Vectors: geometry

Flaky definition

A vector is an object with two properties: direction and magnitude. For example,

- Displacement from a fixed point
- Velocity
- Acceleration
- Force applied by angry customer

Special vector: $\mathbf{0}$

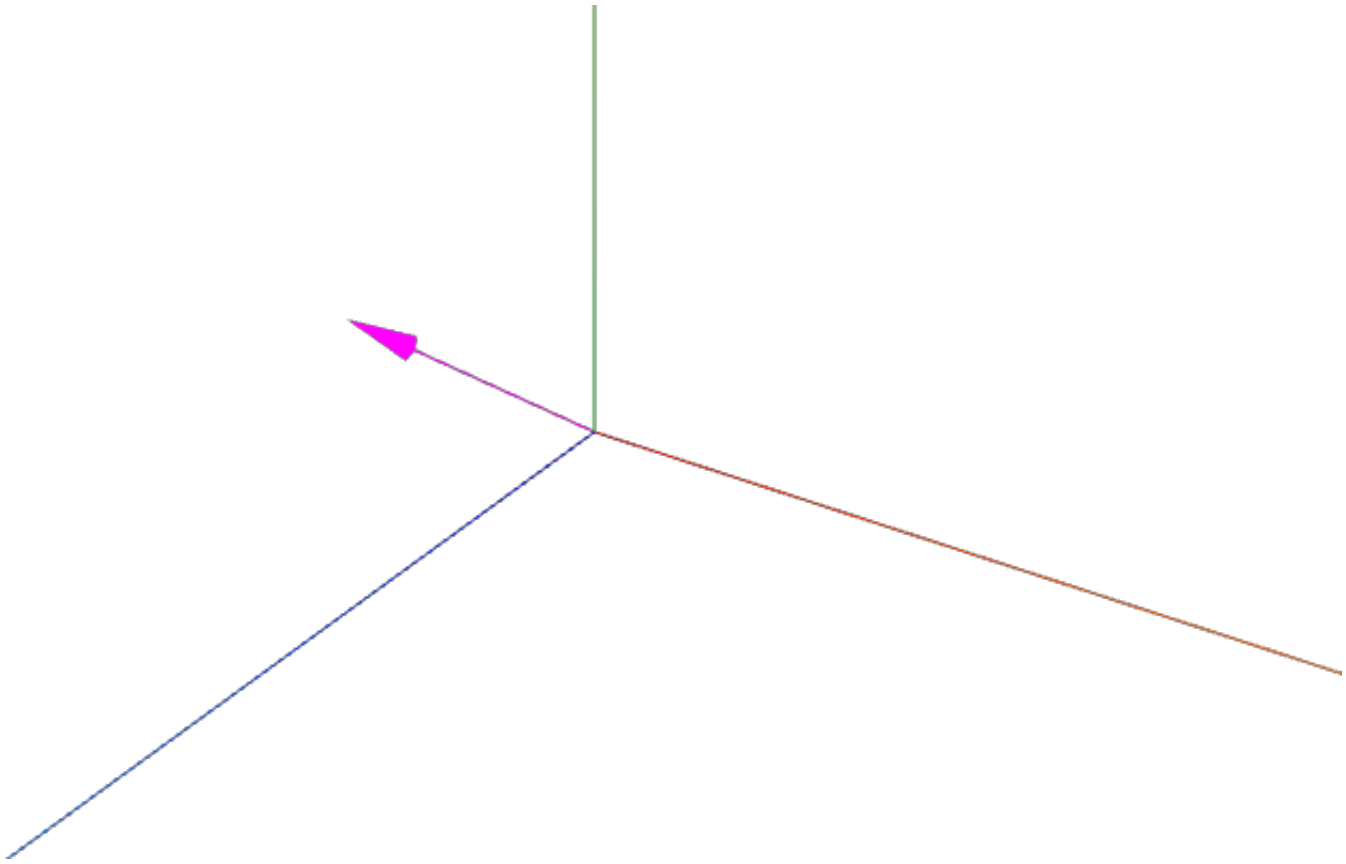
There is one special vector: the 0 vector, written $\mathbf{0}$.

The vector $\mathbf{0}$ has magnitude 0 and "any" direction.

Example: the trivial displacement (go nowhere!).

