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1a. $f(\theta) = \frac{1}{2} \sum_{i=1}^{n} w_i(\theta - x_i)^2{}' = 0$

$0 = \frac{1}{2} \times 2 \sum_{i=1}^{n} w_i(\theta - x_i)$

$\sum_{i=1}^{n} w_i \theta = \sum_{i=1}^{n} w_i x_i$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}}$$

Since $f(\theta)$ is quadratic functions, so

, $f(\theta) = 0$ minimum value when $\sum_{i=1}^{n} w_i \geq 0$

$f(\theta)$ has no minimum when $\sum_{i=1}^{n} w_i < 0$ only maxi-mum

1b. $f(x) = \sum_{i=1}^{d} \max_{s \in \{1,-1\}} s x_i = \sum_{i=1}^{d} |x_i|$

$g(x) = \max_{s \in \{1,-1\}} s \sum_{i=1}^{d} x_i = \left| \sum_{i=1}^{d} x_i \right|$

When $\forall x_i \geq 0$ or < 0, $f(x) = g(x)$

when $\exists x_i < 0 \land \exists x_i > 0$, $f(x) > g(x)$

Therefore, $f(x) \geq g(x)$

1c. For a fix number K, Event $\{X = k\}$, $P(X=k) = (\frac{5}{6})^{k-1} \cdot \frac{1}{6}$

Expected numer of roll till stop $= E|X| = \frac{1}{6} \sum_{k=1}^{\infty} k (\frac{5}{6})^{k-1}$

$\frac{5}{6} E|X| = \frac{1}{6} \sum_{k=1}^{\infty} k (\frac{5}{6})^{k} = \frac{1}{6} \sum_{k=2}^{\infty} (k-1)(\frac{5}{6})^{k-1} = \frac{1}{6} \sum_{k=2}^{\infty} k (\frac{5}{6})^{k-1} - \frac{1}{6} \sum_{k=2}^{\infty} (\frac{5}{6})^{k-1}$

$= E|X| - \frac{1}{6} - \frac{1}{6} \cdot \frac{5}{6} \cdot \frac{1}{1 - \frac{5}{6}}$

$= E|X| - \frac{1}{6} - \frac{5}{6}$

$= E|X| - 1$

so $E|X| = 6$, Total points $= E|X| \cdot (\frac{b}{6} - \frac{a}{6}) = b - a$

1d $L(P) = P^4(1-P)^3$

$\ln(L(P)) = \ln(P^4) + \ln(1-P)^3$

$= 4\ln(P) + 3\ln(1-P)$

$$\frac{d\ln(L(P))}{dP} = \frac{4}{P} - \frac{3}{1-P} = 0$$

when $P = \frac{4}{7}$, $L(P)$ maximum value

so 4 matches 4 H in {H, H, T, H, T, T, H} out of 7 times flips

$\boxed{P = 4/7}$ is exact same number of heads out of total flips to maximize the probabality.

1e. $\nabla f(w) = \sum_{i=1}^{n}\sum_{j=1}^{n} 2(a_i^T w - b_j^T w)(a_i^T - b_j^T) + 2\lambda \sum_{k=1}^{d} W_k^T$

2a. rectangle placement $= \frac{n(n+1)\, n(n+1)}{4} = \frac{n^2(n+1)^2}{4} \Rightarrow O(n^4)$

For one rectangle placement $= O(n^4)$

For Each 6 rectangl placements $= n^4 \times n^4 \times n^4 \times n^4 \times n^4 \times n^4 = \boxed{O(n^{24})}$

2b. Algorithm in dynamic programming, pseudo code below:

```
def MIN_d(i, j):
        if i==n and j=n:
            return 0
        elif i==n and j<>n;
            return j-1+n
        elif i<>n and j==n:
            return i-1+n
        else:
            for i in range(i, n+1):
                for j in range(j, n+1):
                    min_distance = min(min_distance, C(i,j)
                    + MIN_d(i,j))
            return min_distance
```

Therefore the complexity should be $\boxed{O(n^2)}$ with nested for loops

2C $n=1$ $f(n)=1$

$n=2$ $f(n)=2$

$n=3$ $f(n)=4$

$n=4$ $f(n)=8$

$n=5$ $f(n)=16$

$n=n$ $\vdots$ $f(n)=2^{n-1}$

so $f(n)=2^{n-1}$

$f(n)=\underbrace{1+1+1\cdots+1}_{n \text{ times}}$

there are $n-1$ plus signs between n 1s

there are 2^{n-1} ways of choosing where to split the sum, therefore $\boxed{2^{n-1}}$ possible sum

2d. $f(w)=\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i^T w-b_j^T w)^2+\lambda\|w\|_2^2$

$=\underbrace{w^2\sum_{i=1}^{n}\sum_{j=1}^{n}(a_i^T-b_j^T)^2}+\underbrace{\lambda\sum_{k=1}^{d}w_k^2}$

First Process for $O(nd^2)$ since w^2 has been preprocess, we just need $O(d^2)$ to process the second part (because $\sum_{k=1}^{d}\boxed{w^2}$ is preprocessed)