

A Modified Multi-objective Binary Particle Swarm Optimization Algorithm

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Abstract. In recent years a number of works have been done to extend Particle Swarm Optimization (PSO) to solve multi-objective optimization problems, but a few of them can be used to tackle binary-coded problems. In this paper, a novel modified multi-objective binary PSO (MMBPSO) algorithm is proposed for the better multi-objective optimization performance. A modified updating strategy is developed which is simpler and easier to implement compared with standard discrete binary PSO. The mutation operator and dissipation operator are introduced to improve the search ability and keep the diversity of algorithm. The experimental results on a set of multi-objective benchmark functions demonstrate that the proposed MMBPSO is a competitive multi-objective optimizer and outperforms the standard binary PSO algorithm in terms of convergence and diversity.

Keywords: Binary PSO, Multi-objective optimization, Pareto.

1 Introduction

Multi-objective optimization problems (MOPs), which have more than one objective function, are ubiquitous in science and engineering fields such as astronomy science, electronic engineering, automation, artificial intelligence. In MOPs, a unique optimal solution is hard to find due to the contradictory objectives. On the contrary, the ‘trade-off’ solutions, in other words, the non-dominated solutions are preferred. Several approaches have been proposed to deal with multi-objective optimization problems like reducing the problem dimension by combining all objectives into a single objective [1] or optimizing one while the rests being constrained [2]. However, these mentioned methods rely on a priori knowledge of the appropriate weights or constraint values. Furthermore, they are only capable of finding the individual point on the tradeoff curve for each problem solution. As a result, Pareto-based multi-objective methods, which optimize all objectives simultaneously and eliminate the need for determining appropriate weights or formulating constraints, have been current research hotspot. Pareto-based multi-objective methods operate on the concept of ‘Pareto domination’ [3] and the solutions on the curve of Pareto front represent the best possible compromises among the objectives [4]. So, one of the crucial goals in multi-objective optimization is to find a set of optimal solutions that distribute well along the Pareto front.

Particle Swarm Optimization (PSO) was firstly developed by Kennedy and Eberhart in 1995[6]. It is originated by imitating the behavior of a swarm of birds trying to search for food in an unknown area [5]. Owing to its simple arithmetic structure, high convergence speed and excellent global optimization ability, PSOs have been researched and improved to solve various multi-objective optimization problems. However, standard PSO and most of its improved versions work in continuous space, which mean they cannot tackle the binary-coded problems directly. To make up for it, Kennedy extended the PSO and proposed a novel discrete binary PSO (DBPSO) [6]. Based on DBPSO, researchers have introduced binary PSO to solve multi-objective problems. Abdul [8] proposed a multi-objective DBPSO, called BMPSO, to select the cluster head for lengthening the network lifetime and preventing network connectivity degradation. Peng and Xu [7] proposed a modified multi-objective binary PSO combining DBPSO with immune system to optimize the placement of the phasor measurement unit. These works prove that DBPSO-based multi-objective optimizers are efficient in solving MOPs. Nevertheless, the previous works on single objective optimization problems show that the optimization ability of DBPSO is not ideal [9], [10]. So we propose a novel modified multi-objective binary PSO (MMBPSO) in this paper to achieve the better multi-objective search ability and simplified the implementation of algorithm.

The rest of the paper is organized as follows. In Section 2, the brief introduction on DBPSO and a modified binary PSO algorithm is given first, and then the proposed MMBPSO algorithm are described in detail. Section 3 validates the MMBPSO with several benchmark problems, and the optimization performance and comparison are also illustrated. Finally, some concluding remarks are given in Section 4.

2 Modified Multi-objective Binary Particle Swarm Optimization

2.1 Standard Modified Binary Particle Swarm Optimization

Shen and Jiang [11] developed a modified binary PSO (MBPSO) algorithm for feature selection. In MBPSO, the updating formulas are demonstrated as Eq. (1-3).

$$X_i^{k+1} = X_i^k \quad (0 < V_i^{k+1} \leq \alpha) \quad (1)$$

$$X_i^{k+1} = P_i^k \quad (\alpha < V_i^{k+1} \leq (1 + \alpha)/2) \quad (2)$$

$$X_i^{k+1} = P_g \quad ((1 + \alpha)/2 < V_{ij}^{k+1} \leq 1) \quad (3)$$

The parameter α , called static probability, should be set properly. A small value of α can improve the convergent speed of the algorithm but makes MBPSO be trapped in the local optimum easily; while MBPSO with a big α may be ineffective as it cannot utilize the knowledge gained before well [9].

