OLLSCOIL NA hÉIREANN, CORCAIGH

THE NATIONAL UNIVERSITY OF IRELAND, CORK

COLÁISTE NA hOLLSCOILE, CORCAIGH

UNIVERSITY COLLEGE, CORK

Examination Session and Year	Summer 2020
Module Code	ST2054
Module Title	Probability and Mathematical Statistics
Paper Number	1
External Examiner	Dr. Ji Yao
The Head of the Department	Dr. Michael Cronin
Internal Examiners	Dr Liang Chen Dr. Kevin Hayes
Instructions to Candidates	**ANSWER ANY NINE QUESTIONS FOR FULL MARKS**
Duration of Paper	3.0 hours
Special Requirements	**Reading time of fifteen minutes is permitted prior to the commencement of this examination.** **Statistical tables are available.** **A non-programmable calculator may be used provided that it does not contain any information stored by any person prior to the examination.**

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THEN ENSURE THAT YOU HAVE THE CORRECT EXAM PAPER

Question 1 [10 marks]

(i) Let A and B be events with probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$. Show that

$$\frac{1}{12} \leq P(AB) \leq \frac{1}{3},$$

and give examples to show that both extremes are possible. [4]

(ii) Six cups and saucers come in pairs: there are two cups and saucers which are red, two white and two with stars on. If the cups are placed randomly onto the saucers (one each), find the probability that no cup is upon a saucer of the same pattern. [3]

(iii) Let A_r, $r \geq 1$, be events such that $P(A_r) = 1$ for all r. Show that $P\left\{\bigcap_{r=1}^{\infty} A_r\right\} = 1$. [3]

Question 2 [10 marks]

You are lost in the Wonderland. Tourists comprise two-thirds of the visitors to the land, and give a correct answer to requests for directions with probability $\frac{3}{4}$. (Answers to repeated questions are independent, even if the question and the person are the same.) If you ask a local citizen for direction, the answer is always false.

(i) You ask a passer-by whether the exit from the Land is East or West. The answer is East. What is the probability that this is correct? [2]

(ii) You ask the same person again, and receive the same reply. Show the probability that it is correct is $\frac{1}{2}$. [2]

(iii) You ask the same person again, and receive the same reply. What is the probability that it is correct? [2]

(iv) You ask the same person for the forth time, and receive the answer East. Show that the probability it is correct is $\frac{27}{70}$. [2]

(v) Show that, had the forth answer been West instead, the probability that East is nevertheless correct is $\frac{9}{10}$. [2]

Question 3 [10 marks]

(i) The random variable X has the following probability density function (pdf):

$$f(x) \quad = \quad C\exp(-x - e^{-x});\ x \in R$$

Find the value of C. [1]

For the standard normal density $\phi(x)$ and standard normal distribution function $\Phi(x)$, show that:

(ii) $\phi'(x) + x\phi(x) = 0.$ [1]

(iii) $\frac{1}{x} - \frac{1}{x^3} < \frac{1-\Phi(x)}{\phi(x)} < \frac{1}{x} - \frac{1}{x^3} + \frac{3}{x^5}$ [4]

Let X be a positive random variable with density function f and distribution function F. Define the hazard function $H(x) = -\log[1 - F(x)]$ and the hazard rate

$$r(x) = \lim_{h\to} \frac{1}{h} P(X \leq x + h | X > x),\ x \geq 0.$$

Show that:

(iv) $H(x)/x$ increases with x if and only if $[1 - F(x)]^{\alpha} \leq 1 - F(\alpha x)$ for all $0 \leq \alpha \leq 1$, [2]

(v) If $H(x)/x$ increases with x, then $H(x + y) \geq H(x) + H(y)$ for all $x, y \geq 0$. [2]

Question 4 [10 marks]

Let X_1, X_2, X_3 be independent random variables taking values in the positive integers and having mass functions given by $P(X_i = x) = (1 - p_i)p_i^{x-1}$ for $x = 1, 2, 3$.

(i) Find $P(X_1 < X_2)$. [2]

(ii) Find $P(X_1 < X_2 < X_3)$. [4]

(iii) Find $P(X_1 \leq X_2 \leq X_3)$ [4]

Question 5 [10 marks]

(i) Show that if $X_n \xrightarrow{D} X$ then $aX_n + b \xrightarrow{D} aX + b$ for any real a and b. [2]

(ii) If X has zero mean and variance σ^2, show that [2]

$$P(X \geq t) \leq \frac{\sigma^2}{\sigma^2 + t^2}, \text{ for } t > 0$$

(iii) Show that $X_n \xrightarrow{P} 0$ if and only if [4]

$$E\Big(\frac{|X_n|}{1 + |X_n|}\Big) \to 0, \text{ as } n \to \infty.$$

(iv) Let $X_1, X_2, \ldots$ be independent and identically distributed random variables whose common characteristic function ψ satisfies $\psi'(0) = i\mu$. Show that [2]

$$\frac{1}{n}\sum_{j=1}^{n} X_j \xrightarrow{P} \mu.$$

Question 6 [10 marks]

Let X and Y have the joint density function $f(x, y) = cx(y - x)e^{-y}$, $0 \leq x \leq y < \infty$. Find the following:

(i) c. [4]

(ii) $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$. [2]

(iii) $E(X|Y)$. [2]

(iv) $E(Y|X)$. [2]

Question 7 [10 marks]

