Equivalence Testing

Most information in this exercise comes from "Equivalence Testing for Psychological Research: A Tutorial" by Lakens, Scheel, and Isager, and my article "Equivalence Tests: A Practical Primer for t Tests, Correlations, and Meta-Analyses". See these two articles for more examples and information on equivalence testing, and a blog post by Jeff Rouder on using custom Bayesian priors.

Scientists should be able to provide support for the null hypothesis. A limitation of null-hypothesis significance testing is that the null-hypothesis can be rejected, but not accepted. When you perform a statistical test, and the outcome is a *p*-value larger than the alpha level (α), the only formally correct conclusion is that the data are not surprising, assuming the null hypothesis is true. It is not possible to conclude there is *no* effect – our test might simply have lacked the statistical power to detect it. So how can you ever falsify a prediction? In this assignment, we will examine how to provide support for the lack of a meaningful effect. We will do this using both Frequentist and Bayesian statistics.

In Frequentist statistics, we can make a statement about the absence of a meaningful effect using **equivalence testing**. Equivalence testing can be used to test whether an observed effect is surprisingly small, assuming a meaningful effect exists in the population. The test is a simple variation of widely used null hypothesis significance tests (NHST). To understand the idea behind equivalence tests, it is useful to realize the null hypothesis we test against can be any numerical value. When we compare two groups, we often test whether we can reject that the difference between these groups is zero. If you look at Figure 1A on the next page, you can see a point null hypothesis at an effect size of 0, and the two-sided alternative hypothesis of an effect that is different from 0.

Sometimes want to reject other values than zero. Imagine a researcher who is interested in voluntary participation in a national program to train young infants' motor skills. The researcher wants to test whether more boys than girls are brought into the program by their parents. Because the human sex ratio of newborn boys to girls is not exactly 1:1, we should not expect 50% of participants to be boys. On average, 103 boys are born for every 100 girls (United States & Central Intelligence Agency, 2016), so approximately 50.74% of applicants should be boys, and 49.26% should be girls. If boys and girls were exactly equally likely to be brought into the program by their parents, we would not expect a difference of zero, but 50.74% - 49.26% = 1.5% more boys. Rather than testing against a null hypothesis of 0 difference, the researcher tests against a null hypothesis of 0.015.

