A Generalized Language Model in Tensor Space

Tianjin University

Lipeng Zhang, Peng Zhang, Xindian Ma, Shuqin Gu,
Zhan Su, Dawei Song

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Outline

- **Motivation**
- Background
- TSLM basic representation
- Generalization
- Recursive Language Modeling
- Experiment

Motivation

- To construct a high-order based language model

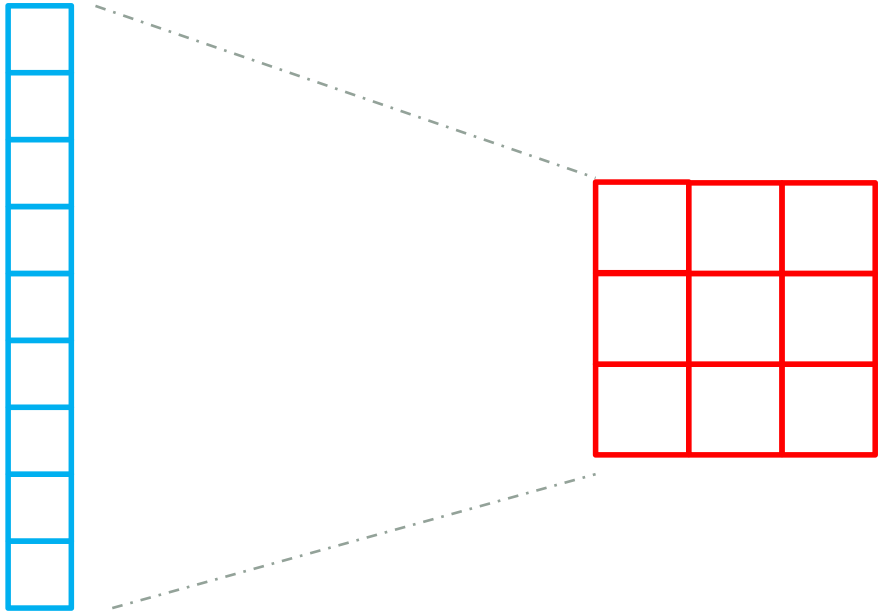
    (not limited to two/three consecutive words in 2/3-order tensor)

- To derive an effective solution and demonstrate such a solution is a general approach for language modeling

    (a high-order tensor contains exponential magnitude of parameters)

Motivation

- Represent the documents as the two order tensors:



For 10000-dimensional vector can be converted into 100×100

Cai, D.; He, X.; and Han, J. 2006. Tensor space model for document analysis. The 29th SIGIR conference on Research and development in information retrieval, 625–626

