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Introduction to Data Science

(BA ZG523 / CSI ZG523)

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Introduction to Data Science
Lecture No. 05 (C.H.: 9, 10)

Previous Lecture:

- Data
- Types of data sources
  - Transactions data
  - Unstructured data
  - Qualitative or expert data
  - Publicly available data
- Sampling
- Types of data elements
  - Continuous
  - Categorical (nominal, ordinal, binary)
- Visualizing data
  - Exploration (plotting) and analysis (basic statistical measurements)
- Missing values
  - Replace, delete, keep

Sampling

- **Population:** The entire group that you want to draw conclusions about.
- **Sample:** A subset of population.
  - A well-chosen sample will contain most of the information about the population.
  - The relation between sample and population should be such that the inferences made from the sample should apply to the population as well.
- **Variable:** It is a characteristic that describes the member of a sample.
  - We will refer to it as “attribute” or “feature”.

Sampling methods

We will study two types:

1. Probabilistic sampling
2. Non probabilistic sampling

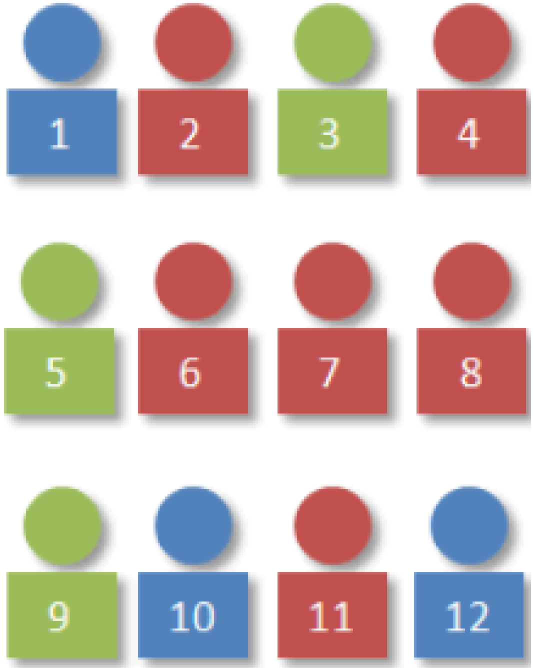
Probabilistic Sampling

- Select the members of population that have equal or non-zero probability.
- Types:
  - Simple sampling
  - Systematic sampling
  - Stratified sampling
  - Cluster sampling

Simple sampling

- A sample of size *n* from a population of size *N* is obtained through **simple random sampling** if every possible sample of size *n* has an equally likely chance of occurring.

- Select 4 members from this group.

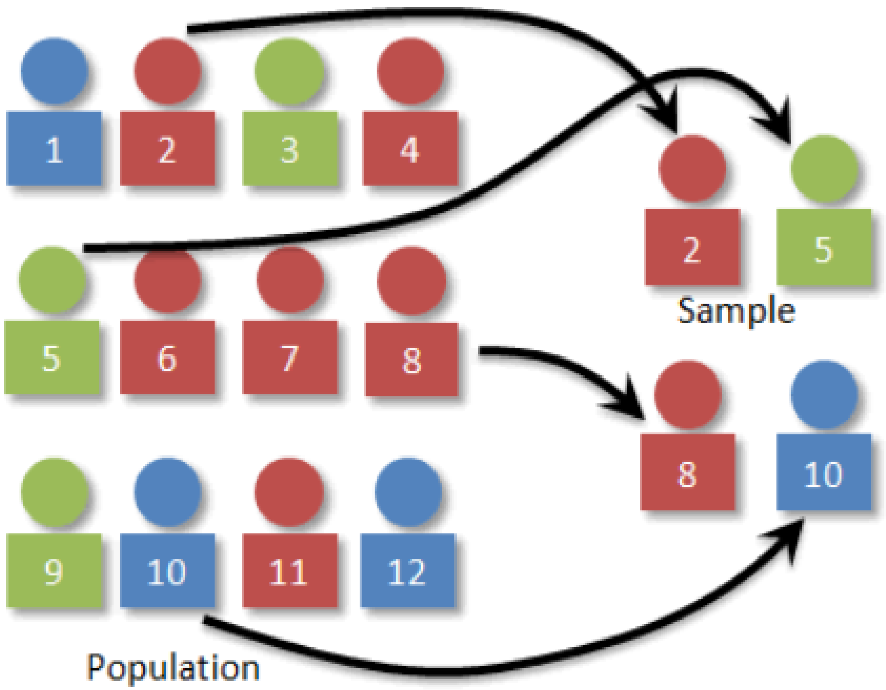


Population [1]

[1] https://faculty.elgin.edu/dkernler/statistics/ch01/1-3.html

Cont.

