Loaded!

Please proceed to the calculus wonderland by pressing the right arrow key or clicking the right arrow that is visible when you move your mouse over the pig.

Instructions

These slides should work with *any* modern browser: IE 9+, Safari 5+, Firefox 9+, Chrome 16+.

- Navigate with arrow keys; you may need to give the window focus by clicking outside the lecture frame (the pig) for key commands described throughout this slide to work properly.
- Press M to see a menu of slides. Press G to go to a specific slide. Press W to toggle scaling of the deck with the window. If scaling is off, slides will be 800 by 600; it is off by default.
- Use left click, middle click, right click or hold A, S, D on the keyboard and move the mouse to rotate, scale, or pan the object.

If your browser or hardware does not support WebGL, interacting with models will be *very* slow (and in general models can get CPU-intensive). Navigate to a slide away from any running model to stop model animation.

Lecture 9

Behind us

Vector functions

Basic calculus with vector functions

Logistics

Homework due Tuesday at 11 PM

Midterm 1 on Tuesday in section

Ahead

Today: polar coordinates

Next: arc length and curvature

Read Sections 10.3, 13.3. *We do not cover everything in lecture or section. In fact, we basically cover nothing in lecture.*

Questions!

How does radar work?

Antenna sweeps around broadcasting signal.

Signal bounces off of objects and comes back.

Radar records the direction and distance to locate the object.

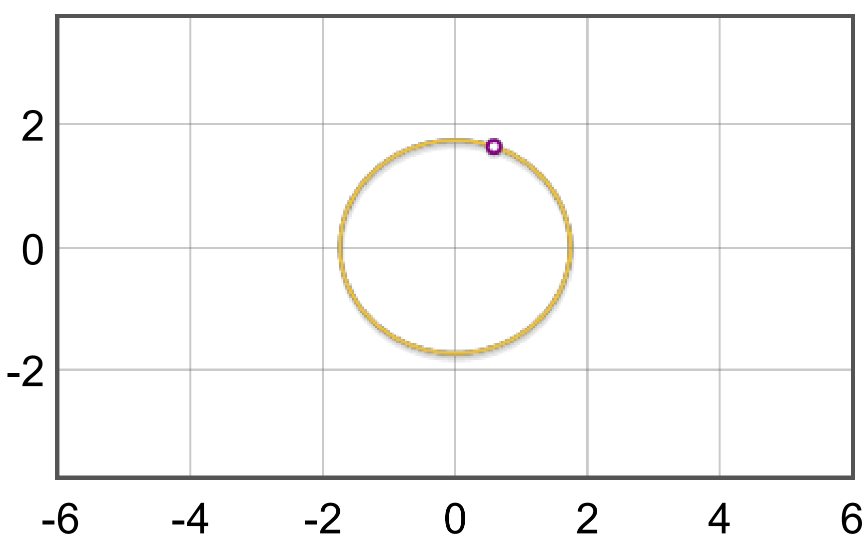
Descartes is cool. So is radar.

The radar uses a different coordinate system . To locate a point in the plane one can:

Find its distance r from the origin.

Find its angle θ counterclockwise from the x-axis.

The pair (r, θ) uniquely determines the location of the point, so it is just as good as the classical Cartesian coordinate pair (x, y).



r	1.7
theta	1

Close Controls

Code name: polar coordinates

We can convert back and forth: $x = r\cos(\theta)$, $y = r\sin(\theta)$ (like sending a code from East Berlin to West Berlin in 1974).

Example: the point with polar coordinates $(r, \theta) = (7, \pi)$ has Cartesian coordinates

$$(x, y) = (7\cos(\pi), 7\sin(\pi)) = (-7, 0).$$

Converting in other direction is more fun: first, $r = \sqrt{x^2 + y^2}$. To get the angle, you could try to use

$$\tan(\theta) = \frac{y}{x}$$

when this makes sense, or you could use one of the formulas $x = r\cos(\theta)$ or $y = r\sin(\theta)$. Use eyeballs and brain(s).

