Lattice Boltzmann Methods for Meandering Rivers

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Subject matter and goal

We have studied the phenomenon of *meandering*, where rivers follow a series of dynamically changing twists and turns at the hand of the continuous forces of erosion and sedimentation[3]. Our research question came down to:

What causes the meandering of rivers?

We hypothesized meandering occurs because rivers flow faster in the outside bend than they do in the inside bend, as particles settle more easily in slow waters while being swept up more easily in fast waters.

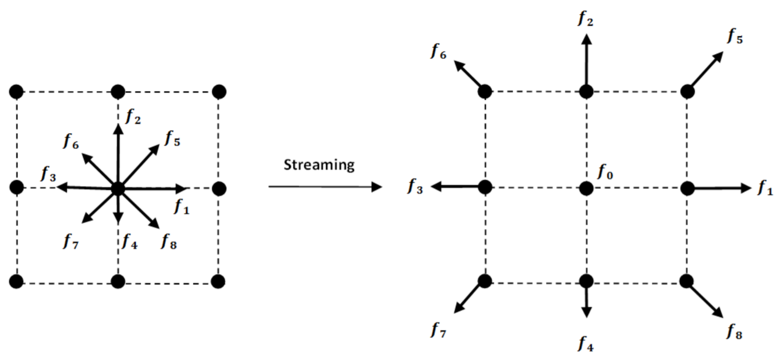
Additionally, we set out to study the correlation between the general speed of the river and its tendency to meander, hypothesizing that the phenomenon occurs more-so in slower rivers.

Figure 1: A meandering river.[5]

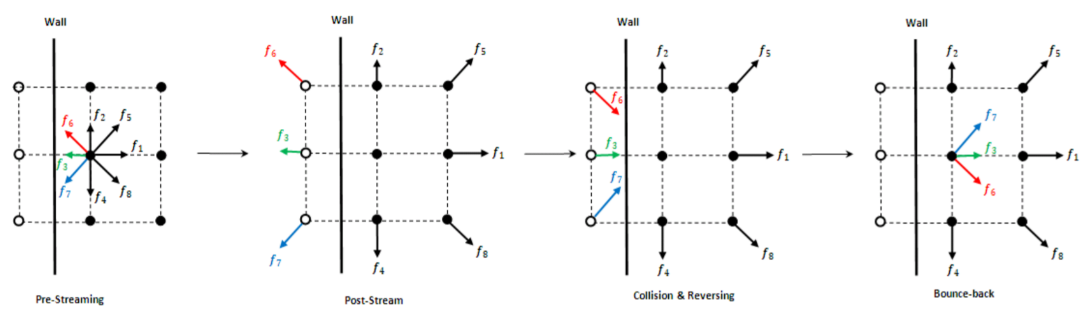
Lattice Boltzmann Method[1]

We chose to utilise the Lattice Boltzmann Methods (LBM), a relatively new and efficient set of lattice-based fluid simulation models, which we modified to include erosion and sedimentation.

The LBM roughly works as follows: Every lattice point $\vec{x}$ is assigned a vector $f \in \mathbb{R}^9$ representing the probabilities of a particle moving to any of the 8 neighbouring positions or not moving at all. These values are redistributed, or *streamed*, to the neighbours in question every time step t.



Any point marked as a wall will *reverse* all particles (or rather their distributions) that enter it.



The second step is the *collision* step, where we re-balance the stream using a local equilibrium function f^{eq}, representing the tendency of the fluid to converge to a state of balance.

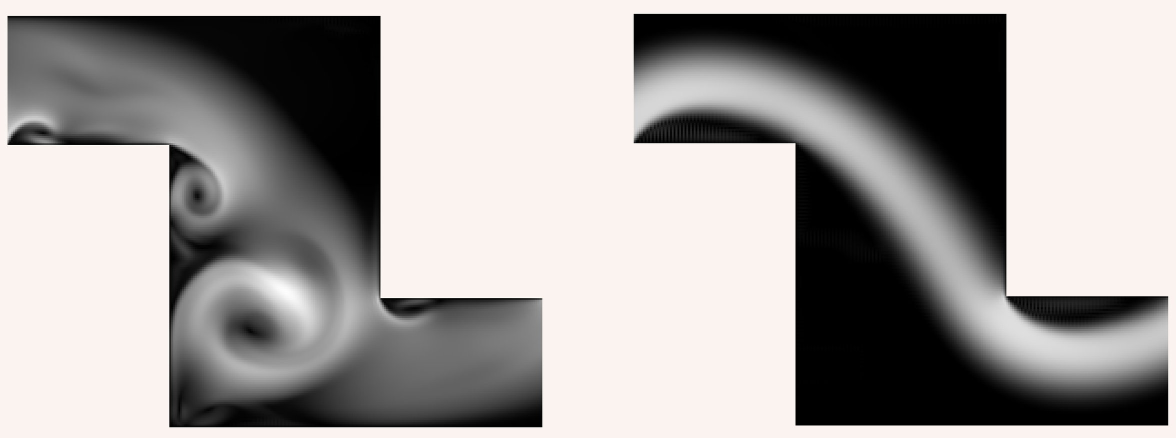


Figure 2: Simulated flow in a rectangular pipe for low (left) and high viscosity (right).

