Loaded!

Please proceed to the calculus wonderland by pressing the right arrow key or clicking the right arrow that is visible when you move your mouse over the pig.

Instructions

These slides should work with *any* modern browser: IE 9+, Safari 5+, Firefox 9+, Chrome 16+.

- Navigate with arrow keys; you may need to give the window focus by clicking outside the lecture frame (the pig) for key commands described throughout this slide to work properly.
- Press M to see a menu of slides. Press G to go to a specific slide. Press W to toggle scaling of the deck with the window. If scaling is off, slides will be 800 by 600; it is off by default.
- Use left click, middle click, right click or hold A, S, D on the keyboard and move the mouse to rotate, scale, or pan the object.

If your browser or hardware does not support WebGL, interacting with models will be *very* slow (and in general models can get CPU-intensive). Navigate to a slide away from any running model to stop model animation.

Lecture 12

You survived!

- You loved it
- It was pretty hard
- We'll do a more detailed post-mortem on Friday

Today: normals, binormals, and osculating planes

Friday: normal and tangential components of acceleration

Read Sections 13.3, 13.4. *We cannot cover everything in lecture or section, but you will need it all for the rest of your lives!*

Questions!

Consider the electron spiraling around the torus.

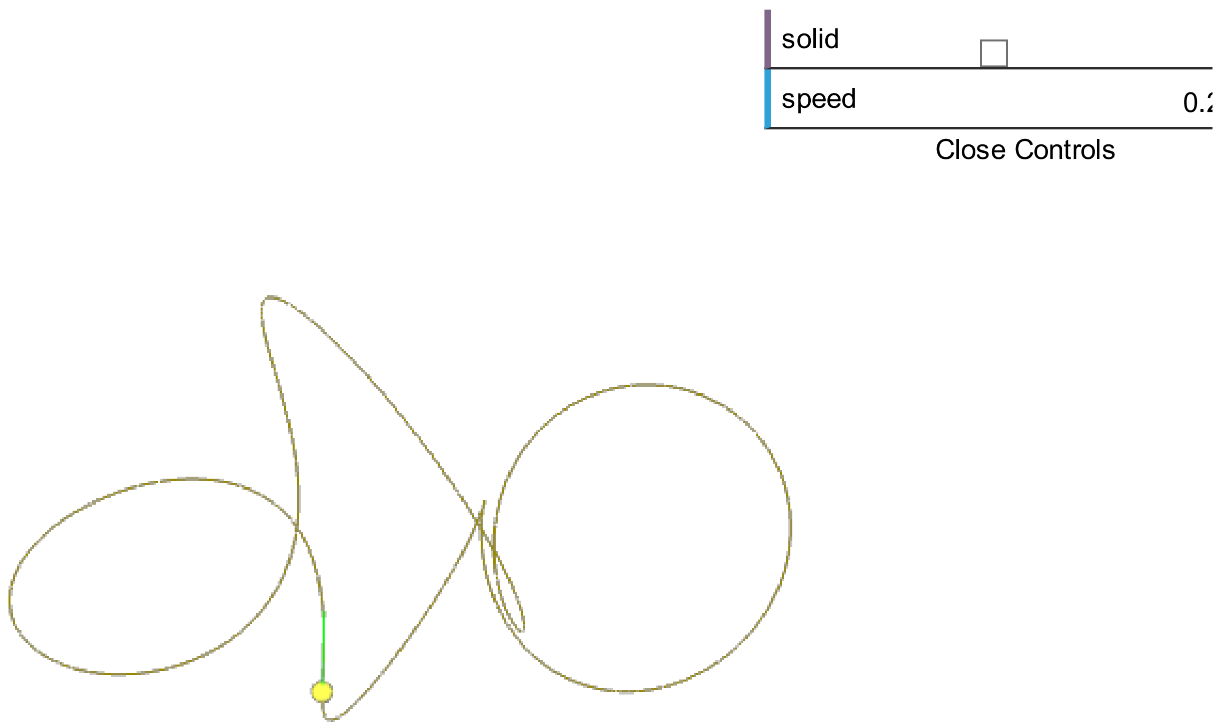
The vector description:

$$\mathbf{f}(t) = \langle \cos(t)(2 - \cos(4t)), \sin(t)(2 - \cos(4t)), \sin(4t) \rangle$$

- What plane best approximates (makes most contact with) the path followed by the electron? (Any guesses?)
- How can we measure the twisting of the path in three-dimensional space?
- Earlier: curvature measures curvature of path, but does not capture direction of curving.

