Synthesis of 3D Printed Materials with View Dependent Appearance

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PRELIMINARY AND INCOMPLETE

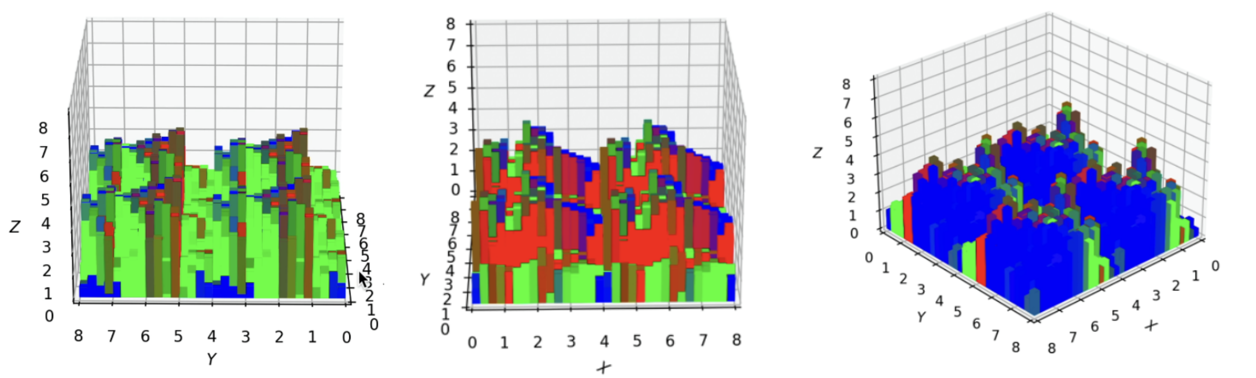

Figure 1. Most recent results as of August 2022. 3D heightfield as viewed from 3 different vantage points, where the desired appearance from each vantage point is green (left), red (middle) and blue (right).

Abstract

We develop a method for the synthesis of 3D materials whose appearance (and the information this appearance conveys) varies based on viewing direction. Our method takes snapshots of a desired material or object from different angles as inputs, where each snapshot shows a different surface color of the material or object, and constructs a 3D printable mesh whose appearance reproduces the given snapshot when observed from the given viewpoint. It accomplishes this through an optimization process that estimates a mesh which minimizes the loss between desired and real appearance/color from each viewpoint. These meshes are then 3D printed and the optimization process tailored to the physical characteristics of the specific 3D printer used.

1. Introduction

[TBD]

2. Related Work

3. Modeling the Problem in 2D

3.1. Problem setup

We seek to represent a 3D printed surface being viewed by a series of cameras. We represent a 3D printed surface as a height field consisting of a series of strips, each with a given height and one-dimensional color. We start by considering just orthogonal cross-sections of this surface. The heights, colors and widths of the series of strips are given by arrays $H = [h_0, ...h_n], 0 \leq h_i \leq k_h$ for some maximum height k_h, $C = [c_0, ...c_n], 0 \leq c_i \leq 1$, and $W = [w_0, ...w_n]$ respectively. For the 2D case, we assume the widths of the strips are constant (i.e., $w_0, ...w_n$ are all equal to k_w). We point orthogonal cameras at this surface. Each camera cam_i consists of an array of rays $R_i = [r_0, ...r_m]$ that are parallel to each other, where each ray r_j is represented by an origin $o_j = (o_j^x, o_j^y)$ and a direction $d_j = (d_j^x, d_j^y)$. These values are shown in Figure 2.

[Note: change occurances to $o_j = (o_j^x, o_j^y)$ and $d_j =$

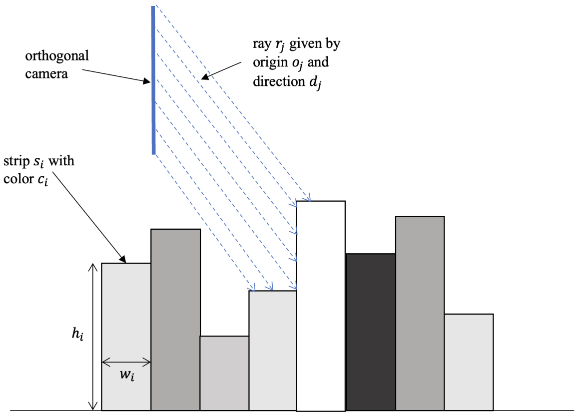

Figure 2. 2D modeling setup

(d^x_j, d^y_j) instead.]