Motivation

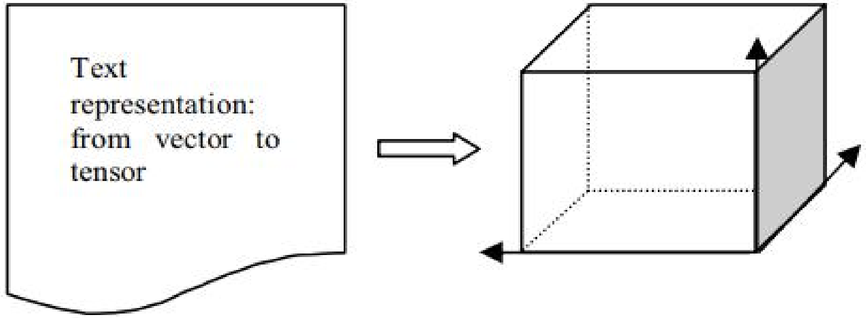


Figure 1. A document is represented as a character level 3-order tensor

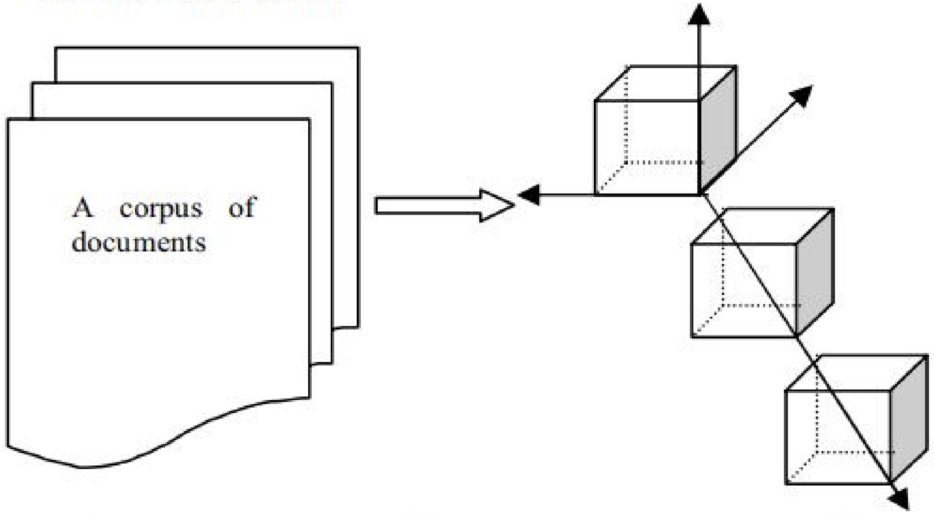


Figure 2. A corpus of documents is represented as a

Represent the text as a 3-order tensor

Liu, N.; Zhang, B.; Yan, J.; and Chen, Z. 2005. Text representation: from vector to tensor. In IEEE International Conference on Data Mining, 725–728

The problem of exponential magnitude of parameters

Tensor Network

Tensor decomposition

High-order tensor based language model

- Consider all the combinatorial relations among words through the interaction among all the dimensions of word vectors.
- Demonstrate that tensor representation is a generalization of the n-gram language model
- Derive a recursive calculation of conditional probability for language modeling via tensor decomposition in TSLM

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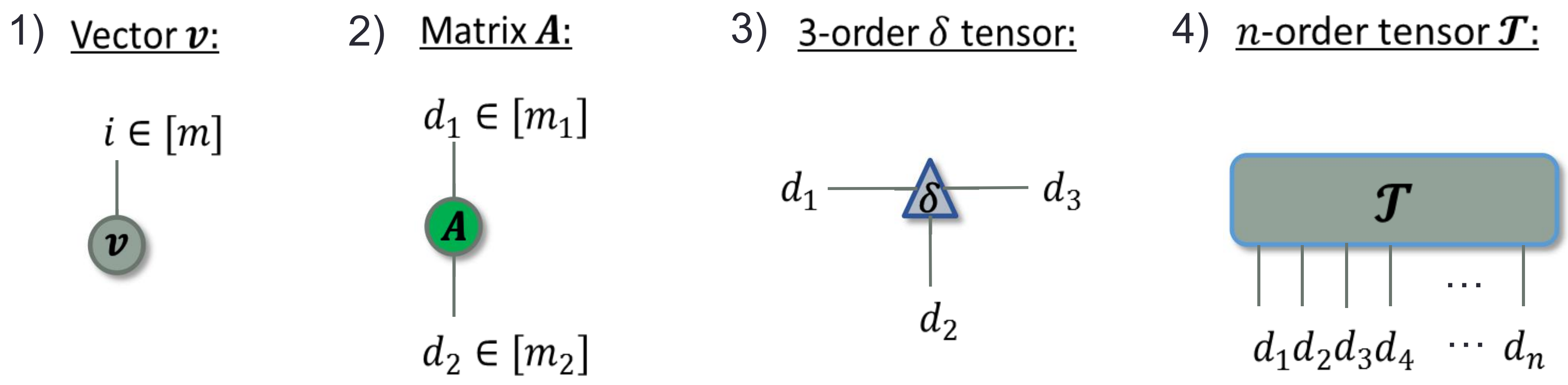
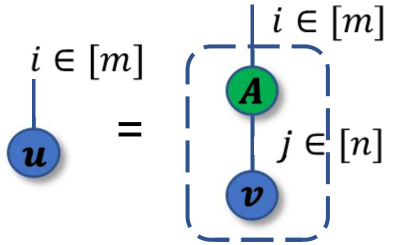
Background

- Tensor and Tensor Networks
  - A tensor : a mutidimensional array
  - The order : the number of indexing entries
  - Tensor product : a fundamental operator