Visualizing changes with vectors, I

Here's a model of the electron path with the unit tangent vector $\mathbf{T}(t)$ attached.



How can we recover more of the curving structure?

Take the derivative of the unit tangent vector!

Definition: the unit normal vector to the parametrized path $\mathbf{f}(t)$ with unit tangent vector $\mathbf{T}(t)$ is defined to be

$$\mathbf{N}(t) = \frac{\mathbf{T}'(T)}{|\mathbf{T}'(t)|}\,.$$

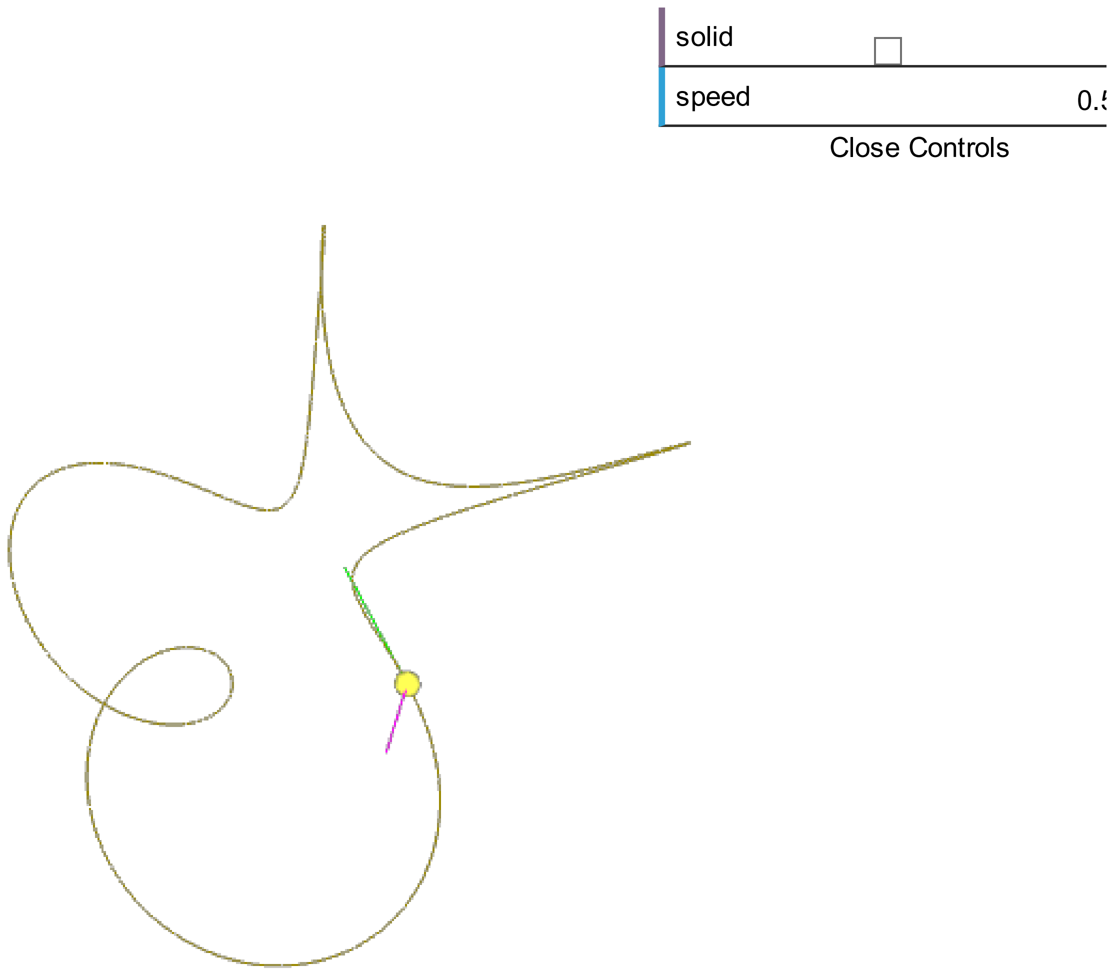
We are normalizing the derivative of the unit tangent so that we can get as close to bare intrinsic geometry as possible.