Most fun:
graphing equations .

Examples

What is the polar equation of a circle of radius a around the origin?

$$r = a$$

What is the polar equation of the line $x = 3$?

$$r\cos(\theta) = 3$$

Try one: find the polar equation of the line $y = 2x + 5$.

Try another one: find the polar equation of the ellipse $\frac{1}{2}\,x^2 + \frac{1}{3}\,y^2 = 1$

Polar graphs are beautiful

Here are some examples

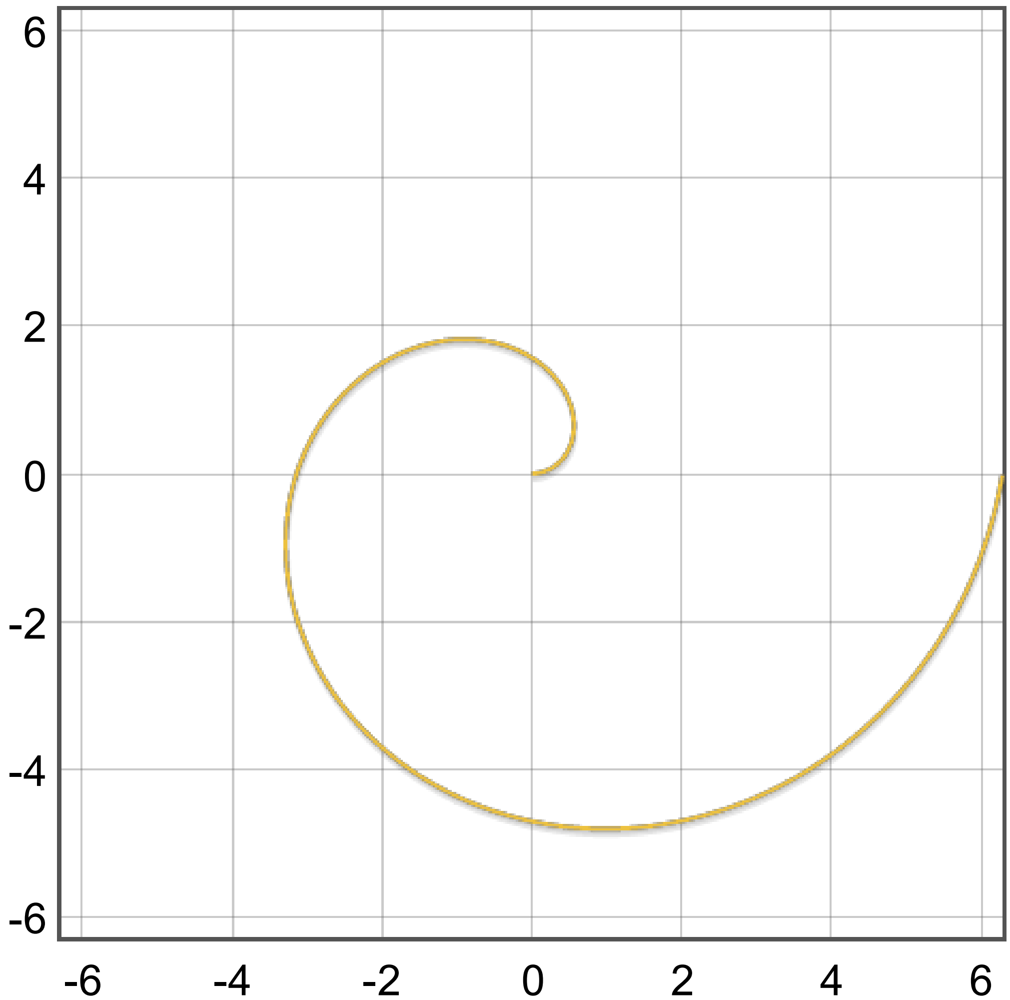
You can play with the parameters and see what happens!