3.2. Ray view

In this setup, the color that is seen by a ray r_j can be fully determined by the heights of the strips and the height of the ray origin. We do this by back-tracing along the angle of the ray from each strip to find boundaries $\theta_0, \dots, \theta_{n+1}$ for each strip, as visualized in 3. In this figure, ray r_0 with origin height oy_0 sees the white strip s_2 as $\theta_2 < oy_0 < \theta_3$, and ray r_1 sees s_3 as $\theta_3 < oy_0 < \theta_4$. For a ray r_j with origin (ox_j, oy_j) and direction (dx_j, dy_j), we can retrieve $\theta_0, \dots, \theta_n$ from heights $h_0, \dots, h_n$ (assuming that the strip s_0 starts with its left edge at x coordinate x_0). We can do this by applying the invertible function t_j to h_i:

$$\theta_i = t_j(h_i) = h_i - \frac{dx_j}{dy_j} \cdot (x_0 - ox_j + (i+1) \cdot w)$$

. The inverse of this function is:

$$h_i = t_j^{-1}(\theta_i) = \theta_i - \frac{dx_j}{dy_j} \cdot (x_0 - ox_j + (i+1) \cdot w).$$

Note that if the camera is instead situated on the right side of the strips and has a left-facing direction, this formula changes slightly (as the boundary points are now on the left corners of the strips rather than the right corners).

3.3. Monotonic θs

In the case that the θs are not naturally monotonically increasing, then we make them monotonic by setting each θ_i value to be equal to $\max\{\theta_0, ...\theta_{i+1}\}$, as visualized in Figure 4.

3.4. Step-wise function for ray-tracing

Let $H(x)$ be the Heaviside step function (or unit step function) where $H(x) = 0$ if $x < 0$ and $H(x) = 1$ if $x \geq 0$. Then,

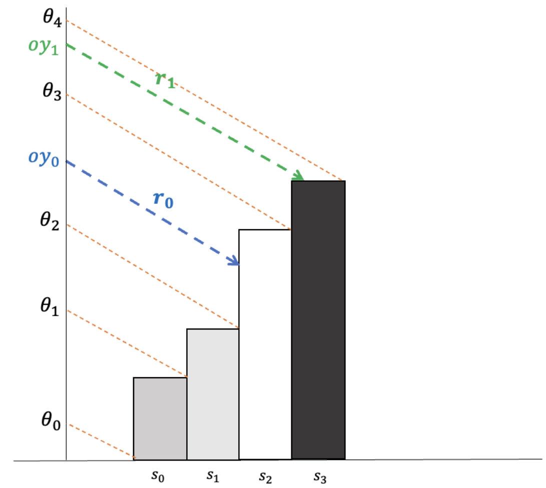

Figure 3. Retrieving strip heights for ray tracing

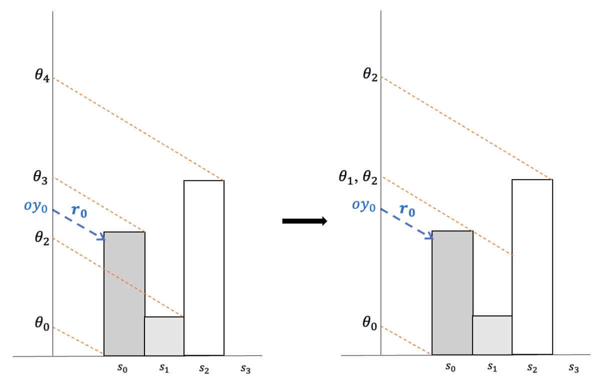

Figure 4. In this case, $\theta_2 < \theta_1$, so we set $\theta_2 = \theta_1$

.

$$\theta_i \leq y < \theta_{i+1} \iff H(y - \theta_i) == 1, H(y - \theta_{i+1}) == 0.$$

Thus:

$$\theta_i \leq y < \theta_{i+1} \iff H(y - \theta_i) - H(y - \theta_{i+1}) == 1.$$

Now, as the θ values are monotonically increasing, for a ray r_j with origin height y_j we are guaranteed that we will have $\theta_i \leq y_j < \theta_{i+1}$ exactly once; in all other cases, either $y_j < \theta_i, \theta_{i+1}$ or $y_j \geq \theta_i, \theta_{i+1}$, in which case the value $H(y - \theta_i) - H(y - \theta_{i+1}) = 0$ (as both values in this subtraction are 0 or both are 1). Thus we can calculate the color seen by ray r_j as

$$\begin{aligned} \mathit{color}(r_j) = \quad & c_0(H(y_j - \theta_0) - H(y_j - \theta_1)) \\ & + c_1(H(y_j - \theta_1) - H(y_j - \theta_2)) \\ & + \dots \\ & + c_n(H(y_j - \theta_n) - H(y_j - \theta_{n+1})). \end{aligned} \tag{1}$$

This formula can be rearranged as

$$\begin{aligned} \mathit{color}(r_j) = \quad & H(y_j - \theta_0) \cdot c_0 \\ & + H(y_j - \theta_1) \cdot (c_1 - c_0) \\ & + \dots \\ & + H(y_j - \theta_n) \cdot (c_n - c_{n-1}) \\ & - H(y_j - \theta_{n+1}) \cdot c_n. \end{aligned} \tag{2}$$

