ST2054 (and ST3068, ST6003) Problem Set 2 Due 4pm 18th of October 2019

Question 1

There are two roads from A to B and two roads from B to C. Each of the four roads is blocked by snow with probability p, independently of the others. Find the probability that there is an open road from A to B given that there is no open route from A to C.

If, in addition, there is a direct road from A to C, this road being blocked with probability p independently of the others, find the required conditional probability.

Question 2

Continue with the Holy Grail War. A relic relevant to an anciant spirit can be used to summon the corresponding spirit as a Servant from the favourable class. However, the relic can be a counterfeit with probability $1 - p$. Suppose due to stress, when the Master has the real relic, there is a probability $1-\theta$ that he casts the incantation correctly and summons a Servant from the targeting class. If the Master does not have a real relic, then he/she randomly summon a Servant from the classes Saber, Lancer and Caster with probability p_1 and p_2 for Saber and Lancer respectively. Again Masters are equally grouped into three teams, denoted as (S_1, S_2), (L_1, L_2) and (C_1, C_2), who want to summon two Sabers, two Lancers and two Casters respectively. Masters in the same team are in favor of Servants from the same class. Assume that Masters carry out the ritual of summoning independently.

(i) What is the probability that a Saber is successfully summoned by Master S_1. Hence, what is the conditional probability that S_1 actually owns a real relic.

(ii) What is the probability that the Master S_1 does not have a real Saber relic when a Saber is not summoned.

(iii) Consider group L_1, L_2, find an expression for the probability that exactly one Lancer is summoned. State any assumptions that are needed for your derivation.

(iv) Find an expression for the probability that exactly one group has a perfectly matched summoning, i.e. both Masters in the same group successfully summon from their favourable class.

Question 3

A fair coin is thrown repeatedly. What is the probability that on the nth throw:

(i) a head appears for the first time?

(ii) the numbers of heads and tails to date are equal?

(iii) exactly two heads have appeared altogether to date?

(iv) at least two heads have appeared to date?

Question 4

A random number N of dices are thrown. Let A_i be the event that $N = i$, and assume that $P(A_i) = 2^{-i}$, $i \geq 1$. The sum of the scores is S. Find the probability that:

(i) $N = 2$ given $S = 4$.

(ii) $S = 4$ given N is even;

(iii) $N = 2$, given that $S = 4$ and the first die showed 1;

(iv) the largest number shown by any die is r, where S is unknown. Hint: Consider the probability $P[M \leq r]$ and $P[M \leq r - 1]$, where M is the largest number shown.

Question 5

An urn contains b blue balls and r red balls. They are removed at random and not replaced. Derive the probability that the first red ball drawn is the $(k+1)$th ball drawn. Find the probability that the last ball drawn is red.

Question 6

Prove that $P(A \cup B \cup C) = 1 - P(A^c | B^c \cap C^c) P(B^c | C^c) P(C^c)$

Question 7

(a) If A is independent of itself, show that $P(A)$ is 0 or 1.

(b) If $P(A)$ is 0 or 1, show that A is independent of all events B.

Question 8

The event A is said to be repelled by the event B if $P(A|B) < P(A)$, and to be attracted by B if $P(A|B) > P(A)$. Show that if B attracts A, then A attracts B, and B^c repels A. If A attracts B, and B attracts C, does A attract C?