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Q3

We use transformation techniques:

(i)

Density Function of X:

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$y = h(x) = e^x$$

$$x = h^{-1}(y) = \ln y$$

We see e^x is a monotonically increasing function.

$$\frac{dx}{dy} = \frac{dh^{-1}(y)}{dy} = \frac{d \ln y}{dy}$$

$$= \frac{1}{y}$$

Density of Y:

$$g(y) = f_X(x) \cdot \frac{dx}{dy}$$

$$= f_X(\log y) \cdot \frac{1}{y}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\log y)^2} \cdot \frac{1}{y}$$

$$= \frac{1}{y\sqrt{2\pi}} e^{-\frac{1}{2}(\log y)^2}$$

(ii)

$$\int_{-\infty}^{\infty} \{1 + a \sin(2\pi \log x)\} f(x)\, dx$$

must equal 1

↓