Cross products: geometry

Sweet Sweet Theorem for $\times$

Given two vectors $\mathbf{a}$ and $\mathbf{b}$ with angle θ

- the vector $\mathbf{a} \times \mathbf{b}$ is $\perp$ to both $\mathbf{a}$ and $\mathbf{b}$;
- the list $\mathbf{a}, \mathbf{b}, \mathbf{a} \times \mathbf{b}$ satisfies the right-hand rule;
- we have $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin(\theta)$;
- $|\mathbf{a} \times \mathbf{b}|$ equals the area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$.

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Ties the cross product to area and orientation.

Do one

Example: find the area of the parallelogram spanned by $\langle 4, 0, 0 \rangle$ and $\langle 3, 4, 0 \rangle$

Compute the cross product!

- Can you predict the direction of the cross product vector without calculating anything?
- Can you predict the magnitude of the cross product without calculating anything?

I did it

Example: find the area of the parallelogram spanned by $\langle 4, 0, 0 \rangle$ and $\langle 3, 4, 0 \rangle$

Compute the cross product!

$$\langle 4, 0, 0 \rangle \times \langle 3, 4, 0 \rangle = \langle 0, 0, 16 \rangle$$

Phew!

Hmmm.... Maybe drawing a picture of this (the parallelogram and the cross product) would help you cement this in your mind.

Exploratory probe

Suppose you model a surface with a computer: you can't do continuous or smooth things, so the surface is modeled as having a ton of faces (like the facets of a demonic gem). These faces might be described with coordinates in $\mathbf{R}^3$. How can you find the normal vector to one of the faces?

How many ways are there to make a normal vector? Is it unique? Can you use them to distinguish "inside" from "outside"?

Use the internet to find out if these things come up outside of math class!

Next: *lines and planes!*