Loaded!

Please proceed to the calculus wonderland by pressing the right arrow key or clicking the right arrow that is visible when you move your mouse over the pig.

Instructions

These slides should work with *any* modern browser: IE 9+, Safari 5+, Firefox 9+, Chrome 16+.

- Navigate with arrow keys; you may need to give the window focus by clicking outside the lecture frame (the pig) for key commands described throughout this slide to work properly.
- Press M to see a menu of slides. Press G to go to a specific slide. Press W to toggle scaling of the deck with the window. If scaling is off, slides will be 800 by 600; it is off by default.
- Use left click, middle click, right click or hold A, S, D on the keyboard and move the mouse to rotate, scale, or pan the object.

If your browser or hardware does not support WebGL, interacting with models will be *very* slow (and in general models can get CPU-intensive). Navigate to a slide away from any running model to stop model animation.

Math 126

Lecture 2

Behind us

Last Time

- Intro
- $\mathbf{R}^3$
- Shapes defined by equations

Quiz Section

- Planes
- Spheres

Questions?

Coming up

vectors, dot products, projections

Read Section 12.2 of the book. *We cannot cover everything in lecture or section, but you will need it all for the rest of your lives!*

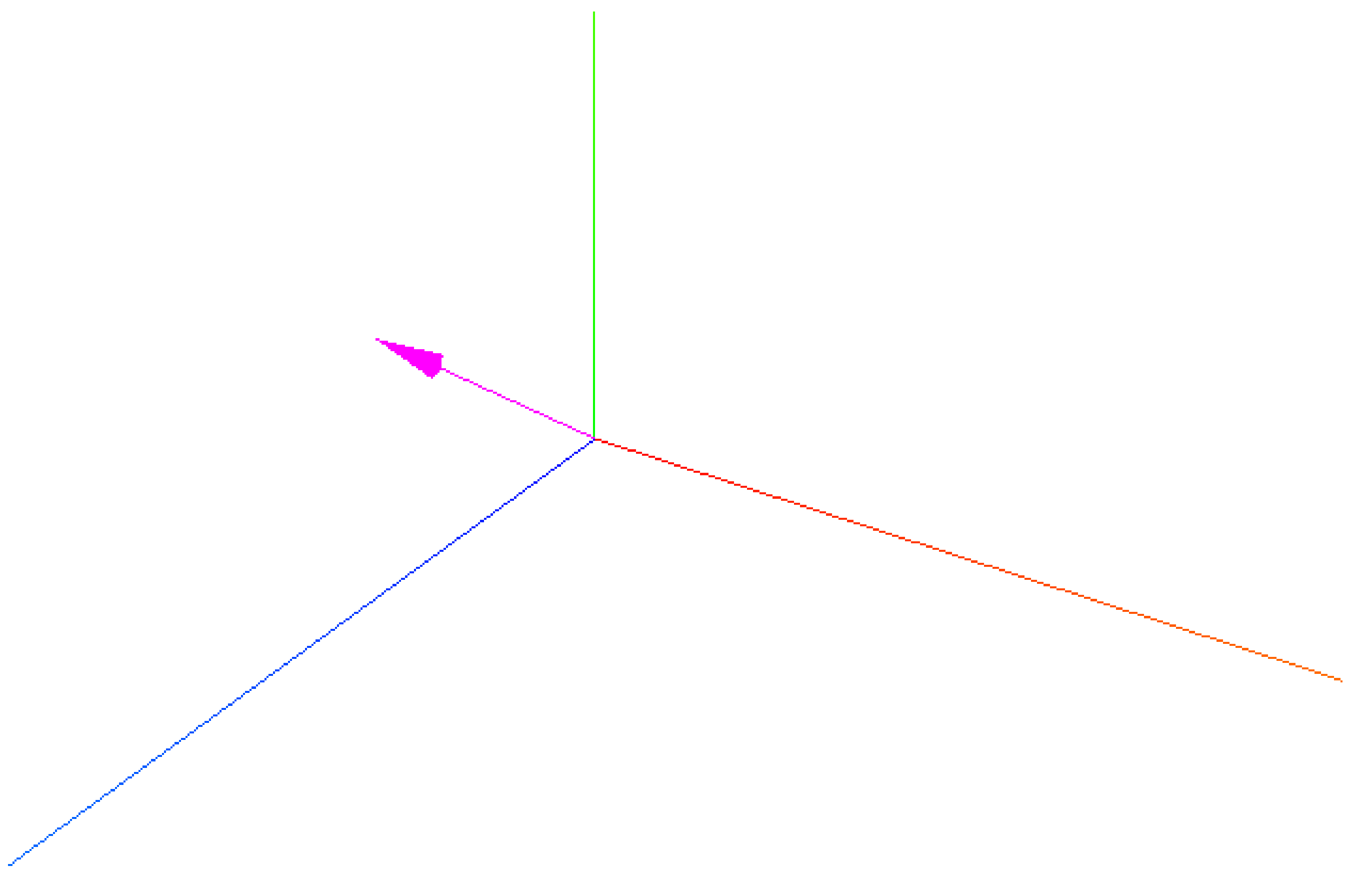
Flaky definition

A vector is an object with two properties: direction and magnitude. For example,

- Displacement from a fixed point
- Velocity
- Acceleration
- Force applied by angry customer

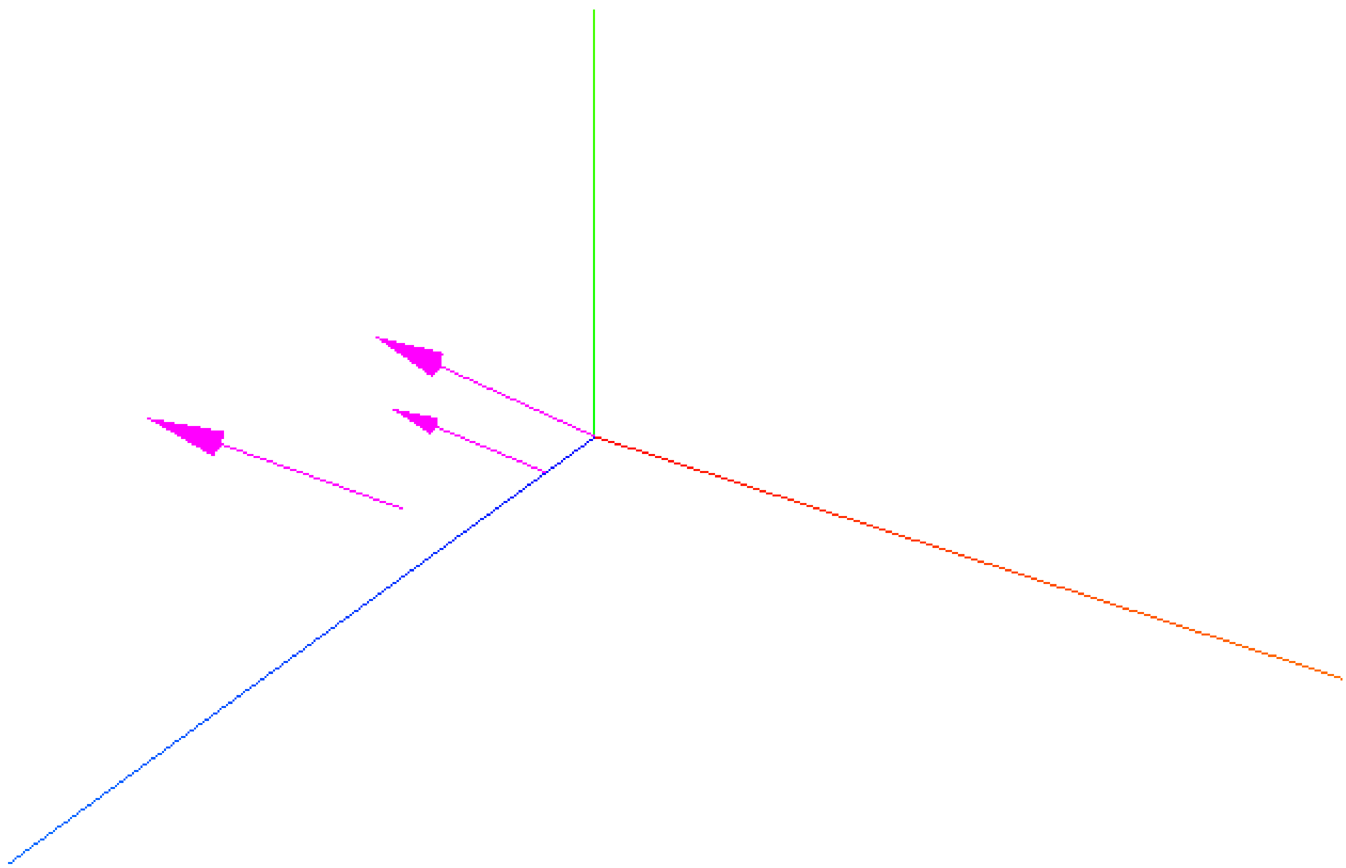
Graphical representation

Draw an arrow:



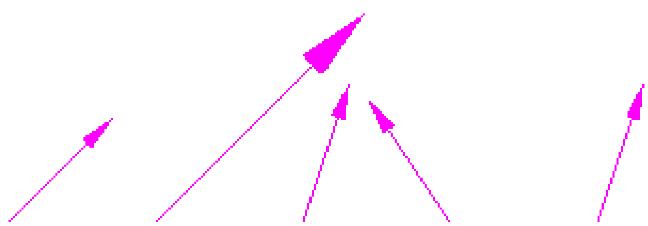
Graphical representation

Important (yet confusing) : two vectors are equivalent if they have the same direction and magnitude. These are all equivalent:



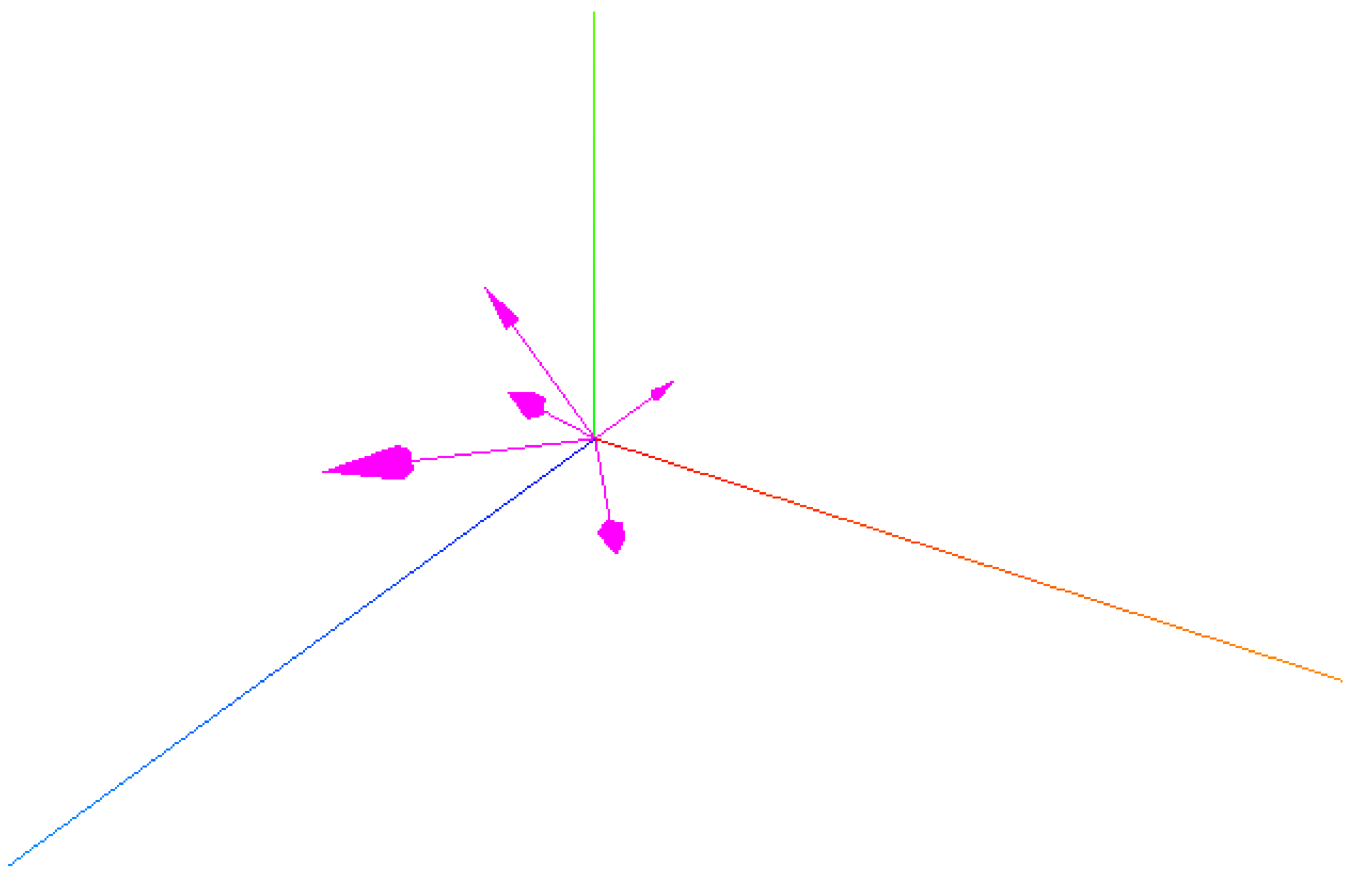
Quick check

Which vectors are equivalent to others in this picture? (Number them 1 through 5 from left to right. For simplicity, they live in a plane.)



