Notes on Fixed Effect Ordered Logit Model

Max Leung

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1 Model

An individual i will choose choice $k \in \{S_{min}, \cdots, S_{max}\}$ (i.e., $y_{it} = k$) if $\tau_{ik} < y^*_{it} \le \tau_{ik+1}$
where $|\{S_{min}, \cdots, S_{min}\}| = K$

The unobservable / latent variable y^*_{it} is modeled as

$$y^*_{it} = \boldsymbol{x}'_{it}\boldsymbol{\beta} + \alpha_i + \varepsilon_{it} \qquad i = 1 \text{ to } N;\ t = 1 \text{ to } T$$

Thus,

$$\begin{aligned}
Pr(y_{it} = k|\boldsymbol{x}_{it}, \alpha_i) &= Pr(\tau_{ik} < y^*_{it} \le \tau_{ik+1}|\boldsymbol{x}_{it}, \alpha_i) \\
&= Pr(\tau_{ik} < \boldsymbol{x}'_{it}\boldsymbol{\beta} + \alpha_i + \varepsilon_{it} \le \tau_{ik+1}|\boldsymbol{x}_{it}, \alpha_i) \\
&= Pr(\tau_{ik} - \boldsymbol{x}'_{it}\boldsymbol{\beta} - \alpha_i < \varepsilon_{it} \le \tau_{ik+1} - \boldsymbol{x}'_{it}\boldsymbol{\beta} - \alpha_i|\boldsymbol{x}_{it}, \alpha_i) \\
&= F(\tau_{ik+1} - \boldsymbol{x}'_{it}\boldsymbol{\beta} - \alpha_i|\boldsymbol{x}_{it}, \alpha_i) - F(\tau_{ik} - \boldsymbol{x}'_{it}\boldsymbol{\beta} - \alpha_i|\boldsymbol{x}_{it}, \alpha_i)
\end{aligned}$$

If ε_{it} follows standard logistic distribution i.e., $F(\varepsilon_{it}|\boldsymbol{x}_{it}, \alpha_i) = logistic(\varepsilon_{it}) := \frac{e^{\varepsilon_{it}}}{1+e^{\varepsilon_{it}}}$, we have

$$= logistic(\tau_{ik+1} - \boldsymbol{x}'_{it}\boldsymbol{\beta} - \alpha_i) - logistic(\tau_{ik} - \boldsymbol{x}'_{it}\boldsymbol{\beta} - \alpha_i)$$

2 Estimation

2.1 BUC Estimation (Baetschmann, Staub, & Winkelmann, 2015)

2.1.1 Strategy

The strategy is to make $K - 1$ copies of the original dataset and then append them together. Each copy (for $k = S_{min} + 1$ to S_{max}) has a new binary dependent variable $\mathbb{1}(y_{it} \ge k)$

As the appended dataset has a binary dependent variable and is a panel dataset (treating duplicated individual as different individual), the model becomes a Fixed Effect (Binary) Logit Model and can be estimated by Conditional Maximum Likelihood (CML) Estimation (see my notes on non-linear panel model for the details of Fixed Effect (Binary) Logit Model and CML).

2.1.2 Log Likelihood Function

Let $LL^k(\boldsymbol{\beta})$ be Log Likelihood Function used in CML estimation of Fixed Effect (Binary) Logit Model (only using $k \in \{S_{min} + 1, \cdots, S_{max}\}$ copy of data in estimation)

$$LL^{BUC}(\boldsymbol{\beta}) = \sum_{k \in \{S_{min}+1, \cdots, S_{max}\}} \sum_{i=1}^{N} ln[\mathbb{P}^k_i(\boldsymbol{\beta})] = \sum_{k \in \{S_{min}+1, \cdots, S_{max}\}} LL^k(\boldsymbol{\beta})$$

3 Reference

Baetschmann, G., Staub, K. E., & Winkelmann, R. (2015). Consistent Estimation of the Fixed Effects Ordered Logit Model. Journal of the Royal Statistical Society, Series A, 178(3), 685-703.