Loaded!

Please proceed to the calculus wonderland by pressing the right arrow key or clicking the right arrow that is visible when you move your mouse over the pig.

Instructions

These slides should work with *any* modern browser: IE 9+, Safari 5+, Firefox 9+, Chrome 16+.

- Navigate with arrow keys; you may need to give the window focus by clicking outside the lecture frame (the pig) for key commands described throughout this slide to work properly.
- Press M to see a menu of slides. Press G to go to a specific slide. Press W to toggle scaling of the deck with the window. If scaling is off, slides will be 800 by 600; it is off by default.
- Use left click, middle click, right click or hold A, S, D on the keyboard and move the mouse to rotate, scale, or pan the object.

If your browser or hardware does not support WebGL, interacting with models will be *very* slow (and in general models can get CPU-intensive). Navigate to a slide away from any running model to stop model animation.

Lecture 7

Behind us

- Cylinders, quadrics
- Horizontal and vertical traces

Ahead

Homework due tomorrow at 11 PM

Today: vector functions and curves

Next time: derivatives and integrals of vector functions

Read Sections 10.1, 10.2, 13.1, and 13.2 of the book. *We will not cover everything in lecture or section. Knowledge implants are impossible.*

Questions!

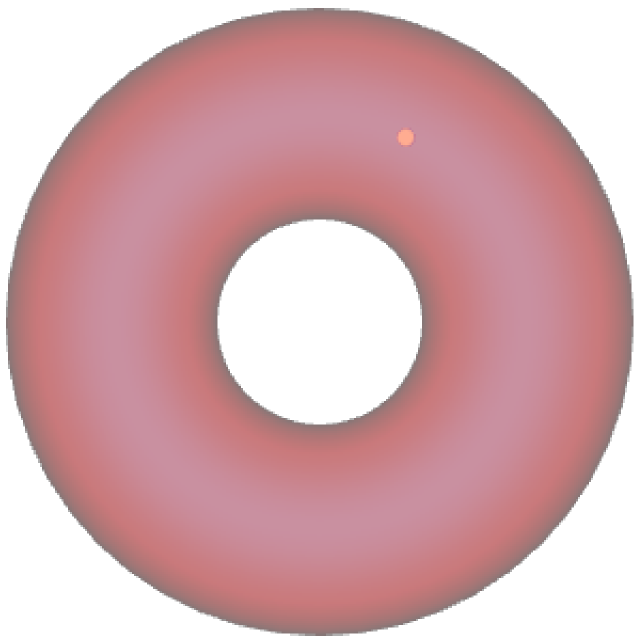
Medium-term question

A fun-loving electron is traveling in a spiral path around the surface of a torus.

> The radius of the torus (i.e., the radius of the circle at the center of the tube) is 2 and the diameter of the circular cross-section is 1.
>
> The electron starts at position $(1, 0, 0)$, travels at a constant angular velocity around the vertical axis of 1 radian per second, and its path winds up and around the torus 4 times before it returns to its starting position.

What is the position and velociy of the electron at time t?

Fun-loving electron in action



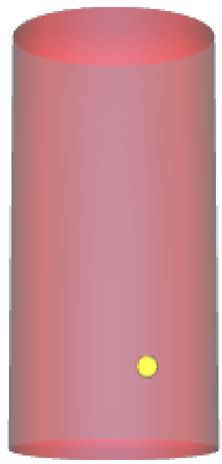
A similar but simpler question

A socially awkward electron is traveling in a spiral path around the surface of a cylinder.

> The cylinder has radius 1 and height 4
>
> The electron starts at position $(1, 0, 0)$, travels counterclockwise at a rotational speed of 1 radian per second.
>
> Its path winds around the cylinder exactly 4 times when it reaches the top.

What is the electron's position at time t?

Awkward electron (loop)



Teach the piglet of calculus

How can we break down the motion into pieces the piglet of calculus can digest?

How would you explain the motion to the piglet?

The piglet needs a precise description in order to predict future positions.

If the piglet fails, it's bacon time. Don't let that happen.

Any ideas?

Parametric description

We can trace the coordinates of the electron as it moves, giving functions

$$x = x(t)$$
$$y = y(t)$$
$$z = z(t)$$

Equivalent formulation: view $\langle x(t), y(t), z(t) \rangle$ as a vector-valued function . *Read section 13.1 for more!*

How do we figure out these functions?

One method: projection

This is a fancy say to saying: ignore some coordinates and try to describe the simpler motion.
We already saw that ignoring coordinates is one way of casting a shadow.

> What happens if we ignore the z coordinate of the electron on the cylinder?
>
> Same as projecting the path into the xy-plane! (Looking down from above.)
>
> What is that projection in this case?
>
> We can try to look at it.

Image in the xy-plane

The projection of the electron into the plane just moves in a circle.

> The radius is 1.
>
> It moves at 1 radian per second.
>
> What are the x and y coordinates as functions of t?

The usual trigonometric formulas give

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Image in the xy-plane

The projection of the electron into the plane just moves in a circle.

- The radius is 1.
- It moves at 1 radian per second.
- What are the x and y coordinates as functions of t?

The usual trigonometric formulas give

$$(x(t), y(t)) = (\cos(t), \sin(t)).$$

What about z?

The key is the timing of the revolutions.

One revolution takes 2π seconds.

It should take four revolutions to get to the top ($z = 4$), so 8π seconds.
Thus, $z(t) = 4 \cdot t/8\pi = t/2\pi$.

Putting it all together:

$$(x(t), y(t), z(t)) = (\cos(t), \sin(t), t/2\pi).$$

A parametric description of the torus

Given two numbers t and u between 0 and 2π, we get a point on a torus of radius 2 and tube radius 1 like this:

$$\begin{aligned} x(t,u) &= \cos(t)(2-\cos(u)) \\ y(t,u) &= \sin(t)(2-\cos(u)) \\ z(t,u) &= \sin(u) \end{aligned}$$

If you fix u, the t-path is a circle around the torus. If you fix t, the u-path is a circle around the tube .

Use this to make a spiral path around the torus that starts at $(1,0,0)$ and winds around the tube 4 times before it returns to its starting point.

Hint:
substitute for u as a function of t to make the two act in concert!

A parametric description of the torus

Given two numbers t and u between 0 and 2π, we get a point on a torus of radius 2 and tube radius 1 like this:

$$\begin{aligned} x(t,u) &= \cos(t)(2-\cos(u)) \\ y(t,u) &= \sin(t)(2-\cos(u)) \\ z(t,u) &= \sin(u) \end{aligned}$$

If you fix u, the t-path is a circle around the torus. If you fix t, the u-path is a circle around the tube .

Use this to make a spiral path around the torus that starts at $(1,0,0)$ and winds around the tube 4 times before it returns to its starting point.

Think about this for next time!

Question

Two tiny cars travel on paths

$$(x, y, z) = (\cos(t), \sin(t), 0)$$
$$(x', y', z') = (0, \cos(t), \sin(t))$$

Will they collide?

Now suppose the second car travels at a different speed so that $(x', y', z') = (0, \cos(\alpha t), \sin(\alpha t))$. For which constants α will the tiny cars collide?

Next time: *derivatives and integrals of vector functions!*