Space

What is reality?

Three dimensional space

What is it?

Here's a picture:

No, really, that's a picture. Is it missing something?

What is reality?

How can we describe this space so that we can calculate things? Get a handle on it? Use it for something?

Predict future positions or motions

Quantify mass, volume, stress

Tell the supplier how much cheese we need for the giant wheel

René Descartes

Je pense, donc je suis.

"I think, therefore I am."

Descartes thought of something brilliant, something that shook the world.

Does anyone know what I am talking about?
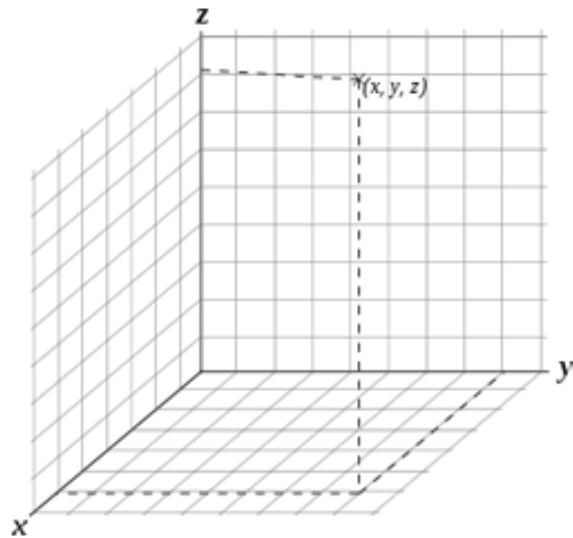
(The pineal gland??)

René Descartes

Descartes discovered *coordinates*

The 3D space of human experience is the set of ordered triples of numbers:

$$\mathbf{R}^3 = \{(x, y, z) | x, y, z \in \mathbf{R}\}$$

Here's a picture you probably recognize.

Numbers breed numbers

We can now calculate distance!

Distance between two points (a, b, c) and (a', b', c') is

$$\sqrt{(a' - a)^2 + (b' - b)^2 + (c' - c)^2}\,.$$

This generalizes the Pythagorean theorem. The book has a good explanation of why it's true. See if you can figure it out (using the Pythagorean theorem) before you read it! If you have already read it, try before reading it again. (You read each section of the book several times, right?)

Numbers breed equations

We can now describe shapes!

What is the set of points at distance 1 from $(0, 0, 0)$?

What shape is the set of points (x, y, z) such that $x + y = z$?

...such that $x^2 + y^2 = z^2$?

...such that $x^2 + y^2 = z$? (How does it differ from the previous one?)

...such that $y = x^2$?

...such that $z = 4$?