Physics 1: Class 4

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1 Acceleration

Let us consider a particle that moves along a straight line and let us suppose that we know the velocities of the particle at the instants t_1 and t_2. We define the *average acceleration* of the particle bewteen t_1 and t_2 as

$$\overline{a} = \frac{v(t_2) - v(t_1)}{t_2 - t_1} \,.$$

In addition, we define the *instantaneous acceleration* of the particle at an instant t by

$$a(t) = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - v(t)}{\Delta t} = \frac{dv}{dt} \,.$$

Since $v(t) = dx/dt$, it follows from the last equation that

$$a(t) = \frac{d}{dt} \left(\frac{dx}{dt} \right) = \frac{d^2 x}{dt^2} \,,$$

where the last term is called the second derivative of the function x at the point t.

Since $a(t)$ is the derivative of the velocity at the instant t, in a v vs t graph, $a(t)$ will be given by the slope of the line that is tangent to the curve at the instant t. On the other hand, since $a(t)$ is the second derivative of the position at the instant t, in an x vs t graph, $a(t)$ will be given by the curvature of the curve at the instant t. Basically, a curve has positive curvature at a point if it has the form $\smile$ and negative curvature if it has the form $\frown$. An inflection point of a curve is a point where the curvature changes its sign. The curvature of the curve at this point is zero.

Exercise 1.1. Let $x(t) = 5t^3 - 10t + 2$ be the position of a particle in meters, where t is measured in seconds. Find the acceleration of the particle at the instant $1\,\mathrm{s}$. *Answer:* $a(1) = 30\,\mathrm{m/s^2}$.

Exercise 1.2. Considering the x vs t graph given in Fig. 1, answer the following:

(i) What are the signs of the acceleration between the instants 1 s and 2 s?

(ii) Is the acceleration negative in some instant between 4 s and 6 s?

(iii) What is the sign of the acceleration when the velocity of the particle attains its maximum value?

(iv) Is there an instant where the acceleration is zero?

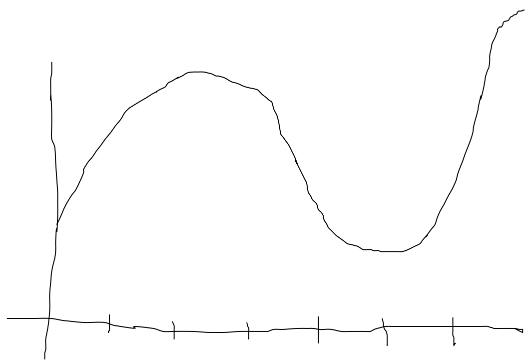

Figure 1: x vs t graph.

2 Constant acceleration

Let us consider a particle that moves in a straight line with constant acceleration a_0, i.e., $a(t) = a_0$ for all $t \geq 0$. We will be interested in obtaining the expression of the position of the particle at an arbitrary instant t.

We begin by observing that, since $a(t)$ is the derivative of the velocity at the instant t, in an analogous way to the case when we obtained the position of a particle from the expression of its velocity, we will have

$$v(t) - v(t_0) = \int_{t_0}^{t} a(t')\, dt' = a_0(t - t_0)\,.$$

Then,

$$v(t) = v_0 + a_0(t - t_0)\,, \tag{1}$$

where $v_0 = v(t_0)$. Now, the displacement of the particle between two instants t_0 and t is given by

$$x(t) - x(t_0) = \int_{t_0}^{t} v(t')\, dt' = v_0(t - t_0) + \frac{a_0}{2}(t^2 - t_0^2) - a_0 t_0 (t - t_0)\,.$$

Hence,

$$x(t) = x_0 + v_0(t - t_0) + \frac{a_0}{2}(t - t_0)^2\,, \tag{2}$$

where $x_0 = x(t_0)$.

From Eqs. (1) and (2) we can obtain other useful equations. For instance, it follows from Eq. (1) that

$$t - t_0 = \frac{v(t) - v_0}{a_0}\,.$$

Using this in Eq. (2), we can obtain that

$$v^2(t) - v_0^2 = 2a_0[x(t) - x_0]\,.$$

Moreover, using the expression of a_0, obtained from Eq. (1), we obtain

$$x(t) - x_0 = \left(\frac{v(t) + v_0}{2}\right)(t - t_0)\,.$$