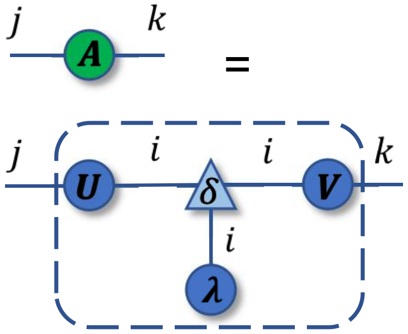
Tensor and Tensor Networks

- Tensor Network is formally represented an undirected and weight graph

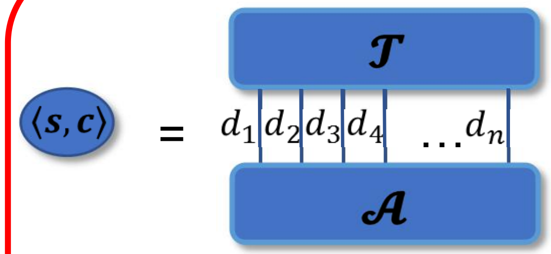


$$\boldsymbol{u} = \boldsymbol{A}\boldsymbol{v}$$

$$u_i = \sum_{j=1}^{n} A_{ij} v_j$$



$$\boldsymbol{A} = \sum_{i=1}^{r} \lambda_i \boldsymbol{u}_i \otimes \boldsymbol{v}_i$$



$$\langle \boldsymbol{s}, \boldsymbol{c} \rangle = \sum_{d_1,\ldots,d_n=1}^{m} \mathcal{T}_{d_1 \ldots d_n} \mathcal{A}_{d_1 \ldots d_n}$$

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Language Modeling by Tensor Space

- How to represent a single word

$$w_i = \sum_{d_i}^{m} \alpha_{i d_i} e_{d_i}$$

- How to represent a original sentence

$$s = w_1 \otimes \cdots \otimes w_n$$

$$s = \sum_{d_1,\cdots,d_n=1}^{m} \mathcal{A}_{d_1,\cdots,d_n} e_{d_1} \otimes \cdots \otimes e_{d_n}$$

Language Modeling by Tensor Space

- Assume that each sentence s_i appears with a probability p_i.
- We can denoted the corpus as:

$$c = \sum p_i s_i$$

$$c = \sum_{d_1 \dots d_n = 1}^{m} \mathcal{T}_{d_1 \dots d_n} \, e_{d_1} \otimes \cdots \otimes e_{d_n}$$

  - The sentence probability:

$$p(s) = \langle s, c \rangle = \sum_{d_1 \dots d_n = 1}^{m} \mathcal{T}_{d_1 \dots d_n} \mathcal{A}_{d_1 \dots d_n}$$

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A Generation of N-Gram Language Mode

- N-gram Language Model
  - N-gram language model: estimate the probability distribution of sentences
  - A sentence's joint probability:

$$p(s) = p(w_1^n)$$

$$p(w_1^n) = p(w_1) \prod_{i=2}^{n} p(w_i | w_1^{i-1})$$

N-Gram Language Model

- The conditional probability can be calculated as:

$$p(w_i|w_1^{i-1}) = \frac{p(w_1^i)}{p(w_1^{i-1})} \approx \frac{count(w_1^i)}{count(w_1^{i-1})}$$

Where the count denotes the frequency statistics in corpus.

How to Prove TSLM as a Generalization of N-Gram

- Three hypotheses
  - The dimension of vector space $m = |V|$
  - The represent of a word is an one-hot vector
  - The corpus:

$$c = \sum p_i |s_i\rangle$$

Compute the joint probability

- N-gram Language Model

  A sentence 's joint probability :

