(A_r) = p$, and $P(A_r \cap A_s) = q$, $r \neq s$, show that $p \geq 1/n$ and $q \leq 2/n$.

Question 5

Describe the underlying probability spaces for the following experiments:

(a) a biased coin is tossed three times;

(b) two balls are drawn without replacement from an urn which originally contained two ultramarine and two vermilion balls;

(c) a biased coin is tossed repeatedly until a head turns up.

Question 6

A traditional fair die is thrown twice. What is the probability that:

(a) a six turns up exactly once?

(b) both numbers are odd?

(c) the sum of the scores is 4?

(d) the sum of the scores is divisible by 3?

Question 7

(a) For any set of events, prove that:

$$P\left\{ \bigcup_{i=1}^{n} E_i \right\} \leq \sum_{i=1}^{n} P(E_i)$$

(b) Prove that

$$P\left\{ \bigcap_{i=1}^{n} A_i \right\} \geq 1 - \sum_{i=1}^{n} P(A_i^c)$$