DEPARTMENT OF COMPUTER SCIENCES

CSC2912 – Numerical Analysis

Tutorial Sheet I

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1. Show that the following equations have at least one solution in the given intervals.
   a. $x \cos x - 2x^2 + 3x - 1 = 0$, $[0.2, 0.3]$ and $[1.2, 1.3]$
   b. $(x - 2)^2 - \ln x = 0$, $[1, 2]$ and $[e, 4]$

2. Find intervals containing solutions to the following equations.
   a. $x - 3 - x = 0$
   b. $4x^2 - e^x = 0$
   c. $x^3 - 2x^2 - 4x + 2 = 0$

3. Show that f(x) is 0 at least once in the given intervals.
   a. $f(x) = 1 - e^x + (e - 1)\sin((\pi/2)x)$, $[0, 1]$
   b. $f(x) = (x - 1)\tan x + x \sin \pi x$, $[0, 1]$

4.
   a. Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = (x - 1)\ln x$ about $x_0 = 1$.
   b. Use $P_3(x)$ to approximate $f(0.5)$. Find an upper bound for error $|f(0.5) - P_3(x)|$ using the error formula, and compare it to the actual error.
   c. Find a bound for the error $|f(x) - P_3(x)|$ in using $P_3(x)$ to approximate $f(x)$ on the interval $[0.5, 1.5]$.
   d.
      i. Approximate

$$\int_{0.5}^{1.5} f(x)dx$$

      ii. Find an upper bound for the error in the approximation above and compare the bound to the actual error.

5. Let $f(x) = (1 - x)^{-1}$ and $x_0 = 0$.
   a. Find the n^{th} Taylor polynomial $P_n(x)$ for $f(x)$ about x_0.
   b. Find a value of n necessary for $P_n(x)$ to approximate $f(x)$ to within 10^{-6} on $[0, 0.5]$.

6. A function $f : [a, b] \rightarrow R$ is said to satisfy a Lipschitz condition with Lipschitz constant L on [a, b], if, for every $x, y \in [a, b]$, we have $|f(x) - f(y)| \leq L|x - y|$. Show that if f satisfies a Lipschitz condition with Lipschitz constant L on an interval $[a, b]$, then $f \in C[a, b]$.

7. Let f be defined on [a, b], $a > 0$, as $f(x) = 1/x$. Show that there exists a c in [a, b] such that $f'(c) = 1/ab$

8. Suppose $f \in C[a, b]$, that x_1 and x_2 are in $[a, b]$.
   a. Show that a number ξ exists between x_1 and x_2 with

$$f(\xi) = \frac{f(x_1) + f(x_2)}{2}$$

   b. Suppose that c_1 and c_2 are positive constants. Show that a number ξ exists between x_1 and x_2

$$f(\xi) = \frac{c_1 f(x_1) + c_2 f(x_2)}{c_1 + c_2}$$

9. Compute the absolute error and relative error in approximations of p by p*.
   a. $p = \pi, p* = 22/7$
   b. $p = \pi, p* = 3.1416$
   c. $p = e, p* = 2.718$
   d. $\sqrt{2}, p* = 1.414$

10. Suppose p* must approximate p with relative error at most 10−3. Find the largest interval in which p* must lie for each value of p.
   a. 150
   b. 900