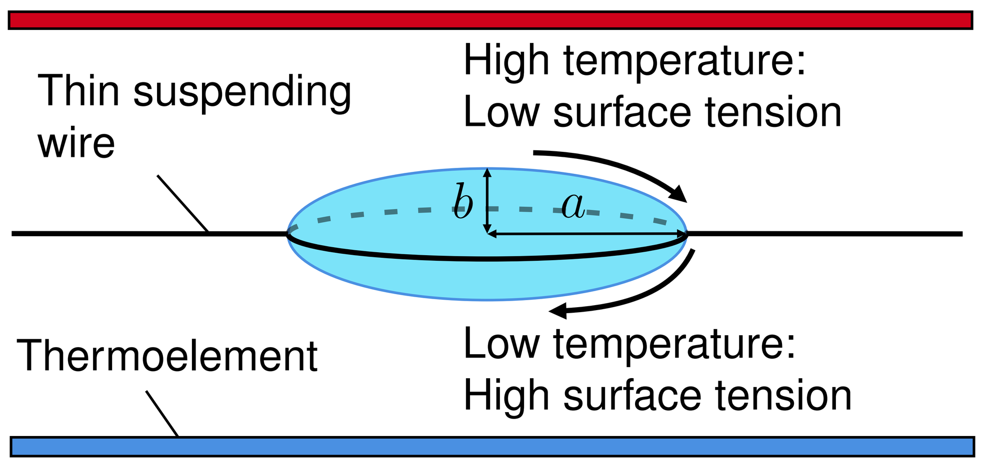
Marangoni instability in isotropic droplets suspended on a circular frame

Maxim A. Shishkin[1,2,3], B.I.Ostrovskii[2,4] and E.S.Pikina[1,2]

[1]Landau Institute for Theoretical Physics of RAS, Chernogolovka, Russia.
[2]Institute of Solid State Physics of RAS, Chernogolovka, Russia.
[3]NRU Higher School of Economics, Moscow, Russia.
[4]FSRC "Crystallography and Photonics" of RAS, Moscow, Russia.

Introduction

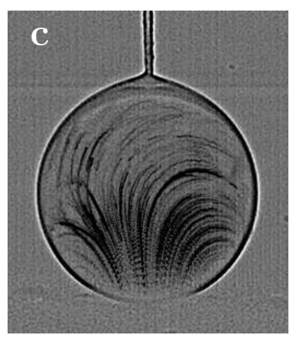


We study theoretically internal flows in small isotropic droplets suspended on the circular frame and placed in a vertical temperature gradient.
The Marangoni convection arises due to surface tension variations at the drop interfaces $\gamma(T) = \gamma_0 - \varsigma T$.
The real drop shape of spherical segments is well approximated by oblate spheroidal when its height is much less than the median radius ($b \ll a$).

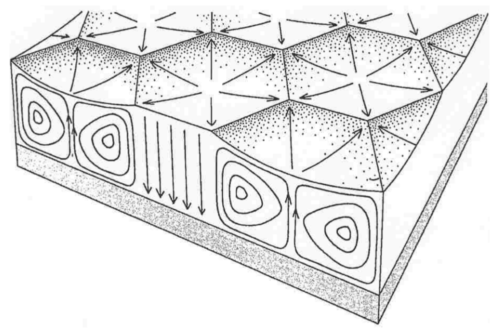
Our research generalizes two well-known problems:

- Stationary thermocapillary convection occurs in a non-threshold way (for the arbitrarily small Marangoni number) due to nonzero curvature of the drop interface.
- However the simple motion becomes unstable for enough large Marangoni number relative to the thermocapillary rolls-convection in analogy with flat layer.

The aim is to determine both stationary motion and critical Marangoni number for different values of the heat conductivity and ellipticity ratio.



Marangoni convection in droplets on superhydrophobic surfaces. *D. Tam et al. Journal of Fluid Mechanics (2009)*



Benard-Marangoni cells. From *Velarde, M. G., and Ch. Normand. Scientific American (1980)*

General stationary solutions for Stokes stream function in the oblate spheroidal coordinates

Axial symmetry

The Stokes stream function formalism
$$\mathbf{v}(z,r) = r^{-1}[\mathbf{e}_\varphi \times \nabla\psi]$$

Small Reynolds number

Small buoyancy force $H \ll H_c = \sqrt{\varsigma/(\rho g \beta)}$

$$\hat{E}^4\psi(z,r) = 0,\ \hat{E}^2 = \nabla^2 - \frac{2}{r}\partial_r$$

$$\begin{pmatrix} r \\ z \end{pmatrix} = \begin{pmatrix} c\sqrt{1+\xi^2}\sqrt{1-u^2} \\ c\xi u \end{pmatrix}$$

