ST2054 (and ST3068, ST6003) Problem Set 5 Due 5pm 31st of January 2020

Question 1

A machine keeps a large number of washers in a drawer. 50% of these washers are of type A; 30% are of type B; and 20% are of type C. Suppose that 10 washers are chosen at random.

(i) Find the probability that exactly 5 type A washers, and one type C washer are chosen.

(ii) Find the probability that only two types of washers are among the chosen ones.

(iii) Find the probability that there are three of one type, three of another type and four of the third tyoe in the chosen 10.

(iv) Find the probability that all three types of washers are among the chosen 10.

Question 2

A factory has produced n robots, each of which is faulty with probability p. To each robot a test is applied which detects the fault, if present, with probability ψ. Let X be the number of faulty robots and Y the number of robots detected as faulty. Assume the usual independence, show that $E[X|Y] = \frac{np(1-\psi)+(1-p)Y}{1-p\psi}$

Question 3

Find the conditional density function and expectation of Y given X when they have joint density function:

(a) $f(x, y) = \lambda^2 e^{-\lambda y}$ for $0 \leq x \leq y \leq \infty$.

(b) $f(x, y) = xe^{-x(y+1)}$ for $x, y \geq 0$.

Question 4

Under a motor insurance policy, the premium is a in the first year. If no claim is made in the first year, the premium drops to da in the second year where $0 < d < 1$, and d is fixed. If no claim is made in the first or second years, he premium in the third year is d^2a. In general, if no claim is made in any of the first r years (r>0), the premium in year $r + 1$ is $d^r a$.

If a claim is made in any year, the premium in that year is unaffected, but the next year's premium reverts to a, and this year is then treated as if it were the first year of the insurance for the purpose of calculating further premiums. Assuming that the probability that no claim will arise in any year is constant and equal to q, show that in year n ($n > 1$) of the policy,

$$P(\text{premium} = d^{n-1}a) = q^{n-1}$$

and

$$P[\text{Premium} = d^{n-j-1}a] = (1-q)q^{n-j-1}, \text{for } 0 < j < n.$$

Hence, or otherwise, calculate the Expected amount of the premium payable on this policy in year n.

Question 5

Suppose that a particle starts at the origin of the Cartesian coordinates system, and moves along the line in jumps of one unit to the left, right, up or down. For each jump, the probability of a jump to the right is p_1, p_2 to the left, p_3 of going up, and p_4 of going down. After n jumps, draw a circle that centred at the origin and pass the particle. Let S be its area. Write down an expression for the expected value for S. You do not need to simplify the expressions.

Question 6

Suppose that X and Y are two random variables, which may be independent, and that $Var(X) = Var(Y)$. Assuming that $0 < Var(X+Y) < \infty$, show that the random variables $(X+Y)$ and $(X-Y)$ are uncorrelated.