

Discussion of:  
Balancing covariates in randomized experiments with the  
Gram–Schmidt Walk design

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## Summary

- Treatment assignment based on covariates  $X$ , in a static experiment, for estimation of sample ATE, evaluated by MSE.
- Observations  $1, \dots, n$ , treatment  $Z_i \in \{-1, 1\}$ , potential outcomes  $Y_i^{-1}, Y_i^1$ ,  $\mu_i = (Y_i^{-1} + Y_i^1)/2$ .
- The ex-ante MSE, given potential outcomes, of the Horovitz-Thompson estimator, for any assignment  $\mathbf{Z}$  with  $E[\mathbf{Z}] = 0$ , equals

$$\frac{4}{n^2} \boldsymbol{\mu}' \text{Cov}(\mathbf{Z}) \boldsymbol{\mu}.$$

- The paper proposes and characterizes an algorithm which aims to trade off two minimization objectives:

$$\sup_{\boldsymbol{\mu}: \|\boldsymbol{\mu}\|^2=1} \boldsymbol{\mu}' \text{Cov}(\mathbf{Z}) \boldsymbol{\mu} \quad (\text{"robustness"})$$

$$\sup_{\boldsymbol{\mu}: \boldsymbol{\mu}=\mathbf{X}\boldsymbol{\theta}, \|\boldsymbol{\theta}\|^2=1} \boldsymbol{\mu}' \text{Cov}(\mathbf{Z}) \boldsymbol{\mu} \quad (\text{"balance"})$$

- The paper provides an elegant treatment, an interesting algorithm, and a range of theoretical characterizations.
- In my discussion, I will reconsider the evaluation criteria used:
  - “Robustness” is used to justify randomization.  
Is this convincing?
  - Robustness is defined, effectively, over an  $L^2$  ball for  $\mu$ .  
Why not over some other set?
  - Balance is defined for the averages of covariates.  
Why not over the entire distribution?

# Randomization and robustness

- Decision theory does not justify randomization:
  - Consider risk functions / Bayes risk / **ex-post** minimax risk for a randomized procedure.
  - Then risk equals the expectation (average) of risk for deterministic decisions, which is worse than the best possible deterministic decision. (e.g. Kasy 2016)
- Adversarial games can justify randomization:
  - “Robustness” formalized as **ex-ante** minimax here (and in the literature since the 1980s).
  - Game against adversarial nature, which chooses  $\mu$  **after** the choice of assignment algorithm, but **before** the realization of randomness.

## Is this a plausible justification of randomization?

- The same “robustness” argument applies to estimation, inference, and prediction:
  - Suppose the data might be generated by one of  $k$  models.
  - Then “robustness” for prediction requires that you report a prediction from one of these models chosen at random.
- “Robustness” only holds ex-ante, not ex post:
  - It requires us to evaluate decisions not just based on what happened, but also based on what could have happened!
  - How much comfort should we derive from the possibility of a counterfactual world where different treatment assignments might have been chosen?
- Also noteworthy:
  - Randomization was introduced to experiments in the 1920s.
  - The rationalization of randomization by ex-ante minimax arguments was only introduced more than 50 years later.

## Minimax over what? Balance of what?

- The minimax objective (robustness) here is defined over an  $L^2$  ball for  $\mu$ ,

$$\sup_{\mu: \|\mu\|^2=1} \mu' \text{Cov}(\mathbf{Z}) \mu.$$

- Why not subject to  $\mu' A \mu \leq 1$  for some other  $A$ ?
- Choice of set to minimax over  $\Leftrightarrow$  choice of prior!  
 $A \Leftrightarrow$  inverse prior covariance matrix.
- Balance here is defined over linear functions only.

$$\|\mathbf{X}' \text{Cov}(\mathbf{Z}) \mathbf{X}\| = 0.$$

- Eliminates bias when  $\mu = X\theta$ .
- But what about non-linear relationships?
- We should balance the entire distribution, not just means! (e.g. Kasy 2016)

Thank you!