Define convenient operators
$\hat{\mathcal{F}} = (1-u^2)\partial_u^2,\ \hat{\mathcal{F}}\mathcal{F}_n = -n(n+1)\mathcal{F}_n$
$\hat{\mathcal{X}} = (1+\xi^2)\partial_\xi^2,\ \hat{\mathcal{X}}\mathcal{X}_n = n(n+1)\mathcal{X}_n$

$$\hat{E}^2 = \frac{\hat{\mathcal{X}} + \hat{\mathcal{F}}}{c^2(\xi^2+u^2)}$$

Solution has been found analytically:
$$\psi = c_1\xi\mathcal{F}_1 + c_2\mathcal{F}_2 + \sum_{N>2} c_N^{(1,2)}(\mathcal{F}_N\mathcal{X}_{N-2}^{(1,2)} + \mathcal{F}_{N-2}\mathcal{X}_N^{(1,2)}) + \sum_{N\geq 1} c_{No}^{(1,2)}\mathcal{X}_N^{(1,2)}\mathcal{F}_N$$

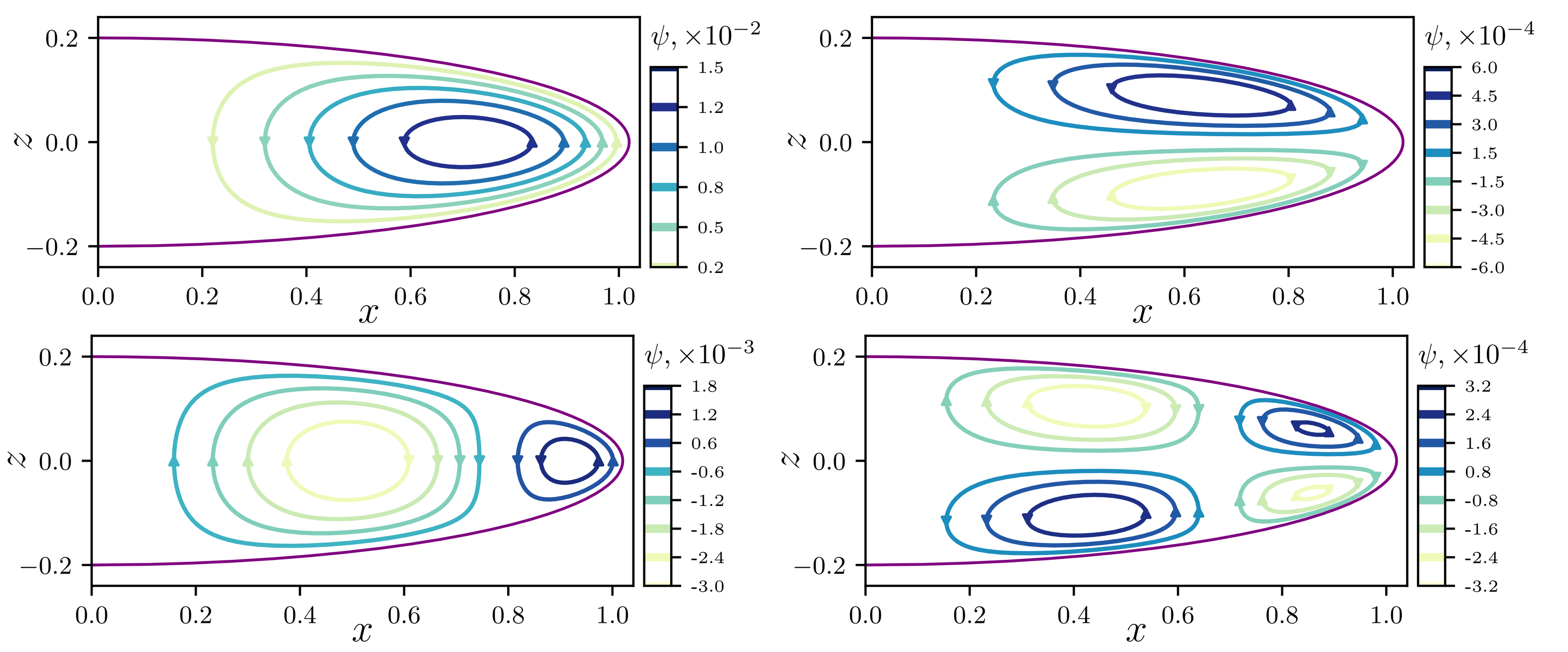


Fig. 1: Streamlines corresponding to the first four basic stream functions for $\xi_0 = 0.2$ (Lengths in a focus distance c)