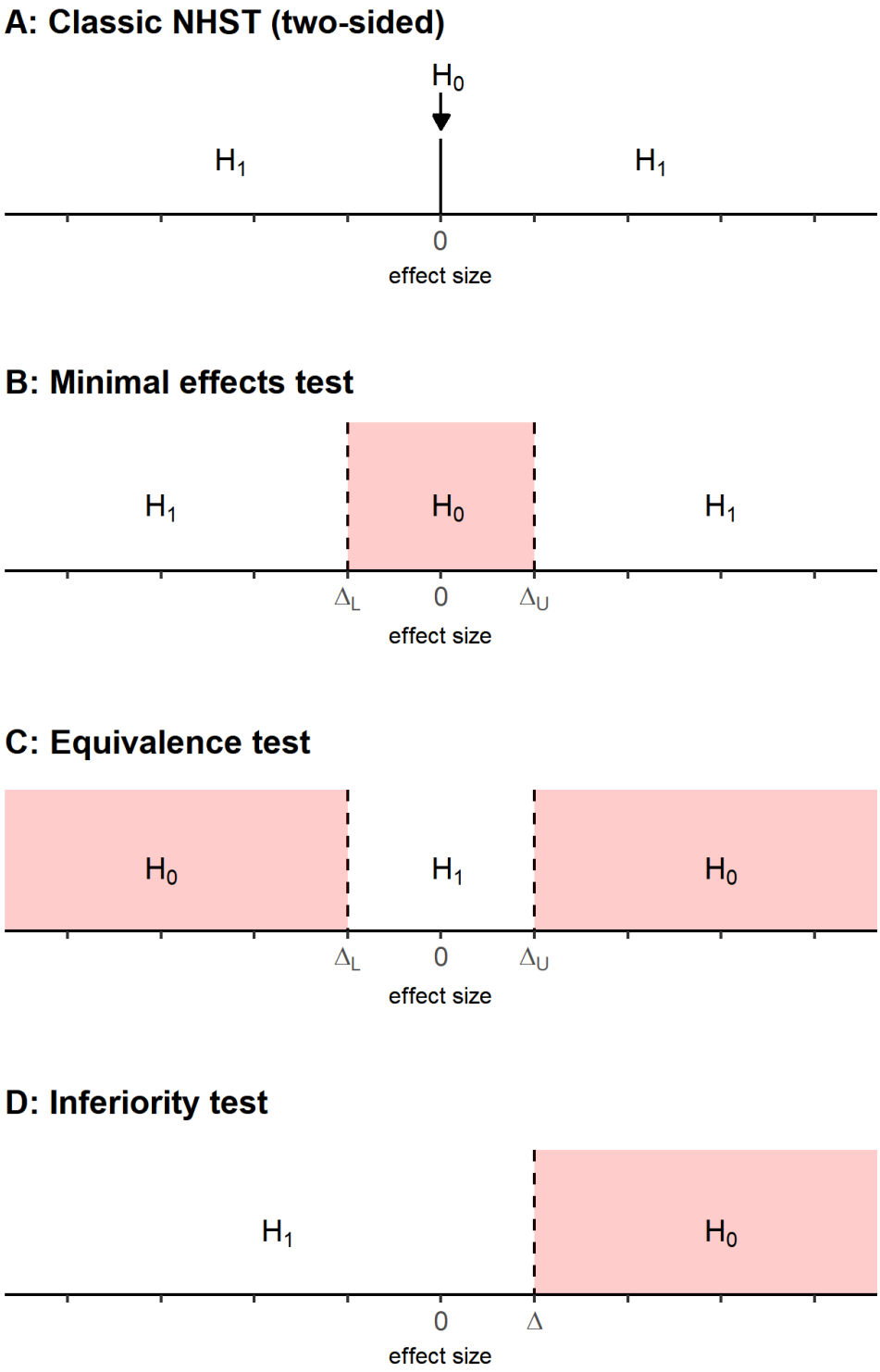


Figure 1: Illustration of null hypotheses (H0) and alternative hypotheses (H1) for different types of significance tests. A) NHST: Tests if the hypothesis (H0) that an effect is equal to 0 can be rejected. B) Minimal effects test: Tests if the hypothesis (H0) that an effect is larger than ΔL and smaller than ΔU can be rejected. C) Equivalence test: Tests if the hypothesis (H0) that an effect is smaller than ΔL or larger than ΔU can be rejected. D) Inferiority test: Tests if the hypothesis (H0) that an effect is larger than Δ can be rejected.

Alternatively, the researcher could decide that even if the true ratio in the population is not exactly 0.015, the null hypothesis should consist of a **range of values** around the difference in proportions of 0.015 that can be considered trivially small. The researcher could for example test if the difference is smaller than -0.005, or larger than 0.035. This test against two bounds, with H0 being a range rather than one value (see Figure 1B), is known as a **minimal effects test** (Murphy, Myors, & Wolach, 2014).

Equivalence tests can be seen as the opposite of minimal effects tests: They examine whether the presence of effects that are large enough to be considered meaningful can be rejected (see Figure 1C). An equivalence test consists of two one-sided tests (and is therefore also called the **TOST** procedure). We test whether the observed effect is statistically larger than the lower equivalence bound (indicated by Δ_L.in Figure 1) and statistically smaller than the upper equivalence bound ((indicated by Δ_U.in Figure 1). In our example, the researcher can perform an equivalence test to examine whether the gender difference in participants is not as large or larger than the **smallest effect size of interest** (**SESOI**). After an extensive discussion with experts, the researcher decides that as long as the difference in proportions is not more extreme than 6%, the gender difference is too small to care about. Given an expected true difference in the population of 0.015, the researcher will test if the observed difference falls outside the boundary values (or **equivalence bounds**) of -0.055 and 0.075 by performing a one-sided test against -0.055 and another one-sided test against 0.075. If differences more extreme than these boundary values can be rejected in two one-sided tests (TOST), the researcher will conclude statistical equivalence, the gender difference will be considered trivially small, and no money will be spent on addressing a gender difference in participation. Both one-sided tests need to be statistically significant to conclude equivalence. It is custom to report only the one-sided test with the largest *p*-value (because if this is significant, so is the other one-sided test) when reporting an equivalence test.

Equivalence testing originates from the field of pharmacokinetics where researchers might want to show that a new, cheaper, drug works just as well as an existing drug. A very simple equivalence testing approach is the 'two-one-sided t-tests' (TOST) procedure (Schuirmann, 1987), which tests whether the observed effect is larger than the lower equivalence bound, and smaller than the upper equivalence bound.

In equivalence tests the null hypothesis is that there *is* a true effect more extreme than a smallest effect size of interest (SESOI), as defined by the upper and lower equivalence bounds (indicated by Δ_U and Δ_L.in Figure 1) That's right – the null-hypothesis is now that there *is* an effect, and we are going to try to reject it (with a $p < 0.05$). The alternative hypothesis is that the effect is anywhere in the *equivalence range*. The equivalence range is a range of effect sizes that are deemed equivalent to the absence of an effect that is

worthwhile to examine (e.g., Δ_L = -0.3 to ΔU = 0.3, where Δ is a difference that can be defined by either standardized differences such as δ, or raw differences such as 1 scale point or 50 milliseconds). If the *p*-value for both tests indicates the observed data is surprising, assuming $-\Delta_L$ or Δ_U are true, we can follow a Neyman-Pearson approach to statistical inferences and reject effect sizes larger than the equivalence bounds.

If we have a clear directional hypothesis we might only be interested in whether an observed effect is not larger than some effect we deem meaningful, which means we can test for inferiority (see Figure 1D), where we test whether the observed effect size is statistically lower than the SESOI (indicated by Δ). In an **inferiority test** we only care about effects is surprisingly smaller than one directional equivalence bound.

When null-hypothesis significance testing and equivalence tests are used, there are four possible outcomes of a study. These four cases are illustrated in the figure below (adapted from Lakens, 2017). A mean difference of Cohen's d = 0.5 (either positive or negative) is specified as a smallest effect size of interest in an independent *t*-test (see the vertical dashed lines at -0.5 and 0.5). Data is collected, and one of four possible outcomes is observed (squares are the observed effect size, thick lines the 90% CI, and thin lines the 95% CI). A 90% confidence interval (1-2α) is used instead of a 95% confidence interval (1-α) because two one-sided tests (each with an alpha of 5%) are performed.