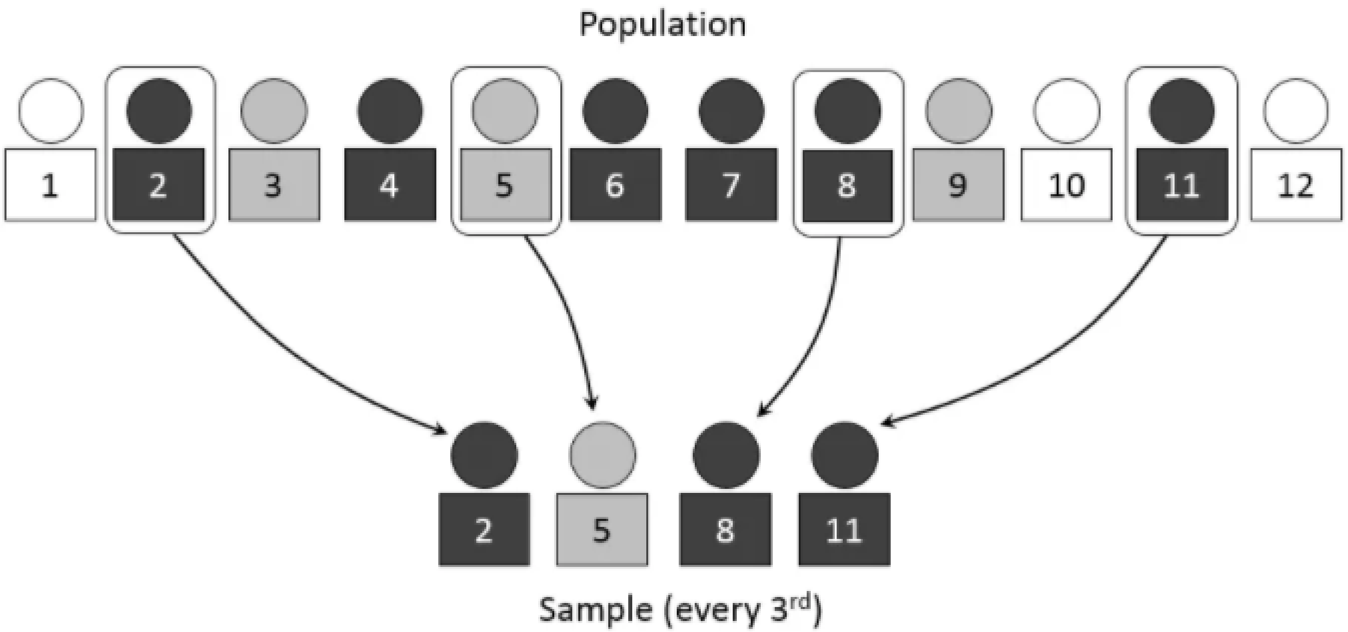
A possible sample from the give population:



Sampled 4 members [1]

Systematic sampling

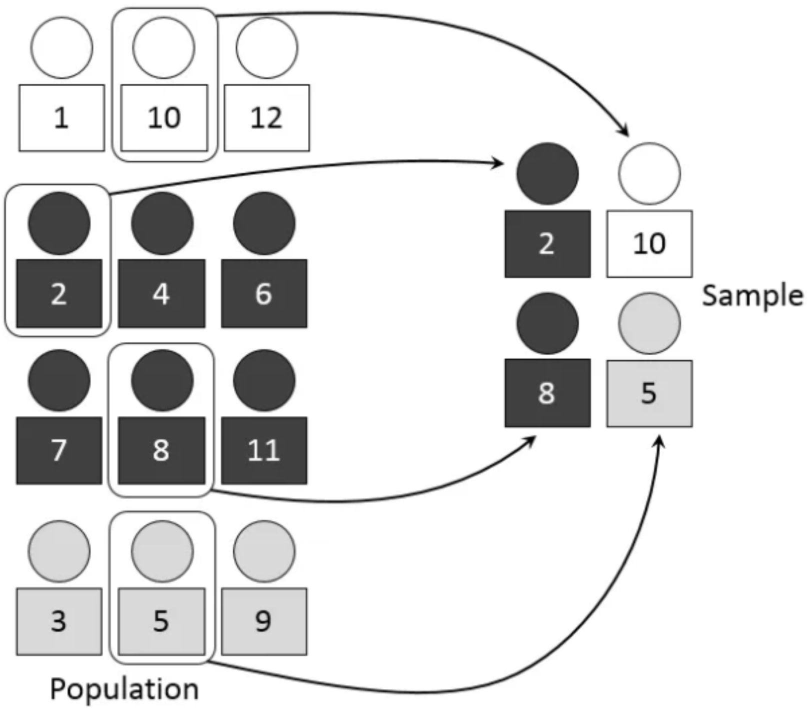
- Pick up the members from the population through a well-defined system to make a Sample.
- Sampling is done based on some given condition.



Source: Wikipedia

Stratified sampling

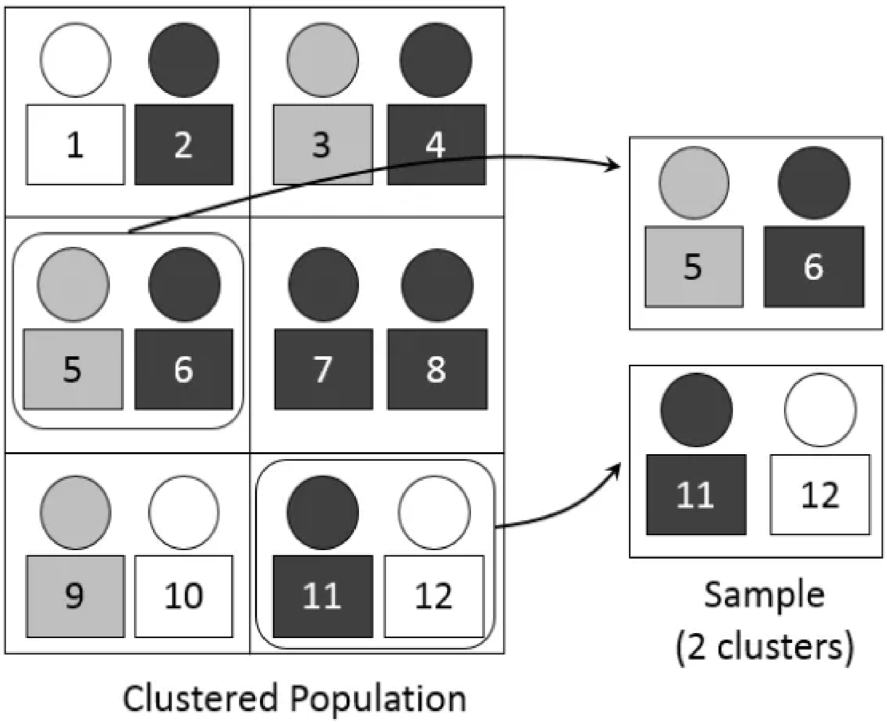
- First, stratify to make an ordered or categorized samples from the population called as **strata**.
- Then, choose members from each stratum for making a sample.



