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SECTION A. ANSWER ALL QUESTIONS [40 Marks]

A.1. Define the following

- A.1.1. Continuity of a function [3 Marks]
- A.1.2. Differentiability of a function [3 Marks]
- A.1.3. Absolute error [3 Marks]
- A.1.4. Relative error [3 Marks]

A.2. State without proof

- A.2.1. The Taylor theorem [3 Marks]
- A.2.2. Mean-value theorem [3 Marks]
- A.2.3. Intermediate value theorem [3 Marks]

A.3. Explain why $f(x) = 3x^2 + 2x - 1$ has exactly one root in [0, 1] [5 Marks]

A.4. Suppose $P_{1,2}(x) = 2x$ and $P_{0,1}(x) = x^2 - 3x + 1$. What is $P_{0,1,2}(0.5)$, if $x_0 = 1.0, x_1 = 1.2$ and $x_2 = 1.4$? [7 Marks]

A.5. Suppose it is known that the function f has a unique root in the interval $[a, b]$ and you want to usethe Bisection method to approximate the solution of f to 10^{-k}. In terms of a, b and k, how many iterations of the Bisection method are required to achieve this? [7 Marks]

SECTION B. ANSWER THREE (3) OF THE FOUR QUESTIONS. ALL QUESTIONS HAVE 20 MARKS

B.1. Given the function defined on R by $f(x) = e^{\frac{x-1}{2}}$

- B.1.1. Derive $P_4(x)$, the fourth Taylor polynomial approximating f at a point $x = 1$.
- B.1.2. Use P to estimate $\frac{1}{\sqrt{e}}$
- B.1.3. What is the error bound for this estimation?
- B.1.4. What is the relative error of this estimation?

B.2. In class we derived the three-point formulae for $f'(x_0), f'(x_1)$ and $f'(x_2)$. Consider a four-point case with $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ and (x_3, y_3). Derive the four-point formulae for $f'(x_0), f'(x_1), f'(x_2)$ and $f'(x_3)$.

B.3. Consider the $f(x) = x^2 e^x$

- B.3.1. Generate $P_4(x)$, the Lagrange polynomial using Newton's divided difference method where $x_0 = 0.2$, and h = 0.2
- B.3.2. Use $P_4(x)$ to estimate $f(0.5)$

B.4. Consider the function defined on R as $f(x) = x^2 e^x$

- B.4.1. Use three-point numerical differentiation formula to estimate $f'(0.4)$ with $h = 0.2$
- B.4.2. Estimate

$$\int_{0.0}^{1.0} f(x)dx$$

Using the composite Simpson rule with $h = 0.2$

********************END OF EXAMINATION*****************