## Question Bank 5

### Question 1

Consider a pair of random variables $X$ and $Y$. Let $Y$ be distributed as a Binomial distribution $Bin(n, X)$, where $X$ has a beta distribution on $[0,1]$ with parameters $a$ and $b$.

**(i)** Describe the probability mass function of $Y$. [ 4 ]

**(ii)** Evaluate $E[Y]$. [ 2 ]

**(iii)** Evaluate $Var(Y)$. [ 2 ]

**(iv)** Specify the distribution of $Y$ if $X$ is uniform. [ 2 ]

### Question 2

**(i)** Derive expressions, in terms of $a, b$ and $c$, for the principal components of a 2-dimensional random vector $X$ with mean $\mu$ and covariance matrix:

$$S = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$$

What are the variances of the first and second principal components? [ 6 ]

**(ii)** A $p-$dimensional random variable $X$ has a $N_p(\mu, \Sigma)$ distribution if its characteristic function is given by

$$\phi_X(t) = e^{it'\mu - \frac{1}{2}t'\Sigma t}.$$

Derive the distribution of the vector $Y = AX$ where $A$ is a $q \times p$ matrix. [ 2 ]

**(iii)** Describe the distribution of the vector $Z = W^{-1}(X - \mu)$, where $W$ is a symmetric matrix such that $W^2 = \Sigma$. [ 2 ]

### Question 3

Let $Y_1, \ldots, Y_n$ be independently and identically distributed random variables, with $Y_i \sim$ Poi$(\mu)$ $\forall\ i$.

**(i)** Derive the probability generating function $G_{Y_1}(t)$ and the moment generating function $M_{Y_1}(t)$ for $Y_1$. [ 4 ]

**(ii)** Let $S = \sum_{i=1}^{n} Y_i$. Show that $S$ follows a Poisson distribution with parameter $n\mu$. [ 3 ]

**(iii)** Let $X$ have the Poisson distribution with parameter $Y_1$, show that the probability generating function for $X + Y_1$ is [ 3 ]

$$G_{X+Y_1}(t) = \exp\{\mu(te^{t-1} - 1)\}$$