Stationary convection

Conduction heat transfer dominates
$\nabla^2 T = 0, T_{air}(r\to\infty) = A_{air}z \implies$
$\implies T_{in} = Az$

Balance of tangent surface forces
$$\tilde{\sigma}_{n\tau} = \mathrm{Ma}\,\partial_{\tilde{\tau}}\tilde{T},\ \mathrm{Ma} = \varsigma A H^2/(\eta\chi)$$

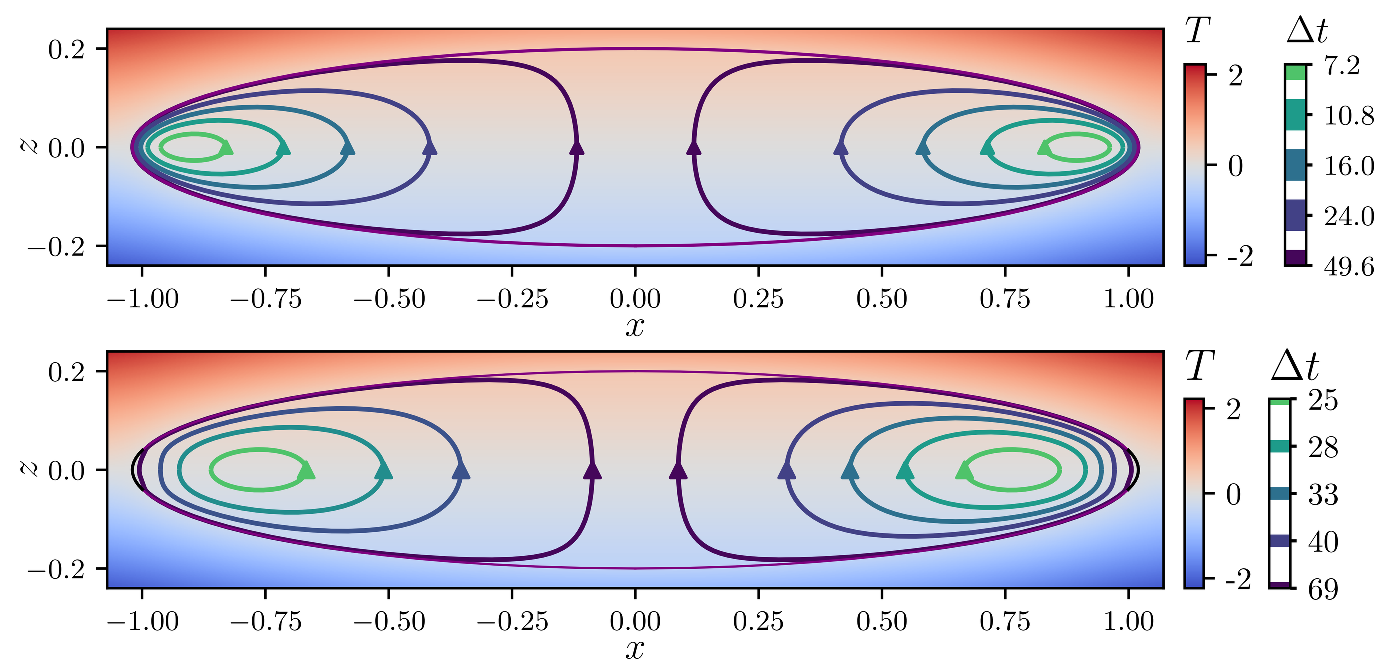


Fig. 2: Dimensionless temperature distribution and streamlines with corresponding circulation period (in $\eta/(2\xi_0\varsigma A)$).
Comparing cases of full-free surface (top) and with small sticking area near the apex (black line at bottom figure).
Note that periods depend only on semiaxes ratio, but not on height of drops. Δt scale is in seconds for η, ς like water for $A \sim 15$ mK/mm.
($\xi_0 = 0.2,\ \kappa = 0.2 = \kappa_{air}/\kappa_{lq}$ is a relative heat conductivity)