(i) If $\boldsymbol{X}$ is a random vector with the $N(\boldsymbol{\mu}, \boldsymbol{V})$ distribution where $\boldsymbol{V}$ is non-singular, show that $\boldsymbol{Y} = (\boldsymbol{X} - \boldsymbol{\mu})\boldsymbol{W}^{-1}$ has the $N(\boldsymbol{0}, \boldsymbol{I})$ distribution, where $\boldsymbol{I}$ is the identity matrix and $\boldsymbol{W}$ is a symmetric matrix satisfying $\boldsymbol{W}^2 = \boldsymbol{V}$. [2]

Let $Z = (X, Y)'$ be a random vector with length 2. Ten data points are observed for X and Y respectively as follows:.

$$\begin{aligned} X &= (0.69,\ 0.11,\ 2.01,\ 2.81,\ 0.76,\ 3.95,\ 2.36,\ 1.23,\ 1.57,\ 3.17) \\ Y &= (1.24,\ 1.88,\ 2.87,\ 3.54,\ 1.38,\ 4.71,\ 1.33,\ 1.45,\ 3.55,\ 3.56) \end{aligned}$$

(ii) Based on these data points, find out the principle components of Z. What is the direction along which the variance of Z is mostly explained? You should clearly demonstrate each step of your calculation for full marks. [8]

Question 8 [10 marks]

If X and Y have joint probability generating function

$$G_{X,Y}(s,t) = E(s^X t^Y) = \frac{\{1 - (p_1 + p_2)\}^n}{\{1 - (p_1 s + p_2 t)\}^n},$$

where $p_1 + p_2 \leq 1$, find the following:

(i) the marginal probability mass function of X, [2]

(ii) the marginal probability mass function of Y, [1]

(iii) the marginal probability mass function of $X + Y$, [3]

(iv) the conditional probability generating function $G_{X|Y}(s|y) = E(s^X|Y = y)$ of X given that $Y = y$. [4]

Hint: $(x + a)^{-n} = \sum_{k=0}^{\infty} {}^{-n}C_k x^k a^{-n-k}$.

Question 9 [10 marks]

Let X and Y be independent $N(0,1)$ variables, and think of (X,Y) as a random point in the plane. Change to polar coordinates (R,θ) such that $R^2 = X^2 + Y^2$, $\tan\theta = Y/X$.

(i) Specify the distribution of R^2 and θ. Justify that R and θ are independent. [2]

(ii) Find the density of R. [1]

(iii) Find $E(X^2/R^2)$. [3]

(iv) Find [4]

$$E\left\{\frac{\min(|X|,|Y|)}{\max(|X|,|Y|)}\right\}$$

Question 10 [10 marks] Stopping distance is a critical measure of the safety performance of a car. An experiment was carried out to compare the stopping distance of three car makers' (A,B,C) coupe line. Cars from each maker were accelerated to 40 MPH and the corresponding stopping distance was then recorded. A random sample of stopping distance was collected for A, B, and C with size 10, 9 and 7 respectively, see the table below.

Maker											Mean	S.D.
A	35.3	35.5	52.2	60.5	36.5	73.6	43.9	39.3	54.5	62.4	49.38	13.35
B	37.3	52.8	36.0	32.5	37.7	29.2	17.8	32.6	47.5		35.9	10.13
C	38.9	45.4	62.5	30.0	58.5	45.1	42.1				46.1	11.2

(i) Calculate a 95% confidence interval for μ for Maker A, assuming σ is 11.34. [1]

(ii) Calculate a 95% confidence interval for σ^2 for Maker A. [3]

(iii) Use the sample means and standard deviations to compute the mean and standard deviation of the pooled data. [2]

(iv) Carry out a hypothesis test that the mean stopping distance of the 3 makers are the same. Carefully describe the ANOVA model and the assumptions underlying your analysis. [4]

Notation, parameters and formulae for distributions

Notation	pdf/pmf $f(x)$	Mean	Variance	MGF $M_x(t)$
Binomial $X \sim BIN(n,p)$ $0 < p < 1$ $q = 1-p$	$\binom{n}{x} p^x q^{n-x}$ $x = 0, 1, \ldots, n$	np	npq	$(pe^t + q)^n$
Bernoulli $X \sim BIN(1,p)$ $0 < p < 1$ $q = 1-p$	$p^x q^{1-x}$ $x = 0, 1$	p	pq	$pe^t + q$
Geometric $X \sim GEO(p)$ $0 < p < 1$ $q = 1-p$	pq^{x-1} $x = 1, 2, \ldots$	$1/p$	q/p^2	$\frac{pe^t}{1-qe^t}$
Negative Binomial–Type I $X \sim NB(k,p)$ $k =$ positive integer, $0 < p < 1$ $q = 1-p$	$\frac{(x-1)!}{(k-1)!(x-k)!} p^k q^{x-k}$ $x = k, k+1, k+2, ...$	$\frac{k}{p}$	$\frac{kq}{p^2}$	$\left(\frac{pe^t}{1-qe^t}\right)^k$
Negative Binomial–Type II $X \sim NB(k,p)$ $k > 0$, $0 < p < 1$ $q = 1-p$	$\frac{\Gamma(k+x)}{\Gamma(x+1)\Gamma(k)} p^k q^x$ $x = 0, 1, 2, ...$	$\frac{kq}{p}$	$\frac{kq}{p^2}$	$\left(\frac{p}{1-qe^t}\right)^k$
Poisson $X \sim POI(\mu)$ $0 < \mu$	$\frac{e^{-\mu}\mu^x}{x!}$ $x = 0, 1, \ldots$	μ	μ	$e^{\mu(e^t-1)}$
Chi-square $X \sim \chi^2(\nu)$ $\nu = 1, 2, \ldots$	$\frac{x^{\nu/2-1}e^{-x/2}}{2^{\nu/2}\Gamma(\nu/2)}$ $0 < x$	ν	2ν	$(1-2t)^{-\nu/2}$
Uniform $X \sim UNIF(a,b)$ $a < b$	$\frac{1}{b-a}$ $a < x < b$	$(a+b)/2$	$(b-a)^2/12$	$\frac{e^{bt}-e^{at}}{(b-a)t}$
Normal $X \sim N(\mu, \sigma^2)$ $0 < \sigma$	$\frac{e^{-[(x-\mu)/\sigma]^2}}{\sqrt{2\pi}\sigma}$	μ	σ^2	$e^{\mu t + \sigma^2 t^2/2}$
Gamma $X \sim GAM(\theta, \kappa)$ $0 < \theta$, $0 < \kappa$	$\frac{e^{-x/\theta}x^{\kappa-1}}{\theta^\kappa \Gamma(\kappa)}$ $0 < x$	$\kappa\theta$	$\kappa\theta^2$	$(1-\theta t)^{-\kappa}$

