ST2054 (and ST3068, ST6003) Problem Set 4 Due 4pm 29th of November 2019

Question 1

Suppose we have the customized Beta-typed random variable X with the corresponding density function $f(x) = C(1+x)^{g_1}(1-x)^{g_2}$, $|x| \leq 1$, $g_1, g_2 \in Z^+$. Now refer to the value of C we calculated in Question 6 in Problem set 3, find the values of g_1 and g_2 such that $E[X] = 0.98$ and $Var(X) = 0.01^2$.

Question 2

In the Holy Grial War, a Servant consumes Mana to cast spells under the commands of the Master while the Master plays as the source of Mana, i.e. the Servant drains Mana from his/her Master.

Arutoria Pendoragon is the Servant of Shirou Emiya. She believes that the total Mana required, denoted as S, for the first day follows a Log-Normal distribution with mean 1.5 million units and variance 2 million. The total Mana she can drain from Shirou during the entire day will be $(1+\theta)E(S)$. Given at the beginning of the first day Arutoria reserves an initial portion of Mana equivalent to $800k$ units to cover the Mana consumption. Suppose one of her ability is to regenerate Mana slowly with an effective rate of 6% per day on all Mana drained from Shirou and the initial Mana she reserves. Calculate the value of θ such that the probability that Arutoria does not run out of Mana at the end of the day is 95%. State any assumptions you make.

Question 3

Let $N(t)$ be a Poisson Process with constant intensity on R.

(a) Find the covariance of $N(s)$ and $N(t)$.

(b) Show that N is continuous in mean square, which is to say that $E[\{N(t+h)-N(t)\}^2] \to 0$ as $h \to 0$.

(c) Prove that N is continuous in probability, which is to say that $P(|N(t+h)-N(t)| > \epsilon) \to 0$ as $h \to 0$, for all $\epsilon > 0$.

(d) Show that N is differentiable in probability but not in mean square.

Question 4

Let X and Y have the joint mass function

$$f(x,y) = \frac{C}{(x+y-1)(x+y)(x+y+1)}$$

(i) Find the marginal mass functions of X and Y, calculate C.

(ii) Find the mass functions of $U = X + Y$ and $V = X - Y$.

Question 5

Let X and Y have joint mass function

$$f(j,k) = \frac{c(j+k)a^{j+k}}{j!k!}, j,k \geq 0$$

where a is a constant. Find c, $P[X = j]$, $P[X + Y = r]$ and $E[X]$.

Question 6

Let X and Y have the joint density $f(x,y) = cx(y-x)e^{-y}$, $0 \leq x \leq y < \infty$. Find c, $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.