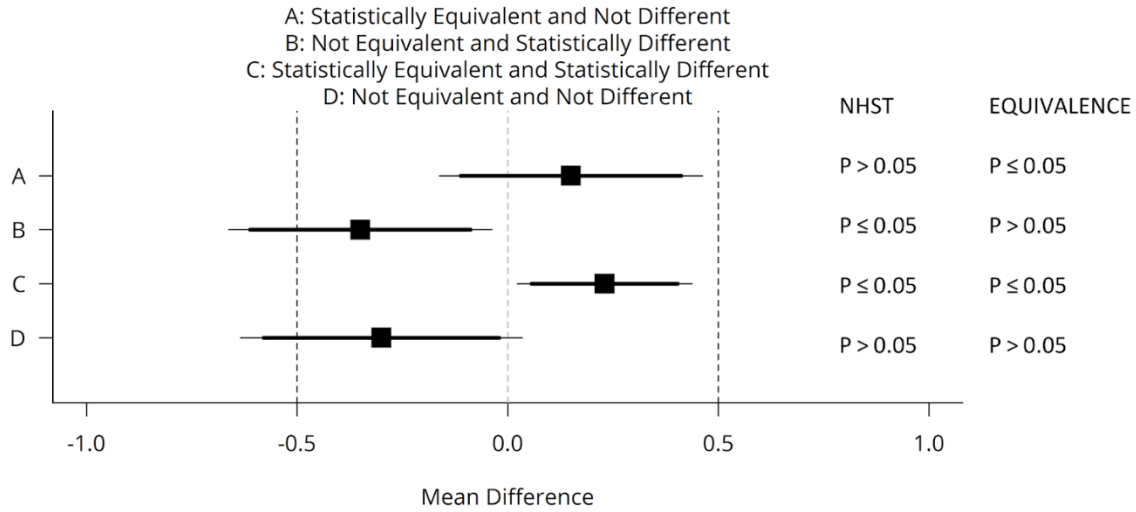


Figure 2. Four possible outcomes when combining NHST and equivalence tests.

We can conclude statistical equivalence if we find the pattern indicated by A: The confidence interval overlaps with 0, but not with the smallest effect size of interest. The *p*-value from the traditional NHST is not significant ($p > 0.05$), and the *p*-value for the

equivalence test is significant ($p \leq 0.05$). Thus, we conclude the effect is statistically equivalent: It does not differ from 0, and it is too small to be considered meaningful. We can also conclude the effect is significant, and that the possibility that the effect is large enough to matter can not be rejected, under pattern B, which means we can reject the null, and the effect might be large enough to care about. Using equivalence tests, we can also observe pattern C: An effect is statistically significant, but also smaller than anything we care about. In other words, we can conclude there is an effect, but it lacks **practical significance** – it is too small to matter in practice. Finally, we can observe pattern D, where we can not reject an effect of 0, nor an effect that is large enough to care about. We thus remain **undecided** – we need more data to draw a conclusion.

Testing for equivalence is just as simple as performing the normal statistical tests you already use today. You don't have to learn any new statistical theory. Given how easy it is to use equivalence tests, and how much they improve your statistical inferences, it is surprising how little they are used, but I'm confident that will change in the future. The TOST procedure entails performing two one-sided tests to examine whether the observed data is surprisingly larger than a lower equivalence boundary (Δ_L), or surprisingly smaller than an upper equivalence boundary (Δ_U):

$$t_L = \frac{\bar{M}_1 - \bar{M}_2 - \Delta_L}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \text{ and } t_U = \frac{\bar{M}_1 - M_2 - \Delta_U}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \quad (1)$$

Where *M* indicates the means of each sample, *n* is the sample size, and σ is the pooled standard deviation:

$$\sigma = \sqrt{\frac{(n_1-1)SD_1^2+(n_2-1)SD_2^2}{n_1+n_2-2}} \quad (2)$$

The formulas are highly similar to the normal formula for the *t*-statistic. The difference between a NHST *t*-test and the TOST procedure is that the lower equivalence boundary Δ_L and the upper equivalence boundary Δ_U are subtracted from the mean difference between groups.

The TOST procedure for a t-test is identical to calculating a 90% CI around the effect, and check whether this 90% CI falls completely within the equivalence range (see Figure 2 above).

In this assignment we will perform equivalence tests using my own TOSTER package in R. It works by entering summary statistics into a function, and proviced all the information you need to perform and report an equivalence test.

Let's assume we performed a study where we have two independent groups, a control and experimental condition, and we are interested in the average score of participant on some validated questionnaire, measured using a 7-point scale. We want to know if there is a difference between means. Before we collect the data, we considered an effect size of d = 0.4 the smallest effect size of interest, and therefore set the lower equivalence bound to d = -0.4 and the upper equivalence bound to d = 0.4. After collecting the data we have 130 participants in the control group and 134 participants in the experimental group. The control group score was *M* = 4.2, *SD* = 1.1, and the experimental group score was *M* = 4.3, *SD* = 1.2.

We now have all the information we need to perform an equivalence test. In R, the function will look like:

```
TOSTtwo(m1=4.2,m2=4.3,sd1=1.1,sd2=1.2,n1=130,n2=134,low_eqbound_d=-
0.4,high_eqbound_d=0.4)
```

TOSTtwo is the function name for the TOST procedure for two independent groups. It needs as input the means (m1 and m2), the standard deviations (sd1 and sd2), the sample sizes in each group (n1 and n2) and you need to specify the equivalence bounds, which for the TOSTtwo function should be in standardized scores (Cohen's d – you can use the TOSTtwo.raw function to enter equivalence bounds in unstandardized mean differences). Note that you need to consistently fill in the information for one group at m1, sd1, and n1, and for the other group as m2, sd2, n2. Also note that the lower equivalence bound is entered as a negative value, because we are testing against d = -0.4.

