MECA

Base : $\boxed{M}$ pouce à gauche : directe :  $\vec{e_1}$.

Produit scal^R : $\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cdot\cos(\vec{u},\vec{v})$.

$\|\vec{v}\| = \sqrt{\vec{v}^2}$ + Pptés $\begin{matrix}\cos a \pm b\\ \sin a \pm b\end{matrix}$ $\|$.

Si $\vec{u} \mathbin{//} \vec{v} \Rightarrow \vec{u}\wedge\vec{v} = \vec{0}$.

$\boxed{M}$ 3 doigt : produit vectoriel : $\vec{u}\wedge\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$

→ forces à distance, contact, ponctuelle, réparties, intérieures

Moment d'une force : $\boxed{\vec{M}_K(\vec{F},A) = \vec{KA}\wedge\vec{F}}$

Formule du transport : $\boxed{\vec{M}_K(\vec{F},A) = \vec{M}_J(\vec{F},A) + \vec{KJ}\wedge\vec{F}}$

$\vec{AB} = \|\vec{AB}\|\cdot\vec{u}$.

$\|\vec{u}\wedge\vec{v}\| = \|\vec{u}\| \times \|\vec{v}\| \times \sin(\vec{u},\vec{v})$

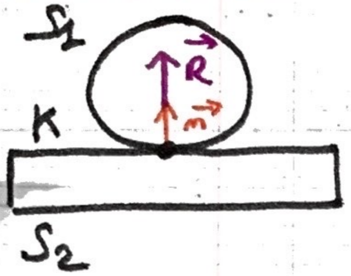
TORSEUR STATIQUE

Résultante : $\boxed{\vec{R}\{F\} = \sum_{i=1}^{n} \vec{F_i}}$

Moment en un point : $\boxed{\begin{array}{l} \vec{M_K}\{F\} = \sum_{i=1}^{n} M_K(\vec{F_i}, A_i) \\ \vec{M_K}\{F\} = \vec{KA} \wedge \vec{F} \end{array}}$

FFTT : $\boxed{\vec{M_K}\{F\} = \vec{M_J}\{F\} + \vec{KJ} \wedge \vec{R}}$

Contact Ponctuel Unilatéral Parfait



→ efforts de contact transmis selon la normale.

$$\boxed{\vec{R} = \|\vec{R}\| . \vec{n}}$$ force de réac°.

$\vec{R_{21}} . \vec{n} > 0$ $\qquad \vec{R_{21}} = N_{21} . \vec{n}$

$$\{ F(S_2 - S_1) \} = \left\{ \begin{matrix} \vec{R_{21}} \\ 0 \end{matrix} \right\}_K .$$

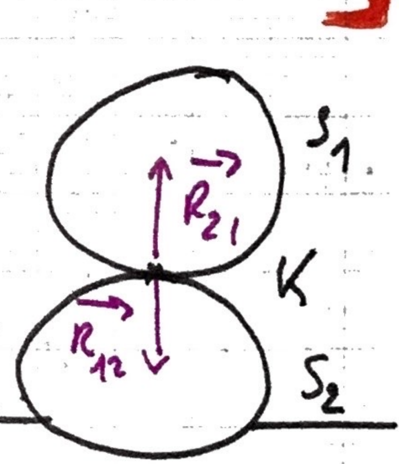
Loi de la Statique : PFS

Principe Fondamental Statique

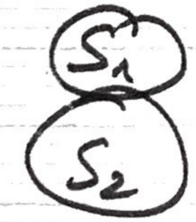
"∃ ref galiléen tq ∀ système matériel S en équilibre, le torseur des ac°s mécaniqs extérieures à ce systm soit NULLE."

$$PFS \Rightarrow \{ F(S_1 - S_2) \} = \{0\} \Rightarrow \left\{ \begin{matrix} \vec{R} = \vec{0} \\ \vec{M_K} = \vec{0} \end{matrix} \right\} .$$

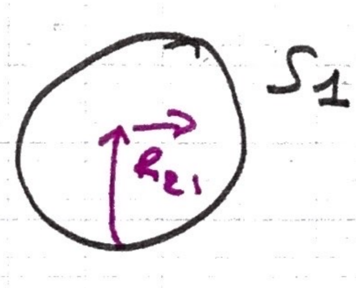
$$\Rightarrow \vec{R_{12}} = -\vec{R_{21}}$$



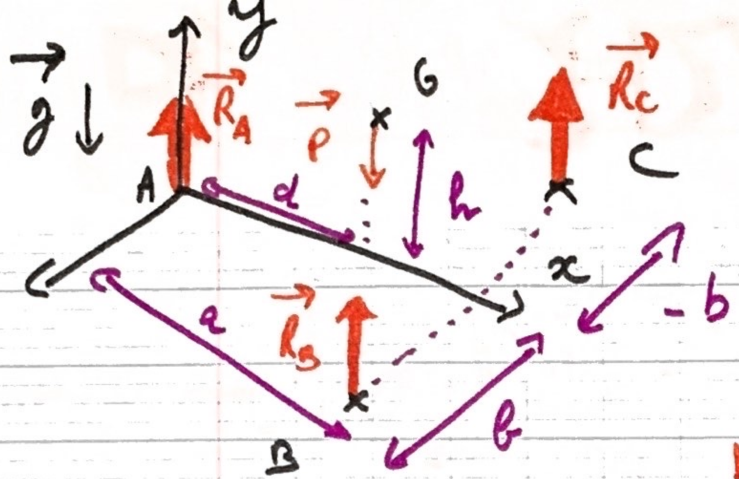
"Isoler système S_1"



Isoler S_1 $\Longrightarrow$



Contact si $\|\vec{R_{21}}\| > 0$.



① Isoler véhicule
Effectuer bilan actions extérieures

② Ecrire équations équilibres : avec PFS.

$$PFS \Rightarrow \{F\ T\text{-}S\} = \{0\} \Rightarrow \begin{cases} \vec{R}\ \{T\text{-}S\} = \vec{0} \\ \vec{M_K}\ \{T\text{-}S\} = \vec{0} \end{cases}$$

(1) • $\vec{R}\{T\text{-}S\} = \vec{P} + \vec{R_A} + \vec{R_B} + \vec{R_C} = \vec{0}$

(2) • $\vec{M_A}\{T\text{-}S\} : \vec{AA} \wedge \vec{R_A} + \vec{AC} \wedge \vec{R_C} + \vec{AB} \wedge \vec{RB} + \vec{AG} \wedge \vec{P} = \vec{0}$.

Projection sur les axes du repère :

$$\begin{matrix} (3) \\ (4) \\ (5) \end{matrix} \begin{cases} 0 + 0 + 0 + 0 = 0 \\ -P + R_A + R_B + R_C = 0 \\ 0 + 0 - 0 + 0 = 0 \end{cases} \quad \vec{AG}\begin{pmatrix} d \\ h \\ 0 \end{pmatrix} \quad \vec{AB}\begin{pmatrix} a \\ 0 \\ b \end{pmatrix} \quad \vec{AC}\begin{pmatrix} a \\ 0 \\ -b \end{pmatrix}$$

$$\cdot\ \vec{AG} \wedge \vec{P} = \begin{pmatrix} 0 \\ 0 \\ -dP \end{pmatrix} \quad \vec{AB} \wedge \vec{R_B} = \begin{pmatrix} -b.R_B \\ 0 \\ a.\ R_B \end{pmatrix} \quad \vec{AC} \wedge \vec{R_C} = \begin{pmatrix} b + R_C \\ 0 \\ a.R_C \end{pmatrix}$$

$$(2) \Rightarrow \begin{pmatrix} 0 \\ 0 \\ -dP \end{pmatrix} + \begin{pmatrix} -b.R_B \\ 0 \\ a.\ R_B \end{pmatrix} + \begin{pmatrix} b\ .\ R_C \\ 0 \\ a.\ R_C \end{pmatrix} = 0$$

$$\Rightarrow \begin{cases} -b.R_B + b..R_C = 0 \\ -dP + aR_B + aR_C = 0 \end{cases} = \begin{cases} R_B - R_C = 0 \\ a(R_B + R_C) = dP \end{cases}$$

③ Actions du sol

$$\begin{cases} 0 = 0 & \rightarrow \text{translat}^{\circ} \text{ OK} \\ R_A + R_B + R_C = P \\ -b(\underbrace{R_B - R_C}_{0=0}) = 0 \\ 0 = 0 \\ a(R_B + R_C) = dP & (8) \end{cases}$$

$R_B = R_C$

- (8) $R_B + R_C = \frac{dP}{a}$ or $\boxed{R_B = R_C}$
- $\Rightarrow 2R_B = \frac{dP}{a} \Rightarrow \boxed{R_B = \frac{dP}{2a} = R_C}$

(6) $\boxed{R_A} = P - 2R_B = P - \frac{2dP}{2a} = P - \frac{dP}{a} = P\left(1 - \frac{d}{a}\right) = \boxed{\left(\frac{a-d}{a}\right)P}$

④ Vérifier conditions de contact : $\boxed{R > 0}$

$R_A = \left(\frac{a-d}{a}\right)P > 0$ & $R_B = R_C = \frac{dP}{2a} > 0$.