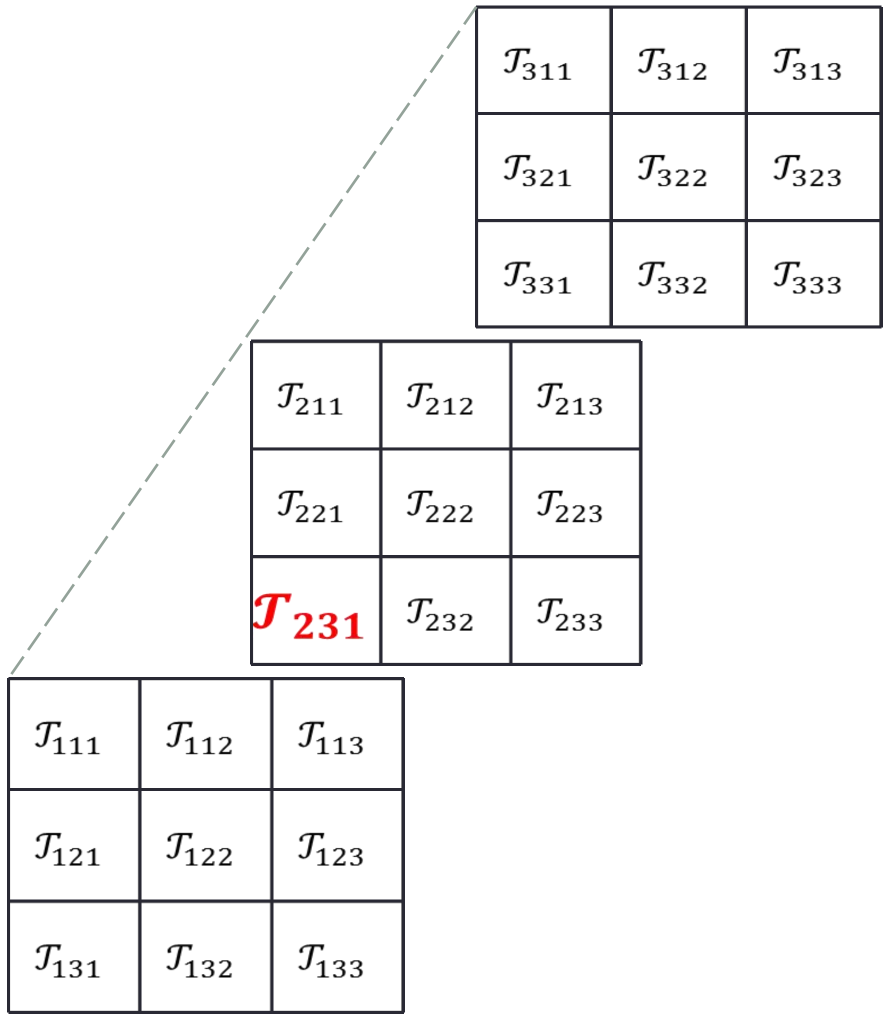
$$p(s) = p(w_1^n)$$

$$p(w_1^n) = p(w_1)\prod_{i=2}^{n} p(w_i|w_1^{i-1}) \quad ①$$

An example

- V={A,B,C}
- The probability of each combination is one element in the tensor

- S_i=(B,C,A)
- $p(S_i) = \mathcal{T}_{231}$



Vocablary V={A,B,C},

$$A = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad C = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$S_i = B \otimes C \otimes A$$

$$p(BCA) = \langle \boldsymbol{\mathcal{T}}, \boldsymbol{\mathcal{A}} \rangle = \mathcal{T}_{231}$$

$$\begin{bmatrix} \mathcal{T}_{311} & \mathcal{T}_{312} & \mathcal{T}_{313} \\ \mathcal{T}_{321} & \mathcal{T}_{322} & \mathcal{T}_{323} \\ \mathcal{T}_{331} & \mathcal{T}_{332} & \mathcal{T}_{333} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{T}_{211} & \mathcal{T}_{212} & \mathcal{T}_{213} \\ \mathcal{T}_{221} & \mathcal{T}_{222} & \mathcal{T}_{223} \\ \color{red}{\mathcal{T}_{231}} & \mathcal{T}_{232} & \mathcal{T}_{233} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \color{red}{\mathbf{1}} & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \mathcal{T}_{111} & \mathcal{T}_{112} & \mathcal{T}_{113} \\ \mathcal{T}_{121} & \mathcal{T}_{122} & \mathcal{T}_{123} \\ \mathcal{T}_{131} & \mathcal{T}_{132} & \mathcal{T}_{133} \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\mathcal{T}} \qquad\qquad \boldsymbol{\mathcal{A}}$$

