Loaded!

Please proceed to the calculus wonderland by pressing the right arrow key or clicking the right arrow that is visible when you move your mouse over the pig.

Instructions

These slides should work with *any* modern browser: IE 9+, Safari 5+, Firefox 9+, Chrome 16+.

- Navigate with arrow keys; you may need to give the window focus by clicking outside the lecture frame (the pig) for key commands described throughout this slide to work properly.
- Press M to see a menu of slides. Press G to go to a specific slide. Press W to toggle scaling of the deck with the window. If scaling is off, slides will be 800 by 600; it is off by default.
- Use left click, middle click, right click or hold A, S, D on the keyboard and move the mouse to rotate, scale, or pan the object.

If your browser or hardware does not support WebGL, interacting with models will be *very* slow (and in general models can get CPU-intensive). Navigate to a slide away from any running model to stop model animation.

Lecture 14

Behind us

- Tangential and normal components of acceleration
- Examples in dimensions 1, 2, 3
- It was awesome

Ahead

Today: functions of multiple variables

Wednesday: partial derivatives and tangent planes

Read Sections 14.1, 14.3, 14.4 (not 14.2 unless you want to have some additional fun). *Adult mathematics means a lot time by yourself.*

Homework due Tuesday at 11 PM

Questions!

How to describe Mt. Rainier?

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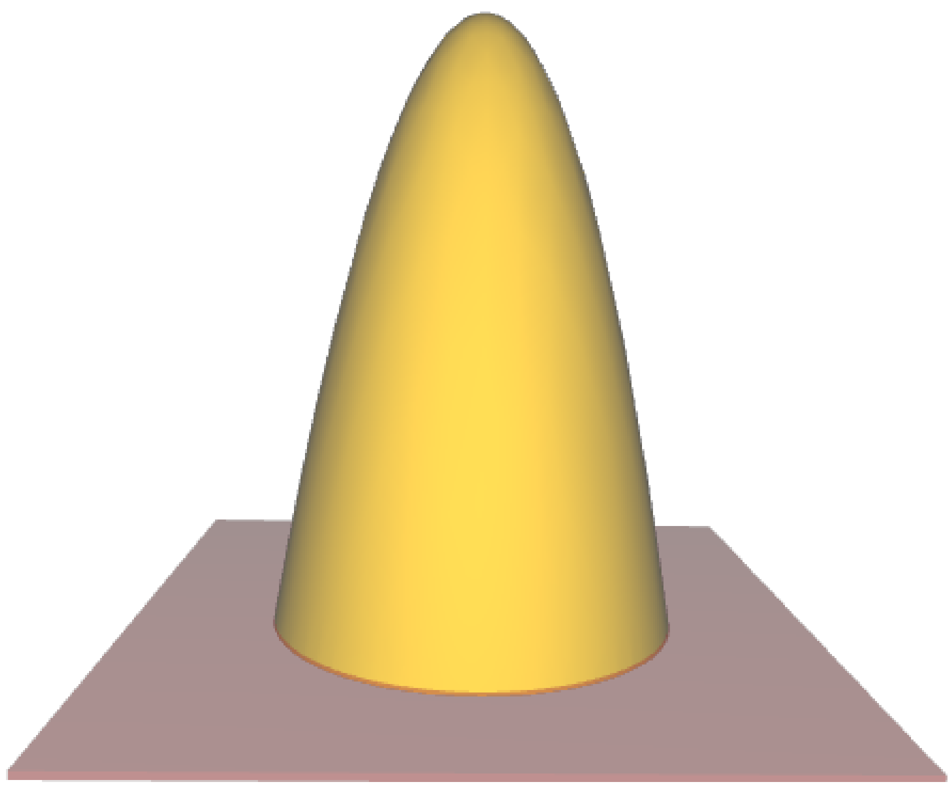
With poetry.

By height above the "ground".

By depth below a giant tarp hovering at constant altitude above the mountain.

"Assume a spherical cow...."

Let's approximate and say that the mountain is shaped roughly like a paraboloid. We can peek under the mountain a bit to see the shadow it casts:



Describe our toy mountain using numbers

For each point in the shadow, record the height of the mountain over that point.

For our toy model, this function is $f(x, y) = 9 - x^2 - y^2$

The general philosophy of functions still works.

> A function takes an input and returns an output .
>
> There need not be a formula. The function could be defined on weird inputs.
>
> For example: my desire for coffee (rated as "small","medium", "large") is a function of my fatigue ("mild", "moderate", "extreme") and my rough Husky card balance ("empty", "some", "lots"). Thus, d(mild,lots)=small and d(extreme,some)=large, but perhaps d(extreme,none)=moderate. Etc.

Our toy picture is a graph

The shape you have recorded is the graph of a function of two variables!