Notation	pdf/pmf $f(x)$	Mean	Variance	MGF $M_x(t)$
Exponential $X \sim EXP(\theta)$ $0 < \theta$	$\frac{e^{-x/\theta}}{\theta}$ $0 < x$	θ	θ^2	$(1-\theta t)^{-1}$
Two parameter Exponential $X \sim EXP(\theta, \eta)$ $0 < \theta$	$\frac{e^{-(x-\eta)/\theta}}{\theta}$ $\eta < x$	$\eta + \theta$	θ^2	$(1-\theta t)^{-1}e^{\eta t}$
Double Exponential $X \sim DE(\theta, \eta)$ $0 < \theta$	$\frac{e^{-\lvert x-\eta\rvert/\theta}}{2\theta}$	η	$2\theta^2$	$(1-\theta^2 t^2)^{-1}e^{\eta t}$
Weibull $X \sim WEI(\theta, \beta)$ $0 < \theta, \beta$	$\frac{\beta x^{\beta-1}e^{-(x/\theta)^\beta}}{\theta^\beta}$	$\theta\Gamma\left(1+\frac{1}{\beta}\right)$	$\theta^2\left[\Gamma\left(1+\frac{1}{\beta}\right)\right.$ $\left.-\Gamma^2\left(1+\frac{1}{\beta}\right)\right]$	*
Beta $X \sim BETA(\alpha, \beta)$ $0 < \alpha, \beta$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$ $0 < x < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta+1)(\alpha+\beta)^2}$ $\left.-\Gamma^2\left(1+\frac{1}{\beta}\right)\right]$	*

Law of Total Probability: $P(A) = \sum_{i=1}^{k} P(A|E_i)P(E_i)$ **when** $\cup_{i=1}^{k} E_i = \Omega$ **and all** E_i **are mutually exclusive.**

Linear Regression:

$$\hat{\beta}_1 = \frac{S_{XY}}{S_{XX}} = \frac{\sum x_i y_i - n\bar{x}\bar{y}}{\sum x_i^2 - n(\bar{x})^2};$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

Analysis of Variance:

$$SST = (\sum_i \sum_j (y_{ij} - \bar{y})^2) = \sum_i \sum_j (y_{ij})^2 - \frac{(y_{..})^2}{n}$$

$$SSB = [\sum_i \frac{(y_{i.})^2}{n_i}] - \frac{(y_{..})^2}{n},$$

where $y_{i.} = \sum_{j=1}^{n_i} y_{ij}$ and $y_{..} = \sum_i \sum_j y_{ij}$.

PERCENTAGE POINTS OF THE F DISTRIBUTION

$\nu_2 \backslash \nu_l$	q	2	3	4	5	6	7	8	10	12	15	20	30	50	∞
1	0.900	49.5	53.6	55.8	57.2	58.2	59.1	59.7	60.5	61.0	61.5	62.0	62.6	63.0	63.3
	0.950	199.	216.	225.	230.	234.	237.	239.	242.	244.	246.	248.	250.	252.	254.
	0.975	800.	864.	900.	922.	937.	948.	957.	969.	977.	985.	993.			
	0.990														
	0.999														
2	0.900	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.39	9.41	9.43	9.44	9.46	9.47	9.49
	0.950	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
	0.975	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	39.4	39.4	39.4	39.5	39.5	39.5
	0.990	99.0	99.2	99.2	99.3	99.3	99.4	100.	100.	100.	100.	100.	100.	100.	99.5
	0.999	999.	999.												
3	0.900	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.23	5.22	5.20	5.18	5.17	5.15	5.13
	0.950	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.79	8.74	8.70	8.66	8.62	8.58	8.53
	0.975	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.4	14.3	14.3	14.2	14.1	14.0	13.9
	0.990	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.2	27.1	26.9	26.7	26.5	26.4	26.1
	0.999	149.	141.	137.	135.	133.	132.	131.	129.	128.	127.	126.	125.	125.	123.
4	0.900	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.92	3.90	3.87	3.84	3.82	3.79	3.76
	0.950	6.94	6.59	6.39	6.26	6.16	6.09	6.04	5.96	5.91	5.86	5.80	5.75	5.70	5.63
	0.975	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.84	8.75	8.66	8.56	8.46	8.38	8.26
	0.990	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.5	14.4	14.2	14.0	13.8	13.7	13.5
	0.999	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.0	47.4	46.8	46.1	45.4	44.9	44.1
5	0.900	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.30	3.27	3.24	3.21	3.17	3.15	3.10
	0.950	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.74	4.68	4.62	4.56	4.50	4.44	4.36
	0.975	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.62	6.52	6.43	6.33	6.23	6.14	6.02
	0.990	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.1	9.89	9.72	9.55	9.38	9.24	9.02
	0.999	37.1	33.2	31.1	29.8	28.8	28.2	27.6	26.9	26.4	25.9	25.4	24.9	24.4	23.8
6	0.900	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.94	2.90	2.87	2.84	2.80	2.77	2.72
	0.950	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.06	4.00	3.94	3.87	3.81	3.75	3.67
	0.975	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.46	5.37	5.27	5.17	5.07	4.98	4.85
	0.990	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.87	7.72	7.56	7.40	7.23	7.09	6.88
	0.999	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.4	18.0	17.6	17.1	16.7	16.3	15.7
7	0.900	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.70	2.67	2.63	2.59	2.56	2.52	2.47
	0.950	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.64	3.57	3.51	3.44	3.38	3.32	3.23
	0.975	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.76	4.67	4.57	4.47	4.36	4.28	4.14
	0.990	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.62	6.47	6.31	6.16	5.99	5.86	5.65
	0.999	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.1	13.7	13.3	12.9	12.5	12.2	11.7
8	0.900	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.54	2.50	2.46	2.42	2.38	2.35	2.29
	0.950	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.35	3.28	3.22	3.15	3.08	3.02	2.93
	0.975	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.29	4.20	4.10	4.00	3.89	3.81	3.67
	0.990	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.81	5.67	5.52	5.36	5.20	5.07	4.86
	0.999	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.5	11.2	10.8	10.5	10.1	9.80	9.33