Compute the joint probability

-

- The sentence s will be represented as:

$$s = \sum_{d_1,\cdots,d_n}^{|V|} \mathcal{A}_{d_1,\cdots,d_n} w_{d_1} \otimes \cdots \otimes w_{d_n}$$

Where

$$\mathcal{A}_{d_1,\cdots,d_n} = \begin{cases} 1, d_k = V.index(w_k) \\ 0, \qquad\qquad\qquad otherwise \end{cases}$$

- The corpus is $c \coloneqq \sum p_i s_i$

$$c = \sum_{d_1 \cdots d_n = 1}^{|V|} \mathcal{T}_{d_1 \cdots d_n} w_{d_1} \otimes \cdots \otimes w_{d_n}$$

- Therefore, the probability of sentence

$$p_i = \langle s_i , c \rangle = \sum_{d_1 \cdots d_n = 1}^{|V|} \mathcal{T}_{d_1 \cdots d_n} \mathcal{A}_{d_1 \cdots d_n}$$

$$= \mathcal{T}_{d_1 \cdots d_n}, d_k = V.index(w_k).$$

Compute the conditional probability

- N-Gram Language Model
  - The conditional probability can be calculated as:

$$p\left(w_i \middle| w_1^{i-1}\right) = \frac{p(w_1^i)}{p(w_1^{i-1})} \approx \frac{count(w_1^i)}{count(w_1^{i-1})}$$

- In TSLM

$$\boxed{\frac{p(w_1^i)}{p(w_1^{i-1})} = \frac{\langle w_1^i, c\rangle}{\langle w_1^{i-1}, c\rangle} \quad ②}$$

- We define $p\left(w_1^i\right) = p(w_1, \cdots w_i)$, in our model, have:

$$p\left(w_1^i\right) = \left\langle w_1^i, c \right\rangle$$

$$= \left\langle w_1 \otimes \cdots \otimes \mathbf{1}\, , \sum_{d_1 \cdots d_n = 1}^{|v|} \mathcal{T}_{d_1 \cdots d_n} w_{d_1} \otimes \cdots \otimes w_{d_n} \right\rangle$$

$$= \sum_{d_1 \cdots d_n = 1}^{|v|} \mathcal{T}_{d_1 \cdots d_n} \left\langle w_1 \otimes \cdots \otimes \mathbf{1}\, , w_{d_1} \otimes \cdots \otimes w_{d_n} \right\rangle$$

$$= \sum_{d_{i+1} \cdots d_n = 1}^{|v|} \mathcal{T}_{d_1 \cdots d_n}\, , d_k = V.index(w_k), \forall k \epsilon [i]$$

We can compute the $p\left(w_1^{i\text{-}1}\right) = \left\langle w_1^{i\text{-}1}, c \right\rangle$ like this.

