Loaded!

Please proceed to the calculus wonderland by pressing the right arrow key or clicking the right arrow that is visible when you move your mouse over the pig.

Instructions

These slides should work with *any* modern browser: IE 9+, Safari 5+, Firefox 9+, Chrome 16+.

- Navigate with arrow keys; you may need to give the window focus by clicking outside the lecture frame (the pig) for key commands described throughout this slide to work properly.
- Press M to see a menu of slides. Press G to go to a specific slide. Press W to toggle scaling of the deck with the window. If scaling is off, slides will be 800 by 600; it is off by default.
- Use left click, middle click, right click or hold A, S, D on the keyboard and move the mouse to rotate, scale, or pan the object.

If your browser or hardware does not support WebGL, interacting with models will be *very* slow (and in general models can get CPU-intensive). Navigate to a slide away from any running model to stop model animation.

Lecture 13

Behind us

- Tangent, normal, binormal vector
- Osculating, normal planes
- Refined grasp on 3-dimensional parametric motion

Homework due Tuesday at 11 PM

Ahead

Today: normal and tangential components of acceleration

Monday: multivariable functions

Read Sections 13.4, 14.1. *Read them. Study them. Love them.*

Questions!

Question

The electron again

The electron is joined by a tiny rubber blob of length and width each equal to 0.1. Usual motion:

$$\mathbf{f}(t) = \langle \cos(t)(2 - \cos(4t)), \sin(t)(2 - \cos(4t)), \sin(4t) \rangle$$

Headwind (resp. tiny monkey) causes rubber blob to change length (resp. width) in proportion to the acceleration a along the path (resp. the acceleration b normal to the path).

$$\Delta(\text{length}) = -0.01a \qquad \Delta(\text{width}) = 0.01b$$

> What are the length and width of the rubber blob at time t?
>
> More basic question: how can we describe the acceleration along the path or perpendicular to the path? What does it mean?

Examples

What is acceleration along and perpendicular to these paths?

$$(t^{45}, 0, 0)$$

$$(t^2 - 1, t^3 - t, 0)$$

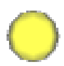
$$(\cos(t), \sin(t), t)$$

Examples

First: $(t^{45}, 0, 0), -1 \leq t \leq 1$ loop

acceleration

Close Controls



Examples

First: $(t^{45}, 0, 0)$

Motion on a string!

The acceleration in the direction of motion is just the vector $\langle 1980t^{43}, 0, 0\rangle$. It always points in the direction of the motion (including negative scalars, so could point in opposite direction).

How is the sign change at $t = 0$ reflected in the motion?