- NB : égalité des réactions car axe de symétrie

⑤ Équa° moment en B.

$\overrightarrow{M_B}\{F\} = \overrightarrow{BB} \wedge \overrightarrow{RB} + \overrightarrow{BA} \wedge \overrightarrow{RA} + \overrightarrow{BC} \wedge \overrightarrow{RC} + \overrightarrow{BG} \wedge \overrightarrow{P}$

⑥ Actions du véhicule sur le sol

- $R_A^V = -R_A$; $R_B^V = -R_B$; $R_C^V = -R_C$.

Couple :

Une act° méca est un couple ssi résultante est nul.

$$\{F\} = \left\{ \begin{matrix} \vec{0} \\ \vec{M} \end{matrix} \right\} \Rightarrow \quad \vec{M}_K\{F\} = M_J\{F\} + \underbrace{\vec{KJ} \wedge \vec{R}}_{0}$$

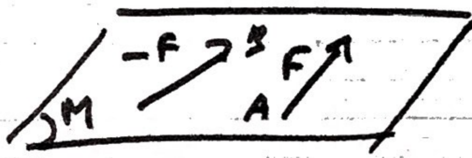
$$\vec{M}_K\{F\} = \vec{M}_J\{F\} = \vec{M}.$$

Un couple tend à imprimer une rotation au système matériel. (vis)

Ppté : Le couple ne varie pas ; couple indépdt point K.

$$\vec{M} = \vec{KA} \wedge \vec{F} + \vec{KB} \wedge -\vec{F} = (\vec{KA} - \vec{KB}) \wedge \vec{F}.$$

$$\boxed{\vec{M} = \vec{BA} \wedge \vec{F}}$$



$$\|\vec{M}\| = \|\vec{BA}\| . \|\vec{F}\| . \sin(\vec{BA}, \vec{F}) = \|\vec{F}\| . d.$$

Pb ds espace ⟶ ds le plan :

↳ si plan symétrie ds espace : réduc° pb slt ds 1 plan.

@ $\{F\} = \left\{ \begin{matrix} R_x \\ R_y \\ 0 \end{matrix} \middle| \begin{matrix} 0 \\ 0 \\ M_z \end{matrix} \right\}.$

L iaisons :

6 degrés de liberté : 3 translat° + 3 rotat°.

- @ poutre : 6 DDL / chaise : 5 DLB + 1 DLA.

- → 1 système soumis à 1 $\vec{F}$ ne transmet pas m de couple de mmt ni de forces svt axes DLB.

Hypothèses : solide csdé rigide & ∅ frottm^t ds liaisons

n_C : DLB / n_S : DLA.

$$\boxed{n_C + n_S = 6}$$

Configurat° plane : $n_C + n_S = 3$.

$$\{T\} = \begin{Bmatrix} R_x & 0 \\ R_y & 0 \\ 0 & M_z \end{Bmatrix}.$$

Liaisons Usuelles.

@ poutre.

- Encastrement : $\{F\}_K = \begin{Bmatrix} R_x & M_x \\ R_y & M_y \\ R_z & M_z \end{Bmatrix}$

- Pivot : $\{F\}_K = \begin{Bmatrix} R_x & 0 \\ R_y & M_y \\ R_z & M_z \end{Bmatrix}$

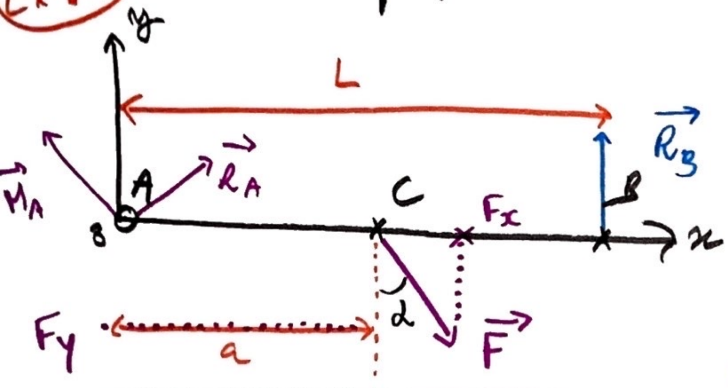
- Glissière : $\{F\}_K = \begin{Bmatrix} 0 & M_x \\ R_y & M_y \\ R_z & M_z \end{Bmatrix}$ @tobogan.

DLB = 0
DLA = R ou M

Tortue

Translat° Rotat°.

Ex 18 — Pb plan $m_c + m_s = 3$



• liaison pivot Az

$\vec{F}\begin{pmatrix} F.\sin\alpha \\ -F.\cos\alpha \\ 0 \end{pmatrix}$

$\vec{R_A}\begin{pmatrix} R_{Ax} \\ R_{Ay} \\ 0 \end{pmatrix}$ pb plan → 0

$\vec{M_A}\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ pivot Az → 0

2) $\vec{R_A} + \vec{F} + \vec{R_B} = \vec{0}$ (I)

$\vec{AC} \wedge \vec{F} + \vec{AB} \wedge \vec{R_B} = \vec{0}$ (II)

$\vec{AC}\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$ $\vec{AB}\begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix}$

(I) $\Rightarrow \begin{cases} R_{Ax} + F\sin\alpha = 0 \\ R_{Ay} - F\cos\alpha + R_B = 0 \end{cases}$

$\vec{AC} \wedge \vec{F} = a\vec{x} \wedge (F\sin\alpha.\vec{x} - F\cos\alpha\,\vec{y})$

$\vec{AC} \wedge \vec{F} = a.F.\sin\alpha\,\vec{x} \wedge \vec{x} - aF\cos\alpha\,\vec{x} \wedge \vec{y}$

$\vec{AC} \wedge \vec{F} = -aF.\cos\alpha.\vec{z}$

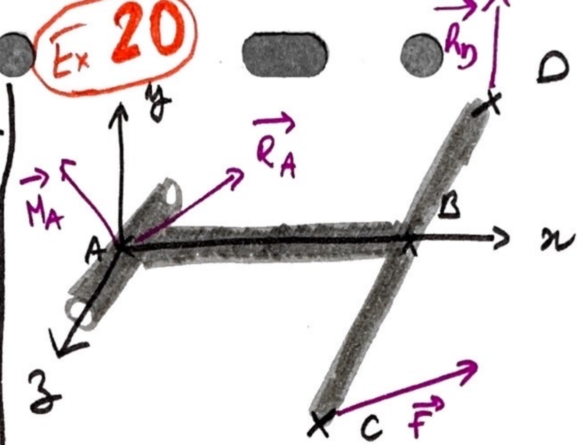
$\vec{AB} \wedge \vec{R_B} = L R_B.\vec{z}$

(II) $\Rightarrow \begin{cases} 0 = 0 \\ 0 = 0 \\ -aF\cos\alpha + L R_B = 0 \end{cases} \Rightarrow \begin{cases} R_{Ax} = -F.\sin\alpha \\ R_B = \dfrac{aF\cos\alpha}{L} \end{cases}$

$R_{Ay} = F\cos\alpha = F\cos\alpha - \dfrac{a\cos\alpha F}{L}$

$R_{Ay} = F.\cos\alpha\left(\dfrac{L-a}{L}\right)$

Ex 20



• pivot axe Az.

