THE UNIVERSITY OF ZAMBIA

School of Natural Sciences

Department of Computer Science

EXAMINATION

NUMERICAL ANALYSIS
CSC2912

Date:	WEDNESDAY, 9TH NOVEMBER 2022
Time:	14:00HRS – 17:00HRS
Duration:	3 HOURS
Venue:	NSLT

Instructions

a) *There are TWO (2) Sections in this Examination*

b) *You are required to answer ALL questions in Section A and answer ONLY THREE (3) of the FIVE (5) questions in Section B*

c) *Indicate your computer number CLEARLY on all the answer booklets you submit.*

$x_2 - x_1 = x_1 - x_0 = h$

SECTION A: ANSWERR ALL QUESTIONS **[40 PTS]**

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1. Let $f: X -> Y$ be a function and x_0 be a point in X. Define the following.
   a. f is continuous at x_0 [4pts]
   b. f is differentiable at x_0 [4pts]
2. Let p* be the approximation of a value p. What range of p* will approximate p to 3 significant figures if p is
   a. 10 [4pts]
   b. 100 [4pts]
3. State the following
   a. Rolle's Theorem [4pts]
   b. Intermediate value theorem [4pts]
   c. Mean-value theorem [4pts]
4. Derive the following:
   a. The three-point formulae for $f'(x_0), f'(x_1), and\ f'(x_2)$, where the interval between the $x_i s$ is h at points $(x_0, y_0), (x_1, y_1) and\ (x_2, y_2)$ [8pts]
   b. The Trapezoidal rule for approximating $\int_{x_0}^{x_2} f(x)dx$ [4pts]

SECTION B: ANSWER THREE OF THE FIVE QUESTIONS. EACH QUESTION CARRIES 20 PTS

$f(b)-$

$-h$ h^2

1.

a. Suppose f is continuous in $[a, b]$, and $f'(x) \neq 0$ for all x in $[a, b]$, show that f has at most one root in [a, b]. **[10pts]**

b. Hence or otherwise, show that the function $f(x) = x^3 + 2x - k$ has exactly one root, regardless of the value of k

[Hint: check the signs of the values of f at k and $-k$ to show existence of at least one root then prove unique] **[10pts]**

2. Let $f: R -> R$ be a function defined as $f(x) = e^{x/2}$.

a. Derive P_4, the 4th Taylor polynomial about $x_0 = 0$. **[10pts]**

b. Use P_4 to approximate $\sqrt{e}$ **[6pts]**

c. What is the absolute error bound for this approximation **[4pts]**

3.

a. Show that the curves $x^3 + 2x - 3 = 0$ and $2x^2 - 5 = 0$ intersect at some point in the interval [-1, 0]. [10pts]

b. Approximate this point to 10^{-3} accuracy using the Newton's method. [10pts]

4.

a. Show that the fixed point of the function $g(x) = 1 + 1/(x + 1)$, has a unique fixed point in the interval [1, 2]. [10pts]

b. Hence use the fixed-point iteration to approximate this fixed point [10pts]

5. The following values are to be used in questions 3 and 4 below.

x	0.2	0.4	0.6	0.8	1.0
f(x)	0.31	0.65	0.94	1.32	1.46

a. **Approximate $f(0.5)$ using Newton's Divided differences method [10pts]**

b. **Approximate**

$$\int_{0.2}^{1.0} f(x)dx$$

Using the composite Simpson's rule, with n = 2. [10pts]

[Ensure that you use all the points in both cases]

***************** END OF EXAMINATION************

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