The vectors f for the next time step are based on a weighted average between the equilibrium state f^{eq} and the state after the streaming step. That weight is decided by the viscosity of the fluid. Think about how water or milk (low viscosity) flow more freely than oil or honey (high viscosity).

Validating our model

Validation was done by comparing fluid flow in a simple horizontal tube with Poisseuille flow, the theoretical solution for flow in such a tube. The horizontal flow speed is given by

$$u_x(y) = -\frac{\nabla P}{\nu}(h - y)y,$$

where ∇P is the pressure gradient and h the height of the tube. ν is the fluid viscosity.[4]

To simulate a pressure gradient using LBM we pivoted the grid velocities to the right every time step. ∇P was calculated using a least squares fit, which can be seen on the right. The small deviation in both velocities is most likely due to LBM being a grid-based approximation.

Furthermore, our model assumes fully Newtonian and incompressible fluids, as does the Poisseuille flow used here.

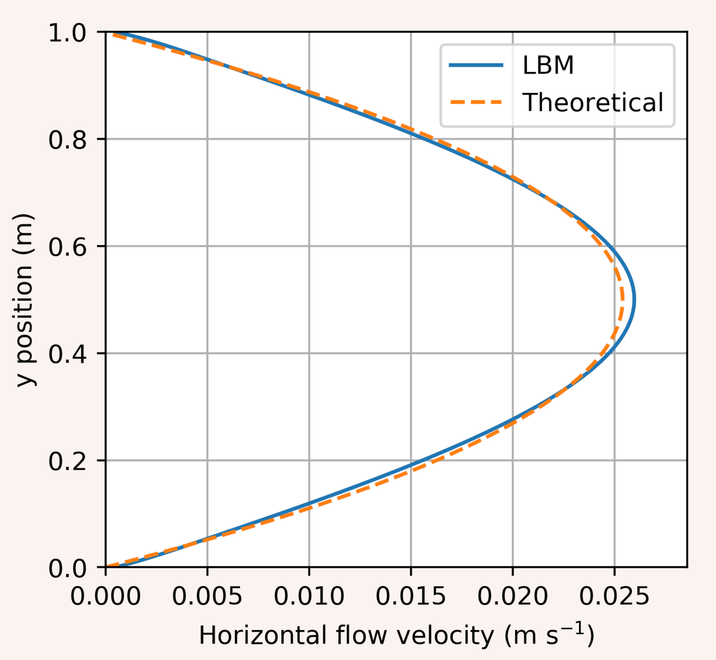
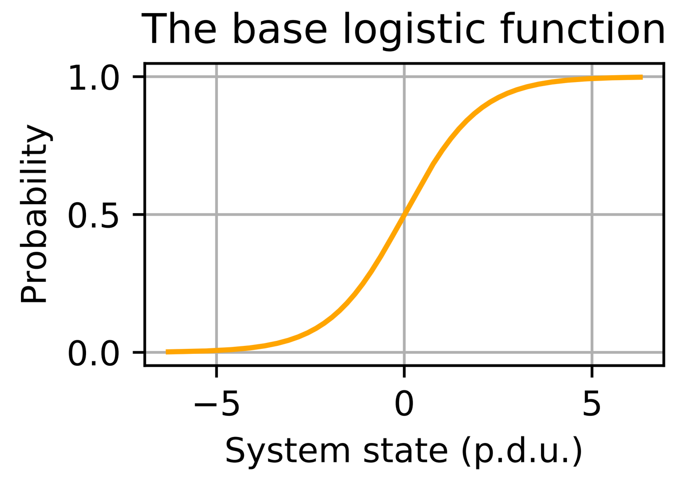


Figure 3: Fluid flow in a straight tube. Here an effective pressure gradient of $\nabla P \approx -0.001$ Pa m^{-1} was applied on a fluid with viscosity $\nu = 0.01$ Pa s.

Sedimentation and Erosion

Corrosion and sedimentation manifest as changes to the wall tiles on the grid, while keeping in mind the balance of the fluid in the system. These changes are based on relevant data from the system state.