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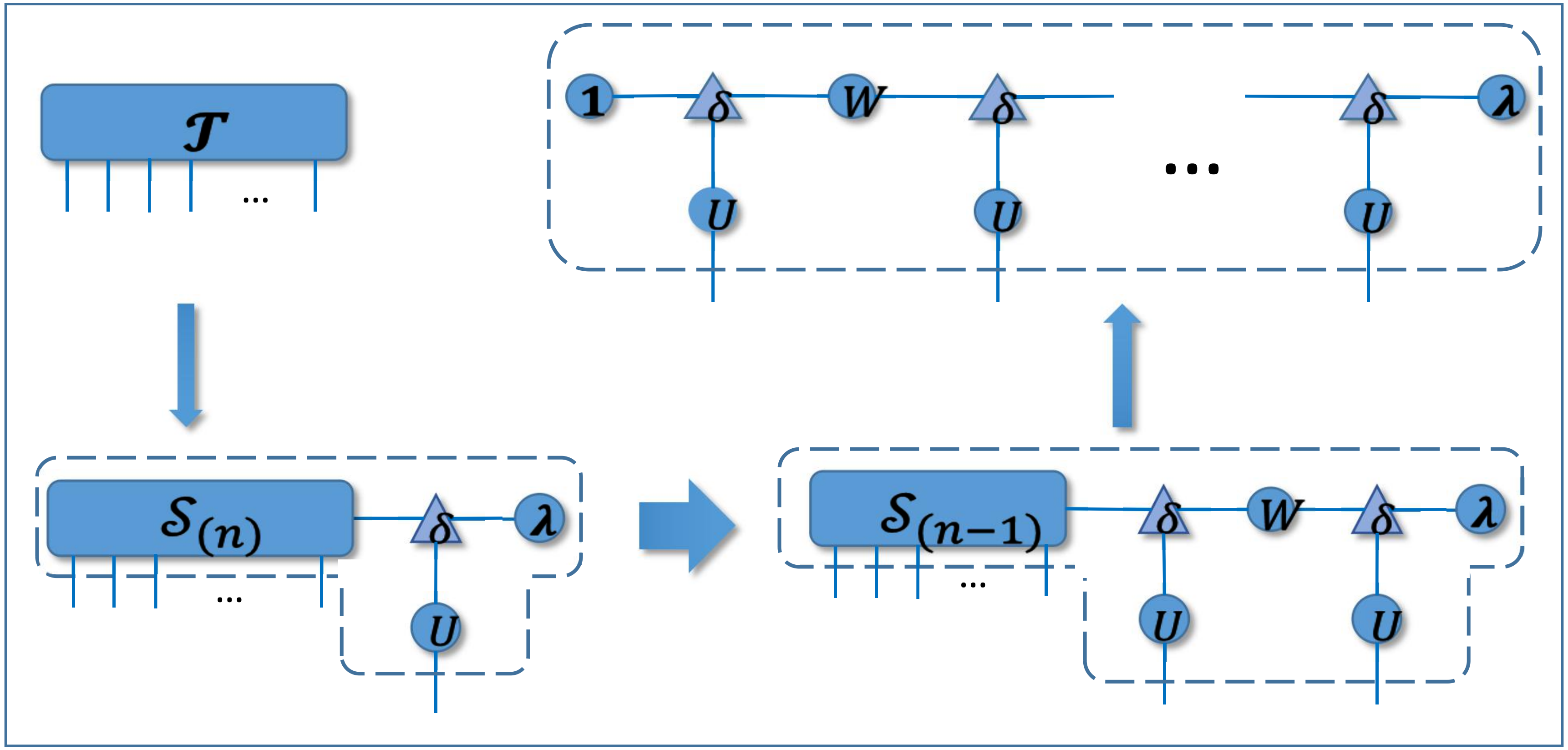
Recursive Language Modeling

- Two hypotheses
  - The dimensions of word vectors is $m \ll |V|$
  - The parameters are the same after each recursive SVD decomposition
  - The corpus : $c = \sum p_i |s_i\rangle$
- The formula of the recursive decomposition about tensor $\mathcal{T}$ is :