Source: Wikipedia

Cluster sampling

- Divide the population into groups call as clusters.
- Use simple random sampling to select the cluster to form a sample. (*mostly common in geography*)



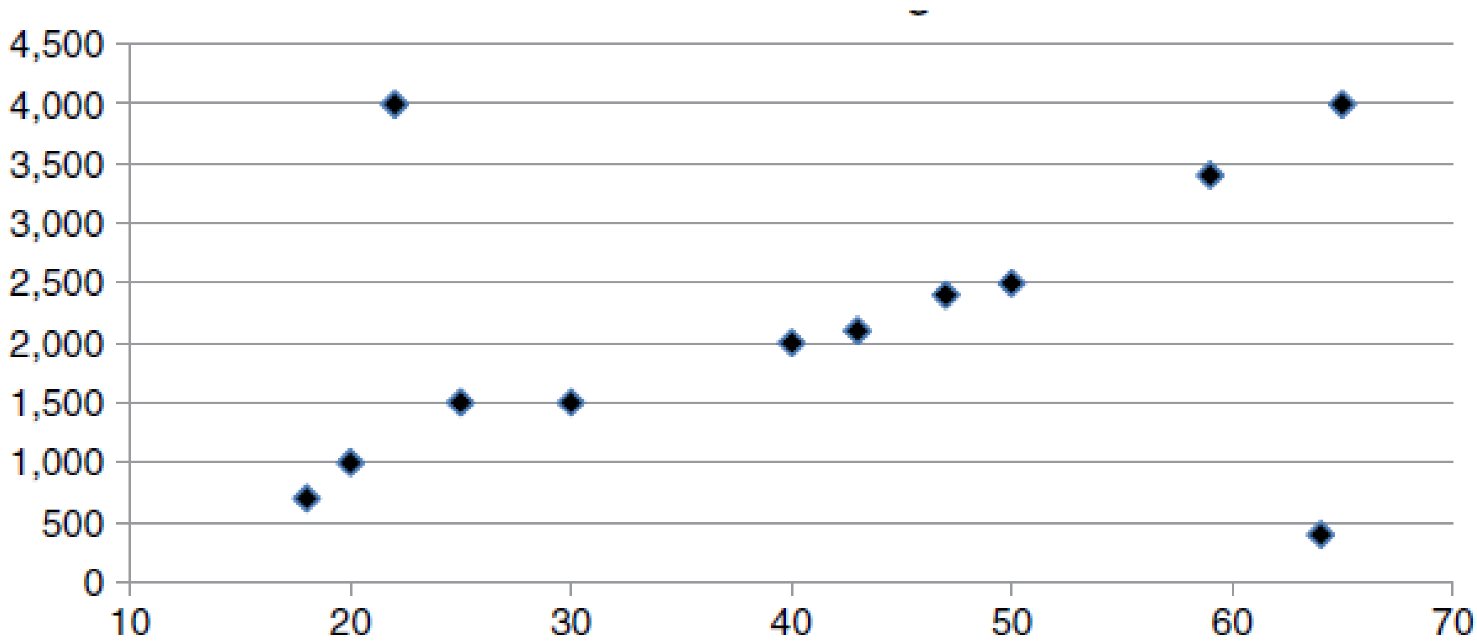
Source: Wikipedia

Outliers

- Outliers are extreme observations that are very dissimilar to the rest of the population.
- Two types of outliers:
  1. Valid observations (e.g., salary of boss is $1 million)
  2. Invalid observations (e.g., age is 300 years)
- Both (1) and (2) are univariate (outlying on 1 dimension).
- Multivariate outliers are observations that are outlying in multiple dimensions.

Cont.

- Multivariate outliers



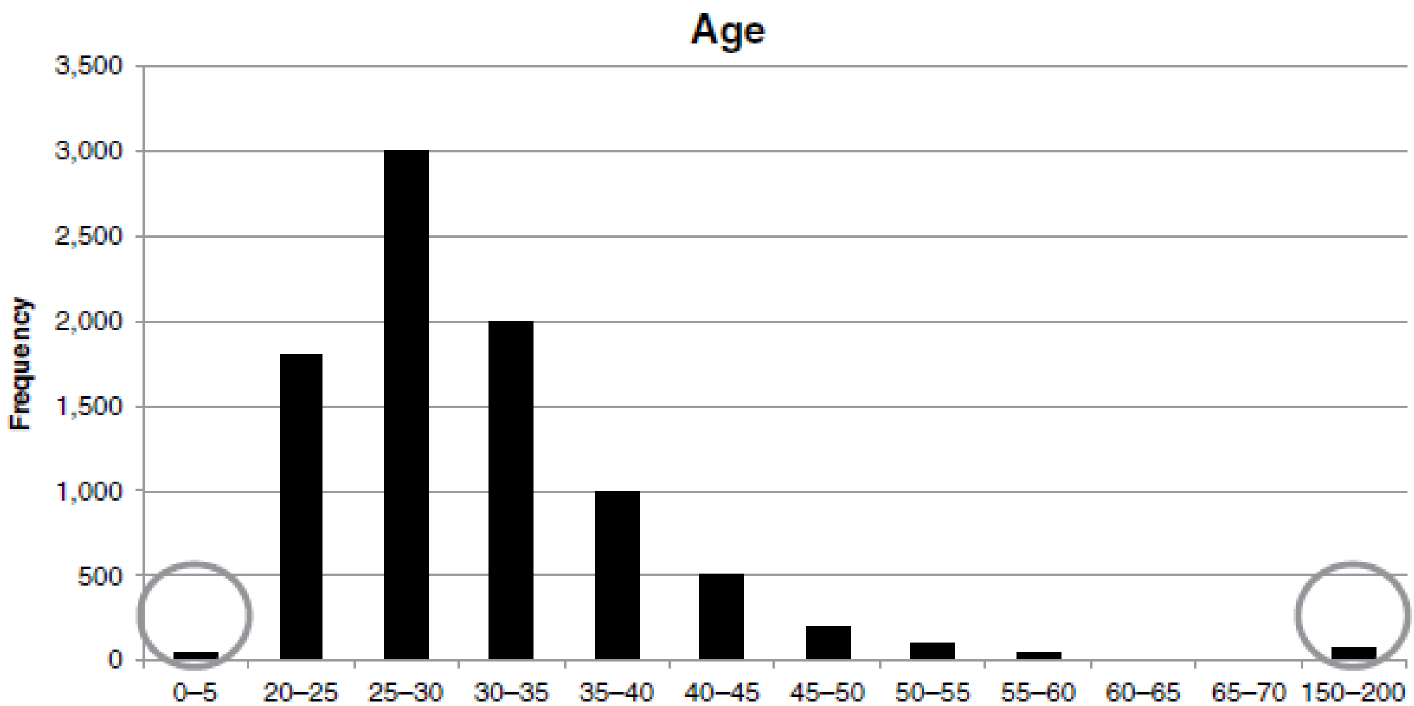
Income and age (Source: T4, Ch.2)

