Notes of elementary physics 1

Max Jauregui

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1 Kinematics in one dimension

1.1 Frames of reference

We are going to study the motion of an object which can move along a straight line. We assume that the dimensions of the object are not important in its motion. Hence, any object will be considered as a *particle*, which is represented by a dot.

In order to study the motion of a particle, we need to define a *frame of reference*, which is composed by a coordinate system and a clock. In the particular case where a particle moves along a straight line, the coordinate system is just composed by a point, labeled as the *origin* of the frame of reference, and an axis, usually called the x*-axis*.

Fixed a frame of reference, the position of a particle at an instant $t \geq 0$ is given by a real number $x(t)$. This defines a function $x : [0, \infty) \to \mathbb{R}$, which, as almost any function in physics, will be assumed to have derivatives of any order.

The international system of units (SI) establishes the second (s) as the unit of time and the meter (m) as the unit of length. In this direction, we will consider that $x(t)$ is the position of a particle measured in meters at an instant t, measured in seconds.

1.2 Average velocity and average speed

Let us consider a particle that moves in a straight line and let us suppose that we know the position of the particle at two distinct instants t_1 and t_2. The *average velocity* of the particle in this interval of time is defined as

$$\overline{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}\,.$$

Introducing the notation $\Delta t = t_2 - t_1$, we can write

$$\overline{v} = \frac{x(t_1 + \Delta t) - x(t_1)}{\Delta t} \,.$$

The unit of average velocity in the SI is meters per second (m/s).

The average velocity of a particle can be positive, negative or zero. For instance, if $x(t_2) = x(t_1)$ (the particle returns to the same position), then $\overline{v} = 0\,\mathrm{m/s}$. In this case, the particle is not necessarily at rest and, consequently, it could have traveled a distance d different from zero. We define the *average speed* of the particle as

$$v_s = \frac{d}{|\Delta t|} \,.$$

The average speed of a particle is always non-negative. The definition of average speed remains the same for the general motion of a particle in three dimensions.

Exercise 1.1. A dog is initially (instant $0\,\mathrm{s}$) in the position $0\,\mathrm{m}$. Then, the dog runs $40\,\mathrm{m}$ to the right and then $20\,\mathrm{m}$ to the left, reaching its final position at the instant $12\,\mathrm{s}$. Find the average velocity and the average speed of the dog in this interval of time. *Answer:* $\overline{v} = 1{,}7\,\mathrm{m/s}$ and $\overline{v}_s = 5\,\mathrm{m/s}$.

1.3 Instantaneous velocity

Let us begin with an example.

Exercise 1.2. A particle moves in a straight line such that it is in the position $1{,}0\,\mathrm{m}$ at instant $1{,}0\,\mathrm{s}$. (i) If the particle is in the position $4{,}0\,\mathrm{m}$ at the instant $2{,}0\,\mathrm{s}$, find the average velocity of the particle between the instants $1{,}0\,\mathrm{s}$ and $1{,}2\,\mathrm{s}$. (ii) Suppose that the particle is in the positions $2{,}3\,\mathrm{m}$ and $1{,}7\,\mathrm{m}$ at the instants $1{,}5\,\mathrm{s}$ and $1{,}1\,\mathrm{s}$ respectively. Find the average velocity of the particle between the instants $1{,}0\,\mathrm{s}$ and $1{,}5\,\mathrm{s}$ and between $1{,}0\,\mathrm{s}$ and $1{,}3\,\mathrm{s}$. *Answer:* (i) $3{,}0\,\mathrm{m/s}$ (ii) $2{,}6\,\mathrm{m/s}$ and $2{,}3\,\mathrm{m/s}$.

The exercise above illustrates that the average velocity between the instants $1\,\mathrm{s}$ and t approximate to $2\,\mathrm{m/s}$ as t approaches $1\,\mathrm{s}$. This motivates to say that the velocity of the particle at the instant $1\,\mathrm{s}$ is $2\,\mathrm{m/s}$. This velocity is called the *instantaneous velocity* and is defined in general, for an arbitrary instant t, by

$$v(t) = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t} \,.$$

The limit on the right hand side of this equation is a very important one and is called the *derivative* of the function x at the point t, denoted by dx/dt. Then, we have

$$v(t) = \frac{dx}{dt}\,.$$

The absolute value of the instantaneous velocity is called *instantaneous speed.*

In this course we will essentially work with polynomial functions. In order to compute the derivative of a polynomial, we will use the following rules:

$$\begin{aligned} \frac{dt^n}{dt} &= nt^{n-1}\,, \quad n \in \mathbb{N}\,; \\ \frac{dc}{dt} &= 0\,, \quad \text{for any constant } c\,. \end{aligned}$$

Exercise 1.3. The position of a particle in meters is given by the expression $x(t) = 5t^2 - 2t + 3$ if t is measured in seconds. Find the instantaneous velocity of the particle at the instant 2 s. *Answer:* 18 m/s.

Having the pairs $(t, x(t))$ for several values of t, we can plot the graph of the function x. This graph is usually called *displacement-time graph* or simply *x vs t graph.* The ratio $r = [x(t + \Delta t) - x(t)]/\Delta t$ is the slope of the segment that connects the points $(t, x(t))$ and $(t + \Delta t, x(t + \Delta t))$. If Δt tends to 0 ($\Delta t \to 0$), the ratio r becomes the slope of the tangent line that passes through the point $(t, x(t))$. Thus, if we have an x vs t graph, the instantaneous velocity at an instant t will be given by the slope of the line that is tangent to the curve at that instant.

Exercise 1.4. Consider the x vs t graph given in Fig. 1. (i) Find the approximate values of the instants where the instant velocity is 0 m/s. (ii) Find the approximate instant where the particle moves from right to left at maximum instantaneous speed.

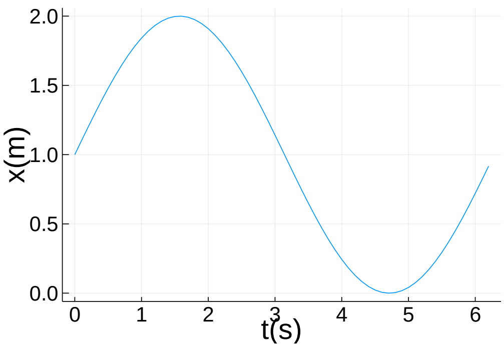

Figure 1: x vs t graph.