PERCENTAGE POINTS OF THE F DISTRIBUTION

$\nu_2 \backslash \nu_l$	q	2	3	4	5	6	7	8	10	12	15	20	30	50	∞
9	0.900	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.42	2.38	2.34	2.30	2.25	2.22	2.16
	0.950	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.14	3.07	3.01	2.94	2.86	2.80	2.71
	0.975	5.71	5.08	4.72	4.48	4.32	4.20	4.10	3.96	3.87	3.77	3.67	3.56	3.47	3.33
	0.990	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.26	5.11	4.96	4.81	4.65	4.52	4.31
	0.999	16.4	13.9	12.6	11.7	11.1	10.7	10.4	9.89	9.57	9.24	8.90	8.55	8.26	7.81
10	0.900	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.32	2.28	2.24	2.20	2.16	2.12	2.06
	0.950	4.10	3.71	3.48	3.33	3.22	3.14	3.07	2.98	2.91	2.84	2.77	2.70	2.64	2.54
	0.975	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.72	3.62	3.52	3.42	3.31	3.22	3.08
	0.990	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.85	4.71	4.56	4.41	4.25	4.11	3.91
	0.999	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.75	8.45	8.13	7.80	7.47	7.19	6.76
11	0.900	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.25	2.21	2.17	2.12	2.08	2.04	1.97
	0.950	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.85	2.79	2.72	2.65	2.57	2.51	2.40
	0.975	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.53	3.43	3.33	3.23	3.12	3.03	2.88
	0.990	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.54	4.40	4.25	4.10	3.94	3.81	3.60
	0.999	13.8	11.6	10.3	9.58	9.05	8.66	8.35	7.92	7.63	7.32	7.01	6.68	6.42	6.00
12	0.900	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.19	2.15	2.10	2.06	2.01	1.97	1.90
	0.950	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.75	2.69	2.62	2.54	2.47	2.40	2.30
	0.975	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.37	3.28	3.18	3.07	2.96	2.87	2.72
	0.990	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.30	4.16	4.01	3.86	3.70	3.57	3.36
	0.999	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.29	7.00	6.71	6.40	6.09	5.83	5.42
13	0.900	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.14	2.10	2.05	2.01	1.96	1.92	1.85
	0.950	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.67	2.60	2.53	2.46	2.38	2.31	2.21
	0.975	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.25	3.15	3.05	2.95	2.84	2.74	2.60
	0.990	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.10	3.96	3.82	3.66	3.51	3.37	3.17
	0.999	12.3	10.2	9.07	8.35	7.86	7.49	7.21	6.80	6.52	6.23	5.93	5.63	5.37	4.97
14	0.900	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.10	2.05	2.01	1.96	1.91	1.87	1.80
	0.950	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.60	2.53	2.46	2.39	2.31	2.24	2.13
	0.975	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.15	3.05	2.95	2.84	2.73	2.64	2.49
	0.990	6.51	5.56	5.04	4.69	4.46	4.28	4.14	3.94	3.80	3.66	3.51	3.35	3.22	3.00
	0.999	11.8	9.73	8.62	7.92	7.44	7.08	6.80	6.40	6.13	5.85	5.56	5.25	5.00	4.60
15	0.900	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.06	2.02	1.97	1.92	1.87	1.83	1.76
	0.950	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.54	2.48	2.40	2.33	2.25	2.18	2.07
	0.975	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.06	2.96	2.86	2.76	2.64	2.55	2.40
	0.990	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.80	3.67	3.52	3.37	3.21	3.08	2.87
	0.999	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.08	5.81	5.53	5.25	4.95	4.70	4.31
16	0.900	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.03	1.99	1.94	1.89	1.84	1.79	1.72
	0.950	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.49	2.42	2.35	2.28	2.19	2.12	2.01
	0.975	4.69	4.08	3.73	3.50	3.34	3.22	3.12	2.99	2.89	2.79	2.68	2.57	2.47	2.32
	0.990	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.69	3.55	3.41	3.26	3.10	2.97	2.75
	0.999	11.0	9.01	7.94	7.27	6.80	6.46	6.19	5.81	5.55	5.27	4.99	4.70	4.45	4.06
17	0.900	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.00	1.96	1.91	1.86	1.81	1.76	1.69
	0.950	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.45	2.38	2.31	2.23	2.15	2.08	1.96
	0.975	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.92	2.82	2.72	2.62	2.50	2.41	2.25
	0.990	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.59	3.46	3.31	3.16	3.00	2.87	2.65
	0.999	10.7	8.73	7.68	7.02	6.56	6.22	5.96	5.58	5.32	5.05	4.77	4.48	4.24	3.85

