THE UNIVERSITY OF ZAMBIA

School of Natural Sciences

Department of Computer Science

FINAL EXAMINATION

CSC 2912

Numerical Analysis

Date:	4TH NOVEMBER 2019
Time:	14:00hrs – 17:00hrs
Duration:	3 Hours
Venue:	NSLT

Instructions

1. There are **two (2) Sections** in this exam, **Section A** and **Section B**.
2. In Section A, **answer all the questions** and in Section B **choose any three (3) questions of the five (5) in any order**.

SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION **[40 Marks]**

1. Define the following
   a. Continuity of a function [3 Marks]
   b. A fixed point of a function [3 Marks]
   c. Differentiability of a function [3 Marks]
   d. Relative error [3 Marks]

2. Concisely, state, without proof, the following theorems
   a. Rolle's theorem [4 Marks]
   b. Mean Value theorem [4 Marks]
   c. Intermediate value theorem [4 Marks]
   d. Taylor's theorem [4 Marks]
   e. Fixed point theorem [4 Marks]

3. Prove that If a function is continuous in the interval $[a, b]$ and differentiable in the open interval (a, b) such that $f'(x) \neq 0$ for x in this interval, then f has at most one root, in the interval $[a, b]$ [8 Marks]

SECTION B: ANSWER THREE (3) OF THE FIVE (5) QUESTIONS. **[60 Marks]**

1.
   a. Show that the graph of the equation $x^3 + 2x - 4 = 0$, crosses the x-axis exactly once in the interval $[1, 2]$ [8 Marks]
   b. Approximate this root to 10^{-4} accuracy using the
      i. Newton-Raphson method. [6 Marks]
      ii. Secant method [6 Marks]

2.
   a. Derive, P_4, the fourth Taylor polynomial for the function $f(x) = e^x$ about the point $x_0 = 0$. [8 Marks]
   b. Use P_4 to approximate $\sqrt{e}$. [4 Marks]
   c. What is the relative error of this approximation? [4 Marks]
   d. Using the remainder term, establish the upper bound of the error introduced by this approximation. [4 Marks]

3. Consider the function g defined below

$$g(x) = 1 + \frac{2}{1+x}$$

   a. Show that g as a unique fixed point in the interval [1, 2] [8 Marks]

b. Show that the fixed point of the g is the root of the function [6 Marks]

$$f(x) = x^2 - 3$$

c. Hence, use the fixed point iteration method to approximate the number $\sqrt{3}$, to 10^{-3} accuracy. [6 Marks]

4. Consider the table of values of the function f at the given points of x

x	0.4	0.8	1.2
f(x)	2.225541	4.953032	11.02318

Approximate $f(0.5)$ using

a. Neville's Iterated method [10 Marks]
b. Newton's Forward Divided Differences [10 Marks]

5. Given the set of points for f below.

x	0.2	0.4	0.6	0.8	1.0
f(x)	1.105171	1.221403	1.349859	1.491825	1.648721

a. Using h = 0.4, approximate
   i. $f'(0.6)$ [4 Marks]
   ii. $f'(0.2)$ [4 Marks]
b. Evaluate

$$\int_{0.2}^{1.0} f(x)dx$$

Using Composite

i. Trapezoidal rule n = 2 [6 Marks]
ii. Simpson rule with n = 2 [6 Marks]

**********END OF EXAMINATION**********