Detecting univariate outliers

- Two standard ways:
  - Visualizing data
  - Statistical measurements
- Visualization based methods:
  - Histogram
  - Box-plot
- Statistical measurements:
  - z-score

Cont.

- Using histogram for outlier detection:

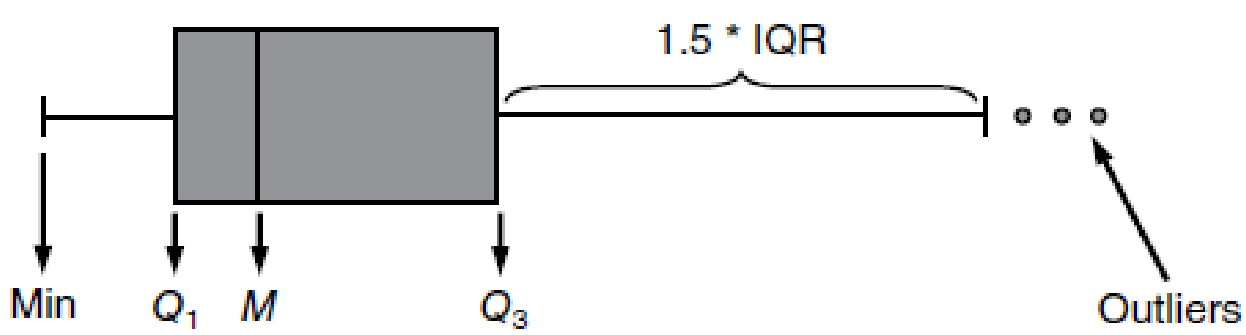


Example: Histogram of age (Source: T4, Ch.2)

Contd.

- Using box-plot for outlier detection:
  - It represents 3 key quartiles of the data: the first quartile (25% of the observations have a lower value), the median (50% of the observations have a lower value), and the third quartile (75% of the observations have a lower value).
  - All three quartiles are represented as a box.
  - The minimum and maximum values are then also added unless they are too far away from the edges of the box.
  - Too far away is then quantified as more than 1.5 * Interquartile Range:

$$IQR = Q3 - Q1$$



Box-plot (Source: T4, Ch.2)

Cont.

- z-score method for outlier detection:
  - Measures how many standard deviation an observation lies away from the mean.

$$z_i = \frac{x_i - \mu}{\sigma}$$

  - Here, μ is the mean of X; x_i is an observation that belongs to X; the denominator is the standard deviation.
  - A practical rule of thumb: Outliers when the absolute value of the *z*-score |z| is greater than 3.
  - Note: z-score relies on the normal distribution.

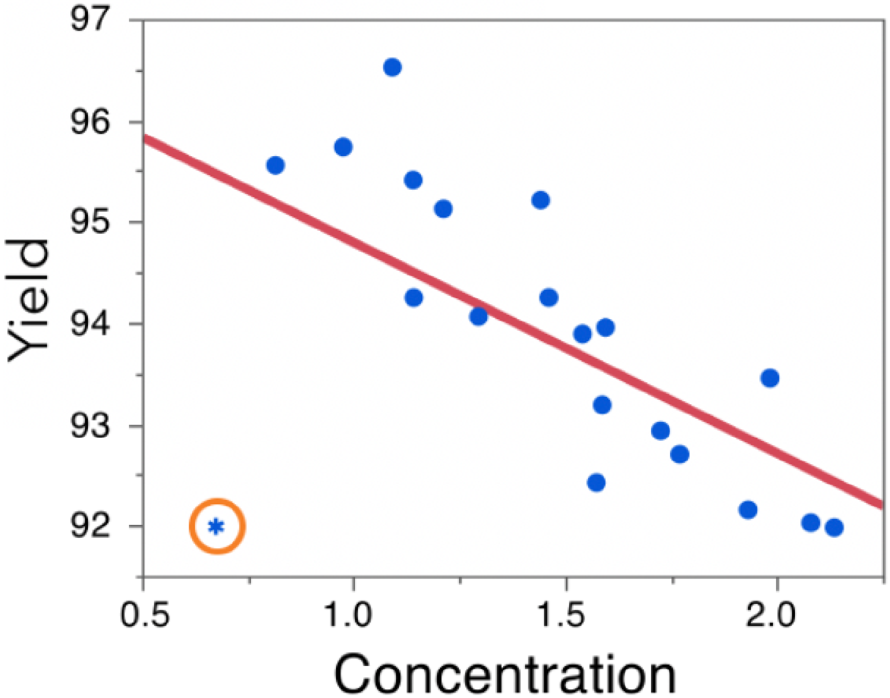
Cont.

Age	Z-Score
30	$(30 - 40)/10 = -1$
50	$(50 - 40)/10 = +1$
10	$(10 - 40)/10 = -3$
40	$(40 - 40)/10 = 0$
60	$(60 - 40)/10 = +2$
80	$(80 - 40)/10 = +4$
...	...
$\underline{\mu = 40}$ $\underline{\sigma = 10}$	$\underline{\mu = 0}$ $\underline{\sigma = 1}$

z-score based outlier detection for Age (Source: T4, Ch.2)

Detecting multivariate outliers

- Multivariate outliers can be detected by fitting regression lines and inspecting the observations with large errors (using, for example, a residual plot).