Graphical representation

Thus, we will always represent vectors as arrows starting at the origin $(0,0,0)$.



Cheerleading

Does a vector have a position?

I can't hear you!

What does it have?

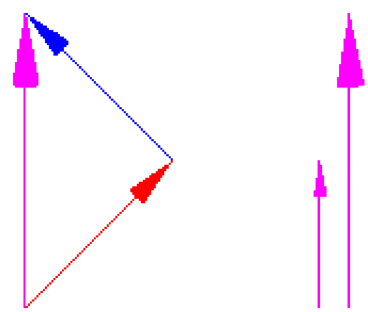
A vector only has

DIRECTION

MAGNITUDE

The magic of vectors

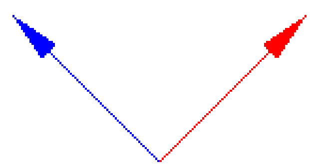
Vectors can be added and scaled .



The magic of vectors

Adding vectors graphically with the triangle rule

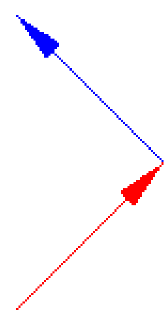
Draw the two vectors to be added using our representation that positions the start at the origin.



The magic of vectors

Adding vectors graphically with the triangle rule

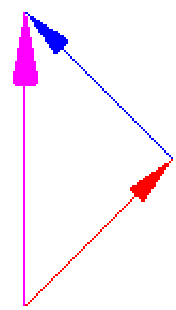
Translate the second one so that it starts at the end of the first one.



The magic of vectors

Adding vectors graphically with the triangle rule

Connect the start of the first with the end of the translated second. We end up with purple as red plus blue



The magic of vectors

Check yourself before you wreck yourself

What is the sum of the displacement vector connecting points p_1 and p_2 and the displacement vector connecting points p_3 and p_4?

Fun

- Let A, B, C, D be the vertices of a square. Choose a specific example if you want. Compute the sum of vectors
$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}.$$
- Let $A_1, A_2, A_3, A_4, A_5, A_6$ be points. Compute the sum
$$\overrightarrow{A_1 A_2} + \overrightarrow{A_2 A_3} + \overrightarrow{A_3 A_4} + \overrightarrow{A_4 A_5} + \overrightarrow{A_5 A_6} + \overrightarrow{A_6 A_1}.$$

Numbers breed vectors

We can also describe vectors using numbers.

The vector connecting $A = (a, b, c)$ to $B = (a', b', c')$ is

$$\overrightarrow{AB} = \langle a' - a, b' - b, c' - c \rangle$$

Note: you must always subtract the coordinates in the same order!

Numbers breed vectors

Any vector is made of three coordinates like that:

$$\mathbf{v} = \langle a, b, c \rangle.$$

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This is the same as the vector running from $(0, 0, 0)$ to the point (a, b, c).

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Its length is

$$||\mathbf{v}|| = \sqrt{a^2 + b^2 + c^2}$$

Brain massage

- Calculate the length of the vector connecting the point $(0, 2, 4)$ to the point $(1, -1, 1)$.
- Consider the vectors $\langle 1, 0, 0\rangle$ and $\langle 0, 1, 0\rangle$. Find a, b, c such that
  $$\langle 1, 0, 0\rangle + \langle 0, 1, 0\rangle = \langle a, b, c\rangle.$$
  Try this for another pair of vectors if you finish early. Rinse and repeat.

Numbers breed vectors

Addition and scaling using numbers:

$$\langle a, b, c\rangle + \langle a', b', c'\rangle = \langle a + a', b + b', c + c'\rangle$$

$$\langle 0, 3, 4\rangle + \langle 1, -1, 0\rangle = \langle 1, 2, 4\rangle$$

$$\gamma\langle a, b, c\rangle = \langle \gamma a, \gamma b, \gamma c\rangle$$

$$3\langle 1, 1, 2\rangle = \langle 3, 3, 6\rangle$$

Use it or lose it

Do the points

$$(1,2,3),(2,3,4),(37,38,40)$$

lie on a single line in $\mathbf{R}^3$?

Find the line containing the largest number of the following points

$$(1,0,1),(0,2,0),(1,2,3),(2,2,4),(3,2,5).$$

Criterion

- Two non-zero vectors $\mathbf{v}$ and $\mathbf{w}$ have the same or opposite direction if

  $$\mathbf{v} = c\mathbf{w}$$

  for some non-zero number c.
- Why is this true?
- Does this help with the problem?

Next time: *dot product* !