Although the update formulas of MBPSO and DBPSO are different, the updating strategy is still the same. In MBPSO, each particle still flies through the search space according to its past optimal experience and the global optimal information of the group. The Eq. (1) is an exhibition of inertia which represents the information that a particle inherited from its previous generation. The Eq. (2) represents particle's cognitive capability which draws the particle to its own best position. The Eq. (3) is the particle's social capacity which leads the particle to move to the best position found by the swarm [12].

2.2 Modified Multi-objective Binary Particle Swarm Optimization

Although MBPSO has been successfully adopted to solve various problems such as numerical optimization problem, feature selection and multidimensional knapsack problem, it is obvious that standard MBPSO cannot tackle Pareto-based multi-objective optimization problems. So we extend MBPSO and propose a novel modified multi-objective binary PSO.

2.2.1 Updating Operator

To achieve the good convergence and diversity performance simultaneously in multi-objective problem, it should very carefully counterpoise the global and local search capability. In original MBPSO, the control parameter α has not the function of adjusting the global search ability which spoils the performance of algorithm in MOPs. To make up of this drawback, we modified the updating operator as Eq. (4-6).

$$X_i^{k+1} = X_i^k \quad (0 < V_i^{k+1} \leq \alpha) \quad (4)$$

$$X_i^{k+1} = P_i^k \quad (\alpha < V_i^{k+1} \leq \beta) \quad (5)$$

$$X_i^{k+1} = P_g \quad (\beta < V_{ij}^{k+1} \leq 1) \quad (6)$$

Here the parameter β can adjust the probability of tracking the two different best solutions.

According to the Eq. (4-6), MMBPSO is easy to stick in the local optimal. For instance, if X_i , P_i and P_g are all equal to "1", X_i will be "1" forever and vice versa. So the dissipation operator and the mutation operator are introduced to keep the diversity and enhance the local search ability.

2.2.2 Dissipation Operator

The dissipation operator, as Eq. (7), is defined as randomly re-initializing a particle with some probability which brings the new particle into the swarm and retains the diversity effectively. Due to the characteristic of randomness, the probability of dissipation operating pd should not be a big value which may destroy the basic updating mechanism of algorithm. Usually pd is set between 0.05 and 0.15 to prevent the loss of optimal information.

$$X_i^{k+1} = \text{Reini}(X_i) \quad (\text{rand} < pd) \quad (7)$$

2.2.3 Mutation Operator

The dissipation operator can greatly improves the diversity, but it may destroy the useful information at the same time as it operates on the level of individual. To further enhance the optimization ability of algorithm, the mutation operation is also introduced which is defined as Eq. (8). Different from the dissipation operator, the mutation operator works on the level of bit with the probability pm . So mutation can improve the local search ability as well as keeping the diversity of algorithm.

$$\text{if rand} < pm \quad X_i^{k+1} = \begin{cases} 1, & \text{if } X_i^{k+1} = 0 \\ 0, & \text{if } X_i^{k+1} = 1 \end{cases} \quad (8)$$

2.2.4 The Updating of Personal Best and Global Best

Each particle is guided by its two best individuals, i.e., the personal best solution (P_i) and the global best solution of swarm (P_g) to perform the search. So the updating of P_i and P_g is very important for the optimization performance of algorithm. In Pareto-based MOPs, the goal of algorithm is to find the diverse non-dominated solutions laid in the Pareto front, which means that attention should be paid to the diversity as well as convergence when we design the algorithm. To realize this goal, the niche count [13] as a density measure is adopted in MMBPSO to select P_g .

Niche count is defined as the number of the particles in the niche. For example, σ_{share} is the niche which indicates the radius of the neighborhood in Figure 1. From Figure 1, we can find that particle B has 4 neighbors while particle A has 8 neighbors, that is, particle B has a less crowded niche than particle A. So particle B is superior to particle A in terms of diversity. In this work, the σ_{share} is calculated as Eq. (9).

$$\sigma_{share} = \frac{\max_2 + \max_1 - \min_2 - \min_1}{N - 1} \quad (9)$$

where \max_i and \min_i are the maximum and minimum values of the two objective functions, and N is the population size.