Source: [2]

[2] https://www.jmp.com/

Cont.

- Alternative methods are clustering or calculating the Mahalanobis distance.
- Note: Although potentially useful, multivariate outlier detection is typically not considered in many modeling exercises due to the typical marginal impact on model performance.

Standardizing Data

- It is a data preprocessing activity targeted at scaling variables to a similar range.
- Why should you standardize data?

Standardizing Data

- It is a data preprocessing activity targeted at scaling variables to a similar range.
- Why should you standardize data?
  - Consider, for example, two variables: gender (coded as 0/1) and income (ranging between $0 and $1 million).
  - When building logistic regression models using both information elements, the coefficient for income might become very small.
  - Hence, it could make sense to bring them back to a similar scale.

Cont.

- Methods:
  - Min/Max standardization
  - z-score standardization
  - Decimal scaling

Contd.

- Min/Max standardization

$$X_{new} = \frac{X_{old} - \min(X_{old})}{\max(X_{old}) - \min(X_{old})}(newmax - newmin) + newmin,$$

  - *newmax* and *newmin* are the newly imposed maximum and minimum (e.g., 1 and 0).

Contd.

- z-score standardization

$$z_i = \frac{x_i - \mu}{\sigma}$$

  - This becomes the standardized value for an observation x_i.

- Decimal scaling
  - Divide by a power of 10 as follows:

$$X_{new} = \frac{X_{old}}{10^n}$$

  - Here, *n* is the number of digits of the maximum absolute value.

Data Quality

- 4 categories:
  - Intrinsic
  - Contextual
  - Representational
  - Accessibility

Cont.

- Intrinsic category
  - **Accuracy**: Data are regarded as correct
  - **Believability**: Data are accepted or regarded as true, real, and credible
  - **Objectivity**: Data are unbiased and impartial
  - **Reputation**: Data are trusted or highly regarded in terms of their source and content

- Contextual category
  - **Value-added:** Data are beneficial and provide advantages for their use
  - **Completeness**: Data values are present
  - **Relevancy**: Data are applicable and useful for the task at hand
  - **Appropriate amount of data**: The quantity or volume of available data is appropriate

Cont.

- Representational category
  - **Interpretability**: Data are in appropriate language and unit and the data definitions are clear
  - **Ease of understanding**: Data are clear without ambiguity and easily comprehended

Cont.

- Accessibility category
  - **Accessibility**: Data are available or easily and quickly retrieved
  - **Security**: Access to data can be restricted and hence kept secure.

High-dimensional Data

- Dimensionality refers to the number of features (attributes) that defines a data point or a data instance.
- Example: In one of our experiments, we saw that the an Iris flower is described by 4 features:
  - Sepal length
  - Sepal width
  - Petal length
  - Petal width

- For Iris dataset, dimensionality is 4.

Cont.

There are some common misunderstandings and wrong definitions:

- Large number of features
  - My question: How much large is “large”? Is it 1000, 10000, 100000000?
- When our computer cannot handle the given data dimensions
  - My question: What if someone else’s computer can?
- There are many such definitions available all over the internet. Do not read such non-scientific definitions.

Cont.

Definition: Let a dataset contain *N* data instances. Each data instance can be described by *d* features. The dataset is called "high-dimensional" data, iff ***d* > *N***.

Example:

- *N*=5, *d* = 10: high-dimensional data
- *N*=100, *d*=101: high-dimensional data
- *N*=1000000, *d*=10000: not high-dimensional data

Question:

- A dataset of 1M images, of size 128×128×3. Will you call this dataset high-dimensional?

Cont.

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Example:

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- N=100, d=101: high-dimensional data
- N=1000000, d=10000: not high-dimensional data

Question:

- A dataset of 1M images, of size 128×128×3. Will you call this dataset high-dimensional? Answer: **No**

Cont.

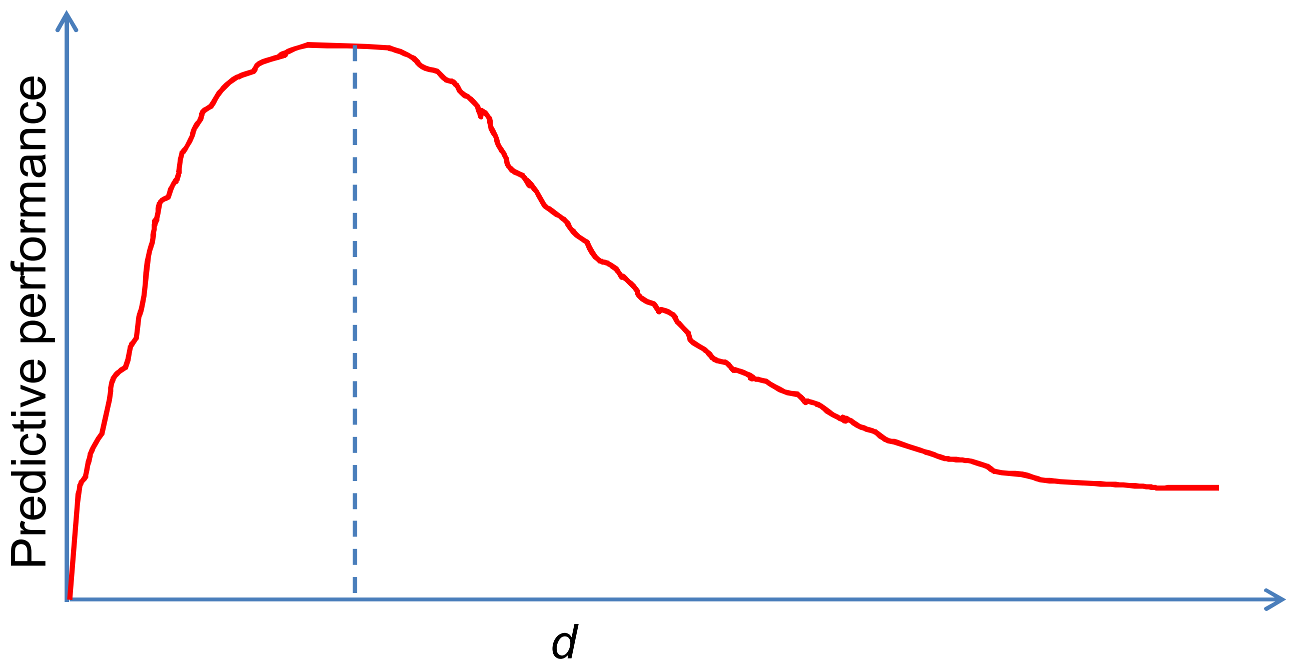
Is high-dimensionality a problem?