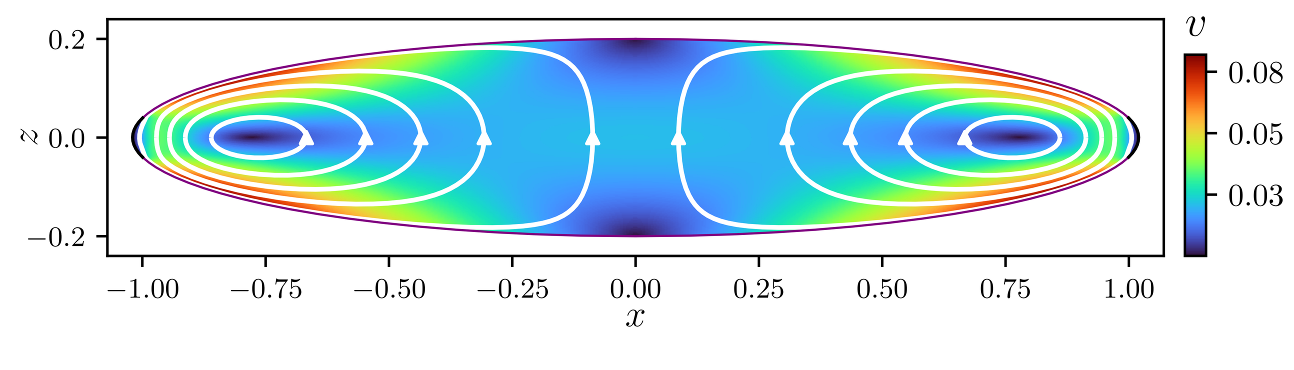


Fig. 3: Modulus of dimensional velocity for case with sticking area. $\mathrm{Ma} = 1$

Marangoni instability of stationary flow

Small critical perturbation $v_c, T_c \propto e^{0\cdot t}$ in the linear instability theory
$$\begin{cases} \hat{E}^4\psi_c = 0 \\ \nabla^2 T_c = (\mathbf{v}_c, \nabla)T_0 \end{cases}$$

The same stationary basis expansion:
$\psi_c = \sum_j c_{cj}\psi_j \implies T_c = \sum_j c_{cj}T_{cj}$,
where the temperature response
$\nabla^2 T_{cj} = (\mathbf{v}[\psi_j], \nabla)T_0$
to each motion is found analytically.

Marangoni boundary condition
$$\sum_j \sigma_{\tau n}^j(u,\xi_0)\, c_{cj} = \mathrm{Ma_c} \sum_j \partial_\tau T_{cj}(u,\xi_0)\, c_{cj}$$
is a *Generalized Eigenvalue Problem* on $\mathrm{Ma_c}$ and coefficient $\{c_{cj}\}$:
$$\hat{\mathfrak{S}}\,|c_c\rangle = \mathrm{Ma_c}\,\hat{\mathcal{G}}\,|c_c\rangle\,,\quad \hat{\mathfrak{S}}, \hat{\mathcal{G}} : \mathbb{R}^\infty \longrightarrow C^\infty(u \in [-1,1])$$