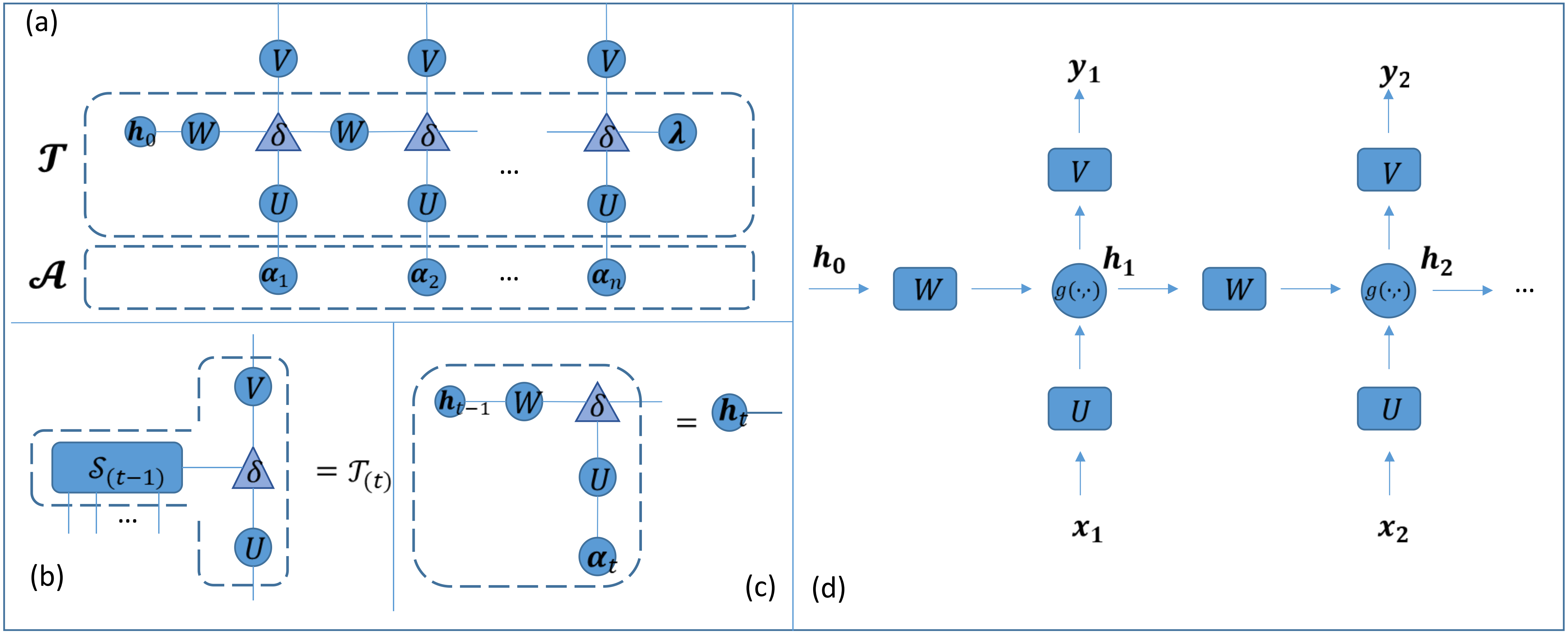
$$\mathcal{T} = \sum_{i=1}^{r} \lambda_i S_{(n),i} \otimes u_i$$

$$S_{(n),k} = \sum_{i=1}^{r} w_{k,i} S_{(n-1),i} \otimes u_i$$

Tensor recursive decomposition



Recursive Language Modeling



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Experimental Result

	PTB				WikiText-2			
Model	**Hidden size**	**Layers**	**Valid**	**Test**	**Hidden size**	**Layers**	**Valid**	**Test**
KN-5(Mikolov and Zweig 2012)	-	-	-	141.2	-	-	-	-
RNN(Mikolov and Zweig 2012)	300	1	-	124.7	-	-	-	-
LSTM(Zaremba, Sutskever, and Vinyals 2014)	200	2	120.7	114.5	-	-	-	-
LSTM(Grave, Joulin, and Usunier 2016)	1024	1	-	82.3	1024	1	-	99.3
LSTM(Merity et al. 2017)	650	2	84.4	80.6	650	2	108.7	100.9
RNN†	256	1	130.3	124.1	512	1	126.0	120.4
LSTM†	256	1	118.6	110.3	512	1	105.6	101.4
TSLM	256	1	**117.2**	**108.1**	512	1	**104.9**	**100.4**
RNN+MoS†(Yang et al. 2018)	256	1	88.7	84.3	512	1	85.6	81.8
TSLM+MoS	256	1	**86.4**	**83.6**	512	1	**83.9**	**81.0**

Table 2: Best perplexity of models on the PTB and WikiText-2 dataset. Models tagged with † indicate that they are reimplemented by ourselves.