- Curse of dimensionality
- Nearest-neighbor problem

Cont.

Curse of dimensionality

- With a fixed number of training samples, the predictive power of a classifier or regressor first increases as d increases, and then decreases with *d*.



Cont.

Nearest-neighbor problem

- Let's look at the following example of determining nearest neighbor (used in *k*-NN, geoinformatics, etc.)

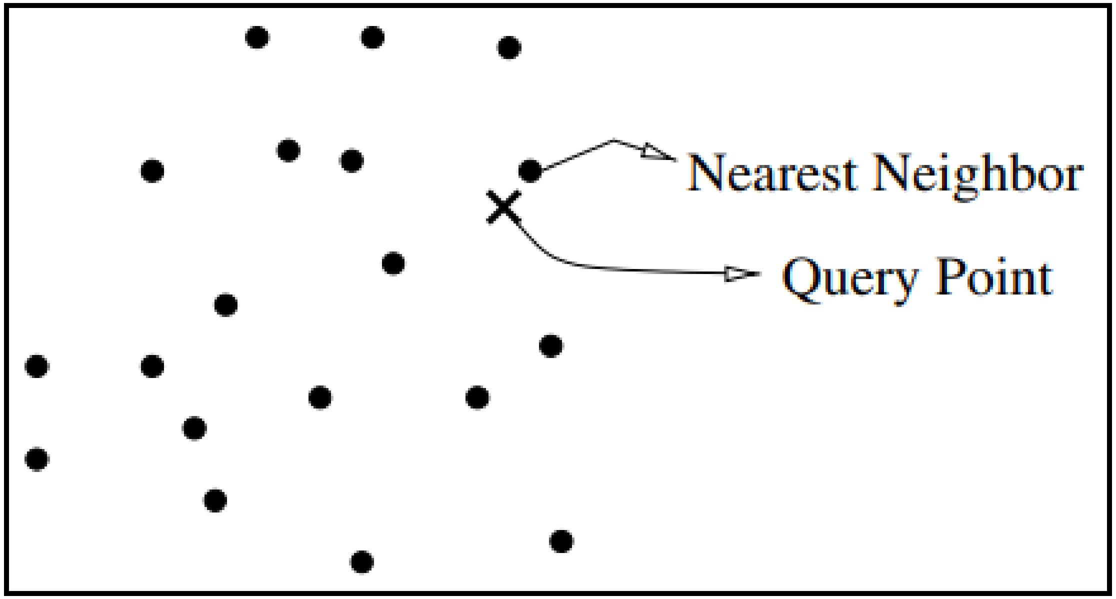


Fig 1: Query point (*Q*), and its neighbor [3]

[3] Beyer *et al.:* https://doi.org/10.1007/3-540-49257-7_15

Cont.

Nearest-neighbor problem

- But, what happens now?

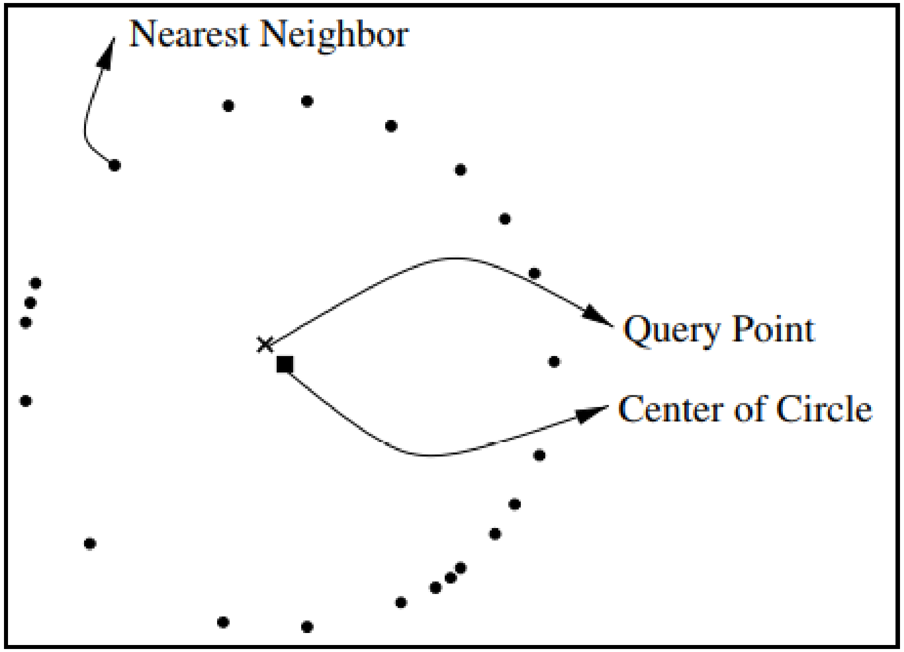


Fig 2: Query point (Q), and its neighbor [3]

[3] Beyer *et al.:* https://doi.org/10.1007/3-540-49257-7_15

Cont.

