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a)

Decision variables:

$$x_{ik} = \begin{cases} 1 & \text{if location } i \text{ is used for the } k^{\text{th}} \text{ FC} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ijk} = \begin{cases} 1 & \text{if FC at location } i \text{ serves DP } j \text{ when there are } k \text{ open FC} \\ 0 & \text{otherwise} \end{cases}$$

Objective function:

$$\begin{aligned} \text{min } & 6 \sum_{i=1}^{10} c_{ij} y_{ij1} \quad \forall j \quad j=1,2,\dots,20 \\ & + 5 \sum_{i=10}^{10} c_{ij} y_{ij2} \quad \forall j \quad j=1,2,\dots,20 \\ & + 9 \sum_{i=1}^{10} c_{ij} y_{ij3} \quad \forall j \quad j=1,2,\dots,20 \end{aligned}$$

c_{ij} is the distance matrix between all FCs and DPs

Constraints:

$$\sum_{i=1}^{20} y_{ijk} \leq x_{jk} \quad \forall j \quad j=1,2,\dots,10 \quad \forall k \quad k=1,2,3$$

$$\sum_{k=1}^3 \sum_{j=1}^{10} x_{jk} = k+1$$

$$\sum_{i=1}^{20} \sum_{j=1}^{10} y_{ijk} = 1 \quad \forall k \quad k=1,2,3$$

$$\sum_{i=1}^{10} x_{jk} \geq \# \quad \forall k=1,2,$$

$$x_{j,k-1}$$

b) Optimal FCs to open at

immediately: FC 1

after 6 years: FC 7

after 11 years: FC 10