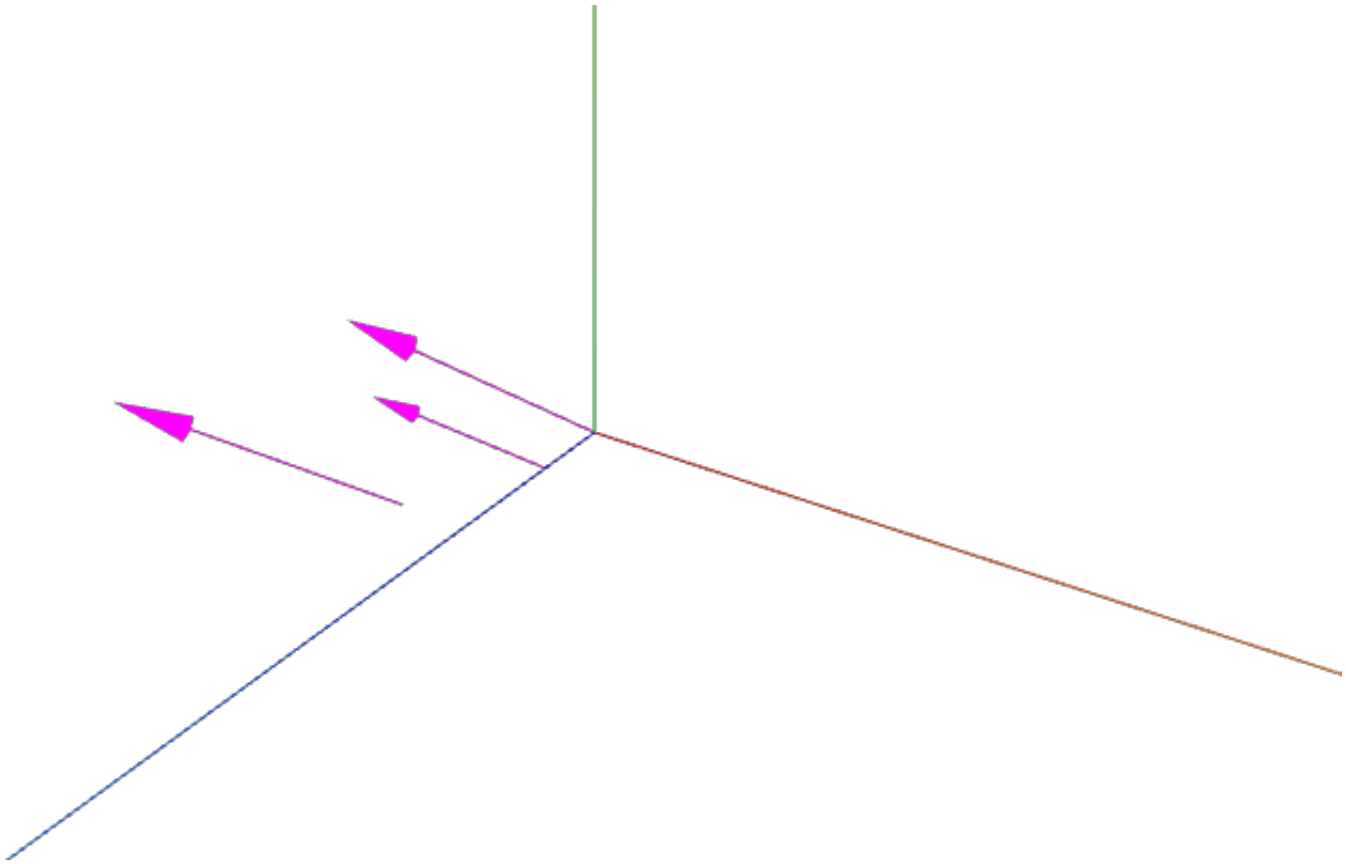
Graphical representation

Draw an arrow:



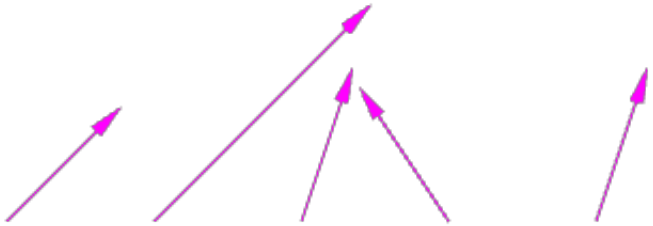
Graphical representation

Important (yet confusing): two vectors are equivalent if they have the same direction and magnitude. These are all equivalent:



