

SIGNAL ANALYSIS

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SIGNAL ANALYSIS

TIME, FREQUENCY, SCALE,
AND STRUCTURE

Ronald L. Allen
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IEEE PRESS



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To Beverley and to the memory of my parents, Mary and R.L. (Kelley).

R.L.A.

To those yet born, who will in some manner—large or small—benefit from the technology and principles described here. To the reader, who will contribute to making this happen.

D.W.M.

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PREFACE

This text provides a complete introduction to signal analysis. Inclusion of fundamental ideas—analogue and discrete signals, linear systems, Fourier transforms, and sampling theory—makes it suitable for introductory courses, self-study, and refreshers in the discipline. But along with these basics, *Signal Analysis: Time, Frequency, Scale, and Structure* gives a running tutorial on functional analysis—the mathematical concepts that generalize linear algebra and underlie signal theory. While the advanced mathematics can be skimmed, readers who absorb the material will be prepared for latter chapters that explain modern mixed-domain signal analysis: Short-time Fourier (Gabor) and wavelet transforms.

Quite early in the presentation, *Signal Analysis* surveys methods for edge detection, segmentation, texture identification, template matching, and pattern recognition. Typically, these are only covered in image processing or computer vision books. Indeed, the fourth chapter might seem like a detour to some readers. But the techniques are essential to one-dimensional signal analysis as well. Soon after learning the rudiments of systems and convolutions, students are invited to apply the ideas to make a computer understand a signal. Does it contain anything significant, expected, or unanticipated? Where are the significant parts of the signal? What are its local features, where are their boundaries, and what is their structure? The difficulties inherent in understanding a signal become apparent, as does the need for a comprehensive approach to signal frequency. This leads to the chapters on the frequency domain. Various continuous and discrete Fourier transforms make their appearance. Their application, in turn, proves to be problematic for signals with transients, localized frequency components, and features of varying scale. The text delves into the new analytical tools—some discovered only in the last 20 years—for such signals. Time-frequency and time-scale transforms, their underlying mathematical theory, their limitations, how they differently reveal signal structure, and their promising applications complete the book. So the highlights of this book are:

- The signal analysis perspective;
- The tutorial material on advanced mathematics—in particular function spaces, cast in signal processing terms;
- The coverage of the latest mixed domain analysis methods.

We thought that there is a clear need for a text that begins at a basic level while taking a *signal analysis* as opposed to *signal processing* perspective on applications.

The goal of signal analysis is to arrive at a structural description of a signal so that later high-level algorithms can interpret its content. This differs from signal processing *per se*, which only seeks to modify the input signal, without changing its fundamental nature as a one-dimensional sequence of numerical values. From this viewpoint, signal analysis stands within the scope of artificial intelligence. Many modern technologies demand its skills. Human–computer interaction, voice recognition, industrial process control, seismology, bioinformatics, and medicine are examples.

Signal Analysis provides the abstract mathematics and functional analysis which is missing from the backgrounds of many readers, especially undergraduate science and engineering students and professional engineers. The reader can begin comfortably with the basic ideas. The book gradually dispenses the mathematics of Hilbert spaces, complex analysis, distributions, modern integration theory, random signals, and analog Fourier transforms; the less mathematically adept reader is not overwhelmed with hard analysis. There has been no easy route from standard signal processing texts to the latest treatises on wavelets, Gabor transforms, and the like. The gap must be spanned with knowledge of advanced mathematics. And this has been a problem for too many engineering students, classically-educated applied researchers, and practising engineers. We hope that *Signal Analysis* removes the obstacles. It has the signal processing fundamentals, the signal analysis perspective, the mathematics, and the bridge from all of these to crucial developments that began in the mid-1980s.

The last three chapters of this book cover the latest mixed-domain transform methods: Gabor transforms, wavelets, multiresolution analysis, frames, and their applications. Researchers who need to keep abreast of the advances that are revolutionizing their discipline will find a complete introductory treatment of time-frequency and time-scale transforms in the book. We prove the Balian-Low theorem, which pinpoints a limitation on short-time Fourier representations. We had envisioned a much wider scope for mixed-domain applications. Ultimately, the publication schedule and the explosive growth of the field prevented us from achieving a thorough coverage of all principal algorithms and applications—what might have been a fourth highlight of the book. The last chapter explains briefly how to use the new methods in applications, contrasts them with time domain tactics, and contains further references to the research literature.

Enough material exists for a year-long university course in signal processing and analysis. Instructors who have students captive for two semesters may cover the chapters in order. When a single semester must suffice, Chapters 1–3, 5, 7, 8, and 9 comprise the core ideas. We recommend at least the sections on segmentation and thresholding in Chapter 4. After some programming experiments, the students will see how hard it is to make computers do what we humans take for granted. The instructor should adjust the pace according to the students' preparation. For instance, if a system theory course is prerequisite—as is typical in the undergraduate engineering curriculum—then the theoretical treatments of signal spaces, the Dirac delta, and the Fourier transforms are appropriate. An advanced course can pick up the mathematical theory, the pattern recognition material in

Chapter 4, the generalized Fourier transform in Chapter 6, and the analog filter designs in Chapter 9. But the second semester work should move quickly to and concentrate upon Chapters 10–12. This equips the students for reading the research literature.

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