Nearest-neighbor problem

- So: the distance between the nearest neighbor (point at minimum distance) and any other point in the space is small.
- Specifically, given a query point *Q*, the minimum and maximum distance between *Q* and any list of *n* random points in the space follows:

$$\lim_{d\to\infty} \frac{dist_{max}(d) - dist_{min}(d)}{dist_{min}(d)} \to 0$$

- As *d* increases, distance function loses its usefulness.

Data Models

- We will mean a data model as a statistical model.
- It provides:
  - Quantitative summary of the data
  - Impose a specific structure on the population
- A trivial model is no model at all.
  - Example: Let say, you are building a new product. You did a market survey on a set of people on ‘how much would they be willing to pay for the new product?’ And you got the following price data:

10, 20, 13, 7, 8, …, 11, 13, 27, …, 10, 7, 4

  - A trivial model could be: you draw the histogram of prices i.e. X-axis will be each discrete price, and Y-axis will be frequency.

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  - A trivial model could be: you draw the histogram of prices i.e. X-axis will be each discrete price, and Y-axis will be frequency.
  - Difficulty: You can’t reliably question like “Will anyone buy my product if I sell it for 30?”

Cont.

Models as expectations

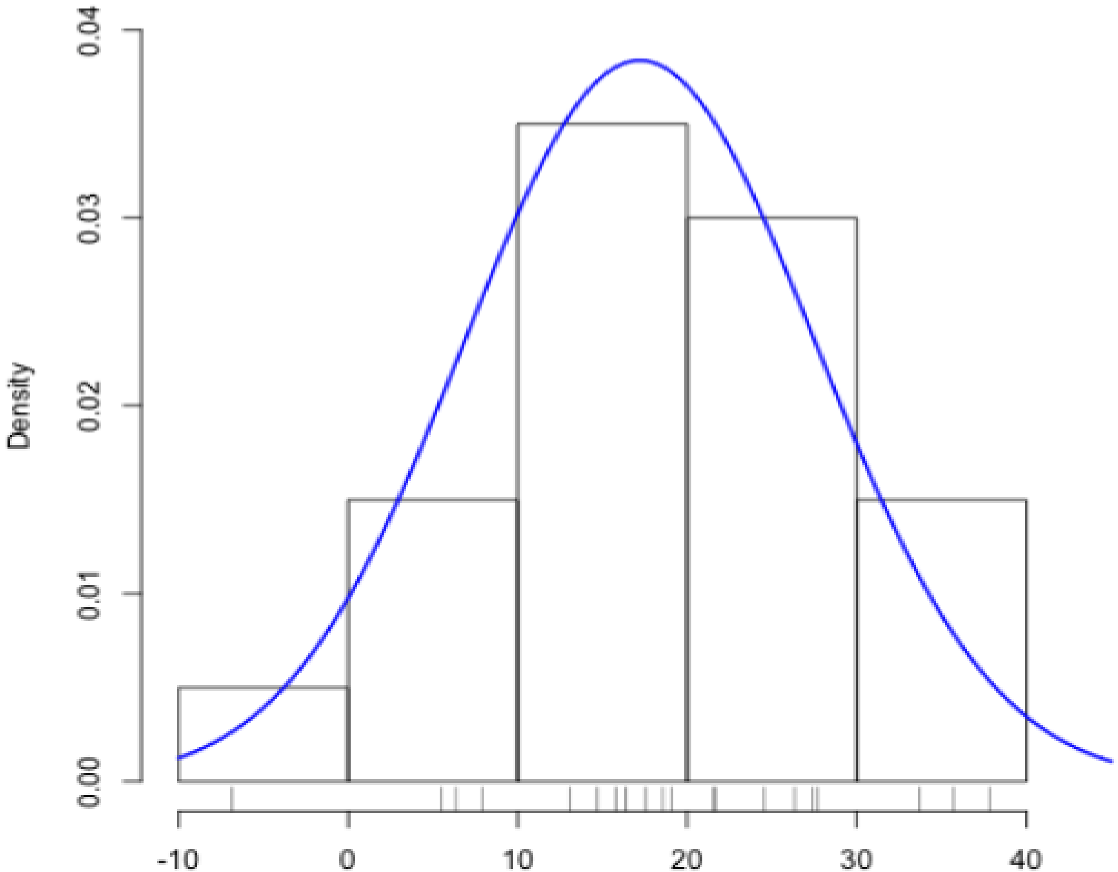
- Imposes a structure on the population
- We will call this structure as a 'distribution'.
- The most common distribution is normal distribution. This is called a normal model, denoted as $\mathcal{N}(\mu, \sigma^2)$
- The probability density function (PDF) is:

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}} \qquad \mu : \text{mean}, \sigma^2 : \text{variance}$$

- We can assume that this distribution generated the obtained data.

Cont.

Models as expectations

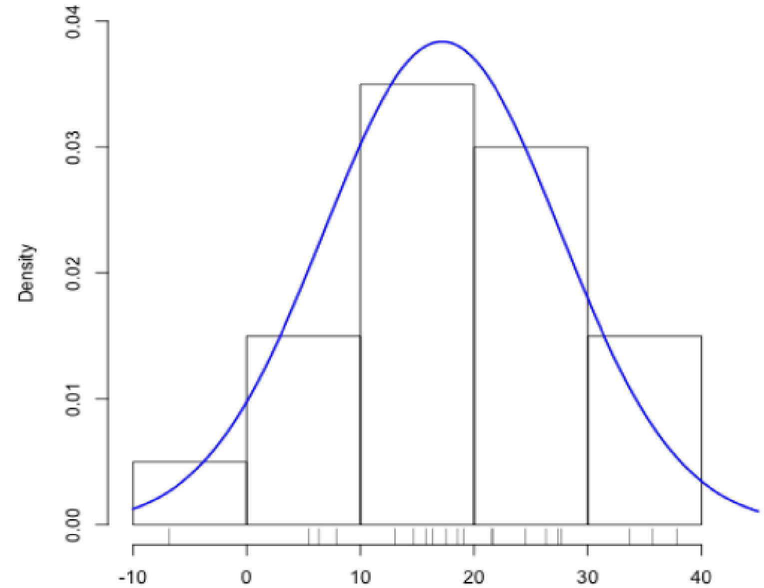


X: price, Y: density (Source: T3, Ch.5)

Cont.