• $\vec{F} = F_x\vec{x} + F_y\vec{y} + F_z\vec{z}$

• $\vec{RD} = (0, R_D, 0)$

• $\vec{R_A} = (R_{Ax}, R_{Ay}, R_{Az})$

• $\vec{M_A} = (M_{Ax}, M_{Ay}, 0)$

PFS $\Rightarrow \begin{cases} \vec{R_A} + \vec{R_D} + \vec{F} = \vec{0} & \text{(I)} \\ \vec{M_A} + \vec{AD} \wedge \vec{RD} + \vec{AC} \wedge \vec{F} = \vec{0} & \text{(II)} \end{cases}$

(I) $\Rightarrow \begin{cases} R_{Ax} + 0 + F_x = 0 \\ R_{Ay} + R_D + F_y = 0 \\ R_{Az} + 0 + F_z = 0 \end{cases}$

$\vec{AD} = (L, 0, -b)$

$\vec{AC} = (L, 0, a)$

• $\vec{AD} \wedge \vec{RD} = (L\vec{x} - b\vec{z}) \wedge \vec{RD}.\vec{y} = bR_D\vec{x} + LR_D.\vec{z}$

• $\vec{AC} \wedge \vec{F} = (L\vec{x} + a\vec{z}) \wedge (F_x.\vec{x} + F_y.\vec{y} + F_z.\vec{z})$

$\vec{AC} \wedge \vec{F} = \begin{pmatrix} -aF_y \\ aF_x - LF_z \\ LF_y \end{pmatrix}$

II $\Rightarrow \begin{cases} M_{Ax} + bR_D - aF_y = 0 & (6) \\ M_{Ay} + aF_x - LF_z = 0 & (7) \\ M_{Az} + LR_D + LF_y = 0 & (8) \end{cases}$

$\Rightarrow R_{Ax} = -F_x$
$R_{Az} = -F_z$
$R_D = -F_y$
$R_{Ay} = 0$

$M_{Ax} = (a+b)F_y$
(6)

$\{T\}_K = \begin{Bmatrix} -F_x & (a+b)F_y \\ 0 & -aF_x + LF_z \\ -F_z & 0 \end{Bmatrix}$

CH [redacted]

· Eϕ : excit°/éjec° e- lx incdte $\nu_1 > \nu_0$

· $W = h\nu_0$ / $h\nu_i = W + \frac{1}{2} m_e v^2$

Méca · PFS : ∃ RG, ∀ SM S en E, TFAME SN.

· démo : $M_K(\vec{F}, A) = \sum \vec{KA_i} \wedge \vec{F}$

· contact si $\|\vec{R}_{21}\| > 0$

· m_S = DLA / m_C = DLB

· PAM : $\mathcal{F}\{S_1 \to S_2\} = - \mathcal{F}\{S_2 \to S_1\}$

EEA · $\boxed{E = P.t}$ $\boxed{P = U.I}$ $\boxed{P = G.U^2}$

· $G = \sigma \times \frac{S}{L}$ & $R = \rho . \frac{L}{S}$ & $U_{AB} = V_A - V_B$

Info · // : quotient DE & % : reste : DE

· miroir, contains, my-len ; 65, 90 ; 97, 122

PM · ⊙ : P Q S P DM S DDM

· ⊙ Prog : ○ QSP ≠ ⊙S & Perturba° : VLMD VP

· ⊙ plane si ⊙ PM, NDP ⊥ à P

· $\psi(x_0, t) = a_0 \cdot \sin(\omega(t - \frac{x}{v}) + \varphi_0)$

· Plane : PM̂ A ont M̂ P.

π/2 · So : EPG a M̂P à I à DD
0
π · PI : RG, PM qNS ∅ $\vec{F}$ SC SER o MRU
π
π/2 · PFD : RG, SV ∀ FA à SM = D ∅ TQM.
-π
· PAR : SI, C Σ_1, Σ_2 . FE par Σ_1 sur Σ_2 M̂ND MS SO ∅ FE Σ_2 sur Σ_1.

M [redacted]

· Bijective : $\forall y \in F, \exists ! x \in E, \; y = f(x)$.

· Injective : $\forall x_1 = x_2 \in E, \Rightarrow f(x_1) = f(x_2)$.

· Surjective : $\forall y \in F, \exists x \in E, \; y = f(x)$.

· Majorée : $\exists M \in \mathbb{R}, \forall n \in \mathbb{N}, U_n \leq M$.

· Croissante : $\forall n \in > \mathbb{N}, \; U_{n+1} \geq U_n$

· CV : $\forall \varepsilon > 0, \exists N \in \mathbb{N}, |U_n - l| < \varepsilon$. $\forall n \in \mathbb{N}$

· DV : $\forall M > 0, \exists N \in \mathbb{N}, U_n > M$. $\forall n \in \mathbb{N}$

· SRN : $\omega_0 + \ldots + \omega_{n-1} = 0$.

· PRV-1 : $-z = \omega_0 \times \ldots \times \omega_{n-1}$

· $\omega^n - z = \prod_{k=0}^{n-1} (\omega - \omega_k)$

· $S_n = \frac{1 - a^{n+1}}{1 - a}$ lim
→ si $a \in]-1, 1[$: $\frac{1}{1-a}$
→ si $a > 1$: $+\infty$
→ si $a \leq -1$: n'a pas lim

· $|x| \geq x$ / $\geq -x$: tous les termes et négatifs.

· ⚠ Dj - Résolu° Equa°

Pptés : $\frac{d_b(\vec{u}+\vec{v})}{dt} = \frac{d_b\vec{u}}{dt} + \frac{d_b\vec{v}}{dt}$

$$\frac{d_b(\lambda\cdot\vec{v})}{dt} = \frac{d\lambda}{dt}\vec{v} + \lambda\cdot\frac{d_b\vec{v}}{dt}$$

$$\frac{d(\vec{u}\cdot\vec{v})}{dt} = \frac{d_b\vec{u}}{dt}\cdot\vec{v} + \vec{u}\cdot\frac{d_b\cdot\vec{v}}{dt}$$

$$\frac{d_b(\vec{u}\wedge\vec{v})}{dt} = \frac{d_b\vec{u}}{dt}\wedge\vec{v} + \vec{u}\wedge\frac{d_b\cdot\vec{v}}{dt}$$

Base de Référence & Base de Projecθ.

Base de réf{ce} : $b_0 = (\vec{x_0}, \vec{y_0}, \vec{z_0})$

Base de projecθ ou mobile : $b_1 = (\vec{u}, \vec{v}, \vec{z_0})$.

Dérivaθ Vectorielle

$$\vec{u} = u_1\cdot\vec{e_1} + u_2\cdot\vec{e_2} + u_3\cdot\vec{e_3}$$

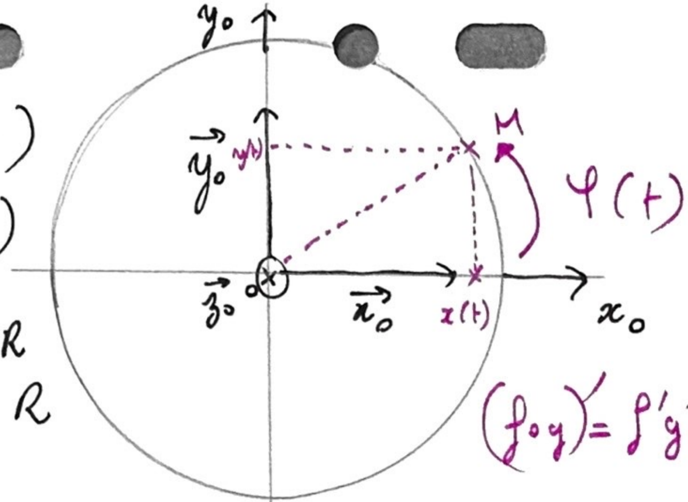
$$\frac{d_b\vec{u}}{dt} = \dot{u}_1\cdot\vec{e_1} + \dot{u}_2\cdot\vec{e_2} + \dot{u}_3\cdot\vec{e_3}$$