$\nu_2 \backslash \nu_l$	q	2	3	4	5	6	7	8	10	12	15	20	30	50	∞
18	0.900	2.62	2.42	2.29	2.20	2.13	2.08	2.04	1.98	1.93	1.89	1.84	1.78	1.74	1.66
	0.950	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.41	2.34	2.27	2.19	2.11	2.04	1.92
	0.975	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.87	2.77	2.67	2.56	2.44	2.35	2.19
	0.990	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.51	3.37	3.23	3.08	2.92	2.78	2.57
	0.999	10.4	8.49	7.46	6.81	6.35	6.02	5.76	5.39	5.13	4.87	4.59	4.30	4.06	3.67
19	0.900	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.96	1.91	1.86	1.81	1.76	1.71	1.63
	0.950	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.38	2.31	2.23	2.16	2.07	2.00	1.88
	0.975	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.82	2.72	2.62	2.51	2.39	2.30	2.13
	0.990	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.43	3.30	3.15	3.00	2.84	2.71	2.49
	0.999	10.2	8.28	7.27	6.62	6.18	5.85	5.59	5.22	4.97	4.70	4.43	4.14	3.90	3.51
20	0.900	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.94	1.89	1.84	1.79	1.74	1.69	1.61
	0.950	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.35	2.28	2.20	2.12	2.04	1.97	1.84
	0.975	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.77	2.68	2.57	2.46	2.35	2.25	2.09
	0.990	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.37	3.23	3.09	2.94	2.78	2.64	2.42
	0.999	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.08	4.82	4.56	4.29	4.00	3.76	3.38
21	0.900	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.92	1.87	1.83	1.78	1.72	1.67	1.59
	0.950	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.32	2.25	2.18	2.10	2.01	1.94	1.81
	0.975	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.73	2.64	2.53	2.42	2.31	2.21	2.04
	0.990	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.31	3.17	3.03	2.88	2.72	2.58	2.36
	0.999	9.77	7.94	6.95	6.32	5.88	5.56	5.31	4.95	4.70	4.44	4.17	3.88	3.64	3.26
22	0.900	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.90	1.86	1.81	1.76	1.70	1.65	1.57
	0.950	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.30	2.23	2.15	2.07	1.98	1.91	1.78
	0.975	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.70	2.60	2.50	2.39	2.27	2.17	2.00
	0.990	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.26	3.12	2.98	2.83	2.67	2.53	2.31
	0.999	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.83	4.58	4.33	4.06	3.78	3.54	3.15
23	0.900	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.89	1.84	1.80	1.74	1.69	1.64	1.55
	0.950	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.27	2.20	2.13	2.05	1.96	1.88	1.76
	0.975	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.67	2.57	2.47	2.36	2.24	2.14	1.97
	0.990	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.21	3.07	2.93	2.78	2.62	2.48	2.26
	0.999	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.73	4.48	4.23	3.96	3.68	3.44	3.05
24	0.900	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.88	1.83	1.78	1.73	1.67	1.62	1.53
	0.950	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.25	2.18	2.11	2.03	1.94	1.86	1.73
	0.975	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.64	2.54	2.44	2.33	2.21	2.11	1.94
	0.990	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.17	3.03	2.89	2.74	2.58	2.44	2.21
	0.999	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.64	4.39	4.14	3.87	3.59	3.36	2.97
25	0.900	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.87	1.82	1.77	1.72	1.66	1.61	1.52
	0.950	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.24	2.16	2.09	2.01	1.92	1.84	1.71
	0.975	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.61	2.51	2.41	2.30	2.18	2.08	1.91
	0.990	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.13	2.99	2.85	2.70	2.54	2.40	2.17
	0.999	9.22	7.45	6.49	5.89	5.46	5.15	4.91	4.56	4.31	4.06	3.79	3.52	3.28	2.89
26	0.900	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.86	1.81	1.76	1.71	1.65	1.59	1.50
	0.950	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.22	2.15	2.07	1.99	1.90	1.82	1.69
	0.975	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.59	2.49	2.39	2.28	2.16	2.05	1.88
	0.990	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.09	2.96	2.81	2.66	2.50	2.36	2.13
	0.999	9.12	7.36	6.41	5.80	5.38	5.07	4.83	4.48	4.24	3.99	3.72	3.44	3.21	2.82

