1. **To do:**

**Optimum model**

The main issue seems to be the standardization. What it seems to do is to is to blur effects of mass and temperature that we know exists. This is evident in the lack of clear hump-shape, with or without size-dependence. And it’s maybe not that strange. Neither the model nor the standardization can currently handle that species may have different widths of their thermal performance curves and live at temperatures of varying distance from the optimum temperature for growth (especially since we have species from different biogeographical regions, which could also covary with the widths). Maybe it’s doing more harm than good (the good being able to pool data in the first place).

I will therefore try and fit models per species in the subset of the data that only has optimum curves. I’ll fit the models with and without interaction to evaluate the support for size-dependence of the thermal optimum, and then summarize the fitted model parameters. For visualization, we could standardize the data predicted maximum, as in Englund and e.g. color code by mass. But the first step is even to see how well we can fit those polynomials to the filtered data, I don’t know on the top of my head how many data points I have per species there. The potentially new result here, compared to Englund, is that we directly model the body size effect rather than controlling for it by selecting similar sized individuals.

1. **Growth rate:**

Below are three options for how we can include growth in this ms:

**a) No growth result at all**

1. Start with the question: How do these rates scale with body mass, within species? Highlight multivariate hierarchical models to overcome some limitations with previous scaling models and the “factorial” data.
2. Skip the W\_inf prediction from scaling and the heat map for the reasons previously discussed (mainly size range) and do not replace it with anything else
3. Focus the main results around the actual intraspecific parameter estimates and how we did not have good intraspecific values for those before. Lift the data and modelling, and the general vs species prediction (and their distributions).
4. Potentially discuss their implications for growth (cite e.g. Marshall and White Tree and their discussion about the mass-exponents), but still focus discussion around “scaling laws” – both temperature and mass, now fully intraspecific.

**b) With growth – predict growth from scaling, validate with growth data**

1. Start with the question: How do these rates scale with body mass, within species? Highlight multivariate hierarchical models to overcome some limitations with previous scaling models and the “factorial” data.
2. The next question we ask is: assuming these rates influence growth, what can we predict about the mass and temperature dependence of growth?
3. So, we basically keep the structure we have alread, but instead of the W\_inf heatmap we plot actual growth rates from a Pütter growth model for some combinations of mass and temperature, given the scaling we find for Cmax and metabolism.
4. Then we pull out the growth data and compare the allometric growth data and fit from growth experiments, as well as the T\_opt result
5. We find that growth rate is predicted to increase with warming in both predictions and growth data.
6. The scaling of Cmax and metabolism does not however predict differences in the size-scaling exponent of growth at different temperatures, suggesting size-dependent effects, as we find in the growth data. The scaling of those rates predicts similar mass-scaling slopes in different temperatures (BUT! The posterior of the interaction coefficient in the growth-allometry model is only about 80% above 0. And, I don’t know how robust that is to model selection. But we do compare different ways of standardizing and find largely consistent results, but I guess we might find that a model without the interaction can be selected, actually).
7. We also find that T\_opt declines with body size.
8. Morita et al 2010 (Oikos), showed that \***if**\* there is an optimum over temperature, it declines with body size, as long as that the mass-exponent for Cmax < metabolism. We do find the latter part, but we actually do not even predict there to be an optimum in the first place, because the activation energy for Cmax is larger than for metabolism. An optimum in growth could however still occur if for instance Cmax has a (lower) optimum than metabolism (or a size-dependent optimum even), but it’s still not clear if we can show that in our data.
9. If we can’t show that, I think one interesting finding here is that the scaling of Cmax and metabolism can only explain growth to a certain degree (and Pütter-type growth models should not be expected to capture the whole picture of the mass and temperature dependence of growth rates). OR, if you really want to believe they can, our results show that in that case, you can’t extrapolate scaling from juveniles to adults. Why? Because we don’t predict an optimum in growth over temperature with our scaling parameters, but we do know that occurs because we have T\_opt data in our paper…

**c) With growth – given the growth rate results, how well can we explain that with scaling data?**

1. Start with the growth rate result. Answer the question: how does growth rate depended on body size and temperature, and how does T\_opt depended on body size? These are interesting questions both from a TSR perspective (not often actual growth rate is quantified, and not often in terms of its size-dependence and temperature dependence. More common to relate growth parameters with environmental variables, at least in fish). It’s also interesting from a general (intraspecific) scaling perspective.
2. Follow up question: how well can we reproduce those growth results using a Putter growth model, which most mechanistic models are, e.g. the West model and the VBGE?
3. To answer that, we collate data on the size- and temperature scaling of metabolism and Cmax, which we assume scale in proportion to the allometric functions in the Pütter model (as in Ursin 1967, White & Marshall, 2018 etc etc). We use those data to predict growth rate and it’s scaling with mass and temperature. (Highlight here that we have nice generic intraspecific data and fit hierarchial models, and don’t rely on between-species data)
4. … And now we can jump back to point 6 in suggestion b) and proceed from there…

**Conclusion:**

I am starting to feel quite stressed about this thesis. Even though it’s not mainly due to this chapter, I still want to try and limit extra work and make a priority list. That said, I think option b) and c) make the ms much stronger and more interesting. And for the thesis, I think it’s nice to show the declining T\_opt and the mass-temperature interaction for growth, because it helps motivate some of the things we discuss and analyze in the modelling papers.

One suggestion is then to follow option c) (which I prefer of the two with growth) and prioritize writing the paper asap – without the optimum/polynomial model. If there’s time in the end, we can go back to it and do the analysis we discussed. The optimum model part is most likely a quite minor part of the story in option “c” anyway, and can probably be added quite easily if we have time. In fact, this might even be the more time-efficient approach, since “fixing” the optimum model might be more important to do if we only write a scaling paper but it can also take quite some time compared to the growth results which are already there.