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Q4
(a) For X_n, $\frac{1}{2}$ chance $=\left(\frac{1}{2}X_{n-1}\right)$, $\frac{1}{2}$ chance $=\left(\frac{1}{2}X_{n-1}+Y_{n-1}\right)$

$$X_n = \begin{cases} \frac{1}{2}X_{n-1} & \text{with prob } \frac{1}{2} \\ \frac{1}{2}X_{n-1}+Y_{n-1} & \text{with prob } \frac{1}{2} \end{cases}$$

Probability mass function of X_n:

$$P(X_n = x_{nk}) = P(X_{n-1} = x_{(n-1)k}) \ldots = P(X_1 = x_{1k})$$

(b)
$$M_{X_n}(t) = E[e^{tX_n}]$$

(1) $= \frac{1}{2}E\left[e^{t(\frac{1}{2}X_{n-1})}\right] + \frac{1}{2}E\left[e^{t(\frac{1}{2}X_{n-1}+Y_{n-1})}\right]$

half chance each

$$\frac{1}{2}E\left[e^{t(\frac{1}{2}X_{n-1}+Y_{n-1})}\right] = \frac{1}{2}E\left[e^{\frac{1}{2}tX_{n-1}}\right]E\left[e^{tY_{n-1}}\right]$$

$$= \frac{1}{2}M_{X_{n-1}}\left(\frac{1}{2}t\right)\cdot\frac{\lambda}{\lambda-t}$$ ~ exponential MGF

(1) $= \frac{1}{2}M_{X_{n-1}}\left(\frac{1}{2}t\right)\left\{1 + \frac{\lambda}{\lambda-t}\right\}$