

# Tutorial 8

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## 1 Introduction

In this tutorial we try to estimate posterior probability. A short description of the ELBO algorithm with some important derivations can be found in the introduction to the theoretical part. In the practical assignment I followed the pseudo code and filled the missing parts of the provided code in order to implement the CAVI algorithm.

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**Algorithm 1** CAVI algorithm

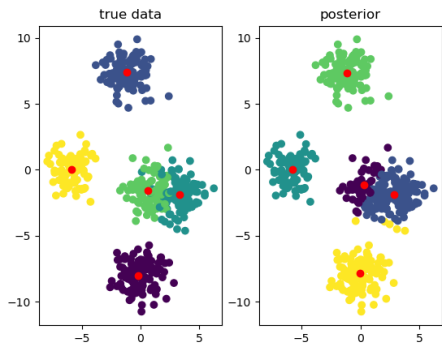
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```
while the ELBO has not converged do
  for  $i \in \{1, \dots, N\}$  do
    Update  $\phi_i$ 
  end for
  for  $k \in \{1, \dots, K\}$  do
    Update  $m_k$ 
    Update  $s_k^2$ 
  end for
  Compute  $ELBO(m, s_k^2, \phi_i)$ 
end while
```

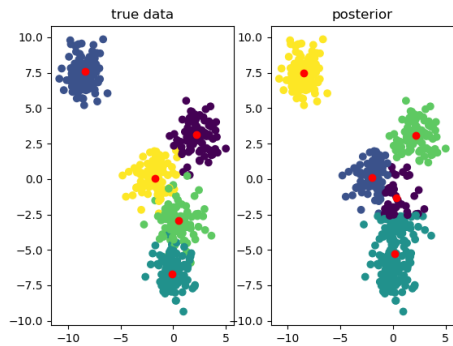
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## 2 Results

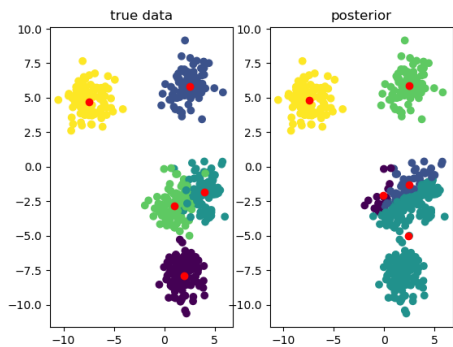
