

# THE UNIVERSITY OF ZAMBIA School of Natural Sciences

Department of Computer Science

# FINAL EXAMINATION

## CSC 2912 NUMERICAL ANALYSIS

Date : 19 NOVEMBER 2018 Time : 14:00hrs - 17:00HRS

Duration: 3 Hours Venue: NSLT

## Instructions

- There are two (2) Sections in this exam, Section A and Section B.
- In Section A, answer all the questions and in Section B choose any three (3) questions.

#### SECTION A: ANSWER ALL QUESTIONS IN THIS SECTION

- 1. Let  $f:R\to R$  be a function defined on R. Define the following: a. Limit L of a f at  $x_0$ b. Continuity of a f at  $x_0$ c. Differentiability of a function at  $x_0$
- Suppose a number p<sub>0</sub> approximates p. Define the following
   a. Absolute error of this approximation
   b. Relative error of this approximation
- State the following theorems
   a. Rolle's theorem
   b. Mean value theorem
- Taylor's theorem
   Suppose p<sub>0</sub>, approximates 100 to 5 significant digits. What is the range of p<sub>0</sub>? [4 Marks]
- Let a function f have a root in [2, 3]. How many iterations of the bisection method are required to approximate the root to 10<sup>-4</sup> accuracy? [4 Marks]

### SECTION B: ANSWER THREE (3) OF THE FOUR (4) QUESTIONS

1.

2.

c. Intermediate value theorem

a. Prove that if a function f is differentiable at a point  $x_0$ , then it is also continuous at  $x_0$ . [10 Marks]
b. Use the mean value theorem to show that for an interval [a,b], where a>0, and  $\left|\frac{1}{x}\right|\leq K, \forall x\in [a,b]$ , [10 Marks]

$$ln\left(\frac{b}{a}\right) \le K|b-a|$$

a. How many iterations are required to approximate √2 in [1, 2] to 10<sup>-4</sup> accuracy using the Bisection method? [10 Marks]
 b. Use the Newton's method to approximate √2 to 10<sup>-4</sup> accuracy. Let p<sub>0</sub> = 1 [10 Marks]

3.

- a. Suppose  $x_0 = 1$ ,  $x_1 = 2$ , and  $x_2 = 3$ , and you are given that  $P_{0,1}(x) = 2x$  and  $P_{0,2}(x) = x^2$ , find  $P_{0,1,2}(2.5)$  [10 Marks]
- b. Given the following data

| × | 1   | 2   | 3  |  |
|---|-----|-----|----|--|
| У | 1.6 | 4.4 | 12 |  |

Use the Newton's divided difference method to interpolate f(1.5)

[10 Marks]

4.

- a. Estimate
  - i. f'(3)
  - ii. f"(2)
- b. Estimate

[8 Marks]

 $\int_{2}^{4} x^{2} dx$ 

using the composite

- i. Trapezoidal rule, with h = 1
- ii. Simpson rule, with n = 2

[12 Marks]