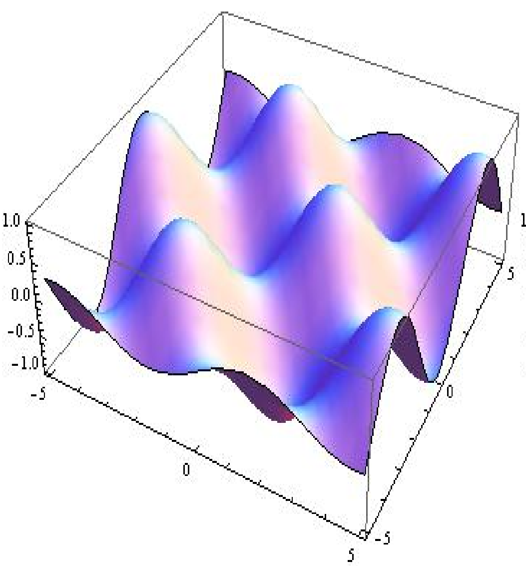
In this toy model, the function is $f(x, y) = 9 - x^2 - y^2$, so the graph is described by $z = 9 - x^2 - y^2$.

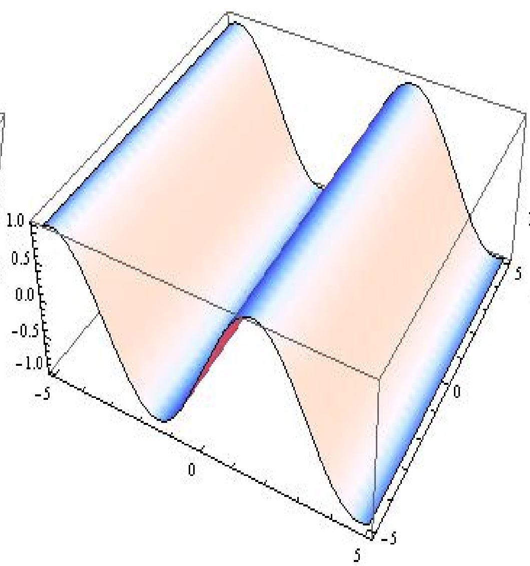
Take a whack at graphing these:

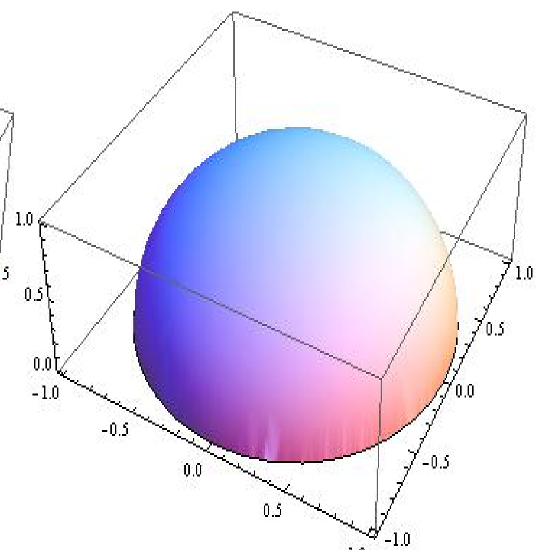
$$f(x, y) = \sin(x)$$
$$f(x, y) = \sin(x)\cos(y)$$
$$f(x, y) = \sqrt{1 - x^2 - y^2}$$

Match the graph with the function

$\sin(x)$ $\sin(x)\cos(y)$ $\sqrt{1-x^2-y^2}$







Enter the domain of the sheep*

*My college roommate studied Akkadian and found this written in Akkadian on a Pepsi can in 1999

Functions of two (or more!) variables have domains just like functions of one variable.

Sometimes, the domain is a natural consequence of the shape of the function.

