THE UNIVERSITY OF ZAMBIA

School of Natural Sciences

Department of Computer Studies

END OF YEAR - FINAL EXAMINATION

NUMERICAL ANALYSIS
CSC 2912

Date:	MONDAY, 5TH SEPTEMBER 2016
Time:	12:00hrs – 14:00hrs
Duration:	3 Hours
Venue:	NSLT

Instructions

a) *There are Two(2) Sections in this Exam, Section A and Section B. Answer ALL questions in Section A and ANY THREE (3) OF The FIVE(5) in Section B IN ANY ORDER.*

b) *Section A has **40 Marks** and all questions in Section B carry the weight of **20 marks** each.*

c) *Each question should start on its separate page or booklet.*

PART A: Answer all questions in this section. [40 Marks]

1. [8 Marks] Define the following
   a. Continuity of a function f at a point x_0
   b. Differentiability of a function f at a point x_0
   c. Relative error of an approximation
   d. Absolute error of an approximation

2. [4 Marks] What is (i) the absolute error (ii) The relative error in the approximating p by $p*$ where $p = \pi$ and $p^* = \frac{22}{7}$

3. [4 Marks] Suppose p^* approximates p to three significant digits. Find the interval in which p^* must lie if p is 150

4. [16 Marks] State, without proof, the following theorems
   a. Rolle's Theorem
   b. Intermediate value theorem
   c. Taylor theorem
   d. Fixed point theorem

5. [8 Marks] Derive the three-point formulas for approximating $f'(x_0)$, $f'(x_1)$ and $f'(x_2)$ using numerical differentiation, given the three points (x_0, y_0), (x_1, y_1) and (x_2, y_2) and that $x_2 - x_1 = x_1 - x_0 = h$

PART B: Answer 3 of the 5 questions in this section. Each question carries 20 marks. [60 Marks]

1.
   a. Generate P_4, the fourth Maclaurin polynomial for $f(x) = e^x$.
   b. Use P_4 to approximate $f(0.5)$
   c. Use the value in b) to approximate the value of the constant e.
   d. What is the absolute error of the approximation of e in c?

2.
   a. Show that the number of iterations, n, required to approximate the root $p \in [a, b]$, of a function f to the accuracy Tol, using the Bisection method is expressed in the following inequality.

$$\frac{(b-a)}{2^n} \leq Tol$$

   b. Hence find the number of iterations required to approximate the root p ∈ [1, 2], of the function f, defined below, to 10^{-2} accuracy.

$$f(x) = \frac{x^3 + x - 1}{x^2}$$

   c. Find this approximation. [Show the table of derivations of successful p_ns]

3.

a. Show that function

$$g(x) = 1 + \frac{1}{x^2}$$

has a fixed point at $p \in [1,2]$ precisely when $f(p) = 0$, where

$$f(x) = x^3 - x^2 - 1$$

b. Hence, approximate the solution to $f(x) = 0$, to 10^{-3} accuracy, using the fixed point iteration. Take $p_0 = 1$. [Show the table of derivations of successive $p_i s$

4. Consider the following table of the function $f(r) = \pi r^2$, giving the area of a circle of radius r.

x	0.4	0.8	1.2
f(x)	0.5026	2.0106	4.5238

a. Use P_2 , the Lagrange polynomial which agrees with f at the given points , to approximate $f(1)$

b. Note that $f'(r) = 2\pi r$ = the circumference of the circle of radius r.

i. Calculate the actual circumference of the circle of radius $r = 0.4$ (Take $\pi = 3.14152$)

ii. Approximate the circumference of the circle of radius 0.4 using the appropriate formula using the points in the table above.

iii. What is the relative error of the approximation?

5.

a. Consider the function defined by the table below

x	0.0000	0.5000	1.0000	1.5000	2.0000
f(x)	1.0000	1.2840	1.6487	2.1170	2.7183

Approximate

$$\int_0^2 f(x)dx$$

Using

i. Composite Trapezoidal rule with h = 1.0

ii. Composite Simpson rule with h = 0.5

*********END OF EXAMINATION*********