Vectors: components

Numbers breed vectors

We can also describe vectors using numbers.

The vector connecting $A = (0, 0, 0)$ to $B = (a, b, c)$ is written as

$$\mathbf{v} = \langle a, b, c \rangle$$

This uses the standard representation of vectors from before: force them to start at $(0, 0, 0)$.

The length of $\mathbf{v} = \langle a, b, c \rangle$ is

$$|\mathbf{v}| = \sqrt{a^2 + b^2 + c^2}$$

Numbers breed vectors

Any vector is made of three coordinates like that:

Using this notation, the vector connecting the point $A = (a, b, c)$ to the point $B = (a', b', c')$

$$\overrightarrow{AB} = \langle a' - a, b' - b, c' - c \rangle.$$

Note: you must always subtract the coordinates in the same order!

Brain massage

- Calculate the length of the vector connecting the point $(0, 2, 4)$ to the point $(1, -1, 1)$.
- Consider the vectors $\langle 1, 0, 0\rangle$ and $\langle 0, 1, 0\rangle$. Find a, b, c such that

  $$\langle 1, 0, 0\rangle + \langle 0, 1, 0\rangle = \langle a, b, c\rangle.$$

  Try this for another pair of vectors if you finish early. Rinse and repeat.

Numbers breed vectors

Addition and scaling using numbers:

$$\langle a, b, c\rangle + \langle a', b', c'\rangle = \langle a + a', b + b', c + c'\rangle$$

$$\langle 0, 3, 4\rangle + \langle 1, -1, 0\rangle = \langle 1, 2, 4\rangle$$

$$\gamma\langle a, b, c\rangle = \langle \gamma a, \gamma b, \gamma c\rangle$$

$$3\langle 1, 1, 2\rangle = \langle 3, 3, 6\rangle$$

Use it or lose it

Do the points

$$(1,2,3),(2,3,4),(37,38,40)$$

lie on a single line in $\mathbf{R}^3$?

Find the line containing the largest number of the following points

$$(1,0,1),(0,2,0),(1,2,3),(2,2,4),(3,2,5).$$

Criterion

- Two non-zero vectors $\mathbf{v}$ and $\mathbf{w}$ have the same or opposite direction if
$$\mathbf{v} = c\mathbf{w}$$
for some non-zero number c.
- Why is this true?
- Does this help with the problem?