Optimization of surfboard thickness.

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Resume Optimizing thickness of the surfboard in order to minimize its bending energy while keeping its buckling strength and mass within the limits.

Key Words FEM · No-linear optimization · Sandwich structure

1 Introduction

Surfboard is one of the most common sandwich structures. It is constructed from 2 main parts (fig.1):
- Core, which is either polyurethane or polystyrene foam,
- Ply, usually made from fiberglass and either polyester or epoxy resin.

When we are looking at the surfboard during its usage, we can see that most of the failures are caused by bending. Usually to reinforce the board for this particular matter, the wooden stringer is being used. However, it increases the weight of the board and similar effect can be achieved also by increasing the thickness of the ply. In this work, the main goal was to reach the balance between the thickness of each component and minimum bending energy.

2 Definition of engineering problem

The main goal is to find the proper thickness of the core and ply. To simplify the problem for the calculations, example of a sandwich structure beam with constant thickness on the whole length was used. In order to check if the calculations are accurate, fine element analysis was performed at the end.

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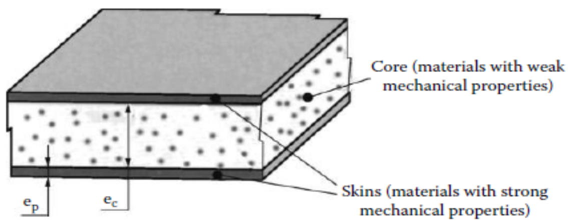

Figure 1. Structure of a sandwich composite.

3 Formulation of optimization problem

The function to minimize is bending energy equation [1]. First constrain which needs to be considered is the mass of the board [5], which should not be higher than 4.5 kg

Main function:

$$W(e_p,e_c) = \int_0^l \frac{M^2}{2EI}dx + \int_0^l \frac{kT^2}{2GS}dx = \tag{1}$$

$$\int_0^l \frac{M^2}{E_p e_p (e_c + e_c)^2}dx + \int_0^l \frac{T^2}{2(e_c + 2e_p)G_c l}dx \tag{2}$$

Constrains:

$$M(e_p,e_c) = \rho_c V \frac{e_c}{e} + \rho_p V \frac{e_p}{e} < 4.5kg \tag{3}$$

$$F(e_p,e_c) = 1.64 e_p l E_p (\frac{E_c e_p}{E_p e_c})^{\frac{1}{2}} < 1080N \tag{4}$$

Bounds:

$$1 < e_p < 3 \tag{5}$$

$$30 < e_c < 70 \tag{6}$$

4 Sensible analysis

After formulating the problems it was necessary to see graphical dependency of the constraints on different thickness. Using python library matplotlib formulas and 3D graphs was generated. It can be easily seen that the mass increases with an increment of both e_p and e_c (fig. 2). However, it is quite interesting that the maximum

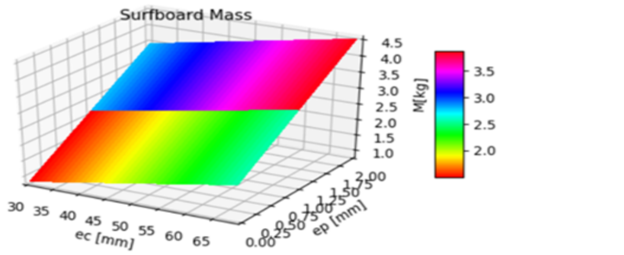

Figure 2. Dependency of surfboard mass from its thickness.

buckling force, decreases with decrement of core thickness (fig.3).

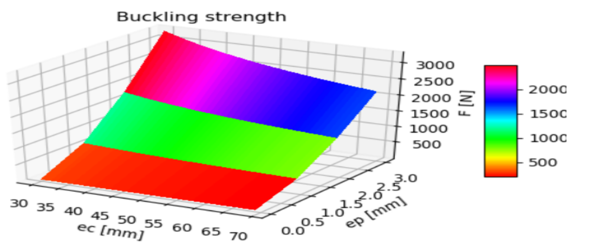

Figure 3. Dependency of surfboard buckling strength from its thickness.

5 Avaliation

To solve the problem, Sequantial Quadratic Programing [4] method with multistart was chosen. The multistart was added to the method in order to make sure that the program is not finding local minimums. The initial values of e_c were varied between 30 and 50 mm and e_p between 1 to 3 mm. The properties of material were either calculated or found in the literature on composite materials.