4

$R_0 = (O, b_0, t)$

$b_0 = (\vec{x_0}, \vec{y_0}, \vec{z_0})$

$\begin{cases} x(t) = \cos(\varphi(t)).R \\ y(t) = \sin(\varphi(t)).R \end{cases}$



$(f\circ g)' = f' g' \circ g$

$\vec{OM} = x(t).\vec{x_0} + y(t).\vec{y_0} = R.\cos(\varphi(t)).\vec{x_0} + R.\sin(\varphi(t)).\vec{y_0}$

$\vec{V}(M/R_0) = R.\dfrac{d_{b_0}}{dt}\left(\cos\varphi(t).\vec{x_0} + \sin\varphi(t).\vec{y_0}\right).$

$\vec{V}(M/R_0) = R.\dfrac{d_{b_0}}{dt}\left(\overbrace{\cos}^{f}(\overbrace{\varphi(t)}^{g}).\vec{x_0}\right) + \dfrac{R\,d_{b_0}}{dt}\left(\sin(\varphi(t)).\vec{y_0}\right)$

$\vec{V}(M/R_0) = -R.\dot{\varphi}.\sin(\varphi t).\vec{x_0} + R.\cos(\varphi t).\dot{\varphi}.\vec{y_0}$

$\vec{V}(M/R_0) = R.\dot{\varphi}\left(-\sin(\varphi t).\vec{x_0} + \cos(\varphi t).\vec{y_0}\right)$

$\vec{u} = \cos\varphi.\vec{x_0} + \sin\varphi.\vec{y_0}$ | $\vec{v} = -\sin\varphi.\vec{x_0} + \cos\varphi.\vec{y_0}$

$b_1 = (\vec{u}, \vec{v}, \vec{z_0})$ $\quad \vec{V}(M/R_0) = R.\dot{\varphi}.\vec{v}$

$\vec{\Gamma}(M/R_0) = \dfrac{d_{b_0}(\vec{V}(M/R_0))}{dt} = \dfrac{d_{b_0}}{dt}(R.\dot{\varphi}.\vec{v})$

$\vec{\Gamma}(M/R_0) = R\left[\dfrac{d_{b_0}}{dt}\dot{\varphi}\,\vec{v} + \dot{\varphi}\,\dfrac{d_{b_0}.\vec{v}}{dt}\right]$

$\vec{\Gamma}(M/R_0) = R\left[\ddot{\varphi}\,\vec{v} + \dot{\varphi}\,\dfrac{d_{b_0}.\vec{v}}{dt}\right]$ $\quad (f.g)' = f'g + g'f$

Avec $\dfrac{d_{b_0}\vec{v}}{dt} = \dfrac{d_{b_0}}{dt}\left(-\sin\varphi(t).\vec{x_0} + \cos\varphi(t).\vec{y_0}\right)$

$\dfrac{d_{b_0}\vec{v}}{dt} = -\dot{\varphi}\cos\varphi(t).\vec{x_0} - \dot{\varphi}\sin\varphi(t).\vec{y_0}$

$\vec{\Gamma}(M/R_0) = R\left[\ddot{\varphi}\,\vec{v} + \dot{\varphi}\left(-\dot{\varphi}\cos(\varphi(t)).\vec{x_0} - \dot{\varphi}\sin\varphi t.\vec{y_0}\right)\right]$

$\vec{\Gamma}(M/R_0) = R\left[\ddot{\varphi}\left(-\sin\varphi.\vec{x_0} + \cos\varphi.\vec{y_0}\right) + \dot{\varphi}\left(-\dot{\varphi}\cos(\varphi t).\vec{x_0} - \dot{\varphi}\sin\varphi t.\vec{y_0}\right)\right]$

$\vec{\Gamma}(M/R_0) = R\left[\left(-\ddot{\varphi}\sin\varphi - \dot{\varphi}^2\cos\varphi\right).\vec{x_0} + \left(\ddot{\varphi}\cos\varphi - \dot{\varphi}^2\sin\varphi\right).\vec{y_0}\right]$

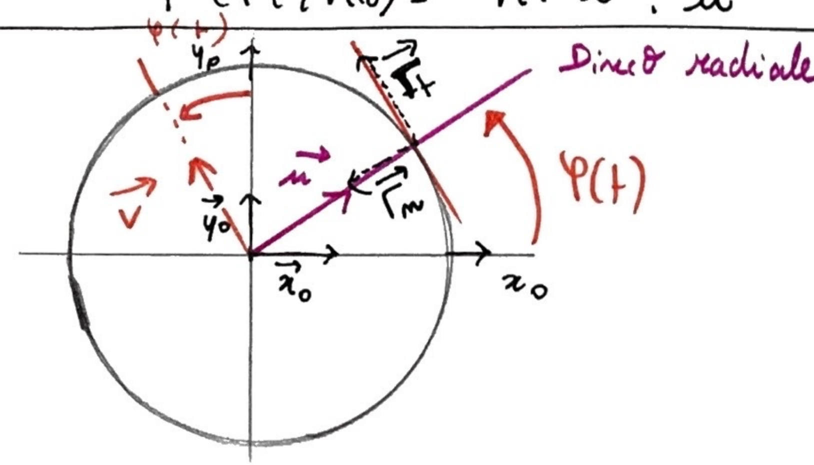
$\vec{\Gamma}(M/R_0) = R\left(\ddot{\varphi}\,\vec{v} - \dot{\varphi}^2\,\vec{u}\right)$

$\vec{\Gamma}(M/R_0) = R(\ddot{\varphi}\,\vec{v}) - R.\dot{\varphi}^2.\vec{u} = \vec{\Gamma_t} - \vec{\Gamma_n}$

$\vec{V}(M/R_0) = R.\dot{\varphi}\,\vec{v}$

$\dot{\varphi} = \omega = cte \Rightarrow \vec{V}(M/R_0) = R.\omega.\vec{v}$

$\vec{\Gamma}(M/R_0) = -R.\omega^2.\vec{u}$



Cas particuliers

1- Vecteur du module cte

Si $\vec{u}$ est module cte Alors $\dfrac{d_b \vec{u}}{dt} \perp \vec{u}$.

DM $\dfrac{d(\vec{u}.\vec{u})}{dt} = \vec{u} . \dfrac{d_b \vec{u}}{dt} + \dfrac{d_b \vec{u}}{dt} . \vec{u} = 2\vec{u} \dfrac{d_b \vec{u}}{dt} = 0.$

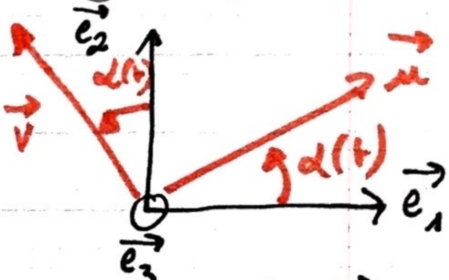
2- Vecteur de direcθ cte

Si $\vec{u}$ est direcθ cte Alors $\dfrac{d_b \vec{u}}{dt} \parallel \vec{u}$.

DM $\dfrac{d_b \vec{u}}{dt} = \dfrac{d\lambda(t)}{dt} \vec{u} + \lambda(t) . \underbrace{\dfrac{d_b \vec{u}}{dt}}_{=0} \Leftrightarrow \dfrac{d_b \vec{u}}{dt} = \dfrac{d\lambda(t)}{dt} . \vec{u}$

3- Vecteur unit^re en rotaθ autour axe fixe

$b_1 = (\vec{e_1}, \vec{e_2}, \vec{e_3})$, $b_2 = (\vec{u}, \vec{v}, \vec{e_3})$.



