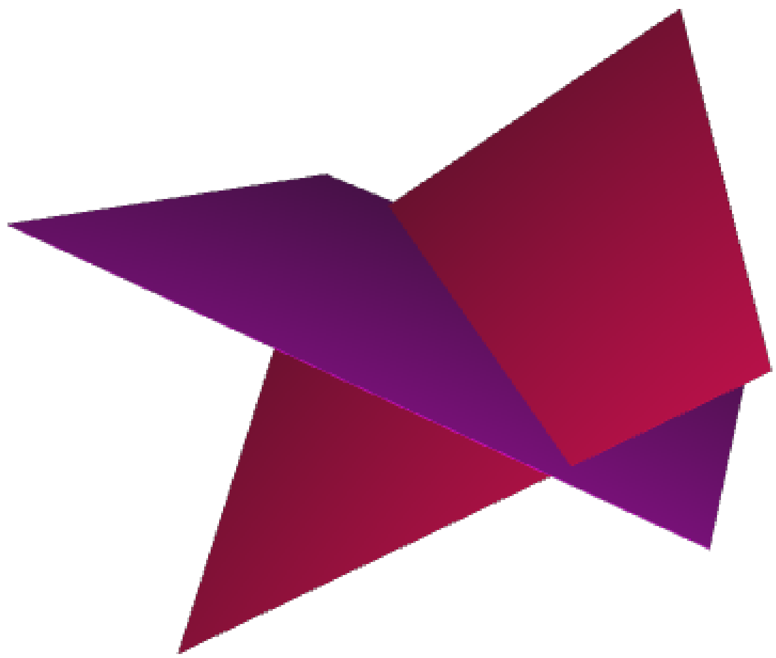
Lines and planes: introduction

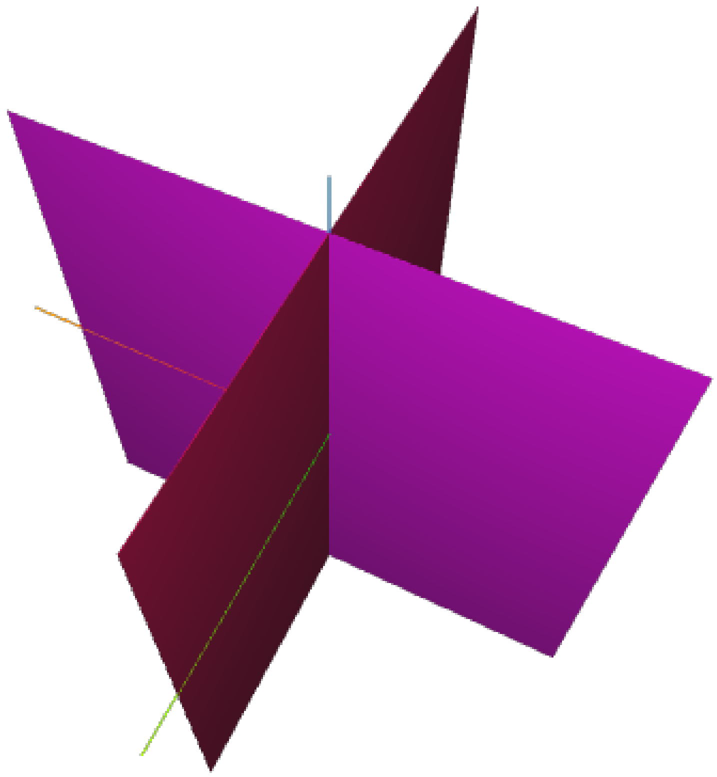
Warm up

Question: how can we describe the line of intersection of two planes?



Warm up

Simpler: what is the intersection of the planes $x = 0$ and $y = 0$



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- What is a plane?
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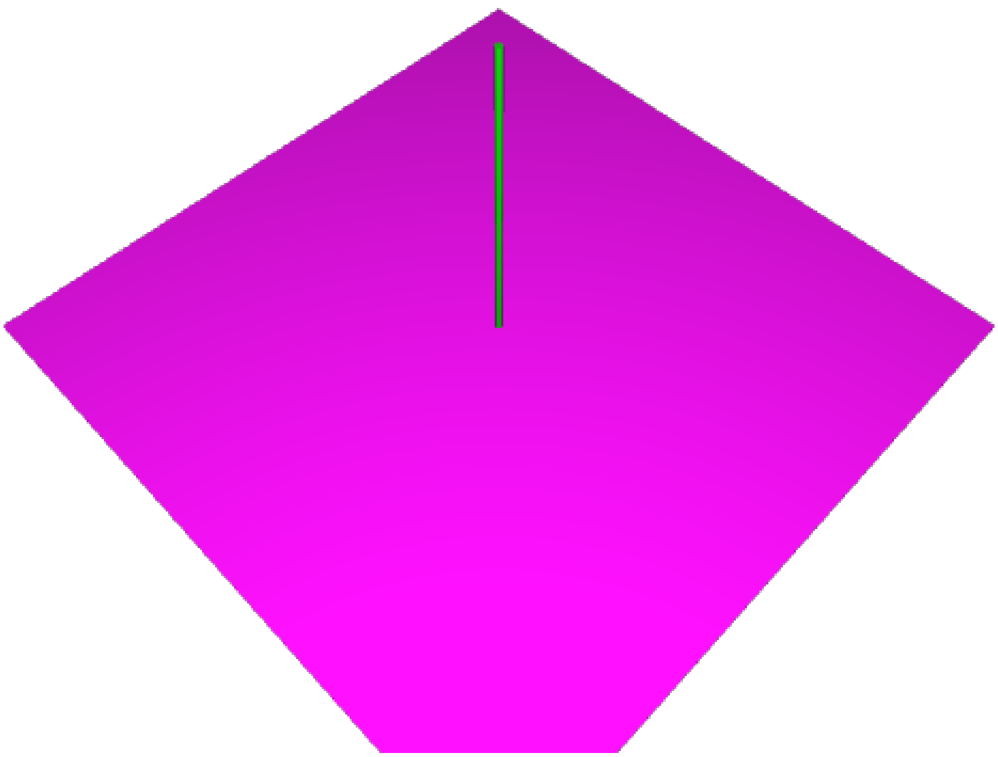

Quick review

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Don't look at the next slide if you don't want to see the answer!

