# THE UNIVERSITY OF ZAMBIA

## School of Natural Sciences

### Department of Computer Science

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**<u>END OF YEAR EXAMINATION</u>**

**CSC 2912**
**NUMERICAL ANALYSIS**

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| Date: | Monday 28th December, 2020 |
| Time: | 14:00hrs – 16:00hrs |
| Duration: | 2 Hours |

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**<u>Instructions</u>**

1. There are **two (2) sections** in this paper.
2. *Answer all the questions in Section A* and choose ***any two (2) questions*** *from Section B*

## SECTION A: ANSWERR ALL QUESTIONS [40 MARKS]

1. Let $f: X -> Y$ be a function and $x_0$ be a point in X. Define the following.
   a. $f$ is continuous at $x_0$ [4 Marks]
   b. $f$ is differentiable at $x_0$ [4 Marks]
2. Let p* be the approximation of a value p. Define the following
   a. Absolute error of approximating $p$ by $p^*$ [4 Marks]
   b. Relative error of approximating $p$ by $p^*$ [4 Marks]
3. State the following
   a. Rolle's theorem [4 Marks]
   b. Intermediate value theorem [4 Marks]
   c. Fixed-point theorem [4 Marks]
4. Suppose $(x_0, y_0)$, $(x_1, y_1)$,and $(x_2, y_2)$, are points for a function $f$. Outline the appropriate approximations of the
   a. three-point formulas for $f'(x_0)$, $f'(x_1)$, and $f'(x_2)$, [6 Marks]
   b. $\int_{x0}^{x2} f(x)dx$ Using the
      i. Trapezoidal rule [3 Marks]
      ii. Simpsons [3 Marks]

## SECTION B: ANSWER TWO OF THE FOUR QUESTIONS. EACH QUESTION CARRIES 30 MARKS

1.
   a. Let $f: X -> Y$ be a function and x0 be a point in $X$.
      i. Show that if f is differentiable on $x_0$, then it is also continuous at $x_0$ [8 Marks]
      ii. Suppose f is continuous in $[a, b] \subseteq X$, and $f'(x) \neq 0$ for all x in $[a, b]$, show that f has at most one root in [a, b]. [Hint: assume f has two roots in $[a, b]$ and show this lead to a contradiction using the Rolle's theorem [6 Marks]
   b. Let $f: R -> R$ be a function defined as $f(x) = e^{x/2}$.
      i. Derive $P_4$, the 4th Taylor polynomial about $x_0 = 0$. [8 Marks]
      ii. Use $P_4$ to approximate $\sqrt{e}$ [4 Marks]
      iii. What is the error bound for this approximation [4 Marks]

2. ✓

a. Show that $f(x) = x^2 - 2$ has a root in $[1, 2]$ [6 Marks]

b. How many iterations are required to approximate $\sqrt{2}$ in [1, 2], to $10^{-3}$accuracy using the Bisection method. [6 Marks]

c. Show that the fixed point of $g(x) = 1 + 1/(x + 1)$, is the root of $f(x) = x^2 - 2$. [8 Marks]

d. Hence use the fixed-point iteration to approximate $\sqrt{2}$ [10 Marks]

The following values are to be used in questions 3 and 4 below.

| x | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|
| f(x) | 0.31 | 0.65 | 0.94 | 1.32 | 1.46 |

3. Approximate $f(0.5)$ using

a. Neville's iterated method [15 Marks]

b. Newton's divided differences [15 Marks]

4. ✓

a. Use the correct three-point formula to find $f'(0.6)$ [8 Marks]

b. Approximate

$$\int_{0.2}^{1.0} f(x)dx$$

c. Using the composite

i. Trapezoidal rule [10 Marks]

ii. Simpsons rule. [12 Marks]

[Ensure you use all the points in both cases]

*************** END OF EXAMINATION**********