When you run this single line of code, you will receive written output, and a figure:

```
Using alpha = 0.05 Welch's t-test was non-significant, t(261.167) =
-0.7061548, p = 0.4807213


Using alpha = 0.05 the equivalence test based on Welch's t-test  was
significant, t(261.167) = 2.545226, p = 0.005748213

TOST results:

  t-value 1   p-value 1 t-value 2    p-value 2      df

1  2.545226 0.005748213 -3.957536 4.882819e-05 261.167


Equivalence bounds (Cohen's d):

  low bound d high bound d

1        -0.4          0.4
```

```
Equivalence bounds (raw scores):
  low bound raw high bound raw
1    -0.4604346      0.4604346

TOST confidence interval:
  Lower Limit 90% CI raw Upper Limit 90% CI raw
1             -0.3337602              0.1337602
```

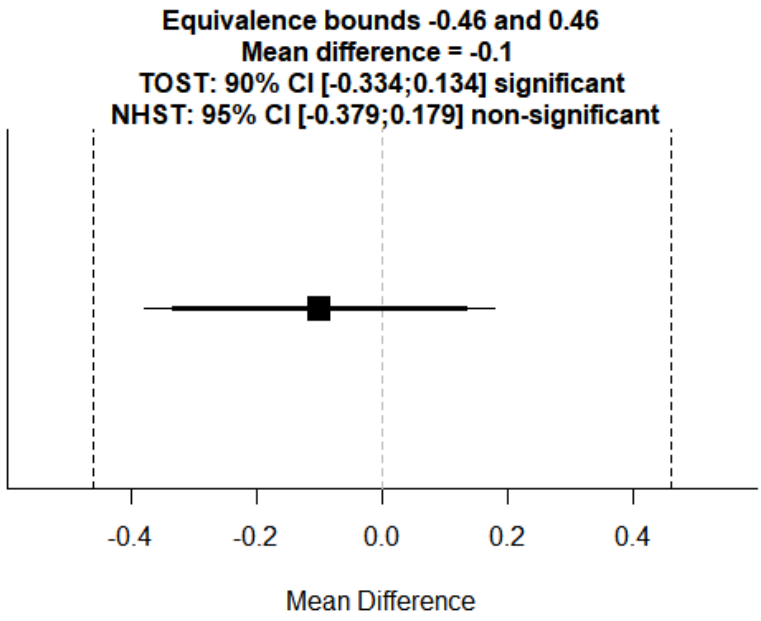
Just like the default *t*-test in R, the TOSTtwo function will by default calculate Welch's t-test (instead of student's t-test). This is actually the recommended default (see Delacre, Lakens, and Leys, 2017), but you can request Student's *t*-test by adding var.equal=TRUE as an argument to the TOSTtwo function.

The TOST results provide the two one-sided tests, one against the lower bound of d = -0.4 yields a *t*-value of 2.545 and a *p*-value of 0.005748213, and the test against the upper bound of d = 0.4 yields a *t*-value of -3.957536 and a *p*-value of 4.882819e-05 (or 0.0000488). The output also provides a written summary (remember that it is custom to only report the largest *p*-value) which after rounding reads: Using alpha = 0.05 the equivalence test based on Welch's *t*-test was significant, $t(261.167) = 2.55$, $p = 0.006$. Because the equivalence test is significant, we can reject effects smaller than d = -0.4 and larger than d =0.4, and therefore reject effects we found large enough to be meaningful when we designed the study.

TOST is performed against equivalence bounds that are considered the smallest effect size of interest. The SESOI can sometimes be determined objectively, for example based on just noticeable differences are clear quantifiable theoretical predictions. In lieu of objective justifications, the SESOI should ideally be based on a cost-benefit analysis (for example, which effect sizes are large enough to be worth studying). Since both costs and benefits are necessarily subjective, the SESOI will vary across researchers, fields, and time. The goal of setting a SESOI is to clearly justify why designing a study that has a high probability of rejecting effects more extreme than the specified equivalence bounds contributes to our knowledge base (see Lakens, 2017, Lakens, Scheel, & Isager, 2018). A SESOI should be chosen such that inferences based on it answer a meaningful question. In the study above, we can conclude equivalence using bounds of d = -0.4 and d =0.4 – but someone else is free to consider an effect size of d = 0.2 meaningful, and design a much larger study to examine such a relatively small effect size.

The picture provided with the output provides the raw equivalence bounds (both raw and standardized equivalence bounds are provided in the output as well) which in this case

translate to mean differences of -0.46 and 0.46. It is good to always look at, and think about, the raw equivalence bounds. These should make sense, even when you chose to use standardized equivalence bounds. We also see the observed mean difference (in this case, -0.1), and the 90% confidence interval (related to the TOST procedure) and the 95% confidence interval (related to the NHST test).



Q1) When the 90% CI around an effect size falls within the equivalence range, the observed effect is statistically smaller than the smallest effect size of interest. Based on your knowledge about confidence intervals, and looking at the picture above, when you lower the equivalence range from -0.4 – 0.4 to -0.3 – 0.3, what is needed for the equivalence test to be significant (assuming the effect size estimate and standard deviation remains the same)?

A) A larger effect size

B) A lower alpha level

C) A larger sample size

D) Lower statistical power

To answer questions 2 through 9, open the EquivalenceTesting.R script. The script will allow you to perform the TOST procedure for the questions below by filling in the summary statistics. To help you along, I have already copied the function and all arguments – you just need to fill in the correct values for each function, and interpret the output.

Q2) Researchers often manipulate something they are interested in. To ensure their manipulation does not inadvertently alter participants' moods, they assess positive and negative emotions using the PANAS. Let's assume in one specific experiment, positive mood in one condition (M_1 = 4.55, SD_1 = 1.05, n_1 = 15) did not differ from the mood in the other condition (M_2 = 4.87, SD_2 = 1.11, n_2 = 15). The researchers conclude: "*Mood did not differ between conditions, t = 0.81, p =.42*". Let's assume we consider any effect larger than *d*=-0.5 and smaller than *d* = 0.5 equivalent (even though *d* = 0.5 is actually a medium effect size!). Use the spreadsheet for the independent *t*-test, and fill in the 8 numbers (note: you need to fill in -0.5, not 0.5, as the lower equivalence bound). Were the authors correct in concluding mood did not differ between conditions, given the equivalence range of -0.5 to 0.5?

Use 0.05 as the alpha level for all questions in this assignment.