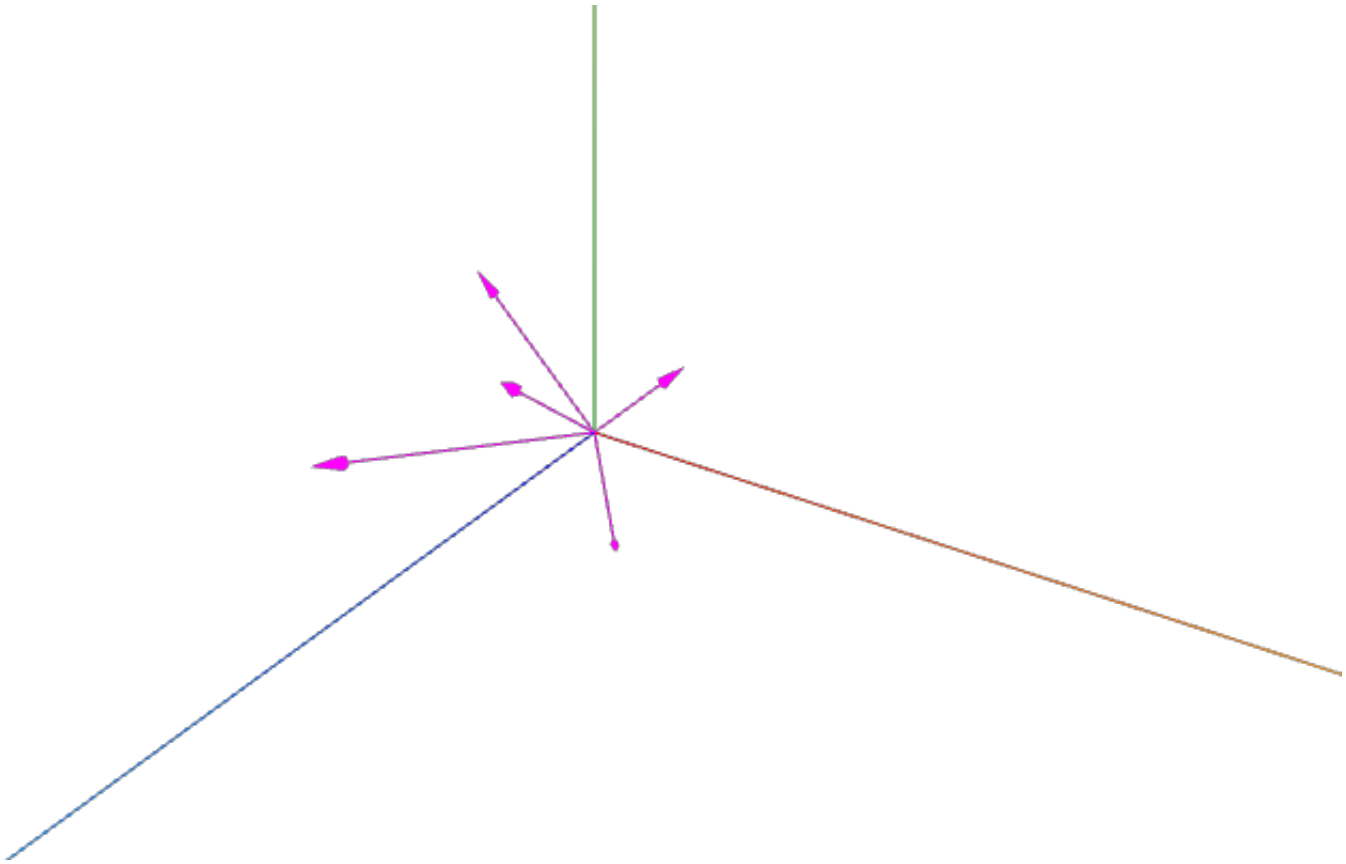
Quick check

Which vectors are equivalent to others in this picture? (Number them 1 through 5 from left to right. For simplicity, they live in a plane.)



Graphical representation

Thus, we will always represent vectors as arrows starting at the origin $(0, 0, 0)$.



Cheerleading

Does a vector have a position?

I can't hear you!

What does it have?

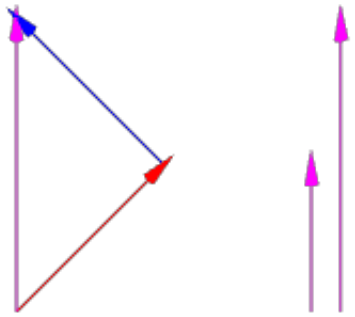
A vector only has

DIRECTION

MAGNITUDE

The magic of vectors

Vectors can be added and scaled.



The magic of vectors

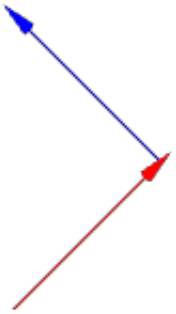
Adding vectors graphically with the triangle rule

Draw the two vectors to be added using our representation that positions the start at the origin.

The magic of vectors

Adding vectors graphically with the triangle rule

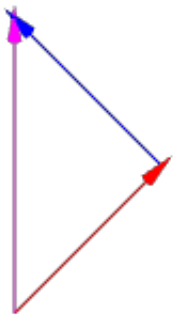
Translate the second one so that it starts at the end of the first one.



The magic of vectors

Adding vectors graphically with the triangle rule

Connect the start of the first with the end of the translated second. We end up with purple as red plus blue



The magic of vectors

Check yourself before you wreck yourself

What is the sum of the displacement vector connecting points p_1 and p_2 and the displacement vector connecting points p_2 and p_3?

Fun

- Let A, B, C, D be the vertices of a square. Choose a specific example if you want. Compute the sum of vectors
$$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}.$$
- Let $A_1, A_2, A_3, A_4, A_5, A_6$ be points. Compute the sum
$$\overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \overrightarrow{A_3A_4} + \overrightarrow{A_4A_5} + \overrightarrow{A_5A_6} + \overrightarrow{A_6A_1}.$$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