$\vec{u} = \cos\alpha(t) . \vec{e_1} + \sin\alpha(t) . \vec{e_2}$

$\dfrac{d_{b_1} \vec{u}}{dt} = -\dot{\alpha}(t) . \sin\alpha(t) . \vec{e_1} + \dot{\alpha}(t) . \cos\alpha(t) . \vec{e_2}$

$\dfrac{d_{b_1} \vec{u}}{dt} = \dot{\alpha}(t) \left[-\sin\alpha(t) . \vec{e_1} + \cos\alpha(t) . \vec{e_2} \right]$

$$\boxed{\dfrac{d_{b_1} \vec{u}}{dt} = \dot{\alpha}(t) . \vec{v}}$$

TH Vecteur UNITAIRE TOURNANT

1- Vecteur du module cte

Si $\vec{u}$ est module cte Alors $\frac{d_{\mathcal{B}}\vec{u}}{dt} \perp \vec{u}$.

DM $\frac{d(\vec{u}.\vec{u})}{dt} = \vec{u}.\frac{d_{\mathcal{B}}\vec{u}}{dt} + \frac{d_{\mathcal{B}}\vec{u}}{dt}.\vec{u} = 2\vec{u}\frac{d_{\mathcal{B}}\vec{u}}{dt} = 0.$

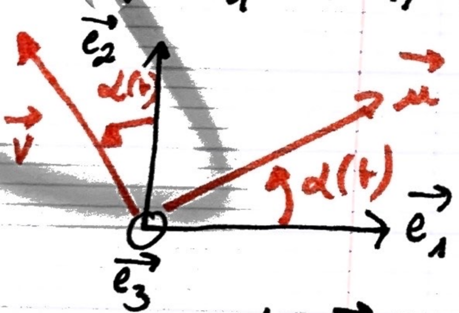
2- Vecteur de direcθ cte

Si $\vec{u}$ est direcθ cte Alors $\frac{d_{\mathcal{B}}\vec{u}}{dt} \parallel \vec{u}$.

DM $\frac{d_{\mathcal{B}}\vec{u}}{dt} = \frac{d\lambda(t)}{dt}\vec{u} + \lambda(t).\underbrace{\frac{d_{\mathcal{B}}\vec{u}}{dt}}_{=0} \Leftrightarrow \frac{d_{\mathcal{B}}\vec{u}}{dt} = \frac{d\lambda(t)}{dt}.\vec{u}$

3- Vecteur unit^R en rotaθ autour axe fixe

$\mathcal{B}_1 = (\vec{e_1}, \vec{e_2}, \vec{e_3})$, $\mathcal{B}_2 = (\vec{u}, \vec{v}, \vec{e_3})$.



$\vec{u} = \cos\alpha(t).\vec{e_1} + \sin\alpha(t).\vec{e_2}$

$\frac{d_{\mathcal{B}_1}\vec{u}}{dt} = -\dot{\alpha}(t).\sin\alpha(t).\vec{e_1} + \dot{\alpha}(t).\cos\alpha(t).\vec{e_2}$

$\frac{d_{\mathcal{B}_1}\vec{u}}{dt} = \dot{\alpha}(t)\,[-\sin\alpha(t).\vec{e_1} + \cos\alpha(t).\vec{e_2}]$

$$\boxed{\frac{d_{\mathcal{B}_1}\vec{u}}{dt} = \dot{\alpha}(t).\vec{v}}$$

TH Vecteur UNITAIRE TOURNANT

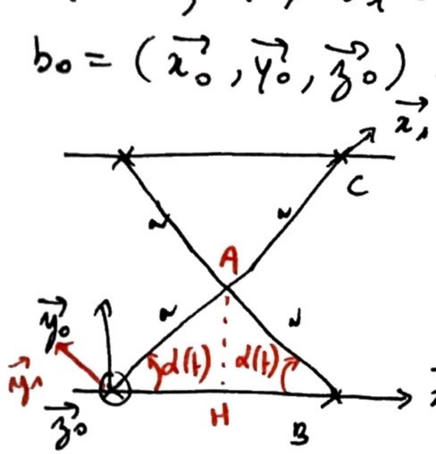
$$\frac{d_{\mathcal{B}_1}\vec{v}}{dt} = -\dot{\alpha}(t).\vec{u}$$

Ex6 Tube élévatrice

$R_0 = (O, b_0)$, $b_0 = (\vec{x_0}, \vec{y_0}, \vec{z_0})$

$R_1 = (O, b_1)$, $b_1 = (\vec{x_1}, \vec{y_1}, \vec{z_0})$

$b_0 = (\vec{x_0}, \vec{y_0}, \vec{z_0}) \xrightarrow{\alpha(t)} b_1 (\vec{x_1}, \vec{y_1}, \vec{z_0})$



$$\boxed{\vec{OB} = 2a.\cos\alpha(t).\vec{x_0}}$$

$$\vec{V}(B,R_0) = \frac{d_{b_0}\vec{OB}}{dt} = -2a.\dot\alpha.\sin\alpha.\vec{x_0}$$

$$\vec{\Gamma}(B/R_0) = \frac{d_{b_0}}{dt}(-2a\dot\alpha.\sin\alpha.\vec{x_0})$$

$(f g)' = f'g + g'f$

$$\boxed{\vec{\Gamma}(B/R_0) = -2a[\ddot\alpha.\sin\alpha + \dot\alpha^2.\cos\alpha.\vec{x_0}]}$$

$\vec{x_1} = \cos\alpha.\vec{x_0} + \sin\alpha.\vec{y_0}$

$\vec{y_1} = -\sin\alpha.\vec{x_0} + \cos\alpha.\vec{y_0}$

Dans R_1, $\boxed{\vec{OA} = a.\vec{x_1}}$

$$\boxed{\vec{V}(A/R_0) = \frac{d_{b_0}\vec{OA}}{dt} = a.\dot\alpha(t).\vec{y_1}}$$

D'après TH VU tournant

$$\vec{\Gamma}(A,R_0) = \frac{d_{b_1}\vec{V}}{dt} = a(\ddot\alpha(t).\vec{y_1} + \vec{y_1}.\dot\alpha(t))$$

$$\boxed{\vec{\Gamma}(A,R_0) = a(\ddot\alpha(t).\vec{y_1} - \dot\alpha^2(t).\vec{x_1})}$$

$\vec{OC} = 2\vec{OA}$ dc $\vec{V}(C/R_0) = \frac{2\, d_{b_0}\vec{OA}}{dt} = 2\vec{V}(A/R_0)$

$$\vec{V_T} = V_T.\vec{y_0} = \vec{V}(C/R_0).\vec{y_0} = 2a.\dot\alpha(t).\vec{y_1}.\vec{y_0}$$

$$\vec{V_T} = 2a.\dot\alpha(t)[-\sin\alpha(t).\vec{x_0} + \cos\alpha t.\vec{y_0}].\vec{y_0}$$

$$\boxed{\vec{V_T} = 2a.\dot\alpha(t).\cos\alpha(t)}$$

$h = 2a(\sin\alpha_{max} - \sin\alpha_{min})$

$T = \frac{\alpha_{max} - \alpha_{min}}{\dot\alpha} = \frac{L}{V}$

$\dot\alpha = cte$ (vitesse angul^R)

$V_{min} = 2a.\dot\alpha.\cos(\alpha_{max})$ $\quad$ $V_{max} = 2a\dot\alpha\cos(\alpha_{min})$

$V_{moy} = \frac{h}{T} = \frac{V_{min} + V_{max}}{2}$

Dérivation Composée

FF de Bout

$$\boxed{\frac{d_b \vec{X}}{dt} = \frac{d_\mathcal{E} \vec{X}}{dt} + \vec{\Omega}(\mathcal{E}/b) \wedge \vec{X}}$$

$b = (\vec{e_1}, \vec{e_2}, \vec{e_3})$; $\mathcal{E} = (\vec{\mathcal{E}_1}, \vec{\mathcal{E}_2}, \vec{\mathcal{E}_3})$

$\vec{\Omega}$: vecteur rotaθ instantanné

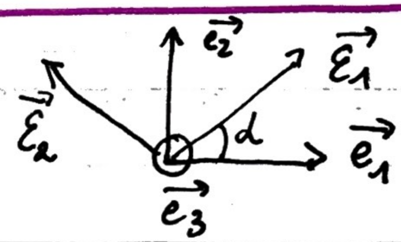
Si $\vec{X}$ cte ds base $\mathcal{E}$: $\frac{d_b \vec{X}}{dt} = \vec{\Omega}(\mathcal{E}/b) \wedge \vec{X}$.

