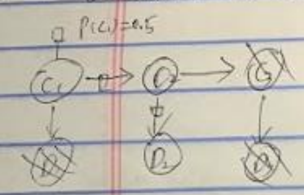
HW 7 SUNID: BENMA

1a.



$$p(C_2=1 \mid D_2=0) = \sum_{C_1} p(C_1)\, p(C_2=1 \mid C_1)\, p(D_2=0 \mid C_2=1)$$

$$= p(C_1=0)\, p(C_2=1 \mid C_1=0)\, p(D_2=0 \mid C_2=1)$$

$$+ p(C_1=1)\, p(C_2=1 \mid C_1=1)\, p(D_2=0 \mid C_2=1)$$

$$= 0.5\,\varepsilon\cdot\eta + 0.5\,(1-\varepsilon)\eta = 0.5\,\eta$$

$$p(C_2=0 \mid D_2=0) = \sum_{C_1} p(C_1)\, p(C_2=0 \mid C_1)\, p(D_2=0 \mid C_2=0)$$

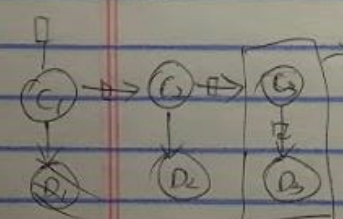
$$= p(C_1=0)\, p(C_2=0 \mid C_1=0)\, p(D_2=0 \mid C_2=0)$$

$$+ p(C_1=1)\, p(C_2=0 \mid C_1=1)\, p(D_2=0 \mid C_2=0)$$

$$= 0.5\,(1-\varepsilon)(1-\eta) + 0.5\,\varepsilon\,(1-\eta) = 0.5\,(1-\eta)$$

$$\mathbb{P}(C_2=1 \mid D_2=0) = \frac{p(C_2=1 \mid D_2=0)}{p(C_2=1 \mid D_2=0) + p(C_2=0 \mid D_2=0)} = \frac{0.5\eta}{0.5\eta + 0.5(1-\eta)} = \eta$$

1b.



$$f(C_2) = \sum_{C_3} p(C_3 \mid C_2)\, p(d_3=1 \mid C_3)$$

$$= p(C_3=0 \mid C_2)\, p(d_3=1 \mid C_3=0)$$

$$+ p(C_3=1 \mid C_2)\, p(d_3=1 \mid C_3=1)$$

$$= \begin{cases} (1-\varepsilon)\eta + \varepsilon(1-\eta) & \text{when } C_2=0 \\ \varepsilon\,\eta + (1-\varepsilon)(1-\eta) & \text{when } C_2=1 \end{cases}$$

$$p(C_2=1 \mid D_2=0, D_3=1) = p(C_2=1 \mid D_2=0)\, f(C_2) = 0.5\eta\,(\varepsilon\eta + (1-\varepsilon)(1-\eta))$$

$$p(C_2=0 \mid D_2=0, D_3=1) = p(C_2=0 \mid D_2=0)\, f(C_2) = 0.5(1-\eta)((1-\varepsilon)\eta + \varepsilon(1-\eta)),$$

$$\mathbb{P} = \frac{0.5\eta(\varepsilon\eta + (1-\varepsilon)(1-\eta))}{0.5\eta(\varepsilon\eta + (1-\varepsilon)(1-\eta)) + 0.5(1-\eta)((1-\varepsilon)\eta + \varepsilon(1-\eta))} = \frac{\varepsilon\eta^2 + (1-\varepsilon)(1-\eta)\eta}{\varepsilon\eta^2 + 2(1-\varepsilon)(1-\eta)\eta + (1-\eta)^2\varepsilon}$$

1 c i. $\mathbb{P}(C_2=1 \mid D_2=0) = \eta = 0.2$

$$\mathbb{P}(C_2=1 \mid D_2=0, D_3=1) = \frac{0.1 \times 0.2^2 + (1-0.1)(1-0.2) \times 0.2}{0.1 \times 0.2^2 + 2(1-0.1)(1-0.2) \times 0.2 + 0.1 \times (1-0.2)^2}$$

$$\approx 0.4157$$

ii When adding sensor $D_3 = 1$, it increase the probability that car was at positions 1 at $t_2 = 2$, when prior senor $D_2 = 0$ indicate that car is very low possibility to be at position 1 because ε is small (0.1) suggesting that car moves ~~slow~~ at low possibility

iii When $\mathbb{P}(C_2=1 \mid D_2=0) = \mathbb{P}(C_2=1 \mid D_2=0, D_3=1)$

$$\Rightarrow \eta = \frac{\varepsilon\eta^2 + (1-\varepsilon)(1-\eta)\eta}{\varepsilon\eta^2 + 2(1-\varepsilon)(1-\eta)\eta + \varepsilon(1-\eta)^2}$$