We thus have a simple formula in terms of $\theta_0, ...\theta_n$, strip colors $c_0, \dots, c_n$ and ray height y_j for the view of ray r_j:

$$\begin{aligned} \mathit{color}(r_j) = \quad & H(y_j - \theta_0) \cdot c_0 - H(y_j - \theta_{n+1}) \cdot c_n \\ & + \sum_{i=0}^{n} H(y_j - \theta_i) \cdot (c_i - c_{i-1}). \end{aligned} \tag{3}$$

3.5. Estimating Heaviside step function as a smooth function

[TBD]

3.6. Ray to color: complete differentiable process

The complete process to get from a ray r_j to a view color consists of the following steps:

1. Apply the differentiable, invertible function t_j to all of the strips heights $h_0, \dots, h_n$ to obtain $\theta_0, ...\theta_1$.
2. Monotonize each θ_i with the function $monotonic(\theta_i) = \max\{\theta_0, ...\theta_i + 1\}$.
3. Calculate the color of the ray with $\mathit{color}(r_j)$ using a differentiable Heaviside estimation.

This establishes a completely differentiable forward process to derive the view color of each ray from a set of strips of given heights and colors. Let this mapping be denoted as $\mathit{forward}_j(S)$ on the set of strips S.

3.7. Gradient descent

We can then perform gradient descent to optimize the heights and colors of strips S with respect to an array of desired ray colors $RC^{desired}$. We define our loss function to be the MSE loss between desired view colors and actual view colors RC^{actual} obtained by applying $\mathit{forward}_j(S)$ for each ray r_j, and use gradient descent to update the height and color of each strip.

4. Modeling the Problem in 3D

[TBD] We

5. Results

5.1. Optimization Ablation Study

Figure 6 shows results from a preliminary optimization ablation study.

[TBC]

Left Camera Desired View	Right Camera Desired View	uniform initial configuration							random initial configuration						
			barrier	clamp	subdivide + barrier	subdivide + barrier + smoothing	subdivide + barrier + pertubations	subdivide + barrier + pertubation + smoothing		barrier	clamp	subdivide + barrier	subdivide + barrier + smoothing	subdivide + barrier + pertubations	subdivide + barrier + pertubation + smoothing
All Black	Green	**0**	0.1784	0.245	0.0171	0.0007	0.04187	0.0001	0.0002	0.2289	0.2638	0.0052	0.0039	0.0112	**0**
Random	Random	0.1456	0.1498	0.1605	0.1078	0.1317	0.08215	0.1633	0.1642	0.1485	0.164	0.0928	0.1541	**0.0925**	0.1644
Red	Random	0.08468	0.1153	0.1185	0.04419	0.06842	0.05405	0.05558	0.08823	0.1154	0.1382	**0.0503**	0.0554	0.05122	0.05416
Red	Stripes	0.1923	0.135	0.3355	0.01196	0.1733	0.02923	0.1774	0.171	0.135	0.3355	0.0892	0.1722	**0.03695**	0.1228
Random	Stripes	0.1588	0.1697	0.2776	0.05448	0.2387	0.08739	0.2269	0.1938	0.1696	0.251	0.06604	0.1942	**0.05681**	0.225

Figure 5. Ablation Study for gradient descent optimization in 2D

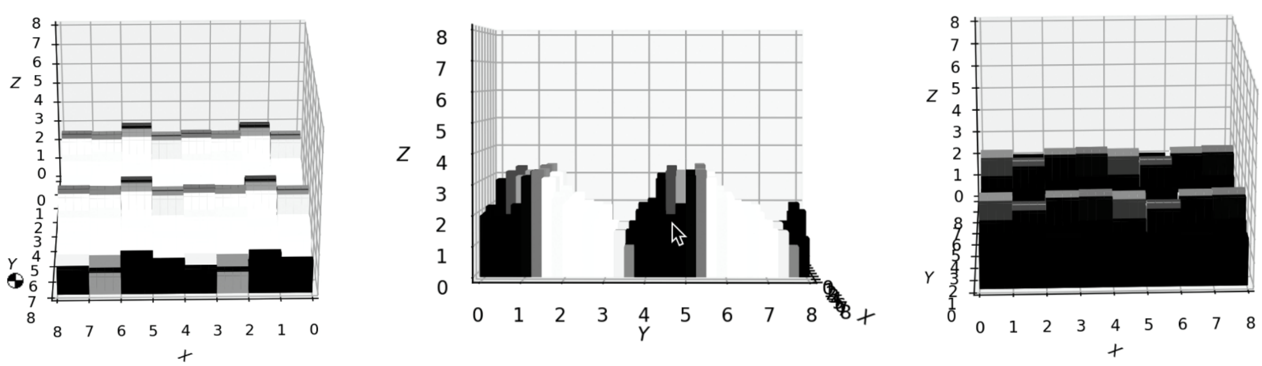

Figure 6. 3D results. We optimized this 3D height field to look completely white or completely back from opposite view directions. The view direction with desired view of completely white is on the left, with completely black is on the right, and the view between these two directions is in the middle.