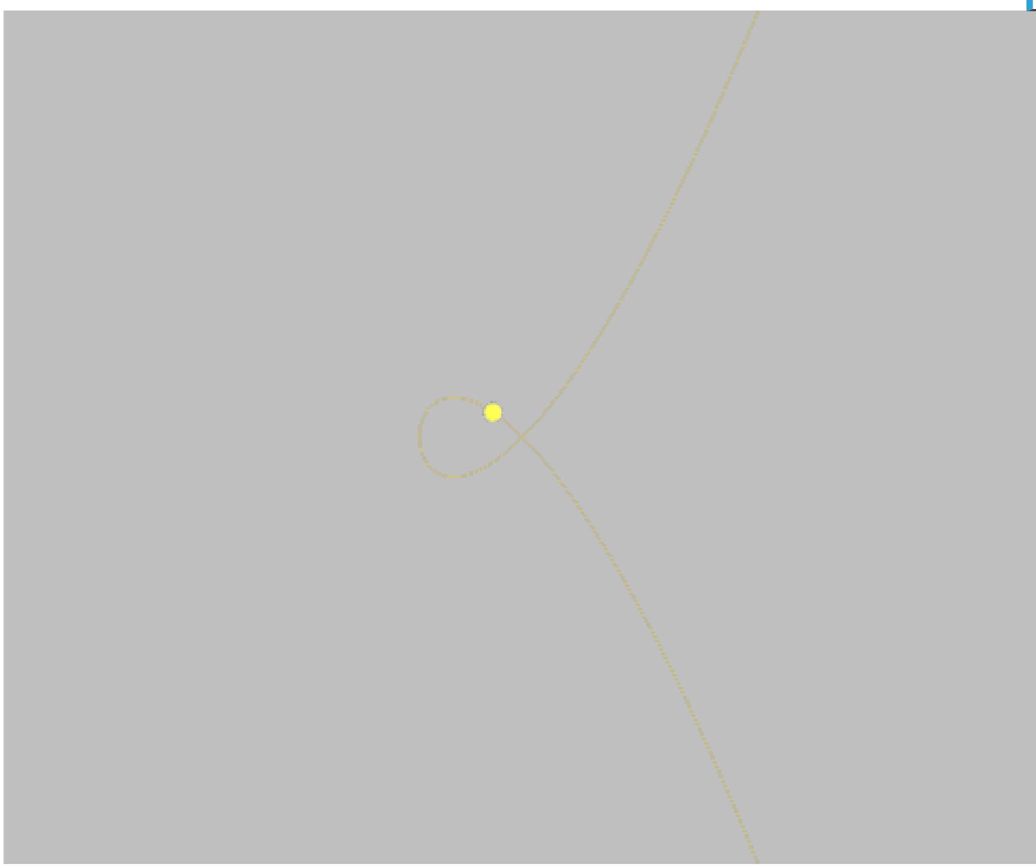
Examples

Second: $(t^2 - 1, t(t^2 - 1), 0), -2 \le t \le 2$ loop

vectors

speed 0.5

Close Controls



Examples

Second: $\boldsymbol{(t^2 - 1, t(t^2 - 1), 0)}$, observations

What we call "turning" looks like increased magnitude of normal component of acceleration.

Tangential component of acceleration can be very negative for a while before things slow down!

What else?

Examples

Second: $(t^2 - 1, t(t^2 - 1), 0)$

To calculate: find the unit tangent and normal vectors, then calculate components of acceleration.

Unit tangent: $\mathbf{T}(t) = \frac{1}{\sqrt{9t^4 - 2t^2 + 1}} \langle 2t, 3t^2 - 1, 0 \rangle$

Unit normal: $\mathbf{N}(t) = \frac{1}{\sqrt{9t^4 - 2t^2 + 1}} \langle 1 - 3t^2, 2t, 0 \rangle$

Acceleration: $\mathbf{a}(t) = \langle 2, 6t, 0 \rangle$

The components of $\mathbf{a}(t)$ in the directions of $\mathbf{T}(t)$ and $\mathbf{N}(t)$:

$$(\mathbf{a}(t) \cdot \mathbf{T}(t))\mathbf{T}(t) = \frac{18t^3 - 2t}{9t^4 - 2t^2 + 1} \langle 2t, 3t^2 - 1, 0 \rangle$$

$$(\mathbf{a}(t) \cdot \mathbf{N}(t))\mathbf{N}(t) = \frac{6t^2 + 2}{9t^4 - 2t^2 + 1} \langle 1 - 3t^2, 2t, 0 \rangle$$

Which component dominates for large t? How about for t near 0? Consistent with observations?

Examples

Do the third: the helix $(\cos(t), \sin(t), t)$

To calculate: find the unit tangent and normal vectors, then calculate components of acceleration.

Unit tangent: $\mathbf{T}(t) =$

Unit normal: $\mathbf{N}(t) =$

Acceleration: $\mathbf{a}(t) =$

The components of $\mathbf{a}(t)$ in the directions of $\mathbf{T}(t)$ and $\mathbf{N}(t)$:

$(\mathbf{a}(t) \cdot \mathbf{T}(t))\mathbf{T}(t) =$

$(\mathbf{a}(t) \cdot \mathbf{N}(t))\mathbf{N}(t) =$

Examples

Do the third: the helix $(\cos(t), \sin(t), t)$

To calculate: find the unit tangent and normal vectors, then calculate components of acceleration.

- Unit tangent: $\mathbf{T}(t) = \frac{1}{\sqrt{2}} \langle -\sin(t), \cos(t), 1 \rangle$
- Unit normal: $\mathbf{N}(t) = \langle -\cos(t), -\sin(t), 0 \rangle$
- Acceleration: $\mathbf{a}(t) = \langle -\cos(t), -\sin(t), 0 \rangle$
- The components of $\mathbf{a}(t)$ in the directions of $\mathbf{T}(t)$ and $\mathbf{N}(t)$:
    - $(\mathbf{a}(t) \cdot \mathbf{T}(t))\mathbf{T}(t) = \mathbf{0}$
    - $(\mathbf{a}(t) \cdot \mathbf{N}(t))\mathbf{N}(t) = \langle -\cos(t), -\sin(t), 0 \rangle$

Why does the acceleration have no tangential component?

(More troubling?) Why does the normal component have no z-component? How can the particle be climbing??

General theory

The procedure is always the same: find unit tangent and unit normal, calculate components.

The book has a discussion of various formulas, deductions using curvature, the product rule, etc.

The upshot: given a path $\mathbf{r}(t)$ with unit tangent $\mathbf{T}(t)$ and unit normal $\mathbf{N}(t)$, we can write the acceleration $\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}$ where

$$a_T = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|} \qquad\qquad a_N = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

Do one: the electron/rubber blob

$$\mathbf{f}(t) = \langle \cos(t)(2-\cos(4t)), \sin(t)(2-\cos(4t)), \sin(4t) \rangle$$

Next time

functions of multiple variables! YEAH!!