Loaded!

Please proceed to the calculus wonderland by pressing the right arrow key or clicking the right arrow that is visible when you move your mouse over the pig.

Instructions

These slides should work with *any* modern browser: IE 9+, Safari 5+, Firefox 9+, Chrome 16+.

- Navigate with arrow keys; you may need to give the window focus by clicking outside the lecture frame (the pig) for key commands described throughout this slide to work properly.
- Press M to see a menu of slides. Press G to go to a specific slide. Press W to toggle scaling of the deck with the window. If scaling is off, slides will be 800 by 600; it is off by default.
- Use left click, middle click, right click or hold A, S, D on the keyboard and move the mouse to rotate, scale, or pan the object.

If your browser or hardware does not support WebGL, interacting with models will be *very* slow (and in general models can get CPU-intensive). Navigate to a slide away from any running model to stop model animation.

Math 126

Lecture 5

Behind us

- Cross product
- Areas of parallelograms

Homework due tomorrow at 11 PM

Today: lines and planes

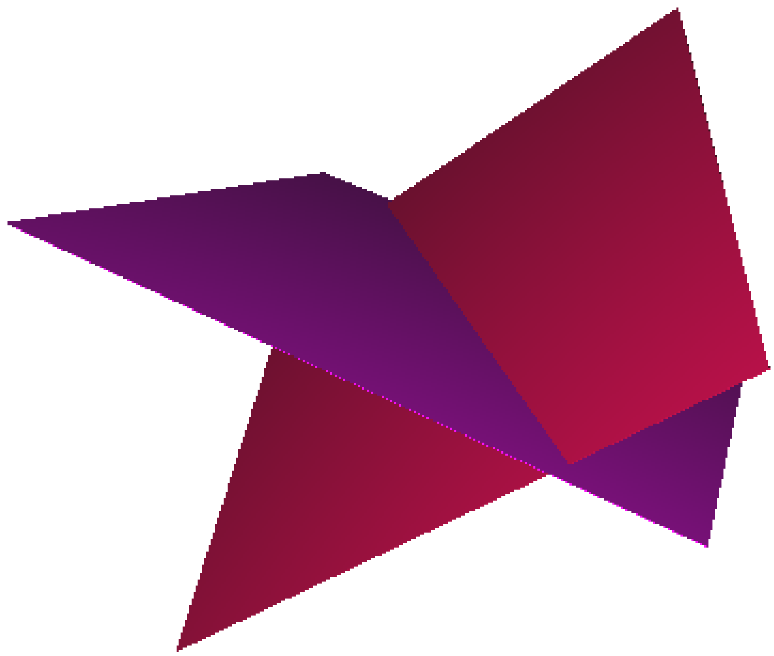
Wednesday: cylinders and quadric surfaces

Read Sections 12.5 and 12.6 of the book. *We will not cover everything in lecture or section. Grow or die!*

Questions!

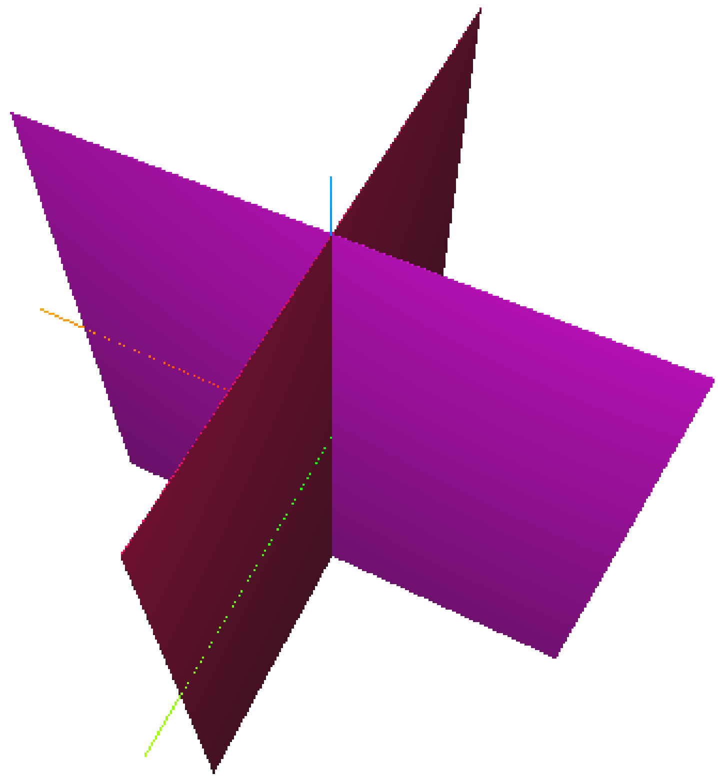
Warm up

Question: how can we describe the line of intersection of two planes?



Warm up

Simpler: what is the intersection of the planes $x = 0$ and $y = 0$



The enemy of my enemy...

What is a plane?

The plane is perpendicular to the line that is perpendicular to it (?!?!!?!)