$\nu_2 \backslash \nu_l$	q	2	3	4	5	6	7	8	10	12	15	20	30	50	∞
27	0.900	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.85	1.80	1.75	1.70	1.64	1.58	1.49
	0.950	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.20	2.13	2.06	1.97	1.88	1.81	1.67
	0.975	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.57	2.47	2.36	2.25	2.13	2.03	1.85
	0.990	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.06	2.93	2.78	2.63	2.47	2.33	2.10
	0.999	9.02	7.27	6.33	5.73	5.31	5.00	4.76	4.41	4.17	3.92	3.66	3.38	3.14	2.75
28	0.900	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.84	1.79	1.74	1.69	1.63	1.57	1.48
	0.950	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.19	2.12	2.04	1.96	1.87	1.79	1.65
	0.975	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.55	2.45	2.34	2.23	2.11	2.01	1.83
	0.990	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.03	2.90	2.75	2.60	2.44	2.30	2.06
	0.999	8.93	7.19	6.25	5.66	5.24	4.93	4.69	4.35	4.11	3.86	3.60	3.32	3.09	2.69
29	0.900	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.83	1.78	1.73	1.68	1.62	1.56	1.47
	0.950	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.18	2.10	2.03	1.94	1.85	1.77	1.64
	0.975	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.53	2.43	2.32	2.21	2.09	1.99	1.81
	0.990	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.00	2.87	2.73	2.57	2.41	2.27	2.03
	0.999	8.85	7.12	6.19	5.59	5.18	4.87	4.64	4.29	4.05	3.80	3.54	3.27	3.03	2.64
30	0.900	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.82	1.77	1.72	1.67	1.61	1.55	1.46
	0.950	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.16	2.09	2.01	1.93	1.84	1.76	1.62
	0.975	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.51	2.41	2.31	2.20	2.07	1.97	1.79
	0.990	5.39	4.51	4.02	3.70	3.47	3.30	3.17	2.98	2.84	2.70	2.55	2.39	2.25	2.01
	0.999	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.24	4.00	3.75	3.49	3.22	2.98	2.59
60	0.900	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.71	1.66	1.60	1.54	1.48	1.41	1.29
	0.950	3.15	2.76	2.53	2.37	2.25	2.17	2.10	1.99	1.92	1.84	1.75	1.65	1.56	1.39
	0.975	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.27	2.17	2.06	1.94	1.82	1.70	1.48
	0.990	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.63	2.50	2.35	2.20	2.03	1.88	1.60
	0.999	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.54	3.32	3.08	2.83	2.55	2.32	1.89
80	0.900	2.37	2.15	2.02	1.92	1.85	1.79	1.75	1.68	1.63	1.57	1.51	1.44	1.38	1.24
	0.950	3.11	2.72	2.49	2.33	2.21	2.13	2.06	1.95	1.88	1.79	1.70	1.60	1.51	1.32
	0.975	3.86	3.28	2.95	2.73	2.57	2.45	2.35	2.21	2.11	2.00	1.88	1.75	1.63	1.40
	0.990	4.88	4.04	3.56	3.26	3.04	2.87	2.74	2.55	2.42	2.27	2.12	1.94	1.79	1.49
	0.999	7.54	5.97	5.12	4.58	4.20	3.92	3.70	3.39	3.16	2.93	2.68	2.41	2.16	1.72
100	0.900	2.36	2.14	2.00	1.91	1.83	1.78	1.73	1.66	1.61	1.56	1.49	1.42	1.35	1.21
	0.950	3.09	2.70	2.46	2.31	2.19	2.10	2.03	1.93	1.85	1.77	1.68	1.57	1.48	1.28
	0.975	3.83	3.25	2.92	2.70	2.54	2.42	2.32	2.18	2.08	1.97	1.85	1.71	1.59	1.35
	0.990	4.82	3.98	3.51	3.21	2.99	2.82	2.69	2.50	2.37	2.22	2.07	1.89	1.74	1.43
	0.999	7.41	5.86	5.02	4.48	4.11	3.83	3.61	3.30	3.07	2.84	2.59	2.32	2.08	1.62
120	0.900	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.65	1.60	1.54	1.48	1.41	1.34	1.19
	0.950	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.91	1.83	1.75	1.66	1.55	1.46	1.25
	0.975	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.16	2.05	1.94	1.82	1.69	1.56	1.31
	0.990	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.47	2.34	2.19	2.03	1.86	1.70	1.38
	0.999	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.24	3.02	2.78	2.53	2.26	2.02	1.54
∞	0.900	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.60	1.55	1.49	1.42	1.34	1.26	1.00
	0.950	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.83	1.75	1.67	1.57	1.46	1.35	1.00
	0.975	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.05	1.94	1.83	1.71	1.57	1.43	1.00
	0.990	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.32	2.18	2.04	1.88	1.70	1.52	1.00
	0.999	6.91	5.42	4.62	4.10	3.74	3.47	3.27	2.96	2.74	2.51	2.27	1.99	1.73	1.00