6 Algorithm

As it was mentioned before Sequential Quadratic Programming algorithm with multistart was used to calculate this problem (fig.4). After calculating the constraints, Jacobian matrix is calculated in order to discover monotonicity of the functions. Next, df/dx is calculated and added to the initial value or value from previous iteration $x_t =_{xt-1} +dx$. If x_t is within the constraints and bounds and KKT conditions are satisfying, the solution is saved into the array and program runs again with new initial values. If not, next iteration is made. After minimum is found for all different initial conditions, the minimum is chosen from the array as the final solution [3].

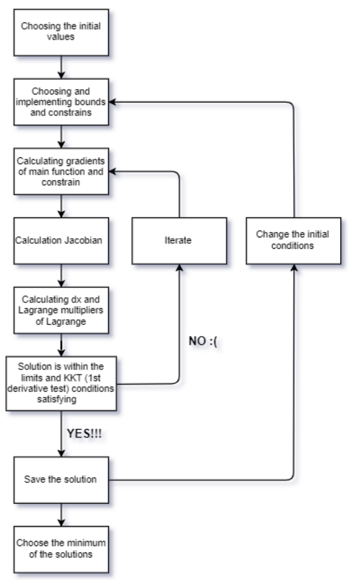

Figure 4. SQP algoritm, used in the calculations.

7 Results

After getting the results it was quite clear, that the method is finding the proper values which are global, not local minimum, because even though the multistart with different initial values was used, the variations of results were not bigger than 5.% The minimum value of the function was about 70 MPa , while using e_c=33 mm and e_p=2.75 mm (fig.5). The constrain values for this thicknesses are equal to:

- Mass = 4.5 kg
- Buckling strength = 2485 N

These values are in the limits of the constrains. Mass is maximal and the buckling strength more then 50% higher than the minimal. However, it is quite interesting that the maximum buckling force, decreases with decrement of core thickness. After getting these results, the CAD model of the board was created and the stress and strain were calculated using FEM simulation. First, the simple beam was calculated and the results were quite close to the ones calculated by the formula and were equal to 82 MPa (fig. 6). It is interesting, there were

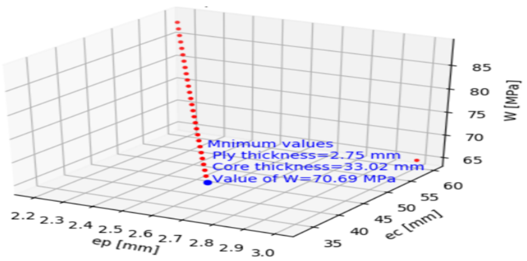

Figure 5. Graph of the result for one of the initial values.

almost no stress in the core part of the beam and the biggest stresses appeared on the supports.

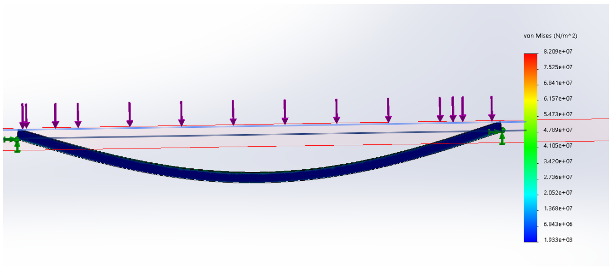

Figure 6. FE analysis of the sandwich structure beam.

After that the simulation on the real board with the surfer on it was run and results were much smaller then in the beam, about 35.4 MPa in the boarder of fixed surface and unfixed one (fig.7).

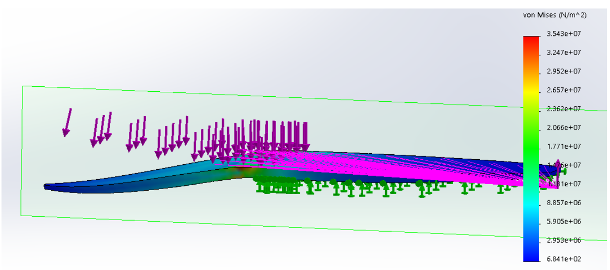

Figure 7. FE analysis of the sandwich structure surfboard.

8 Conclusion

The optimization method in this case was very close to the FEM solution. The difference between this values may be the result of CAD program using different evaluation methods. The biggest problem is implementing these values in the real life in the shaping room. The core is usually being made by CNC but glassing is made by hands. Even if one uses the vacuum method for the glassing, it is still hard to reach thickness accuracy up to 0.1 mm. What is a bit concerning is that the mass of the board is almost out of the limits, where the buckling strength is much higher than required. What can be done is slightly decrease the mass, sacrificing bending strength a bit, but reducing the weight.

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