Examples

$r = \theta^{\text{power}}, 0 \leq \theta \leq \text{multiple} \cdot \pi$

power	1
multiple	2

Close Controls

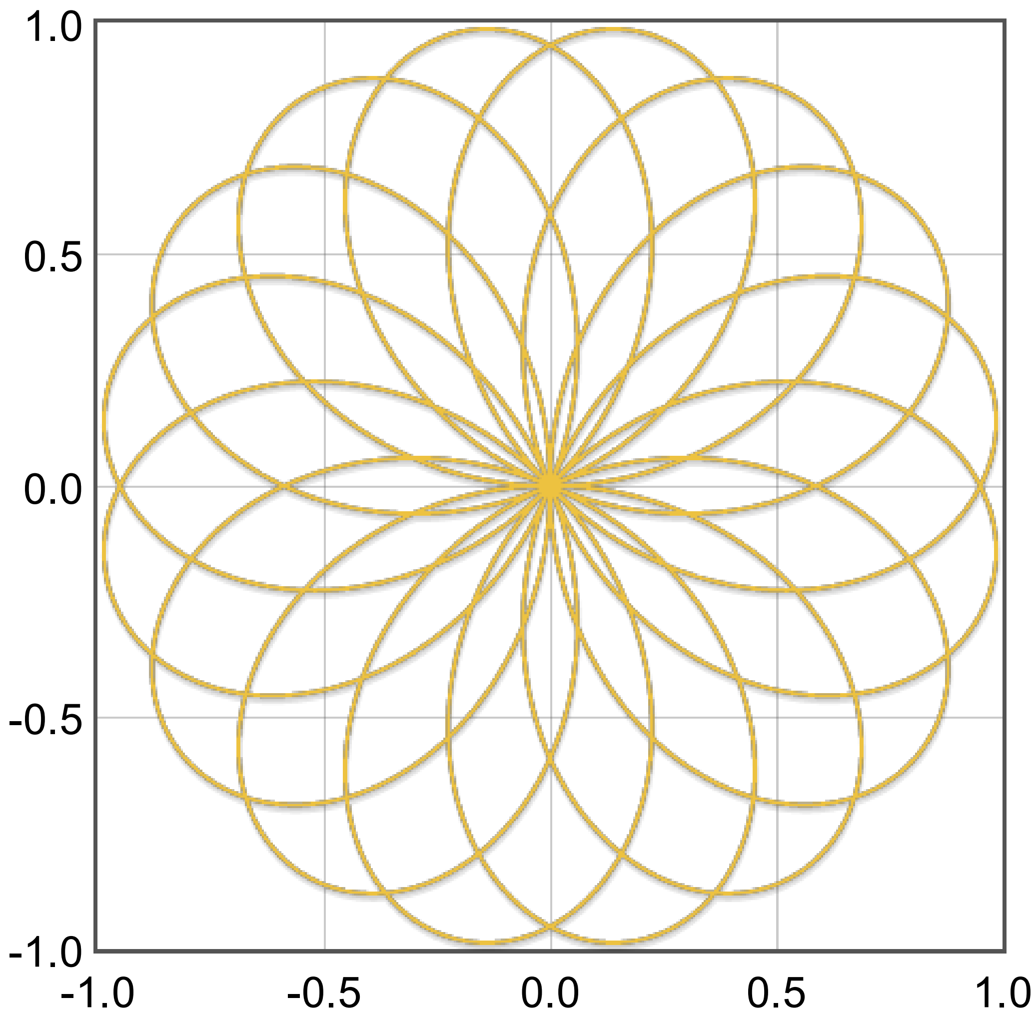


Examples

$r = \sin(\text{love} \cdot \theta),\, 0 \leq \theta \leq \text{multiple} \cdot \pi$