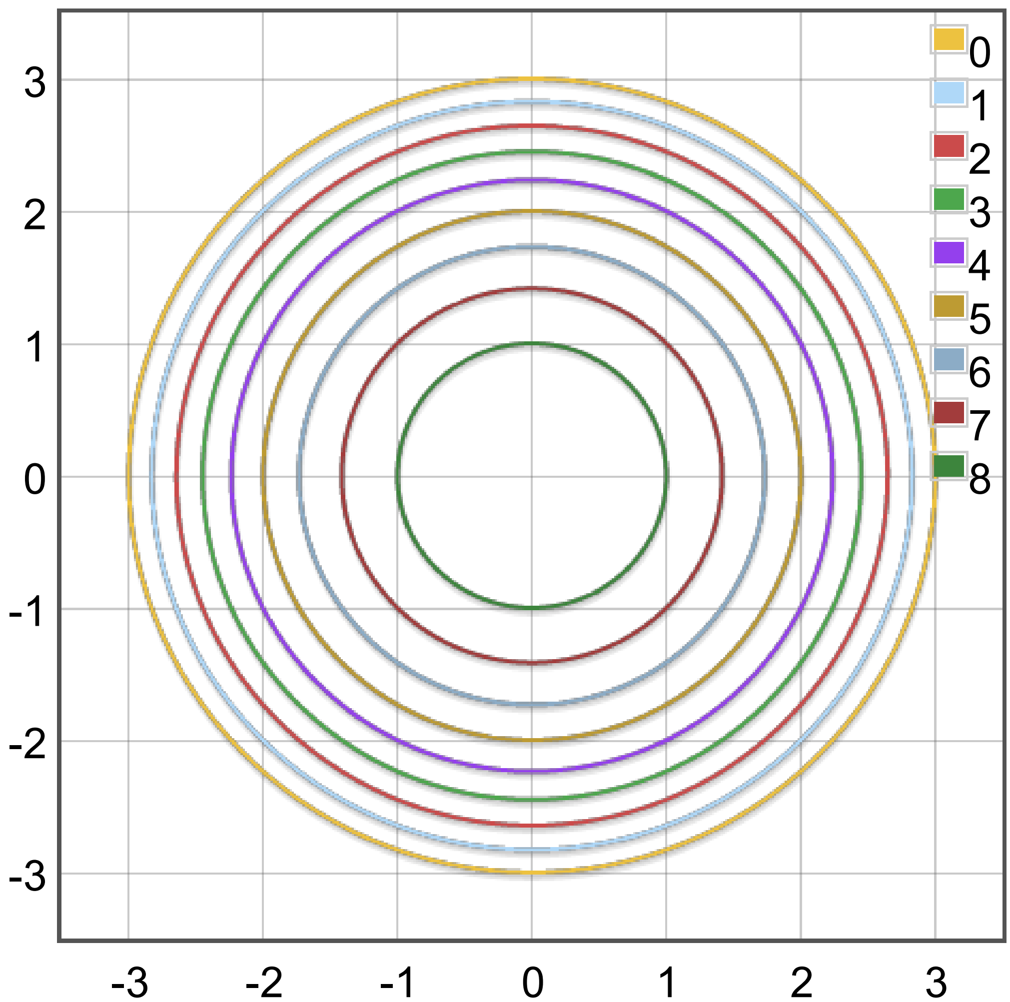
Usually, the domain is specified in advance.

What are the natural domains of the following functions?

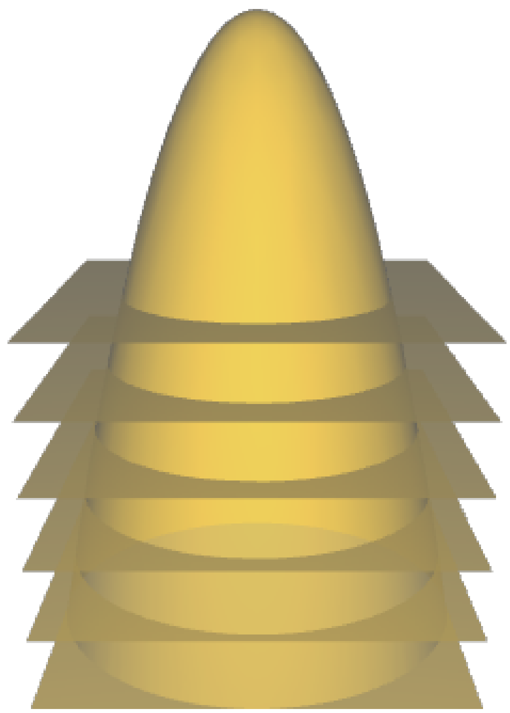
$$f(x, y) = 9 - x^2 - y^2$$
$$f(x, y) = \sin(x) + \cos(|y| + \cos(x^{2012}))$$
$$f(x, y) = \ln(x + y)$$

A mountaineer cannot lift mountains

She needs to have a map, like this one, with level curves .