In contrast to the *deterministic* nature of LBM, we decided to have these factors be based on random chance, based on a logistic activation function as seen below.



For sedimentation the logical choice was a negative correlation with the length of the *macroscopic velocity* $\vec{u}(\vec{x}, t)$, where

$$\vec{u}(\vec{x}, t) = \sum_i f_i(\vec{x}, t)\vec{e}_i.$$

Erosion on the other hand turned out to be more complicated. It ought to be based on the momentum of the fluid flow when colliding with a wall, but the so-called *no-slip condition*, which assumes fluid will have zero velocity at the boundary, makes this difficult to execute in practice. Multiple approaches have been attempted, among which the *momentum exchange*[2] $|F|$ with F defined as

$$F = \sum_i \vec{e}_i\big(f_i(\vec{x}, t+1) - f_{i^*}(\vec{x} + \vec{e}_i, t)\big)$$

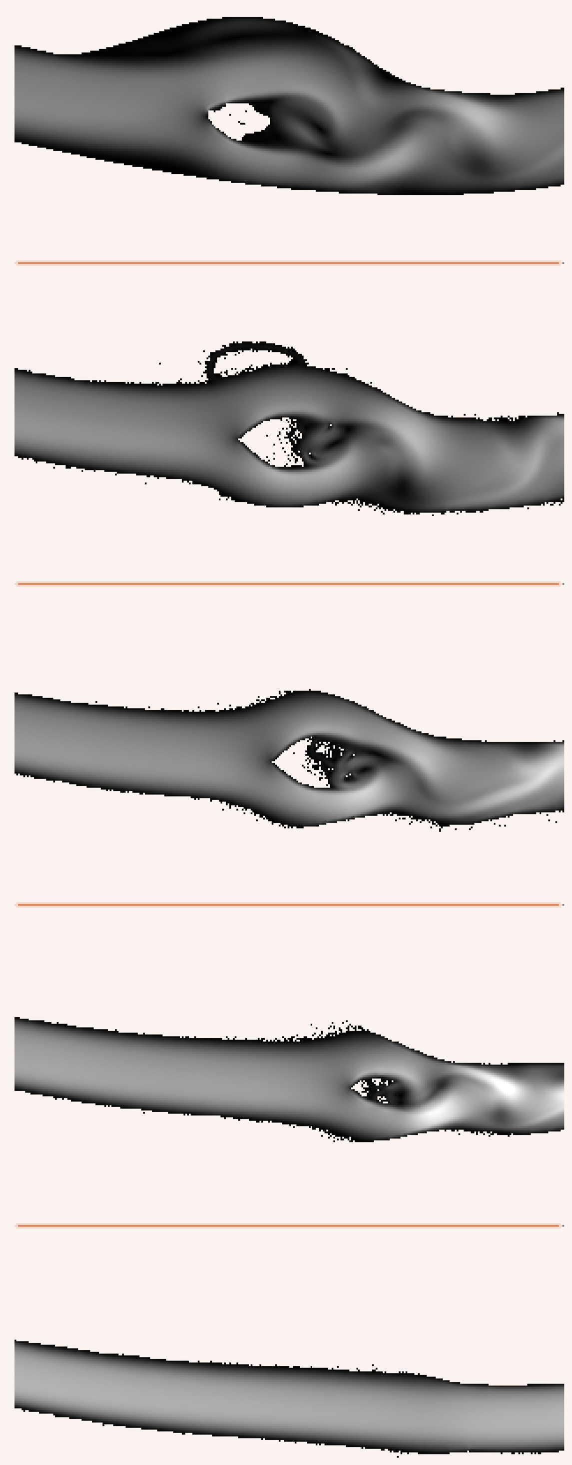


Figure 4: Effects of sedimentation and erosion within the model.