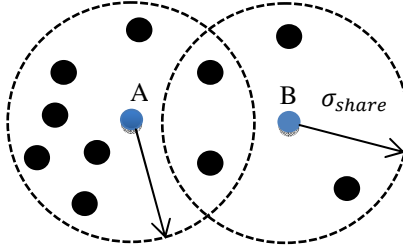


Fig. 1. An example of niche count

During each iteration process, the non-dominated solution set is sorted according to the niche count. P_g for each generation is randomly chosen among top 10% “less crowded” non-dominated particles in the set. To encourage MMBPSO to search the whole space and find more non-dominated solutions, P_i is replaced by the non-dominated current particle.

3 Experiments

3.1 Benchmark Functions and Performance Metrics

To test the performance of the proposed MMBPSO, five well-known benchmark functions, i.e., ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6 [15] are adopted in the paper. All problems have two objective functions and without any constraint. Multi-objective optimizer is designed to achieve two goals: 1) convergence to the Pareto-optimal set and 2) maintenance of diversity in solutions of the Pareto-optimal set. These two tasks cannot be measured adequately with one performance metric. So the

convergence metric Y proposed in [15] and the diversity metric S proposed in [14] are adopted to evaluate the performance of MMBPSO.

3.2 Experimental Results

MMBPSO was applied to optimize the 5 benchmark functions, and each function was run 30 times independently. For a comparison, BMPSO [8] and DBPSO with the same P_g and P_i updating strategy are also used to solve these benchmarks. The population size and the maximum generation of all algorithms are 200 and 250 respectively, and each decision variable is coded to 30 bits. The other parameter settings of 3 algorithms are shown in Table 1. The optimization results are listed in the Table 2-3 and are drawn in Fig.2 -3 as well.

The experimental results of convergence metric in Fig. 2 demonstrates that the proposed MMBPSO finds better solutions which are more closely related to the real Pareto front. For functions ZDT1, ZDT2, ZDT3 and ZDT6, MMBPSO has no difficulty to reach P^* ; but for function ZDT4, the performance of MMBPSO is not ideal due to the 21^9 different local Pareto-optimal fronts in the search space. However, MMBPSO has much better convergence values than BMPSO, MDBPSO in all 5 benchmark functions. The metrics of diversity drawn in Fig. 3 also show that MMBPSO is better than the other two algorithms on all functions.

To evaluate the performances of 3 algorithms more exactly and clearly, Fig. 4 plots the founded Pareto front of MMBPSO, MDBPSO and BMPSO on all functions, which displays that MMBPSO outperforms MDBPSO and BMPSO.

Table 1. Parameters settings of MMBPSO, MDBPSO and BMPSO

Algorithm	Parameters
MMBPSO	$\alpha=0.55, \beta=0.775, pd=0.1, pm=0.001;$
MDBPSO	$c_1 = 2.0, c_2 = 2.0, \omega = 0.8, v_i \in [-5,5];$
BMPSO [10]	$c_1 = 2.0, c_2 = 2.0, \omega = 0.8, v_i \in [-5,5].$