NORMAL CUMULATIVE DISTRIBUTION FUNCTION

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

STUDENT'S t PERCENTAGE POINTS

ν	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.325	0.577	1.000	1.376	2.414	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.500	0.816	1.061	1.604	1.886	2.920	4.303	6.965	9.925	22.327
3	0.277	0.476	0.765	0.978	1.423	1.638	2.353	3.182	4.541	5.841	10.215
4	0.271	0.464	0.741	0.941	1.344	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.457	0.727	0.920	1.301	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.453	0.718	0.906	1.273	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.449	0.711	0.896	1.254	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.447	0.706	0.889	1.240	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.445	0.703	0.883	1.230	1.383	1.833	2.262	2.821	3.250	4.297
10	0.260	0.444	0.700	0.879	1.221	1.372	1.812	2.228	2.764	3.169	4.144
11	0.260	0.443	0.697	0.876	1.214	1.363	1.796	2.201	2.718	3.106	4.025
12	0.259	0.442	0.695	0.873	1.209	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.441	0.694	0.870	1.204	1.350	1.771	2.160	2.650	3.012	3.852
14	0.258	0.440	0.692	0.868	1.200	1.345	1.761	2.145	2.624	2.977	3.787
15	0.258	0.439	0.691	0.866	1.197	1.341	1.753	2.131	2.602	2.947	3.733
16	0.258	0.439	0.690	0.865	1.194	1.337	1.746	2.120	2.583	2.921	3.686
17	0.257	0.438	0.689	0.863	1.191	1.333	1.740	2.110	2.567	2.898	3.646
18	0.257	0.438	0.688	0.862	1.189	1.330	1.734	2.101	2.552	2.878	3.610
19	0.257	0.438	0.688	0.861	1.187	1.328	1.729	2.093	2.539	2.861	3.579
20	0.257	0.437	0.687	0.860	1.185	1.325	1.725	2.086	2.528	2.845	3.552
21	0.257	0.437	0.686	0.859	1.183	1.323	1.721	2.080	2.518	2.831	3.527
22	0.256	0.437	0.686	0.858	1.182	1.321	1.717	2.074	2.508	2.819	3.505
23	0.256	0.436	0.685	0.858	1.180	1.319	1.714	2.069	2.500	2.807	3.485
24	0.256	0.436	0.685	0.857	1.179	1.318	1.711	2.064	2.492	2.797	3.467
25	0.256	0.436	0.684	0.856	1.178	1.316	1.708	2.060	2.485	2.787	3.450
26	0.256	0.436	0.684	0.856	1.177	1.315	1.706	2.056	2.479	2.779	3.435
27	0.256	0.435	0.684	0.855	1.176	1.314	1.703	2.052	2.473	2.771	3.421
28	0.256	0.435	0.683	0.855	1.175	1.313	1.701	2.048	2.467	2.763	3.408
29	0.256	0.435	0.683	0.854	1.174	1.311	1.699	2.045	2.462	2.756	3.396
30	0.256	0.435	0.683	0.854	1.173	1.310	1.697	2.042	2.457	2.750	3.385
35	0.255	0.434	0.682	0.852	1.170	1.306	1.690	2.030	2.438	2.724	3.340
40	0.255	0.434	0.681	0.851	1.167	1.303	1.684	2.021	2.423	2.704	3.307
45	0.255	0.434	0.680	0.850	1.165	1.301	1.679	2.014	2.412	2.690	3.281
50	0.255	0.433	0.679	0.849	1.164	1.299	1.676	2.009	2.403	2.678	3.261
55	0.255	0.433	0.679	0.848	1.163	1.297	1.673	2.004	2.396	2.668	3.245
60	0.254	0.433	0.679	0.848	1.162	1.296	1.671	2.000	2.390	2.660	3.232
∞	0.253	0.431	0.674	0.842	1.150	1.282	1.645	1.960	2.326	2.576	3.090

CHI-SQUARED PERCENTAGE POINTS

ν	0.1%	0.5%	1.0%	2.5%	5.0%	10.0%	12.5%	20.0%	25.0%	33.3%	50.0%
1	0.000	0.000	0.000	0.001	0.004	0.016	0.025	0.064	0.102	0.186	0.455
2	0.002	0.010	0.020	0.051	0.103	0.211	0.267	0.446	0.575	0.811	1.386
3	0.024	0.072	0.115	0.216	0.352	0.584	0.692	1.005	1.213	1.568	2.366
4	0.091	0.207	0.297	0.484	0.711	1.064	1.219	1.649	1.923	2.378	3.357
5	0.210	0.412	0.554	0.831	1.145	1.610	1.808	2.343	2.675	3.216	4.351
6	0.381	0.676	0.872	1.237	1.635	2.204	2.441	3.070	3.455	4.074	5.348
7	0.598	0.989	1.239	1.690	2.167	2.833	3.106	3.822	4.255	4.945	6.346
8	0.857	1.344	1.646	2.180	2.733	3.490	3.797	4.594	5.071	5.826	7.344
9	1.152	1.735	2.088	2.700	3.325	4.168	4.507	5.380	5.899	6.716	8.343
10	1.479	2.156	2.558	3.247	3.940	4.865	5.234	6.179	6.737	7.612	9.342
11	1.834	2.603	3.053	3.816	4.575	5.578	5.975	6.989	7.584	8.514	10.341
12	2.214	3.074	3.571	4.404	5.226	6.304	6.729	7.807	8.438	9.420	11.340
13	2.617	3.565	4.107	5.009	5.892	7.042	7.493	8.634	9.299	10.331	12.340
14	3.041	4.075	4.660	5.629	6.571	7.790	8.266	9.467	10.165	11.245	13.339
15	3.483	4.601	5.229	6.262	7.261	8.547	9.048	10.307	11.037	12.163	14.339
16	3.942	5.142	5.812	6.908	7.962	9.312	9.837	11.152	11.912	13.083	15.338
17	4.416	5.697	6.408	7.564	8.672	10.085	10.633	12.002	12.792	14.006	16.338
18	4.905	6.265	7.015	8.231	9.390	10.865	11.435	12.857	13.675	14.931	17.338
19	5.407	6.844	7.633	8.907	10.117	11.651	12.242	13.716	14.562	15.859	18.338
20	5.921	7.434	8.260	9.591	10.851	12.443	13.055	14.578	15.452	16.788	19.337
21	6.447	8.034	8.897	10.283	11.591	13.240	13.873	15.445	16.344	17.720	20.337
22	6.983	8.643	9.542	10.982	12.338	14.041	14.695	16.314	17.240	18.653	21.337
23	7.529	9.260	10.196	11.689	13.091	14.848	15.521	17.187	18.137	19.587	22.337
24	8.085	9.886	10.856	12.401	13.848	15.659	16.351	18.062	19.037	20.523	23.337
25	8.649	10.520	11.524	13.120	14.611	16.473	17.184	18.940	19.939	21.461	24.337
26	9.222	11.160	12.198	13.844	15.379	17.292	18.021	19.820	20.843	22.399	25.336
27	9.803	11.808	12.879	14.573	16.151	18.114	18.861	20.703	21.749	23.339	26.336
28	10.391	12.461	13.565	15.308	16.928	18.939	19.704	21.588	22.657	24.280	27.336
29	10.986	13.121	14.256	16.047	17.708	19.768	20.550	22.475	23.567	25.222	28.336
30	11.588	13.787	14.953	16.791	18.493	20.599	21.399	23.364	24.478	26.165	29.336
35	14.688	17.192	18.509	20.569	22.465	24.797	25.678	27.836	29.054	30.894	34.336
40	17.916	20.707	22.164	24.433	26.509	29.051	30.008	32.345	33.660	35.643	39.335
45	21.251	24.311	25.901	28.366	30.612	33.350	34.379	36.884	38.291	40.407	44.335
50	24.674	27.991	29.707	32.357	34.764	37.689	38.785	41.449	42.942	45.184	49.335
55	28.173	31.735	33.570	36.398	38.958	42.060	43.220	46.036	47.610	49.972	54.335
60	31.738	35.534	37.485	40.482	43.188	46.459	47.680	50.641	52.294	54.770	59.335

