**ST2054 (and ST3068, ST6003) Problem Set 3 Due 4pm 8th of November 2019**

**Question 1**

Now let the Holy Grial War begin. Suppose a Caster servant can curse her target with the "Immolate" spell. The Immolate will periodically cause damage on the target for every 5 seconds over the next 60 second. The first damage incurred as soon as the target is cursed by the spell. Assume the Caster does not stop casting other spells (excluding Immolate) when the Immolate is in effect and all her spells can be casted instantly. Suppose every time Immolate causes damage, it has a rate of 3% to trigger the effect of "burning mind" that enrages the Caster and her next spell casted is guaranteed to be a critical strike, which deals twice its normal damage. Denote as $X$ the number of critical strikes when the Immolate is effective on the target.

**(i)** Specify the distribution of X and its corresponding probability function. State any assumptions you made.

**(ii)** Find an approximated value for the probability that the Caster gets enraged for at least twice during the period of the curse.

**(iii)** Compare your result obtained in (ii) with the corresponding theoritical value. Make comments on the goodness of the approximation.

**Question 2**

In your pocket is a random number $N$ of coins, where $N$ has the Poisson distribution with parameter $\lambda$. You toss each coin once, with heads showing with probability $p$ each time. Show that the total number of heads has the Poisson distribution with parameter $\lambda p$.

**Question 3**

For what values of the constant $C$ do the following define mass functions on the positive integers $1, 2, \dots$?

**(a)** $f(x) = C2^{-x}$.

**(b)** $f(x) = C2^{-x}/x$.

**(c)** $f(x) = Cx^{-2}$.

**(d)** $f(x) = C2^{x}/x!$.

**Question 4**

For a random variable $X$ having each of the four mass functions in Question 3, find:

**(i)** $P(X > 1)$.

**(ii)** the most probable value of $X$.

**(iii)** the probability that $X$ is even.

### Question 5

**(a)** Show that, if $X$ is a binomial or Poisson random variable, then the mass function $f(k) = P(X = k)$ has the property that $f(k-1)f(k+1) \leq f(k)^2$.

**(b)** Show that, if $f(k) = 90/(\pi k)^4$, $k \geq 1$, then $f(k-1)f(k+1) \geq f(k)^2$.

**(c)** For those discrete random variables we have covered so far, is there a random variable such that its mass function $f$ satisfies $f(k)^2 = f(k-1)f(k+1)$, $k \geq 1$.

### Question 6

Let us build a customized Beta-typed random variable $X$ with the corresponding density function $f(x) = C(1+x)^{g_1}(1-x)^{g_2}$, $|x| \leq 1$, $g_1, g_2 \in Z^+$. Find the value of $C$ such that $f(x)$ is proper.

### Question 7

For the standard normal density function $f(x)$, show that $f'(x) + xf(x) = 0$. Hence show that for $x > 0$,

$$\frac{1}{x} - \frac{1}{x^3} < \frac{1 - F(x)}{f(x)} < \frac{1}{x} - \frac{1}{x^3} + \frac{3}{x^5}$$