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What shape is that?

Piglet of calculus conjectures

Any plane is just the set of endpoints of vectors perpendicular to a fixed one! So just fix a vector $\mathbf{u}$ and let

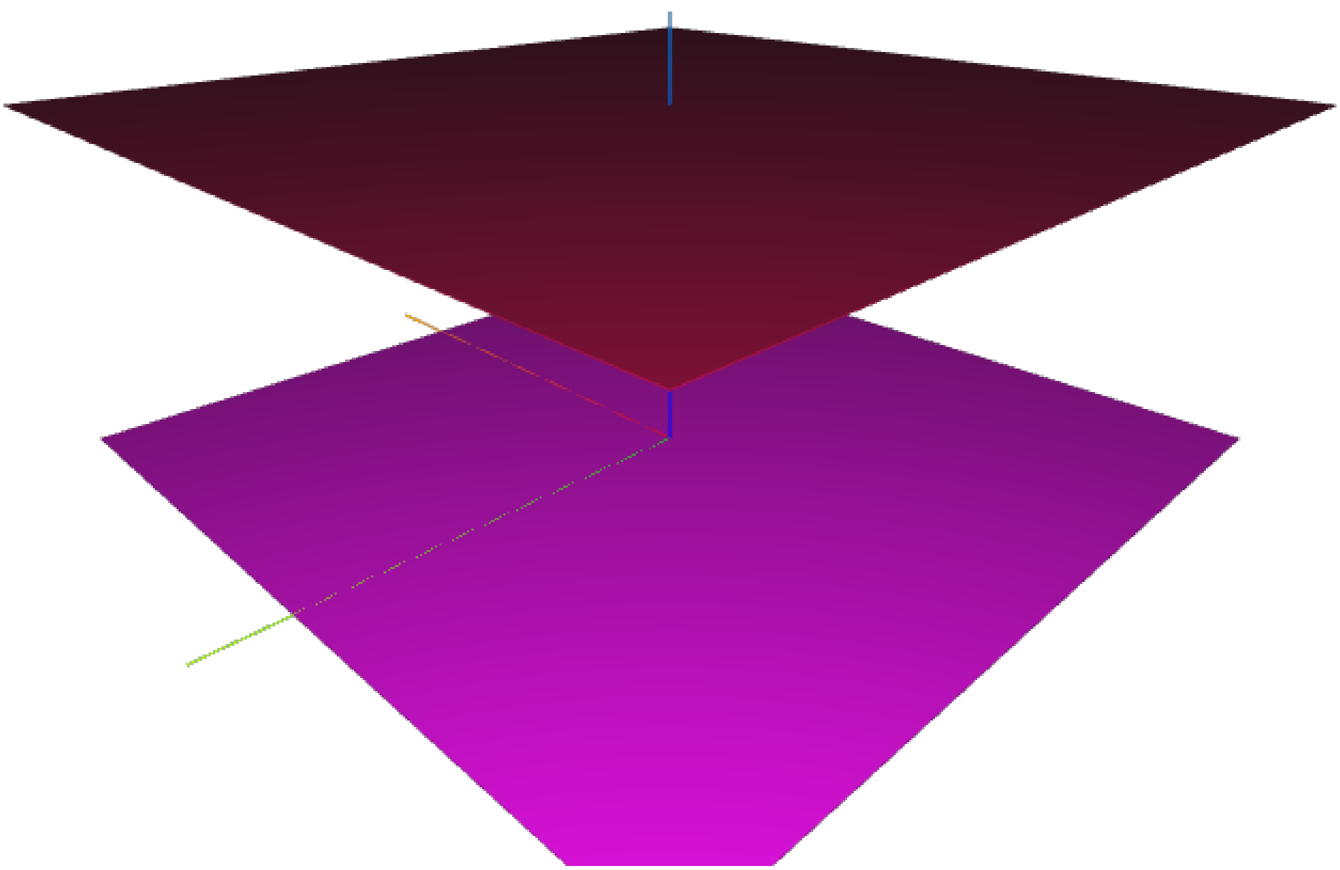
$$P_{\mathbf{u}} = \{\mathbf{v} \text{ such that } \mathbf{v} \cdot \mathbf{u} = 0\}.$$

For example, the xy-plane is the set of endpoints of vectors perpendicular to $\langle 0, 0, 1 \rangle$

Does it work? Can the piglet of calculus go to sleep now?

Conundrum: translation

- This is OK if the plane can be anchored like vectors can at $(0, 0, 0)$.
- If not, we have to take what we just did and translate it in space (i.e., move it away from $(0, 0, 0)$).
- This is just like making the plane $z = 4$ by translating the xy-plane up 4 units: the plane $z = 4$ is not the set of endpoints of vectors perpendicular to $\mathbf{k}$, just a parallel translation of it.



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