A) Yes

B) No

Q3) If we increase the sample size in the above example to 150 participants in each condition, and assuming the means and standard deviations remain the same, which conclusion would we draw?

A) Equivalent: The difference in mood is not statistically significant, and it is statistically equivalent.

B) Undetermined: The difference in mood is not statistically significant, and it is not statistically equivalent.

C) Not zero, and not meaningful: The difference in mood is statistically significant, and it is statistically equivalent.

D) Not zero, and meaningful: The difference in mood is statistically significant, and it is not statistically equivalent.

When should you consider an effect too small to be meaningful? This is clearly a subjective choice, and what you consider meaningful can change over time. Ideally, you can determine the equivalence range either based on practical considerations or theoretical considerations. For example, if an advertisement campaign increases sales, but not enough to earn back the cost of the advertising campaign, you might decide an advertising campaign has an effect smaller than your smallest effect size of interest.

Alternatively, you might believe based on available theories that a delay in response selection in a Stroop experiment where people respond verbally is larger than 20 milliseconds.

The choice for a smallest effect size of interest is also relevant when performing a power analysis. Because it is often unclear which effect size you can expect (after all, if you already knew the effect size, you would not need to do the experiment!) researchers sometimes make sure they have sufficient power for the smallest effect size they find interesting. Although it might be difficult to decide upon a SESOI, you might find it much easier to decide upon a maximum sample size you are willing to collect, and look at the effect size you can reject in an equivalence test. This can be examined using power analyses. Just as with a normal t-test, you can perform a power analysis for an equivalence test, which is very useful when designing a study.

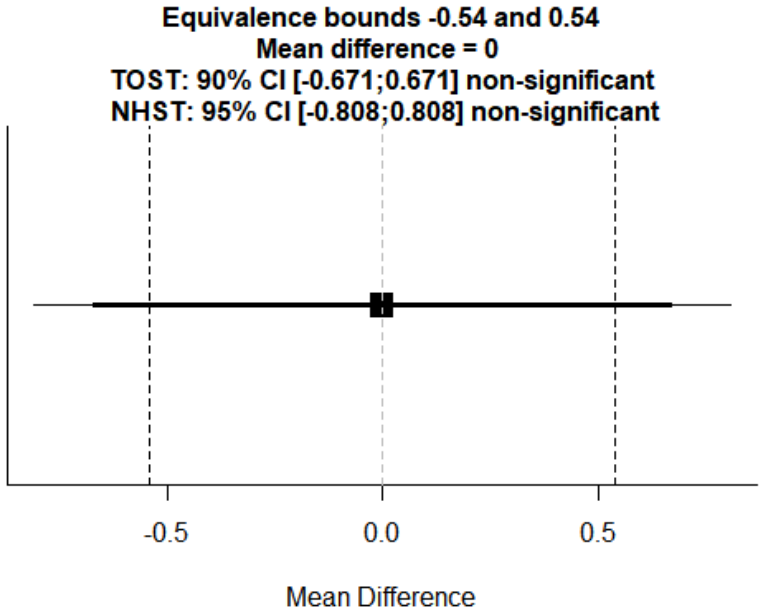
We used an equivalence range of *d*=-0.5 to *d* = 0.5 in question 2, even with a sample size of 15 participants in each condition. One might wonder if that sample size actually gave us sufficient statistical power to detect equivalence. The TOSTER package also has functions for power analysis. The powerTOSTtwo function can be used to calculate the required sample size, the observed power, or the required equivalence bounds to achieve a desired power given a sample size. If we run the function:

```
powerTOSTtwo(alpha = 0.05, N = 15, low_eqbound_d = -0.5,
high_eqbound_d = 0.5)
```

we are calculating the power we have when we have 15 participants in each group (N =15), bounds of -0.5 and 0.5, and we use an alpha of 0.05. The function returns the following information:

```
The statistical power is 0 % for equivalence bounds of -0.5 and 0.5.
```

This might look like a mistake. How can we have 0 power? The answer becomes obvious if we look at when would happen in the study described in question 2 when we would have observed exactly the same means in both groups, so a mean difference of 0 (but the same sample sizes in each group, and the same standard deviations). This hypothetical scenario is plotted in the figure below:



We see the 90% confidence interval is so wide (due to the small sample size and given the standard deviations) that it will ALWAYS include either the lower or the upper equivalence bound. The 90% confidence interval is simply so wide that it will never for within the equivalence bounds. This, there is 0 probability that we can conclude equivalence. The sample size is simply too small. We can see power analysis is very important in general, but especially when performing equivalence tests.

Q4) We might wonder how wide our bounds would need to be, to have decent power to conclude equivalence. Let's aim for 90% power, and use an alpha of 0.01 for our test. The powerTOSTtwo function can be used to calculate how wide our equivalence bounds would need to be, to have 90% power with 15 participants in each condition and an alpha of 0.01 (this power analysis assumes the true effect size is 0 – more advanced power analyses can be performed with PASS software). Which equivalence bounds should a researcher use if they want 90% power with 15 participants and an alpha of 0.01? Round the answer to two digits after the decimal.

A) d = -1.07 and d = 1.07

B) d = -1.20 and d = 1.20

C) d = -1.32 and d = 1.32

D) d = -1.45 and d = 1.45

You can see that with such a small sample, we can only reject effect sizes that are very large (d > 1). Is it interesting to perform a study where you can only reject effects that are

very large? It depends. Most effect sizes studied in for example psychology, but also many other social sciences, are much smaller. Asking 'can we reject very large effects' is therefore not very interesting (unless a theory explicitly predicts only very large effects, obviously!).

Q5) The most common use of power analysis is to determine the sample size needed to design a study with high power to detect a significant effect. If we want to have 90% power, use an alpha of 0.01, and use equivalence bounds of d = -0.5 and d = 0.5, how many participants in each condition should we collect? Use the code in the R file.

