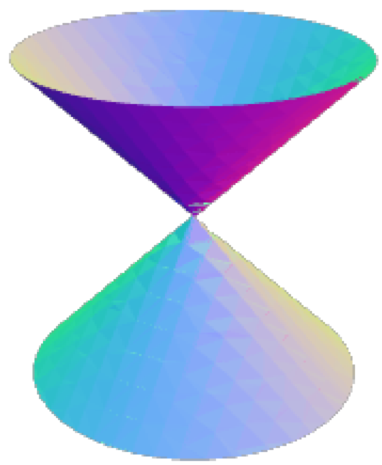
# Surfaces

# Vague question

**How can we relate equations to the shapes of their zero loci?**

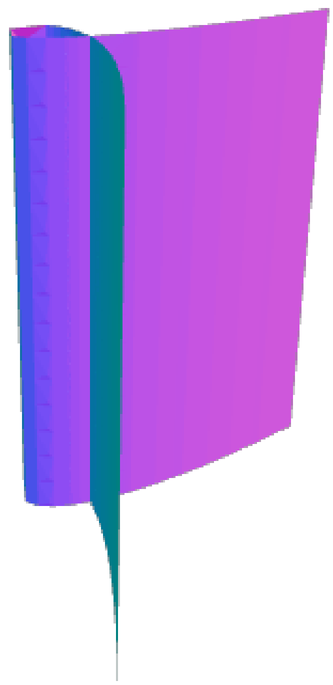
# A menagerie of shapes

Cone: $x^2 + y^2 = z^2$



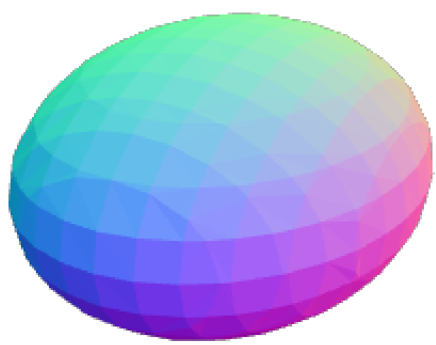
# A menagerie of shapes

Freaky cylinder: $y^2 = x^2(x-1)$



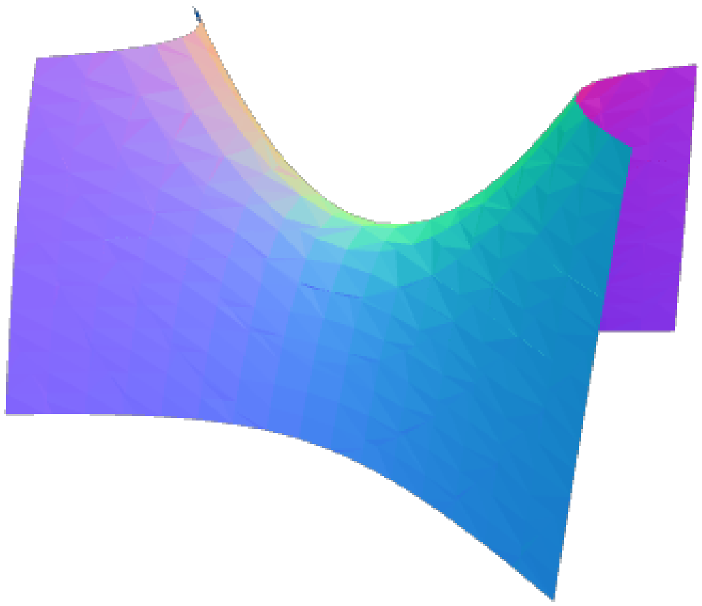
# A menagerie of shapes

Ellipsoid: $\frac{1}{2}x^2 + \frac{1}{3}y^2 + z^2 = 1$



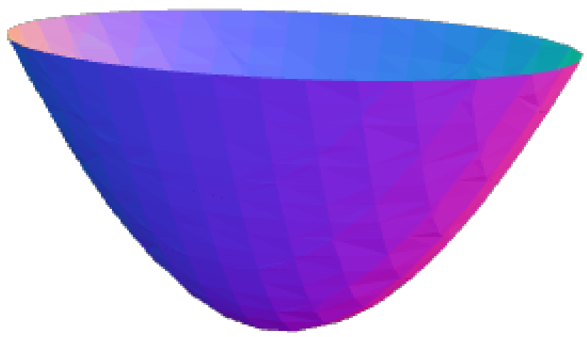
# A menagerie of shapes

Hyperbolic paraboloid: $\frac{1}{9}\,x^2 - \frac{1}{4}\,y^2 = z$



# A menagerie of shapes

Elliptic paraboloid: $\frac{1}{9}\,x^2 + \frac{1}{4}\,y^2 = z$



# Key idea

- Slice the shape with planes and reassemble the pieces.
- This idea recurs throughout the study of geometry (even by professionals!).
- Subdividing, solving, reassembling is also how computers graph these things.