Results

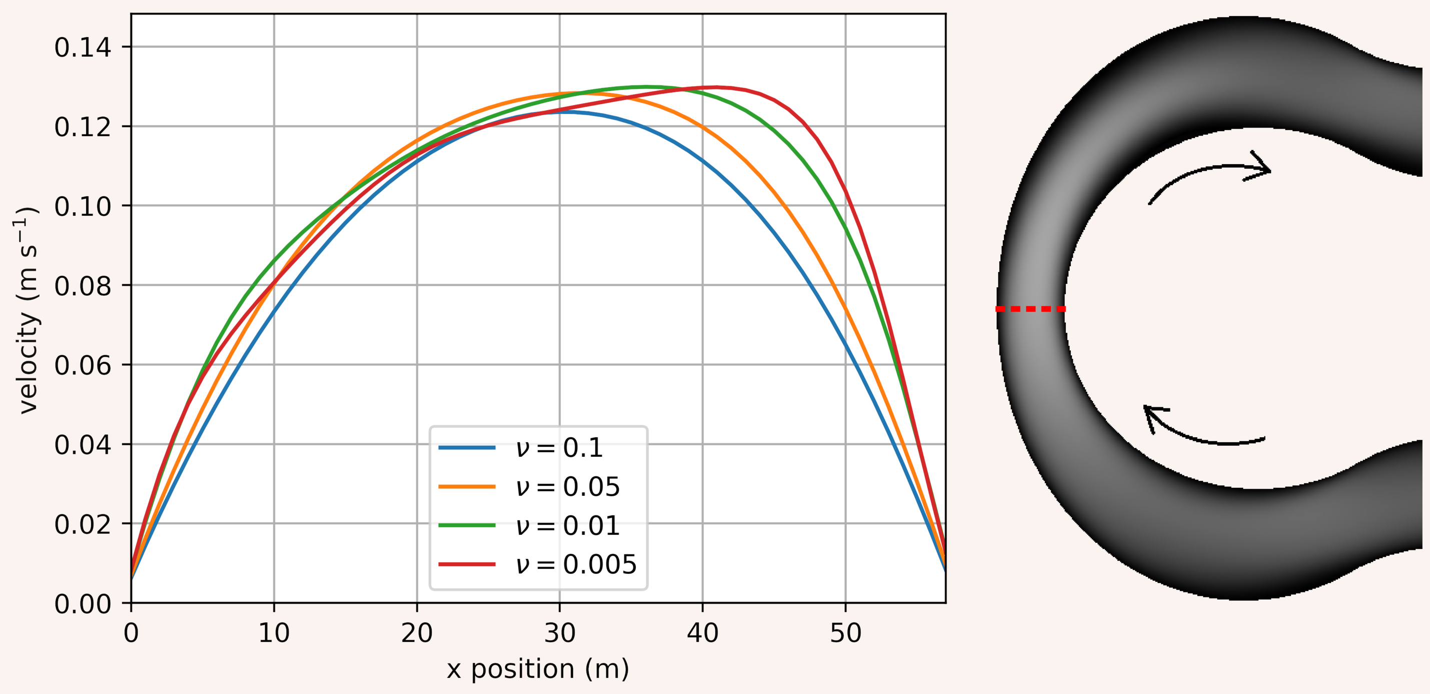


Figure 5: Upwards flow velocity over the x position for different viscosities. The sample position is indicated by the red line in the river shown on the right, where the flow is indicated by the arrows.

Unfortunately, we were not able to reproduce the phenomenon of meandering within our model. In fact, analysing the simulated river within a curve, as shown in Figure 5, revealed that the flow in the outer area of the bend was *less* strong than in the inner one, contradicting our hypothesis.

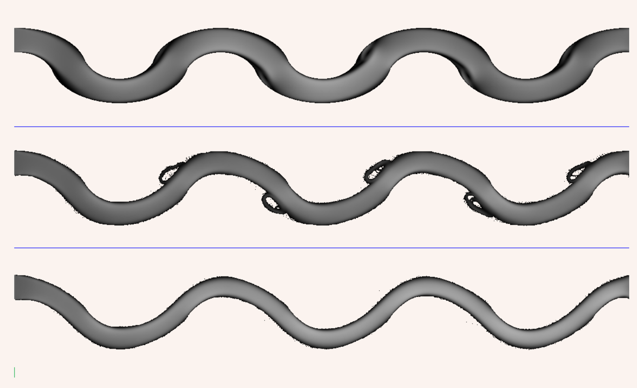


Figure 6: The evolution of a simulated river. It becomes smaller over time.

There are a few possible explanations as to why this happened. Firstly, our model is 2 dimensional. This means that it lacks any information on relevant factors such as the depth of the river or vertical flow.

Secondly, our implementation is numerically unstable at low viscosities, which limited us to relatively high viscosities in comparison to water, where $\nu = 10^{-4}$.

Sources

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2. A. Caiazzoa, M. Junk, *Boundary forces in lattice Boltzmann: Analysis of Momentum Exchange algorithm*, Sciencedirect, April 2008, Pages 1415-1423
3. R. Charlton, *Fundamentals Of Fluvial Geomorphology*, January 2007
4. D. Darmofal, *Poiseuille Flow Through a Duct in 2-D*, MIT Aerodynamics Lecture Notes 2005
5. F. Lanting, *Meandering river, Tambopata, Peru*, Mint Images/Science Photo Library, www.sciencephoto.com (used without explicit permission)