Recall: $E_1, E_2, \cdots E_n$.

If

$$P[E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_m}] = P[E_{i_1}] P(E_{i_2}) \cdots P[E_{i_m}]$$

for $m = 2$, or, 3, or 4, -- or n.

Example: Reliability of complex system.

1) A system with 10 components, each works independently.

P[Each component stays working] = $\boxed{0.95}$ ✓

Find Prob. system fails.

$$P[\text{system fails}] = 1 - P[\text{all work}]$$

$$= 1 - P[\text{all 10 work}]$$

$$= 1 - P[C_1 \cap C_2 \cdots \cap C_{10}]$$

$$= 1 - [P(C_1) \times P(C_2) \cdots P(C_{10})]$$

$$= 1 - 0.95^{10} = 0.4.$$

(2) A system with 1000 components.

P[each stays work] = 0.999

$P[\text{fails}] = 1 - 0.999^{1000} = 0.6323$

(3) A system with 1000 components.

P[each stays work] = 0.9999

$P[\text{fails}] = 1 - 0.9999^{1000} = 0.095$

An experiment has only two outcomes.

$\left\langle \begin{array}{l} S \\ F \end{array} \right.$ toss a coin $\left\langle \begin{array}{l} H \\ T. \end{array} \right.$

We call this experiment a Bernalli trial.

Repeating a Bernalli trial by n times.

Each experiment has only two outcomes.

$$P[S] = p$$
$$P[F] = q = 1 - p \Rightarrow p + q = 1.$$

We assume the n repetitions are mutually indep. $P(S)$ or $P(F)$ remains the same for all n.

We call this as a Binomial Trial /Distribution.

Let us denote as X the # of successes in these n trials.

$\underline{X}$ is out first example of a random variable (R.V.)

In this Binomial trial, the values for X that can be taken are.

$$X = \{\underbrace{0, 1, 2, \cdots, n}\}$$

$P[X=k]$, k can be $0, 1, 2, \cdots n$.

it means getting k successes

" $n-k$ failures.

$$\underbrace{\overset{p}{S}\ \overset{p}{S}\ S\ S \cdots \overset{p}{S}}_{k}\ \underbrace{\overset{1-p}{F}\ \overset{1-p}{}\cdots \overset{1-p}{F}}_{n-k}$$

indep. indep.

$$p^k \underbrace{(1-p)\cdots(1-p)}_{n-k} = \boxed{p^k(1-p)^{n-k}}$$

${}^nC_k$ = total number of this kind of pattern.

$$\boxed{P(X=k) = {}^nC_k\, p^k (1-p)^{n-k}}$$ ***.

$X = \{0, 1, 2, \cdots n\} = r$.

$P(\Omega) = 1.$

$$P[\underbrace{X=0}_{E_1} \cup \underbrace{X=1}_{E_2} \cup \underbrace{X=2}_{E_3} \cdots \cup \underbrace{X=n}_{E_n}] = 1.$$

$E_1 \cap E_2 = \phi. \Rightarrow \{E_i\}_{i=1,\ldots n}$ is a partition of Ω.

$= P[X=0] + P[X=1] + \cdots + P(X=n) = 1.$

$= \sum_{k=0}^{n} P[X=k] = 1.$ (1)

If a (discrete) R.V. X satisfies (1),

$\Rightarrow$ a proper dist^n.

Recall $\boxed{P(X=k) = {}^nC_k\, p^k (1-p)^{n-k}.}$

$\boxed{\sum P(X=k) = 1}$ ②

Binomial formula

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + {}^3C_1 a^2 b + {}^3C_2 ab^2 + b^3$

$$\boxed{(a+b)^n = \sum_{k=0}^{n} {}^nC_k \, a^k b^{n-k}} \quad (3)$$

Use ~~B~~ (3) to prove (2) given $X \sim Bin(n,p)$

Replace $a = p$, $b = 1-p$ in (3)

$LHS = (p+1-p)^n = 1^n = 1$

$RHS = \sum_{k=0}^{n} {}^nC_k \, p^k (1-p)^{n-k}$ #

Prob. Distⁿ: specification of probability values is called prob. distⁿ. for the R.V. X.

E.g. Manufacturing process.

sample of n items each hour,

test all n item & count # of failure (X)

Rule: say $n = 20$, if $X \leq 3$, carry on

if $X > 3$, stop

Find the prob. of failure / stop

This is equivalent to $P[\boxed{X > 3}]$

$= P[X \geq 4]$

$= 1 - P[X \leq 3]$

$= 1 - P[X=0) - P(X=1)$

$- P(X=2)$

$- P(X=3)$

$n = 20$ $= 0.0159$.

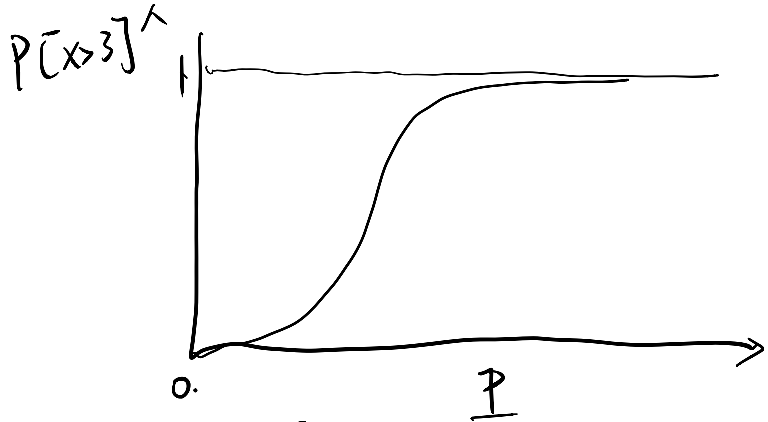
Given $p = 0.05$ $P[X > 3] = 0.0159$.

$\downarrow$.

$p = 0.1$. $P[X > 3] = 0.1330$

$P[X > 3](p)$ as a f^n. w.r.t. p,

this a increasing f^n.



Curve of P[X>3](p)