A) 87

B) 105

C) 127

D) 254

Q6) Change the equivalence range to -0.1 and 0.1. To be able to reject effects outside such a very narrow equivalence range, you'll need a large sample size. With an alpha of 0.01, and a desired power of 0.9 (or 90%), how many participants would you need in each group?

A) 2165

B) 2604

C) 3155

D) 6310

You can see it takes a very large sample size to have high power to reliably reject very small effects. This should not be surprising – it also requires a very large sample size to *detect* small effects!

Q7) You can do equivalence tests for all tests. The TOSTER package has functions for t-tests, correlations, differences between proportions, and meta-analyses. Let's do an equivalence test for a meta-analysis. Hyde, Lindberg, Linn, Ellis, and Williams (2008) report that effect sizes for gender differences in mathematics tests across the 7 million students in the US represent trivial differences, where a trivial difference is specified as

an effect size smaller then d = 0.1. The present a table with Cohen's d and se is reproduced below:

Grades	**d + se**
Grade 2	0.06 +/- 0.003
Grade 3	0.04 +/- 0.002
Grade 4	-0.01 +/- 0.002
Grade 5	-0.01 +/- 0.002
Grade 6	-0.01 +/- 0.002
Grade 7	-0.02 +/- 0.002
Grade 8	-0.02 +/- 0.002
Grade 9	-0.01 +/- 0.003
Grade 10	0.04 +/- 0.003
Grade 11	0.06 +/- 0.003

For grade 2, when we perform an equivalence test with boundaries of d = -0.1 and d = 0.1, using an alpha of 0.01, which conclusion can we draw? Use the TOSTER function TOSTmeta, and enter the alpha, effect size (ES), standard error (se), and equivalence bounds.

A) Equivalent: The difference in test scores is not statistically significant, and it is statistically equivalent.

B) Undetermined: The difference in test scores is not statistically significant, and it is not statistically equivalent.

C) Not zero, and not meaningful: The difference in test scores is statistically significant, and it is statistically equivalent.

D) Not zero, and meaningful: The difference in test scores is statistically significant, and it is not statistically equivalent.

Q8) Olson, Fazio, and Hermann (2007) reported correlations between implicit and explicit measures of self-esteem, such as the IAT, Rosenberg's self-esteem scale, a feeling thermometer, and trait ratings. In Study 1 71 participants completed the self-esteem

measures. Because no equivalence bounds are mentioned, we can see which equivalence bounds the researchers would have 50% power to detect (the bounds a study has 50% power for, is related to the bounds that can just be detected with $p < .05$, see Lakens, Scheel, and Isager, 2018 or this blog post). Use the powerTOSTr function (which is used for correlations, and when the equivalence bounds are set based on *r*). Which boundaries do we have 50% power for, with an alpha of 0.05, and 71 participants? Round the bounds to 2 digits after the decimal.

A) r = -0.19 and r = 0.19

B) r = -0.21 and r = 0.21

C) r = -0.27 and r = 0.27

D) r = -0.32 and r = 0.32

Q9) The correlations observed by Olson et al (2007), Study 1, are presented in the table below (significant correlations are flagged by an asterisk).

Measure	IAT	Rosenberg	Feeling thermometer	Trait ratings
IAT	-	-.12	-.09	-.06
Rosenberg		-	.62*	.09
Feeling thermometer			-	.29*
Trait ratings				-

We can test each correlation for equivalence, for example the correlation between the IAT and the Rosenberg self-esteem scale of -0.12, given 71 participants. When you test all 4 non-significant correlations (-0.12, -0.09, -0.06, and 0.09) for equivalence, using an alpha of 0.05 and equivalence bounds of $r = -0.2$ and $r = 0.2$, how many are statistically equivalent?

A) 0

B) 1

C) 2

D) 3

Supporting the null with Bayes Factors

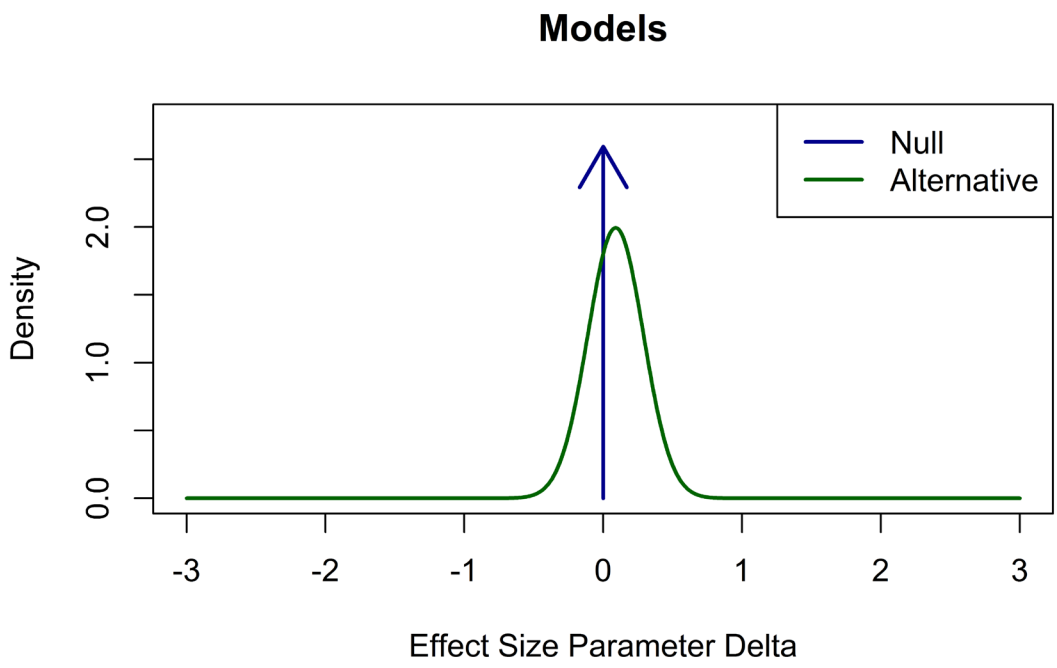
Bayesian statistics, through its reliance on likelihoods, allows you to express the relative support for one hypothesis over the other hypothesis. You can conclude both that the alternative hypothesis is more plausible than the null model, or vice versa. Here we will use r code by Jeff Rouder to decide upon priors, calculate the posterior based on observed data, and calculate the Bayes Factor for a one-sample *t*-test.

