Data Representation

Problem Solving

Tirtharaj Dash

BITS Pilani, K.K. Birla Goa Campus

tirtharaj@goa.bits-pilani.ac.in

September 18, 2020

Question Given two matrices

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 4 & 5 \\ 6 & 7 & 7 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{bmatrix}$$

What can you say about these two matrices?

A. **B** is inverse of **A**, but not vice-versa.
B. **A** is inverse of **B**, but not vice-versa.
C. **A** and **B** are inverse of each other.
D. None of the above.

Q & A II

Answer $\mathbf{A}$ and $\mathbf{B}$ are inverse of each other, since $\mathbf{AB} = \mathbf{I} = \mathbf{BA}$. Here, $\mathbf{AB}$ denotes the matrix multiplication of $\mathbf{A}$ and $\mathbf{B}$. $\mathbf{I}$ is the identity matrix.

Q & A III

Question What is the Frobenius norm of the following matrix?

$$\mathbf{B} = \begin{bmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{bmatrix}$$

- (A) -14.0357
- (B) -10
- (C) 10
- (D) 14.0357

Q & A IV

Answer $||\mathbf{B}||_F = \left(\sum_{i,j} b_{i,j}^2\right)^{\frac{1}{2}} = 14.0357$

Q & A V

Question For $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$. Which of the following are the two eigenvalues of $\mathbf{A}$?

A. $0, -5$
B. $5, -5$
C. $0 + i, -5 + i$
D. None of the above

Q & A VI

Answer Characteristic equation is $\det(\mathbf{A} - \lambda \mathbf{I}) = 0$. Roots of this equation are 0, 5.

Q & A VII

Question Let **A** be a 2 matrix. Let the trace of **A** is -1, and its determinant is -30. Its eigenvalues are:

Q & A VIII

Answer $tr(\mathbf{A})$ is the sum of the diagonal elements of $\mathbf{A}$. There are the following relationships between eigenvalues of a matrix with trace and determinant:

- Sum of eigenvalues is the trace of the matrix
- Product of eigenvalues is the determinant of the matrix

So, $\lambda_1 + \lambda_2 = -1$, and $\lambda_1 \times \lambda_2 = -30$. Solving these, we get: $\lambda_1 = 5, \lambda_2 = -6$.

Q & A IX

Question Find out the eigenvalues for the matrix

$$\mathbf{A} = \begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix}$$

Answer The eigenvalues are the roots of of the equation

$$\det\left(\begin{bmatrix}-6 & 3\\ 4 & 5\end{bmatrix} - \lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right) = 0$$

That is

$$\det\left(\begin{bmatrix}-6-\lambda & 3\\ 4 & 5-\lambda\end{bmatrix}\right) = 0$$

or, $(-6-\lambda)(5-\lambda) - 3 \times 4 = 0$ or, $\lambda = -7, 6$.

Q & A XI

Question Find out the eigenvectors of $\mathbf{A}$ for the eigenvalues you obtained in the previous question.

Q & A XII

Answer Let $\mathbf{v} = [x, y]^\mathsf{T}$ be the eigenvector. Now, we have to solve

$$\mathbf{Av} = \lambda \mathbf{v}$$

or,

$$\begin{bmatrix} -6 & 3 \\ 4 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix}$$

So, we get two equations for $\lambda = 6$:

$$-6x + 3y = 6x$$
$$4x + 5y = 6y$$

Simplifying this we get $y = 4x$. So, we can fix: $x = 1, y = 4$.
(Similarly, find the eigenvectors for $\lambda = -7$.)