NMEP

as a Linear Combination of
Math 54

(with lots of coefficients set to 0)

1 Vectors

1.1 Lists of numbers

Some vectors (but only **some**) are lists of numbers:

$$\begin{bmatrix} 1 & 2 & 3 & \cdots & n \end{bmatrix}$$

For now, we will focus on this type of vectors, but remember that it is a very particular special case.

1.2 Typical notation

We use $\overrightarrow{x}$ to unambiguously denote x as a vector variable. However, we will also use bold $\mathbf{x}$ (and simply x in hand-writing), whenever the nature of x is obvious from the context.

We use $\overrightarrow{x} \in \mathbb{R}^n$ or, more generally, $\overrightarrow{x} \in \mathbb{R}^{n \times 1}$ to denote that vector $\overrightarrow{x}$ has n elements. Ideally get used to thinking of n as the **height** of the vector and not its length. Also note that $\mathbb{R}$ in this notation means that this vector is particularly a list of real numbers. For non-numerical vectors we sometimes have no choice but to be verbose and type out that "$\overrightarrow{x}$ is a vector of n elements'.

NOTE: You will sometimes hear engineers abuse Computer Science slang and say "length" of a vector to describe the number of its elements. They are all going to math jail.

For numerical ($\mathbb{R}^n$) vectors, *norm* is defined as the euclidean distance of their endpoint from the origin. The notation for norm of $\overrightarrow{x}$ is $\|\overrightarrow{x}\|$. Sometimes, especially in hand-writing, we use just $|\overrightarrow{x}|$ (this notation conflicts with the absolute value and so is discouraged).

$$\|\overrightarrow{x}\| = \| \begin{bmatrix} 1 & 2 & 3 & \cdots & n \end{bmatrix} \| \coloneqq \sqrt{1^2 + 2^2 + 3^2 + \ldots + n^2}$$

NOTE: People who prefer geometry to abstract algebra sometimes say *length* to refer to norm. They have connections in the math department and so do not go to math jail.
Engineers are left confused and slighted.

NOTE: Norm is also defined for non-numerical vectors. **Euclidean distance is just the most common special case of the definition of norm.** In fact, other norms can be used for $\mathbb{R}^n$ too.
See https://en.wikipedia.org/wiki/Norm_(mathematics)

We write vectors horizontally (slyly pretending that they are *row vectors*) only if we just want to list the vector's contents. In all other cases (e.g. in equations) we write vectors vertically (as *column vectors*).

NOTE: We generally never use row vectors. The horizontal notation is just more convenient in writing. It still represents **column vectors**.

$$\overrightarrow{x} \in \mathbb{R}^{n \times 1} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

For alternative notation, check out https://en.wikipedia.org/wiki/Vector_notation

1.3 Numerical vector operations

$\overrightarrow{x}$ and $\overrightarrow{y}$ have an equal number of elements. Otherwise we cannot add or subtract them, or find their dot product.

$$\overrightarrow{x}, \overrightarrow{y} \in \mathbb{R}^n$$

Addition:

$$\overrightarrow{x} + \overrightarrow{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} 16 \\ 13 \\ 22 \\ \vdots \\ y_n \end{bmatrix} := \begin{bmatrix} 17 \\ 15 \\ 25 \\ \vdots \\ x_n + y_n \end{bmatrix}$$

Subtraction:

$$\overrightarrow{x} - \overrightarrow{y} = \begin{bmatrix} 20 \\ 13 \\ 3 \\ \vdots \\ x_n \end{bmatrix} - \begin{bmatrix} 16 \\ 2 \\ 22 \\ \vdots \\ y_n \end{bmatrix} := \begin{bmatrix} 4 \\ 11 \\ -19 \\ \vdots \\ x_n - y_n \end{bmatrix}$$

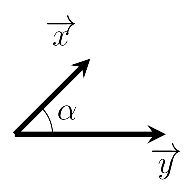
Dot product:
There are two equivalent (proof omitted) definitions/views of dot product.

The algebraic view:

$$\overrightarrow{x} \cdot \overrightarrow{y} = \begin{bmatrix} 4 \\ 8 \\ 16 \\ \vdots \\ x_n \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 5 \\ -7 \\ \vdots \\ y_n \end{bmatrix} := 4 + 40 + -112 + \ldots + x_n y_n = \sum_{i=0}^{n} x_n y_n$$

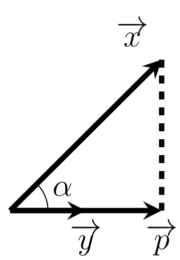
$$\begin{bmatrix} 5 \\ 9 \\ 17 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 6 \\ -8 \end{bmatrix} = 10 + 54 - 136 = -72$$

The geometric view:



$$\overrightarrow{x} \cdot \overrightarrow{y} := \|\overrightarrow{x}\|\|\overrightarrow{y}\|\cos(\alpha)$$

In particular, if $\|\overrightarrow{y}\| = 1$, the picture is even more convenient. This is related to vector projections which will be discussed later.



$$\boxed{\overrightarrow{x} \cdot \overrightarrow{y} = \|\overrightarrow{x}\|\cos(\alpha) = \|\overrightarrow{p}\|}$$

In three dimensions ($\overrightarrow{x}, \overrightarrow{y} \in \mathbb{R}^3$), there is also a notion of a *cross product*:

$$\overrightarrow{x} \times \overrightarrow{y} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} := \begin{bmatrix} yc - zb \\ za - xc \\ xb - ya \end{bmatrix}$$