
Question Bank 2

Question 1

There are n urns of which the r^{th} urn contains $r - 1$ red balls and $n - r$ magenta balls. You pick an urn at random and remove two balls at random without replacement. Find the probability that:

- (i) the second ball is magenta;
- (ii) the second ball is magenta, given that the first is magenta. (Hint: $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=1}^n i = \frac{n(n+1)}{2}$)

Question 2

A Beacon Tower was built at the top of the mountain to give alarm by smoke, e.g. Gondor alarmed the invasion of the Orcs to the Rohan in The Lord of the Rings. If invading enemy forces were spotted during the daytime, the patrol in one tower would burn wolf manure immediately. The smoke (also known as wolf smoke) could then be visible to the next tower, where another wolf smoke would be set. Information of an invasion was therefore delivered from one tower to the next.

Alpha1, Beta2, Charlie3 and Delta4 are four beacon towers built strategically such that none of any three towers lie in a straight line. The visibility between any two towers can be blocked by thick fog with a probability p . Fog between each pair take place independently. If any of the tower starts the wolf smoke, it can only be seen by another with no fog in between. If Alpha1 spots an invasion, what is the probability that

- (i) Delta4 is aware of the invasion given fog between Delta4 and Alpha1.
- (ii) Delta4 is aware of the invasion given fog between Beta2 and Charlie3.
- (iii) Delta4 is aware of the invasion given fog between Beta2 and Alpha1.
- (iii) Delta4 is aware of the invasion.

Question 3

On a flight from Cork to Shanghai, n students are going to spend summer in China to celebrate finishing the leaving cert. Given there are 32 counties in China and suppose each student is visiting only one county independently.

- (i) What is the probability that at least two students are heading to the same county.

Zhejiang and Jiangsu are two adjacent counties in the southeast of China, also known as JiangZhe area altogether. Suzhou and Hangzhou are the capital cities of the corresponding counties respectively. Countless natural attractions and representative ancient Chinese garden-style architecture have made both cities world famous, with the proverb "Up above there is paradise, down below there are Suzhou and Hangzhou" by Chengda Fan (1126-1193 AC) in Song Dynasty and poems such as "At sunrise riverside flowers redder than fire, in spring green waves grow as blue as sapphire" by Letian Bai (772-846 AC) in Tang Dynasty.

(ii) Randomly select two students, what is the probability that both students are going to the Zhejiang province?

(iii) Randomly select two students, what is the probability that both students are travelling to the JiangZhe area?

(iv) While passing the Chinese customs control, each student's destination is recorded one by one. What is the probability that there are two students going to the JiangZhe area?

Question 4

State the definition of two independent events. Let A and B be independent events, show that A^c, B are independent events, and deduce that A^c, B^c are independent.