# Example: $\frac{1}{9}x^2 + \frac{1}{4}y^2 = z$

We can make a horizontal trace (horizontal slice) at $z = 6$.

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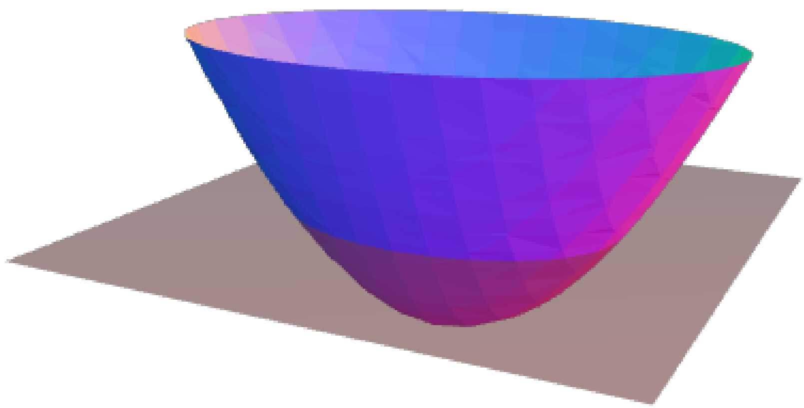

# Example: $\frac{1}{9}x^2 + \frac{1}{4}y^2 = z$

We can make a horizontal trace (horizontal slice) at $z = 6$.

- Equation for the curve in the horizontal plane:
$$\frac{1}{9}x^2 + \frac{1}{4}y^2 = 6.$$
- What shape is this?
- What shape will a general horizontal trace have?

# Example: $\frac{1}{9}x^2 + \frac{1}{4}y^2 = z$

We can also make a vertical trace (vertical slice) at $y = 0$.

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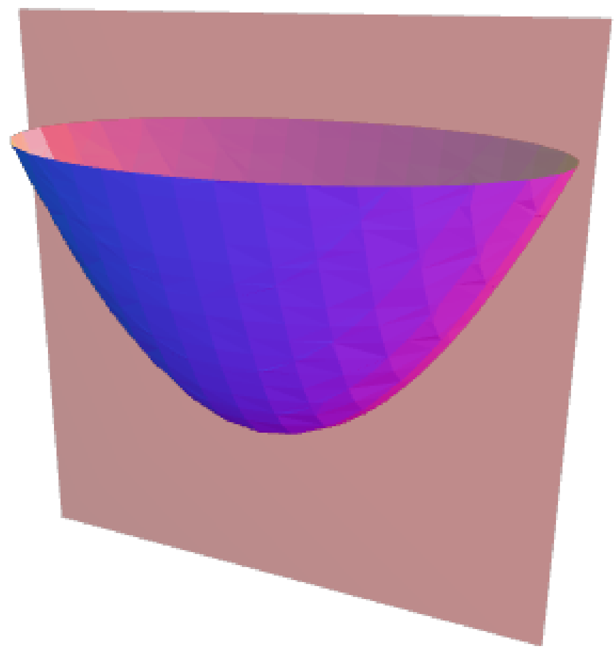


# Example: $\frac{1}{9}\,\mathbf{x}^2 + \frac{1}{4}\,\mathbf{y}^2 = \mathbf{z}$

We can make a vertical trace (vertical slice) at $y = 0$.

- Equation for the curve in the vertical plane:
$$\frac{1}{9}\,x^2 = z.$$
- What shape is this?
- What shape will a general vertical trace have?

# Sketching the shape

- Choose some horizontal traces and sketch them
- Choose some vertical traces and sketch them
- Hope for the best

# Demonstration

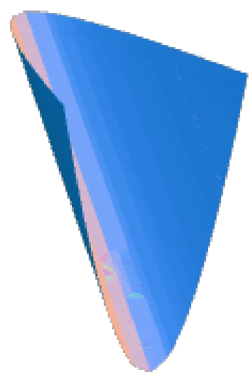
Let's try the equation $x^2 + y - z = 0$.

- General horizontal trace: $y = -x^2 + a$, $a$ a constant.
- General $xz$-plane vertical trace: $z = x^2 + b$, $b$ a constant.
- General $yz$-plane vertical trace: $z = y + c$, $c$ a constant.
- Draw some!

# Demonstration

## Final assembled product

Cosmic taco: $x^2 + y - z = 0$.



# Practice

- Calculate horizontal and vertical traces for the hyperbolic paraboloid: $\frac{1}{9}x^2 - \frac{1}{4}y^2 = z$
- How do the traces help you to sketch the object?
- How do the traces help you to tell objects apart?

# New shapes to consider

Apply the techniques we've been discussing to to draw sketches of the solutions to these equations in three variables.

- $x^2 + y^2 - z^2 = 1$
- $z^2 - x^2 - y^2 = 1$
- $x^2 + y^2 = 1$
- $x^2 - 2x + y^2 - z^2 = 0$