Q1.

(a), (b) & (d) are identically true.

(c) is true if and only if $A \subseteq C$.

Q2.

Lay out the masters in order, say SSLLCC., and there are in total 6! ways as the masters are distinct individuals, hence order matters. Students also score for $\frac{6!}{2^3}$

Consider the complement event that no perfect matched summoning. The only possible results would then be LLCCSS, CCSSLL, by inspection, there are a further eight arrangements in which the first pair of cups is either SW or WS, the second pair is either RS or SR, and the third either RW or WR. Hence the required prob. of no match $= \frac{10}{6!}$ $\Rightarrow$ Prob[At least one match]

$$= 1 - \frac{10}{6!}$$

Also score for $1 - \frac{10}{6!/2^3}$

Q3. According to the De Morgan's law and Q7,

$$P[\bigcap_{r=1}^{\infty} A_r] = \lim_{n\to\infty} P[\bigcap_{r=1}^{n} A_r] = \lim_{n\to\infty}[1 - P[(\bigcap_{r=1}^{n} A_r)^c]]$$

$$= 1 - \lim_{n\to\infty} P(\bigcup_{r=1}^{n} A_r^c) \geq 1 - \lim_{n\to\infty} \sum_{r=1}^{n} P[A_r^c] = 1.$$

Q4. $1 = P[\bigcup_{1}^{n} A_r] = \sum_r P(A_r) - \sum_{r<s} P[A_r \cap A_s)$

$$= np - \tfrac{1}{2} n(n-1) q$$

$\Rightarrow \quad np = 1 + \tfrac{1}{2} n(n-1) q \geq 1 \qquad p \geq \frac{1}{n}$

Also $\tfrac{1}{2} n \cancel{(n-1)} q = np - 1 \leq \cancel{n-1}$.

$$q \leq \frac{2}{n}$$

Q5.

(a) Possible outcome space $= \{H, T\} \times \{H, T\} \times \{H, T\}$
$= \{H, T\}^3$

the corresponding prob. space consists of probs for each possible outcome.

An expression for $h$ heads and $3-h$ tails is
$p^h (1-p)^{3-h}$.

(b) possible outcome space $= \{U, V\} \times \{U, V\}$
$= \{U, V\}^2$

$P(UU) = P(VV) = \frac{2}{4} \times \frac{1}{3} = \frac{2}{12}$

$P(UV) = P(VU) = \frac{2}{4} \times \frac{2}{3} = \frac{1}{3}$.

(c) sample space $= \{T^n H : n \geq 0\}$ and $\{T^n : n \to \infty\}$

~~The~~ $P[T^n H] = (1-p)^n p$

$P[T^{n\to\infty}] = \lim_{n\to\infty} (1-p)^n = 0$ if $p \neq 0$.

Q6.

(a) $P = 2 \times \frac{1}{6} \times \frac{5}{6} = \frac{5}{18}$

(b) $P = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(c) $P[S=4) = P(1,3) + P(2,2) + P(3,1)$

$= \frac{3}{36}$

(d) $P(S/3) = P(S=3) + P(S=6) + P(S=9) + P[S=12)$

$= P[1,2] + P(2,1)$

$+ P(1,5) + P(2,4) + P(3,3) + P(4,2) + P(5,1)$

$+ P(3,6) + P(4,5) + P(5,4) + P(6,3)$

$+ P(6,6)$

$= \frac{12}{36} = \frac{1}{3}$

Q7.

(a) Use induction.

$n=1$, $P(E_1) = P(E_1)$, holds.

Let $m \geq 1$ and assume that "$\leq$" holds for $n \leq m$.

Then

$$P\left(\bigcup_1^{m+1} A_i\right) = P\left(\bigcup_1^{m} A_i\right) + P(A_{m+1}) - P\left(\bigcup_1^{m}(A_i \cap A_{m+1})\right)$$

$$\leq P\left(\bigcup_1^{m} A_i\right) + P(A_{m+1}) \leq \sum_1^{m+1} P(A_i)$$

(b). $$P\left(\bigcap_1^{n} A_i\right) = P\left(\left(\bigcup_1^{n} A_i^c\right)^c\right) = 1 - P\left[\bigcup_1^{n} A_i^c\right]$$

$$\geq 1 - \sum_1^{n} P(A_i^c)$$