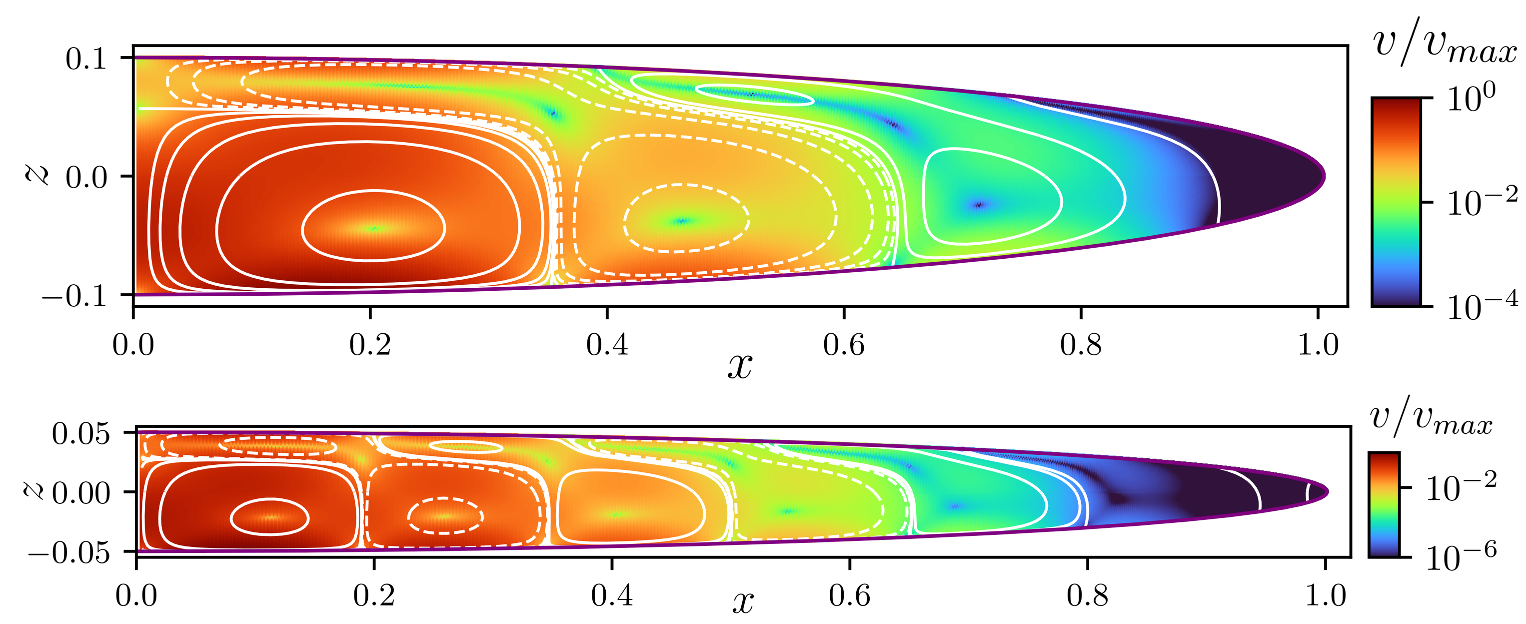


Fig. 4: Velocity modulus of critical flows with streamlines for $\xi_0 = 0.1, \mathrm{Ma_c} = 107$ and for $\xi_0 = 0.05, \mathrm{Ma_c} = 101$ ($\kappa = 0.1$).

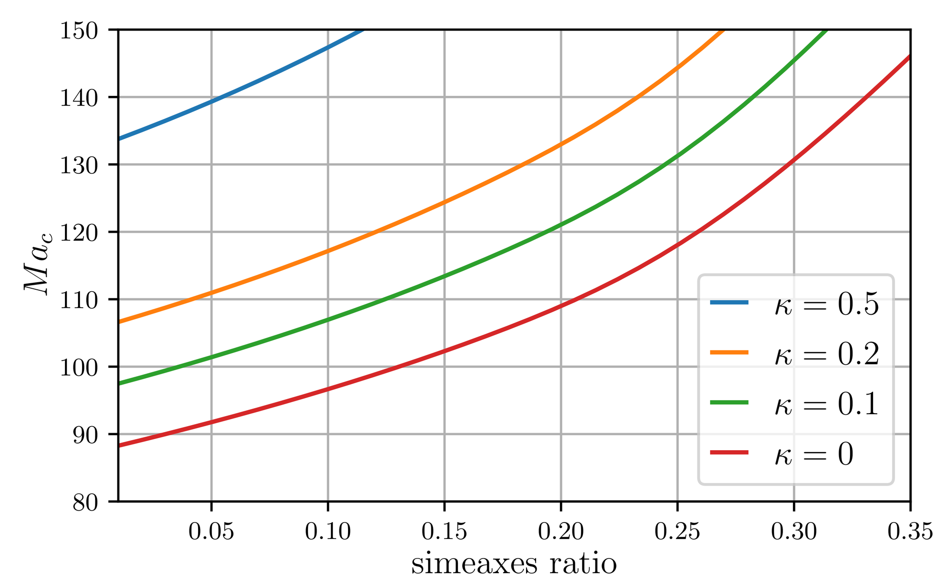


Fig. 5: The critical Marangoni numbers lines for different values of the air-liquid relative heat conductivity κ.
As far as critical perturbation are very small near the apex (see Fig.4), the stationary convective heat transfer has a small influence on value $\mathrm{Ma_c}$, but for semiaxes ratio $b/a \gtrsim 0.2$ the full drop is "apex" and the convection term in the heat transfer should be taken into account to determine $\mathrm{Ma_c}$.

Conclution

We have developed the theory of stationary thermocapillary convection in oblate drops in a vertical temperature gradient. The limit of stability $\mathrm{Ma_c}$ was also determined for $b/a \lesssim 0.2$. Note that the proposed experimental scheme excludes Rayleigh convection in the surrounding air due to heating from above.

Acknowledgements

This work is funded by Russian Science Foundation (Grant 18-12-00108).

Contact information:
Maxim Shishkin
bakolavro@ya.ru

References

Manuscript "Circulating Marangoni flows within droplets in smectic films" by E. S. Pikina, M. A. Shishkin, K. S. Kolegov, B. I. Ostrovskii and S. A. Pikin is submitted to PRE and is available at http://arxiv.org/abs/2207.02652