Ptés : $\vec{\Omega}(b/b) = \vec{0}$ | $\vec{\Omega}(b_3/b_1) = \vec{\Omega}(b_3/b_2) + \vec{\Omega}(b_2/b_1)$

$\vec{\Omega}(b_1/b_2) = -\vec{\Omega}(b_2/b_1)$.

Rotaθ autour d'un axe : $\boxed{\vec{\Omega}(b_2/b_1) = \dot{\alpha}(t).\vec{e_3}}$

$b_1 = (\vec{e_1}, \vec{e_2}, \vec{e_3}) \xrightarrow{\alpha(t)} b_2 = (\vec{\mathcal{E}_1}, \vec{\mathcal{E}_2}, \vec{e_3})$



$$\frac{d_{b_1} \vec{X}}{dt} = \frac{d_{b_2} \vec{X}}{dt} + \vec{\Omega}(b_2/b_1) \wedge \vec{X}$$

Posons $\vec{X} = \vec{e_3}$

$$\underbrace{\frac{d_{b_1} \vec{e_3}}{dt}}_{\text{"0}} = \underbrace{\frac{d_{b_2} \vec{e_3}}{dt}}_{\text{0"}} + \vec{\Omega}(b_2/b_1) \wedge \vec{e_3}$$

$\Rightarrow \vec{\Omega}(b_2/b_1) \wedge \vec{e_3} = \vec{0}$

$\vec{\Omega}(b_2/b_1) = \lambda . \vec{e_3}$

- $\frac{d_{b_1} \vec{\mathcal{E}_1}}{dt} = \frac{d_{b_2} \vec{\mathcal{E}_1}}{dt} + \vec{\Omega}(b_2/b_1) \wedge \vec{\mathcal{E}_1}$: $\# = 0 + \vec{\Omega}(b_2/b_1) \wedge \vec{\mathcal{E}_1}$

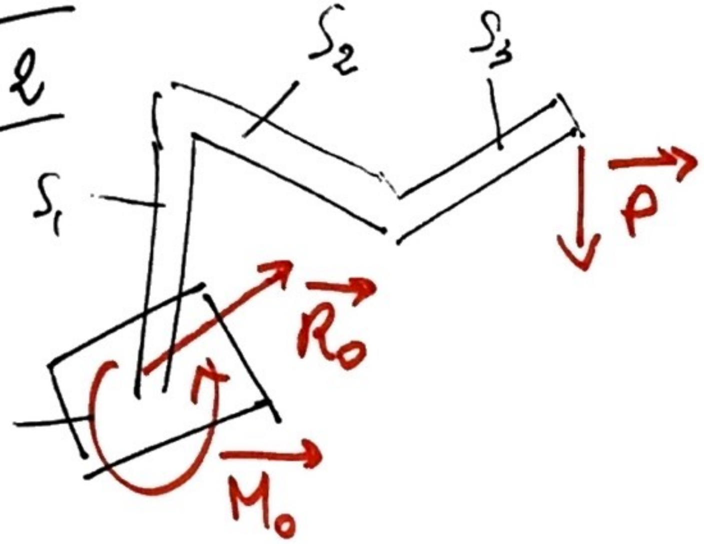
TH Vecteur Unitaire Tournant

$$\frac{d_{b_1} \vec{\mathcal{E}_1}}{dt} = \dot{\alpha} \vec{\mathcal{E}_2} = \vec{\Omega}(b_2/b_1) \wedge \vec{\mathcal{E}_1}$$

$$\dot{\alpha} \vec{\mathcal{E}_2} = \lambda . \vec{e_3} \wedge \vec{\mathcal{E}_1} = \lambda . \vec{\mathcal{E}_2}$$

$\Rightarrow \dot{\alpha} = \lambda$ $\Rightarrow$ $\boxed{\vec{\Omega}(b_2/b_1) = \dot{\alpha} . \vec{e_3}}$.

Ex 2



Isoler l'appareil

$\Updownarrow$

ce q se passe de sol

ou si $\exists$ bâti.

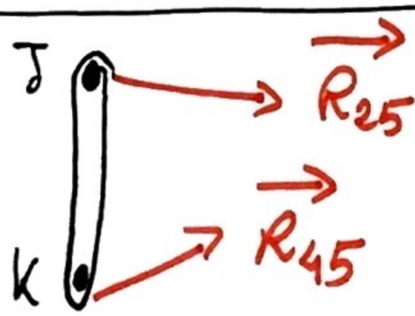
$$\vec{R_0} + \vec{P} = \vec{0}$$

$$\vec{R_0} = \begin{pmatrix} 0 \\ 0 \\ -P \end{pmatrix}$$

$$\vec{M_0} + \vec{OC} \wedge \vec{P} = \vec{0}$$

$$\vec{M_0} = -\vec{OC} \wedge \vec{P}$$

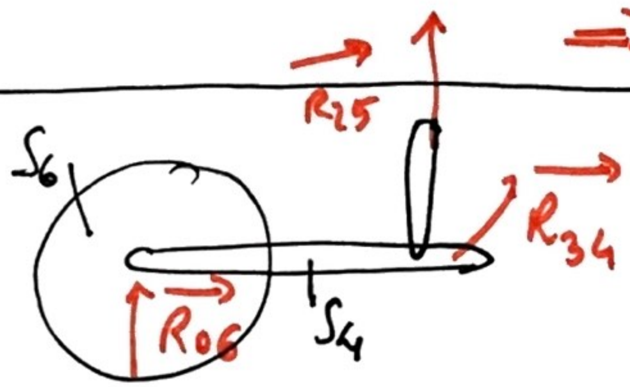
Ex 3



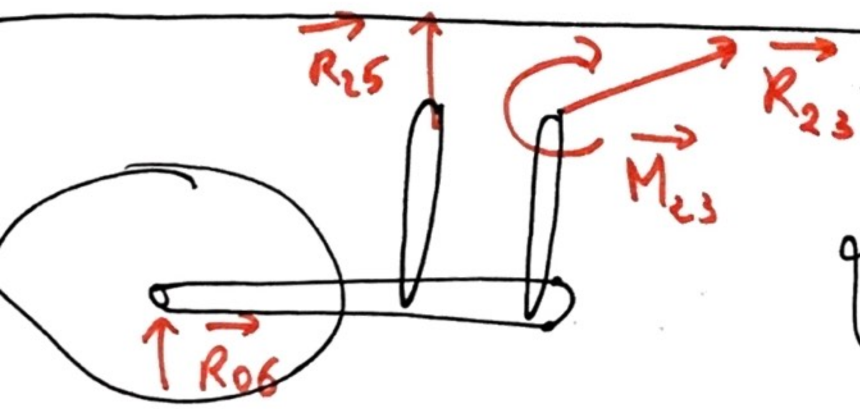
$$\vec{R_{25}} \left| \begin{matrix} R_{25x} \\ R_{25y} \\ 0 \end{matrix} \right. \qquad \vec{M_{25}} \left| \begin{matrix} 0 \\ 0 \\ 0 = M_{25z} \end{matrix} \right.$$

$$\vec{KJ} \wedge \vec{R_{25}} = \vec{0} \Rightarrow \vec{KJ} \,//\, \vec{R_{25}}$$

$$\Rightarrow X_{25} = 0$$



$$\begin{cases} \vec{R_{25}} + \vec{R_{06}} + \vec{R_{34}} = \vec{0} \\ \vec{EJ} \wedge \vec{R_{25}} + \vec{EA} \wedge \vec{R_{06}} = \vec{0} \end{cases}$$

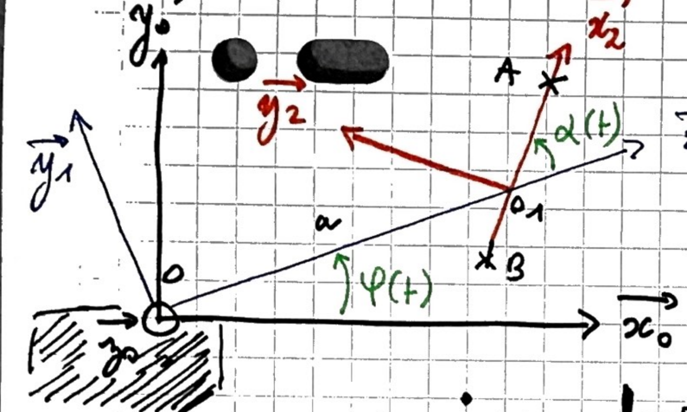


$$\begin{cases} \vec{R_{06}} + \vec{R_{25}} + \vec{R_{23}} = \vec{0} \\ \vec{M_{23}} + \vec{DA} \wedge \vec{R_{06}} + \vec{DJ} \wedge \vec{R_{25}} = \vec{0} \end{cases}$$

Isoler ens système :
Réac°s & Moments ⇔ sol. & bâti

DS.