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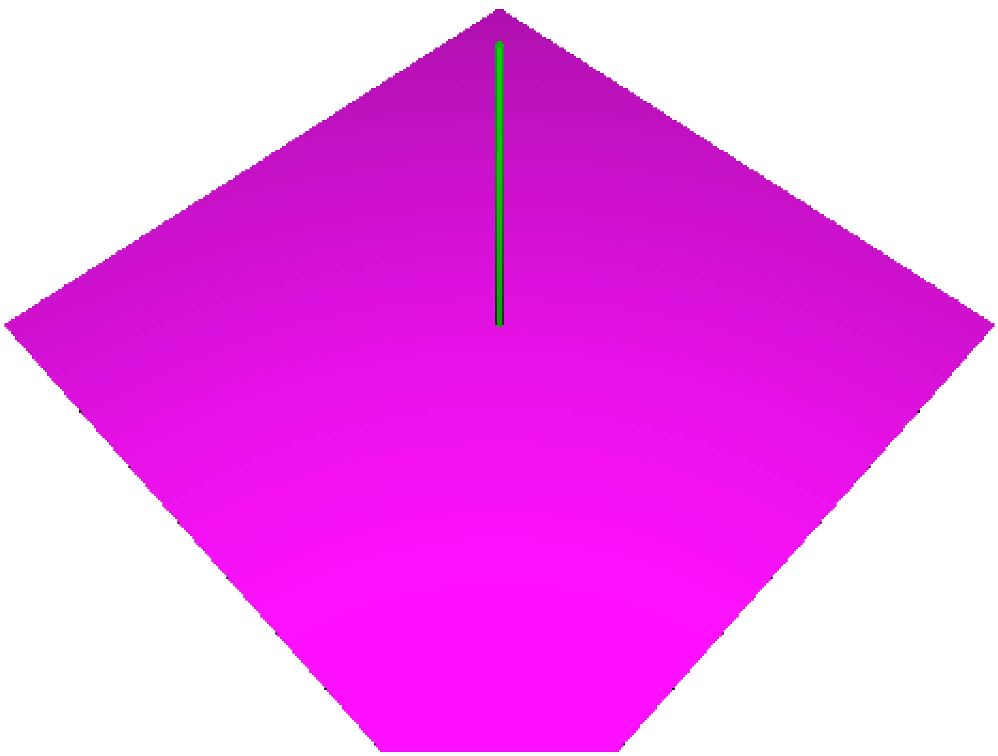

Quick review

How can you tell if two vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular?

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Don't look at the next slide if you don't want to see the answer!

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What shape is that?

Piglet of calculus conjectures

Any plane is just the set of endpoints of vectors perpendicular to a fixed one! So just fix a vector $\mathbf{u}$ and let

$$P_{\mathbf{u}} = \{\mathbf{v} \text{ such that } \mathbf{v} \cdot \mathbf{u} = 0\}.$$

For example, the xy-plane is the set of endpoints of vectors perpendicular to $\langle 0, 0, 1 \rangle$

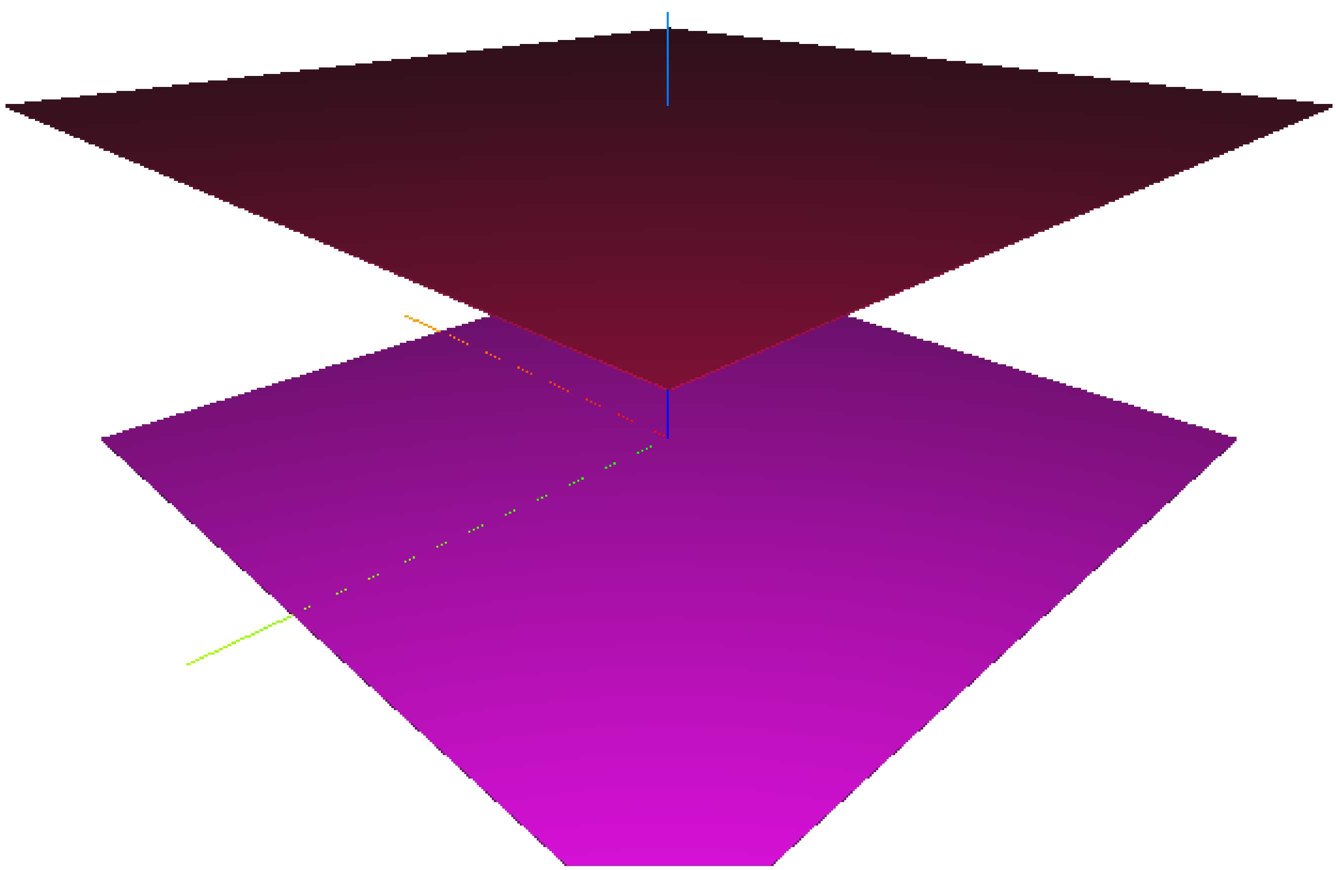
Does it work? Can the piglet of calculus go to sleep now?