Models as expectations

- Now we can answer questions.



X: price, Y: density (Source: T3, Ch.5)

- What is the probability that the product can be sold for 30? (*Homework!*)

Cont.

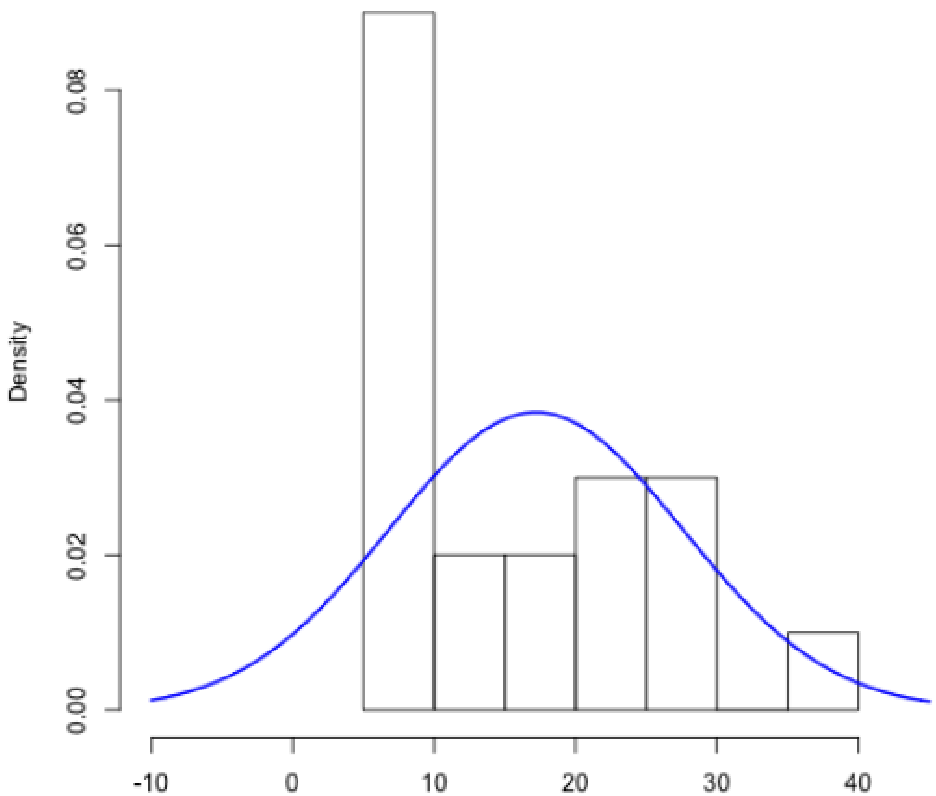
Here are the steps:

- You have the mathematical function for the density.
- Plug in the value of 30 for *x* in the PDF and obtain the answer.
- The mean and variance are the mean and variance of the data sampled during the market survey.

Cont.

Issues with normal model

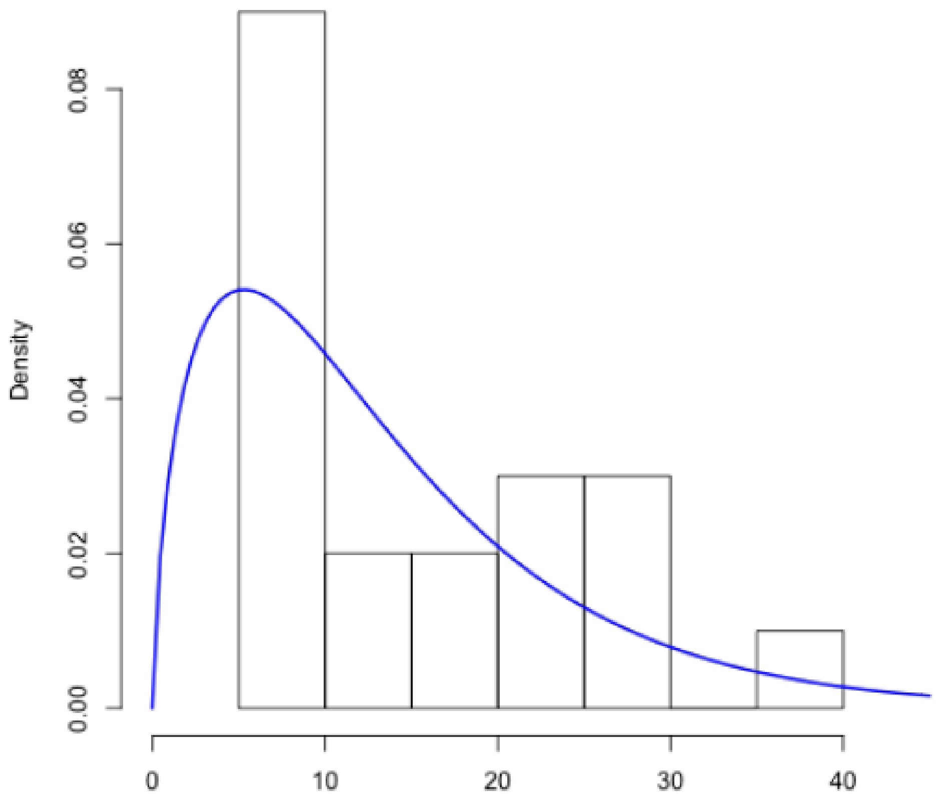
- Assumptions: Normal model will fit the data. What if it does not.



X: price, Y: density (Source: T3, Ch.5)

Cont.

Solution 1: Get a different model (*e.g.* gamma distribution)



X: price, Y: density (Source: T3, Ch.5)

Cont.

Solution 2: Get different or more data: perform the survey again to collect more data or replace the existing data. Getting more data is better.