CHI-SQUARED PERCENTAGE POINTS

ν	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.708	0.936	1.323	1.642	2.354	2.706	3.841	5.024	6.635	7.879	10.828
2	1.833	2.197	2.773	3.219	4.159	4.605	5.991	7.378	9.210	10.597	13.816
3	2.946	3.405	4.108	4.642	5.739	6.251	7.815	9.348	11.345	12.838	16.266
4	4.045	4.579	5.385	5.989	7.214	7.779	9.488	11.143	13.277	14.860	18.467
5	5.132	5.730	6.626	7.289	8.625	9.236	11.070	12.833	15.086	16.750	20.515
6	6.211	6.867	7.841	8.558	9.992	10.645	12.592	14.449	16.812	18.548	22.458
7	7.283	7.992	9.037	9.803	11.326	12.017	14.067	16.013	18.475	20.278	24.322
8	8.351	9.107	10.219	11.030	12.636	13.362	15.507	17.535	20.090	21.955	26.125
9	9.414	10.215	11.389	12.242	13.926	14.684	16.919	19.023	21.666	23.589	27.877
10	10.473	11.317	12.549	13.442	15.198	15.987	18.307	20.483	23.209	25.188	29.588
11	11.530	12.414	13.701	14.631	16.457	17.275	19.675	21.920	24.725	26.757	31.264
12	12.584	13.506	14.845	15.812	17.703	18.549	21.026	23.337	26.217	28.300	32.910
13	13.636	14.595	15.984	16.985	18.939	19.812	22.362	24.736	27.688	29.819	34.528
14	14.685	15.680	17.117	18.151	20.166	21.064	23.685	26.119	29.141	31.319	36.123
15	15.733	16.761	18.245	19.311	21.384	22.307	24.996	27.488	30.578	32.801	37.697
16	16.780	17.840	19.369	20.465	22.595	23.542	26.296	28.845	32.000	34.267	39.252
17	17.824	18.917	20.489	21.615	23.799	24.769	27.587	30.191	33.409	35.718	40.790
18	18.868	19.991	21.605	22.760	24.997	25.989	28.869	31.526	34.805	37.156	42.312
19	19.910	21.063	22.718	23.900	26.189	27.204	30.144	32.852	36.191	38.582	43.820
20	20.951	22.133	23.828	25.038	27.376	28.412	31.410	34.170	37.566	39.997	45.315
21	21.991	23.201	24.935	26.171	28.559	29.615	32.671	35.479	38.932	41.401	46.797
22	23.031	24.268	26.039	27.301	29.737	30.813	33.924	36.781	40.289	42.796	48.268
23	24.069	25.333	27.141	28.429	30.911	32.007	35.172	38.076	41.638	44.181	49.728
24	25.106	26.397	28.241	29.553	32.081	33.196	36.415	39.364	42.980	45.559	51.179
25	26.143	27.459	29.339	30.675	33.247	34.382	37.652	40.646	44.314	46.928	52.620
26	27.179	28.520	30.435	31.795	34.410	35.563	38.885	41.923	45.642	48.290	54.052
27	28.214	29.580	31.528	32.912	35.570	36.741	40.113	43.195	46.963	49.645	55.476
28	29.249	30.639	32.620	34.027	36.727	37.916	41.337	44.461	48.278	50.993	56.892
29	30.283	31.697	33.711	35.139	37.881	39.087	42.557	45.722	49.588	52.336	58.301
30	31.316	32.754	34.800	36.250	39.033	40.256	43.773	46.979	50.892	53.672	59.703
35	36.475	38.024	40.223	41.778	44.753	46.059	49.802	53.203	57.342	60.275	66.619
40	41.622	43.275	45.616	47.269	50.424	51.805	55.758	59.342	63.691	66.766	73.402
45	46.761	48.510	50.985	52.729	56.052	57.505	61.656	65.410	69.957	73.166	80.077
50	51.892	53.733	56.334	58.164	61.647	63.167	67.505	71.420	76.154	79.490	86.661
55	57.016	58.945	61.665	63.577	67.211	68.796	73.311	77.380	82.292	85.749	93.168
60	62.135	64.147	66.981	68.972	72.751	74.397	79.082	83.298	88.379	91.952	99.607

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