$AB = 2a$ (1) **TH BOUT**



1) $\vec\Omega(b_1/b_0) = \dot\varphi(t)\,\vec z_0$ | $\vec\Omega(b_2/b_1) = \dot\alpha(t)\,\vec z_0$

$\vec\Omega(b_2/b_0) = \vec\Omega(b_2/b_1) + \vec\Omega(b_1/b_0) = (\dot\alpha + \dot\varphi)\,\vec z_0$

2) Vecteurs vitesses / accélérations :

* $\vec{OO_1} = \|\vec{OO_1}\|\,\vec x_1 = a\,\vec x_1$

$$\vec V(O_1/R_0) = \frac{d_{b_0}\vec{OO_1}}{dt} = \frac{d_{b_0}(a\vec x_1)}{dt} = a\,\frac{d_{b_0}\vec x_1}{dt}$$

$$\boxed{\vec V(O_1/R_0) = a\dot\varphi\,\vec y_1}$$

$$\vec\Gamma(O_1/R_0) = \frac{d\vec V(O_1/R_0)}{dt} = a\ddot\varphi\,\vec y_1 + a\dot\varphi\,\frac{d_{b_0}\vec y_1}{dt}$$

$$\vec\Gamma(O_1/R_0) = a\ddot\varphi\,\vec y_1 - a\dot\varphi^2\,\vec x_1$$

* $\vec{OA} = \vec{OO_1} + \vec{O_1A} = a\vec x_1 + r\vec x_2$

$$\vec V(A/R_0) = a\dot\varphi\,\vec y_1 + r\,\frac{d_{b_0}\vec x_2}{dt}$$

$$\frac{d_{b_0}\vec x_2}{dt} = \frac{d_{b_2}\vec x_2}{dt} + \vec\Omega(b_2/b_0)\wedge\vec x_2$$

$$\frac{d_{b_0}\vec x_2}{dt} = \vec\Omega(b_2/b_0)\wedge\vec x_2 = (\dot\alpha+\dot\varphi)\,\vec z_0\wedge\vec x_2 = (\dot\alpha+\dot\varphi)\,\vec y_2.$$

$$\vec V(A/R_0) = a\dot\varphi\,\vec y_1 + r(\dot\alpha+\dot\varphi)\,\vec y_2$$

$$\vec\Gamma(A/R_0) = \frac{d_{b_0}\vec V(A/R_0)}{dt}$$

$$\vec\Gamma(A/R_0) = a\ddot\varphi\,\vec y_1 + a\dot\varphi\,\frac{d_{b_0}\vec y_1}{dt} + r(\ddot\alpha+\ddot\varphi)\,\vec y_2 + r(\dot\alpha+\dot\varphi)\,\frac{d_{b_0}\vec y_2}{dt}$$

$$\vec\Gamma(A/R_0) = a\ddot\varphi\,\vec y_1 - a\dot\varphi^2\,\vec x_1 + r(\ddot\alpha+\ddot\varphi)\,\vec y_2 - r(\dot\alpha+\dot\varphi)^2\,\vec y_2$$

* Déterminer $\vec V$ & $\vec\Gamma$ p/r à R_1.

$\vec{OA} = \vec{OO_1} + \vec{O_1A} = a\vec x_1 + r\vec x_2$

$$\vec V(A/R_1) = r\,\frac{d_{b_0}\vec x_2}{dt} = r\,\dot\alpha\,\vec y_2$$

$$\vec\Gamma(A/R_1) = r\,[\ddot\alpha\,\vec y_2 - \dot\alpha^2\,\vec x_2]$$

* $\vec{O_1B} = \vec{OO_1} + \vec{O_1B} = \vec{OO_1} - \vec{O_1A}$

$$\vec V(B/R_1) = -r\,\dot\alpha\,\vec y_2.$$

$$\vec\Gamma(B/R_1) = -r\,[\ddot\alpha\,\vec y_2 + \dot\alpha^2\,\vec x_2].$$

$$\begin{cases} \dot{\varphi} = cte \\ \dot{\alpha} = cte \end{cases}$$

$$\vec{\Gamma}(A/R_0) = a\ddot{\varphi}\vec{y_1} - a\dot{\varphi}^2\vec{x_1} + x(\ddot{\alpha}+\ddot{\varphi})\vec{y_2} - x(\dot{\alpha}+\dot{\varphi})^2\vec{x_2}.$$

$$\vec{\Gamma}(A/R_0) = -a\dot{\varphi}^2\vec{x_1} - x(\dot{\alpha}+\dot{\varphi})^2\vec{x_2}$$

$$\|\vec{\Gamma_A}\| = \sqrt{\vec{\Gamma_A}\cdot\vec{\Gamma_A}}$$

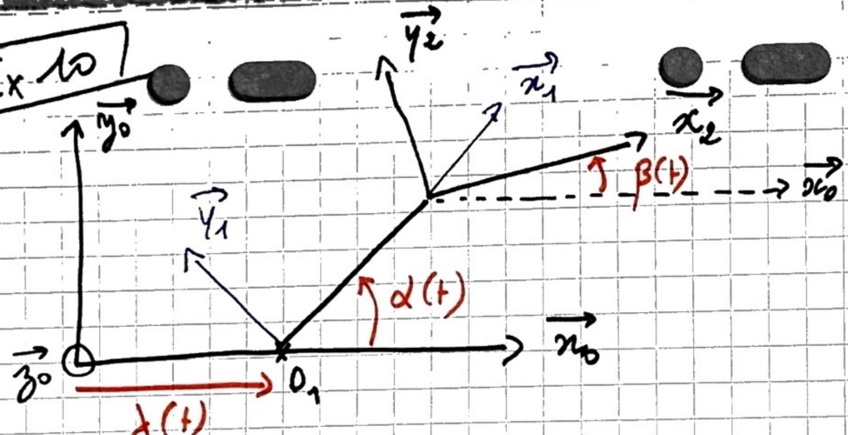
$$\|\vec{\Gamma_A}\| = \sqrt{(-a\dot{\varphi}^2\vec{x_1} - x(\dot{\alpha}+\dot{\varphi})^2\vec{x_2})(-a\dot{\varphi}^2\vec{x_1} - x(\dot{\alpha}+\dot{\varphi})^2\vec{x_2})}$$

$$\|\vec{\Gamma_A}\| = \sqrt{(a^2\dot{\varphi}^4\vec{x_1}\vec{x_1} + x^2(\dot{\alpha}+\dot{\varphi})^4\vec{x_2}\vec{x_2} + xa\dot{\varphi}^2(\dot{\alpha}+\dot{\varphi})^2\vec{x_1}\vec{x_2} + xa\dot{\varphi}^2(\dot{\alpha}+\dot{\varphi})^2\vec{x_1}\vec{x_2}}$$

$$\|\vec{\Gamma_A}\| = \sqrt{a^2\dot{\varphi}^4 + x^2(\dot{\alpha}+\dot{\varphi})^4 + 2ax\,\dot{\varphi}^2(\dot{\alpha}+\dot{\varphi})^2\cos\alpha}$$

$$\alpha = 0 \Rightarrow \Gamma_{max} = a\dot{\varphi}^2 + x(\dot{\alpha}+\dot{\varphi})^2$$