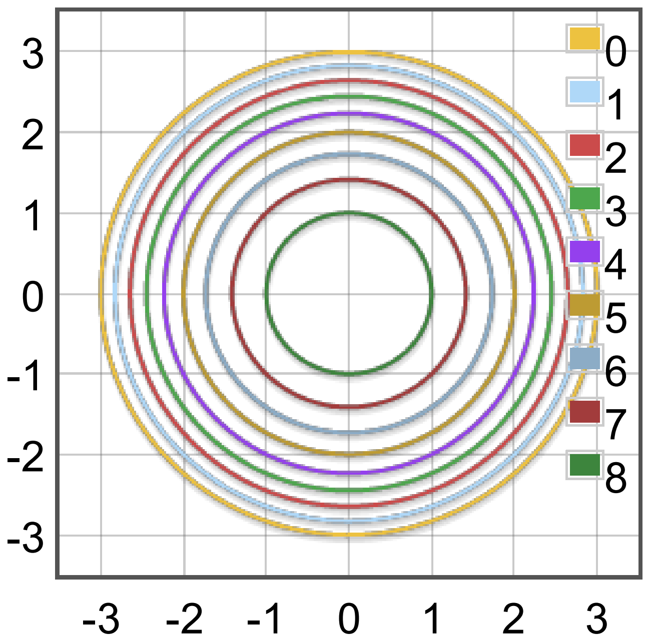


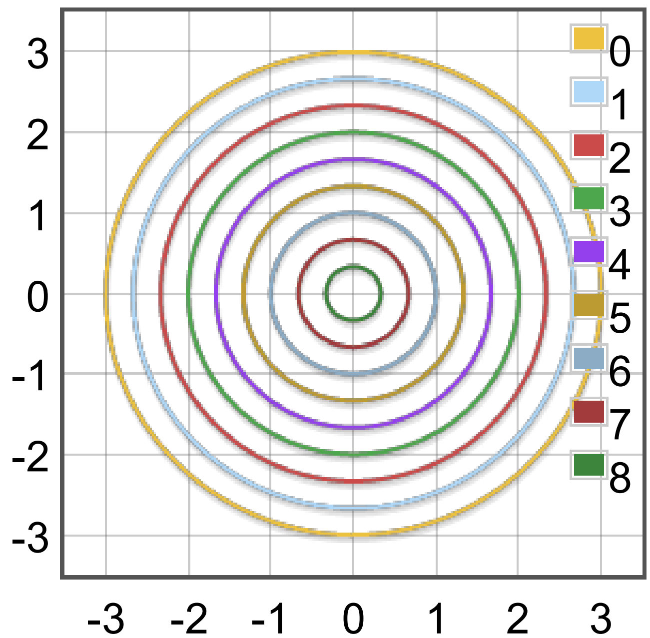
The level curves are horizontal traces!



Telling things apart

»Which of these contour maps corresponds to a circular cone?





»What is the function $f(x, y)$ whose graph in the region $0 \leq z \leq 9$ is a circular cone with base radius 3 and vertex at $(0, 0, 9)$?

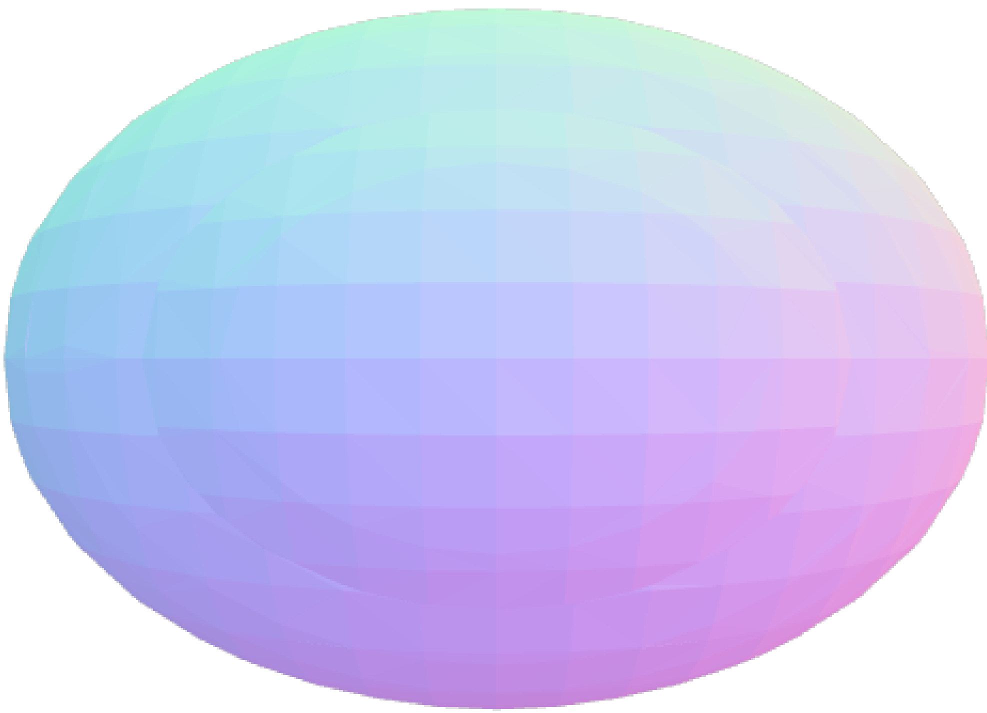
People really do this

Here's a link to a map of Mt. Rainier from 1924 in the US national atlas.

What does it mean when the level curves are bunched together?

What about more variables?

We can plot level surfaces for functions of three variables (but it's rather hard to visualize). Here's an example with $f(x, y, z) = x^2 + y^2 + 2z^2$.



Next time: *partial derivatives will blow your minds.*