

Q3 (i)

$$\int_{-\infty}^{\infty} f(x) dx$$

$$+ \int_{-\infty}^{\infty} a \sin(2\pi \log x) f(x) dx$$

Because \sin is an odd function
 $\sin(-x) = -\sin(x)$

$$= 1 + 0 = \boxed{1}$$

For k^{th} moment of f_a equals:

$$\int_{-\infty}^{\infty} x^k f_a(x)$$

$$= \int_{-\infty}^{\infty} x^k f(x) dx + \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} a \sin(2\pi y) e^{i y - \frac{1}{2} y^2} dy$$

Because \sin is odd

$$= \int_{-\infty}^{\infty} x^k f(x) dx$$

Same moment as log-distribution