Let's take a look at the pre-cognition experiments by Daryl Bem (2011). In the first study, 100 participants classified the future position of pictures, before the computer had randomly chosen where the pictures would appear. For erotic pictures, this difference was statistically significant (see the result section below).

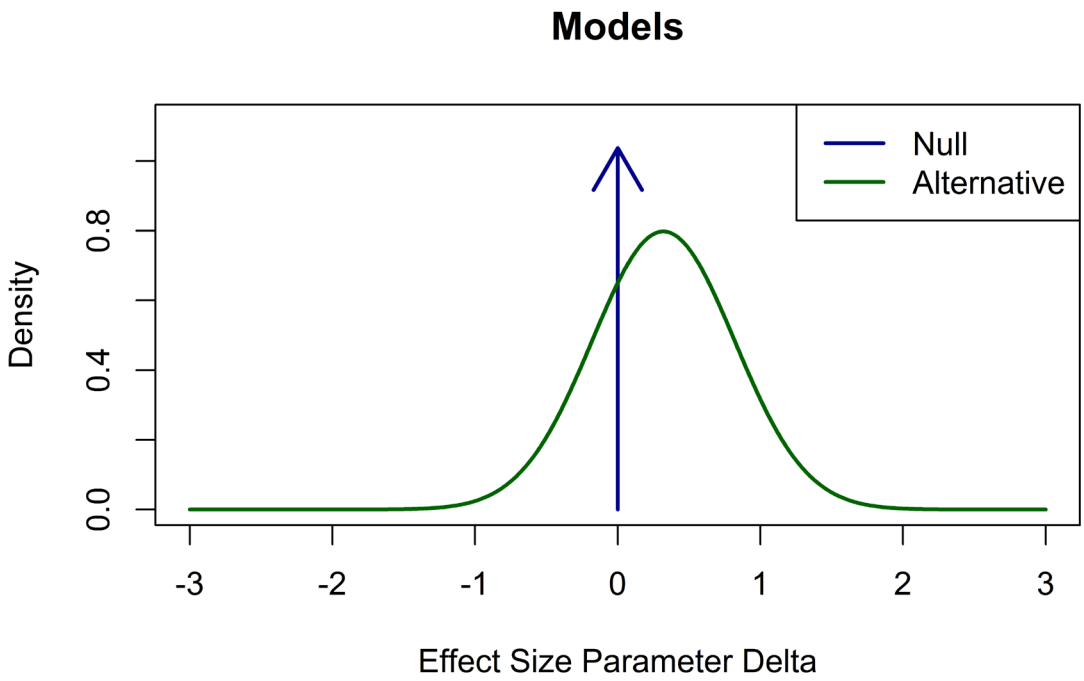
Results and Discussion

Across all 100 sessions, participants correctly identified the future position of the erotic pictures significantly more frequently than the 50% hit rate expected by chance: 53.1%, $t(99) = 2.51$, $p = .01$, $d = 0.25$.[3] In contrast, their hit rate on the nonerotic pictures did not differ significantly from chance: 49.8%, $t(99) = -0.15$, $p = .56$. This was true across all types of nonerotic pictures: neutral pictures, 49.6%; negative pictures, 51.3%; positive pictures, 49.4%; and romantic but nonerotic pictures, 50.2%.

We will calculate a Bayes Factor for this experiment. To do so, we need to choose a prior for the null-model, and a prior for the alternative. The null-prior we will use is known as a point prior of $d_z = 0$. Because Bayesian priors are normally distributions, and a $d_z = 0$ point prior means the density at this point goes up to infinity (which is illustrated by an arrow in the figures below). For the alternative hypothesis, we'll use a normal distribution for the effect size, which means we need to specify a mean and standard deviation that reflects our prior. We don't know which prior Daryl Bem had for the expected effect size, but with $N = 100$, an experiment has 90% power to observe a d_z of 0.32. So let's assume a prior belief of an effect size around d_z of 0.32. We also need to specify the standard deviation of the normal distribution – the higher the standard deviation, the wider the distribution. With a standard deviation of 0.2, the prior looks like:



With a standard deviation of 0.5, the prior looks like the graph below:

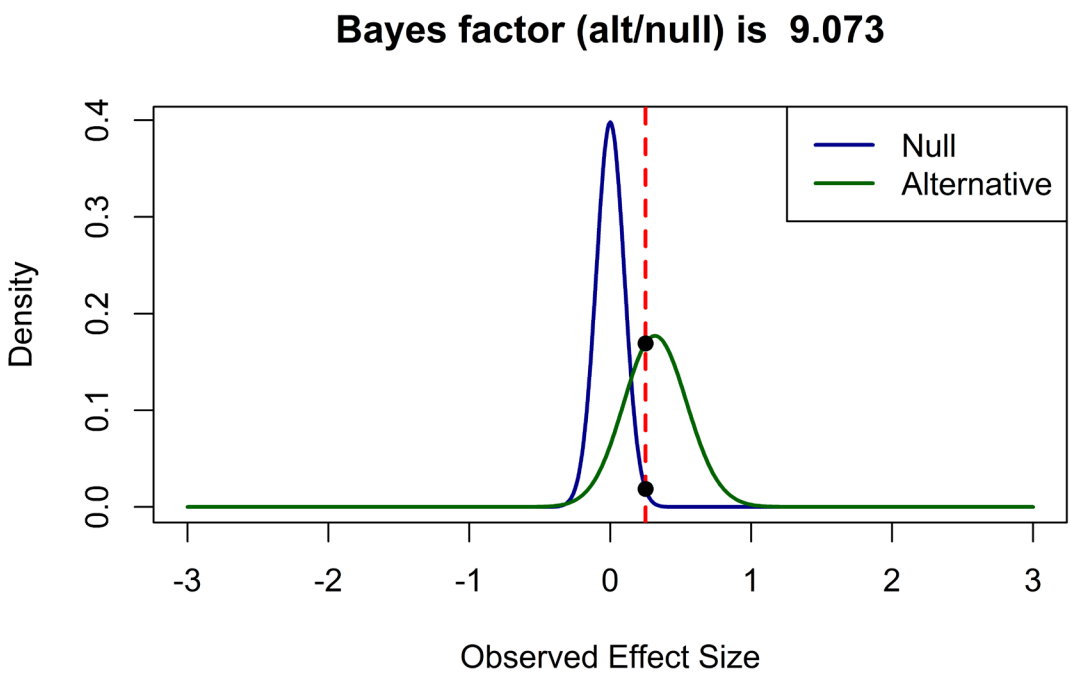


Because pre-cognition effects are very unlikely to be so large as to include $d_z = 1$, and are much more likely small, let's use a *sd* = 0.2 for the prior here (but remember: this is your

prior, so there are no right or wrong answers, as long as the prior correctly represents your belief!).

Open the 'Bayes Factor One-Sided T-test.R' script. Daryl Bem observed an effect size of d_z = 0.25 with N = 100. In line number 4 (see the black line numbers on the left of the source window) you can enter the sample size, and in line 5 you can fill in the **observed** effect size d_z. In lines 7 and 8 you can fill in the **expected** effect size for the prior (0.32), and the **expected** standard deviation for the prior (0.2).

Run the code, which will create two plots (remember you can browse through plots with the blue left and right arrows in the Rstudio interface). The first is a plot for the prior (the top Figure on the previous page), and the second plot looks like the figure below.



The vertical red dotted line indicates our observed effect size. The blue line gives the posterior for the null model, and the green line gives the posterior for the alternative model. We can see that given the observed effect size of d_z = 0.25, the data is more likely under the alternative model than under the null model, and the Bayes Factor tells us that given this data and our prior, the alternative model has become 9.073 times more likely.

Bayes Factors can also be used to provide support for the null hypothesis. Let's take a look at the reported effect for the remaining stimulus types in the study. Performance on

these trials did not differ from guessing average, $t(99) = -0.15$. This corresponds to a Cohen's d_z of -0.015.

Q11) In line 5, change the observed effect size to -0.015. Run the code, using the same prior. What can we conclude? The title of the figure provides the Bayes Factor for the alternative over the null (BF_{10}). Remember that a Bayes Factor of 1 means the data is equally likely under both models – values smaller than $BF_{10} = 1$ mean the data are more likely under the null hypothesis. You can reverse the BF_{10} to reflect how much more likely the data is under the null compared to under the alternative (BF_{01}) by computing 1/BF. When the Bayes factor is smaller than 0.333 or larger than 3, the Bayes Factor can be interpreted as modest support. When the Bayes factor is smaller than 0.1 or larger than 10 the Bayes Factor can be interpreted as strong support. When the Bayes factor falls within the 0.333 to 3 range, it is considered too weak support for either hypothesis to draw a conclusion about the data.

A) The data is more likely under the alternative model than under the null model, with a BF_{10} of 0.147

B) The data is more likely under the alternative model than under the null model, with a BF_{10} of 14.70

C) The data is more likely under the null model than under the alternative model, with a BF_{10} of 0.147

D) The data is more likely under the null model than under the alternative model, with a BF_{10} of 14.70

In 2015 Bem and colleagues published a meta-analysis of pre-cognition effects. In this meta-analysis, the authors argue pre-cognition has a meta-analytic effect size of Hedges' $g = 0.09$.

Q12) In line 7, change the prior to an effect size of dz_prior = 0.09. Given this prior, can we still conclude that the Bayes Factor provides support for the null model for the remaining conditions, where the statistical test was $t(99) = -0.15$, $d_z = -0.015$?

A) With a $BF_{10} = 7.365$, the data now actually provide support for the alternative model, compared to the null model.

B) With a $BF_{10} = 0.405$, we can no longer conclude the data provide support for the null model, compared to the alternative model.

C) With a BF_{10} = 0.405, the data provide support for the null model, compared to the alternative model.

Conclusion

It is important to be able to provide support for the null-hypothesis, if you want to be able to test theories that predict no effect, or when you want to be able to falsify theory that predict an effect. According to Popper, falsifiability is the demarcation criteria between science and pseudoscience, so being able to falsify hypotheses is very important. According to Lakatos, even though we rarely outright reject our hypotheses, we enter a degenerative research line when our alternative hypothesis is rejected. Finally, being able to conclude an effect is too small to be worthwhile to examine enables us to improve our statistical inferences.

You will always have to make assumptions about the alternative hypothesis, either by specifying an equivalence region consisting of a range of effect sizes you find meaningful, or by specifying a prior distribution. You can use either Frequentist or Bayesian approaches to test absence of evidence (or even both!). Don't simply conclude that a $p > 0.05$ means there is no effect – instead, provide quantitative arguments for the conclusion that there is no effect by using equivalence tests or Bayes Factors.

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