Why is this called a normal vector?

> Because $\mathbf{T}(t) \cdot \mathbf{T}'(t) = 0$, as you read in the book (and we saw earlier in class).

Visualizing changes with vectors, II

Here's a model of the electron path with the unit tangent and unit normal both attached.



Try an example

Consider the helix

$$\mathbf{h}(t) = \langle \cos(t), \sin(t), t \rangle$$

- Calculate the unit tangent vector $\mathbf{T}(t)$
- Calculate the unit normal vector $\mathbf{N}(t)$

But wait, there's more!

Take the cross product now for a free third vector!

Definition: the binormal vector at time t is the unit vector

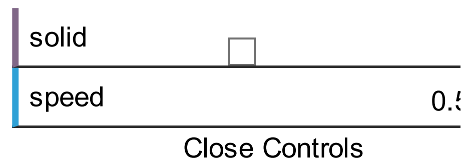
$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t).$$

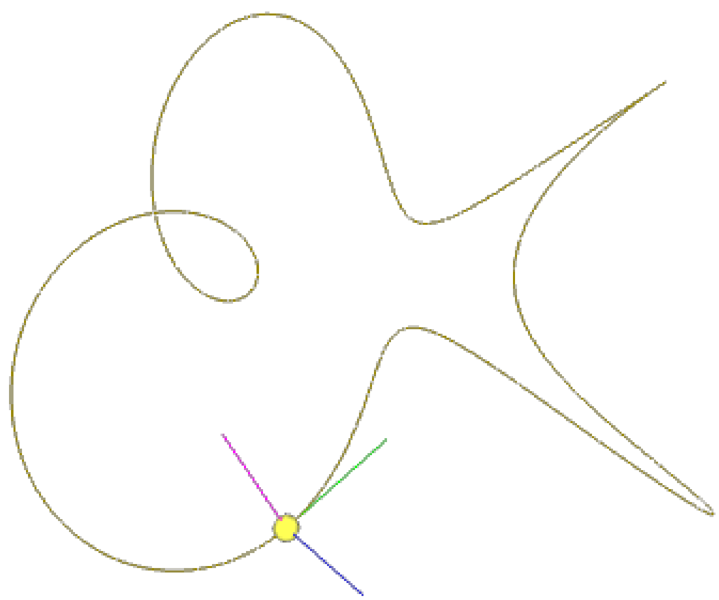
Together, the vectors $\mathbf{T}(t), \mathbf{N}(t), \mathbf{B}(t)$ form a triad of unit vectors satisfying the right-hand rule. This is called a *frame* .

> Complete your unit tangent and unit normal to get the full frame for the helix $\mathbf{h}(t) = \langle \cos(t), \sin(t), t \rangle$

Visualizing changes with vectors, III

Here's a model of the electron path with the unit tangent , unit normal , and binormal vectors all attached. Observe that the unit normal explains the changes in direction of the unit tangent. The binormal is a bit more mysterious at the moment.





What does this do for us?

We can find the normal plane and the osculating plane!

Osculating: kiss

You probably never thought you would be finding planes that kiss curves in your calculus class.

Math is fun

The osculating plane is the plane spanned by $\mathbf{T}$ and $\mathbf{N}$. The normal plane is the plane spanned by $\mathbf{N}$ and $\mathbf{B}$.

Do one: calculate the osculating plane and the normal plane to the helix at time t.

More problems to think about

- What are the osculating and normal planes of a parametric curve $(x(t), y(t), 0)$?
- Does the osculating plane depend upon the parametrization? E.g., what about the crazy helix $(\cos(t^2), \sin(t^2), t^2)$?
- How do the osculating planes of the electron's path on the torus relate to the tangent planes of the torus?
- How awesome is this?

Next time: *acceleration++!*