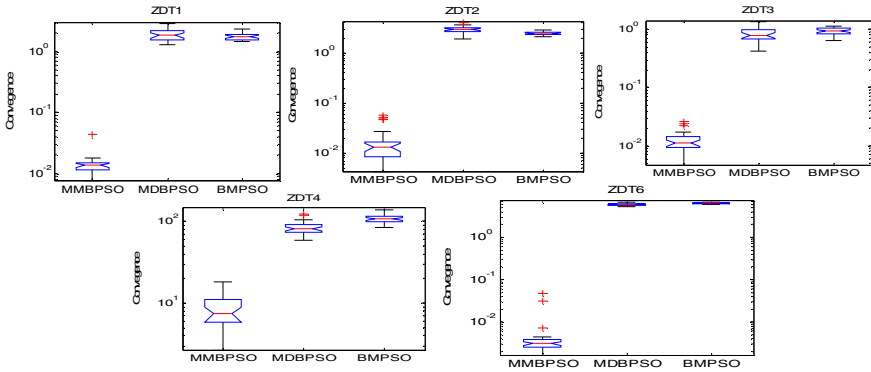


Fig. 2. Box plots of the convergence metric obtained by MMBPSO, MDBPSO and BMPSO

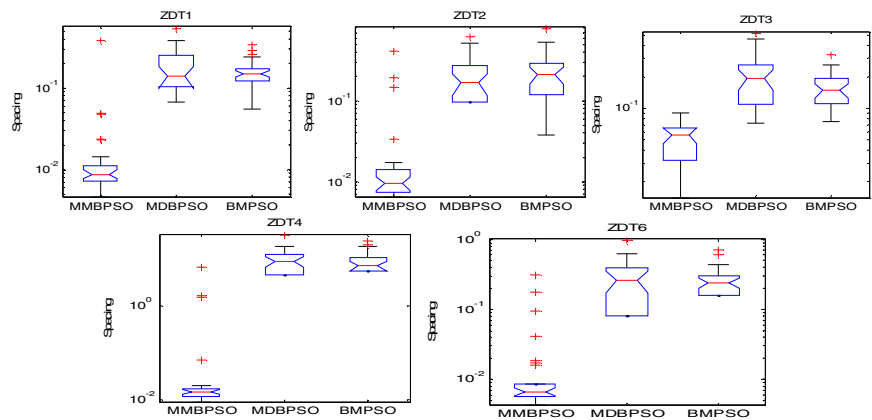


Fig. 3. Box plots of the distance metric obtained by MMBPSO, MDBPSO and BMPSO

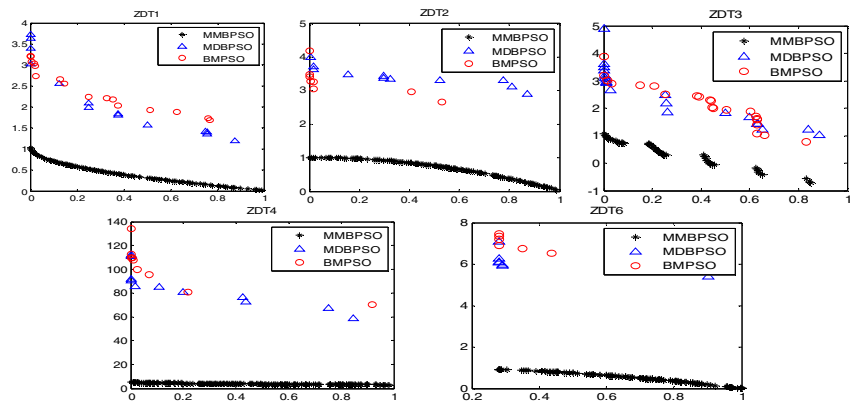


Fig. 4. The founded Pareto front of MMBPSO, MDBPSO and BMPSO on ZDT series functions

Table 2. The results of convergence metric γ

Algorithm		MMBPSO	MDBPSO	BMPSO
ZDT1	Mean	1.41E-02	1.88E+00	1.77E+00
	Variance	3.68E-05	1.60E-01	5.12E-02
ZDT2	Mean	1.74E-02	2.94E+00	2.45E+00
	Variance	2.12E-04	1.93E-01	4.23E-02
ZDT3	Mean	1.23E-02	8.26E-01	9.23E-01
	Variance	2.43E-05	5.91E-02	2.19E-02
ZDT4	Mean	8.39E+00	8.30E+01	1.08E+02
	Variance	1.44E+01	2.34E+02	1.49E+02
ZDT6	Mean	4.90E-03	6.08E+00	6.40E+00
	Variance	3.80E-05	1.37E-01	4.98E-02

Table 3. The results of diversity metric S

Algorithm		MMBPSO	MDBPSO	BMP SO
ZDT1	Mean	2.46E-02	1.82E-01	1.59E-01
	Variance	4.70E-03	1.20E-02	3.80E-03
ZDT2	Mean	3.48E-02	2.03E-01	2.26E-01
	Variance	6.80E-03	2.20E-02	2.31E-02
ZDT3	Mean	5.05E-02	2.08E-01	1.57E-01
	Variance	4.87E-04	1.33E-02	3.30E-03
ZDT4	Mean	3.92E-01	9.23E+00	8.35E+00
	Variance	1.59E+00	3.72E+01	2.83E+01
ZDT6	Mean	2.74E-02	2.71E-01	2.47E-01
	Variance	4.10E-03	4.17E-02	2.25E-02

4 Conclusion

In this paper, a novel multi-objective modified binary particle swarm optimization is proposed. Compared with DBPSO, the proposed MMBPSO developed an improving updating strategy which is simpler and more easily to implement. The mutation operator and dissipation operator are introduced to improve its search ability and keep the diversity of algorithm. The modified global best and local best solutions updating strategy help MMBPSO converge to the Pareto front better. Five well-known benchmark functions were adopted for testing the proposed algorithm. The experimental results proved that the proposed MMBPSO can find better solutions than MDBPSO and BMP SO. Especially, the superior of MMBPSO to MDBPSO demonstrated the advantages of the developed updating strategy in terms of convergence and diversity.

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