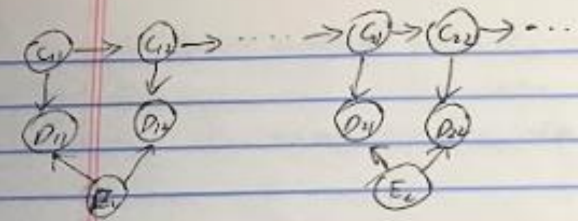
$$\Rightarrow \varepsilon\eta + (1-\varepsilon)(1-\eta) = \varepsilon\eta^2 + 2(1-\varepsilon)(1-\eta)\eta + \varepsilon(1-\eta)^2$$

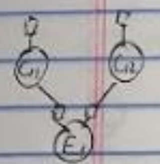
$$2\varepsilon(2\eta^2 - 3\eta + 1) = 2\eta^2 - 3\eta + 1$$

$$2\varepsilon = 1$$

$\Rightarrow \varepsilon = 0.5$ when car move at 0.5 ~~prob~~ probability ~~as~~ it does matter if the $D_3 = 1$ sensor read the position

5a.





$$\mathbb{P}(C_1 = c_{11} \mid E_1 = e_1) \Rightarrow p(c_{11}, c_{12} \mid e_1)$$

$$\mathbb{P}(C_{11} = c_{11} \mid E_1 = e_1) \propto p(c_{11}) p_N(e_{11}; \|a_1 - c_{11}\|, \sigma^2)$$

$$p(c_{11}, c_{12} \mid e_{11}, e_{12}) = p(c_{11}, c_{12})\, p(e_{11}, e_{12} \mid c_{11}, c_{12})$$

$$= p(d_{11} = e_{11} \mid c_{11})\, p(d_{12} = e_{12} \mid c_{12})$$
$$+\, p(d_{11} = e_{12} \mid c_{11})\, p(d_{12} = e_{11} \mid c_{12})$$

$$= p_N(e_{11}; \|a_1 - c_{11}\|, \sigma^2)\, p_N(e_{12}; \|a_1 - c_{12}\|, \sigma^2)$$
$$+\, p_N(e_{12}; \|a_1 - c_{11}\|, \sigma^2)\, p_N(e_{11}; \|a_1 - c_{12}\|, \sigma^2)$$

$$\mathbb{P}(C_{11}, C_{12} \mid E_1 = e_1) \propto p(c_{11}, c_{12} \mid e_1)$$
$$= p(c_{11})\, p(c_{12}) \big(p_N(e_{11}; \|a_1 - c_{11}\|, \sigma^2)\, (p_N(e_{12}; \|a_1 - c_{12}\|, \sigma^2)$$
$$+\, p_N(e_{12}; \|a_1 - c_{11}\|, \sigma^2)\, p_N(e_{11}; \|a_1 - c_{12}\|, \sigma^2) \big)$$

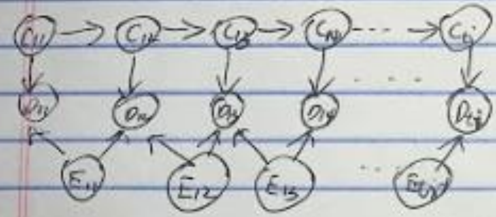
5b. $\mathbb{P}(C_{11} = c_{11}, \ldots, C_{1k} = c_{1k} \mid E_1 = e_1) \propto$ ~~$p(c_{11} = c_1) p(c_{12} = c_2)$~~ $p(c_{11}) p(c_{12}) \cdots p(c_{1k}) \times$

$$\cdot \underbrace{\sum \left(p_N(e_{11}; \|a_1 - c_{11}\|, \sigma^2) \cdots p_N(e_{1k}; \|a_k - c_{1k}\|, \sigma^2) \right)}_{\text{constant}}$$

so we will need to maximize $p(c_{11}) p(c_{12}) \cdots p(c_{1k})$, which has $k!$ permutations, so for all k cars ($c_{11} \cdots c_{1k}$) there's no specific observations, so maximize value of $\mathbb{P}(C_{11} = c_{11}, \cdots C_{1k} = c_{1k} \mid E_1 = e_1)$ is at lease $k!$ ways to permute

5C. for the factor graph of $\mathbb{P}(C_{11}=c_{11}\cdots C_{1K}=c_{1K}|E_1=e_1,\cdots E_T=e_T)$
since from $1, 2, \cdots T$ time steps, we can led to K motions of Variables elemination, so k is the tree width

5d



$X \in (1, 2 \cdots k)$, where $X \in$ Domain

weight$(x) = \prod_{i=1}^{k} f_i(x)$

$$P(C_{ti}|e_1 \cdots e_T) \propto f_1(C_t)\, p(e_T|C_{ti}) = \sum_{1}^{k} P(C_t)\, P(C_t|C_{t-1})\, P(e_t|C_{ti})$$
$$\propto \prod_{i=1}^{K} f(X_{C_{ti}}) \cdot f(X_{C_{ti-1}})$$

```
def p():
    for i in range(0,T)
        for j in range(1, k)
            z = p(C_t|C_{t-1})·p(C_{t+1}|C_{t-2})...p(C_{t+k}|C_{t+k-1})+P(e_t|C_t)
        return z
```

Therefore complexity is $O(k!T) \to O(T)$