Ex 10



(*) $\vec{OO_1} = \|\vec{OO_1}\|.\vec{x_0} = \lambda(t).\vec{x_0}$

$\vec{V}(O_1/R_0) = \dot\lambda(t).\vec{x_0} + \lambda(t).\dot{\vec{x_0}} = \dot\lambda(t).\vec{x_0}$

$\vec{\Gamma}(O_1/R_0) = \ddot\lambda(t).\vec{x_0}$

$$\left\{T_c(S_1/R_0)\right\}_{O_1} = \left\{\begin{matrix}\vec{\Omega}(b_1/b_0)\\ \vec{V}(O_1/R_0)\end{matrix}\right\}_{O_1} = \left\{\begin{matrix}\dot\alpha(t).\vec{z_0}\\ \dot\lambda(t).\vec{x_0}\end{matrix}\right\}_{O_1}$$

$\vec{V}(A/R_0) = \vec{V}(O_1/R_0) + \vec{\Omega}(b_1/b_0) \wedge \vec{O_1A}$

$\vec{V}(A/R_0) = \dot\lambda(t).\vec{x_0} + a\dot\alpha\,\vec{y_1}$

$$\left\{T_c(S_2/R_0)\right\}_A = \left\{\begin{matrix}\vec{\Omega}(b_2/b_0)\\ \vec{V}(A/R_0)\end{matrix}\right\}_A = \left\{\begin{matrix}\dot\beta(t).\vec{z_0}\\ \dot\lambda(t).\vec{x_0} + a\dot\alpha\,\vec{y_1}\end{matrix}\right\}$$

$\vec{V}(B/R_0) = \vec{V}(A/R_0) + \vec{\Omega}(b_2/b_0) \wedge \vec{AB}$

$\vec{V}(B/R_0) = \dot\lambda(t).\vec{x_0} + a\dot\alpha\,\vec{y_0} + c\dot\beta\,\vec{y_2}$

$$\left\{T_c(S_2/R_1)\right\} = \left\{\begin{matrix}\vec{\Omega}(b_2/b_1)\\ \vec{V}(A/R_1)\end{matrix}\right\}$$

$\vec{\Omega}(b_2/b_0) = \vec{\Omega}(b_2/b_1) + \vec{\Omega}(b_1/b_0)$

$\dot\beta\,\vec{z_0} = \vec{\Omega}(b_2/b_1) + \dot\alpha\,\vec{z_0}$

$\vec{\Omega}(b_2/b_1) = (\dot\beta - \dot\alpha)\vec{z_0}$

$\vec{V}(A/R_1) = \vec{0}$ (car dérivée $a\vec{x_1} = \vec{0}$)

Éléments de la cinématique du solide

$$\boxed{\vec{V}(B/R_0) = \vec{V}(A/R_0) + \vec{\Omega}(b_1/b_0) \wedge \vec{AB}}$$ FFCS

DÉMO

formule fond cinémtq Solide

⊛ Torseur cinématique

$$\vec{H}(B) = \vec{H}(A) + \vec{S} \wedge \vec{AB}$$

champ antisymétriq

vecteur indépendant point

$$\{F\}_B = \begin{Bmatrix} \vec{S} \\ \vec{H}(B) \end{Bmatrix}_B$$

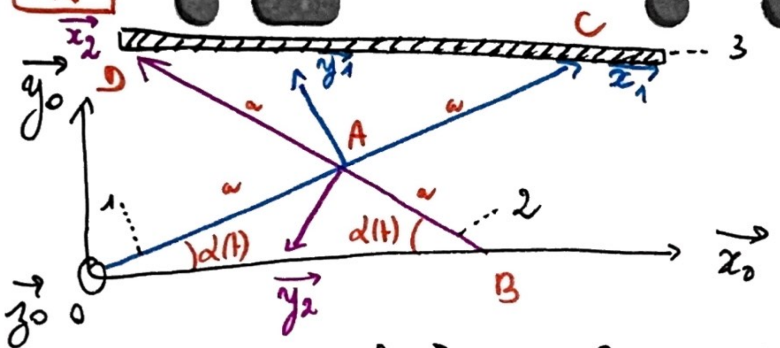
$$\boxed{\{T_C(S_1/R_0)\}_K = \begin{Bmatrix} \vec{\Omega}(b_1/b_0) \\ \vec{V}(K/R_0) \end{Bmatrix}_K}$$

⊛ **Solide au repos** : $\{T_C\} = \begin{Bmatrix} \vec{0} \\ \vec{0} \end{Bmatrix}$

⊛ **Mvt translaᵒ** : $\{T_C\}_K = \begin{Bmatrix} \vec{0} \\ \vec{V}(K/R_0) \end{Bmatrix}_K$

⊛ **Rotaᵒ autour axe** : $\{T_C(S_1/R_0)\}_K = \begin{Bmatrix} \vec{\Omega}(S_1/R_0) \\ \vec{0} \end{Bmatrix}_K$

12



1) $\left\{T_C(S_1/R_0)\right\}_O = \left\{\begin{array}{l}\vec{\Omega}(b_1/b_0) \\ \vec{V}(O/R_0)\end{array}\right\}_O = \left\{\begin{array}{c}\dot{\alpha}.\vec{z_0} \\ \vec{0}\end{array}\right\}_O$

2) $\vec{V}(A/R_0) = \vec{V}(O/R_0) + \vec{\Omega}(b_1/b_0) \wedge \vec{OA}$

$\vec{V}(A/R_0) = \vec{0} + \dot{\alpha}\,\vec{z_0} \wedge a\,\vec{x_1} = a\dot{\alpha}\,\vec{y_1}$.

$\vec{V}(B/R_0) = \vec{V}(A/R_0) + \vec{\Omega}(b_2/b_0) \wedge \vec{AB}$

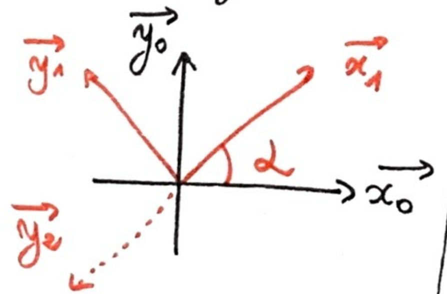
$\vec{V}(B/R_0) = a\dot{\alpha}\,\vec{y_1} - \dot{\alpha}\,\vec{z_0} \wedge -a\,\vec{x_2} = a\dot{\alpha}\,\vec{y_1} + \vec{y_2}$

$\vec{V}(D/R_0) = \vec{V}(A/R_0) + \vec{\Omega}(b_2/b_0) \wedge \vec{AD}$

$\left\{T_C(S_2/R_0)\right\}_A = \left\{\begin{array}{l}\vec{\Omega}(b_2/b_0) \\ \vec{V}(A/R_0)\end{array}\right\}_A = \left\{\begin{array}{c}\boxed{-\dot{\alpha}.\vec{z_0}} \\ a\dot{\alpha}\,\vec{y_1}\end{array}\right\}_A$ inclinaison entre $\vec{x_0}$ et $\vec{x_2}$ $(\pi-\alpha)$

$\vec{y_1} = -\sin\alpha.\vec{x_0} + \cos\alpha.\vec{y_0}$

$\vec{y_2} = -\sin\alpha.\vec{x_0} - \cos\alpha.\vec{y_0}$



$\vec{V}(B/R_0) = -2a\dot{\alpha}.\sin\alpha.\vec{x_0}$

$\vec{V}(D/R_0) = 2a\dot{\alpha}\cos\alpha\,\vec{y_0}$

5) $\left\{T_C(S_3/R_0)\right\}_D = \left\{\begin{array}{l}\vec{\Omega}(S_3/R_0) \\ \vec{V}(D/R_0)\end{array}\right\}_D = \left\{\begin{array}{c}\vec{0} \\ 2a\dot{\alpha}\cos\alpha\,\vec{y_0}\end{array}\right\}_D$

6) Nature du mvt:

Tige ①: circul$^{\circ}$ / Tige ②: mvt général (rota$^{\circ}$ & transla$^{\circ}$) / Tige ③: transla$^{\circ}$ rectiligne