love	1.6
multiple	10

Close Controls

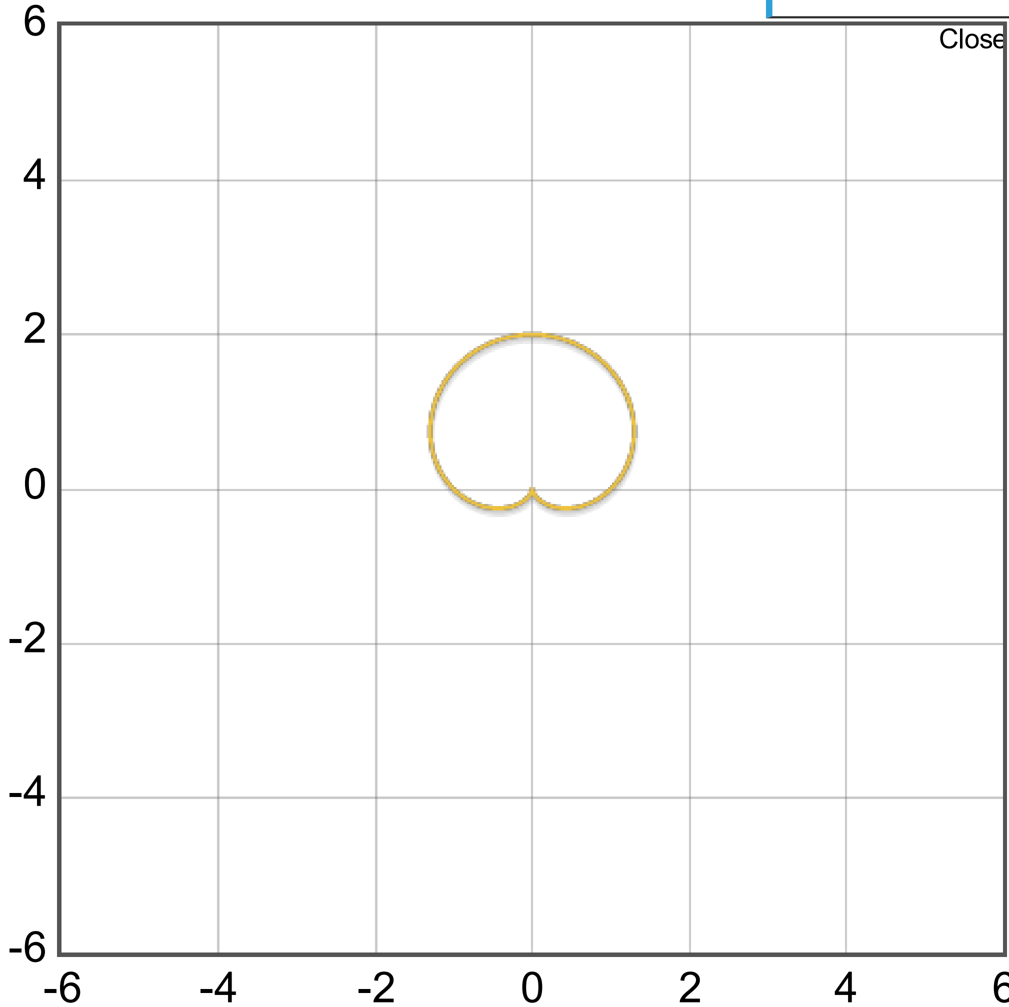


Examples

$$r = c + \sin(\text{love} \cdot \theta),\ 0 \leq \theta \leq \text{multiple} \cdot \pi$$

love	1
multiple	2
c	1

Close Controls



Try some the other way!

Example: the Cartesian equation of the curve with polar equation

$$r = \sin(\theta)$$

is

$$x^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{4}.$$

Trick: multiply both sides by r, yielding $r^2 = r\sin\theta$, then use $r^2 = x^2 + y^2$ and $r\sin\theta = y$.

- What is the Cartesian equation corresponding to $r = \sin(\theta) + \cos(\theta)$?
- How about $r = \cos(\theta)$?
- Or $r^2 = \sec(\theta)$?

Next time: *arc length and curvature!*