Conundrum: translation

This is OK if the plane can be anchored like vectors can at $(0, 0, 0)$.

If not, we have to take what we just did and translate it in space (i.e., move it away from $(0, 0, 0)$).

This is just like making the plane $z = 4$ by translating the xy-plane up 4 units: the plane $z = 4$ is not the set of endpoints of vectors perpendicular to $\mathbf{k}$, just a parallel translation of it.



Let's do one together

Describe the plane $x - 3y + 47z - 28 = 0$ using vectors.

Normal vector: $\langle 1, -3, 47 \rangle$. How did I know?

Trick: always just use the coefficients of x, y, and z

To translate: find one solution by eyeballs. A solution: $(-16, 1, 1)$.

So the plane is the set of endpoints of vectors $\mathbf{v}$ such that

$$(\mathbf{v} - \langle -16, 1, 1 \rangle) \cdot \langle 1, -3, 47 \rangle = 0.$$

Another way to say it: it is what you get when you take the set of vectors perpendicular to $\langle 1, -3, 47 \rangle$ and translate them all by $\langle -16, 1, 1 \rangle$ (and then just keep the set of endpoints)

Practice

Describe the plane $3x - 4y - 5z = 6$ using vectors.

Who cares?

Using this approach, you can prove that any plane is the set of solutions of a linear equation in x, y, z (see book!).

This gives us a way to get a grip on the intersection of two planes.

Example

Describe the intersection of the planes $x - 2y - z = 0$ and $2x - y + z = 6$.

Perpendicular vectors: $\langle 1, -2, -1 \rangle$ and $\langle 2, -1, 1 \rangle$

Common solution: $(2, 0, 2)$

Thus, the line of intersection is the set of vectors $\mathbf{v}$ such that

$$(\mathbf{v} - \langle 2, 0, 2 \rangle) \cdot \langle 1, -2, -1 \rangle = 0$$

and

$$(\mathbf{v} - \langle 2, 0, 2 \rangle) \cdot \langle 2, -1, 1 \rangle = 0.$$

The vector $\mathbf{v} - \langle 2, 0, 2 \rangle$ is perpendicular to both: cross product!

The line is just the endpoints of vectors of the form

$$\langle 2, 0, 2 \rangle + t \langle 1, -2, -1 \rangle \times \langle 2, -1, 1 \rangle,$$

where t ranges over all scalars.
Parametric equations!

Last step: expand cross product

To describe $\langle 2,0,2\rangle + t\langle 1,-2,-1\rangle \times \langle 2,-1,1\rangle$, let's expand:

$$\langle 1,-2,-1\rangle \times \langle 2,-1,1\rangle = \langle -3,-3,3\rangle$$

So the line is given by the endpoints of the vectors $\{\langle 2,0,2\rangle + t\langle -3,-3,3\rangle\}$

Parametric form: $(x,y,z) = (2-3t, -3t, 2+3t)$

As t varies, this traces out the line of intersection.

More practice

- Describe the intersection of $3x + 4y + 5z = 6$ and $y + z = 0$.
- Describe the intersection of $3x + 4y + 5z = 6$ and $6x + 8y + 10y = 12$.
- Describe the intersection of $3x + 4y + 5z = 6$ and $9x + 12y + 15z = 17$.

Next time: *cylinders and quadrics!*