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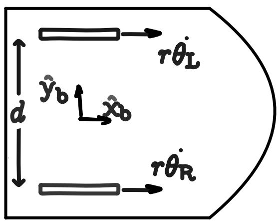
DIFFERENTIAL DRIVE KINEMATICS

INVERSE KINEMATICS: BODY TWIST TO WHEEL SPEEDS



Differential drive robot sketch

$r \rightarrow$ wheel radius

$\theta_L \rightarrow$ left wheel angle

$\theta_R \rightarrow$ right wheel angle

$d \rightarrow$ wheel track / distance

Body twist $\mathcal{V}_b \longrightarrow \mathcal{V}_b = (\omega_{bz}, v_{bx}, v_{by})$

Represents velocities (linear, angular) in the body frame.

The relationship between the body twist and the wheel speeds is given by the matrix H:

$$\begin{bmatrix} \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} = H\,\mathcal{V}_b = \begin{bmatrix} -d/2r & 1/r & 0 \\ d/2r & 1/r & 0 \end{bmatrix} \begin{bmatrix} \omega_{bz} \\ v_{bx} \\ v_{by} \end{bmatrix} \quad [1]$$

No wheel speed can cause the robot to displace in $\hat{y}$.

ODOMETRY: WHEEL ROTATION → DISPLACEMENT

We can solve the same equation as in the inverse kinematics for the body twist:

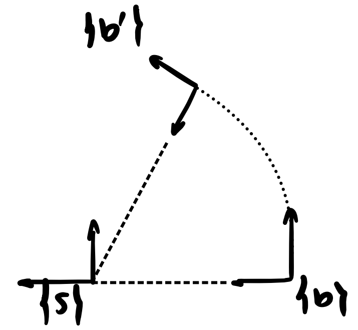
$$\mathcal{V}_b = H^{\dagger} \begin{bmatrix} \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix} = r \begin{bmatrix} -1/(2d) & 1/(2d) \\ 1/2 & 1/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_L \\ \dot{\theta}_R \end{bmatrix}$$

Left pseudoinverse of H

If a constant twist is applied, we can integrate it to get the displacement, $T_{bb'}$. If the twist doesn't have an angular component then the displacement is:

$$T_{bb'} = T(0, v_{bx} \cdot \Delta t, v_{by} \cdot \Delta t) \qquad [2]$$

If the twist has an angular component then we can find the center of rotation {s} aligned with {b}



It is evident that x_s in the frame {b} is 0.

y_s = radius of the arc

for any arc: $\quad r\,d\theta = \Delta x \rightarrow r = \frac{\Delta x}{\Delta \theta}$

So y_s in the {b} frame is $\frac{v_{bx}}{\omega_{bz}}$ $\quad (\Delta t = 1)$

Then $\quad T_{sb} = T\left(0, 0, -\frac{v_{bx}}{\omega_{bz}}\right)$

Now, to perform the transform from {b} to {b'} we can just rotate by $\theta \rightarrow \quad T_{ss'} = T(\Delta\theta, 0, 0)$.

Performing the inverse transform, T_{bs}, we get back to the body frame. Note $T_{bs} = T_{b's'}$ because of the careful orientation of {s}.

Chaining all the transforms:

$$T_{bb'} = T_{bs}\, T_{ss'}\, T_{s'b'} \qquad [3]$$

Which yields the relative movement of the robot for a constant twist applied for one time unit.

We can integrate the relative movement by composing the transforms:

$$T_{wb_{t+1}} = T_{wb_t} \cdot T_{bb'} \qquad [4]$$

REFERENCES

Modern Robotics: Mechanics, Planning and Control, Lynch and Park, Cambridge University Press, 2017.

Sensing, Navigation and Machine Learning for Robotics course notes, Matthew Elwin, 2024.