Experience

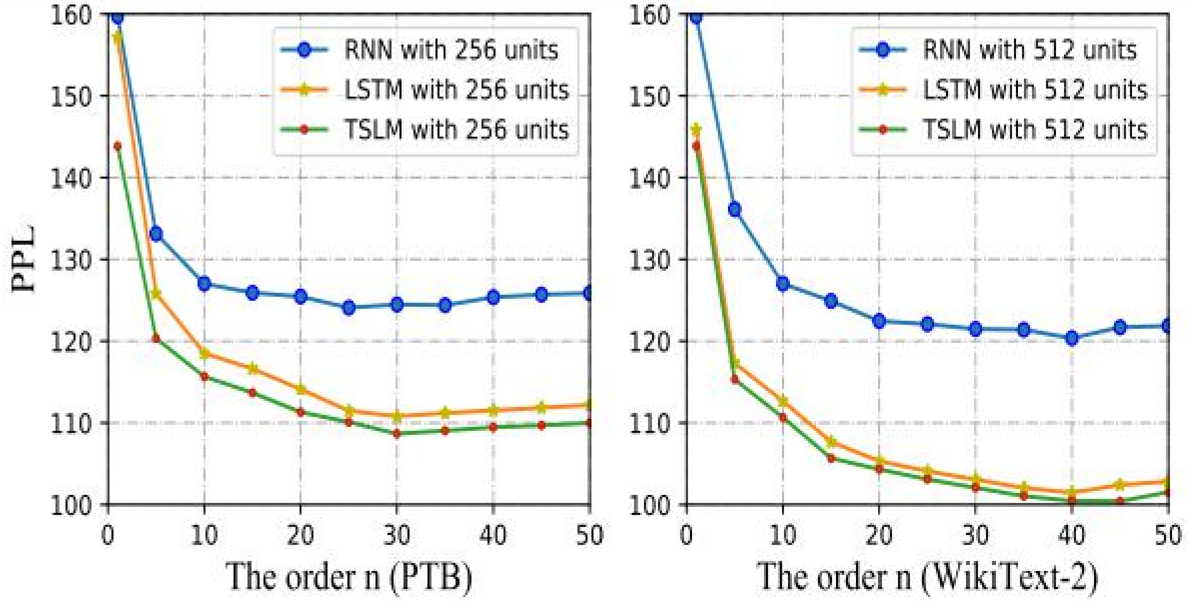

Figure 4: Perplexity (PPL) with different max length of sentences in corpus.

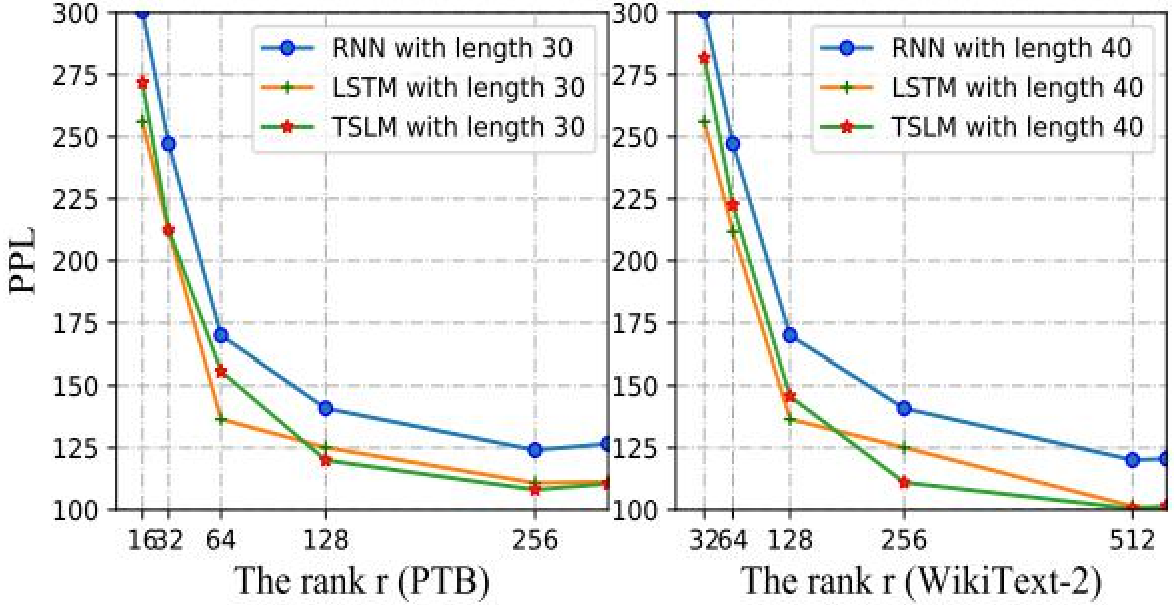

Figure 5: Perplexity (PPL) with different hidden sizes.

Future Work

- Achieve text generation by using TSLM
- Further interpreted in the neural network by tensor network
- Further explore the potential of tensor network for language model