Question 1

Let $\{A_i : i \in I\}$ be a collection of sets. Prove "De Morgan's Law":

(i)

$$\left(\underset{i}{\cup} A_i\right)^c = \underset{i}{\cap} A_i^c$$

(ii)

$$\left(\underset{i}{\cap} A_i\right)^c = \underset{i}{\cup} A_i^c$$

Question 2

A conventional know-out tournament (such as that Wimbledon) begins with 2^n competitors and has n rounds. There are no play-offs for the positions $2, 3, \ldots, 2^n - 1$, and the initial table of draws is specified. Give a concise description of the sample space of all possible outcomes.

Question 3

A fair coin is tossed repeatedly. Show that, with probability one, a head turns up sooner or later. Show similarly that any given finite sequence of heads and tails occurs eventually with probability one.

Question 4

A biased coin is tossed repeatly. Each time there is a probability p of a head turning up. Let p_n be the probability that an even number of heads has occurred after n tosses (zero is an even number).

(i) What is the value of p_0, explain why.

(ii) Show that $p_n = p(1 - p_{n-1}) + (1 - p)p_{n-1}$ if $n \geq 1$. Solve this difference equation.

Question 5

Let A and B be the events with probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$. Show that $\frac{1}{12} \leq P(AB) \leq \frac{1}{3}$, and give examples to show that both extremes are possible. Find corresponding bounds for $P(A \cup B)$.