As we can observe in the figures below, the algorithm could quite well estimate the posterior probability of the model. Some harder examples where clusters heavily overlap, show higher error and the predicted posterior does not correspond that well with the ground truth. From the other hand, centroids of clearly separated classes are always correctly predicted. ELBO values vary between -4000 and -2000 depending on the run. In the first *four* runs I forced the algorithm to run for at least 20 steps by adding one more condition to the *while* loop, whereas in the 5<sup>th</sup> and 6<sup>th</sup> experiment, I just kept the original convergence condition.



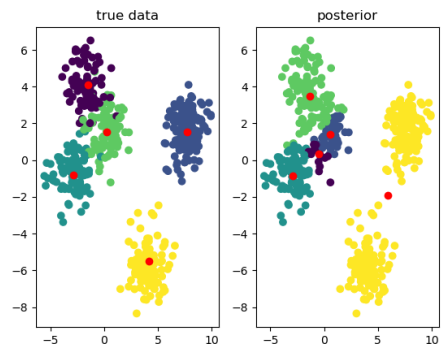
(a) Experiment 1



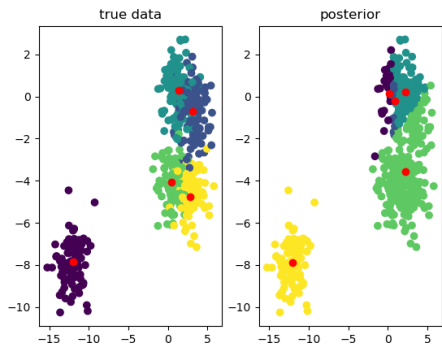
(b) Experiment 2



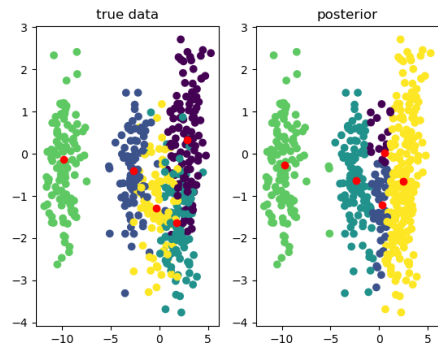
(c) Experiment 3



(d) Experiment 4

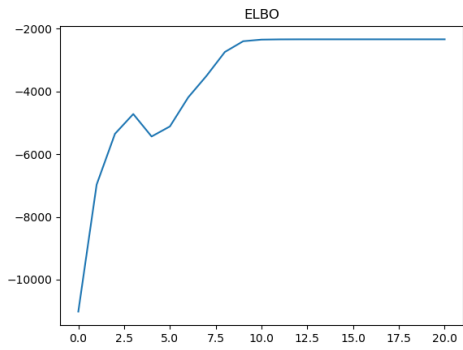


(e) Experiment 5

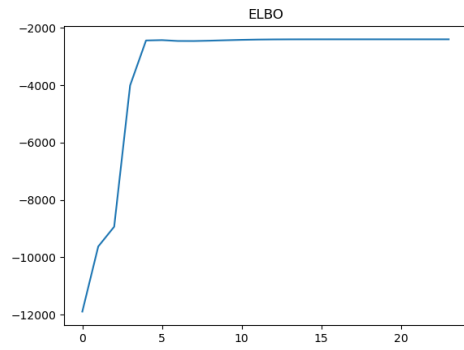


(f) Experiment 6

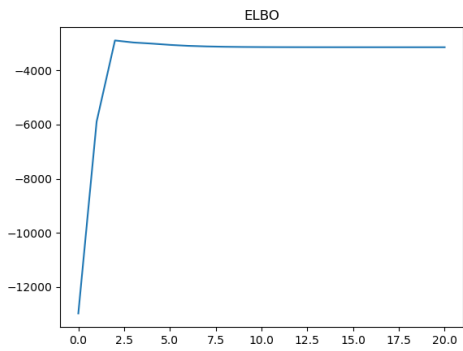
Figure 1: Posterior and true data



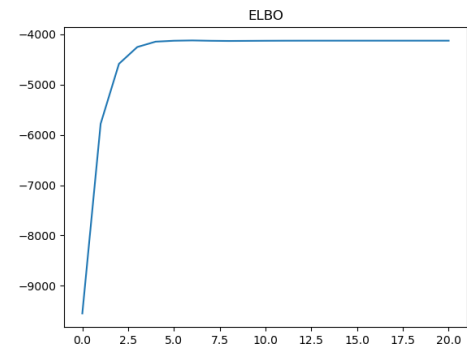
(a) Experiment 1



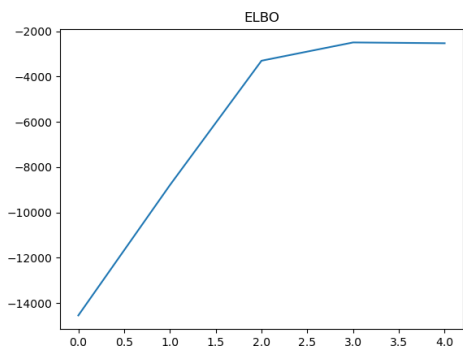
(b) Experiment 2



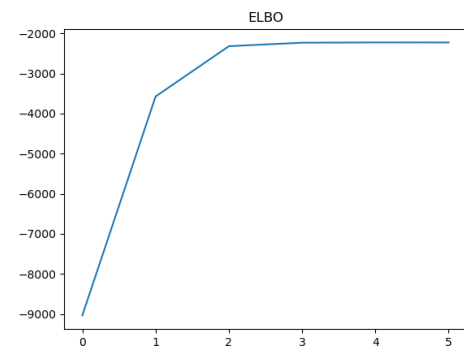
(c) Experiment 3



(d) Experiment 4



(e) Experiment 5



(f) Experiment 6

Figure 2: ELBO values over the iterations